



Generalized Feistel Networks with Optimal Diffusion

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Generalized Feistel Networks with Optimal Diffusion

Léo Perrin

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Dagstuhl 2018 (seminar-18021)



Technical University of Denmark



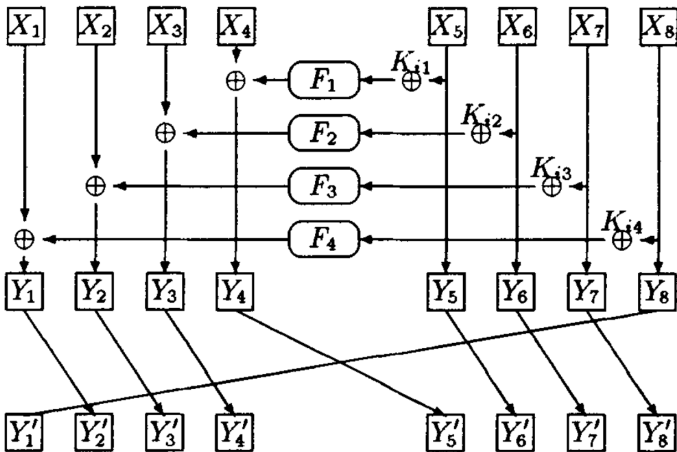
In this talk

- A new type of generalized Feistel Networks
- Linear layer design
- Wide block cipher/sponge permutation blueprint
- Fibonacci numbers!

Outline

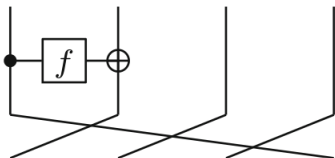
- 1 Introduction
- 2 Observations on GFNs**
- 3 Multi-Rotating Feistel Network (MRFN)
- 4 Possible Applications
- 5 Conclusion

First GFN

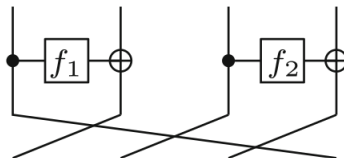


Source: *Generalized Feistel networks*, K. Nyberg (1996)

Basic GFN



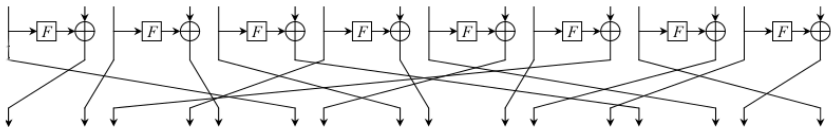
Type-I GFN



Type-II GFN

Source: *Generalized Feistel networks revisited*, A. Bogdanov, K. Shibutani (2013)

Improved GFN



Source: *TWINE: A Lightweight, Versatile Block Cipher*, T. Suzaki, K. Minematsu, S. Morioka, and E. Kobayashi

Diffusion in Generalized Feistel networks

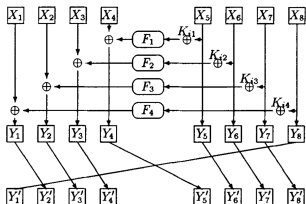
How long does it take for each input word to influence each output word?

The state consists of $2b$ branches.

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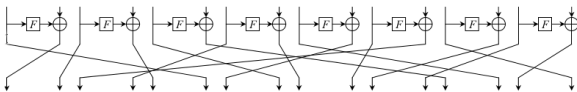
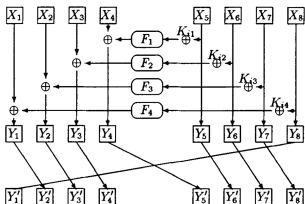
Nyberg/Type-II GFN:

$\approx 2b$ rounds

Diffusion in Generalized Feistel networks

How long does it take for each input word to influence each output word?

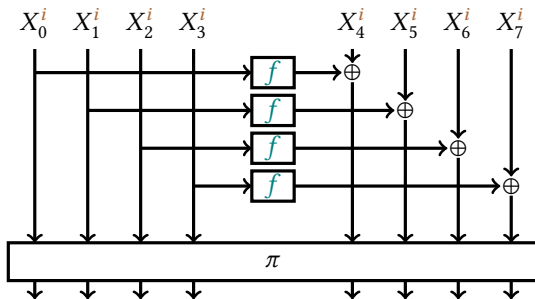
The state consists of $2b$ branches.



