



# S-Box Reverse-Engineering: Boolean Functions, American/Russian Standards, and Butterflies

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► To cite this version:

Léo Perrin. S-Box Reverse-Engineering: Boolean Functions, American/Russian Standards, and Butterflies. CECC 2018 - Central European Conference on Cryptology, Jun 2018, Smolenice, Slovakia. pp.1-99. hal-01953348

**HAL Id: hal-01953348**

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Submitted on 12 Dec 2018

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# S-Box Reverse-Engineering

Boolean Functions, American/Russian Standards, and Butterflies

Léo Perrin

Based on joint works with Biryukov, Canteaut, Duval and Udovenko

June 6, 2018

CECC'18



# Outline

- 1 Building Blocks for Symmetric Cryptography
- 2 Statistics and Skipjack
- 3 TU-Decomposition and Kuznyechik
- 4 The Butterfly Permutations and Functions
- 5 Conclusion

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# Symmetric Cryptography

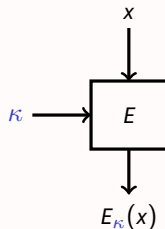
There are many **symmetric** algorithms! Hash functions, MACs...

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## Definition (Block Cipher)

- Input:  $n$ -bit block  $x$
- Parameter:  $k$ -bit key  $\kappa$
- Output:  $n$ -bit block  $E_{\kappa}(x)$
- Symmetry:  $E$  and  $E^{-1}$  use the same  $\kappa$

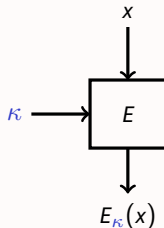


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### Properties needed:

Diffusion

Confusion

No cryptanalysis!

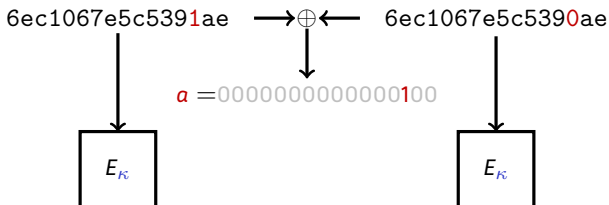
## No Cryptanalysis?

Let us look at a typical cryptanalysis technique: the **differential attack**.

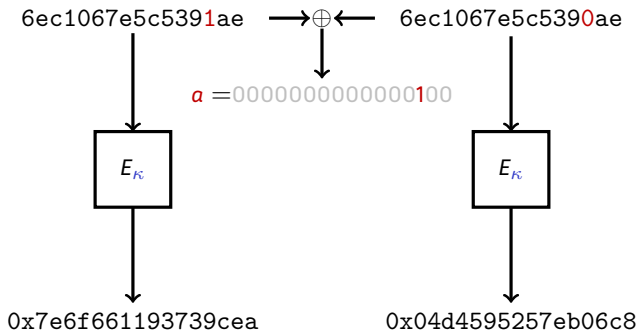




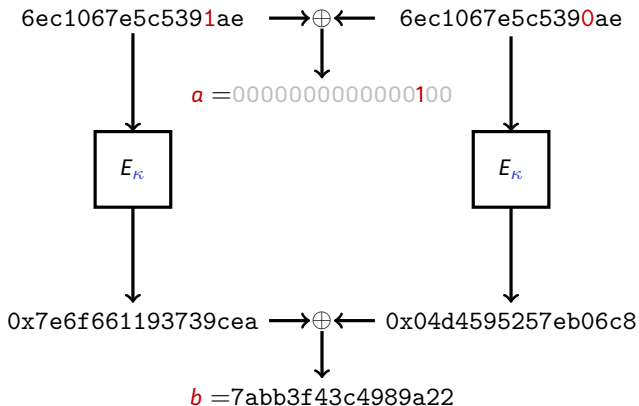
## Differential Attacks



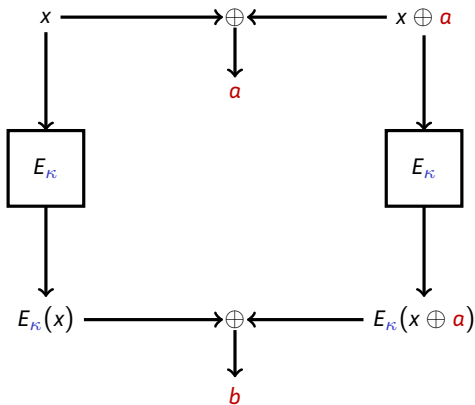
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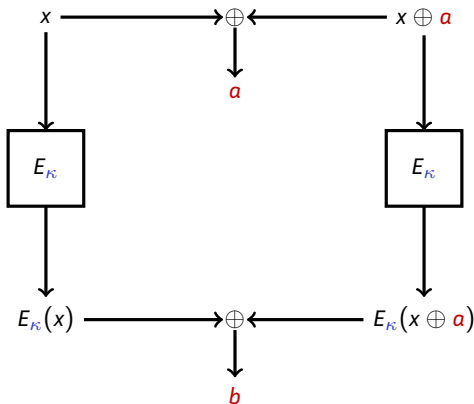
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### Differential Attack

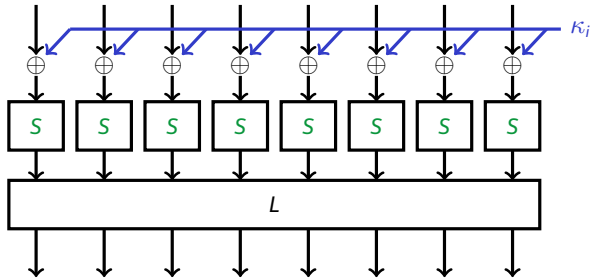
If there are many  $x$  such that  $E_{\kappa}(x) \oplus E_{\kappa}(x \oplus a) = b$ , then the cipher is **not secure**.

