

A Trust-Region based Sequential Linear Programming Approach for AC Optimal Power Flow Problems

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Abstract

This study proposes a new trust-region based sequential linear programming algorithm to solve the AC optimal power flow (OPF) problem. The OPF problem is solved by linearizing the cost function, power balance and **engineering constraints of the system**, followed by a trust-region to control the validity of the linear model. To alleviate the problems associated with the infeasibilities of a linear **approximation**, a feasibility restoration phase is introduced. This phase uses the original nonlinear constraints to quickly locate a feasible point when the linear **approximation** is infeasible. The algorithm follows convergence criteria to satisfy the first order optimality conditions for the original OPF problem. Studies on standard IEEE systems and large-scale Polish systems show an acceptable quality of convergence to a set of best-known solutions and a substantial improvement in computational time, with linear scaling proportional to the network size.

Keywords: Nonlinear programming, optimal power flow, sequential linear programming, trust-region method.

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1. Introduction

The OPF problem optimizes the total operating cost to support efficient operation of power systems while satisfying system constraints for a nominal state [1]. In practice one needs to solve a security-constrained OPF (SC-OPF) problem which takes into account the possibility of a sudden failure of a single component (generator, transmission line, transformer, etc) in the system. This is known as the N-1 security criterion [1, 2]. The OPF problem without security constraints has been extensively investigated in the literature (see, for instance [2], and references therein). This paper addresses the OPF problem for simplicity, but the benefits of our approach extend to the context of the SC-OPF problem as well. It is well-known that the OPF problem is nonlinear and nonconvex in nature, potentially having multiple equilibrium points. Hence searching for a global solution is in principle NP-hard (cf.[2, 3, 4, 5]). Electricity market clearing strategies are mainly based on nodal prices, which are the dual variables of power balance constraints of the OPF problem. This highlights the importance of the convexity and scalability features for any algorithm to use in OPF calculations [1, 6]. In addition to this, real-world OPF problems involve very large numbers of decision variables. This makes them challenging for a solution technique, both in terms of memory and computational time requirements. Consequently there is a great need for computationally efficient techniques which can handle the nonconvex AC network constraints.

In the context of OPF, solution approaches, such as linear programming (LP) [6, 7, 8], quadratic programming (QP) [9], Lagrangian relaxation [10], and interior-point (IP) methods [11] have been extensively investigated in the literature. It is worth noting that, among all these approaches, IP methods have emerged as a promising direct solution approach for OPF problems. IP methods have proven to be a viable computational alternative for the solution of large-scale OPF problems [12]. The primal-dual logarithmic barrier IP method and its predictor-corrector variant are known to be efficient for OPF solution algorithms due to their superior computational efficiency [13]. We refer to [14]

31 and references therein for a detailed survey of other solution approaches for the
32 OPF problem.

33 Many convexification approaches for power flow constraints have been pro-
34 posed to make the AC-OPF problem computationally tractable. One of the
35 widely used techniques in the last decade is semidefinite relaxation (SDR) which
36 can find the global optimal solution of the OPF problem for radial networks
37 under mild operating conditions [15, 16, 17, 18, 19]. However, in the case of
38 meshed networks, SDR possesses a relaxation gap and necessitates the use of
39 virtual phase shifters to recover bus voltage angles [20]. This can be an ex-
40 pensive task in practice. Furthermore, semidefinite programs do not scale well
41 for large-size power systems [21]. To circumvent the scalability issue associated
42 with SDR, recently a second-order cone relaxation (SOCR) has been introduced.
43 The SOCR enhances the computational performance, enabling the application
44 of the technique for OPF problems in large-scale power networks [22, 23]. Based
45 on SOCR, two different power flow formulations are considered in the literature,
46 namely the bus injection model [21] and the branch flow model [22, 23]. Re-
47 cently, the work in [24] introduced additional linear cuts in the branch flow
48 framework to guarantee the exactness of SOCR for active distribution power
49 networks. Similarly, an improved quadratic convex relaxation is proposed in
50 [21] as an extension of SDR, in which voltage magnitudes are coupled with volt-
51 age angles using additional polyhedral constraints. This improves the relaxation
52 gap in comparison to SOCR without sacrificing the computational performance.
53 However, a significant relaxation gap still persists in many power system cases
54 [21]. Solution approaches based on the global optimization philosophy, such as
55 convex envelopes [25] and decomposition methods [26] have also been reported
56 in the literature.

