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Asymmetric Mean Reversion of Bitcoin Price Returns

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Abstract

Non-linearity is characterised by an asymmetric mean-reverting property, which has been found to be inherent in the short-term return dynamics of stocks. In this paper, we explore as to whether cryptocurrency returns, as represented by Bitcoin, exhibit similar asymmetric reverting patterns for minutely, hourly, daily and weekly returns between June 2010 and February 2018. We identify several differences in the behaviour of Bitcoin price returns in the pre- and post-$1,000 sub-periods and evidence of asymmetric reverting patterns in the Bitcoin price returns under all the ANAR models employed, regardless of the data frequency considered. We also present evidence indicating stronger reverting behaviour of negative price returns in terms of both reverting speed and magnitude compared to positive returns and evidence of positive serial correlation with prior positive price returns. Finally, we also investigated asymmetries in Bitcoin price return series' persistence by employing higher order ANAR models, finding evidence of a higher persistence of positive returns than negative returns, a result that further supports the existence of asymmetric reverting behaviour in the Bitcoin price returns.

Keywords: Digital Currencies; Cryptocurrency; Bitcoin; Short-horizon stock returns; Asymmetric reverting patterns.

1. Introduction

The rapid growth of cryptocurrencies has been a point of concern for regulatory authorities and policy-makers alike. The sharp appreciation in the price of some cryptocurrencies has been associated with numerous questions and discussions surrounding the fundamental propellant, generating accusations based on the presence of a substantial pricing bubble within a number of individual cryptocurrency markets (Corbet et al. [2017]). The price appreciation of a number of cryptocurrencies, with particular emphasis on Bitcoin, reached unprecedented levels in late 2017 and early 2018, as investors continued to purchase significant volumes. Evidence of stock market overreaction has been associated for a significant period of time with stock market price mean reversion (Nam et al.

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[2002]). Such overreaction is derived from the overreaction hypothesis, which is a theory developing on crowd reaction and indeed overreaction to good and bad news, and as to whether short-term profits can be generated from investor panic based on overreaction to news events. Further, such investor overreaction can be determined from the assumption that a stock's price will tend to move to the average price over time, also defined as mean reversion. Traders often find significant use in differing moving averages when making predictions as to whether an asset is either under-valued or over-valued, and such perceptions can be influenced at the differing frequencies at which investor's are investigating asset prices, whether it be at very high-frequency as measured in second, minutes and hours, or indeed at a relatively higher frequency when measured in days, weeks and months. In this context, it was identified that uneven reverting patterns in stock markets is not justified under the time-varying rational expectation hypothesis which had been tested by Engle et al. [1987] where the expectation of the excess holding yield on a long bond is postulated to depend on it conditional variance. Nam et al. [2002] could not then reject the validity of stock market overreaction as a relevant explanation of contrarian profitability within stock market porfolios. Therefore, it is important to analyse whether similar effects are found to be present in cryptocurrency markets. This is particularly important as recent research surrounding cryptocurrencies continues to focus on potential irregularities (Gandal et al. [2018]; Griffin and Shams [2018]). Nam et al. [2006] investigated nonlinearity in short-term stock market price dynamics between 1962 and 2003 at a daily and weekly frequency to find evidence indicating strong reverting patterns where negative returns revert more frequently and sizeably than positive returns. This indicates that investors exhibit asymmetric reactions to both good and bad news, therefore opposing the time-varying rational expectation hypothesis' assumption of a positive relationship between future volatility and risk premium, as portfolios of stocks that fall in price are theoretically most likely to outperform portfolios of stocks that increase in price. This is an important behavioural feature to investigate in cryptocurrency markets, a market that has obtained thorough scrutiny during its short life span.

Examining mean reversion behaviour in cryptocurrencies can shed further light on the authenticity of the cryptocurrency pricing dynamics which we have witnessed, with emphasis on the sharp increase in the price of Bitcoin to $20,000 in 2017. Since early 2018, academics and regulators alike have begun to investigate as to whether there is evidence of pricing irregularities in cryptocurrencies, or more disturbingly, market manipulation. Evidence of mean reverting behaviour could potentially indicate that losing cryptocurrency portfolios could theoretically outperform winning cryptocurrency portfolios in the same manner as the three indices and thirty Dow Jones stocks investigated by Nam et al. [2006]. However, the transfer of such a theoretical assumption to cryptocurrency markets could be muddied somewhat not only by the presence of failing ICOs or indeed ICOs designed to defraud investors (a prospective member of the losing portfolio), but also through price manipulation that has been accused during the sharp increases in cryptocurrency prices (which would constitute that of a winning portfolio, albeit fraudulent).

Consequently, in this paper, we utilise the multiple frequencies of price return data to analyse and investigate as to whether mean reversion is present in cryptocurrency markets in the same manner as stock markets, not only to further develop knowledge about the maturing financial product, but to identify as to whether there exist any abnormalities that may further support such accusations of pricing abnormalities or indeed market manipulation.

Overall, we identify several differences in the behaviour of Bitcoin price returns in the pre- and post-$1,000 sub-periods. More specifically, it is shown that while over the pre-$1,000 sub-period the average price returns are positive, over the post-$1,000 sub-period the average price returns equal zero, and the price returns of the pre-$1,000 sub-period have a higher standard deviation and display higher excess kurtosis than the returns in the post-$1,000 sub-period, irrespective of the data frequency considered. Moreover, when considering the pre-$1,000 sub-period, the price returns of Bitcoin are positively skewed, while the price returns of the post-$1,000 sub-period are mostly negatively skewed. We also find evidence of asymmetric reverting patterns in the Bitcoin price returns under all the ANAR models employed, regardless of the data frequency considered. More specifically, we identify stronger reverting behaviour of negative price returns in terms of both reverting speed and magnitude compared to positive returns, irrespective of the period considered, although the reverting pattern tends to become more symmetrical as we consider lower data frequencies. In addition, there is evidence of positive serial correlation with prior positive price returns over the entire sample period as well as over the pre- and post-$1,000 sub-periods. However, serial correlation decreases with prior negative price returns and can be either positive, negative or zero, a fact that further highlights asymmetric behaviour between positive and negative returns. Through the use of an EGARCH model for the conditional volatility in order to capture leverage effects, we find that the asymmetric reverting pattern is still observed, even when allowing for time-varying conditional heteroskedasticity in return dynamics, with stronger overall asymmetric behaviour for the post-$1,000 sub-period as compared to the pre-$1,000 sub-period, though, especially for minutely data. Finally, when analysing asymmetries in Bitcoin price return series' persistence by employing higher order ANAR models, we find evidence of higher persistence of positive returns than negative returns over both the entire period and the two sub-periods under examination, a result that further supports the existence of asymmetric reverting behaviour in the Bitcoin price returns.

The remainder of this paper is as follows. Section 2 presents an overview of the main research to date associated with market dynamics in cryptocurrency markets and their efficiency along with key research based on mean reversion. Section 3 describes the data while Section 4 presents the methodology employed. The empirical findings are discussed in section 5. Finally, some concluding remarks are given in section 6.

2. Previous Literature

In a thorough systematic analysis of the main literature based on the major topics that have been studied with regards to cryptocurrencies, Corbet et al. [2018] found that, as of early 2018, the main areas that have been researched with regards to cryptocurrencies include cybercriminality, diversification, and market efficiency. Cryptocurrencies' market efficiency can be measured through a host of progressive factors including the existence of a new futures exchange, liquid cross-currency indices and the relative reduction of intra-day volatility although daily volatility remains high. Studies of the market efficiency of cryptocurrencies include those of Urquhart [2016], Bariviera et al. [2017], Nadarajah and Chu [2017], Brauneis and Mestel [2018], Cheah et al. [2018], Khuntia and Pattanayak [2018], Sensoy [2018], Tiwari et al. [2018], and Vidal-Tomás and Ibañez [2018], among others. More specifically, in an early study of the efficiency of Bitcoin, Urquhart [2016] used a battery of robust tests to find that Bitcoin returns are significantly inefficient over their selected full sample, but when dividing the same sample, Bitcoin presented evidence of becoming more efficient. Later, Khuntia and Pattanayak [2018] examined the evolving return predictability in Bitcoin and showed that market efficiency evolves with time in a manner consistent with the Adaptive Market Hypothesis. While examining the time-varying efficiency of Bitcoin in terms of US dollars and Euros, Sensoy [2018] also showed that both markets have become more informationally efficient over time. This finding is further echoed by the work of Tiwari et al. [2018] who investigated the informational efficiency of Bitcoin using a battery of computationally efficient long-range dependence estimators and also found that the market is informational efficient. Yet, Vidal-Tomás and Ibañez [2018] examined the semi-strong efficiency of Bitcoin in the Bitstamp and Mt.Gox and found Bitcoin to be unaffected by monetary policy news, highlighting the absence of any kind of control on Bitcoin. More recently, Brauneis and Mestel [2018] extended the existing literature on the efficiency of cryptocurrency markets by performing various tests on efficiency of several cryptocurrencies, and additionally linked efficiency to measures of liquidity when the authors found that cryptocurrencies become more efficient as liquidity increases. On the other hand, the studies of Jiang et al. [2017] and Cheah et al. [2018] found contradictory results to those listed above. More specifically, in an attempt to examine the time-varying long-term memory in the Bitcoin market through a rolling window approach and by employing the efficiency index of Sensoy and Hacihasanoglu [2014], Jiang et al. [2017] found a high degree of inefficiency ratio and that the Bitcoin market does not become more efficient over time. Similarly, Cheah et al. [2018] modelled cross market Bitcoin prices as long-memory processes in order to study dynamic interdependence in a fractionally cointegrated VAR framework and found long memory in both the individual markets and that the system of markets depicting non-homogeneous informational inefficiency. Moreover, Nadarajah and Chu [2017] investigated the market price efficiency of cryptocurrencies by means of five different tests on Bitcoin returns and concluded that the returns do not satisfy the efficient market hypothesis. However, the authors showed that a simple power transformation of the Bitcoin returns do satisfy the efficient market hypothesis through the use of eight different tests.