## Basic Block Cipher Structure

How do we build block ciphers that prevent such attacks (as well as others)?

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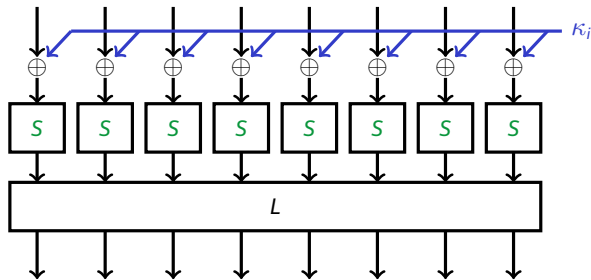
How do we build block ciphers that prevent such attacks (as well as others)?





## Basic Block Cipher Structure

How do we build block ciphers that prevent such attacks (as well as others)?



### Substitution-Permutation Network

Such a block cipher iterates the round function above several times. **S** is the Substitution **Box** (S-Box).

## The S-Box (1/2)

$\pi' = (252, 238, 221, 17, 207, 110, 49, 22, 251, 196, 250, 218, 35, 197, 4, 77, 233, 119, 240, 219, 147, 46, 153, 186, 23, 54, 241, 187, 20, 205, 95, 193, 249, 24, 101, 90, 226, 92, 239, 33, 129, 28, 60, 66, 139, 1, 142, 79, 5, 132, 2, 174, 227, 106, 143, 160, 6, 11, 237, 152, 127, 212, 211, 31, 235, 52, 44, 81, 234, 200, 72, 171, 242, 42, 104, 162, 253, 58, 206, 204, 181, 112, 14, 86, 8, 12, 118, 18, 191, 114, 19, 71, 156, 183, 93, 135, 21, 161, 150, 41, 16, 123, 154, 199, 243, 145, 120, 111, 157, 158, 178, 177, 50, 117, 25, 61, 255, 53, 138, 126, 109, 84, 198, 128, 195, 189, 13, 87, 223, 245, 36, 169, 62, 168, 67, 201, 215, 121, 214, 246, 124, 34, 185, 3, 224, 15, 236, 222, 122, 148, 176, 188, 220, 232, 40, 80, 78, 51, 10, 74, 167, 151, 96, 115, 30, 0, 98, 68, 26, 184, 56, 130, 100, 159, 38, 65, 173, 69, 70, 146, 39, 94, 85, 47, 140, 163, 165, 125, 105, 213, 149, 59, 7, 88, 179, 64, 134, 172, 29, 247, 48, 55, 107, 228, 136, 217, 231, 137, 225, 27, 131, 73, 76, 63, 248, 254, 141, 83, 170, 144, 202, 216, 133, 97, 32, 113, 103, 164, 45, 43, 9, 91, 203, 155, 37, 208, 190, 229, 108, 82, 89, 166, 116, 210, 230, 244, 180, 192, 209, 102, 175, 194, 57, 75, 99, 182).$

*The S-Box  $\pi$  of the latest Russian standards, Kuznyechik (BC) and Streebog (HF).*

## The S-Box (2/2)

### Importance of the S-Box

If  $S$  is such that

$$S(x) \oplus S(x \oplus a) = b$$

does not have many solutions  $x$  for all  $(a, b)$  then the cipher may be proved secure against differential attacks.

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does not have many solutions  $x$  for all  $(a, b)$  then the cipher may be proved secure against differential attacks.

In **academic** papers presenting new block ciphers, the choice of  $S$  is carefully explained.

## S-Box Design

- AES S-Box
- Inverse (other)
- Exponential
- Math (other)
- SPN
- Misty
- Feistel
- Lai-Massey
- Pseudo-random
- Hill climbing
- Unknown

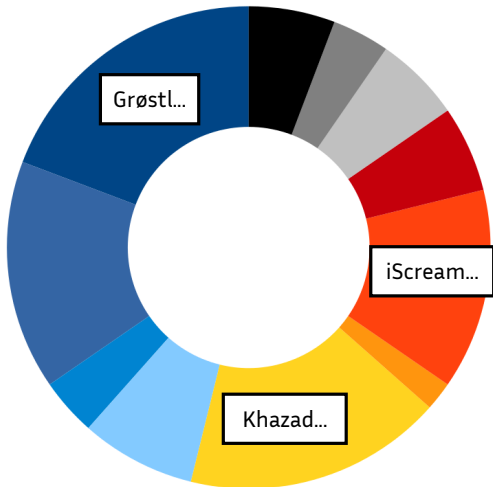
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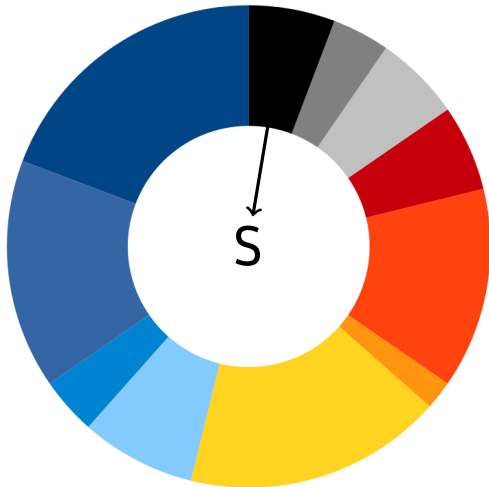
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## S-Box Reverse-Engineering

