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INCREASE-DECREASE GAME UNDER IMPERFECT COMPETITION IN TWO-STAGE ZONAL POWER MARKETS PART I: CONCEPT ANALYSIS

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28 November 2018

This paper is part I of a two-part paper. It proposes a two-stage game to analyze imperfect competition of producers in zonal power markets with a day-ahead and a real-time market. We consider strategic producers in both markets. They need to take both markets into account when deciding what to bid in each market. The demand shocks between these markets are modeled by several scenarios. The two-stage game is formulated as a Twostage Stochastic Equilibrium Problem with Equilibrium Constraints (TS-EPEC). Then it is further reformulated as a two-stage stochastic Mixed-Integer Linear Program (MILP). The solution of this MILP gives the Subgame Perfect Nash Equilibrium (SPNE). To tackle multiple SPNE, we design a procedure which _nds all SPNE with di_erent total dispatch costs. The proposed MILP model is solved using Benders decomposition embedded in the CPLEX solver. The proposed MILP model is demonstrated on the 6-node and the IEEE 30-node example systems.

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Keywords Two-stage game, Zonal pricing, Two-stage equilibrium problem with equilibrium constraints, Wholesale electricity market

JEL Classification C61, C63, C72, D43, L13, L94

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Increase-Decrease Game under Imperfect Competition in Two-stage Zonal Power Markets – Part I: Concept Analysis☆
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Abstract

This paper is part I of a two-part paper. It proposes a two-stage game to analyze imperfect competition of producers in zonal power markets with a day-ahead and a real-time market. We consider strategic producers in both markets. They need to take both markets into account when deciding what to bid in each market. The demand shocks between these markets are modeled by several scenarios. The two-stage game is formulated as a Two-stage Stochastic Equilibrium Problem with Equilibrium Constraints (TS-EPEC). Then it is further reformulated as a two-stage stochastic Mixed-Integer Linear Program (MILP). The solution of this MILP gives the Subgame Perfect Nash Equilibrium (SPNE). To tackle multiple SPNE, we design a procedure which finds all SPNE with different total dispatch costs. The proposed MILP model is solved using Benders decomposition embedded in the CPLEX solver. The proposed MILP model is demonstrated on the 6-node and the IEEE 30-node example systems.

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1. Introduction

Liberalized electricity markets are settled in multiple stages: a day-ahead market, a real-time market and sometimes an intra-day market in-between. It is well-known from the seminal work by [1] that sequential trading can have significant influence on competition. Sequential trading is particularly relevant for Europe, which is divided into zones with uniform day-ahead prices. The European day-ahead market neglects transmission constraints inside zones, while all transmission constraints are considered in real-time [2]. Overloading of intra-zonal transmission lines are relieved by counter-trading in the real-time market. The difference between the representations of the transmission constraints gives different

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prices in the two stages. This creates an arbitrage opportunity for producers. As shown by [3–6], a producer located at an export-constrained node can increase its payoff by increasing its sales in the day-ahead market and then buy back power at a lower price in the real-time market. This strategy is called the increase-decrease (inc-dec) game. The inc-dec game contributed to the electricity crisis in California and to that California and other markets in US switched from zonal to nodal pricing which takes into account all transmission constraints in both day-ahead and real-time markets [6]. The game has also been observed in the British electricity market [7].

In power markets, two-stage games are often modeled by Two-stage Stochastic Equilibrium Problems with Equilibrium Constraints (TS-EPEC). The solution of such a TS-EPEC gives the Subgame Perfect Nash equilibrium (SPNE) - an outcome that is sequentially rational and where no producer can increase its payoff by changing its bid unilaterally. TS-EPECs allow us to model the structure of the two-stage markets where the outcome of one stage affects and is affected by the outcome of another stage. Strategic bidding of producers in a forward market and in a spot market is modeled using a TS-EPEC in [8] and [9]. The model in [8] assumes that the producers competing in the forward market are price-takers. This assumption ignores that the producers can influence the price in the forward market through their bids. The strategic bidding of producers in the forward market is considered in [9] but the network constraints are ignored. A recent study in [10] analyzes the impact of transparency in the forward market on the strategic behavior of producers in both forward and spot markets. The studies [8–10] analyze the two-stage power markets without network constraints. Authors in [11] investigate optimality conditions of the TS-EPEC model under the condition that the demand in the spot market has a finite distribution. The sufficient conditions for the existence of SPNE under no uncertainty is analyzed in [12]. The studies [11, 12] analyze the two-stage power markets under nodal pricing.

Various researchers have analyzed competition in zonal markets. Authors in [13] and [14] approximate the two stages of the game by one stage. The study in [15] considers a non-market based redispatch without the inc-dec game. Authors in [4] consider a competitive market and analyze the imperfections caused by arbitrage opportunities. The study in [5] considers both imperfect competition and arbitrage opportunities but their analysis is limited to two-node networks. In previous paper [16], we evaluate designs of a zonal power market with imperfect competition. The two-stage game is formulated as a TS-EPEC and then the TS-EPEC is reformulated as a Mixed-Integer Bilinear Program (MIBLP). Unfortunately, the MIBLP model is computationally burdensome and hard to solve.

The contributions of this paper are as follows: (a) it proposes a zonal real-time market which is reminiscent of the real-time markets in the Nordic countries. The proposed zonal market is formulated by including additional constraints to the primal minus dual formulation of a nodal real-time market model. (b) Based on the proposed zonal market, this paper proposes a two-stage game to analyze imperfect competition and the inc-dec game in the two-stage zonal markets. The two-stage game is formulated as a TS-EPEC. Then the TS-EPEC is reformulated as a MILP model. (c) The MILP model may result in multiple SPNE. To tackle this situation, we develop an iterative procedure that finds all SPNE that have different total dispatch costs. The proposed MILP model and the iterative procedure are demonstrated on the 6-node and the modified IEEE 30-node example systems. The impacts of the inc-dec game are carefully analyzed.

The rest of the paper is organized as follows. The symbols used in the mathematical models are presented in Section 2. Section 3 derives the MILP model of the two-stage game in the zonal power market and in the nodal power market. We use the nodal two-stage power market as our benchmark model. The method to tackle multiple SPNE is explained in section 4. An illustrative example and a case study are presented in Sections 5 and 6, respectively. Section 7 concludes the paper.

2. Nomenclature

The main symbols are presented below. Additional symbols are introduced throughout the text.

Indices

u	Producer, $u = 1, ..., U$
n	Power system node, $n = 1, ..., N$
z	Zone, $z = 1, ..., Z$
k	Transmission line, $k = 1, ..., K$
l	Inter-zonal line, $l = 1, ..., L$
s	Net demand deviation scenario, $s = 1, ..., S$
a	Bidding action of producer, $a = 0, ..., A$
i,(j)	Bidding strategy for real-time (day-ahead) market, $i = 1, ..., I$ ($j = 1, ..., J$)
r	Day-ahead bid combination, $r = 1, ..., A^U$

Parameters (upper-case letters)

$H_{k,n}$	Nodal PTDF matrix,
$H'_{l,z}$	Zonal PTDF matrix,
C_u	Marginal cost of producer u,
C_u^{up}	Marginal up-regulation cost of producer u,
C_u^{dn}	Marginal down-regulation cost of producer u,
G_u	Capacity of producer u,
F_k	Capacity of transmission line k,
$\bar{F}_l$	Capacity of inter-zonal line l,
D_n	Net demand at node n,
$\bar{W}_{n,s}$	Wind production at node n,
$\Delta W_{n,s}$	Deviation in net-demand at node n,
σ_s	Probability of scenario s,
$B_{u,a}$	Step size of day-ahead bid action a,
$\hat{B}_{u,a}$,($\tilde{B}_{u,a}$)	Step size of up-regulation (down-regulation) bid action a,

Variables (lower-case letters)

$\hat{c}_u$	Price bid of producer u,
$\hat{c}_u^{up}, (\hat{c}_u^{dn})$	Up-regulation (down-regulation) price bid of producer u,
$x_{u,a}$	Binary variable for day-ahead bidding decision of producer u in action a,
$x_{u,a}^{up}, (x_{u,a}^{dn})$	Binary variable for up-regulation (down-regulation) bidding decision of producer u,
g_u	Day-ahead dispatch level of producer u,
$g_{u,s}^{up}, (g_{u,s}^{dn})$	Up (down) regulation provided by producer u in scenario s,
$v_{n,s}$	Wind spillage at node n in scenario s,

$\rho_{n,s}, (\omega_z)$	Real-time (day-ahead) market price at node n (in zone z),
$\phi_{u,s}$	Real-time profit of producer u in scenario s,
π_u	Day-ahead profit of producer u.