Extensive research has also been conducted on the price dynamics and volatility of cryptocurrencies. For instance, Urquhart [2017] found evidence of significant price clustering in the market for Bitcoin. Moreover, Katsiampa [2017] compared several GARCH-type models and found that the Component GARCH model fits Bitcoin price returns better than its counterparts, while Phillip et al. [2018] employed the stochastic volatility model to examine the price volatility of several cryptocurrencies. In addition, while studying the general behavioural aspects of cryptocurrencies, Corbet et al. [2018] examined the reaction of a broad set of digital assets to US Federal Fund interest rates and quantitative easing announcements to find a broad range of differing volatility responses and feedback dependent on the type of crytocurrency investigated and as to whether the cryptocurrency was mineable or not. With regards to the statistical properties of the Bitcoin market, Bariviera et al. [2017] found that Hurst exponents changed significantly during the first years of existence of Bitcoin, tending to stabilize in recent times, while Alvarez-Ramirez et al. [2018] found that the market of Bitcoin presents asymmetric correlations with respect to increasing and decreasing price trending, with the former trend linked to anti-persistence of returns dynamics.

Several studies have also examined the existence of bubbles in cryptocurrency markets. For instance, Cheah and Fry [2015] showed that Bitcoin prices are prone to speculative bubbles, while Cheung et al. [2015] identified several short-lived bubbles as well as three large bubbles during the period 2011 through 2013 lasting between 66 and 106 days. While utilising the bubble identification methodology of Phillips et al. [2011], Corbet et al. [2017] also found clear evidence of periods in which Bitcoin and Ethereum were experiencing bubble phases.

With regards to product diversification, Dyhrberg [2016] showed that Bitcoin can be used as a hedge against stocks in the Financial Times Stock Exchange Index and against the US dollar in the short-term, therefore, Bitcoin was found to possess some of the same hedging abilities as gold and can be included in the variety of tools available to market analysts to hedge market-specific risk. More recently, Urquhart and Zhang [2018] assessed the relationship between Bitcoin and currencies at the hourly frequency and found that Bitcoin can be an intraday hedge for the CHF, EUR and GBP, but acts as a diversifier for the AUD, CAD and JPY. The authors also found that Bitcoin is a safe haven during periods of extreme market turmoil for the CAD, CHF and GBP, supporting the results of Corbet et al. [2018], who found evidence of the relative isolation of Bitcoin, Ripple and Litecoin from a broad variety of other financial assets, and of Baur et al. [2017], who found that Bitcoin is uncorrelated with traditional asset classes in periods of financial turmoil. On the other hand, Bouri et al. [2017], using a dynamic conditional correlation model, examined as to whether Bitcoin could act as a hedge and safe have for four major world stock indices, bond, oil, gold, the general commodity index and the US dollar index and found that it is a poor hedge and is suitable for diversification purposes only. Moreover, Corbet et al. [2018] found that the introduction of Bitcoin futures actually destabilised the popular cryptocurrency market and that such futures were not an effective hedging mechanism. The authors also found that price discovery was driven by uninformed investors in the spot market and not that of the futures market, adding further evidence that Bitcoin is a speculative asset rather than a currency, and that the introduction of Bitcoin futures still rendered the cryptocurrency as a highly speculative asset than a currency. Nevertheless, Bitcoin has been found to be the least risky cryptocurrency along with Litecoin when compared to their counterparts (Gkillas and Katsiampa [2018]).

Another issue related to cryptocurrencies that has received a lot of attention by academics and the media alike is that of fraud and market manipulation. Although many regulatory authorities such as the International Monetary Fund (IMF) have expressed their satisfaction with the product's development and the benefits that are contained within its continued growth (e.g., in April 2018, Christine Lagarde, Head of the International Monetary Fund stated that 'policymakers should keep an open mind work toward an even-handed regulatory framework that minimises risks which allowing the creative process to bear fruit[1]'), the Securities and Exchange Commission (SEC) in 2018 have backtracked on earlier positivity to warn of the inherent potential for spoofing and other market manipulation techniques[2]. Fraud in cryptocurrency markets has to date taken multiple forms. Such regulatory bodies have continued to develop on these broad accusations, focusing specifically on ICOs that have been designed to defraud investors where organisers have little or no intention of developing a financial product that will perform to the standard listed in provided white papers and advertisement. Another form of fraud that has been linked to the growth in cryptocurrencies has been fraudulent cross-border transactions and tax evasion (Slattery [2014]; Levin et al. [2015]). Fraud has also been experienced at the exchange level. The largest examples included Mt. Gox[3] in 2014, Bitfinex[4] in 2016 and Coincheck [5] in 2018. Gandal et al. [2018] investigated the relationship between observed 'suspicious' trading activity on the Mt. Gox exchange theft of approximately 600,000 Bitcoins, demonstrating that this activity was most likely a significant contributory factor during the sharp increase in the price of Bitcoin from $150 to $1,000 in late 2013 as presented in Figure 1. The authors found that trading volumes on all Bitcoin exchanges increased substantially

[1] An Even-handed Approach to Cryptocurrencies, IMF blogpost written by Christine Lagarde, Head of the International Monetary Fund available at: https://blogs.imf.org/2018/04/16/an-even-handedapproach-to-crypto-assets/

[2] US Securities and Exchange Commission, Public Statement, Statement on Potentially Unlawful Online Platforms for Trading Digital Assets, Available at: https://www.sec.gov/news/publicstatement/enforcement-tm-statement-potentially-unlawful-online-platforms-trading

[3] Mt. Gox was a bitcoin exchange based in Shibuya, Tokyo, Japan. Launched in July 2010, it quickly became the largest bitcoin intermediary and the world's leading bitcoin exchange. In February 2014, Mt. Gox suspended trading, closed its website and exchange service, and filed for bankruptcy protection from creditors. In April 2014, the company began liquidation proceedings. Mt. Gox announced that approximately 850,000 bitcoins belonging to customers and the company were missing and likely stolen, an amount valued at more than $450 million at the time.

[4] 120,000 Bitcoin were stolen from the exchange's multi-signature wallets, in the amount equivalent to approximately $78 million then and $840 million now. The exchange did not pay out any compensations, but issued its own debt token, BFX, which was fully exchanged for dollars at a one-to-one rate in early April 2017.

[5] 523 million NEM were stolen from one of the exchcange's last 'hot' wallets, which was equivalent to $500 million then, or $95 million now. It is the largest 2018 hack to date, and the second or third in the history of cryptocurrency.

on days denoted to contain such suspicious activity, which leads the authors to demonstrate that this same price increase was generated by one single actor or agent. Such research have sharply focused the attention of regulators across multiple jurisdictions. On the other hand, Griffin and Shams [2018] investigated whether Tether, a cryptocurrency that is pegged to the US dollar, influenced other cryptocurrencies during the price appreciations of 2017 and 2018 using algorithms to analyse blockchain data. The authors found significant evidence indicating that purchases with Tether are timed during market downturns and were found to be associated with sharp appreciations of the price of Bitcoin. Specifically, less than 1% of the hours denoted as contained substantial Tether transactions were found to be directly associated with 50% of the increase of Bitcoin and 64% of the increase in value of other top cryptocurrencies, which cannot be explained by investor demand proxies. But dealing with such issues could be quite problematic. Hendrickson and Luther [2017] found that although some countries have considered the outright ban of Bitcoin, the success of such a ban is reliant on the severity of the punishments that are associated with such misuse. It would appear as though in some jurisdictions, the solution to any detrimental behaviour in cryptocurrency markets and associated solutions would be very much reliant on trial and error. This pricing behaviour must be considered when analysing the mean reversion of cryptocurrency prices. Spoofing and market manipulation has broadly associated with price increases for prolonged periods of time (Gandal et al. [2018]; Griffin and Shams [2018]). The identification of such issues of market manipulation continue to damage the repuation and integrity of cryptocurrency markets and exchanges, generating potential price declines (Dechow et al. [1996]). The creation of new exchanges, including derivatives exchanges has also been found to generate an increasing number of opportunities for price manipulation (Jarrow [1994]).

Further pricing abnormalities and potential disruption has been identified through some emerging issues relating to cryptocurrencies. On the 9th of January 2018, the camera manufacturer Kodak announced that it was entering the crytocurrency market through the creation of KODAKOne, described as a revolutionary new image rights management and protection platform secured in the blockchain. Kodak announced that its development seamlessly registers, manages and monetizes creative assets for the photographic community (Corbet et al. [2018]). Shares increased from over $3 per share to over $12 in less than one week. Such announcements have also attracted the attention of regulators. Jay Clayton, the chairman of the Securities and Exchange Commission (SEC), said that the agency was 'looking closely at the disclosures of public companies that shift their business models to capitalize on the perceived promise of distributed ledger technology.' On the 17th of July, Kodak proceeded to distance itself from the company behind a 'Kashminer' mining scheme where investors could rent mining hardware which could be used to mine future Kodak cryptocurrency. To date, KodakCoin has yet to be established. Even without the creation of an ICO, the announcement of a cryptocurrency related plan has potentially incorporated cryptocurrency speculation into the share price of a publicly traded company. This is a point of concern for regulators and policymakers alike.