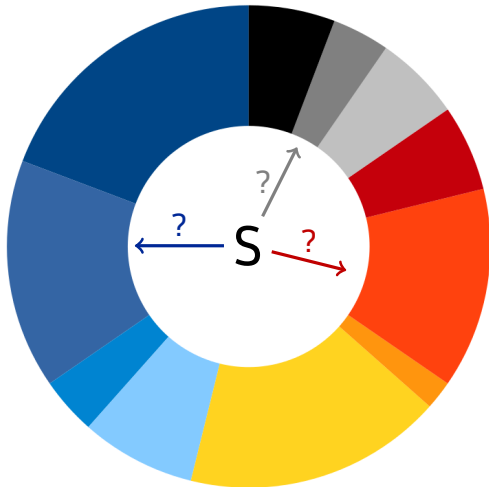
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**A malicious designer can easily hide a structure in an S-Box.**

To keep an advantage in implementation (WB crypto)...  
... or an advantage in cryptanalysis (backdoor).

## Motivation (2/3)

### Definition (Kleptography)

The study of trapdoored cryptography is called **kleptography** (term introduced by Jung and Young).

### S-Box based backdoors in the literature

- Rijmen, V., & Preneel, B. (1997). *A family of trapdoor ciphers*. FSE'97.
- Patterson, K. (1999). *Imprimitive Permutation Groups and Trapdoors in Iterated Block Ciphers*. FSE'99.
- Blondeau, C., Civino, R., & Sala, M. (2017). *Differential Attacks: Using Alternative Operations*. eprint report 2017/610.
- Bannier, A., & Filiol, E. (2017). *Partition-based trapdoor ciphers*. InTech'17.

## Motivation (3/3)

Even without malicious intent, an unexpected structure can be a problem.

⇒ We need tools to *reverse-engineer* S-Boxes!

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## Summary



We can recover parts of the design process of an S-Box using some statistics.

- 1 The two tables (basics of Boolean functions for cryptography)
- 2 A statistical tool based on the two tables
- 3 Application to NSA's Skipjack



# The Two Tables

Let  $S : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$  be an S-Box.

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### Definition (DDT)

The *Difference Distribution Table* of  $S$  is a matrix of size  $2^n \times 2^n$  such that

$$\text{DDT}[a, b] = \#\{x \in \mathbb{F}_2^n \mid S(x \oplus a) \oplus S(x) = b\}.$$

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## Definition (LAT)

The *Linear Approximations Table* of  $S$  is a matrix of size  $2^n \times 2^n$  such that

$$\text{LAT}[a, b] = \#\{x \in \mathbb{F}_2^n \mid x \cdot a = S(x) \cdot b\} - 2^{n-1}.$$

## Example

$$S = [4, 2, 1, 6, 0, 5, 7, 3]$$

The **DDT** of  $S$ .

$$\begin{bmatrix} 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\ 0 & 0 & 4 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\ 0 & 4 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \end{bmatrix}$$

The **LAT** of  $S$ .

$$\begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 & 0 & 0 & 2 & -2 \\ 0 & 2 & 2 & 0 & 0 & 2 & -2 & 0 \\ 0 & 2 & 0 & 2 & 0 & -2 & 0 & 2 \\ 0 & 2 & 0 & -2 & 0 & -2 & 0 & -2 \\ 0 & -2 & 2 & 0 & 0 & -2 & -2 & 0 \\ 0 & 0 & -2 & 2 & 0 & 0 & -2 & -2 \\ 0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 \end{bmatrix}$$

## Coefficient Distribution in the DDT

If an  $n$ -bit S-Box is bijective, then its DDT coefficients behave like **independent** and identically distributed random variables following a Poisson distribution:

$$\Pr [\text{DDT}[a, b] = 2z] = \frac{e^{-1/2}}{2^z} .$$

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- Always even,  $\geq 0$
- Typically between 0 and 16.
- Lower is better.

## Coefficient Distribution in the LAT

If an  $n$ -bit S-Box is bijective, then its LAT coefficients behave like **independent** and identically distributed random variables following this distribution:

$$\Pr [\text{LAT}[a, b] = 2z] = \frac{\binom{2^{n-1}}{2^{n-2+z}}}{\binom{2^n}{2^{n-1}}}.$$

## Coefficient Distribution in the LAT

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$$\Pr [\text{LAT}[a, b] = 2z] = \frac{\binom{2^{n-1}}{2^{n-2+z}}}{\binom{2^n}{2^{n-1}}}.$$

- Always even, signed.
- Typically between -40 and 40.
- Lower absolute value is better.



## Looking Only at the Maximum

$\delta$	$\log_2 (\Pr [\max(\text{DDT}) \leq \delta])$
14	-0.006
12	-0.094
10	-1.329
8	-16.148
6	-164.466
4	-1359.530

**DDT**

$\ell$	$\log_2 (\Pr [\max(\text{LAT}) \leq \ell])$
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34	-1.008
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**LAT**

Probability that the maximum coefficient in the DDT/LAT of an 8-bit permutation is at most equal to a certain threshold.

