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TEACHER CHANGE IN POST-16 MATHEMATICS: A MULTIPLE CASE ANALYSIS OF TEACHERS IN THE ZONE OF ENACTMENT

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This research focuses on the use of teaching resources that support an ambitious student-centred approach to learning mathematics. A multiple case study involving three teachers of post-16 mathematics is presented to develop new insights into the process of teacher change. Data sources include classroom observations in addition to teacher and student interviews. Using the concept of Zone of Enactment (ZoE) and teacher self-efficacy as a theoretical framework this research reveals the importance of teachers being aware of, and able to reflect on, the ZoE and their self-efficacy.

INTRODUCTION

This paper explores the implementation of an ambitious approach to teaching mathematics in the context of post-16 mathematics (A-level) in England. *Ambitious teaching* is conceptualized here as providing rich learning environments in which students are required to use their powers of reasoning and problem-solving; there is a strong component of dialogue and the teacher is responsive and adaptive to the learning process (Stylianides & Stylianides, 2014). Research involving three teachers is presented to examine how practitioners respond to, and enact, an approach designed to encourage ambitious teaching. Using the concept of Zone of Enactment (ZoE) as a theoretical framework particular attention is paid to the potential for teacher change.

This research is motivated by the need to investigate the use of resources (tasks and teacher support materials) developed by the Cambridge Mathematics Education Project (CMEP)[1]. Since 2012, CMEP has been developing resources to support and inspire more ambitious teaching of A-level mathematics. Through an investigation of classroom practices, this study aims to improve understanding of teachers' experiences using CMEP resources and make a broader contribution to knowledge on classroom instruction.

BACKGROUND

The General Certificate of Education Advanced Level (A-level) is an established two-year course taken predominantly by 16–19-year-olds in England, Wales and Northern Ireland. Funded by the UK Department for Education for an initial period of 3.5 years, CMEP aims to enhance A-level maths through the development of resources that help make it a rich, coherent and stimulating experience for students and teachers. This research forms part of a research and development sub-project of CMEP, involving the iterative analysis and refinement of support materials to promote the faithful implementation of tasks developed by CMEP (see Major, Watson, & Kimber, 2015). By developing a deeper understanding of teachers' experience using CMEP resources, we aim to develop ways to encourage teachers to embed CMEP pedagogy in their practice.

[1] Note that CMEP has been renamed Underground Mathematics. The resources and teacher notes are now available to all at www.undergroundmathematics.org

Theoretical framework

We use Spillane's notion of a *Zone of Enactment* (ZoE) as a framework for teacher change (Spillane, 1999). This acknowledges the existence of patterns of practice in mathematics teaching, where teachers adopt and individualise historically- and culturally-developed classroom practices. ZoE is an adaptation of Vygotsky's *Zone of Proximal Development* (ZPD; Vygotsky, 1978) which describes the progress that can be made by an individual acting on their own, compared with the progress that can be made by the individual working with a knowledgeable other. In terms of teaching reform, ZoE means the extent to which a teacher will engage with (and commit to) a change in practice, given that there is support to do this and the extent of the change is not so large that it represents too radical a departure from existing practice.

ZoE, conceptually, provides an overarching model of change and suggests mechanisms and processes within a notion of change. To fully appreciate the significance of ZoE we need a theory of process. For this research, we adopt a social cognitive approach for the purpose of understanding teacher change and barriers to change (Bandura, 1986). Social Cognitive Theory (SCT), like Vygotsky's ZPD, acknowledges the social and cognitive dimensions of learning (specifically that individuals learn as part of participation and interaction as well as through the construction of meaning and knowledge). A key component of SCT is self-efficacy, which is the belief an individual has in their ability to be successful in a domain (Bandura, 1997).

Before elaborating on the role of self-efficacy in relation to the ZoE, we explain what we mean by change in relation to practice. We use the polarized characterizations of teacher-centred and student-centred teaching, although we acknowledge that dichotomizing practice as either teacher- or student-centred is not without its limitations. In our terminology, these strong, almost oppositional, characterizations reflect very subtle shifts in mathematical authority.

