New Results on Quantum Symmetric Cryptanalysis

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Outline

- Introduction
  On Quantum-Safe Symmetric Cryptography

- Efficient Quantum Collision Search
  joint work with A. Chailloux and A. Schrottenloher
  [Asiacrypt17]

- On Modular Additions
  joint work with X. Bonnetain

Symmetric Cryptography

Classical Cryptography

Enable secure communications even in the presence of malicious adversaries.

Asymmetric (e.g. RSA) *(no key exchange/computationally costly)*

Security based on well-known hard mathematical problems (e.g. factorization).

Symmetric (e.g. AES) *(key exchange needed/efficient)*

Ideal security defined by generic attacks ($2^{|K|}$).

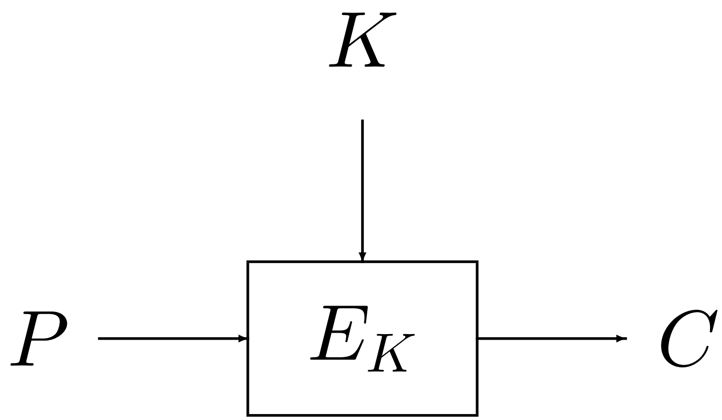
Need of continuous security evaluation (cryptanalysis).

$\Rightarrow$ Hybrid systems! (e.g. in SSH)

Symmetric primitives

- Block ciphers, (stream ciphers, hash functions..)

Message decomposed into blocks, each transformed by the same function E_K.



E_K is composed of a round transform repeated through several similar rounds.

Generic Attacks on Ciphers

- Security provided by an ideal block cipher defined by the best generic attack:
  exhaustive search for the key in $2^{|K|}$.

- Recovering the key from a secure cipher must be infeasible.

  $\Rightarrow$ typical key sizes $|K| = 128$ to 256 bits.

Cryptanalysis: Foundation of Confidence

> Any attack better than the generic one
> is considered a “break”.

- Proofs on symmetric primitives need to make unrealistic assumptions.
- We are often left with an empirical measure of the security: cryptanalysis.
- Security redefinition when a new generic attack is found (e.g. accelerated key search with bicliques [BKR 12])

Current scenario

- Competitions (AES, SHA-3, eSTREAM, CAESAR).
- New needs: lightweight, FHE-friendly, easy-masking.
  $\Rightarrow$ Many good proposals/candidates.
- How to choose?
- How to be ahead of possible weaknesses?
- How to keep on trusting the chosen ones?

Cryptanalysis: Foundation of Confidence

When can we consider a primitive as secure?

- A primitive is secure as far as no attack on it is known.
- The more we analyze a primitive without finding any weaknesses, the more reliable it is.

Design new attacks + improvement of existing ones:

- essential to keep on trusting the primitives,
- or to stop using the insecure ones!

On weakened versions

If no attack is found on a given cipher, what can we say about its robustness, security margin?

The security of a cipher is not a 1-bit information:

- Round-reduced attacks.
- Analysis of components.

$\Rightarrow$ determine and adapt the security margin.

On high complexities

When considering large keys, sometimes attacks breaking the ciphers might have a very high complexity far from practical *e.g.*. 2^{120} for a key of 128 bits.

Still dangerous because:

- Weak properties not expected by the designers.
- Experience shows us that attacks only get better.
- Other existing ciphers without the "ugly" properties.

- When determining the security margin: find the highest number of rounds reached.

Post-Quantum
Symmetric Cryptography

Post-Quantum Cryptography

Adversaries have access to **quantum computers.**

Asymmetric (e.g. RSA):

Shor's algorithm: Factorization in polynomial time

$\Rightarrow$ current systems not secure!

Solutions: lattice-based, code-based cryptography...

Symmetric (e.g. AES):

Grover's algorithm: Exhaustive search from $2^{|K|}$ to $2^{|K|/2}$.

Double the key length for equivalent ideal security.

We don't know much about cryptanalysis of current ciphers when having quantum computing available.

Post-Quantum Cryptography

Problem for present existing long-term secrets.
$\Rightarrow$ start using quantum-safe primitives NOW.