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## What is Skipjack? (1/2)

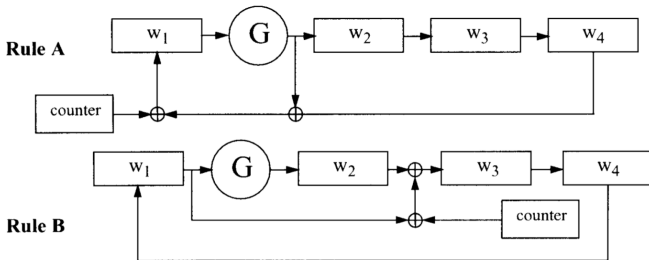
Type Block cipher

Bloc 64 bits

Key 80 bits

Authors NSA

Publication 1998



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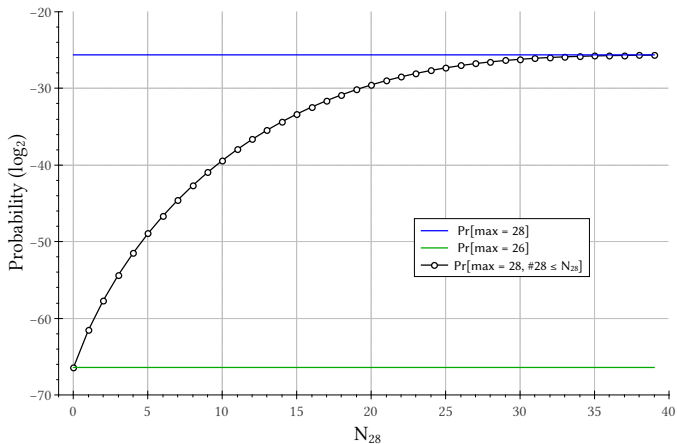
- Skipjack was supposed to be secret...
- ... but eventually published in 1998.
- Skipjack was to be used by the *Clipper Chip*,
- It uses an  $8 \times 8$  S-Box ( $F$ ) specified only by its LUT.

## Reverse-Engineering $F$

For Skipjack's  $F$ ,  $\max(\text{LAT}) = 28$  and  $\#28 = 3$ .

## Reverse-Engineering F

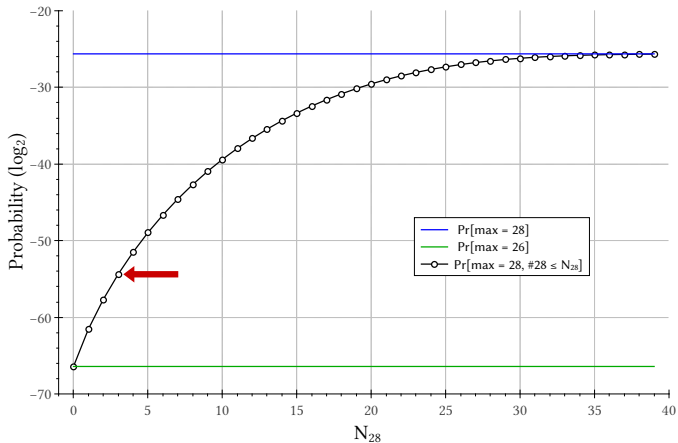
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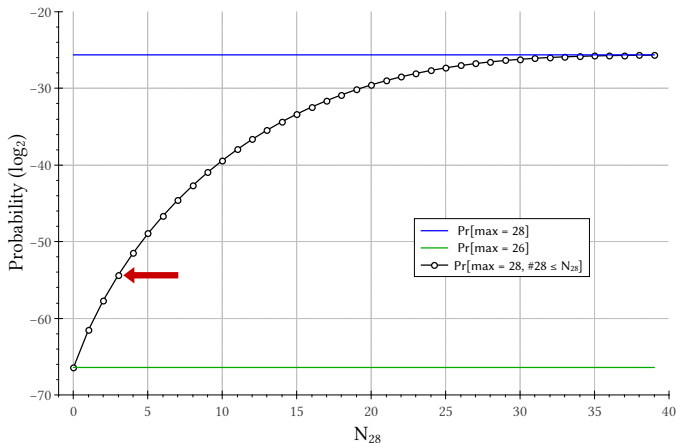
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$$\text{Pr} [\max(\text{LAT}) = 28 \text{ and } \#28 \leq 3] \approx 2^{-55}$$

## What Can We Deduce?

- $F$  has not been picked uniformly at random.
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- Its linear properties were optimized (though poorly).

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**The S-Box of Skipjack was built  
using a dedicated algorithm.**

## Timeline

**Jun 98** Declassification of Skipjack

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**1987** Initial design of Skipjack

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- Sep 95** Schneier published his thoughts on “alleged Skipjack”, including the result of a FOIA request
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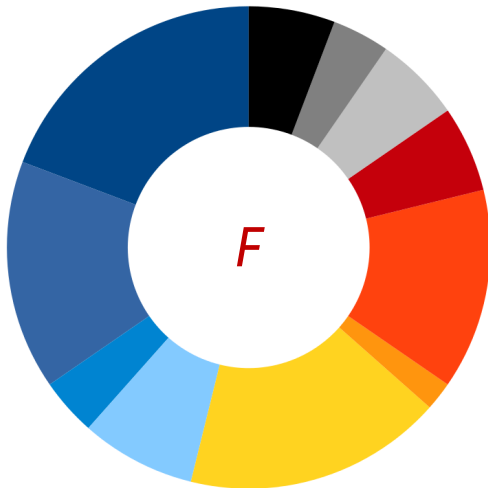