3. Mathematical Model

We consider a two-stage market where the first stage is the zonal day-ahead market and the second stage is the zonal real-time market. We assume that oligopoly producers participate in both markets. Competition in the two-stage electricity market is modeled as a two-stage game under uncertainty. The structure of the two-stage game is illustrated in Fig. 1. In the first stage, each producer decides its day-ahead bid by taking into account the Nash equilibrium (NE) in the zonal real-time market and its competitors' predicted day-ahead bids. The market operator forecasts the net-demand and clears the zonal day-ahead market. In the second stage, each producer decides its regulation bid by taking into account the given day-ahead dispatch results and expectations for its competitors' regulation bids. The market operator collects the regulation bids from all producers and it clears the zonal real-time market with the actual net-demand. We want to find the SPNE in this two-stage game. The most straightforward way of solving for a SPNE is to use backward induction, i.e. to solve the game backwards. Thus, we start with the last stage, the zonal real-time market.

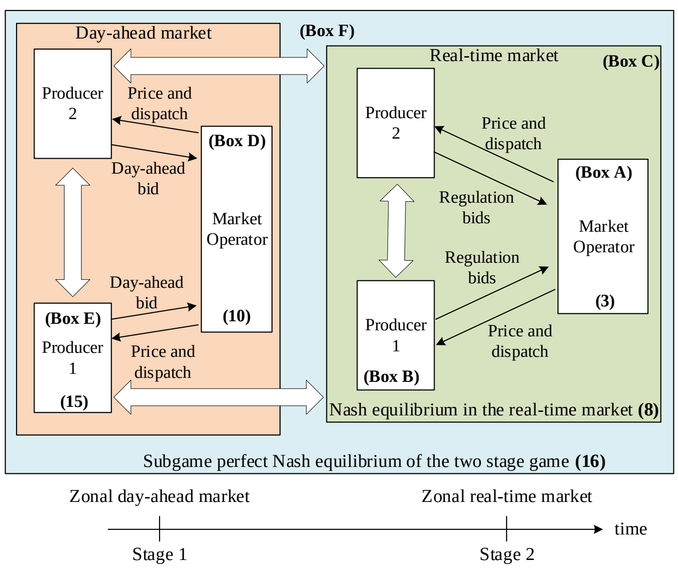

Figure 1: Roadmap of the mathematical derivations in this paper, the numbers in parenthesis refer to the mathematical models of each box, $\Longleftrightarrow$: Strategic interactions, $\rightarrow$: Information exchange between the market operator and producers

3.1. Nash equilibrium in the zonal real-time market

The Nash equilibrium between producers is reached when no producer wants to deviate unilaterally from the chosen bidding strategy. This is formulated in (1).

$$\mathbb{E}_s[\phi_{u,s}] \geq \mathbb{E}_s[\phi_{u,s}^{(i)}] \quad \forall u, i \tag{1}$$

Constraint (1) ensures that for each producer the expected real-time profit in the chosen strategy $\mathbb{E}_s[\phi_{u,s}]$ is always greater than or equal to the expected real-time profit in all alternative strategies $\mathbb{E}_s[\phi_{u,s}^{(i)}]$ while holding its competitors' strategies fixed.

In the nodal real-time market, the market operator runs a bid-based, security-constrained economic dispatch in order to dispatch the regulation bids. This is modeled in (2).

$$\underset{g_{u,s}^{up}, g_{u,s}^{dn}, v_{n,s}}{\text{Minimize}} \sum_{s,u} \sigma_s (\hat{c}_u^{up} g_{u,s}^{up} - \hat{c}_u^{dn} g_{u,s}^{dn}) \tag{2a}$$

Subject to:

$$\sum_u (g_u + g_{u,s}^{up} - g_{u,s}^{dn}) = \sum_n (v_{n,s} + D_n - \Delta W_{n,s}) : (\alpha_s) \ \forall s \tag{2b}$$

$$F_k - \sum_n H_{k,n} (\sum_{n:u} (g_u + g_{u,s}^{up} - g_{u,s}^{dn}) - v_{n,s} - D_n + \Delta W_{n,s}) \geq 0 : (\mu_{k,s}), \ \forall k, s \tag{2c}$$

$$0 \leq g_{u,s}^{up} \leq G_u - g_u : (\kappa_{u,s}, \beta_{u,s}) \ \forall u, s \tag{2d}$$

$$0 \leq g_{u,s}^{dn} \leq g_u : (\psi_{u,s}, \varphi_{u,s}) \ \forall u, s \tag{2e}$$

$$0 \leq v_{n,s} \leq \bar{W}_n + \Delta W_{n,s} : (\theta_{n,s}, \chi_{n,s}) \tag{2f}$$

Problem (2) is a Linear Program (LP). Objective function (2a) minimizes the total regulation cost while satisfying energy balance constraint (2b) and transmission constraint (both upper and lower bounds of a transmission line), regulation and wind spillage limits (2c),(2d)-(2e) and (2f), respectively. g_u is the given dispatch level in the day-ahead market. The Lagrange multipliers of each constraint are given in parenthesis.

In this study, we consider a zonal real-time market where the producers are paid a marginal zonal price, if the system imbalance and the direction of its accepted regulation bid is in the same direction. Otherwise they are paid with its bid price[1]. The proposed zonal real-time market is illustrated in Fig. 2.

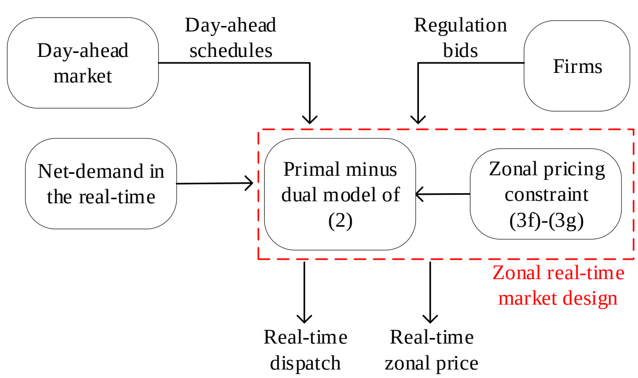

Figure 2: The proposed zonal real-time market considered in this study

[1] This zonal real-time market is similar to the real-time markets in the Nordic countries, where most bids are paid a zonal price, but bids that are accepted in the counter-trading process, which relaxes overloaded lines, are paid as bid.

Our proposed zonal real-time market is based on duality theory and the real-time zonal prices and quantities are calculated in one stage. To formulate it, we use the primal minus dual model [17]. We write the primal minus dual model of (2) and add constraint (3f) which gives a uniform price inside a zone. Moreover, we add constraint (3g) to ensure that all producers make nonnegative profit in the zonal real-time market. Our proposed zonal real-time market is represented by (**Box A**) in Fig 1 and its LP model is set out in (3).

$$\underset{\Pi}{\text{Maximize}} \sum_{s,u} \sigma_s(-\hat{c}_u^{up} g_{u,s}^{up} + \hat{c}_u^{dn} g_{u,s}^{dn}) - \sum_s (\alpha_s (\sum_u g_u - \sum_n (D_n - \Delta W_{n,s})) +$$
$$\sum_k \mu_{k,s}(F_k - \sum_n H_{k,n}(\sum_{u:n} g_u - D_n + \Delta W_{n,s})) + \sum_u (\beta_{u,s}(G_u - g_u) + \varphi_{u,s} g_u) +$$
$$\sum_n \chi_{n,s}(W_n + \Delta W_{n,s})) \tag{3a}$$