Each of the above listed issues have developed and become substantial within then ten years that cryptocurrencies have existed. Nevertheless, despite the extensive research conducted on cryptocurrencies, no previous study has examined the mean reversion property in cryptocurrency markets. The mean reversion property is frequently observed in stock markets. In fact, stock market overreaction has been associated with mean reversion (Nam et al. [2002]) and, since cryptocurrencies behave more like assets rather than currencies, it is therefore important to analyse whether similar effects are found to be present in cryptocurrency markets. Mean reversion has been identified across a number of different regions and financial products, with differing evidence provided for such causes and effects. For instance, Poterba and Summers [1988], while investigating transitory components in stock prices across eighteen countries, found evidence of positive autocorrelation in returns over short horizons and negative autocorrelations over longer time horizons, although at the time, the authors could not eliminate the possibility that disparities between prices and fundamental values could also explain the results. More recently, Mukherji [2011], using a non-parametric bootstrap method, showed that large and small company stocks experience significant mean reversion in returns for differing periods of time between one and five years in duration between the years 1926 and 1966. However, between 1966 and 2007, large companies experienced significant mean reversion at the five-year level and a shorter time horizon for smaller companies, therefore indicating that mean reversion persists, and particularly so for smaller companies. Investigating as to whether mean reversion in stock behaves differently in bull and bear markets, Cuñado et al. [2010] found significant differences with mean reversion more prevalent in bull market periods, while Serban [2010] confirmed that mean reversion was not just applicable to stock markets and could actually generate a more profitable strategy when investing in mean reversion and momentum phenomenon in foreign exchange markets. Mean reversion has been found to hold across some markets for significant time periods using a range of investigative procedures, such as that of South-east Asian stock markets (Malliaropulos and Priestley [1999]; Wang et al. [2015]). Emerging market stock indices have also been found to incorporate evidence of mean reversion which has been found to be capable of producing contrarian profits under certain trading strategies (Akarim and Sevim [2013]).

To the best of the authors' knowledge, this is the first study on mean reversion in cryptocurrency markets. While evidence of asymmetric mean reversion will not isolate specific evidence of market manipulation, it can help to further develop our understanding of this product and provide beneficial information for investors, policy-makers and regulators alike as we seek to further develop our knowledge of this product.

3. Data

In our analysis we have used data spanning a number of differing frequencies to produce a thorough analysis of time-varying mean aversion. Data is used between midnight on 20 July 2010 and 22 February 2018. Although Bitcoin is broadly described as being created in 2009, complete and thorough data at a minutely basis is best available in the period after 20 July 2010. The weekly return series is constructed by computing the geometric average of seven consecutive daily returns as the market for cryptocurrencies is open throughout the entire week. All the return series are computed as percentage returns. Table 2 outlines the key summary statistics of each dataset used, with 3,994,142 minutely observations used at the highest frequency analysis.

Bitcoin has existed for less than a decade, with it's trading initiation largely associated with the middle of the subprime market collapse in the United States and the beginning of the European sovereign crisis. Corbet et al. [2017] found that the pricing behaviour of Bitcoin appeared to have entered a consistent bubble-phase in the period after the price passed $1,000, which occurred on 1 January 2017. While considering that there have been a number of significant periods of continuous price appreciation in the period denoted as being in a bubble, it is important to investigate as to whether there are differences in mean reversion behaviour between both of the stated periods. Our analysis further investigates as to whether such mean reverting behaviour differs due to the frequency of data being investigated, whether it be minutely, hourly, daily or weekly. The symmetry of any identified reverting patterns provides important information about the functionality and efficiency of such a relatively youthful financial product. We must note that negative return shocks in equity markets have been associated with increased risk premiums, which in turn reduce the concurrent price, therefore generating another negative return (Nam et al. [2006]). Further, using data from eighteen countries, Poterba and Summers [1988] found that their estimates implied the existence of positive autocorrelation in returns over short horizons and negative autocorrelation over longer horizons. Fama and French [1988] had found that a slowly mean-reverting component of stock prices tends to induce negative autocorrelation in these returns. It is important to understand as to whether similar effects occur within cryptocurrency markets.

4. Methodology

We follow the approach used by Nam et al. [2006] who investigate asymmetric mean reversion of short-term stock returns, that are found to evolve through the non-linear autoregressive process described as:

$$r_t = \mu + \phi^- r_{t-1} + \epsilon_t, r_{t-1} < 0, \tag{1}$$

$$r_t = \mu + \phi^+ r_{t-1} + \epsilon_t, r_{t-1} \geq 0, \tag{2}$$

where $|\phi^-|< 1$ and $|\phi^+|< 1$ holds for the stationarity condition of r_t. Return serial correlation is measured by ϕ^- when $r_t < 0$, while it is measured by ϕ^+ when $r_t > 0$. Serial correlation under a prior negative return is less than serial correlation under a prior positive return, that is $\phi^+ > \phi^-$. The implications of this condition include: 1) both ϕ^+ and ϕ^- measure the reverting speed of r_t under a prior positive and negative return, where $\phi^+ > \phi^-$ implies that a negative return reverts on average more quickly than does the same magnitude of a positive return; and 2) $\phi^+ > \phi^-$ measures the relative reverting magnitude of a positive and negative return [6]. To capture the asymmetric reverting behaviour of Bitcoin returns, similar to the work of Nam et al. [2006], we utilise a univariate first-order asymmetric nonlinear autoregressive model for the return series r_t to investigate asymmetric reverting properties at the minutely, hourly, daily and weekly frequencies respectively. The first model we use is specified as:

$$r_t = \mu + [\phi_1 + \rho_1 D_1 (r_{t-1} < 0)] + \varepsilon_t \tag{3}$$

where D_1 is an indicator function specified for a dummy variable that takes a value of one if $r_{t-1} < 0$, or zero otherwise. $\phi_1 + \rho_1 D_1$ represents serial correlation with the above model allowing for the autocorrelation coefficient of Bitcoin returns[7] to vary along with sign of r_{t-1}. We confirm asymmetric reverting patterns to incorporate two, three, four and even five consecutive price decreases through the analysis of the following specifications:

$$r_t = \mu + [\phi_1 + \rho_1 D_2 (r_{t-1} < 0, r_{t-2} < 0)] + \varepsilon_t \tag{4}$$

$$r_t = \mu + [\phi_1 + \rho_1 D_3 (r_{t-1} < 0, r_{t-2} < 0, r_{t-3} < 0)] + \varepsilon_t \tag{5}$$

$$r_t = \mu + [\phi_1 + \rho_1 D_4 (r_{t-1} < 0, r_{t-2} < 0, r_{t-3} < 0, r_{t-4} < 0)] + \varepsilon_t \tag{6}$$

$$r_t = \mu + [\phi_1 + \rho_1 D_5 (r_{t-1} < 0, r_{t-2} < 0, r_{t-3} < 0, r_{t-4} < 0, r_{t-5} < 0)] + \varepsilon_t \tag{7}$$

where D_2, D_3, D_4 and D_5 are dummy variables. D_2 takes a value of one if r_{t-1} and r_{t-2} are both negative or zero otherwise. D_3, D_4 and D_5 take a value of one if all three, four and five prior returns are negative respectively. As stated in the baseline model, $\rho_1 < 0$ confirms that a negative return exhibits a relatively stronger asymmetry in reverting patterns. The estimation results of the models

[6] Both of these implications are proven in Nam et al. [2006] when considering that loser stocks outperform winning stocks in contrarian literature and short-term contrarian profits can be a consequence of a trading strategy exploiting the asymmetric property which was found to be intrinsic in the dynamic process of short-horizon stock returns.

[7] Serial correlation is measured by $\phi_1 + \rho_1$ if $r_{t-1} < 0$ or ϕ_1 if $r_{t-1} \geq 0$

focusing on two, three, four and five consecutive negative returns provide important information. With regards to stocks, time-varying rational expectation hypothesis assumes a positive relationship between the future volatility and risk premium sourced in the intertemporal relationship between future volatility and expected returns in the asymmetric mean-reverting behaviour of stock returns (Nam et al. [2006]). This is relevant when considering the stock-market overreaction hypothesis, where assumptions of a negative relationship between current asset price and risk premiums, negative returns should be followed by another negative return, which further implies that empirical results should not estimate that $\phi_1 < 0$.

To capture heteroskedasticity of return dynamics, we utilise the exponential GARCH (EGARCH) model to capture leverage effects which is specified for modelling conditional variance h_t:

$$log(h_t) = \alpha_0 + \frac{\Phi(L)}{\Psi(L)} g(v_t) \tag{8}$$

$$g(v_t) = \theta v_t + \gamma \left[|v_t| - E\, |v_t| \right] \tag{9}$$

where $\Phi(L)$ and $\Psi(L)$ are the finite-order polynomials of order q and p, and the standardised residual v_t is defined as $v_t = \frac{\varepsilon_t}{\sqrt{h_t}}$ for the estimated residual ε_t. Since the EGARCH model specifies the log of h_t in the variance equation, it does not require any positivity restrictions on parameters to ensure nonnegativity of h_t. The value of $g(v_t)$ is a function of both the magnitude and sign of v_t. The term $\gamma \left[|v_t| - E\, |v_t| \right]$ represents the magnitude effects and θv_t, the sign effect of the standardized residual on the conditional variance. The magnitude effect can be thought of as capturing the volatility clustering, and the sign effect as capturing the asymmetric effect of return shocks on volatility.

