L'insoutenable légèreté du chiffrement

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(travaux communs avec C. Beierle, G. Leander et Y. Rotella)

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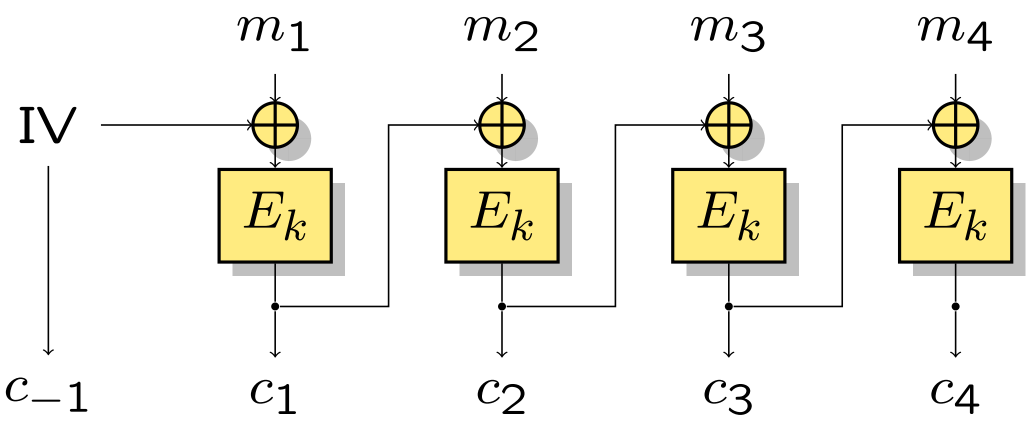
Symmetric Encryption Schemes

For encrypting messages of an arbitrary length:

- use a transformation operating on n-bit blocks (block cipher)
- chain the blocks with a mode of operation (CBC, CTR...)



Typical block size:

$$n \in \{128, 64\}$$

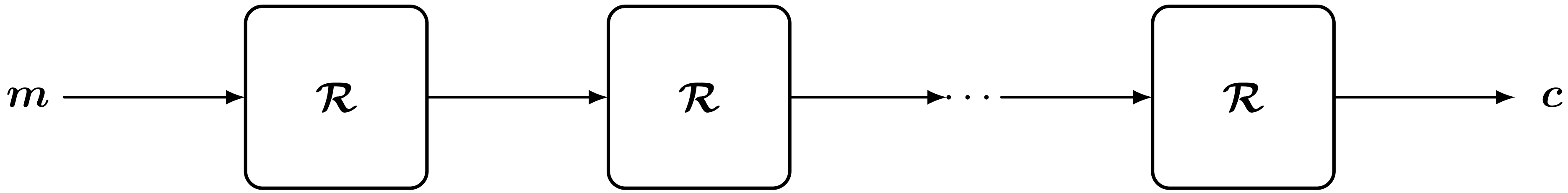
What is a block cipher?

$$E_k : \{0,1\}^n \longrightarrow \{0,1\}^n, \quad n \in \{64, 128\}$$

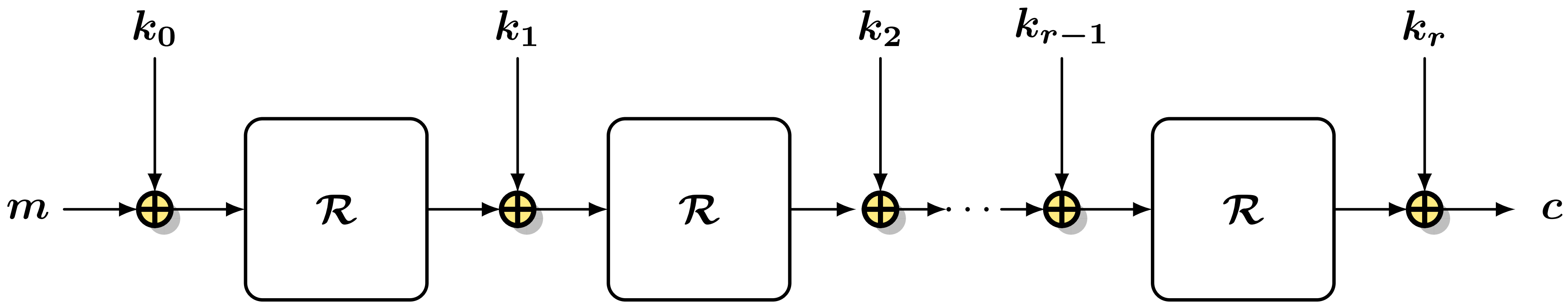
- indistinguishable from a set of randomly chosen permutations of $\{0,1\}^n$
- implementable

$\rightarrow$ Contradiction!