Subject to:

$$\text{Constraints } (2b) - (2f): \quad (\lambda_s^A, \lambda_{k,s}^B, \lambda_{u,s}^C, \lambda_{u,s}^D, \lambda_{u,s}^E, \lambda_{u,s}^F, \lambda_{n,s}^G, \lambda_{n,s}^H) \tag{3b}$$

$$-\sigma_s \hat{c}_u^{up} + \alpha_s - \sum_{n:u,k} H_{k,n}\mu_{k,s} + \kappa_{u,s} - \beta_{u,s} = 0: \quad (\lambda_{u,s}^I), \ \forall u, s \tag{3c}$$

$$\sigma_s \hat{c}_u^{dn} - \alpha_s + \sum_{n:u,k} H_{k,n}\mu_{k,s} + \psi_{u,s} - \varphi_{u,s} = 0: \quad (\lambda_{u,s}^J), \ \forall u, s \tag{3d}$$

$$-\alpha_s + \sum_k H_{k,n}\mu_{k,s} + \theta_{n,s} - \chi_{n,s} = 0: \quad (\lambda_{n,s}^K), \ \forall n, s \tag{3e}$$

$$\rho'_{z,s} = (\alpha_s - \sum_k H_{k,n}\mu_{k,s})/\sigma_s: \quad (\lambda_{n,s}^L), \forall n \in z, \forall s \tag{3f}$$

$$\phi_{u,s} \geq 0: (\lambda_{u,s}^M), \forall u, s \tag{3g}$$

$$\mu_{k,s}, \kappa_{u,s}, \beta_{u,s}, \psi_{u,s}, \varphi_{u,s}, \theta_{n,s}, \chi_{n,s} \geq 0: \quad (\lambda_{k,s}^N, \lambda_{u,s}^O, \lambda_{u,s}^P, \lambda_{u,s}^Q, \lambda_{u,s}^R, \lambda_{n,s}^S, \lambda_{n,s}^T) \tag{3h}$$

$$\phi_{u,s} = (\overline{R}_{z,s}\beta_{u,s}(G_u - g_u) + \underline{R}_{z,s}\varphi_{u,s} g_u)/\sigma_s + C_u^{dn} g_{u,s}^{dn} - C_u^{up} g_{u,s}^{up} + \hat{c}_u^{up} g_{u,s}^{up} - \hat{c}_u^{dn} g_{u,s}^{dn}, \quad \forall u, s \tag{3i}$$

The set of decision variables in (3) is $\Pi = \{g_{u,s}^{up},\ g_{u,s}^{dn},\ v_{n,s},\ \alpha_s,\ \mu_{k,s},\ \kappa_{u,s},\ \beta_{u,s}\ ,\psi_{u,s},\ \varphi_{u,s},\theta_{n,s},\ \chi_{n,s},\ \rho'_{z,s},\ \phi_{u,s}\}$. (3c)-(3h) are the constraints of the dual of problem (2). $\rho'_{z,s}$ is the real-time zonal price. The Lagrange multipliers related to each constraint are given in parenthesis. Note that in constraint (3b), Lagrange multipliers given in parenthesis are assigned to the constraints (2b)-(2f) in the context of optimization model (3). The original profit function in this zonal real-time market is formulated in (4).

$$\phi_{u,s} = g_{u,s}^{up}(\sum_{z:u} \overline{R}_{z,s}(\sum_{n:u} \rho_{n,s} - \hat{c}_u^{up}) + \hat{c}_u^{up} - C_u^{up}) + g_{u,s}^{dn}(\sum_{z:u} \underline{R}_{z,s}(\hat{c}_u^{dn} - \sum_{n:u} \rho_{n,s}) + C_u^{dn} - \hat{c}_u^{dn}) \tag{4}$$

Parameters $\overline{R}_{z,s}$ and $\underline{R}_{z,s}$ are the indicators of the imbalance at each zone in each scenario. $\overline{R}_{z,s}$ is set to 1 if there is deficit of generation in zone z in scenario s. Otherwise it is set to 0. $\underline{R}_{z,s}$ is set to 1 if there is excess of generation in zone z in scenario s. Otherwise it is set to 0. The nodal real-time price is calculated as $\rho_{n,s}$=$(\alpha_s - \sum_k H_{k,n}\mu_{k,s})\ /\sigma_s$. From the stationary conditions of (2), (shown in (3c), (3d)) and the complementary slackness conditions for (2d) and (2e), the original profit function in (4) is reformulated as in (3i).

Optimization problem (3) is linear so the Karush-Kuhn-Tucker (KKT) conditions are both necessary and sufficient [18]. The stationary, dual feasibility and strong duality conditions of (3) are derived in (5a)-(5l), (5m) and (5n), respectively.

$$-\sigma_s \hat{c}_u^{up} + \lambda_s^A - \sum_{n:u}\sum_k H_{k,n}\lambda_{k,s}^B + \lambda_{u,s}^C - \lambda_{u,s}^D + \lambda_{u,s}^M \hat{c}_u^{up} - \lambda_{u,s}^M C_u^{up} = 0,\ \forall u, s \tag{5a}$$

$$\sigma_s \hat{c}_u^{dn} - \lambda_s^A + \sum_{n:u}\sum_k H_{k,n}\lambda_{k,s}^B + \lambda_{u,s}^E - \lambda_{u,s}^F + \lambda_{u,s}^M C_u^{dn} - \lambda_{u,s}^M \hat{c}_u^{dn} = 0,\ \forall u, s \tag{5b}$$

$$\lambda_s^A - \sum_k H_{k,n}\lambda_{k,s}^B - \lambda_{n,s}^G + \lambda_{n,s}^H = 0,\ \forall n, s \tag{5c}$$

$$\sum_n (\lambda_{n,s}^L/\sigma_s - \Delta W_{n,s} - \lambda_{n,s}^K) + \sum_u (\lambda_{u,s}^I - \lambda_{u,s}^J) = 0,\ \forall s \tag{5d}$$

$$-F_k + \sum_n H_{k,n}(\Delta W_{n,s} - D_n + \lambda_{n,s}^K + \lambda_{n,s}^L/\sigma_s + \sum_{u:n}(g_u + \lambda_{u,s}^J - \lambda_{u,s}^I)) + \lambda_{k,s}^N = 0,\ \forall k, s \tag{5e}$$

$$\lambda_{u,s}^I + \lambda_{u,s}^O = 0,\ \forall u, s \tag{5f}$$

$$g_u - G_u - \lambda_{u,s}^I + \sum_{z:u}\overline{R}_{z,s}(G_u - g_u)\lambda_{u,s}^M/\sigma_s + \lambda_{u,s}^P = 0,\ \forall u, s \tag{5g}$$

$$\lambda_{u,s}^J + \lambda_{u,s}^Q = 0,\ \forall u, s \tag{5h}$$

$$g_u + \lambda_{u,s}^J - \sum_{z:u}\underline{R}_{z,s} g_u \lambda_{u,s}^M/\sigma_s - \lambda_{u,s}^R = 0,\ \forall u, s \tag{5i}$$

$$\lambda_{n,s}^K + \lambda_{n,s}^S = 0,\ \forall n, s \tag{5j}$$

$$\lambda_{n,s}^T - \lambda_{n,s}^K - (W_n + \Delta W_{n,s}) = 0,\ \forall n, s \tag{5k}$$

$$\sum_{n\in z}\lambda_{n,s}^L = 0 : (\delta_{z,s}^{(32)}),\ \forall z, s \tag{5l}$$

$$\lambda_{k,s}^B, \lambda_{u,s}^C, \lambda_{u,s}^D, \lambda_{u,s}^E, \lambda_{u,s}^F, \lambda_{n,s}^G, \lambda_{n,s}^H, \lambda_{u,s}^M, \lambda_{k,s}^N, \lambda_{u,s}^O, \lambda_{u,s}^P, \lambda_{u,s}^Q, \lambda_{u,s}^R, \lambda_{n,s}^S, \lambda_{n,s}^T \geq 0 \tag{5m}$$

$$\begin{aligned}
&\sigma_s \sum_u (-\hat{c}_u^{up} g_{u,s}^{up} + \hat{c}_u^{dn} g_{u,s}^{dn}) - (\alpha_s(\sum_u g_u - \sum_n (D_n - \Delta W_{n,s})) + \sum_k \mu_{k,s}(F_k - \\
&\sum_n H_{k,n}(\sum_{n:u} g_u - D_n + \Delta W_{n,s})) + \sum_u (\beta_{u,s}(G_u - g_u) + \varphi_{u,s} g_u) + \sum_n \chi_{n,s}(W_n + \Delta W_{n,s})) - \\
&(\lambda_s^A(\sum_u g_u - \sum_n (D_n - \Delta W_{n,s})) + \sum_k \lambda_{k,s}^B(F_k - \sum_n (H_{k,n}(\sum_{n:u} g_u + \Delta W_{n,s} - D_n))) + \\
&\sum_u (\lambda_{u,s}^D(G_u - g_u) + \lambda_{u,s}^F g_u - \lambda_{u,s}^I \hat{c}_u^{up}\sigma_s + \lambda_{u,s}^J \hat{c}_u^{dn}\sigma_s) + \sum_n \lambda_{n,s}^H(W_n + \Delta W_{n,s})) = 0,\ \forall s
\end{aligned} \tag{5n}$$