57 In the aforementioned approaches, LP methods can be an attractive can-
58 didate for OPF problems due to their inherent scalable nature. Recent works
59 [6, 27] have used successive linear programming (SLP)¹ principles to demon-

¹The words ‘successive’ or ‘sequential’ are used interchangeably in the context of linear

60 strate this fact. Specifically, [6] has shown the scalability of LP tools against
 61 the well-known IP solver IPOPT [28] as well as the nonlinear optimization solver
 62 KNITRO [29]. In [5], rectangular form of complex quantities is used to formulate
 63 the power flow model, which disregards quasi-linear relationships of active power
 64 and bus voltage angles, and the reactive power and voltage magnitudes [5]. In
 65 addition, that formulation results in nonconvex voltage limit constraints which
 66 need additional slack variables in order to be linearized.

67 Note that SLP approaches can suffer from poor approximation of the original
 68 OPF problem due to lack of any globalization strategy. An SLP approach
 69 starting at an arbitrary point far from a solution to the original OPF may not
 70 converge to a feasible solution. In such circumstances, trust-region (TR) based
 71 methods have proven to be a viable alternative; see for instance TR-SE [30],
 72 TR-IP [31], [32] and TR-QP [33]. In TR methods, an approximation problem
 73 is solved within a small radius (called the *trust-region*). This enables a good
 74 approximation for the original OPF to be obtained at each solution step within
 75 the given trust-region.

76 This paper proposes a synergistic approach based on a trust-region method
 77 and SLP for the OPF problem. Our approach is very much inspired by the
 78 recently proposed successive linearization scheme of [6] and the trust-region
 79 implementation [31]. However, our work differs in the following ways:

- 80 • Unlike [6], we use the polar form of complex quantities. This assists in
 81 capturing the quasi-linear relationship between active power and bus volt-
 82 age angles, and the reactive power and voltage magnitudes for the original
 83 OPF problem.
- 84 • In addition compared to [6], we propose a trust-region radius constraint
 85 to improve the validity of the linear approximations in subsequent SLP
 86 steps.

programming approximation schemes. This work prefers to use the phrase ‘sequential linear programming’.

- Further, compared to [31], instead of a penalty reformulation, we propose a simple feasibility restoration phase based on the original nonlinear constraints, in order to avoid infeasibilities of intermediate linearizations.

In brief, this work uses first-order Taylor series to construct a local linear model for the original OPF problem. A trust-region constraint is designed to ensure the validity of the constructed linear model, that is, to ensure that the original nonlinear constraints are satisfied. This is then integrated in an iterative procedure to optimize bus voltage magnitudes and angles, and active and reactive power generation. This trust-region sequential linear program (TR-SLP) terminates in a finite number of iterations, returning an OPF solution satisfying the convergence criteria (see Section 3.4). The performance of TR-SLP is tested on various benchmark IEEE and Polish systems against the SLP approach in [6], NLP solvers IPOPT [28] and KNITRO [29]. The results of TR-SLP demonstrate an acceptable quality of convergence to the best-known solution for the considered benchmark systems.

The paper presents the OPF problem formulation in Section 2, followed by the algorithm of TR-SLP in Section 3. Section 4 presents the numerical results on various IEEE networks. Finally, the paper is concluded in Section 5.

2. Mathematical Formulation

In this section, we first present the network model for a general power system and formulate the AC-OPF problem. Then, a linear programming (LP) approximation of the AC-OPF problem is derived using first-order Taylor series. This linear approximation is later embedded in an iterative procedure to form the TR-SLP algorithm (see Section 3).