If cryptocurrency return dynamics present evidence of strong asymmetric reverting patterns, there should also exist an asymmetry in return persistence which provides useful information about how long a positive and negative return shock persists. A quicker reversion of negative returns implies that positive returns tend to persist longer than negative returns, which is better captured with higher orders ANAR models such as:

$$r_t = \mu + (\phi_1 + \rho_1 D_2) r_{t-1} + (\phi_2 + \rho_2 D_2) r_{t-2} + \varepsilon_t \tag{10}$$

$$r_t = \mu + (\phi_1 + \rho_1 D_3) r_{t-1} + (\phi_2 + \rho_2 D_3) r_{t-2} + (\phi_3 + \rho_3 D_3) r_{t-3} + \varepsilon_t \tag{11}$$

$$r_t = \mu + (\phi_1 + \rho_1 D_4) r_{t-1} + (\phi_2 + \rho_2 D_4) r_{t-2} + (\phi_3 + \rho_3 D_4) r_{t-3} + (\phi_4 + \rho_4 D_4) r_{t-4} + \varepsilon_t \tag{12}$$

where D_2, D_3 and D_4 are dummy variables. D_2 takes a value of one if r_{t-1} and r_{t-2} are both negative, or zero otherwise. Likewise, D_3 and D_4 takes a value of one only if all three and four prior returns are negative respectively. While persistence of positive returns is measured by $\sum \phi_j$, persistence of negative returns is measured by $\sum(\phi_j + \rho_j)$ in both models. If a higher persistence of positive returns is expected, for example a situation where $\sum \phi_j > \sum(\phi_j + \rho_j)$, then $\sum(\rho_j)$ should be negative in both models.

5. Results

5.1. Data analysis

As mentioned in section 3, in this study we have used minutely, hourly, daily and weekly data from 20th July 2010 to 22nd February 2018 for Bitcoin price returns, with the weekly return series being computed as the geometric average of seven consecutive daily price returns. Since the price behaviour of Bitcoin appeared to have entered a consistent bubble-phase in the period after the price exceeded \$1,000, which occurred on 1st January 2017, we not only study the entire sample period but also analyse the pre-\$1,000 sub-period (20th July 2010 through 1st January 2017) separately from the post-\$1,000 sub-period (1st January 2017 through 22nd February 2018) in order to examine whether there exists different mean aversion behaviour in the Bitcoin price returns during the two sub-periods.

Table 1 reports the number of observations of same sign return series found between two and fifteen consecutive periods. It can be noticed that for the entire sample period, irrespective of the data frequency considered, the number of negative returns is considerably smaller than the number of positive returns (with the only exception being the case of hourly data for fifteen consecutive periods where there is no observation for either positive or negative returns). Similar results are obtained for all four data frequencies for the pre-\$1,000 sub-period as well as for the post-\$1,000 sub-period, although in the latter sub-period as the number of consecutive periods increases there is no observation for either positive or negative returns of hourly, daily or weekly data. These results suggest asymmetric reverting behaviour between positive and negative returns, with negative price returns reverting more rapidly than positive price returns reverting to negative returns. This finding further implies that, as negative returns revert more quickly, positive returns persist more than negative returns.

Insert Table 1 about here

Summary statistics for the different data frequencies of Bitcoin price returns over the entire period as well as over the two sub-periods are presented in Table 2. We notice that over both the entire period and the pre-\$1,000 sub-period, irrespective of the data frequency considered, the average price returns are positive. More specifically, the average returns are equal to 1% for minutely returns during the pre-\$1,000 period and 0.1% for the returns of any other frequency over the pre-\$1,000 period or of any frequency over the entire sample period. Nevertheless, over the post-\$1,000 sub-period the average price returns equal zero. Moreover, the price returns of both the pre-\$1,000 sub-period as well as the entire period have a higher standard deviation than the returns in the post-\$1,000 sub-period. Furthermore, when considering the entire sample period, the pre-\$1,000 sub-period or the weekly returns of the post-\$1,000 sub-period, the price returns of Bitcoin are positively skewed, indicating that Bitcoin has a longer right tail, and hence large positive price returns are more commonly observed than large negative returns. However, the opposite result holds for the minutely, hourly and daily price returns of the post-\$1,000 sub-period, which are negatively skewed, suggesting that large negative returns are more commonly observed than large positive returns. It can also be noticed that irrespective of the data frequency or period considered the price returns of Bitcoin are leptokurtic as a result of excess kurtosis, with the returns during the pre-\$1,000 sub-period displaying higher excess kurtosis than the returns during the post-\$1,000 sub-period, though. The above results suggest that the Bitcoin price returns are non-normal irrespective of the data frequency or period under examination, a result which is consistent with previous studies (e.g., Katsiampa [2017]; Phillip et al. [2018]).

Insert Table 2 about here

5.2. Estimation results of ANAR models

Table 3 presents the estimation results of the ANAR models (3)-(7) for the Bitcoin price returns over the entire sample period as well as over the pre- and post-\$1,000 sub-periods. As discussed in the previous section, serial correlation is captured by $\phi_1 + \rho_1$ if $r_{t-1} < 0$ or by ϕ_1 if $r_{t-1} \geq 0$, while the asymmetric reverting behaviour is measured by ρ_1.

Insert Table 3 about here

By examining the estimation results for the entire period, we notice that, irrespective of the data frequency considered, ϕ_1 is positive and statistically significant at the 1% level under any model, with the only exception being the case of weekly data under model (3), where ϕ_1 is not significant at any conventional level. Moreover, ρ_1, which captures asymmetric reverting patterns if different from zero, is found negative under all ANAR models, regardless of the data frequency considered, a fact that suggests stronger reverting behaviour of negative price returns in terms of both reverting speed and magnitude compared to positive returns (Nam et al. [2006]). It can also be noticed that

ρ_1 is statistically significant at the 1% level under all the models considered, except for models (3) and (5) in the case of weekly returns.

Next we examine the estimation results of the pre- and post-\$1,000 sub-periods separately. We notice that when examining the pre-\$1,000 sub-period, the estimation results are very similar to those obtained over the entire period, with ϕ_1 being positive and statistically significant at the 1% level irrespective of the data frequency considered under any model, apart from model (3) for weekly data, where ϕ_1 is not significant at any conventional level, and with ρ_1 being negative and statistically significant at the 1% level under all the models regardless of the data frequency, except for models (3) and (5) in the case of weekly returns. In the post-\$1,000 sub-period, we find once again positive ϕ_1 parameter estimates, which are all now statistically significant at the 1% level for all data frequencies and for all models considered. In addition, ρ_1 is found once again negative and statistically significant at the 1% level under all the models considered irrespective of the data frequency, except for model 5 in the case of weekly returns. Consequently, similar to the negative estimated values over the entire sample period, the negative estimates for ρ_1 in both sub-periods confirm asymmetric reverting patterns of Bitcoin price returns, with negative price returns reverting more quickly than positive returns. It is worth noting that, when comparing the two sub-periods, the magnitude of ρ_1 is greater in absolute terms in the post-\$1,000 sub-period under any model for hourly returns, under models (5) and (6) for minutely returns, under models (3) and (5) for daily returns and under model (3) for weekly returns, while the opposite result holds for minutely returns under models (3), (4) and (7), where the magnitude of ρ_1 is larger in absolute terms in the pre-\$1,000 sub-period. This finding suggests that the asymmetric reverting behaviour seems to be overall stronger after Bitcoin's price exceeded \$1,000, which occurred on 1st January 2017, and seems to be overall consistent with the results presented in Table 1, especially when considering hourly, daily and weekly returns, but the opposite seems to hold when considering minutely returns under models (3), (4) and (7). It can be noticed, though, that the estimates for ρ_1, although negative, are close to zero in the case of daily and weekly returns irrespective of the period considered, a fact that implies that the reverting pattern tends to become more symmetrical as we consider lower data frequencies.

Moreover, combining the aforementioned results of $\phi_1 > 0$ and $\phi_1 < 0$, we can conclude that serial correlation is positive with prior positive price returns over the entire sample period as well as over the pre- and post-\$1,000 sub-periods. However, serial correlation decreases with prior negative price returns and can be either positive, negative or zero, a fact that further highlights asymmetric behaviour between positive and negative returns. More specifically, over the entire sample period and over the pre-\$1,000 sub-period, with prior negative returns serial correlation remains positive under any model in the case of weekly returns or under model (3) in the case of hourly and daily returns, becomes zero under models (4) and (5) in the case of daily returns, and becomes negative under models (4)-(7) in the case of hourly returns and under any model in the case of minutely returns. Similar results are also obtained over the post-$1,000 sub-period in the case of minutely and hourly returns under any model and in the case of daily returns under model (3), where with prior negative returns serial correlation is negative under any model for minutely returns and under models (4)-(7) for hourly returns, and positive under model (3) in the case of hourly and daily returns. However, with prior negative returns serial correlation is now negative under models (4) and (5) in the case of daily returns as well as under model (3) in the case of weekly returns , and zero under models (4) and (5) in the case of weekly returns.

5.3. Estimation results of ANAR-EGARCH models

In this sub-section, we investigate the asymmetric reverting pattern of the Bitcoin price returns while also allowing for conditional heteroskedasticity by employing the EGARCH model for the conditional volatility in order to capture leverage effects. By allowing for time-varying conditional variance in our analysis, our results can lead to a more reliable economic interpretation (Nam et al. [2006]), while the EGARCH model captures the asymmetric effect of return shocks on volatility.

The maximum likelihood estimation results of the ANAR models (3)-(5) for the conditional mean equation combined with the EGARCH specification for the conditional variance equation can be found in Tables 4 and 5 for the different data frequencies. When examining the entire sample period, we notice that $\phi_1 > 0$ is positive and statistically significant at the 1% level under all the three models for any data frequency, while $\rho_1 > 0$ is negative and statistically significant at the 1% level under all the three models for minutely, hourly and daily data as well as under models (3) and (4) for weekly data. Consequently, the asymmetric reverting behaviour is still observed and significant (except for model (5) in the case of weekly data) when allowing for time-varying conditional variances.