We approximate the regulation bids by a set of discrete values [19]. We assume that each producer selects its price-bid from an individual set of discrete values. This means that the same price bid cannot be selected by two or more producers and we do not need a rationing rule. The discrete approximation is modeled in (6) using binary variables $x_{u,a}^{up}$ and $x_{u,a}^{dn}$.

$$\hat{c}_u^{up} = \sum_a \hat{B}_{u,a} x_{u,a}^{up} C_u^{up}, \qquad \hat{c}_u^{dn} = \sum_a \tilde{B}_{u,a} x_{u,a}^{dn} C_u^{dn} \tag{6}$$

Using this approximation, the bidding problem of a producer in the zonal real-time market is to maximize the expected real-time profit ($\mathbb{E}_s[\phi_{u,s}]$=$\sum_s \sigma_s \phi_{u,s}$) subject to (3b)-(3i), (5) and (6). This profit maximization problem is represented by (**Box B**) in Fig. 1. It has six sets of bilinear terms after applying discrete approximation (6): **(i)** $x_{u,a}^{up} g_{u,s}^{up}$ in (3i) and (5n), **(ii)** $x_{u,a}^{dn} g_{u,s}^{dn}$ in (3i) and (5n), **(iii)** $\lambda_{u,s}^{I} x_{u,a}^{up}$ in (5n), **(iv)** $\lambda_{u,s}^{J} x_{u,a}^{dn}$ in (5n), **(v)** $\lambda_{u,s}^{M} x_{u,a}^{up}$ in (5a) and **(vi)** $\lambda_{u,s}^{M} x_{u,a}^{dn}$ in (5b). These bilinear terms are in the form of the product of continuous variables and binary variables. These types of bilinearities can be linearized using McCormick reformulation [20]. Using McCormick reformulation, bilinear term $x_{u,a}^{up} g_{u,s}^{up}$ is linearized in (7) where $\vartheta_{u,a,s}$ is a new variable. This technique is applied to other bilinear terms which has the same form.

$$g_{u,s}^{up} + G_u(x_{u,a}^{up} - 1) \le \vartheta_{u,a,s} \le g_{u,s}^{up} \tag{7a}$$

$$0 \le \vartheta_{u,a,s} \le G_u x_{u,a}^{up} \tag{7b}$$

Since each producer chooses its regulation bid from a discrete set, the set of its alternative strategies $\{\hat{c}_u^{up,(i)}$, $\hat{c}_u^{dn,(i)}\}$ can be formed by different combinations of binary variables $x_{u,a}^{up}$ and $x_{u,a}^{dn}$. We can calculate each producer's expected profit in all alternative strategies while holding its competitors' strategies fixed. This enables us to replace the objective function of each producer's bidding problem by (1). Accordingly, the problem of each producer is transformed into a system of Mixed-Integer Linear Constraints (MILC). Similarly, the MILC model of all other producers are formed. Solving all those MILCs together gives us the Nash equilibrium in the zonal real-time market given the day-ahead dispatch decisions. This is represented by (**Box C**) in Fig. 1 and formulated as a feasibility problem in (8).

$$\begin{aligned}
&\text{Find } \Theta = \{x_{u,a}^{up}, x_{u,a}^{dn}, \hat{c}_u^{up}, \hat{c}_u^{dn}, \lambda_s^{A}, \lambda_{k,s}^{B}, \lambda_{u,s}^{C}, \lambda_{u,s}^{D}, \lambda_{u,s}^{E}, \lambda_{u,s}^{F}, \lambda_{n,s}^{G}, \lambda_{n,s}^{H}, \lambda_{u,s}^{I}, \lambda_{u,s}^{J}, \lambda_{n,s}^{K}, \lambda_{n,s}^{L},\\
&\lambda_{u,s}^{M}, \lambda_{k,s}^{N}, \lambda_{u,s}^{O}, \lambda_{u,s}^{P}, \lambda_{u,s}^{Q}, \lambda_{u,s}^{R}, \lambda_{n,s}^{S}, \lambda_{n,s}^{T}\} \cup \{x_{u,a}^{up}, x_{u,a}^{dn}, \hat{c}_u^{up}, \hat{c}_u^{dn}, \lambda_s^{A}, \lambda_{k,s}^{B}, \lambda_{u,s}^{C}, \lambda_{u,s}^{D}, \lambda_{u,s}^{E}, \lambda_{u,s}^{F},\\
&\lambda_{n,s}^{G}, \lambda_{n,s}^{H}, \lambda_{u,s}^{I}, \lambda_{u,s}^{J}, \lambda_{n,s}^{K}, \lambda_{n,s}^{L}, \lambda_{u,s}^{M}, \lambda_{k,s}^{N}, \lambda_{u,s}^{O}, \lambda_{u,s}^{P}, \lambda_{u,s}^{Q}, \lambda_{u,s}^{R}, \lambda_{n,s}^{S}, \lambda_{n,s}^{T}\}^{(i)} \cup \Pi \cup \Pi^{(i)} && \text{(8a)}\\
&\text{Such that}\\
&\text{Nash equilibrium constraint (1)} && \text{(8b)}\\
&\text{Constraints } (3b)-(3i), (5), (6) && \text{(8c)}\\
&\text{Linerization of } \textbf{(i)} - \textbf{(vi)} \text{ as in (7)} && \text{(8d)}\\
&\text{Constraints } (3b)^{(i)}-(3i)^{(i)}, (5)^{(i)}, (6)^{(i)},\ \forall i && \text{(8e)}\\
&\text{Linerization of } \textbf{(i)}^{(i)} - \textbf{(vi)}^{(i)} \text{ as in (7)},\ \forall i && \text{(8f)}
\end{aligned}$$

Here (8c), (8d) and (8e), (8f) are written for the Nash equilibrium strategy and for the alternative strategies, respectively.

3.2. The SPNE of the two-stage game

Each producer bids to the zonal day-ahead market given the day-ahead bids of its competitors and considering the Nash equilibrium in the zonal real-time market (feasibility problem (8)). The SPNE between producers is reached when no producer wants to deviate unilaterally from the chosen bidding strategy. This is formulated in (9).

$$\pi_u + \mathbb{E}_s[\phi_{u,s}] \geq \pi_u^{(j)} + \mathbb{E}_s[\phi_{u,s}^{(i),(j)}] \quad \forall u, i, j \tag{9}$$

Here π_u and $\pi_u^{(j)}$ are the day-ahead profit of producer u in the chosen strategy and in alternative day-ahead strategy (j), respectively. It can be expressed as $\pi_u = (\sum_{z:u} \omega_z - C_u)g_u$. Here ω_z is the day-ahead price in zone z and $\sum_{z:u} \omega_z$ represents the day-ahead price in zone z where producer u is located.