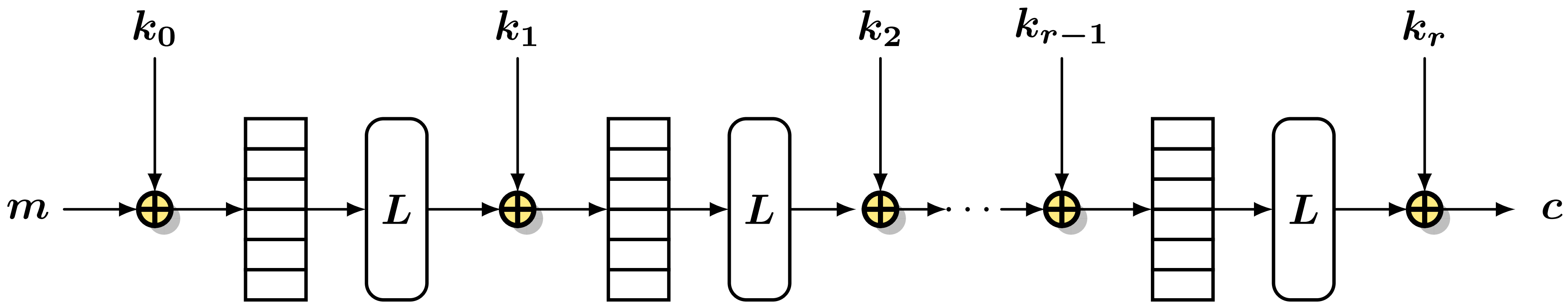
Iterated block ciphers



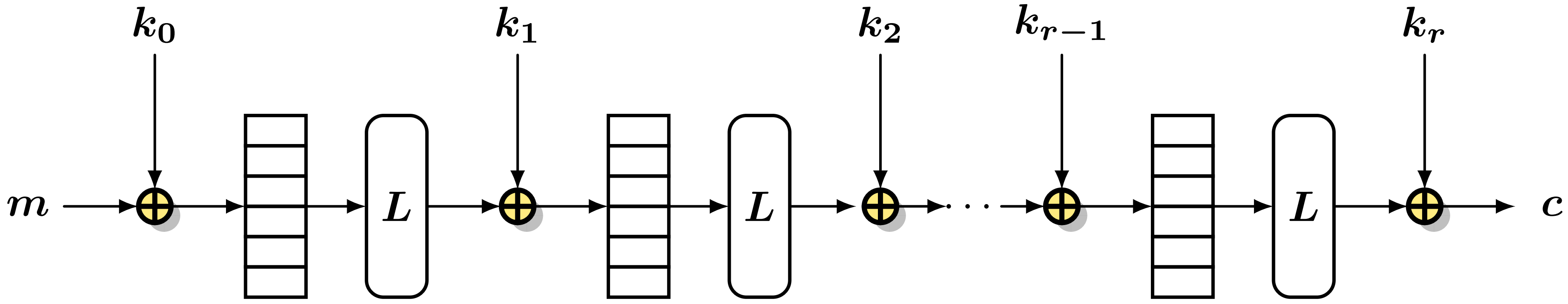
Iterated block ciphers



Iterated block ciphers

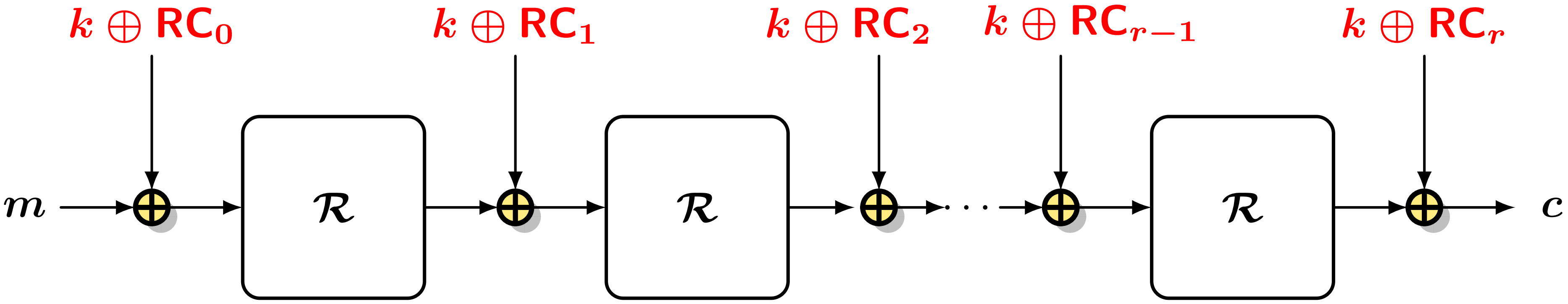


Lightweight block ciphers



- lighter nonlinear functions
- lighter diffusion layers
- simpler key schedules

Lightweight key schedules



where $\mathbf{RC}_0, \mathbf{RC}_1, \ldots, \mathbf{RC}_r$ are fixed round-constants.

Examples:

- PrintCipher [Knudsen et al. 10]
- LED [Guo et al. 11]
- Prince [Borghoff et al. 12]
- Scream and iScream [Grosso et al. 14]