Insert Tables 4 and 5 about here

By examining the two sub-periods, we find similar behaviour for the Bitcoin price returns over both the pre- and post-$1,000 sub-periods, with the estimated ϕ_1 and ρ_1 parameters being positive and negative, respectively. Moreover, both estimated parameters are found statistical significant at the 1% level for the minutely, hourly and daily data, while for the weekly returns ρ_1 is significant at the 1% level under models (3) and (4) over the pre-$1,000 sub-period and under model (3) over the post-$1,000 sub-period. The asymmetric reverting behaviour is thus also observed when studying the two sub-periods separately while allowing for time-varying conditional heteroskedasticity. In addition, it is worth noting that the estimation results indicate stronger asymmetric behaviour for the post-$1,000 sub-period as compared to the pre-$1,000 sub-period for minutely data under any model, for hourly and daily data under models (4) and (5), and for weekly data under model (5), as a result of higher in absolute terms estimates of ρ_1 over the post-$1,000 sub-period for minutely returns under all three models, for hourly and daily returns under models (4) and (5), and for weekly returns under model (5).

5.4. Estimation results of higher order ANAR models

When return series display strong asymmetric reverting behaviour, there should also exist asymmetries in their persistence, but these can better be analysed using higher order, instead of first order, ANAR models (Nam et al. [2006]). Tables 6, 7 and 8 thus report the estimation results of the ANAR models (10)-(12) allowing for conditional heteroskedasticity and leverage effects by employing the EGARCH specification for the conditional variance , which were presented in section 4, for the different data frequencies over the entire period, the pre-$1,000 sub-period and the post-$1,000 sub-period, respectively. In each model persistence of positive returns is measured by $\sum \phi_j$, while persistence of negative returns is captured by $\sum(\phi_j + \rho_j)$.

Insert Tables 6 through 8 about here

When estimating higher order ANAR models for the entire period (Table 6), we notice that $\sum \phi_j$ is positive and $\sum \rho_j$ is negative in all the models and for all data frequencies. Moreover, we notice that $\sum \phi_j > \sum(\phi_j + \rho_j)$ and hence a higher persistence of positive returns than negative returns is found. This result further supports the existence of asymmetric reverting behaviour in the Bitcoin price returns. Similar results are obtained for the returns over both the pre- (Table 7) and post- (Table 8) $1,000 sub-periods, where $\sum \phi_j$ is positive and $\sum \rho_j$ is negative in all the models and for all data frequencies. What is more, for both sub-periods we have that $\sum \phi_j > \sum(\phi_j + \rho_j)$, a result that suggests once again a higher persistence of positive returns than negative returns and thus the existence of asymmetric reverting behaviour in the Bitcoin price returns in both sub-periods.

6. Conclusion

In this paper, by employing ANAR models, we examined whether Bitcoin price returns exhibit asymmetric reverting patterns for minutely, hourly, daily and weekly returns between June 2010 and February 2018. We also investigated whether Bitcoin's pricing behaviour has differed during two sub-periods, namely before and after reaching the price of $1,000 on 1st January 2017, as the price behaviour of Bitcoin appeared to have entered a consistent bubble-phase since then.

We have contributed to the literature in several ways. First of all, to the best of the authors' knowledge, this is the first study to examine whether asymmetric reverting patterns exist in cryptocurrency markets. Secondly, and most importantly, though, according to the results, we found evidence of asymmetric reverting patterns in the Bitcoin price returns under all the ANAR models employed, regardless of the data frequency considered. More specifically, we found stronger reverting behaviour of negative price returns in terms of both reverting speed and magnitude compared to positive returns, irrespective of the period considered, although the reverting pattern tends to become more symmetrical as we consider lower data frequencies. The above result is of particular relevance when compared to similar results presented for equity markets. The reverting behaviour of negative pricing returns in equity markets has not been found to be justified by the positive relationship between future volatility and risk premium, which was identified to be a key component of the time-varying rational expectation hypothesis. Negative return shocks have been associated with increased risk premiums, which in turn reduces the concurrent price, therefore generating another negative return. More research is necessary to investigate the relationship between cryptocurrency implied volatility and risk premiums which will be supported by the provision of substantial data from cryptocurrency derivatives exchanges.

In addition, we found evidence of positive serial correlation with prior positive price returns over the entire sample period as well as over the pre- and post-$1,000 sub-periods. However, it was found that serial correlation decreases with prior negative price returns and can be either positive, negative or zero, a fact that further highlights asymmetric behaviour between positive and negative returns. Thirdly, we also examined the asymmetric reverting behaviour of the Bitcoin price returns while also allowing for conditional heteroskedasticity by employing the EGARCH model for the conditional volatility in order to capture leverage effects, and found that the asymmetric reverting pattern is still observed, even when allowing for time-varying conditional heteroskedasticity in return dynamics, with stronger overall asymmetric behaviour for the post-$1,000 sub-period as compared to the pre-$1,000 sub-period, though, especially for minutely data. Finally, we also investigated asymmetries in Bitcoin price return series' persistence by employing higher order ANAR models and found higher persistence of positive returns than negative returns over both the entire period and the two sub-periods under examination, a result that further supports the existence of asymmetric reverting behaviour in the Bitcoin price returns.

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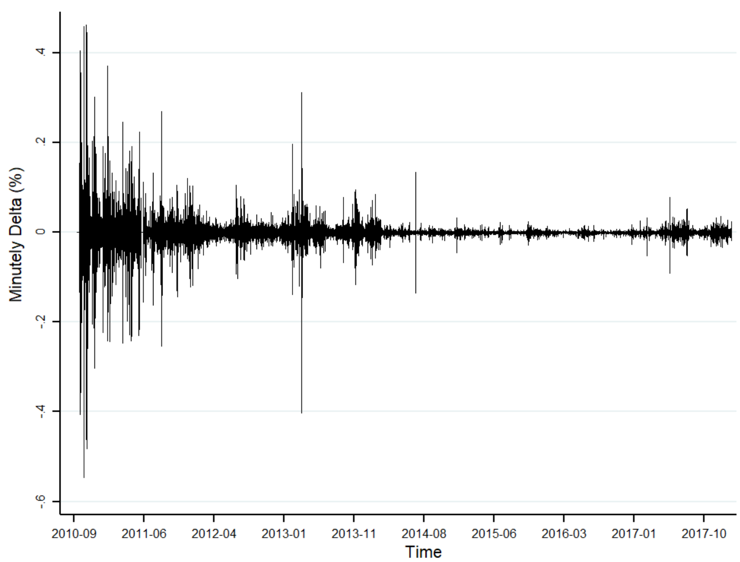
Figure 1: Price and volatility of Bitcoin, 2010-2018



Note: The upper panel represents the minutely price of Bitcoin, while the lower panel represents the minutely price volatility. Data is presented between midnight on 20 July 2010 and 22 February 2018. 3,994,142 observations are presented.

Table 1: Number of observations for the consecutive same sign

	Returns (Full Period)				Returns (Period pre-$1,000)				Returns (Period post-$1,000)			
Consecutive Ret.	Minutely	Hourly	Daily	Weekly	Minutely	Hourly	Daily	Weekly	Minutely	Hourly	Daily	Weekly
2 Cons. +ive	604,089	16,777	968	193	424,167	13,973	812	165	179,922	2,804	156	28
3 Cons. +ive	315,067	8,210	564	149	206,216	6,781	474	127	108,851	1,429	90	22
4 Cons. +ive	177,013	3,984	330	119	108,721	3,278	280	102	68,292	706	50	17
5 Cons. +ive	104,250	1,920	194	99	60,304	1,580	168	85	43,946	340	26	14
6 Cons. +ive	63,503	925	114	85	34,673	766	102	74	28,830	159	12	11
7 Cons. +ive	39,615	450	72	73	20,466	386	67	64	19,149	64	5	9
8 Cons. +ive	25,103	215	47	62	12,298	192	44	55	12,805	23	3	7
9 Cons. +ive	16,064	107	31	54	7,418	97	30	49	8,646	10	1	5
10 Cons. +ive	10,388	57	20	47	4,492	53	20	43	5,896	4	0	4
11 Cons. +ive	6,767	28	13	40	2,724	27	13	37	4,043	1	0	3
12 Cons. +ive	4,438	10	9	35	1,640	10	9	33	2,798	0	0	2
13 Cons. +ive	2,923	4	6	31	1,001	4	6	30	1,922	0	0	1
14 Cons. +ive	1,938	1	5	27	614	1	5	27	1,324	0	0	0
15 Cons. +ive	1,293	0	4	24	375	0	4	24	918	0	0	0
2 Cons. -ive	538,626	11,686	450	49	384,723	9,709	383	43	153,903	1,977	67	6
3 Cons. -ive	267,060	4,836	190	17	179,163	4,045	166	16	87,897	791	24	1
4 Cons. -ive	143,298	1,976	78	6	91,219	1,677	71	6	52,079	299	7	0
5 Cons. -ive	80,392	794	32	3	48,839	696	32	3	31,553	98	0	0
6 Cons. -ive	46,408	312	15	1	27,039	281	15	1	19,369	31	0	0
7 Cons. -ive	27,193	126	8	0	15,282	119	8	0	11,911	7	0	0
8 Cons. -ive	16,068	48	4	0	8,695	46	4	0	7,373	2	0	0
9 Cons. -ive	9,488	12	1	0	4,906	12	1	0	4,582	0	0	0
10 Cons. -ive	5,788	3	0	0	2,911	3	0	0	2,877	0	0	0
11 Cons. -ive	3,557	0	0	0	1,707	0	0	0	1,850	0	0	0
12 Cons. -ive	2,191	0	0	0	1,013	0	0	0	1,178	0	0	0
13 Cons. -ive	1,367	0	0	0	620	0	0	0	747	0	0	0
14 Cons. -ive	839	0	0	0	377	0	0	0	462	0	0	0
15 Cons. -ive	514	0	0	0	232	0	0	0	282	0	0	0

Note: Data is presented between midnight on 20 July 2010 and 22 February 2018.