From Cuban's (1993) descriptors: teacher-centred teaching typically features teacher exposition, explanation or demonstration, followed by student practice using routine exercises or problems and followed by review and (summative) assessment. Student-centred teaching involves students working collaboratively on open-ended, non-routine or unfamiliar tasks. Secondary school mathematics teaching in England is considered to be predominantly teacher-centred (Ofsted, 2008; 2012), including in post-16 contexts (Pampaka et al, 2012).

We use the idea of ambitious teaching to reflect a range of ideas and concepts in relation to reform-oriented student-centred teaching. Povey and Burton (1999) describe this in terms of mathematical authority or authorship, in other words, who is the "author" of the mathematics? Is this the teacher (in the case of teacher-centred teaching) or the students (in the case of student-centred teaching)? Cuban's (2009) recent analysis in the US identified a type of practice that he describes as teacher-centred progressivism. This is where teaching is predominantly and prevalently teacher-centred, but features some characteristics of student-centred practice (such as groupwork). The pedagogical approach remains teacher-centred, however, as authority and authorship is with the teacher. In this paper, we use the following terms: (i) teacher-centred (progressivism) to represent the current state of things, and, (ii) ambitious teaching (student-centred) as a reform model.

Now, let us consider self-efficacy. Tschannen-Moran and Woolfolk Hoy (2001) define self-efficacy, in the context of teaching, as the belief a teacher has in their capability to support learners

in their class to make progress. Teaching self-efficacy is developed through successful experiences and overcoming challenges, and partially reflects underlying knowledge and skills as well as a response to social norms. Bandura (1997) explains how individuals develop skills in a particular area until they are almost routinized, before which they are able to operate within their self-efficacy. It has been shown that more efficacious teachers tend to innovate and deviate from social norms, while lower self-efficacy teachers tend to follow norms i.e. prevalent teacher-centred practices (Nie et al., 2012). In order for reforms to be successfully implemented and sustained, teachers need to be self-efficacious in the reform approach. According to Bandura, the main source of self-efficacy is enactive mastery experience (Bandura, 1997). Within the context of our study, this would involve having success in implementing ambitious pedagogical strategies and being able to attribute success to those strategies. Professional learning or 'change' must, therefore, involve some degree of enactive mastery experience (Tschannen-Moran & McMaster, 2009).

We analyse teachers' attempts to change their approach, from traditional or teacher-centred practice to the ambitious approach suggested by the CMEP materials. This we characterise as within their ZoE; there is an invitation to try out new approaches but with guidance using support materials (which have been designed to make crossing the ZoE achievable and not too distant from existing practice). The knowledgeable 'other' acts indirectly through teacher support materials that were designed by an 'expert'. While this expert is not present in person during the lesson, when the teachers use the materials we assume that the knowledgeable other acts indirectly through the materials. This assumption is supported by Bandura's (1997) conception of observational learning and development of self-efficacy vicariously, where behaviour need not necessarily be observed directly but represented in text-based guidance materials.

By undertaking an exploratory case study, we can better understand what is happening in the ZoE in order to develop and enhance the support and guidance that accompanies CMEP tasks. We reiterate here, that when we refer to teacher-centred teaching this is more than simply chalk, talk and textbook exercises. While such lessons might include groupwork and investigations, the feature that makes them teacher-centred is that the teacher attempts to reduce the cognitive demand in the lesson so that students can progress easily through the tasks. In contrast, the aim of the CMEP tasks is to offer higher levels of demand so that students have to think deeply about the mathematics and, as such, we characterise this as student-centred given that the mathematical authority is transferred to the students (Watson, Major & Kimber, in preparation).

METHODOLOGY

A multiple case study was undertaken to ask:

How do different teachers experience using materials to promote ambitious teaching of A-level mathematics? What happens in the Zone of Enactment (ZoE)?