## Timeline

- 1987** Initial design of Skipjack
- Aug 90** (CRYPTO) Gilbert et al. use linear relations for key recovery (FEAL)
- Aug 91** (CRYPTO) Attack against FEAL using linear relations between key, plaintext and ciphertext
- May 92** (EUROCRYPT) Other attack against FEAL using linear relations between key, plaintext and ciphertext
- Aug 92** The S-Box ("F-table") of Skipjack is changed
- Jul 93** "interim report" on Skipjack published by external cryptographers
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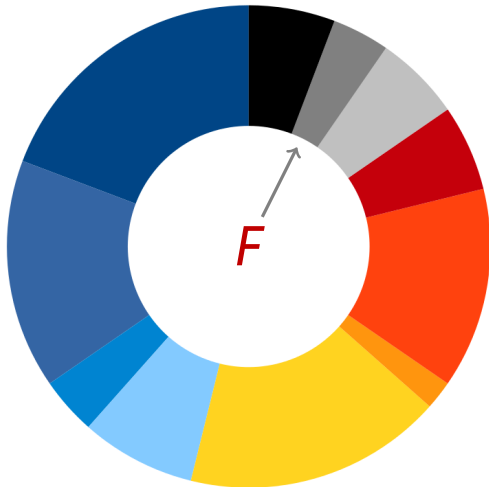
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We can recover an **actual decomposition** using patterns in the LAT.

- 1 Our target, the S-Box of Kuznyechik and Streebog
- 2 TU-decomposition: what is it and how to apply it to Kuznyechik

## Kuznyechik/Stribog

### Stribog

**Type** Hash function

**Publication** 2012

### Kuznyechik

**Type** Block cipher

**Publication** 2015



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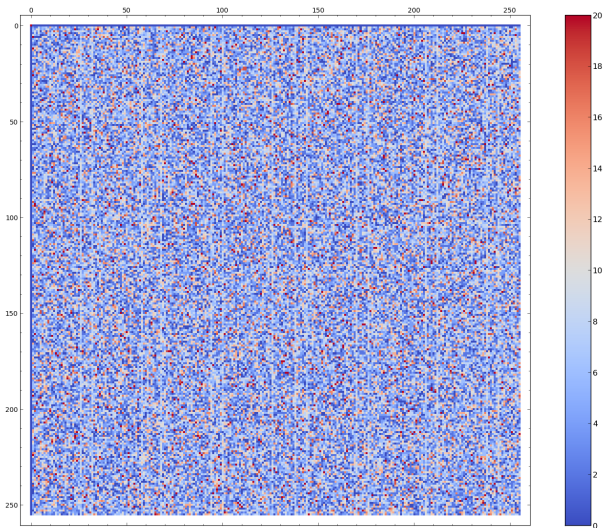
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## Common ground

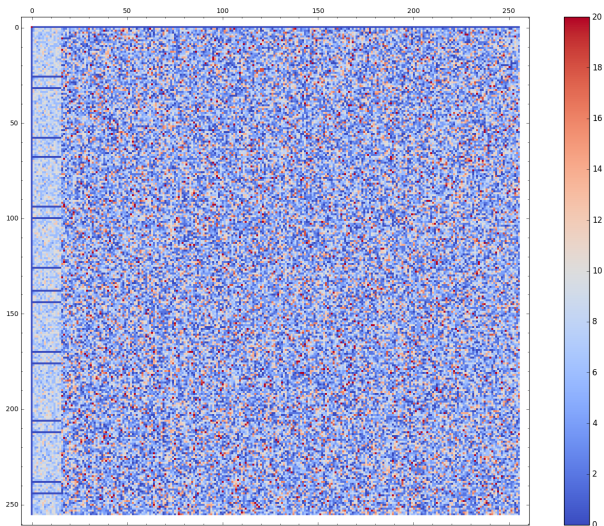
- Both are standard symmetric primitives in Russia.
- Both were designed by the FSB (TC26).
- Both use the same  $8 \times 8$  S-Box,  $\pi$ .

## The LAT of $\pi$

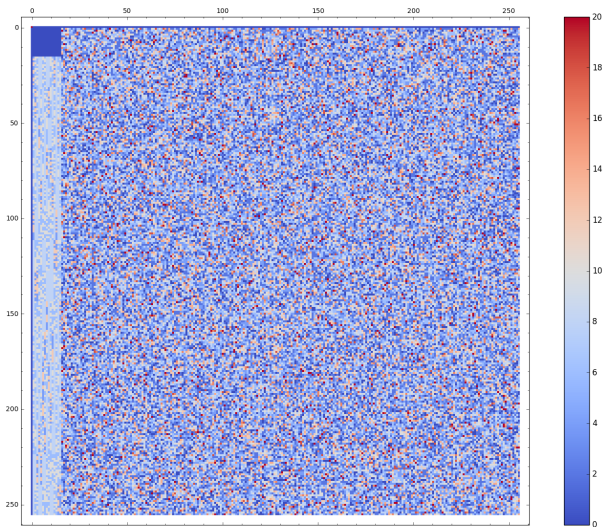




## The LAT of $\eta$ (reordered columns)



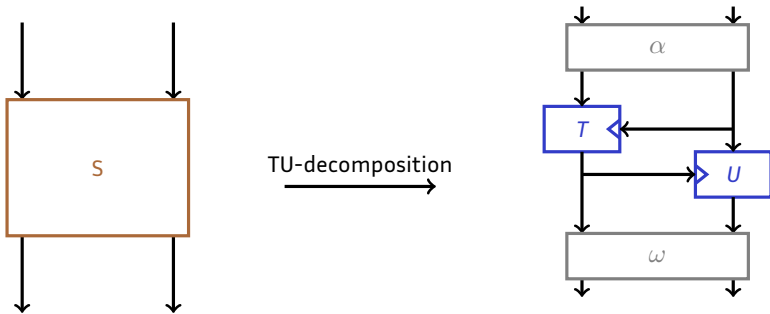
## The LAT of $\eta \circ \pi \circ \mu$