2.1. Network Model

We define \mathcal{N} and \mathcal{L} as the set of buses and the set of transmission lines of the power system respectively, where $|\mathcal{N}| = N$ and $|\mathcal{L}| = L$. Further, let \mathcal{G} ($|\mathcal{G}| = G$) be the set of generators which are connected to a subset of \mathcal{N} . To formulate

the OPF problem, we use the polar form of the complex bus voltage $v \in \mathbb{C}^N$ and its i^{th} element $v_i = V_i e^{j\delta_i}$, where V_i is the voltage magnitude and δ_i is the phase angle of the voltage phasor v_i at bus $i \in \mathcal{N}$. Complex power generation is denoted by $s^G \in \mathbb{C}^G$ such that $s_g^G = P_g^G + jQ_g^G$ for generator $g \in \mathcal{G}$, where P_g^G and Q_g^G are the active and reactive power generation respectively. These two vectors (v and s^G) are the decision variables of the OPF problem. The parameters involved in the formulation are defined below.

The standard π -model is applied for modeling transmission lines. For the transmission line $l \in \mathcal{L}$, let $Y \in \mathbb{C}^L$ be the branch admittance vector, having components $Y_l = g_{l(i,j)} + jb_{l(i,j)}$, where $g_{l(i,j)}$ and $b_{l(i,j)}$ are the series conductance and susceptance respectively. Similarly, $b_{l(i,j)}^{\text{sh}} \in \mathbb{R}$ is the line charging susceptance for transmission line l . Complex power demand is characterized by $s^D \in \mathbb{C}^N$ such that $s_i^D = P_i^D + jQ_i^D$, where P_i^D and Q_i^D are the active and reactive power demand respectively at bus i .

2.2. AC-OPF Problem Formulation

The objective function of the OPF problem is generally formulated as the generation cost minimization. The constraints are formulated to satisfy the power balance at each bus, the generation capacity margins, and network constraints, *namely* power flow limits and voltage bounds.

The quadratic cost function for generator g in the system is represented below.

$$C_g = c_{2,g} (P_g^G)^2 + c_{1,g} P_g^G + c_{0,g}, \quad \forall g \in \mathcal{G} \quad (1)$$

where $c_{2,g}$, $c_{1,g}$ and $c_{0,g}$ denote the coefficients of quadratic, linear, and constant terms of the cost function, respectively. Then the complete OPF can be formulated as a NLP problem to optimize the total operating cost of the system:

$$\begin{aligned} \min_{\substack{\delta_i, V_i, \\ P_g^G, Q_g^G}} \quad & \sum_{g \in \mathcal{G}} C_g \\ \text{s.t.} \quad & \sum_{g \in \mathcal{G}(i)} P_g^G - V_i^2 \sum_{j \in \mathcal{N}(i)} g_{l(i,j)} \end{aligned} \quad (2a)$$

$$+ V_i \sum_{j \in \mathcal{N}(i)} V_j [g_{l(i,j)} \cos(\delta_{i,j}) - b_{l(i,j)} \sin(\delta_{i,j})] = P_i^D, \forall i \in \mathcal{N} \quad (2b)$$

$$\sum_{g \in \mathcal{G}(i)} Q_g^G - V_i^2 \sum_{j \in \mathcal{N}(i)} (b_{l(i,j)} + b_{l(i,j)}^{\text{sh}}/2) + V_i \sum_{j \in \mathcal{N}(i)} V_j [b_{l(i,j)} \cos(\delta_{i,j}) + g_{l(i,j)} \sin(\delta_{i,j})] = Q_i^D, \forall i \in \mathcal{N} \quad (2c)$$

$$I_{l(i,j)}^2 \leq (I_l^{\max})^2, \quad i, j \in \mathcal{N}, \quad \forall l \in \mathcal{L} \quad (2d)$$

$$I_{l(j,i)}^2 \leq (I_l^{\max})^2, \quad i, j \in \mathcal{N}, \quad \forall l \in \mathcal{L} \quad (2e)$$

$$I_{l(i,j)}^2 = I_{l(i,j)}^A V_i^2 + I_{l(i,j)}^B V_j^2 - 2V_i V_j [I_{l(i,j)}^C \cos(\delta_{i,j}) - I_{l(i,j)}^D \sin(\delta_{i,j})] \quad (2f)$$