A case study was considered suitable as this facilitates systematic research that investigates real-world occurrences in their real-world settings (Robson, 2011). Three teachers were involved - *Sophie*, *Bek* and *Tom*. All have at least five years' experience of teaching A-level mathematics. This research complies with the British Educational Research Association's ethical guidelines.

'Two-way functions' and associated teacher support materials

The same CMEP task, 'Two-way functions', was used by each teacher. This task is designed to

develop ideas about the properties of functions and to expand the set of functions students can work with. Students can use a combination of algebraic and graphical representations to attempt the task. Teacher support materials[2] place the main CMEP task in the context of a sequence of preliminary, main and follow-up tasks designed to be used during three separate (but not necessarily successive) lessons. Guidance includes a suggested way of using the task, descriptions of mathematical behaviour to look out for and questions to prompt student reflection. Details of overarching mathematical ideas, connections that students might make, common issues and misconceptions are given. Teacher prompts/questions are also suggested.

Data collection and analysis

Multiple sources of data are used: *(1)* semi-structured teacher interviews (5-20 minutes) before and following lessons; *(2)* focus groups (4-6 students, approx 15 minutes) to establish students' opinions; *(3)* classroom observations (stationary and tracking cameras; audio from teacher lapel mics and recorders on students' desks); *(4)* discussions with teachers videoed during later CMEP workshops. The analytical process centred on observed lessons and involved the project team reviewing the video of the lesson together with associated transcripts. The formation of a case description for lessons, using a theoretical proposition, was iteratively tested. Data triangulation was then effected by developing and elaborating explanations from interview transcripts.

FINDINGS

Sophie's response characterized her ZoE experience in terms of the difference between teaching pre- and post-16 classes, describing how CMEP lessons felt more like those she would teach at Key Stage 3 (students aged 11-14, when there is generally greater opportunity for investigative work and open-ended problem-solving). Of the three teachers, the way in which Sophie presented the lesson was most consistent with the approach suggested in the CMEP support materials. This is evident by Sophie using many of the questions or prompts suggested. As a result, Sophie is considered to have used the support materials to assist her in crossing the ZoE. Students corroborated that the approach in the CMEP lesson was different from usual, commenting, *"...we usually just work through with Miss, get the understanding and then go on to questions at the end so it was different"*. The design of the task was also noted as helping students to resolve the complexity in the task for themselves:

> "... going along the rows and seeing that the numerator was telling them the roots, whereas the denominators were telling them the asymptotes… usually they would have a whole series of questions to do individually..."

Sophie was the most self-efficacious of the three teachers. This can be assumed because she was the most willing to implement innovative approaches to teaching. As we observed in the lesson, and as supported by comments expressed following the lesson (see below), Sophie demonstrated a degree of mastery with the approach suggested by the CMEP support materials. In the following extract from the post-lesson interview Sophie demonstrates her confidence in regards to how she implemented the CMEP activity. This is further demonstrated by Sophie's reaction to her contingent behaviour during the lesson and, in particular, how she responded to unexpected student behaviour. This contingent behaviour supports our conclusion about Sophie's high level of self-efficacy in the CMEP approach:

[2] www.undergroundmathematics.org/thinking-about-functions/two-way-functions-teacher-notes

> "I really enjoyed it. I thought the students were having really good discussions and the task had a really good structure so they could get started straight away and knew what the goal was. Actually having said that they couldn't necessarily write down answers straightaway but that was good because they all had to start by discussing which was good."

Sophie exhibits two characteristics in response to using the CMEP task. The first is the recognition of her ZoE, i.e. that there is a difference between her usual approach and that suggested by the CMEP materials. The second is Sophie's level of self-efficacy in the suggested approach and the confidence with which she used the tasks with her class. As outlined below, the other two case study teachers demonstrated different characteristics.

Bek had been involved in several CMEP teacher workshops. She described her usual approach to teaching in the following way:

> "I talk for a bit and then give an activity and so I try and give something which isn't just chalk and talk and text book exercises every lesson but which is maybe a short part of the lesson rather than a whole lesson. And I don't tend to do that much group work where I divide them up into groups like that. I do do some, but it's generally a more informal grouping as well, or it will just be for a short portion of the lesson. One of the things that I'll do for question practice is getting the white boards and then getting them up and working in pairs and then swapping them round."