Nyberg/Type-II GFN:
 $\approx 2b$ rounds

TWINE-like GFN: $\approx 2 \log_2(b)$ rounds

General Vue



Optimal Diffusion

The best we can achieve is for X_0^0 to influence ϕ_{i+2} branches at round i , where

$$\phi_0 = 0, \phi_1 = 1, \phi_{i+2} = \phi_{i+1} + \phi_i.$$

Diffusion in GFNs

| <i>b</i> | 8 | 16 | 32 | 64 | 128 | .. | 2048 |
|-----------------------|----|----|----|-----|-----|----|------|
| Nyberg Type-II/Nyberg | 16 | 32 | 64 | 128 | 256 | | 4096 |
| TWINE-like | 6 | 8 | 10 | 12 | 14 | | 22 |
| Optimal | 6 | 8 | 9 | 11 | 12 | | 18 |

Number of rounds for full diffusion.

- Can we reach the Fibonacci-based bound?
- Can we have an easy to implement π ?

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Yes (for both)

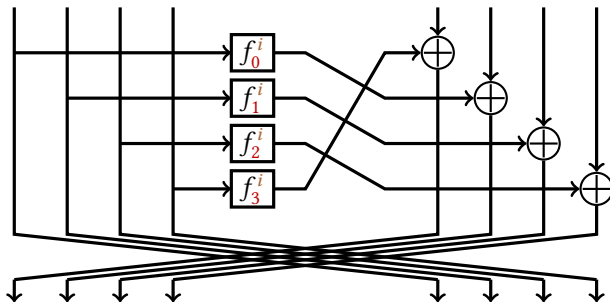
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General Structure

- Number of branches: $2b$
- Number of rounds: r
- w -bit permutations f_j^i ($i < r, j < b$)
- Sequence s^i of rotations of b words.

The round i of a MRFN with $b = 4$ and $s^i = 1$ is:



Some Observations

- Both a Feistel network and a GFN
- π is very simple (1 word-wise rotation per round)
- Round function depends on the round index.
- Interesting case: $s^i = \phi_i$.

Some Observations

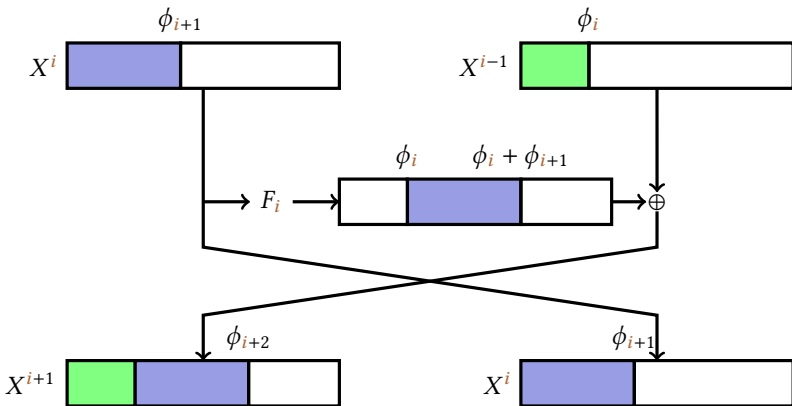
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Fibonacci Case

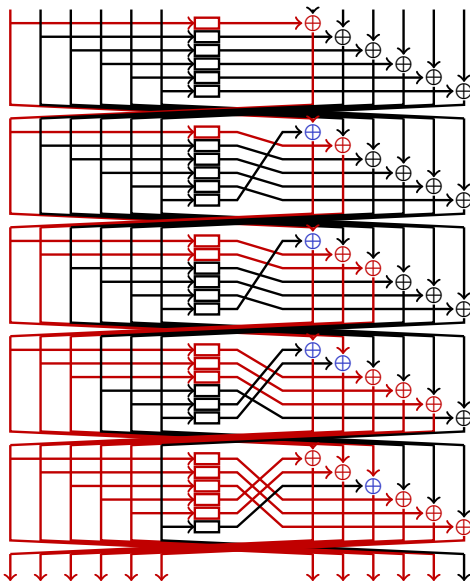
A MRFN with $s^i = \phi_i$ has optimal diffusion.

