Flow simulations in geology-based Discrete Fracture Networks

Géraldine Pichot, Patrick Laug, Jocelyne Erhel, Romain Le Goc, Caroline Darcel, Philippe Davy, Jean-Raynald de Dreuzy

Flow simulations in geology-based Discrete Fracture Networks

Géraldine Pichot
(joint work with Patrick Laug, Jocelyne Erhel, Romain Le Goc, Caroline Darcel, Philippe Davy, Jean-Raynald de Dreuzy)

1. Motivation

The underground is a reservoir of natural resources (water, oil and gas, heat,...) and a potential warehouse storage solution. Using these resources and storage facilities in a sustainable way requires a good understanding of the physical, chemical and biological processes happening there. Also, the geometry of the subsurface couples these processes together. Here, numerical models are very useful: they reduce the costs and risks of *in situ* experiments and allow long-term predictions.

2. Geometric model

At first, we need to build a geometric model of the subsurface. Observations and experiments (investigations of drill cores and drill cuttings, lineament characterization, measurements of reflection seismics, transient electromagnetic soundings, ... [1]) give data for soil characterization and fracture locations. Fractures cannot be neglected as they critically impact the response of subsurface systems through their complex organisation over a broad range of scales. However, the information collected via the measurements are local. To derive an image of the full geometry, numerical models come into play. It is quite challenging to process tremendous amounts of noisy data, scattered and at different scales. Nevertheless, hydrogeologists are able to derive models for the subsurface, and especially for fracture systems [2]. The one we rely on is based on a spatial organization of fractures satisfying two main regimes: a dilute regime for the smallest fractures, where they can grow independently of each other, and a dense regime for which the density distribution is controlled by the mechanical interactions between fractures [3, 4]. In these models, called UFM (likely Universal Fracture Model), the fracture size distribution matches the observations and large fractures inhibit the smaller ones, creating T-termination configurations. Models mix spatial dimensions as fracture networks consist in a large number of 2D fractures interconnected in the 3D space [5]. These models are implemented in a dedicated software, called `UFMLab`. UFM networks may contain millions of fractures.

3. Flow problem

Let us focus on a simple process, which is the flow of water within the fractures. The surrounding rock matrix is assumed impervious. The associated problem is governed by the Poiseuille's law and the mass conservation equation in each fracture combined with continuity conditions at the intersections between fractures. The transmissivity within the fractures is assumed heterogeneous and modeled by random fields whose parameters are given by the experiments.

4. Mesh generation

The next step is to build a mesh of the domain. To easily enforce the continuity conditions at the intersections, we request a mesh which takes the intersections into account, either with matching or non-matching grids at the intersections. However, even in the non-matching cases, the intricate configurations of the fracture systems make it difficult for standard planar meshers to generate a mesh of the geometry. The idea is then to discretize first the border and intersections to build a geometric model, valid for the planar mesher. It implies automatic corrections to build valid curve discretizations. These corrections are implemented in the software `BLSURF_FRAC` [1]. Then a planar mesher is called, for example, `BL2D` [6] or `BAMG` [7]. Up to now, the largest network successfully meshed with this technique contains 508,339 fractures which generate 1,031,231 intersections. It is meshed in 15 minutes using the combination of `BLSURF_FRAC` and `BL2D`, with 4 threads on a PC Intel Core i7 4 cores CPU @ 2.90 GHz, 32GB RAM memory. The final mesh contains 8,112,299 of triangles and 1,630,682 of intersection edges. Notice that the computational time can be improved on a computer with more RAM memory as memory swapping was detected during this numerical experiment. Also, we identified some sequential parts of `BLSURF_FRAC` that can be parallelized.

5. Numerical methods and implementation

The next step is to solve the flow problem efficiently. We propose to solve this problem using a Mixed Hybrid Finite Elements Method (MHFEM) [8, 9, 10]. Possible non-matching grids at the intersections between fractures are handled by using suitable Mortar conditions [11, 12, 13, 14, 15]. In both, the matching and non-matching grids cases, we end up with a linear system whose matrix is symmetric positive definite. It is a sparse matrix with a L-shape structure. The linear system can be solved either with a direct solver (Cholesky factorization) or with an iterative solver (preconditioned conjugate gradient or multigrid). A way to improve the computational time of iterative solvers is to use domain decomposition techniques [16, 17]. We developed a Matlab code, called `NEF-Flow`, which implements the MHFEM either for matching or non-matching meshes, with sink/source terms and contrasts in transmissivities. Efficiency is obtained using Matlab vectorization. For example, we solved the flow problem in the largest network described above in 21 minutes on a PC Intel Core i7 4 cores CPU @ 2.90 GHz, 32GB RAM memory with a direct solver. Here again memory swapping was detected. Further numerical experiments are now performed with iterative solvers to reduce memory requirements. Also to keep an accurate solution with less triangles, we are investigating the use of a higher order hybridizable discontinuous Galerkin technique, the HHO method [18], combined with *a posteriori* error estimates [19] in order to refine only the fractures that carry most of the flow.