Table 2: Summary statistics for minutely, daily, hourly and weekly Bitcoin returns

	Minutely	Hourly	Daily	Weekly
Full Period				
Observations	3,994,142	66,597	2,773	400
Minimum	-0.647%	-0.782%	-2.780%	-0.489%
Maximum	1.833%	2.810%	18.988%	2.713%
Mean	0.001%	0.001%	0.001%	0.001%
Standard Deviation	0.393	0.035	0.006	0.002
Skewness	46.217	17.189	11.938	4.573
Kurtosis	17,571.90	1,087.78	309.34	32.66
Period pre $1,000 {20/7/10 - 1/1/17}				
Observations	3,380,014	57,190	2,356	342
Minimum	-0.647%	-0.782%	-2.780%	-0.489%
Maximum	1.833%	2.810%	18.988%	2.713%
Mean	0.010%	0.001%	0.001%	0.001%
Standard Deviation	0.423	0.036	0.006	0.003
Skewness	43.969	17.089	11.835	4.377
Kurtosis	15,550.69	1,025.24	289.85	29.36
Period post $1,000 {1/1/17 - 22/2/18}				
Observations	614,128	9,407	418	58
Minimum	-0.086%	-0.274%	-1.258%	-0.352%
Maximum	0.082%	0.185%	1.547%	0.533%
Mean	0.000%	0.000%	0.000%	0.000%
Standard Deviation	0.002	0.019	0.004	0.001
Skewness	-0.223	-0.327	-0.040	0.311
Kurtosis	82.82	15.20	5.04	5.96

Note: Data is presented between midnight on 20 July 2010 and 22 February 2018.

Table 3: Parameter estimates of ANAR methodologies for minutely, hourly daily and weekly investigations

	Total Sample			Pre-$1,000			Post-$1,000		
	μ	ϕ_j	ρ_j	μ	ϕ_j	ρ_j	μ	ϕ_j	ρ_j
Minutely									
D_1	0.002	0.123	-0.129	0.002	0.074	-0.095	0.002	0.074	-0.077
	(9.12)	(27.07)	(-27.54)	(10.97)	(26.37)	(-26.58)	(4.31)	(6.01)	(-6.44)
D_2	0.002	0.074	-0.087	0.002	0.077	-0.088	0.002	0.069	-0.085
	(6.54)	(13.42)	(-14.97)	(5.71)	(10.98)	(-12.07)	(6.32)	(16.47)	(-19.39)
D_3	0.001	0.070	-0.085	0.001	0.069	-0.082	0.001	0.069	-0.089
	(5.70)	(9.53)	(-10.76)	(5.05)	(7.27)	(-8.03)	(4.80)	(14.27)	(-16.93)
D_4	0.001	0.071	-0.090	0.001	0.072	-0.088	0.001	0.069	-0.093
	(5.42)	(7.45)	(-8.53)	(4.96)	(5.56)	(-6.22)	(3.37)	(11.72)	(-13.96)
D_5	0.001	0.074	-0.098	0.001	0.078	-0.098	0.001	0.068	-0.096
	(5.28)	(6.01)	(-6.99)	(4.93)	(4.44)	(-5.10)	(2.64)	(9.52)	(-11.42)
Hourly									
D_1	0.000	0.128	-0.127	0.000	0.130	-0.127	0.001	0.203	-0.165
	(4.98)	(25.87)	(-25.45)	(4.72)	(24.34)	(-23.56)	(4.91)	(7.09)	(2.91)
D_2	0.000	0.108	-0.120	0.000	0.107	-0.115	0.000	0.114	-0.143
	(2.42)	(34.43)	(-33.57)	(2.33)	(29.72)	(-28.11)	(0.65)	(26.88)	(-29.82)
D_3	0.000	0.103	-0.132	0.000	0.101	-0.127	0.000	0.107	-0.154
	(4.91)	(25.13)	(-25.54)	(4.79)	(21.61)	(-21.50)	(1.04)	(19.35)	(-21.50)
D_4	0.000	0.106	-0.140	0.000	0.105	-0.134	0.000	0.110	-0.177
	(5.81)	(18.81)	(-17.80)	(5.65)	(16.17)	(-14.91)	(1.38)	(14.29)	(-15.31)
D_5	0.000	0.111	-0.145	0.000	0.112	-0.143	0.000	0.106	-0.168
	(6.48)	(13.81)	(-11.77)	(6.28)	(12.06)	(-10.29)	(1.63)	(9.62)	(-8.23)
Daily									
D_1	0.000	0.003	-0.002	0.000	0.003	-0.002	0.000	0.005	-0.003
	(4.11)	(4.37)	(-2.86)	(3.45)	(4.22)	(-2.54)	(3.76)	(21.00)	(13.94)
D_2	0.000	0.003	-0.003	0.000	0.003	-0.003	0.000	0.002	-0.003
	(3.39)	(10.73)	(-8.89)	(3.23)	(9.66)	(-7.49)	(1.09)	(6.68)	(-7.97)
D_3	0.000	0.003	-0.003	0.000	0.003	-0.003	0.000	0.002	-0.005
	(4.69)	(10.44)	(-7.18)	(4.51)	(9.48)	(-6.03)	(1.42)	(5.89)	(-6.55)
Weekly									
D_1	0.000	0.002	-0.001	0.000	0.002	-0.001	0.000	0.002	-0.004
	(1.98)	(1.37)	(-1.52)	(2.00)	(1.37)	(-1.48)	(3.45)	(7.07)	(4.55)
D_2	0.000	0.002	-0.001	0.000	0.002	-0.001	0.000	0.001	-0.001
	(1.95)	(5.42)	(-2.97)	(1.89)	(4.92)	(-2.66)	(0.58)	(3.77)	(-2.45)
D_3	0.000	0.002	-0.001	0.000	0.002	-0.001	0.000	0.001	-0.001
	(1.93)	(6.80)	(-1.59)	(1.85)	(6.34)	(-1.45)	(0.70)	(3.55)	(-1.16)

Note: The first model we use is specified as: $r_t = \mu + [\phi_1 + \rho_1 D_1 (r_{t-1} < 0)] + \varepsilon_t$, where D_1 is an indicator function specified for a dummy variable that takes a value of one if $r_{t-1} < 0$, or zero otherwise. $\phi_1 + \rho_1 D_1$ represents serial correlation with the above model allowing for the autocorrelation coefficient of stock returns to vary along with sign of r_{t-1}. We confirm asymmetric reverting patterns to incorporate two, three, four and even five consecutive price decreases through the analysis of the following specifications: $r_t = \mu + [\phi_1 + \rho_1 D_2 (r_{t-1} < 0, r_{t-2} < 0)] + \varepsilon_t$; $r_t = \mu + [\phi_1 + \rho_1 D_3 (r_{t-1} < 0, r_{t-2} < 0, r_{t-3} < 0)] + \varepsilon_t$; $r_t = \mu + [\phi_1 + \rho_1 D_4 (r_{t-1} < 0, r_{t-2} < 0, r_{t-3} < 0, r_{t-4} < 0)] + \varepsilon_t$; and $r_t = \mu + [\phi_1 + \rho_1 D_5 (r_{t-1} < 0, r_{t-2} < 0, r_{t-3} < 0, r_{t-4} < 0, r_{t-5} < 0)] + \varepsilon_t$ where D_2, D_3, D_4 and D_5 are dummy variables. D_2 takes a value of one if r_{t-1} and r_{t-2} are both negative or zero otherwise. D_3, D_4 and D_5 take a value of one if all three, four and five prior returns are negative respectively. As stated in the baseline model, $\phi_1 < 0$ confirms that a negative return exhibits a relatively stronger asymmetry in reverting patterns.

Table 4: Parameter estimates of EGARCH specifications for minutely and hourly methodologies

	Full Period			Pre-\$1000			Post-\$1000		
	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
Minutely									
μ	0.0005	0.0005	0.0008	0.0003	0.0002	0.0002	0.0002	0.0001	0.0004
	(11.31)	(11.18)	(4.12)	(4.45)	(2.59)	(2.84)	(4.33)	(2.20)	(4.37)
ϕ_1	0.0788	0.0777	0.0242	0.0282	0.0264	0.0262	0.031	0.0288	0.0278
	(11.77)	(11.04)	(7.89)	(2.77)	(22.87)	(18.27)	(25.67)	(20.36)	(16.45)
ρ_1	-0.0277	-0.0028	-0.0254	-0.0323	-0.0299	-0.0303	-0.0369	-0.0338	-0.0347
	(-4.59)	(-4.49)	(-8.23)	(-30.27)	(-24.91)	(-19.36)	(-27.94)	(-22.50)	(-17.65)
a_0	0.0002	0.0003	0.0006	0.0006	0.0005	0.0006	0.0006	0.0006	0.0006
	(4.65)	(5.22)	(5.01)	(4.98)	(4.58)	(4.28)	(4.95)	(4.55)	(4.74)
θ	0.0037	0.0036	0.0035	0.0021	0.0022	0.0413	0.0002	0.0022	0.0243
	(5.39)	(5.02)	(12.10)	(5.77)	(5.28)	(11.64)	(5.76)	(5.27)	(11.86)
γ	0.991	0.9941	0.9706	0.9812	0.9777	0.9564	0.9872	0.9769	0.9641
	(26.98)	(26.14)	(46.26)	(31.83)	(30.43)	(38.56)	(31.05)	(30.05)	(37.16)
Hourly									
μ	0.0002	0.0002	0.0002	0.0003	0.0002	0.0002	0.0001	0.0001	0.0000
	(9.55)	(2.22)	(2.83)	(9.47)	(4.33)	(2.43)	(4.07)	(9.11)	(2.69)
ϕ_1	0.0345	0.0059	0.0058	0.0313	0.0056	0.0055	0.0143	0.0071	0.0069
	(10.93)	(40.27)	(33.89)	(10.78)	(34.31)	(28.98)	(25.16)	(26.18)	(20.67)
ρ_1	-0.0234	-0.0063	-0.0069	-0.0229	-0.0057	-0.0063	-0.0027	-0.0081	-0.0076
	(-7.86)	(-38.44)	(-31.65)	(-7.83)	(-32.08)	(-27.33)	(-14.03)	(-25.97)	(-17.09)
a_0	0.0001	0.0002	0.0002	0.0001	0.0002	0.0002	0.0003	0.0002	0.0002
	(23.90)	(4.57)	(4.46)	(22.54)	(4.39)	(4.41)	(11.23)	(9.11)	(9.21)
θ	0.2007	0.0962	0.0984	0.2493	0.1031	0.1049	0.0988	0.0972	0.1002
	(43.50)	(42.26)	(42.65)	(40.44)	(38.94)	(39.88)	(17.48)	(18.82)	(18.07)
γ	0.7667	0.8782	0.8749	0.746	0.8751	0.872	0.8909	0.8837	0.8018
	(35.27)	(37.32)	(36.97)	(28.18)	(34.23)	(34.21)	(15.92)	(19.62)	(18.62)