From this we see how Bek recognises that elements of her practice are teacher-centred and that this pedagogy is central to student learning.

Bek is a confident and experienced teacher. Therefore it was assumed beforehand that she has a high level of self-efficacy. Previous research tells us that self-efficacious teachers are more likely to innovate, but instead Bek absorbed the materials into her existing practice. This is not to say that the observed lesson had teacher-centred and traditional structures. Rather, that some of her interventions characterized the lesson as teacher-centred. Bek was prepared to let students work on the task for an extended period in the lesson for almost 30 minutes. However, Bek's interventions were motivated by a focus on completing the task and a need to make the task less challenging. For example, when she approached a group of students who said that they were struggling, Bek responded with examples and explanations rather than exploring how the students were thinking about the task. In the post-lesson interview she expressed concern about the level of challenge:

> "...saying that the x axis is an asymptote and finding something that passes through the origin, is quite a big ask for a group like that… Quite a few times I was going 'shall we just change that bit?'"

Bek acknowledged the CMEP approach was different to her usual approach to teaching while at the same time stating that she was "fine" using it. Bek said she did not use the support materials beyond choosing a preliminary task, instead adapting the ideas into her usual approach. When some students completed the task in a way that Bek judged to be superficial, she asked the groups to swap their work to compare approaches and prompt a more sophisticated response. Bek attributed this technique to prior CMEP workshops, although it is also suggested in the support materials. In a later interview, Bek acknowledged she hadn't read the teacher support materials in detail:

> "I wish I'd read this document [teacher support materials] slightly more carefully before I did it. Things like, allow individual thinking time at the start of the activity, I didn't do that... "

When questioned if there were any challenges in using the resources, Bek commented:

> "Not really. I think the main thing would be not really realising how long the activities are going to last for a class and so ways of dealing with that, either with ways of getting them to engage more deeply, or other things to look at which I think I sort of did"

In summary, Bek appears self-efficacious but her actions did not suggest that she recognized a ZoE. Furthermore, Bek does not recognise the potential ways in which more profound levels of learning can take place with the CMEP materials. In not recognizing the learning opportunities provided by CMEP tasks, Bek does not perceive a change in practice or a ZoE: she sees the materials as supplementary to teaching the course content.

Tom used the support materials and followed the guidance much more closely than Bek. While concerned about the attainment and confidence of his class, Tom was surprised about the level of mathematical discussion that took place during the preliminary task:

> "… there is a sizable minority in there who will have just failed their AS-level [the first component of a full A-level] and yet, the level of discussion they were able to keep going with that work with each other, and the outcomes [of the discussion] that I could see on the whiteboards was quite impressive"

Despite being pleased with the resilience shown by some students, Tom noted that many used tables of values to help them sketch graphs rather than thinking about behaviour of functions. Additionally, despite being impressed with how the students had tackled the preliminary task, half-way through the main task lesson Tom commented to the observer, *"...it's like this group [points to group of students] has taken… six months step backwards"*. Tom also described the task as being *"a little more scary"* and wanting to *"get them used to that".*

Tom did recognise a ZoE in relation to the CMEP materials. That there was a difference between his usual approach to teaching and the approach suggested in the support materials. It was evident, from observation of the lesson and the post-lesson interview, that Tom recognized the challenge in terms of a ZoE. The following example illustrates his affective responses:

> "…it wasn't long into that individual time and that silence that I started to feel really uncomfortable and I realise that perhaps I need to do this more often because it was apparent from my unease that they are always working together and so there isn't that time for them just to work individually on whatever it is they're doing whatever the nature of the task."