Fibonacci Case

At round 0, X_0^0 has touched the first $\phi_1 = 1$ branches of one side.



Example with 12 branches



$$\phi_0 = 0$$

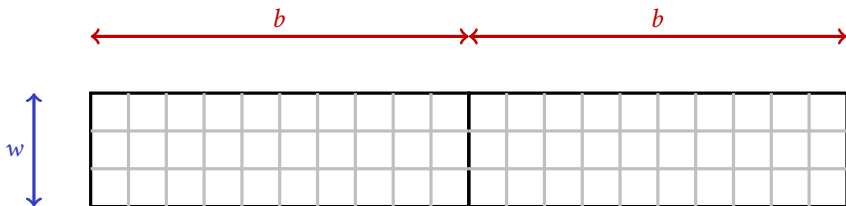
$$\phi_1 = 1$$

$$\phi_2 = 1$$

$$\phi_3 = 2$$

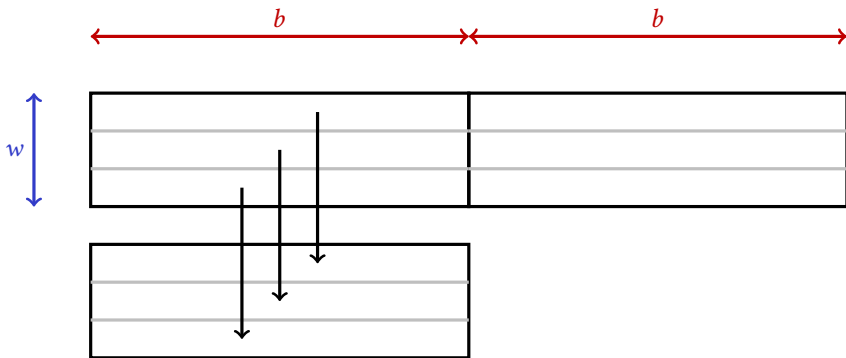
$$\phi_4 = 3$$

Implementation



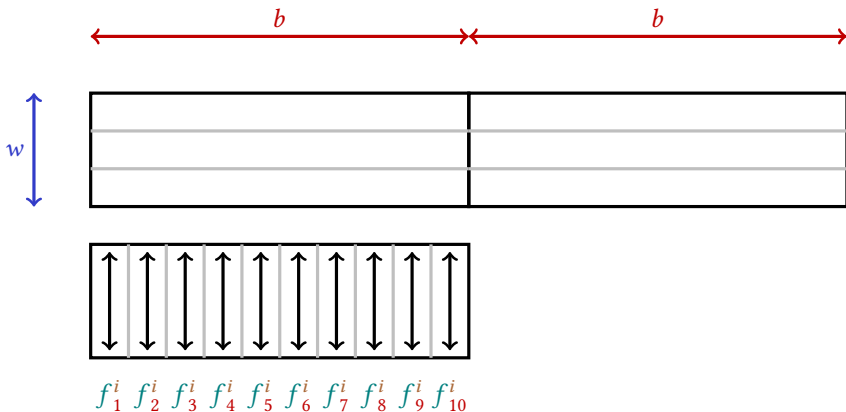
VRound function operating on $2bw$ bit internal state.