Important tasks:

- Conceive the cryptanalysis algorithms for evaluating the security of symmetric primitives in the P-Q world.

- Use them to evaluate and design symmetric primitives for the P-Q world.

Quantum Symmetric Cryptanalysis

Some recent results on Q-symmetric crytanalysis:

3-R Feistel [Kuwakado-Morii10], Even-Mansour [Kuwakado-Morii12], Mitm [Kaplan14], Related-Key [Roetteler-Steinwandt15], Diff-lin [Kaplan-Leurent-Leverrier-NP16], Simon's[Kaplan-Leurent-Leverrier-NP16], FX [Leander-May17], parallel multi-preim. [Banegas-Bernstein17], Multicollision [Hosoyamada-Sasaki-Xagawa17],
AEZ [Bonnetain17]...

Collision Search
w. A. Chailloux & A. Schrottenloher

Collision Search Problem

Given a random function $H : \{0,1\}^n \rightarrow \{0,1\}^n$, find $x, y \in \{0,1\}^n$ with $x \neq y$ such that $H(x) = H(y)$.

Many applications: *i.e.* generic attacks on hash functions.

(Multi-preimage search can be seen as a particular case).

Best known algorithms

	Time	Queries	Memory
Pollard's rho	$2^{n/2}$	$2^{n/2}$	$poly(n)$
Parallelization (2^s)	$2^{n/2-s}$	$2^{n/2}$	2^s

	Time	Queries	Qubits
Grover	$2^{n/2}$	$2^{n/2}$	$poly(n)$
BHT	$2^{2n/3}*$	$2^{n/3}$	$poly(n)*$
Ambainis	$2^{n/3}$	$2^{n/3}$	$2^{n/3}$

Open Questions

Challenge 1: Find an algorithm for collision and/or element distinctness which gives a searching speedup greater than merely a square-root factor over the number of available processing qubits[a]

[a] Grover and Rudolph, *How significant are the known collision and element distinctness quantum algorithms?* 2004.

Considered Model

- The same one as in all the previous quantum algorithms BUT we limit the amout of quantum memory available to a small amount $poly(n)$.

- Available small quantum computers seems like the most plausible scenario.

- We are interested in the theoretical algorithm and we did not take into account implementation aspects.

Starting Point: BHT Algorithm

- Optimal number of queries,
- $poly(n)$ qbits,
- But time?

BHT: Summarized procedure

- Build a list L of size $2^{n/3}$ elements (classic memory),

- Exhaustive search for finding one element that collides: With AA, the number of iterations is $(\frac{2^n}{2^{n/3}})^{1/2} = 2^{n/3}$.

  Testing the membership with L for the superposition of states costs $2^{n/3}$ with n qbits:

$$\text{Time: } 2^{n/3} + 2^{n/3}(1 + 2^{n/3}) \approx 2^{2n/3}$$

Can we improve this?

Lets build the list L with distinguished points
e.g. $H(x_i) = 0^u || z$, for $z \in \{0,1\}^{n-u}$.

The cost of building the list is bigger: $2^{n/3+u/2}$.
The setup of AA is bigger: $2^{u/2}$
The membership test stays the same: $|L| = 2^{n/3}$
BUT The number of iterations is smaller: $2^{n/3-u/2}$

Time: $2^{n/3+u/2} + 2^{n/3-u/2}(2^{u/2} + 2^{n/3}) \approx 2^{2n/3-u/2} + 2^{n/3+u/2}$

With optimal parameters

The cost will be optimized for a certain size of L: $2^v \neq 2^{n/3}$.

Time: $2^{v+u/2} + 2^{\frac{n-v-u}{2}}(2^{u/2} + 2^v)$

For $v = n/5$, $u = 2n/5$: Time: $\tilde{O}(2^{2n/5})$

For multiple preimage search, the algorithm is similar, but we only keep in L the distinguished points amongst the already given ones.

Comparison

	Time	Queries	Qubits	Classic Memory
Pollard	$2^{n/2}$	$2^{n/2}$	0	$poly(n)$
Grover	$2^{n/2}$	$2^{n/2}$	$poly(n)$	0
BHT	$2^{2n/3}$	$2^{n/3}$	$poly(n)$	$2^{n/3}$
Ambainis	$2^{n/3}$	$2^{n/3}$	$2^{n/3}$	0
New algorithm	$2^{2n/5}$	$2^{2n/5}$	$poly(n)$	$2^{n/5}$