## The TU-Decomposition

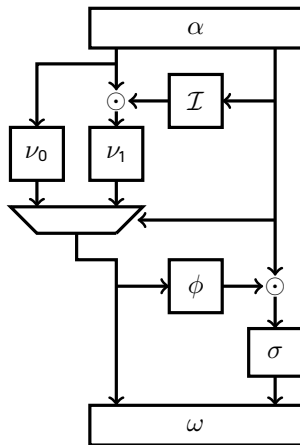
### Definition

The **TU-decomposition** is a decomposition algorithm working against S-Boxes with vector spaces of zeroes in their LAT.



$T$  and  $U$  are mini-block ciphers;  $\mu$  and  $\eta$  are linear permutations.

# Final Decomposition Number 1



$\odot$  Multiplication in  $\mathbb{F}_{2^4}$

$\alpha$  Linear permutation

$\mathcal{I}$  Inversion in  $\mathbb{F}_{2^4}$

$\nu_0, \nu_1, \sigma$   $4 \times 4$  permutations

$\phi$   $4 \times 4$  function

$\omega$  Linear permutation

## Hardware Performance

Structure	Area ( $\mu m^2$ )	Delay (ns)
Naive implementation	3889.6	362.52
Feistel-like	1534.7	61.53
Multiplications-first	1530.3	54.01
Feistel-like (with tweaked MUX)	1530.1	46.11

## Conclusion for Kuznyechik/Stribog?

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strange Feistel...**

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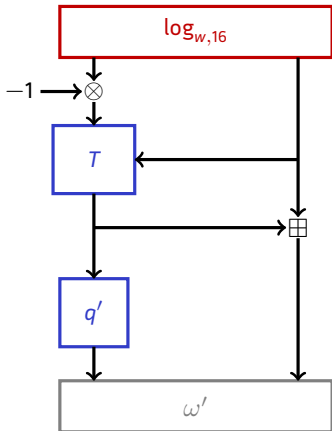
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- The last standard of Belarus (BelT) uses an 8-bit S-box,
- somewhat similar to  $\pi$ ...
- ... based on a **finite field exponential!**

## Final Decomposition Number 2 (!)



	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$T_0$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$T_1$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$T_2$	0	1	2	3	4	5	6	7	8	9	a	b	c	d	f	e
$T_3$	0	1	2	3	4	5	6	7	8	9	a	b	c	f	d	e
$T_4$	0	1	2	3	4	5	6	7	8	9	a	b	f	c	d	e
$T_5$	0	1	2	3	4	5	6	7	8	9	a	f	b	c	d	e
$T_6$	0	1	2	3	4	5	6	7	8	9	f	a	b	c	d	e
$T_7$	0	1	2	3	4	5	6	7	8	f	9	a	b	c	d	e
$T_8$	0	1	2	3	4	5	6	7	f	8	9	a	b	c	d	e
$T_9$	0	1	2	3	4	5	6	f	7	8	9	a	b	c	d	e
$T_a$	0	1	2	3	4	f	6	7	8	9	a	b	c	d	e	
$T_b$	0	1	2	3	f	5	6	7	8	9	a	b	c	d	e	
$T_c$	0	1	2	f	4	5	6	7	8	9	a	b	c	d	e	
$T_d$	0	1	f	3	4	5	6	7	8	9	a	b	c	d	e	
$T_e$	0	1	f	2	3	4	5	6	7	8	9	a	b	c	d	e
$T_f$	0	f	1	2	3	4	5	6	7	8	9	a	b	c	d	e

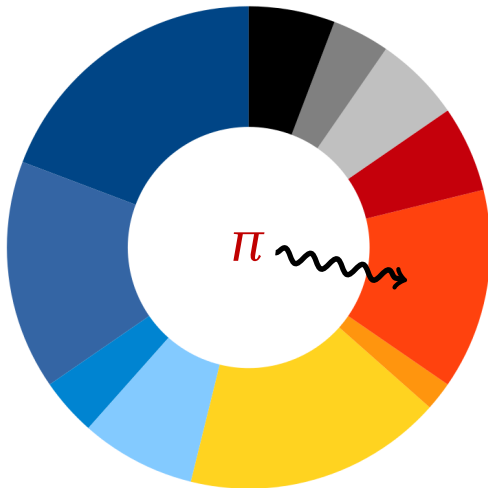
## Conclusion on Kuznyechik/Streebog

- AES S-Box
- Inverse (other)
- Exponential
- Math (other)
- SPN
- Misty
- Feistel
- Lai-Massey
- Pseudo-random
- Hill climbing
- Unknown