Implementation



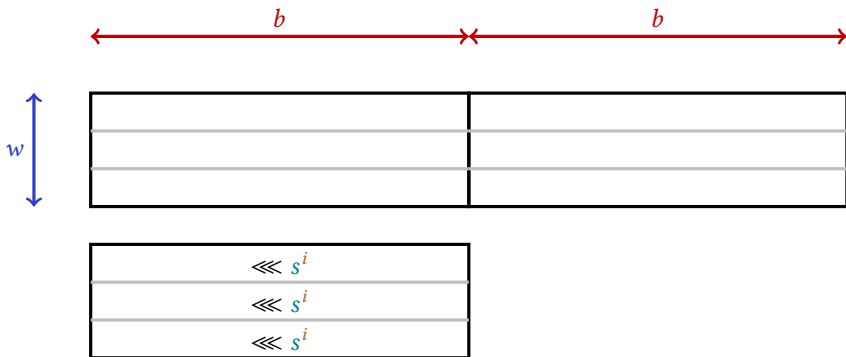
1. copy

Implementation



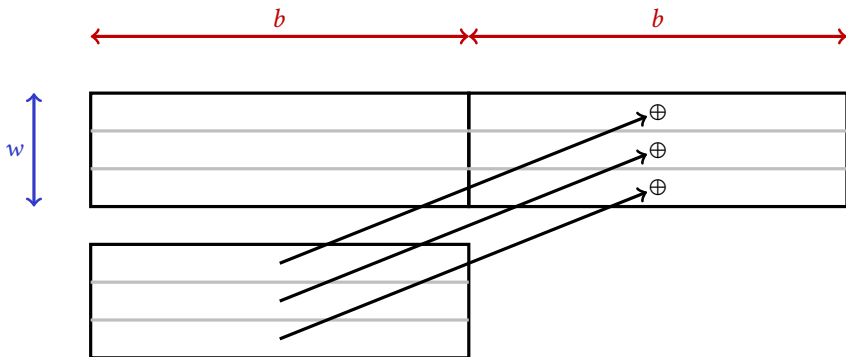
2. parallel layer of f^i

Implementation



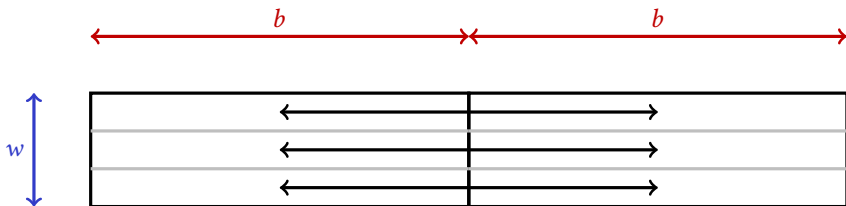
3. rotations

Implementation



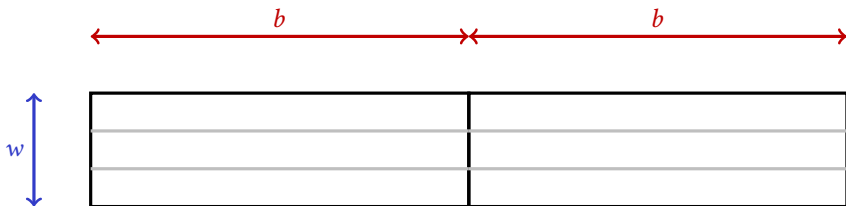
4. XOR

Implementation



5. swap

Implementation



6. finished!

Some Observations

- s^i and $s^i + (-\ell)^i \pmod b$ are equivalent
- if $\gcd(s^i, b) \neq 1$ for all i , **no full diffusion!**
- Importance of the choice of $\{s^i\}_{i \geq 0}$

Security

- If $s^i = \phi_i$, then full diffusion in $\approx \Lambda(n)$ rounds, where $\Lambda(x) = i$ if $\phi_{i-1} < x \leq \phi_i$ (optimal).
- If $s^{2^i} = 0$ and $i_{2^{i+1}} = 2^i$, then full diffusion in $\approx 2 \log_2(n)$ rounds (like TWINE).
- Both are quickly safe from miss-in-the-middle based impossible differential attacks and MitM!

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Open Problem 1

Differential/Linear bound?

Open Problem 2

Choice of $\{s^i\}_{i \geq 0}$?