- Midori [Banik et al. 15]
- Skinny and Mantis [Beierle et al. 16]...

Invariant attacks [Todo-Leander-Sasaki 16]

Principle:

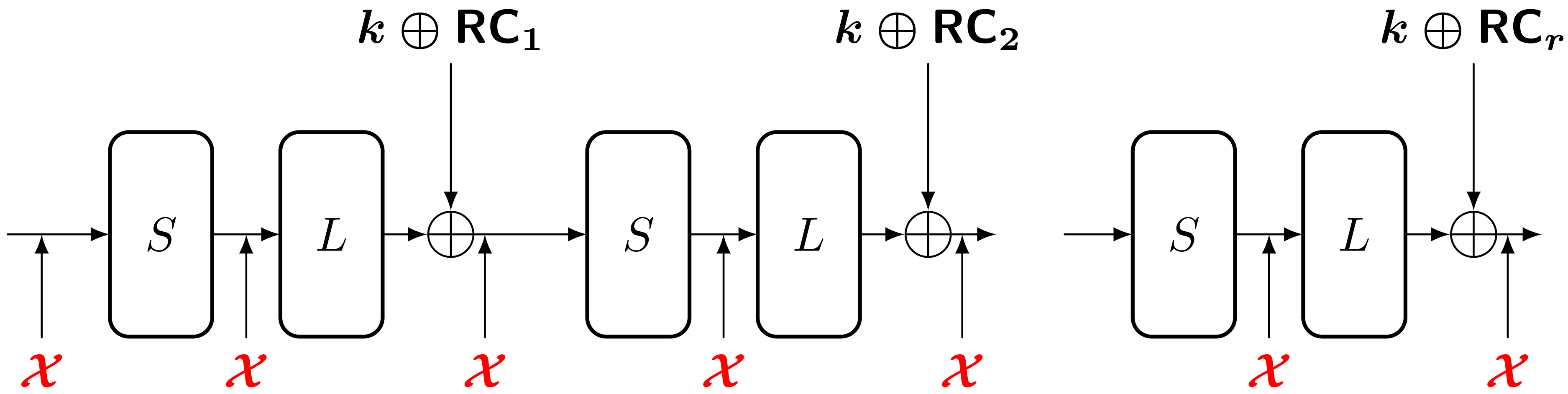
Exhibit a set $\boldsymbol{\mathcal{X}}$ of inputs invariant under $\boldsymbol{E_k}$ for many weak keys.

Ex: Invariant subspace for Midori-64 [Guo et al. 16]

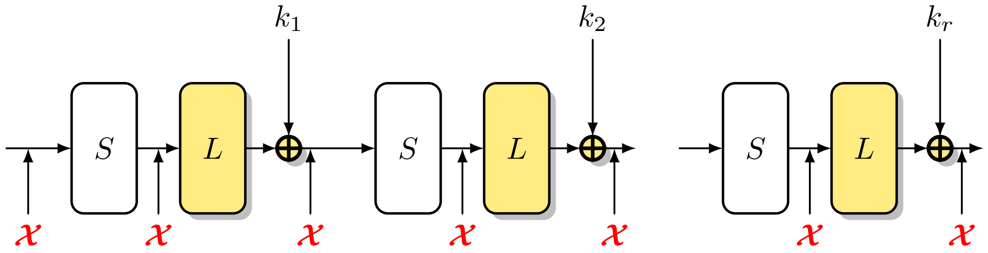
$\boldsymbol{\mathcal{X}} = \{\mathbf{8}, \mathbf{9}\}^{\mathbf{16}}$ is invariant under $\boldsymbol{E_k}$, for any 128-bit key $\boldsymbol{k} \in \{\mathbf{0}, \mathbf{1}\}^{\mathbf{32}}$.