$$V_i \in [V_i^{\min}, V_i^{\max}], \quad \forall i \in \mathcal{N} \quad (2g)$$

$$P_g^G \in [P_g^{G,\min}, P_g^{G,\max}], \quad \forall g \in \mathcal{G} \quad (2h)$$

$$Q_g^G \in [Q_g^{G,\min}, Q_g^{G,\max}], \quad \forall g \in \mathcal{G} \quad (2i)$$

where $\delta_{i,j} = \delta_i - \delta_j$; constraints (2b) and (2c) represent the active and reactive power balance at each bus; $\mathcal{G}(i)$ and $\mathcal{N}(i)$ are the set of generators connected at bus i , and the set of buses connected to bus i by transmission lines, respectively; and constraints (2d) and (2e) constrain the maximum current flow through each transmission line. Here, (2f) models the apparent current flow from bus i to bus j through transmission line l , where

$$\begin{aligned} I_{l(i,j)}^A &= g_{l(i,j)}^2 + \left(b_{l(i,j)} + b_{l(i,j)}^{\text{sh}}/2 \right)^2, \\ I_{l(i,j)}^B &= g_{l(i,j)}^2 + b_{l(i,j)}^2, \\ I_{l(i,j)}^C &= g_{l(i,j)}^2 + b_{l(i,j)} \left(b_{l(i,j)} + b_{l(i,j)}^{\text{sh}}/2 \right) \text{ and} \\ I_{l(i,j)}^D &= b_{l(i,j)} b_{l(i,j)}^{\text{sh}}/2; \end{aligned}$$

136 The physical laws of power flow have been considered in modeling these con-
137 straints. Constraint (2g) bounds the engineering limits of the voltage at each
138 bus; and (2h) and (2i) bound the active and reactive power generation capabili-
139 ties of each generator respectively; and $(\cdot)^{\min}$ and $(\cdot)^{\max}$ indicate the lower and
140 upper bound of the decision variables, respectively. The optimization problem

141 consists of $2(N + G)$ number of variables to optimize subject to the variable
 142 bounds and $2(N + L)$ number of constraints.

143 2.3. LP Formulation

The nonlinearity in the aforementioned OPF problem comes from equations (1), (2b), (2c) and (2f). In our proposed iterative procedure (TR-SLP), the nonlinear terms in these equations are linearized by applying first-order Taylor series approximations evaluated at the solution of the previous iteration. Assume the decision variable vector pertaining to the NLP problem (2) as

$$x := [\delta_1, \dots, \delta_N, V_1, \dots, V_N, P_1^G, \dots, P_G^G, Q_1^G, \dots, Q_G^G]^T \in \mathbb{R}^{2(N+G)}.$$

144 where $(\cdot)^T$ is the transpose operator. Then, the partial derivatives of (1), (2b),
 145 (2c) and (2f) are used to compute the Jacobian matrices as follows.

$$J^{C,k-1} = \left[\mathbf{0}_{2N}^T, \frac{\partial C_1}{\partial P_1^G}, \dots, \frac{\partial C_G}{\partial P_G^G}, \mathbf{0}_G^T \right] \Big|_{x^{k-1}} \quad (3a)$$

$$P_i^N = V_i^2 \sum_{j \in \mathcal{N}(i)} g_{l(i,j)} - V_i \sum_{j \in \mathcal{N}(i)} V_j [g_{l(i,j)} \cos(\delta_{i,j}) - b_{l(i,j)} \sin(\delta_{i,j})], \quad \forall i \in \mathcal{N} \quad (3b)$$

$$J_i^{P,k-1} = \left[\frac{\partial P_i^N}{\partial \delta_1}, \dots, \frac{\partial P_i^N}{\partial \delta_N}, \frac{\partial P_i^N}{\partial V_1}, \dots, \frac{\partial P_i^N}{\partial V_N}, -e_{G,i}^T, \mathbf{0}_G^T \right] \Big|_{x^{k-1}}, \quad \forall i \in \mathcal{N} \quad (3c)$$