Price and dispatch in the zonal day-ahead market are decided by an economic dispatch problem. It is represented by (**Box D**) in Fig. 1 and formulated in (10).

$$\underset{g_u}{\text{Minimize}} \sum_u \hat{c}_u g_u \tag{10a}$$

Subject to:

$$\sum_u g_u = \sum_n D_n \ : (\xi) \tag{10b}$$

$$\bar{F}_l - \sum_z H'_{l,z}(\sum_{u:z} g_u - \sum_{n:z} D_n) \geq 0 \ : (\gamma_l), \ \forall l \tag{10c}$$

$$0 \leq g_u \leq G_u : \ (\eta_u, \nu_u), \ \forall u \tag{10d}$$

The dispatch cost in the zonal day-ahead market is minimized in (10a) considering the energy balance constraint (10b), the inter-zonal transmission limits (10c) and the generation limits (10d). The stationary, dual feasibility and strong duality conditions of (10) are derived in (11a), (11b) and (11c), respectively.

$$-\hat{c}_u + \xi - \sum_{z:u,l} H'_{l,z}\gamma_l + \eta_u - \nu_u = 0, \ \forall u \tag{11a}$$

$$\gamma_l, \eta_u, \nu_u \geq 0 \tag{11b}$$

$$-\sum_u \hat{c}_u g_u - (\xi \sum_n -D_n + \sum_l \gamma_l(\bar{F}_l - \sum_z H'_{l,z}(\sum_{n:z} -D_n)) + \sum_u \nu_u G_u) = 0 \tag{11c}$$

Similar to the zonal real-time market, we approximate the day-ahead bids by a set of discrete values [19]. This is modeled by binary variables $x_{u,a}$ in (12).

$$\hat{c}_u = \sum_a B_{u,a} x_{u,a} C_u \tag{12}$$

The zonal price in the day-ahead market can be calculated as $\omega_z = \xi - \sum_l H'_{l,z}\gamma_l$. From (11a) and the complementary slackness conditions for (10d), the day-ahead profit function is formulated in (13).

$$\pi_u = \nu_u G_u + g_u C_u(\sum_a B_{u,a} x_{u,a} - 1), \ \forall u \tag{13}$$

Bilinear term $g_u x_{u,a}$ appears both in (13) and (11c). It is linearized using the McCormick reformulation as explained before. After this linearization, since the day-ahead dispatch decision (g_u) is a variable, we face nine sets of bilinear terms in the resulting model. These are **(vii)** $\beta_{u,s} g_u$ in (5n) and (3i), **(viii)** $\varphi_{u,s} g_u$ in (5n) and (3i), **(ix)** $\mu_{k,s} \sum_{n,n:u} H_{k,n} g_u$ in (5n), **(x)** $\lambda^B_{k,s} \sum_{n,n:u} H_{k,n} g_u$ in (5n), **(xi)** $\lambda^D_{u,s} g_u$ in (5n), **(xii)** $\lambda^F_{u,s} g_u$ in (5n), **(xiii)** $\lambda^M_{u,s} g_u$ in (5g) and (5i), **(xiv)** $\alpha_s \sum_u g_u$ in (5n) and **(xv)** $\lambda^A_s \sum_u g_u$ in (5n). The bilinear terms in **(xiv)** and **(xv)** are reformulated using (10b). Since $\sum_u g_u = \sum_n D_n$ in (10b), we replace

$\sum_u g_u$ by $\sum_n D_n$ in **(xiv)** and **(xv)**. The following lemma helps in linearizing bilinear terms **(vii)**-**(xiii)**.

Lemma 1
Day-ahead dispatch quantity g_u is a discrete variable.

Proof. Since (a) Parameters (G_u, D_n, F_k) are given in (10), (b) price bids are flat and (c) price bids are selected from an individual and finite set of discrete values, the corresponding dispatch interactions are also selected from a finite set of discrete values.[2] □

Using Lemma 1, g_u is formulated by binary variables $y_{u,r}$ in (14).

$$g_u = \sum_r E_{u,r} y_{u,r},\ \forall u \text{ and } \sum_r y_{u,r} \leq 1,\ \forall u \tag{14}$$

Here parameter $E_{u,r}$ is the day-ahead dispatch of producer u in bid combination r. Suppose there are U producers in the zonal day-ahead market and each producer has A bid alternatives, then we have A^U bid combinations and index r is defined as $r = 1, ..., A^U$. Parameter $E_{u,r}$ is calculated by solving problem (10) for all A^U bid combinations. This calculation can be performed in parallel.

Since g_u is a discrete variable, the bilinear terms in **(vii)**-**(xiii)** can be linearized using the McCormick reformulation. The bidding problem of a producer in both zonal day-ahead and zonal real-time market is represented by (**Box E**) in Fig. 1 and set out in (15).

$$\underset{\Psi}{\text{Maximize}}\ \pi_u + \mathbb{E}_s[\phi_{u,s}] \tag{15a}$$

Subject to:

$$\text{Constraints } (8), (10b) - (10d), (11), (12), (13), (14) \tag{15b}$$

$$\text{Linerization of } g_u x_{u,a} \text{ and } \mathbf{(vii)} - \mathbf{(xiii)} \text{ as in } (7) \tag{15c}$$

The set of decision variables in (15) is $\Psi = \{g_u,\ \xi,\ \gamma_l,\ \eta_u,\ \nu_u,\ x_{u,a},\ \hat{c}_u,\ \pi_u,\ y_{u,r}\} \cup \Theta$.

Inequality (9) is reformulated in (16c) by introducing the nonnegative slack variable ζ_u. ζ_u represents the deviation from the SPNE. The SPNE of the two-stage bidding game is found when $\sum_u \zeta_u$ equals to zero. The SPNE problem is represented by (**Box F**) in Fig. 1 and its formulation can be written as the MILP model in (16).

$$\underset{\Phi}{\text{Minimize}} \quad \sum_u \zeta_u \tag{16a}$$

Subject to:

$$\zeta_u \geq 0,\ \forall u \tag{16b}$$

$$\pi_u + \mathbb{E}_s[\phi_{u,s}] - \pi_u^{(j)} - \mathbb{E}_s[\phi_{u,s}^{(i),(j)}] + \zeta_u \geq 0\ \forall u, i, j \tag{16c}$$

$$\mathbb{E}_s[\phi_{u,s}] \geq \mathbb{E}_s[\phi_{u,s}^{(i),(j)}]\ \ \forall u, i, j \tag{16d}$$

$$\text{Constraints } (15b), (15c) \tag{16e}$$

$$\text{Constraints } (15b)^{(j)}, (15c)^{(j)},\ \forall j \tag{16f}$$

[2]This lemma means that the zonal day-ahead economic dispatch model is originally a discretely-constrained LP model.The LP model we assumed for deriving the KKT conditions is the relaxation of the discretely-constrained LP model with zero duality gap.

The set of decision variables in (16) is $\Phi = \{\zeta_u\} \cup \{g_u,\, \xi,\, \gamma_l,\, \eta_u,\, \nu_u,\, x_{u,a}, \hat{c}_u,\, \pi_u,\, y_{u,r}\}^{(j)}$ $\cup\ \Psi$.

We use a nodal two-stage market as our benchmark in this study. Mathematically, we model it as a zonal system with one node per zone. We replace indices l and z by indices k and n, respectively. Parameters $\bar{F}_l$ and $H'_{l,z}$ are replaced by, parameters F_k and $H_{k,n}$, respectively.

4. Tackling Multiple SPNE

The stochastic MILP model in (16) may have one, multiple or no optimal solution (note that every optimal solution where $\sum_u \zeta_u = 0$ is a SPNE). The number of SPNE is dependent on the number of producers and the number of the bidding strategies available for each producer. In this study, our approach is to find all SPNE which have different total dispatch costs. To implement this, after a SPNE has been found, we add constraint (17) to the stochastic MILP model in (16). TDC and $TDC^{(q)}$ represent the total dispatch cost in the new SPNE and already found SPNE q, respectively. ϵ is a very small positive scalar.

$$TDC = \sum_{s,u} \sigma_s(\hat{c}_u^{up} g_{u,s}^{up} - \hat{c}_u^{dn} g_{u,s}^{dn}) + \sum_u \hat{c}_u g_u \tag{17a}$$

$$|TDC - TDC^{(q)}| \geq \epsilon,\ \forall q \tag{17b}$$

Constraint (17b) is not linear. We linearize it in (18) by introducing binary variable t_q as in [21]. Constraint (18) ensures that any SPNE with the same total dispatch cost will not be considered again. Ξ is a sufficiently large scalar.

$$-\epsilon + \Xi t_q \geq TDC - TDC^{(q)} \geq \epsilon - (1 - t_q)\Xi,\ \forall q \tag{18}$$

The whole procedure to find all SPNE with different total dispatch costs is outlined in Algorithm 1.