Like Bek, Tom had been involved in several CMEP workshops so was aware of the principles and philosophy underpinning the CMEP approach. He described the aims and what learning might look like when using the CMEP materials. Yet he expressed some reservations about implementing the approach suggested in the support materials with students who had limited mathematical confidence. Overall, Tom recognised the ZoE in relation to implementing the CMEP resource but his level of self-efficacy, as a result of the challenges he believed he faced with the group, was probably undermined.

DISCUSSION

Sophie showed an awareness of the existence of the ZoE, and was able to reflect upon it. It is this awareness that allowed Sophie to bridge, to a certain degree, the difference between her existing A-level teaching approach and that proposed by CMEP. This demonstrates a high-level of self-efficacy. It is highly likely that Sophie believed that students would make progress when she introduced the CMEP tasks. It would be valuable to explore why this was the case. Bek, in contrast,

showed little or no awareness of a ZoE as she integrated the CMEP tasks into her existing practice with the lesson being more teacher-centred and thus traditional. Bek was self-efficacious, although this efficacy was in relation to more orthodox teaching methodology. The fact that she did not fully engage with the support material supports the view. It is interesting that Bek saw CMEP-style activities as extension or enrichment work, not as a fundamental to learning in-depth and in making connections. The case of Bek suggests that self-efficacy is not enough in recognising the existence of a ZoE. Indeed, it seems that it may be necessary for teachers to recognise the nature of learning in the context of ambitious teaching approaches to recognise the ZoE.

Tom was aware of a ZoE and recognized the differences in pedagogy and teaching approach required. He found the experience of using CMEP tasks, however, challenging. Tom was concerned with both the challenges of formative and summative assessment and the effectiveness of student collaborative work. This may have been related to the group of students Tom worked with, since he embraced the way the support materials suggested managing group work. What undermined Tom's attempt to cross the ZoE was his level of self-efficacy in the approach, as this was related to the fact that the group he was teaching was not necessarily highly-confident/high-attaining.

Of course, we acknowledge this research has limitations. It features one task implemented by each teacher and it is a small-scale study. We also accept that teachers adopt similar prevalent teacher-centred practices that might have some student-centred features. These teacher-centred progressivist practices are, we venture to say, the *standard* approach to teaching A-level mathematics in England. Furthermore, it is not unreasonable to suggest that these practices are prevalent in mathematics teaching more widely. The introduction of alternative, ambitious teaching approaches invites a change in these practices. The CMEP tasks are designed to provide learning activities that help students to engage more deeply in the mathematics and where the mathematical authority and authorship can shift from teacher to student. The support materials are designed to support teachers in effecting this change.

For this change to be effective teachers need to recognise a ZoE in relation to their existing pedagogy and practice in addition to that suggested in the CMEP materials. ZoE means the extent to which a teacher will engage with (and commit to) a change in practice, given that there is support to do this and the extent of the change is not so large that it represents too radical a departure from existing practice. As we have shown, self-efficacy has an important role in engagement and commitment to change. Not only do teachers need to recognise the importance of the new approach, they need to be confident that they will be successful in implementing it. Our case study of Sophie shows that it is possible for teachers to recognise the ZoE and use support materials to implement suggested approaches. On the other hand, further professional development and support might be necessary to help teachers understand the underlying learning principles and to reflect on the learning in their own teaching. Other teachers may need support and encouragement to help develop self-efficacy in using ambitious teaching approaches with more challenging groups. This has implications for the design of professional development.

CONCLUSION

We conclude that the ZoE is an important concept in the analysis of teacher change and propose that teachers' awareness of the ZoE, in the context of their teaching, is a necessary condition for

them to be able to change their approach. If teachers do not recognise the ZoE, then it is likely that they will not use the support available to them. This research also reveals that self-efficacy is an important aspect of this.

Teachers need to have the confidence to bridge the gap between their existing teaching approaches and ambitious teaching. Through our ongoing research into how teachers use the support materials we will continue to pay close attention to the mechanisms, processes and responses at the site of individual teacher change. Using ZoE has provided a useful conceptual framework for theorising teacher change and will help to inform the design and formative evaluation of teacher support materials in the future.

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