$$Q_i^N = V_i^2 \sum_{j \in \mathcal{N}(i)} (b_{l(i,j)} + b_{l(i,j)}^{\text{sh}}/2) - V_i \sum_{j \in \mathcal{N}(i)} V_j [b_{l(i,j)} \cos(\delta_{i,j}) + g_{l(i,j)} \sin(\delta_{i,j})], \quad \forall i \in \mathcal{N} \quad (3d)$$

$$J_i^{Q,k-1} = \left[\frac{\partial Q_i^N}{\partial \delta_1}, \dots, \frac{\partial Q_i^N}{\partial \delta_N}, \frac{\partial Q_i^N}{\partial V_1}, \dots, \frac{\partial Q_i^N}{\partial V_N}, \mathbf{0}_G^T, -e_{G,i}^T \right] \Big|_{x^{k-1}}, \quad \forall i \in \mathcal{N} \quad (3e)$$

$$J_{l(i,j)}^{I,k-1} = \left[\frac{\partial I_{l(i,j)}^2}{\partial \delta_1}, \dots, \frac{\partial I_{l(i,j)}^2}{\partial \delta_N}, \frac{\partial I_{l(i,j)}^2}{\partial V_1}, \dots, \frac{\partial I_{l(i,j)}^2}{\partial V_N}, \mathbf{0}_{2G}^T \right] \Big|_{x^{k-1}},$$

$$i, j \in \mathcal{N}, \quad \forall l \in \mathcal{L} \quad (3f)$$

146 where $\mathbf{0}_{(\cdot)} = \{0\}^{(\cdot)}$ and $e_{G,i} \in \{0,1\}^G$, in which the g^{th} element is 1 if gen-
 147 erator $g \in \mathcal{G}(i)$, or is 0 otherwise. P_i^N and Q_i^N denote the sum of active and

148 reactive power extractions from bus i respectively; $(\cdot)^{k-1}$ denote the value of the
 149 decision variable/vector (\cdot) at the $(k-1)^{\text{th}}$ iteration. Equations (3a), (3c), (3e)
 150 and (3f) represent the Jacobian matrices of (1), (2b), (2c) and (2f) respectively,
 151 which are originally nonlinear. At the k^{th} iteration of TR-SLP, those Jacobian
 152 matrices in (3) are updated based on the solution of the previous $(k-1)^{\text{th}}$ it-
 153 eration. Finally, the LP approximation of the OPF problem (2) to be solved at
 154 the k^{th} iteration, obtained based on the solution of the $(k-1)^{\text{th}}$ iteration, can
 155 be deduced as follows.

$$\text{LP}(x^{k-1}) \left\{ \begin{array}{l} \min_x J^{C,k-1}(x - x^{k-1}) + \sum_{g \in \mathcal{G}} C_g|_{x^{k-1}} \\ \text{s.t. } J_i^{\text{P},k-1}(x - x^{k-1}) + P_i^{\text{N}}|_{x^{k-1}} - \sum_{g \in \mathcal{G}(i)} P_g^{\text{G},k-1} = -P_i^{\text{D}}, \forall i \in \mathcal{N} \\ J_i^{\text{Q},k-1}(x - x^{k-1}) + Q_i^{\text{N}}|_{x^{k-1}} - \sum_{g \in \mathcal{G}(i)} Q_g^{\text{G},k-1} = -Q_i^{\text{D}}, \forall i \in \mathcal{N} \\ J_{l(i,j)}^{\text{I},k-1}(x - x^{k-1}) + I_{l(i,j)}^2|_{x^{k-1}} \leq (I_l^{\text{max}})^2, i, j \in \mathcal{N}, \forall l \in \mathcal{L} \\ J_{l(j,i)}^{\text{I},k-1}(x - x^{k-1}) + I_{l(j,i)}^2|_{x^{k-1}} \leq (I_l^{\text{max}})^2, i, j \in \mathcal{N}, \forall l \in \mathcal{L} \\ (2\text{g}) - (2\text{i}) \end{array} \right. \quad (4)$$