[1]http://pages.saclay.inria.fr/patrick.laug/logiciels/logiciels.html

Acknowledgments. The author thanks the organizers of the workshop for their kind invitation. The author thanks the MFO and all its staff for the extremely good working and living conditions provided at Oberwolfach.

References

[1] Svensk Kärnbränslehantering AB (SKB), Site description of Forsmark at completion of the site investigation phase. SDM-Site Forsmark, *TR-08-05*, Stockholm (2008).

[2] J. Maillot, P. Davy, R. Le Goc, C. Darcel and J.-R. de Dreuzy. Connectivity, permeability, and channeling in randomly distributed and kinematically defined discrete fracture network models. *Water Resources Research*, 52 (11), 8526–8545, (2016).

[3] P. Davy, R. Le Goc, C. Darcel, O. Bour, J.-R. de Dreuzy and R. Munier. A likely universal model of fracture scaling and its consequence for crustal hydromechanics. *Journal of Geophysical Research: Solid Earth*, 115 (B10) (2010).

[4] P. Davy, R. Le Goc and C. Darcel. A model of fracture nucleation, growth and arrest, and consequences for fracture density and scaling. *Journal of Geophysical Research: Solid Earth*, 118 (4), 1393–1407 (2013).

[5] V. Martin, J. Jaffré and J. E. Roberts. Modeling fractures and barriers as interfaces for flow in porous media, *SIAM J. Sci. Comput.*, 26 (5), pp. 1667-1691 (2005).

[6] H. Borouchaki, P. Laug and P. L. George. Parametric surface meshing using a combined advancing-front – generalized-Delaunay approach. *International Journal for Numerical Methods in Engineering*, 49 (1-2), 233–259 (2000).

[7] F. Hecht, BAMG: Bidimensional Anisotropic Mesh Generator (2006).

[8] D. N. Arnold and F. Brezzi. *Mixed and nonconforming finite element methods: postprocessing, and error estimates*, RAIRO M2AN, vol. 19, 732 (1985).

[9] F. Brezzi and M. Fortin. *Mixed and Hybrid Finite Element Methods*, Springer-Verlag, New York, 1991.

[10] J. E. Roberts and J. M. Thomas. *Mixed and hybrid methods, Handbook of numerical analysis*, Vol. 2, P.G. Ciarlet and J.L. Lions Ed., North Holland, p. 523 (1991).

[11] T. Arbogast and Z. Chen, On the Implementation of Mixed Methods as Nonconforming Methods for Second-Order Elliptic Problems, *Mathematics of Computation*, 211-64, pp. 943–972 (1995).

[12] T. Arbogast, L. C. Cowsar, M. F. Wheeler and I. Yotov, *Mixed finite element methods on non-matching multiblock grids*, *SIAM J. Numer. Anal.*, 37(4), pp. 1295-1315 (2000).

[13] J. Maryška, O. Severýn and M. Vohralík. *Numerical simulation of fracture flow with a mixed-hybrid FEM stochastic discrete fracture network model*, *Computational Geosciences*, 8, pp. 217234 (2004).

[14] G. Pichot, J. Erhel and J.-R. de Dreuzy. A mixed hybrid Mortar method for solving flow in discrete fracture networks, *Applicable Analysis*, 89-10, pp. 1629-1643 (2010).

[15] G. Pichot, J. Erhel, and J.-R. de Dreuzy. A generalized mixed hybrid mortar method for solving flow in stochastic discrete fracture networks. *SIAM J. Sci. Comput.*, 34(1):B86–B105 (2012).

[16] B. Poirriez. Study and implementation of a domain decomposition method for the modelisation of flow in 3D fracture networks, PhD Thesis, University of Rennes 1 (2011).

[17] G. Pichot, B. Poirriez, J. Erhel, and J.-R. De Dreuzy. A Mortar BDD method for solving flow in stochastic discrete fracture networks. *Domain Decomposition Methods in Science and Engineering XXI, Lecture Notes in Computational Science and Engineering (LNCSE)*, 98 (2014).

[18] D. A. Di Pietro and A. Ern. A Hybrid High-Order locking-free method for linear elasticity on general meshes. *Comput. Meth. Appl. Mech. Engrg.*, 283:1–21 (2015).

[19] A. Ern and M. Vohralík. Polynomial-degree-robust a posteriori estimates in a unified setting for conforming, nonconforming, discontinuous Galerkin, and mixed discretizations. *SIAM J. Numer. Anal.*, 53, pp. 10581081 (2015).