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GFN-based Linear Layers

- Use linear $\{f^i\}_{i \geq 0}; s^i = \phi_i$
- n -bit block divided into $2b$ branches of w bits uses:

$$\underbrace{\frac{w^2}{2}}_{f_j^i} \times b \times \underbrace{2 \log_2(b)}_r \text{ XORs .}$$

f layer

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f_{layer}

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f layer

- If we fix w to a small value, then the number of XORs scales with $n \log_2(n)$ rather than n^2 .
- Practical gains even for $n = 256$:

Improvements to the Linear Layer of LowMC: A Faster Picnic, with Angela Promitzer, Sebastian Ramacher and Christian Rechberger (2017/448)

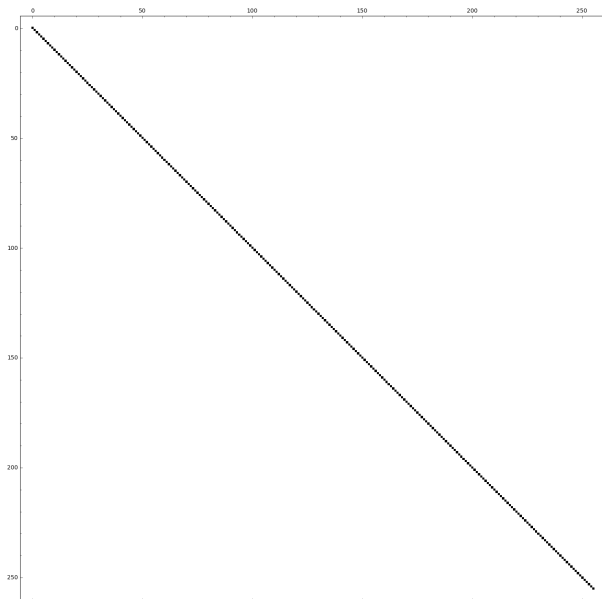
Example of Linear Layer