156 It should be noted that (4) is tightly-coupled to the original OPF problem (2)
 157 at the evaluated point x^{k-1} .

158 **3. Trust-Region based Sequential Linear Programming Algorithm**

159 This section first introduces components such as trust-region LP formulation,
 160 feasibility restoration phase and step acceptance/rejection criterion. Then the
 161 pseudo-code of the main algorithm TR-SLP comprising all these components
 162 is presented. For ease of explanation, the AC-OPF problem (2) is represented

163 using a generic NLP form as follows:

$$\text{NLP} \left\{ \begin{array}{l} \min_x f(x) \\ \text{s.t. } h(x) = 0 \\ c(x) \leq 0 \\ x^{\min} \leq x \leq x^{\max} \end{array} \right. \quad (5)$$

164 where f represents the objective function (2a); h represents the set of equality
 165 constraints which include (2b) and (2c); c represents the set of inequality con-
 166 straints which include (2d) and (2e); and x^{\min} and x^{\max} in (5) represent the
 167 variable bounds (2g)-(2i).

168 3.1. Trust-Region Linear Program

169 At the k^{th} iteration, the LP(x^{k-1}) approximates the original OPF prob-
 170 lem (2) at x^{k-1} . However, it may be a very poor representation of (2) if
 171 $\|x^k - x^{k-1}\|$ is not sufficiently small. To circumvent this issue, we consider
 172 bounding x^{k-1} variations within a small closed region called the trust-region
 173 Δ^k . Specifically, we add a trust-region radius constraint to the LP approxima-
 174 tion (4) and form the following optimization problem.

$$\text{TR-LP}(x^{k-1}, \Delta^k) \left\{ \begin{array}{l} \min_d f(x^{k-1}) + [\nabla f(x^{k-1})]^T d \\ \text{s.t. } h(x^{k-1}) + [\nabla h(x^{k-1})]^T d = 0 : \lambda_h^k \\ c(x^{k-1}) + [\nabla c(x^{k-1})]^T d \leq 0 : \lambda_c^k \\ \max(x^{\min} - x^{k-1}, -\Delta^k) \leq d \\ d \leq \min(x^{\max} - x^{k-1}, \Delta^k) \end{array} \right. \quad (6)$$

where the decision variable vector $d := x - x^{k-1}$ and $\Delta^k > 0 \in \mathbb{R}^{2(N+G)}$ is the TR radius. Here, $\nabla f(x^{k-1})$, $\nabla h(x^{k-1})$ and $\nabla c(x^{k-1})$ represent the first-order partial derivatives of $f(x)$, $h(x)$ and $c(x)$ with respect to x , evaluated at x^{k-1} as in (4), respectively; λ_h^k and λ_c^k are the Lagrange multipliers of the equality (h) and inequality (c) constraints, respectively, with $\lambda^k = [(\lambda_h^k)^T \ (\lambda_c^k)^T]^T$. The solution d^k of the above optimization problem is used as a step to define the

new solution approximation, *i.e.* $x^k = x^{k-1} + d^k$ (see Section 3.3). The Karush-Kuhn-Tucker (KKT) conditions for (6) are:

$$h(x^{k-1}) + [\nabla h(x^{k-1})]^T d^k = 0 \quad (7a)$$

$$c(x^{k-1}) + [\nabla c(x^{k-1})]^T d^k \leq 0 \quad (7b)$$

$$\nabla f(x^{k-1}) + \nabla c(x^{k-1})\lambda_c^k + \nabla h(x^{k-1})\lambda_h^k = 0 \quad (7c)$$

$$\left(c(x^{k-1}) + [\nabla c(x^{k-1})]^T d^k \right) \lambda_c^k = 0 \quad (7d)$$

$$\lambda_c^k \geq 0 \quad (7e)$$

Equations (7) will be satisfied at every successful TR-LP computation.

It should be noted that a smaller TR radius may cause constraint infeasibilities or may reduce the speed of convergence. Similarly, a larger TR radius will weaken the validity of linear models that represent nonlinear constraints in (2). Therefore Δ^k is modified at each step of the algorithm (step 6 of Algorithm 1), the modification depending on the improvement in optimality.