Algorithm 1: Solution procedure for finding all SPNE with different dispatch costs

Step 1: Construct parameter $E_{u,r}$.;
for *r=1 to* A^U **do**
  Set $\hat{c}_u = \hat{c}_u^{(r)}$;
  Solve model (10) and store $E_{u,r} = g_u$;
end
Step 2: Find all SPNE with different dispatch costs;
repeat
  Solve MILP model (16) and update $q = q + 1$;
  Add constraint (18) to MILP model (16);
until $(\sum_u \zeta_u > 0)$;

5. Illustrative Example

We use the 6-node system shown in Fig. 3 to illustrate producers' bidding behavior in the two-stage zonal market.

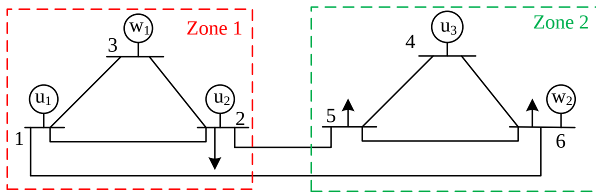

Figure 3: Single line diagram of 6-node example system, u_1,u_2, u_3: Conventional producers, W_1, W_2: Wind farms

The proposed model in (16) is solved using the Benders decomposition embedded in the CPLEX solver of the GAMS platform. In order to reduce the solution time, the GUSS facility of GAMS is used for calculating $E_{u,r}$ in Step 1 of Algorithm 1. The data related to the system is shown in Table 1. To have a zonal system with different zonal prices, the transmission capacities of the lines between nodes 1-2, 2-5, and 1-6 are set to 35 MW, 70 MW and 65 MW, respectively. The transmission capacity of the other lines is set to 100 MW. The market operator sets the flow limits between two zones at 120 MW for the zonal day-ahead dispatch. Two wind farms are connected to nodes 3 and 6 which generates at 30 MWh in the zonal day-ahead market. The deviation from the wind generation ($\Delta W_{n,s}$) is assumed to be between -9 MWh and 9 MWh and this interval is sampled by 7 scenarios showed in Table 2. We assume that each producer has 3 bidding actions for day-ahead price bids with 0%, 10% mark-up and 10% mark-down. The permissible up-regulation and down-regulation bids have 0%, 10%, 20% mark-up and 0%, 10%, 20% mark-down, respectively.

Table 1: Producer and load data for the 6-node system

u	C_u ($/MWh)	C_u^{up} ($/MWh)	C_u^{dn} ($/MWh)	G_u (MW)	Load	D_n (MWh)
u_1	12.5	25	7.5	150	n_2	120
u_2	11.5	21	6.5	250	n_5	100
u_3	13.5	23	8.5	150	n_6	60

Table 2: The net-demand deviation scenarios

	s_1	s_2	s_3	s_4	s_5	s_6	s_7
$\Delta W_{n,s}$	9	6	3	0	-3	-6	-9

Algorithm 1 is employed in the 6-node system. Two SPNE which have different total dispatch costs are found in the zonal two-stage market and one SPNE is found for the benchmark model. They are reported in Table 3.

Table 3: The subgame perfect Nash equilibrium in the 6-node system

	Zonal SPNE-1			Zonal SPNE-2			Nodal SPNE		
	u_1	u_2	u_3	u_1	u_2	u_3	u_1	u_2	u_3
$\hat{c}_u$ (\$/MWh)	11.25	12.65	14.85	12.5	12.65	14.85	12.5	12.65	13.5
$\hat{c}_u^{up}$ (\$/MWh)	25	25.2	27.6	25	25.2	27.6	25	25.2	27.6
$\hat{c}_u^{dn}$ (\$/MWh)	6	5.2	8.5	6	5.2	8.5	6.75	6.5	7.65

Table 3 shows that export-constrained producer u_1 chooses a day-ahead bid to be the cheapest producer in the market in both zonal SPNE. In zonal SPNE-1 it chooses a day-ahead bid which is lower than its marginal cost, which is consistent with the inc-dec game. In zonal SPNE-2 it bids its marginal cost to the zonal day-ahead market. These bidding decisions make producer u_1 the cheapest producer in the market and it is dispatched at full capacity in the zonal day-ahead market. Table 4 shows that producers u_2 and u_3 are dispatched 60 MWh and 10 MWh, respectively. The zonal price in the day-ahead market becomes 12.65 \$/MWh in zone 1 and 14.85 \$/MWh in zone 2. Table 5 shows that the day-ahead profit of u_1, u_2 and u_3 is 22.5 \$/h, 69 \$/h and 13.5 \$/h, respectively. Note that in both zonal SPNE-1 and zonal SPNE-2, the market outcomes (i.e. dispatch quantities, prices, profits) are the same. However, the total dispatch costs in zonal SPNE-1 and in zonal SPNE-2 are different since the bids are different.

Table 4: The dispatch in zonal day-ahead and zonal real-time markets in 6-node system

	g_u (MWh)	$(g_{u,s}^{up}, g_{u,s}^{dn})$ (MWh,MWh)						
		s_1	s_2	s_3	s_4	s_5	s_6	s_7
u_1	150	(0,59.6)	(0,56.4)	(0,53.2)	(0,**50**)	(0,**46.8**)	(0,**44.5**)	(0,**43**)
u_2	60	(**41.6**,0)	(**44.4**,0)	(**47.2**,0)	(**50**,0)	(52.8,0)	(54.6,0)	(55.7,0)
u_3	10	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(1.9,0)	(5.3,0)

Table 5: The day-ahead profit and the total expected profit of producers in 6-node system

	Zonal SPNE-1 and -2			Nodal SPNE		
	u_1	u_2	u_3	u_1	u_2	u_3
π_u (\$/h)	22.5	69	13.5	0	155.3	0
$\pi_u + \mathbb{E}_s[\phi_{u,s}]$ (\$/h)	79.3	276.8	15.8	3.9	166	11.8

The day-ahead dispatch overloads line 1-2 by 29.2 MW. Node 1 is the export constrained node and node 2 is the import-constrained node. The market operator dispatches the down-regulation bid of producer u_1 and the up-regulation bids of producer u_2 to relive this overloading. The counter-traded volumes in each scenario are shown in bold in Table 4. In contrast, no lines are overloaded in the benchmark case so no inc-dec game is observed. The profit in the zonal real-time market is calculated as in (4). The zonal prices and the profit of the producers in the zonal real-time market are presented in Table 6. Note that there is no imbalance in s_4 therefore no real-time price is calculated in s_4 (the payments to the producers are settled by their bid-prices in s_4).

Table 6: The zonal prices and the profits of each producer in the zonal real-time market in 6-node system

		s_1	s_2	s_3	s_4	s_5	s_6	s_7
$\rho'_{z,s}$	z_1	6	6	6	-	25.2	25.5	25.2
(\$/MWh)	z_2	8.5	8.5	8.5	-	27.6	27.6	27.6
$\phi_{u,s}$	u_1	89.4	84.6	79.8	75	70.2	66.6	64.5
(\$/h)	u_2	174.7	186.5	198.2	210	221.8	229.3	233.9
	u_3	0	0	0	0	0	8.7	24.4

Table 5 shows the total expected profit of producers in zonal SPNE-1, zonal SPNE-2 and nodal SPNE. In zonal SPNE-1 and zonal SPNE-2, the total expected profit of producers u_1, u_2 and u_3 is 79.3 \$/h, 276.8 \$/h and 15.8 \$/h, respectively. In nodal SPNE, the total expected profit of producers u_1, u_2 and u_3 is 3.9 \$/h, 166 \$/h, 11.8 \$/h, respectively. We see that playing the inc-dec game increases u_1's total profit by 19.3 times as compared to the benchmark case.

The dispatch costs in zonal SPNE-1, zonal SPNE-2, nodal SPNE and in the competitive bidding case where all producers bid their true marginal costs in the nodal two-stage market are illustrated in Table 7. Table 7 shows that employing zonal pricing increases the total dispatch cost by 695.7 \$/h (24.2%) in zonal SPNE-1 or 883.2 \$/h (30.8%) in zonal SPNE-2 as compared to the benchmark case. The increase in the dispatch cost is due to the inc-dec game which increases the dispatch cost in the zonal real-time market by more than 9 times as compared to the benchmark. We see that when the producers submit their strategic bids which do not correspond to their true marginal costs, the total dispatch cost in the benchmark increases by 8.7% as compared to the competitive bidding case where all producers bid their true marginal costs in the nodal two-stage market.