Note: Values in parentheses are the asymptotic t-statistics. Data is presented between midnight on 20 July 2010 and 22 February 2018. we utilise the exponential GARCH (EGARCH) model to capture leverage effects which is specified for modelling conditional variance h_t: $log(h_t) = \alpha_0 + \frac{\Phi(L)}{\Psi(L)} g(v_t)$ and $g(v_t) = \theta v_t + \gamma \left[|v_t| - E\,|v_t| \right]$ where $\Phi(L)$ and $\Psi(L)$ are the finite-order polynomials of order q and p, and the standardised residual v_t is defined as $v_t = \frac{\varepsilon_t}{\sqrt{h_t}}$ for the estimated residual ε_t. Since the EGARCH model specifies the log of h_t in the variance equation, it does not require any positivity restrictions on parameters to ensure nonnegativity of h_t. The value of $g(v_t)$ is a function of both the magnitude and sign of v_t. The term $\gamma \left[|v_t| - E\,|v_t| \right]$ represents the magnitude effects and θv_t, the sign effect of the standardized residual on the conditional variance.

Table 5: Parameter estimates of EGARCH specifications for daily and weekly methodologies

	Full Period			Pre-$1000			Post-$1000		
	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
Daily									
μ	0.0002	0.0003	0.0004	0.0001	0.0001	0.0004	0.0000	0.0005	0.0004
	(14.36)	(9.36)	(9.83)	(17.77)	(15.50)	(8.93)	(9.16)	(3.72)	(3.53)
ϕ_1	0.0453	0.0526	0.0943	0.0209	0.0449	0.0467	0.0145	0.0111	0.0307
	(25.07)	(21.45)	(19.28)	(10.46)	(50.54)	(30.15)	(9.01)	(6.56)	(5.66)
ρ_1	-010322	-0.1117	-0.1377	-0.0283	-0.02411	-0.0113	-0.0139	-0.03091	-0.1098
	(-22.21)	(-22.68)	(-26.02)	(-45.03)	(-18.96)	(-18.97)	(-8.55)	(-3.51)	(-7.42)
a_0	0.0129	0.0098	0.0099	0.0106	0.0098	0.0099	0.3481	0.0928	0.095
	(7.91)	(13.20)	(5.70)	(8.83)	(6.92)	(14.51)	(6.80)	(4.31)	(7.18)
θ	0.3884	0.1081	0.0886	0.1982	0.2132	0.0929	0.0633	0.0491	0.0915
	(21.76)	(38.74)	(28.76)	(15.62)	(26.38)	(27.06)	(1.99)	(1.61)	(2.51)
γ	0.8515	0.7408	0.1108	0.5797	0.4638	0.1144	0.2883	0.2849	0.3148
	(35.03)	(34.96)	(43.91)	(28.51)	(48.09)	(41.27)	(3.73)	(5.69)	(7.07)
Weekly									
μ	0.0001	0.0000	0.0000	0.0001	0.0003	0.0000	0.0000	0.0000	0.0000
	(4.32)	(1.40)	(1.52)	(5.58)	(1.38)	(1.04)	(4.78)	(2.70)	(2.22)
ϕ_1	0.0139	0.0169	0.0181	0.0192	0.0164	0.0123	0.0197	0.0097	0.0093
	(4.55)	(9.32)	(3.98)	(8.75)	(3.08)	(7.35)	(9.97)	(4.37)	(2.20)
ρ_1	-0.0226	-0.0203	-0.0175	-0.0127	-0.0101	-0.0056	-0.0094	-0.0097	-0.0181
	(-4.13)	(-2.66)	(-1.24)	(-7.92)	(-2.99)	(-1.40)	(-5.16)	(-1.62)	(-1.54)
a_0	0.029	0.0801	0.0068	0.0234	0.0556	0.0602	0.0339	0.044	0.0458
	(5.52)	(5.33)	(4.04)	(4.29)	(12.19)	(2.16)	(5.15)	(1.98)	(2.29)
θ	1.0262	1.0709	1.2319	0.8456	0.9253	0.9095	0.657	0.6858	0.9215
	(10.52)	(13.66)	(22.08)	(7.00)	(2.84)	(5.25)	(3.14)	(1.56)	(1.97)
γ	0.2359	0.0714	0.0216	0.2695	0.0834	0.1195	0.4261	0.261	0.1795
	(3.51)	(5.65)	(2.06)	(4.21)	(3.00)	(4.11)	(0.86)	(1.13)	(1.99)

Note: Values in parentheses are the asymptotic t-statistics. Data is presented between midnight on 20 July 2010 and 22 February 2018. we utilise the exponential GARCH (EGARCH) model to capture leverage effects which is specified for modelling conditional variance h_t: $log(h_t) = \alpha_0 + \frac{\Phi(L)}{\Psi(L)} g(v_t)$ and $g(v_t) = \theta v_t + \gamma \left[|v_t| - E\,|v_t| \right]$ where $\Phi(L)$ and $\Psi(L)$ are the finite-order polynomials of order q and p, and the standardised residual v_t is defined as $v_t = \frac{\varepsilon_t}{\sqrt{h_t}}$ for the estimated residual ε_t. Since the EGARCH model specifies the log of h_t in the variance equation, it does not require any positivity restrictions on parameters to ensure nonnegativity of h_t. The value of $g(v_t)$ is a function of both the magnitude and sign of v_t. The term $\gamma \left[|v_t| - E\,|v_t| \right]$ represents the magnitude effects and θv_t, the sign effect of the standardized residual on the conditional variance.

Table 6: Parameter estimates of the persistence EGARCH methodology for total period of investigation (2010-2018)

Total	Minutely			Hourly			Daily			Weekly		
	Model 4	Model 5	Model 6	Model 4	Model 5	Model 6	Model 4	Model 5	Model 6	Model 4	Model 5	Model 6
μ	0.0021	0.0022	0.0021	0.0008	0.0008	0.0007	0.0007	0.0012	0.0017	0.0000	0.0000	0.0000
	(4.51)	(4.37)	(4.13)	(4.00)	(4.08)	(4.31)	(4.98)	(6.21)	(6.32)	(2.33)	(2.45)	(2.10)
ϕ_1	0.0165	0.0164	0.0165	0.0144	0.0145	0.0144	0.0031	0.0031	0.0032	0.0014	0.0014	0.0014
	(30.12)	(30.13)	(30.13)	(27.40)	(27.41)	(27.40)	(4.09)	(4.17)	(4.21)	(2.05)	(2.06)	(2.07)
ϕ_2	0.0089	0.0083	0.0084	0.0032	0.0029	0.0029	0.002	0.0026	0.0030	0.0006	0.0004	0.0004
	(13.48)	(10.02)	(10.02)	(9.03)	(6.80)	(6.84)	(1.69)	(2.02)	(2.04)	(1.69)	(1.79)	(1.80)
ϕ_3		0.0011	0.0012		0.0005	0.001		0.0009	0.0008		0.0013	0.0005
		(10.80)	(10.03)		(1.11)	(1.65)		(2.63)	(1.61)		(2.79)	(0.74)
ϕ_4			0.0003			0.0009			0.0004			0.0010
			(2.47)			(2.30)			(1.79)			(1.85)
ρ_1	-0.0161	-0.0160	-0.0161	-0.0135	-0.0136	-0.0136	-0.0019	-0.0019	-0.002	-0.0058	-0.006	-0.0061
	(-29.29)	(-29.24)	(-29.29)	(-25.79)	(-25.80)	(-25.79)	(-2.58)	(-2.59)	(-2.62)	(-1.43)	(-2.43)	(-1.45)
ρ_2	-0.0075	-0.0073	-0.0075	-0.002	-0.0026	-0.0025	-0.0004	-0.0002	-0.002	-0.0032	-0.0039	-0.004
	(-11.37)	(-8.61)	(-8.62)	(-5.06)	(-5.51)	(-5.58)	(-1.19)	(-1.41)	(-1.43)	(-1.69)	(-2.71)	(-2.72)
ρ_3		0.0041	-0.0010		-0.0014	-0.0008		-0.0006	-0.0007		-0.0023	-0.0035
		(-4.92)	(-7.69)		(-2.31)	(-1.19)		(-2.07)	(-1.99)		(-2.30)	(-2.39)
ρ_4			-0.0012			-0.0013			-0.0015			-0.0034
			(-8.04)			(-2.41)			(-1.18)			(-1.26)
$\sum(\rho_j)$	0.0254	0.0258	0.0264	0.0176	0.0179	0.0192	0.0051	0.0066	0.0074	0.0020	0.0031	0.0033
$\sum(\phi_j)$	-0.0236	-0.0192	-0.0258	-0.0155	-0.0176	-0.0182	-0.0023	-0.0027	-0.0062	-0.0090	-0.0122	-0.017
$\sum(\phi_j+\rho_j)$	0.0018	0.0066	0.0006	0.0021	0.0003	0.0010	0.0028	0.0039	0.0012	-0.0070	-0.0091	-0.0137

Note: Values in parentheses are the asymptotic t-statistics. Data is presented between midnight on 20 July 2010 and 22 February 2018. Minutely, hourly, daily and weekly analyses are based on estimation of the methodologies: Model 4: $r_t = \mu + (\phi_1 + \rho_1 D_2)r_{t-1} + (\phi_2 + \rho_2 D_2)r_{t-2} + \varepsilon_t$; Model 5: $r_t = \mu + (\phi_1 + \rho_1 D_3)r_{t-1} + (\phi_2 + \rho_2 D_3)r_{t-2} + (\phi_3 + \rho_3 D_3)r_{t-3} + \varepsilon_t$; and Model 6: $r_t = \mu + (\phi_1 + \rho_1 D_4)r_{t-1} + (\phi_2 + \rho_2 D_4)r_{t-2} + (\phi_3 + \rho_3 D_4)r_{t-3} + (\phi_4 + \rho_4 D_4)r_{t-4} + \varepsilon_t$ respectively.