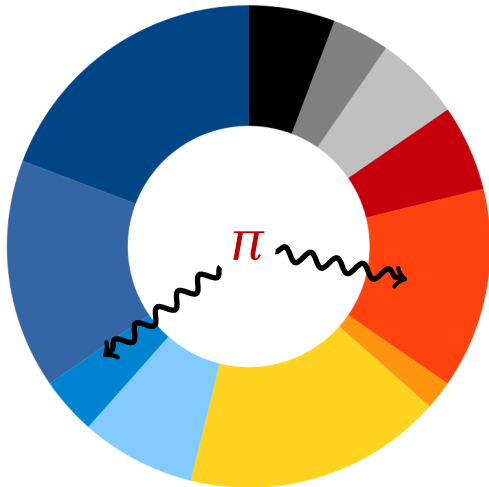
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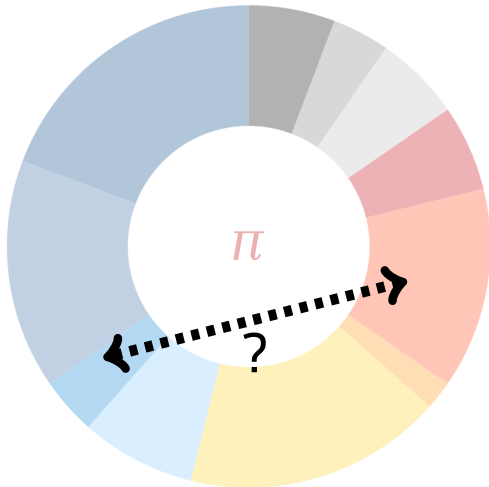
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# Outline

- 1 Building Blocks for Symmetric Cryptography
- 2 Statistics and Skipjack
- 3 TU-Decomposition and Kuznyechik
- 4 The Butterfly Permutations and Functions**
- 5 Conclusion

## Summary



We can obtain new mathematical results using reverse-engineering techniques.

- 1 The big APN problem and its only known solution
- 2 Decomposing and generalizing this solution as butterflies



## NSUCRYPTO (Olympiad in Cryptography)

Siberian Student's Olympiad in Cryptography with International participation — 2014

Second round

NSUCRYPTO

November 17-24



Task 2. «An APN Permutation»

*“Try to find an APN permutation on 8 variables or prove that it doesn't exist.”*

<https://nsucrypto.nsu.ru/>

# The Big APN Problem

## Definition (APN function)

A function  $S : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$  is **Almost Perfect Non-linear (APN)** if

$$S(x \oplus a) \oplus S(x) = b$$

has 0 or 2 solutions for all  $a \neq 0$  and for all  $b$ .

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## Big APN Problem

Are there APN permutations operating on  $\mathbb{F}_2^n$  where  $n$  is even?

## Dillon et al.'s Permutation

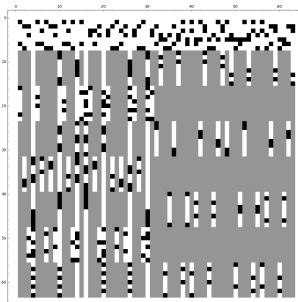
### Only One Known Solution!

For  $n = 6$ , Dillon et al. found an APN permutation.

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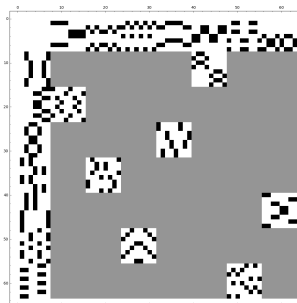
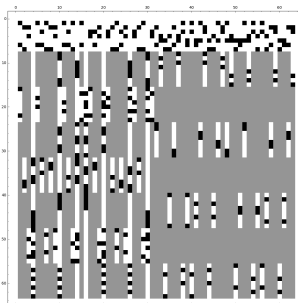
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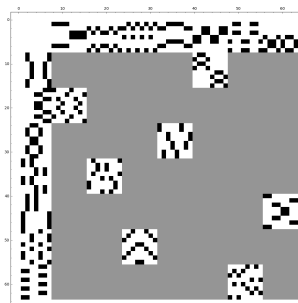
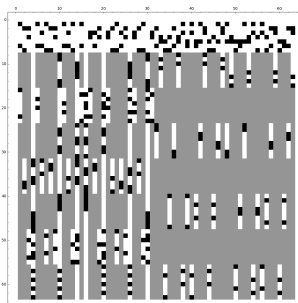
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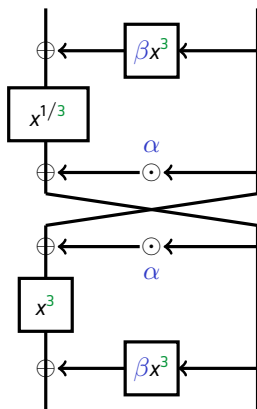
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It is possible to make a TU-decomposition!

## On the Butterfly Structure

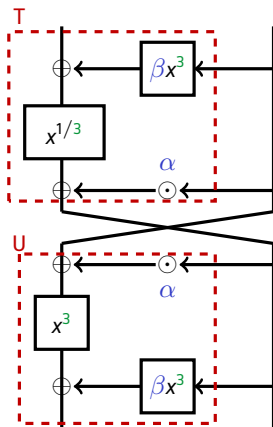


Definition (Open Butterfly  $H_{\alpha,\beta}^3$ )

This permutation is an **open butterfly**.



## On the Butterfly Structure



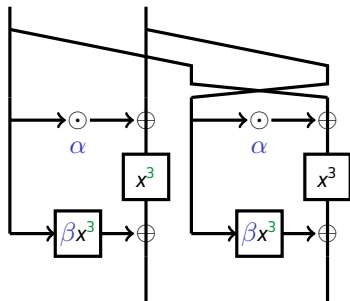
**Definition (Open Butterfly  $H_{\alpha,\beta}^3$ )**

This permutation is an **open butterfly**.