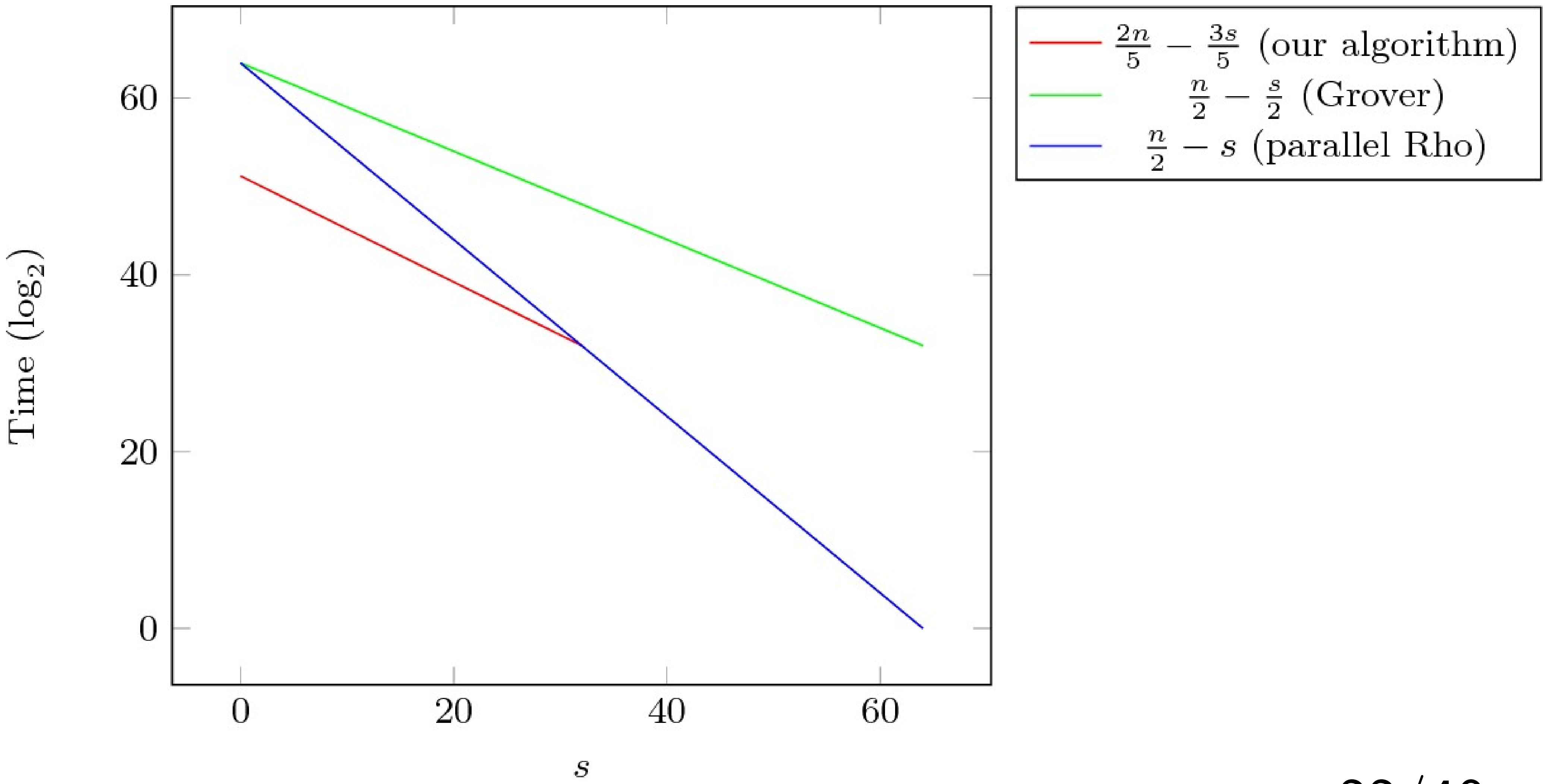
Parallelization

With 2^s n-qbit registers and "external" parallelization we can achieve:

Time: $2^{v+u/2-s} + 2^{\frac{n-v-u}{2}-s/2}(2^{u/2} + 2^v)$

Our theoretical algorithm seems more efficient than classical parallelization/Beal up to $s = n/4$

Comparison example: n=128



Example of Applications

- 1. Hash functions: Collision and Multi-preimages time from $2^{n/2}$ to $2^{2n/5}$ and $2^{3n/7}$.

  Ex.- time and queries for $n = 128$:

  rho= 2^{64}, ours= $2^{51.2}$ (with less than 1GB classical)

Conclusion 1

We solved challenge 1 for Grover and Rudolph 2004: new efficient collision search algorithm with small quantum memory.

Many applications in symmetric cryptograhy.