Table 7: The dispatch cost (DC) in the day-ahead (DA) and in the real-time (RT) markets in 6-node system, CB: Competitive bidding in nodal two-stage market

	DC (\$/h)				DC (\$/h)		
	DA	RT	Total		DA	RT	Total
Zonal SPNE-1	2595	972.1	3567.1	Zonal SPNE-2	2782	972.1	3754.6
Nodal SPNE	2770.3	101.1	2871.4	CB	2550	74.6	2642.6

6. Modified IEEE 30-node system

The IEEE 30-node example system in [22] is modified and used in this study. We consider 5 competing producers. The system is split into three zones as in [23]. To have a zonal system with different zonal prices, the capacity of the lines between nodes 1-2 and 27-28 are changed from 130 MW and 65 MW to 55 MW. The load at each node presented in [22] is increased by 50%. For the zonal day-ahead market dispatch, the market operator sets the flow limits between zones 1-2, 1-3 and 2-3 to 66 MW, 70 MW and 80 MW, respectively. The data related to the producers is presented in Table 8.

Table 8: Producer data for the modified IEEE 30-node system

	Node	C_u (\$/MWh)	C_u^{up} (\$/MWh)	C_u^{dn} (\$/MWh)	G_u (MW)
u_1	22	23	31.5	12.5	100
u_2	27	20.5	30.5	11.5	100
u_3	13	23.5	33.5	14.5	100
u_4	1	26.5	36.5	17.5	100
u_5	2	25	35.5	16.5	100

Three wind farms are connected to nodes 6, 10 and 17. Each wind farm generates at 27 MWh in the zonal day-ahead market. The deviation from the day-ahead wind production ($\Delta W_{n,s}$) is assumed between -7.9 MWh and 7.9 MWh, and this interval is sampled by 11 scenarios. We consider price bids with the same mark-up and mark-down values as in the 6-node system.

Using algorithm 1, we find four SPNE which have different dispatch costs in the zonal two-stage market and two SPNE in the benchmark model. They are shown in Table 9.

Table 9: The subgame perfect Nash equilibria in the modified IEEE 30-node system

	Zonal SPNE-1			Zonal SPNE-2		
	$\hat{c}_u$ (\$/MWh)	$\hat{c}_u^{up}$ (\$/MWh)	$\hat{c}_u^{dn}$ (\$/MWh)	$\hat{c}_u$ (\$/MWh)	$\hat{c}_u^{up}$ (\$/MWh)	$\hat{c}_u^{dn}$ (\$/MWh)
u_1	25.3	34.65	10	25.3	34.65	10
u_2	22.55	30.5	9.2	22.55	30.5	9.2
u_3	21.15	40.2	11.6	23.5	40.2	11.6
u_4	23.85	36.5	14	23.85	36.5	14
u_5	27.5	39.05	16.5	27.5	39.05	16.5
	Zonal SPNE-3			Zonal SPNE-4		
	$\hat{c}_u$ (\$/MWh)	$\hat{c}_u^{up}$ (\$/MWh)	$\hat{c}_u^{dn}$ (\$/MWh)	$\hat{c}_u$ (\$/MWh)	$\hat{c}_u^{up}$ (\$/MWh)	$\hat{c}_u^{dn}$ (\$/MWh)
u_1	25.3	34.65	10	25.3	34.65	10
u_2	22.55	30.5	9.2	22.55	30.5	9.2
u_3	21.15	40.2	11.6	23.5	40.2	11.6
u_4	26.5	36.5	14	26.5	36.5	14
u_5	27.5	39.05	16.5	27.5	39.05	16.5
	Nodal SPNE-1			Nodal SPNE-2		
	$\hat{c}_u$ (\$/MWh)	$\hat{c}_u^{up}$ (\$/MWh)	$\hat{c}_u^{dn}$ (\$/MWh)	$\hat{c}_u$ (\$/MWh)	$\hat{c}_u^{up}$ (\$/MWh)	$\hat{c}_u^{dn}$ (\$/MWh)
u_1	25.3	37	10	23	37.8	10
u_2	22.55	36.6	9.2	22.55	36.6	9.2
u_3	25.85	40.2	13.05	25.85	40.2	13.05
u_4	29.15	43.8	15.75	29.15	43.8	15.75
u_5	22.5	39.05	14.85	25	42.6	14.85

Table 9 shows that in all zonal SPNE, the producers u_3 and u_4, the export-constrained producers, choose a day-ahead bid which is lower than producers u_1 and u_5, respectively. The dispatch quantities in both markets are the same for all zonal SPNE, as shown in Table 10. We observe that producers u_3 and u_4 are dispatched at their full capacity. However, producers u_1 and u_5 are dispatched, below their full capacity, at 30 MW and 15 MW, respectively. The zonal prices in the zonal day-ahead market are 27.5 \$/MWh in zone 1, 26.4 \$/MWh in zone 2 and 25.3 \$/MWh in zone 3. Table 11 shows that the day-ahead profit of producers u_1, u_2, u_3, u_4 and u_5 is 69 \$/h, 480 \$/h, 300 \$/h, 100 \$/h and 37.5 \$/h, respectively.

The day-ahead zonal dispatch overloads the intra-zonal lines between nodes 1-2 and nodes 12-13 by 28.55 MW and 35 MW. To relieve this overloading, the market operator dispatches the up-regulation bid of producer u_5 and the down-regulation bids of producers u_3 and u_4. The counter-traded volumes in each scenario is illustrated in bold fonts in Table 10. In both nodal SPNE, the day-ahead dispatch does not overload any transmission lines in the network and no inc-dec game is observed. The total expected profit both in all zonal SPNE and in all nodal SPNE is presented in Table 11. In all zonal SPNE, the total expected profit of u_1, u_2, u_3, u_4 and u_5 is 69 \$/h, 480 \$/h, 401.5 \$/h, 187,8 \$/h and 250.8 \$/h, respectively. In nodal SPNE-1 and nodal SPNE-2, the total expected profit of u_1, u_2, u_3, u_4 and u_5 is 485.3 \$/h, 203 \$/h, 112.8 \$/h, 44.5 \$/h and 425.4 \$/h, respectively. We see that playing the inc-dec game increases the total profit of export-constrained producers u_3 and u_4 by 2.6 times and 3.2 times, respectively.

Table 10: The dispatch in zonal day-ahead and zonal real-time markets in the modified IEEE 30-node system

	g_u (MWh)	$(g^{up}_{u,s}, g^{dn}_{u,s})$ (MWh,MWh)				
		s_1	s_2	s_3	s_4	s_5
u_1	30	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
u_2	100	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
u_3	100	(0,35)	(0,35)	(0,35)	(0,35)	(0,35)
u_4	100	(0,30.6)	(0,29.6)	(0,28.4)	(0,27.3)	(0,26.2)
u_5	15	(**41.9**,0)	(**45.6**,0)	(**49.2**,0)	(**52.8**,0)	(**56.5**,0)
	$(g^{up}_{u,s}, g^{dn}_{u,s})$ (MWh,MWh)					
	s_6	s_7	s_8	s_9	s_{10}	s_{11}
u_1	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
u_2	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
u_3	(0,**35**)	(0,**35**)	(0,**35**)	(0,**35**)	(0,**35**)	(0,**35**)
u_4	(0,**25.1**)	(0,**24**)	(0,**22.8**)	(0,**21.7**)	(0,**20.6**)	(0,**19.5**)
u_5	(**60.1**,0)	(63.7,0)	(67.3,0)	(70.9,0)	(74.6,0)	(78.2,0)

The dispatch costs in all zonal and nodal SPNE and in the competitive bidding case where all producers bid their true marginal costs in the nodal two-stage market are illustrated in Table 12. Table 12 shows that employing zonal pricing increases the total dispatch cost by 13-19.2% as compared to the benchmark case according to the SPNE selection. We observe that when the producers submit their strategic bids which do not correspond to

Table 11: The day-ahead profit and the total expected profit in the modified IEEE 30-node system

	Zonal SPNE-1, -2, -3 and -4					Nodal SPNE-1 and -2				
	u_1	u_2	u_3	u_4	u_5	u_1	u_2	u_3	u_4	u_5
π_u (\$/h)	69	480	300	100	37.5	485.3	198	112.8	1.1	416
$\pi_u + \mathbb{E}_s[\phi_u, s]$ (\$/h)	69	480	401.5	187.8	250.8	485.3	203	112.8	44.5	425.4

their true marginal costs, the total dispatch cost in the benchmark increases by 4.3-4.6% as compared to the competitive bidding case where all producers bid their true marginal costs in the nodal two-stage market.