Table 7: Parameter estimates of the persistence EGARCH methodology before Bitcoin breached $1,000

Total	Minutely			Hourly			Daily			Weekly		
	Model 4	Model 5	Model 6	Model 4	Model 5	Model 6	Model 4	Model 5	Model 6	Model 4	Model 5	Model 6
μ	0.0032	0.0032	0.0033	0.0001	0.0000	0.0000	0.0006	0.0006	0.0007	0.0000	0.0000	0.0000
	(3.91)	(3.74)	(3.99)	(3.65)	(3.22)	(3.18)	(3.97)	(3.91)	(3.73)	(2.91)	(2.73)	(2.68)
ϕ_1	0.0019	0.0018	0.0019	0.015	0.0147	0.0147	0.0031	0.0032	0.0031	0.0015	0.0015	0.0016
	(29.36)	(29.31)	(29.34)	(25.90)	(25.98)	(25.90)	(3.85)	(3.87)	(3.88)	(2.04)	(2.05)	(2.05)
ϕ_2	0.0011	0.001	0.001	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003	0.0006	0.0005	0.006
	(13.08)	(9.81)	(9.87)	(8.79)	(6.82)	(6.87)	(1.94)	(1.77)	(1.79)	(1.39)	(1.89)	(1.89)
ϕ_3		0.0013	0.0016		0.0004	0.0008		0.0011	0.0008		0.0014	0.0006
		(10.95)	(10.55)		(0.73)	(1.27)		(2.53)	(1.57)		(2.80)	(1.77)
ϕ_4			0.0005			0.001			0.0004			0.0011
			(3.01)			(1.17)			(1.71)			(1.80)
ρ_1	-0.0181	-0.0172	-0.0185	-0.0138	-0.0138	-0.0138	-0.0019	-0.0018	-0.0018	-0.0006	-0.006	-0.0006
	(-28.58)	(-25.42)	(-28.52)	(-24.24)	(-24.24)	(-24.24)	(-2.31)	(-2.29)	(-2.31)	(-1.39)	(-1.40)	(-1.40)
ρ_2	-0.0084	-0.0092	-0.0091	-0.0027	-0.0033	-0.0032	-0.0004	-0.0002	-0.0002	-0.0003	-0.0004	-0.0004
	(-11.25)	(-9.69)	(-8.66)	(-5.97)	(-6.06)	(-6.19)	(-1.84)	(-1.30)	(-1.34)	(-1.54)	(-1.62)	(-1.62)
ρ_3		-0.001	-0.0015		-0.0014	-0.0009		-0.0005	-0.0004		-0.0003	-0.0004
		(-7.24)	(-9.53)		(-1.93)	(-1.13)		(-1.72)	(-1.54)		(-1.29)	(-1.39)
ρ_4			-0.0011			-0.0001			-0.0001			-0.0004
			(-6.23)			(-0.92)			(-1.12)			(-1.28)
$\sum(\rho_j)$	0.0030	0.0041	0.005	0.0154	0.0154	0.0168	0.0034	0.0046	0.0046	0.0021	0.0034	0.0093
$\sum(\phi_j)$	-0.0265	-0.0274	-0.0302	-0.0165	-0.0185	-0.0180	-0.0023	-0.0025	-0.0025	-0.0009	-0.0067	-0.0018
$\sum(\phi_j + \rho_j)$	-0.0235	-0.0233	-0.0252	-0.0011	-0.0031	-0.0012	0.0011	0.0021	0.0021	0.0012	-0.0033	0.0075

Note: Values in parentheses are the asymptotic t-statistics. Minutely, hourly, daily and weekly analyses are based on estimation of the methodologies: Model 4: $r_t = \mu + (\phi_1 + \rho_1 D_2)r_{t-1} + (\phi_2 + \rho_2 D_2)r_{t-2} + \varepsilon_t$; Model 5: $r_t = \mu + (\phi_1 + \rho_1 D_3)r_{t-1} + (\phi_2 + \rho_2 D_3)r_{t-2} + (\phi_3 + \rho_3 D_3)r_{t-3} + \varepsilon_t$; and Model 6: $r_t = \mu + (\phi_1 + \rho_1 D_4)r_{t-1} + (\phi_2 + \rho_2 D_4)r_{t-2} + (\phi_3 + \rho_3 D_4)r_{t-3} + (\phi_4 + \rho_4 D_4)r_{t-4} + \varepsilon_t$ respectively.

Table 8: Parameter estimates of the persistence EGARCH methodology after Bitcoin breached $1,000

Total	Minutely			Hourly			Daily			Weekly		
	Model 4	Model 5	Model 6	Model 4	Model 5	Model 6	Model 4	Model 5	Model 6	Model 4	Model 5	Model 6
μ	0.003	0.0037	0.0035	0.0012	0.0012	0.0012	0.0002	0.0002	0.0002	0.0000	0.0000	0.0000
	(3.75)	(3.11)	(3.83)	(3.96)	(3.96)	(3.97)	(9.24)	(9.31)	(9.34)	(3.17)	(3.12)	(3.09)
ϕ_1	0.0075	0.0074	0.0075	0.0247	0.0248	0.0247	0.0051	0.0051	0.0051	0.0050	0.0050	0.0050
	(6.06)	(6.02)	(6.09)	(5.62)	(5.74)	(5.68)	(14.45)	(14.56)	(14.54)	(2.77)	(2.77)	(2.77)
ϕ_2	0.0041	0.0038	0.0038	0.0093	0.0028	0.0028	0.0004	0.0006	0.0006	0.0004	0.0003	0.0003
	(8.78)	(6.33)	(4.90)	(2.19)	(2.54)	(2.55)	(2.25)	(2.53)	(2.51)	(2.10)	(1.57)	(1.56)
ϕ_3		0.0043	0.0024		0.0013	0.0016		0.0003	0.0003		0.0001	0.0001
		(1.79)	(2.30)		(2.16)	(2.26)		(1.87)	(1.56)		(2.24)	(2.02)
ϕ_4			0.0004			0.0006			0.0001			0.0002
			(3.52)			(0.84)			(1.25)			(2.31)
ρ_1	-0.0072	-0.0071	-0.0071	-0.0013	-0.0024	-0.0019	-0.0019	-0.0018	-0.0018	-0.0016	-0.0016	-0.0017
	(-5.81)	(-5.83)	(-5.82)	(-6.54)	(-4.52)	(-5.12)	(-3.41)	(-3.32)	(-3.18)	(-4.22)	(-4.15)	(-4.11)
ρ_2	-0.0012	-0.0048	-0.0049	-0.0019	-0.0012	-0.0011	-0.0008	-0.0002	-0.0003	-0.0003	-0.0003	-0.0002
	(-24.27)	(-7.91)	(-7.91)	(-4.08)	(-2.16)	(-2.16)	(-2.23)	(-1.63)	(-1.74)	(-1.67)	(-1.55)	(-1.55)
ρ_3		-0.0012	-0.0006		-0.0018	-0.0004		-0.0017	-0.0021		-0.0021	-0.0022
		(-18.07)	(-7.20)		(-2.49)	(-0.49)		(-2.64)	(-2.91)		(-2.22)	(-2.20)
ρ_4			-0.001			-0.0037			-0.0014			-0.0010
			(-11.26)			(-3.21)			(2.24)			(-2.74)
$\sum(\rho_j)$	0.0116	0.0155	0.0141	0.0340	0.0289	0.0297	0.0055	0.0060	0.0061	0.0054	0.0054	0.0056
$\sum(\phi_j)$	-0.0084	-0.0131	-0.0136	-0.0032	-0.0054	-0.0071	-0.0027	-0.0037	-0.0056	-0.0019	-0.004	-0.0051
$\sum(\phi_j + \rho_j)$	0.0032	0.0024	0.0005	0.0308	0.0235	0.0226	0.0028	0.0023	0.0005	0.0035	0.0014	0.0005

Note: Values in parentheses are the asymptotic t-statistics. Minutely, hourly, daily and weekly analyses are based on estimation of the methodologies: Model 4: $r_t = \mu + (\phi_1 + \rho_1 D_2) r_{t-1} + (\phi_2 + \rho_2 D_2) r_{t-2} + \varepsilon_t$; Model 5: $r_t = \mu + (\phi_1 + \rho_1 D_3) r_{t-1} + (\phi_2 + \rho_2 D_3) r_{t-2} + (\phi_3 + \rho_3 D_3) r_{t-3} + \varepsilon_t$; and Model 6: $r_t = \mu + (\phi_1 + \rho_1 D_4) r_{t-1} + (\phi_2 + \rho_2 D_4) r_{t-2} + (\phi_3 + \rho_3 D_4) r_{t-3} + (\phi_4 + \rho_4 D_4) r_{t-4} + \varepsilon_t$ respectively.