**Lemma**

*Dillon's permutation is affine-equivalent to  $H_{w,1}^3$ , where  $\text{Tr}(w) = 0$ .*

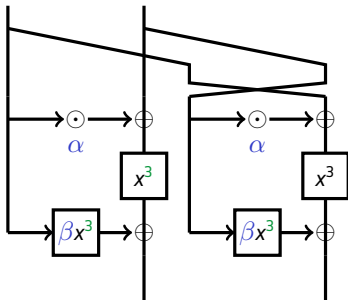
## Closed Butterflies



Definition (Closed butterfly  $V_{\alpha, \beta}^3$ )

This quadratic function is a **closed butterfly**.

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**Lemma (Equivalence)**

*Open and closed butterflies with the same parameters are CCZ-equivalent.*

## Some Properties of Butterflies

### Theorem (Properties of butterflies)

Let  $V_{\alpha,\beta}^3$  and  $H_{\alpha,\beta}^3$  be butterflies operating on  $2n$  bits,  $n$  odd. Then:

- $\deg(V_{\alpha,\beta}^3) = 2$ ,
- if  $n = 3$ ,  $\text{Tr}(\alpha) = 0$  and  $\beta + \alpha^3 \in \{\alpha, 1/\alpha\}$ , then
 
$$\max(\text{DDT}) = 2, \max(\mathcal{W}) = 2^{n+1} \text{ and } \deg(H_{\alpha,\beta}^3) = n + 1,$$
- if  $\beta = (1 + \alpha)^3$ , then
 
$$\max(\text{DDT}) = 2^{n+1}, \max(\mathcal{W}) = 2^{(3n+1)/2} \text{ and } \deg(H_{\alpha,\beta}^3) = n,$$
- otherwise,
 
$$\max(\text{DDT}) = 4, \max(\mathcal{W}) = 2^{n+1} \text{ and } \deg(H_{\alpha,\beta}^3) \in \{n, n + 1\}$$
 and  $\deg(H_{\alpha,\beta}^3) = n$  if and only if
 
$$1 + \alpha\beta + \alpha^4 = (\beta + \alpha + \alpha^3)^2.$$

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## Open Problem

### Cellular Message Encryption Algorithm

From Wikipedia, the free encyclopedia

In [cryptography](#), the **Cellular Message Encryption Algorithm** (**CMEA**) is a [block cipher](#) which was used for securing [mobile phones](#) in the [United States](#). CMEA is one of four cryptographic primitives specified in a [Telecommunications Industry Association](#) (TIA) standard, and is designed to [encrypt](#) the control channel, rather than the voice data. In 1997, a group of cryptographers published attacks on the [cipher](#) showing it had several weaknesses which give it a trivial effective strength of a 24-bit to 32-bit cipher.<sup>[1]</sup>

#### CMEA

##### General

**Designers** [James A. Reeds III](#)

**First published** 1991

##### Cipher detail

**Key sizes** 64 bits

**Block sizes** 16-64 bits

**Rounds** 3

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### A hidden structure!

CMEA uses an 8-bit (non-bijective) S-Box... With a TU-decomposition!

**What is its actual structure?**

## Conclusion

- 1 Cryptographers use mathematics but mathematicians could also use crypto!



## Conclusion

- 1 Cryptographers use mathematics but mathematicians could also use crypto!
- 2 If you **design** a cipher, **justify** every step of your design.

## Conclusion

- 1 Cryptographers use mathematics but mathematicians could also use crypto!
- 2 If you **design** a cipher, **justify** every step of your design.
- 3 If you **choose** a cipher, **demand** a full design explanation.

## The Last S-Box

14	11	60	6d	e9	10	e3	2	b	90	d	17	c5	b0	9f	c5
d8	da	be	22	8	f3	4	a9	fe	f3	f5	fc	bc	30	be	26
bb	88	85	46	f4	2e	e	fd	76	fe	b0	11	4e	de	35	bb
30	4b	30	d6	dd	df	df	d4	90	7a	d8	8c	6a	89	30	39
e9	1	da	d2	85	87	d3	d4	ba	2b	d4	9f	9c	38	8c	55
d3	86	bb	db	ec	e0	46	48	bf	46	1b	1c	d7	d9	1b	e0
23	d4	d7	7f	16	3f	3	3	44	c3	59	10	2a	da	ed	e9
8e	d8	d1	db	cb	cb	c3	c7	38	22	34	3d	db	85	23	7c
24	d1	d8	2e	fc	44	8	38	c8	c7	39	4c	5f	56	2a	cf
d0	e9	d2	68	e4	e3	e9	13	e2	c	97	e4	60	29	d7	9b
d9	16	24	94	b3	e3	4c	4c	4f	39	e0	4b	bc	2c	d3	94
81	96	93	84	91	d0	2e	d6	d2	2b	78	ef	d6	9e	7b	72
ad	c4	68	92	7a	d2	5	2b	1e	d0	dc	b1	22	3f	c3	c3
88	b1	8d	b5	e3	4e	d7	81	3	15	17	25	4e	65	88	4e
e4	3b	81	81	fa	1	1d	4	22	0	6	1	27	68	27	2e
3b	83	c7	cc	25	9b	d8	d5	1c	1f	e5	59	7f	3f	3f	ef