Open question: is it possible to meet the optimal $2^{n/3}$ in time with small quantum memory? (Quantum random walks, quantum learning graphs...?)

On Modular Additions
with X. Bonnetain

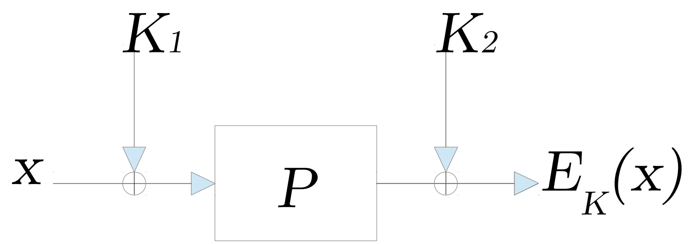
Quantum cryptanalysis: Simon's algorithm

Simon's problem: Given $f : \{0,1\}^n \to \{0,1\}^n$ such that $\exists s \mid f(x) = f(y) \iff [x = y$ or $x \oplus y = s]$, find s.

- Classical complexity: $\Omega(2^{n/2})$.
- Quantum complexity [Simon 94]: $O(n)$.

Simon's algorithm in Symmetric Cryptography

- Even-Mansour cipher [Even Mansour 97]: $DT > 2^n$



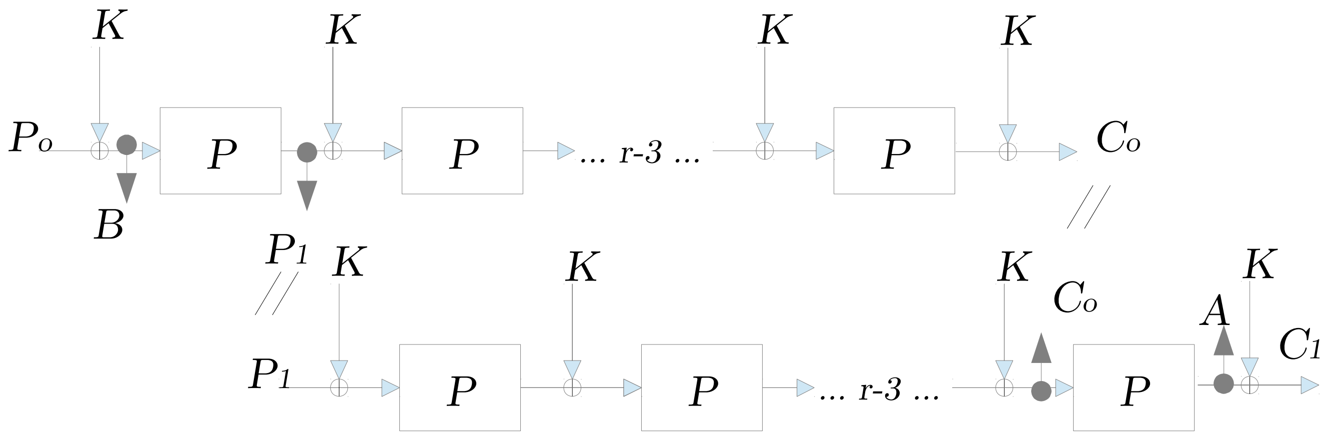
$$f(x) = E_K(x) \oplus P(x) \rightarrow f(x) = f(x \oplus K_1)$$

Simon's algo. on $f \Rightarrow K_1$ in $\mathcal{O}(n)$ [Kuwakado Morii 12] (Q2)

- Related-key attacks [Roetteler Steinwandt 15]
- 3-round Feistel [Kuwakado Morii 10]
- LWR, modes of operation for authentication (CBC-MAC, PMAC, OCB..), some CAESAR candidates [KLLN-P 16b]

Simon's algorithm and Slide attacks

- Classical: $\mathcal{O}(2^{n/2})$ [Biryukov Wagner 99]



- Quantum: Simon $\mathcal{O}(n)$ [KLLN-P 16b]

$$f : \{0,1\} \times \{0,1\}^n \to \{0,1\}^n$$

$$b, x \mapsto \begin{cases} P(E_K(x)) \oplus x & \text{if } b = 0, \\ E_K(P(x)) \oplus x & \text{if } b = 1. \end{cases}$$

$f(x) = f(x \oplus (1||K))$

Simon's algorithm in Symmetric Cryptography

Some (NOT ALL) primitives secure in the classical world become completely broken in the superposition model.

This does not seems a priori to imply that these primitives are unsafe in other settings.

Tweaking to resist Simon's algo. in Q2?