■ $n = 256$

■ $w = 4$

■ $b = 32$

$i = 0$





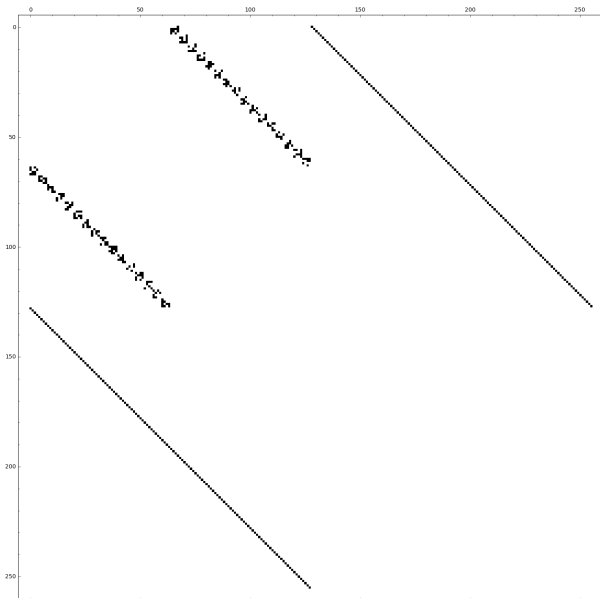
Example of Linear Layer

■ $n = 256$

■ $w = 4$

■ $b = 32$

$i = 1$



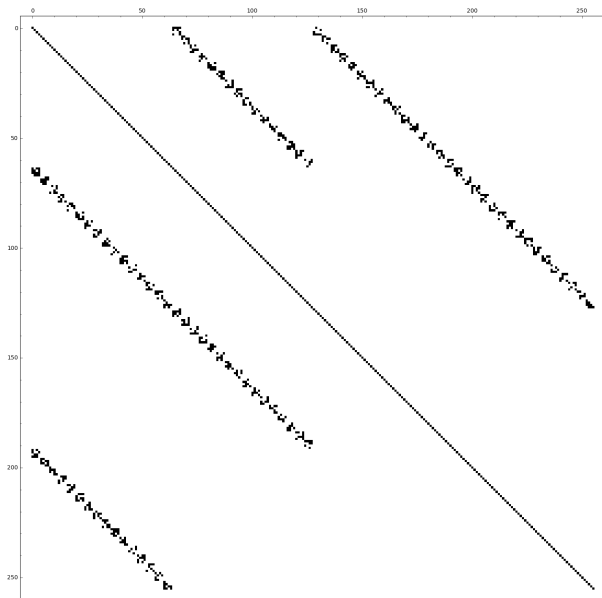
Example of Linear Layer

■ $n = 256$

■ $w = 4$

■ $b = 32$

$i = 2$





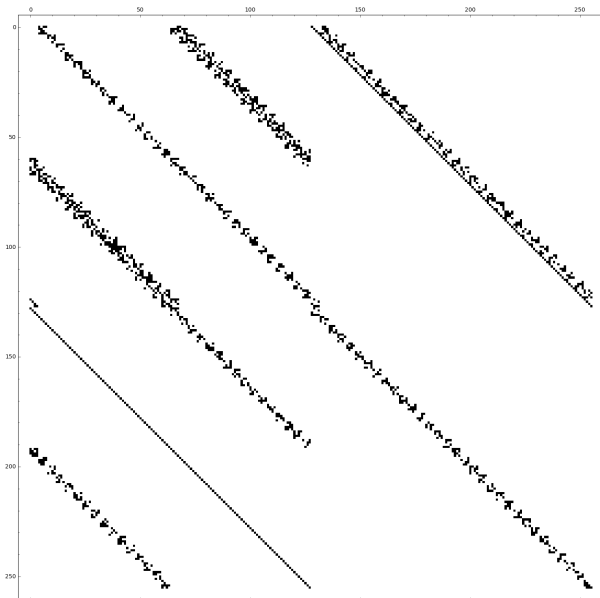
Example of Linear Layer

■ $n = 256$

■ $w = 4$

■ $b = 32$

$i = 3$



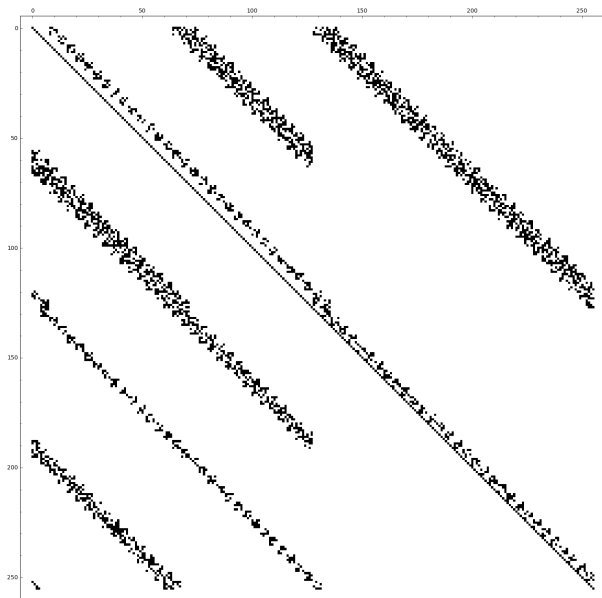
Example of Linear Layer

■ $n = 256$

■ $w = 4$

■ $b = 32$

$i = 4$



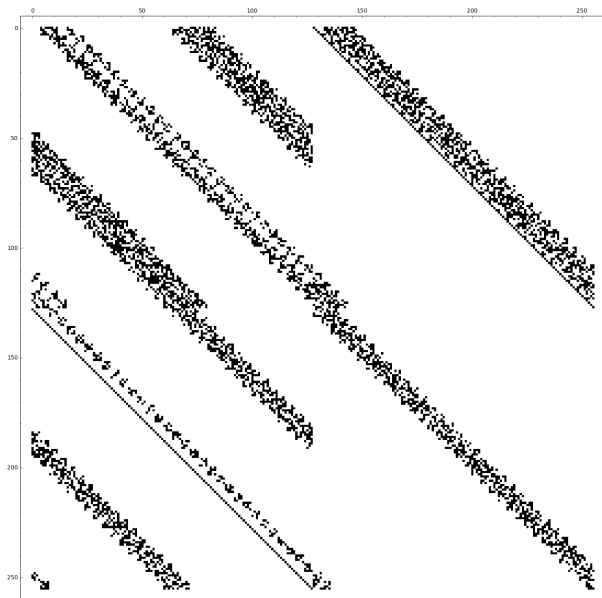
Example of Linear Layer

■ $n = 256$

■ $w = 4$

■ $b = 32$

$i = 5$



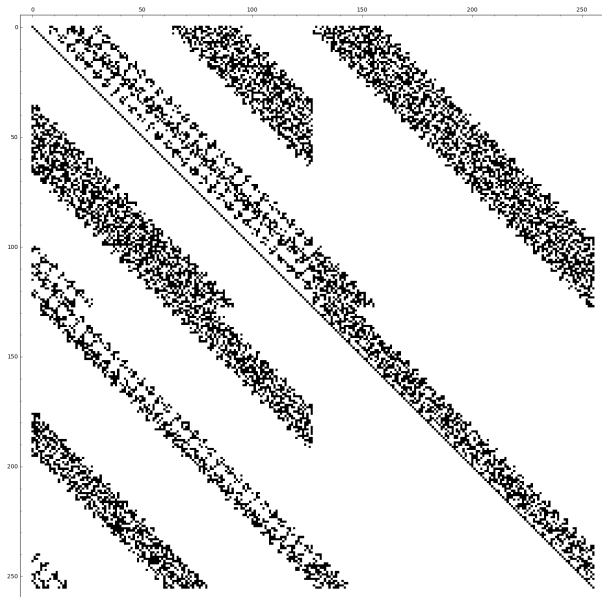
Example of Linear Layer

■ $n = 256$

■ $w = 4$

■ $b = 32$

$i = 6$



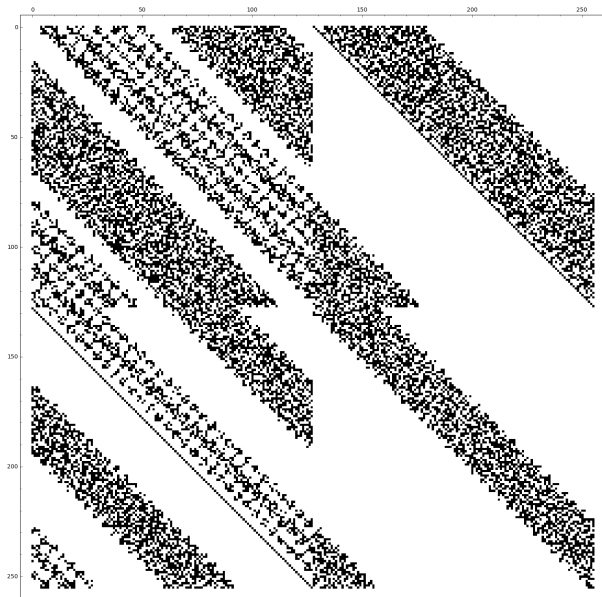