For $\boldsymbol{k} = (\mathbf{1100110011001100}, \mathbf{0011001100110011})$,
$\boldsymbol{m} = \mathbf{9999999999999999}$ leads to the ciphertext $\boldsymbol{c} = \mathbf{8999999988988989}$

Using the same invariant for all layers in an iterated cipher



Finding an $\mathcal{X}$ invariant under all linear layers



$$L(\mathcal{X}) \oplus k_i = \mathcal{X} \text{ and } L(\mathcal{X}) \oplus k_j = \mathcal{X} \Rightarrow \mathcal{X} \oplus k_i = \mathcal{X} \oplus k_j$$

$\mathcal{X}$ is invariant under addition of any $(\mathbf{RC}_i \oplus \mathbf{RC}_j)$

Proposition.

$$\mathbf{LS}(\mathcal{X}) = \{a \in \{0,1\}^n \ : \ (a \oplus \mathcal{X}) = \mathcal{X}\}$$

- $\mathbf{LS}(\mathcal{X})$ is a linear space
- $\mathbf{LS}(\mathcal{X})$ is invariant under L
- $\mathbf{LS}(\mathcal{X})$ contains all $(\mathbf{RC}_i \oplus \mathbf{RC}_j)$

Condition on the existence of invariant sets

$$D := \{(\mathsf{RC}_i \oplus \mathsf{RC}_j),\ \ 0 \le i < j \le r\}$$

$W_L(D) :=$ smallest subspace invariant under L which contains D .

Problem.

Is there a set $\mathcal{X} \subset \{0,1\}^n$ such that $S(\mathcal{X}) = \mathcal{X}$ and
$\mathcal{X}$ is invariant under addition of any element in $W_L(D)$?

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No if $W_L(D) = \{0,1\}^n$

Some lightweight ciphers with $n = 64$

Skinny-64-64.

$$D = \{\mathsf{RC}_1 \oplus \mathsf{RC}_{17},\ \mathsf{RC}_2 \oplus \mathsf{RC}_{18},\ \mathsf{RC}_3 \oplus \mathsf{RC}_{19},\ \mathsf{RC}_4 \oplus \mathsf{RC}_{20},\ \mathsf{RC}_5 \oplus \mathsf{RC}_{21}\}$$

$$\dim W_L(D) = 64$$

The round-constants and L guarantee that the attack does not apply.

Prince.

$$D = \{\mathsf{RC}_1 \oplus \mathsf{RC}_2,\ \mathsf{RC}_1 \oplus \mathsf{RC}_3,\ \mathsf{RC}_1 \oplus \mathsf{RC}_4,\ \mathsf{RC}_1 \oplus \mathsf{RC}_5,\ \alpha\}.$$

$$\dim W_L(D) = 56$$

Midori-64.

$$\dim W_L(D) = 16$$

Maximizing the dimension of $W_L(d)$

$$W_L(d) = \langle L^t(d), t \in \mathbb{N} \rangle \ .$$

Theorem. There exists d such that $\dim W_L(d) = k$ if and only if k is the degree of a divisor of the minimal polynomial of L.

$$\Rightarrow \max_{d \in \mathbb{F}_2^n} \dim W_L(d) = \deg \mathsf{Min}_L$$

For some lightweight ciphers

LED.

$$\text{Min}_L(X) = (X^8 + X^7 + X^5 + X^3 + 1)^4 (X^8 + X^7 + X^6 + X^5 + X^2 + X + 1)^4$$

There exist some d such that $\dim W_L(d) = 64$

Prince.

$$\text{Min}_L(X) = X^{20} + X^{18} + X^{16} + X^{14} + X^{12} + X^8 + X^6 + X^4 + X^2 + 1$$

$$\max_d \dim W_L(d) = 20$$

Midori.

$$\text{Min}_L(X) = (X+1)^6 \Rightarrow \max_d \dim W_L(d) = 6$$

Some conclusions on lightweight cryptography

- standardization process launched by NIST in December
- risky
- clarifies the design criteria.