- In [Alagic Russell 17] several proposals. Most efficient: replace xor by modular additions.
- Hidden shift problem in $\mathbb{Z}/(N)$.
- No algorithm in polynomial time: Kuperberg in $2^{O(\sqrt{n})}$
- Up to what point do primitives resist?

Motivation and results

- 5. Dimensionate symmetric primitives
- 1. More precise evaluation of Kuperberg's algorithm complexity+improvement
- 2. Example of application with Poly1305
- 3. What about parallel modular additions?
- 4. New Quantum attacks (Feistel's slide, FX)

Improvement and Simulation

- Our improvement: all the bits with one iteration. $O(n^2 2^{\sqrt{2\log_2(3)n}}) \Rightarrow O(n 2^{\sqrt{2\log_2(3)n}})$

- Our simulations give: $0.7 \times 2^{1.8\sqrt{n}}$ for recovering full s. Code available: ask Xavier Bonnetain if interested. `xavier.bonnetain@inria.fr`

Application example with Poly1305

Poly1305 in the superposition model.

Two 128-bit keys (r, k), 128-bit nonce n, message m array of 128-bit blocks, output 128-bit tag.
$\text{Poly1305-AES}_{(r,k,n)}(m_1, \ldots, m_q) =$
$\left(\sum_{i=1}^{q}(m_{q-i+1} + 2^{128})r^i \mod (2^{130} - 5)\right) + \text{AES}_k(n)$

Access to:
$Poly_n^2 : |m_1\rangle\,|m_2\rangle\,|0\rangle \mapsto |m_1\rangle\,|m_2\rangle \left|\text{Poly1305-AES}_{(r,k,n)}(m_1, m_2)\right\rangle ,$

Superposition-Poly1305

We denote

$$F(x) = \text{Poly1305-AES}_{(r,k,n)}(1, x)$$
$$= \left(f(x) \mod (2^{130} - 5)\right) + \text{AES}_k(n) \text{ and}$$
$$G(x) = \text{Poly1305-AES}_{(r,k,n)}(0, x)$$
$$= \left(g(x) \mod (2^{130} - 5)\right) + \text{AES}_k(n),$$

which satisfy, for the same nonce, $F(x) = G(x + r)$.

As $f(x) = xr + r^2 + 2^{128}(r + r^2))$, $g(x) = xr + 2^{128}(r + r^2)$ and $f(x) = g(x + r)$.

Apply Kuperberg to find the hidden shift r.

Superposition-Poly1305

Two issues:

- One nonce, one query to both $F(x)$ and $G(x)$: we can compute $(1, x)$ and $(0, x)$ in superposition in one register and call the oracle $Poly_n^2$ on it.
- We cannot sample all group elements: consider 2^{18} possible intervals for r of size 2^{106}: $r \in [2^{106}c, 2^{106}(c+1))$ for $c \in [0, 2^{18})$ and the functions $f(x)$ and $g(x + 2^{106}c)$. Bad element with pb 2^{-21}. Apply Kuperberg to each interval: 2^{20}. Complexity: 2^{38} for r (thanks to our improvement!).

Algorithm for Parallel Modular Additions?

- HSP problem for groups product of cyclic groups
- Recurrent problem in symmetric cryptography
- Kuperberg not optimal

Simon meets Kuperberg

Algorithm for solving the case of p modular additions of words of w, matching Simon's $(w = 1)$ and Kuperberg's $(p = 1)$

- First Idea: Kuperberg's variant- better worst-case gain
- Second Idea: $p + 1$ equations always gain p zeros
- Combining both: best method depends on parameters and thresholds.

New Quantum Attacks

- Advanced slide attacks on Feistel ciphers
- Attacks on Feistel ciphers with non-invertible functions
- FX construction (quantum [Leander-May17]) with modular additions

Conclusion 2

- Improved Kuperberg's algorithm and new algorithm for parallel modular additions.
- State size needed for a 128-bit security.
  at least 5200 bits (but for FX) $\Rightarrow$ not very realistic.
- Might be better to just avoid vulnerable constructions, or try different patches (if we are concerned by superposition attacks).
- *Superposition*-Poly1305 broken implies that Poly1305 is not safe in the superposition model.

Final Conclusion

Open problems

- Optimal collision time $2^{n/3}$?
- $\alpha-$XOR problem.
- Algebraic attacks .
- Boomerang attacks.
- FSE Stevens: Quantum cryptanalysis of SHA-2?
- AES quantum evaluation- on going work.
- Generic key-length extensions?
- What about state size? ...

Symmetric Quantum Cryptanalysis[1]

Lots of things to do !

[1]Thanks to X. Bonnetain, A. Chailloux and A. Schrottenloher for their help with the slides