Example of Linear Layer

■ $n = 256$

■ $w = 4$

■ $b = 32$

$i = 7$



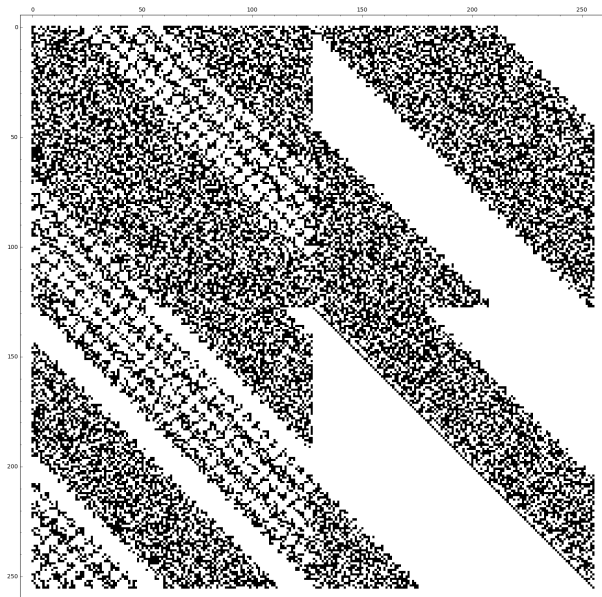
Example of Linear Layer

■ $n = 256$

■ $w = 4$

■ $b = 32$

$i = 8$



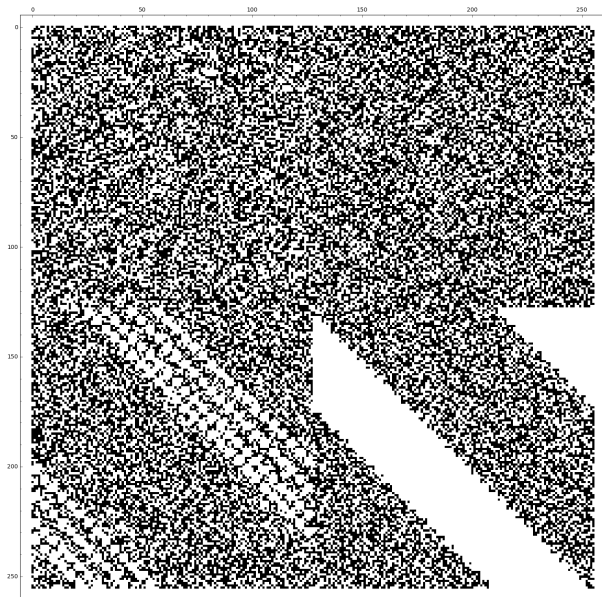
Example of Linear Layer

■ $n = 256$

■ $w = 4$

■ $b = 32$

$i = 9$



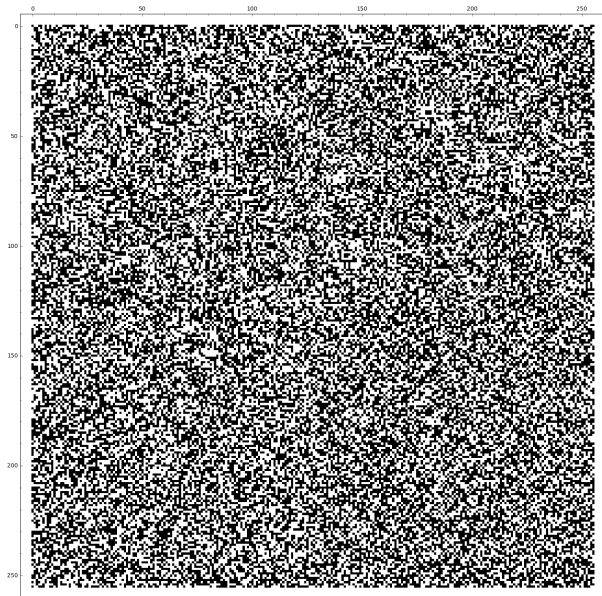
Example of Linear Layer

■ $n = 256$

■ $w = 4$

■ $b = 32$

$i = 10$



Sponge function?

■ $n = 384$, with $b = 64$ and $w = 3$

■ $f_j^i(x) = \chi_3(x \oplus c_j^i)$

■ $s^{2i} = 0, s^{2i+1} = 2^i$ for $0 \leq i < 2 \log_2(b) = 12$, then repeat (4? times):

$$s = \{0, 1, 0, 2, 0, 4, 0, 8, 0, 16, 0, 32\}$$

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Efficiency estimates

On 64-bit processors, for each round:

- 3 word copies
- 3 word-wise AND
- 3+3+3 word-wise XORs
- **Maybe** safe for **48 rounds** if ≥ 8 active f functions/round on average.

Other?

- MiMC-like construction where $f_j^i(x) = (x + c_j^i)^3$ (what Arnab just presented).

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- You tell me!

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Conclusion

Fun stuff happens when we allow the use of different permutations in each round!

Open problems

- 1 What are good sequences of rotations?
- 2 How to bound number of active f functions?
- 3 What can we use it for?
- 4 What happens in other structures (SPN? ARX?) when the linear layers are round-dependent?

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Thank you!