Table 12: The dispatch cost (DC) in the day-ahead (DA) and in the real-time (RT) markets in the modified IEEE 30-node system, CB: Competitive bidding in nodal two-stage market

	DC (\$/h)				DC (\$/h)		
	DA	RT	Total		DA	RT	Total
Zonal SPNE-1	7926.5	1588.6	9515.1	Zonal SPNE-2	8161.6	1589.6	9751.2
Zonal SPNE-3	8191.5	1590.6	9782.1	Zonal SPNE-4	8426.5	1591.6	10018.1
Nodal SPNE-1	8210.9	186.1	8397	Nodal SPNE-2	8230.9	186.1	8417
CB	7919	128.5	8047				

Our numerical analysis reveals two types of players in two-stage zonal markets. Producers located in export-constrained nodes u_1 in 6-node system and u_3, u_4 in the modified IEEE 30-node system behave as strategic arbitrageurs. They submit bids below their marginal costs in the zonal day-ahead market. As in the inc-dec game, they overload lines for which capacity constraints are neglected in the zonal day-ahead auction. In the zonal real-time market, they get dispatched to relax the overloaded lines. There are also other producers which submit a price bid higher than their marginal cost to exercise standard market power in both zonal day-ahead and zonal real-time markets. The proposed MILP model in this paper can successfully detect all of these strategic behaviors.

7. Conclusion

This paper proposes a two-stage game to analyze imperfect competition in two-stage zonal markets. We formulate the two-stage game as a MILP model. Our proposed model may have multiple SPNE. We design an iterative approach to find all SPNE for which dispatch costs are different. The developed MILP model and the iterative approach are demonstrated on the 6-node and the modified IEEE 30-node systems. Our numerical results show that the inc-dec game can be used by export-constrained producers to increase their profits drastically, by several hundred percent. Moreover, zonal pricing with the inc-dec game can increase the dispatch costs by 10-30% in comparison to nodal pricing.

The inc-dec game is not due to the lack of competition, it is due to the misrepresentation of transmission constraints in the day-ahead market. One possible remedy is to improve the representation of transmission constraint in the day-ahead market. This could be done by dividing electricity markets into a larger number of zones.

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References

[1] B. Allaz, J.-L. Vila, Cournot competition, forward markets and efficiency, Journal of Economic Theory 59 (1) (1993) 1 – 16. doi:https://doi.org/10.1006/jeth.1993.1001.

[2] A. Kumar, S. Srivastava, S. Singh, Congestion management in competitive power market: A bibliographical survey, Electric Power Systems Research 76 (1) (2005) 153 – 164. doi:https://doi.org/10.1016/j.epsr.2005.05.001.

[3] S. M. Harvey, W. W. Hogan, Nodal and zonal congestion management and the exercise of market power, Harvard University.
URL http://www.lmpmarketdesign.com/papers/zonal_jan10.pdf

[4] P. Holmberg, E. Lazarczyk, Comparison of congestion management techniques: Nodal, zonal and discriminatory pricing, Energy Journal 36 (2) (2015) 145 – 166. doi:10.5547/01956574.36.2.7.

[5] J. Dijk, B. Willems, The effect of counter-trading on competition in electricity markets, Energy Policy 39 (3) (2011) 1764–1773. doi:https://doi.org/10.1016/j.enpol.2011.01.008.

[6] Z. Alaywan, T. Wu, A. D. Papalexopoulos, Transitioning the california market from a zonal to a nodal framework: an operational perspective, in: IEEE PES Power Systems Conference and Exposition, 2004., Vol. 2, 2004, pp. 862–867. doi:10.1109/PSCE.2004.1397468.

[7] K. Neuhoff, B. Hobbs, D. M. Newbery, Congestion management in european power networks: Criteria to assess the available options, Discussion Papers of DIW Berlin 1161, DIW Berlin, German Institute for Economic Research (2011).
URL https://EconPapers.repec.org/RePEc:diw:diwwpp:dp1161

[8] D. Zhang, H. Xu, Two-stage stochastic equilibrium problems with equilibrium constraints: modeling and numerical schemes, Optimization 62 (12) (2013) 1627–1650. doi:10.1080/02331934.2011.632418.

[9] D. Zhang, S. Kim, A two stage stochastic equilibrium model for electricity markets with forward contracts, in: Proc. 2010 11th International Conference on Probabilistic Methods Applied to Power Systems (PMAPS), 2010, pp. 194–199. doi:10.1109/PMAPS.2010.5528998.

[10] E. Moiseeva, M. R. Hesamzadeh, D. W. Bunn, D. R. Biggar, The strategic effects of lack of transparency in forward contracting by generators with market power, PESD Working Paper Series 106, Stanford University (2018).
URL https://fsi-live.s3.us-west-1.amazonaws.com/s3fs-public/wp_106_moisseeva_et_al_30_april_2018.pdf

[11] R. Henrion, W. Römisch, On m-stationary points for a stochastic equilibrium problem under equilibrium constraints in electricity spot market modeling, Applications of Mathematics 52 (6) (2007) 473–494. doi:10.1007/s10492-007-0028-z.

[12] A. Gupta, R. Jain, K. Poolla, P. Varaiya, Equilibria in two-stage electricity markets, in: Proc. 2015 54th IEEE Conference on Decision and Control (CDC), 2015, pp. 5833–5838. doi:10.1109/CDC.2015.7403136.

[13] R. Green, Nodal pricing of electricity: how much does it cost to get it wrong?, Journal of Regulatory Economics 31 (2) (2007) 125–149. doi:10.1007/s11149-006-9019-3.

[14] M. Bjørndal, K. Jörnsten, Benefits from coordinating congestion managementthe nordic power market, Energy Policy 35 (3) (2007) 1978 – 1991. doi:https://doi.org/10.1016/j.enpol.2006.06.014.

[15] D. Ruderer, G. Zöttl, The impact of transmission pricing in network industries, Cambridge Working Papers in Economics 1230, Faculty of Economics, University of Cambridge (2012).
URL https://ideas.repec.org/p/cam/camdae/1230.html

[16] M. R. Hesamzadeh, P. Holmberg, M. Sarfati, Simulation and evaluation of zonal electricity market designs, IFN Working Papers 1211, Research Institute Of Industrial Economics (IFN) (2018).
URL https://www.ifn.se/publikationer/working_papers/2018/1211

[17] C. Ruiz, A. J. Conejo, S. A. Gabriel, Pricing non-convexities in an electricity pool, IEEE Transactions on Power Systems 27 (3) (2012) 1334–1342. doi:10.1109/TPWRS.2012.2184562.

[18] S. Gabriel, A. Conejo, J. Fuller, B. Hobbs, C. Ruiz, Complementarity Modeling in Energy Markets, International Series in Operations Research & Management Science, Springer New York, 2012.

[19] M. V. Pereira, S. Granville, M. H. C. Fampa, R. Dix, L. A. Barroso, Strategic bidding under uncertainty: a binary expansion approach, IEEE Transactions on Power Systems 20 (1) (2005) 180–188. doi:10.1109/TPWRS.2004.840397.

[20] A. Gupte, S. Ahmed, M. S. Cheon, S. Dey, Solving mixed integer bilinear problems using milp formulations, SIAM Journal on Optimization 23 (2) (2013) 721–744. doi:10.1137/110836183.

[21] L. Liberti, S. Cafieri, F. Tarissan, Reformulations in Mathematical Programming: A Computational Approach, Springer Berlin Heidelberg, Berlin, Heidelberg, 2009, pp. 153–234. doi:10.1007/978-3-642-01085-9_7.

[22] University of Washington, 30 bus power flow test case (1993).
URL http://www.ee.washington.edu/research/pstca/

[23] P. Kachore, M. V. Palandurkar, Ttc and cbm calculation of ieee-30 bus system, in: 2009 Second International Conference on Emerging Trends in Engineering Technology, 2009, pp. 539–542. doi:10.1109/ICETET.2009.227.