3.2. Feasibility Restoration

In practice, $\text{TR-LP}(x^{k-1}, \Delta^k)$ can be infeasible due to the following two reasons: *i)* The constraint gradients $[\nabla h(x^{k-1})]^T$ can become degenerate at the point x^{k-1} , leading to infeasible linearized constraints. Then the system $[\nabla h(x^{k-1})]^T d = -h(x^{k-1})$ simply has no solution. *ii)* If the trust-region is too small, the TR-LP may be infeasible. In such circumstances, the linear constraint, $h(x^{k-1}) + [\nabla h(x^{k-1})]^T d = 0$, cannot be satisfied within the trust-region radius Δ^k of x^{k-1} .

Feasibility restoration (NLP-FR) searches for a feasible point by solving the following problem, so that the next TR-LP subproblem to be solved will be

feasible.

$$\text{NLP-FR} \left\{ \begin{array}{l} \min_{\substack{x, s_c, \\ s_h^+, s_h^-}} s_c + s_h^+ + s_h^- \\ \text{s.t.} \quad c(x) - s_c \leq 0 \\ h(x) - s_h^+ + s_h^- = 0 \\ x^{\min} \leq x \leq x^{\max} \\ s_c, s_h^+, s_h^- \geq 0 \end{array} \right. \quad (8)$$

where s_c , s_h^+ , s_h^- are slack variables used to relax the inequality and equality constraints respectively.

If the NLP-FR cannot find a solution with zero objective value, then the OPF problem (2) is declared as infeasible. Otherwise, we have found a feasible point x^k , which is used to compute the step-size $d^k := x^k - x^{k-1}$.

3.3. Step Acceptance/Rejection Criterion

To accept or reject the new step-size d^k and update the trust-region radius Δ^k for the next TR-SLP iteration, we compute the ratio ρ^k between predicted and actual reduction in the cost function (2a).

Let d^k be a solution of TR-LP(x^{k-1}, Δ^k). Then the predicted reduction in the objective is

$$\Delta\phi_{\text{pre}}^k = [\nabla f(x^{k-1})]^T d^k. \quad (9)$$

In order to take into account any constraint violations, as well as the actual value of the objective of the NLP (5), the following merit function is defined:

$$\phi(x^k) = f(x^k) + (\nu_h^k)^T |h(x^k)| + (\nu_c^k)^T \max\{c(x^k), 0\}, \quad (10)$$

where $\nu_h^k \in \mathbb{R}_+^{n_h}$ and $\nu_c^k \in \mathbb{R}_+^{n_c}$ are penalty factors for equality and inequality constraints respectively. These are derived in each iteration k based on (11a) and (11b) using dual variables λ_h^k and λ_c^k as follows.

$$\nu_h^k = \max\{\nu_h^{k-1}, \lambda_h^k\} \quad (11a)$$

$$\nu_c^k = \max\{\nu_c^{k-1}, \lambda_c^k\} \quad (11b)$$

$$\nu_{h,m}^0 = \frac{\|\nabla f(x^0)\|_2}{\|\nabla h_m(x^0)\|_2}, \quad \forall m \in \{1, \dots, n_h\} \quad (11c)$$

$$\nu_{c,m}^0 = \frac{\|\nabla f(x^0)\|_2}{\|\nabla c_m(x^0)\|_2}, \quad \forall m \in \{1, \dots, n_c\} \quad (11d)$$

Further, (11c) and (11d) are used to calculate the penalty factors for the first iteration, where h_m and c_m are the m^{th} equality and inequality constraints respectively; n_h and n_c are the number of equality and inequality constraints respectively. The actual reduction in the objective is

$$\Delta\phi_{\text{act}}^k = \phi(x^k) - \phi(x^{k-1}). \quad (12)$$

The ratio ρ^k is then defined as

$$\rho^k = \frac{\Delta\phi_{\text{act}}^k}{\Delta\phi_{\text{pre}}^k} \quad (13)$$

Then,

$$\Delta^{k+1} = \begin{cases} \alpha_1 \Delta^k & \text{if } \rho^k \leq 0 \\ \alpha_2 \Delta^k & \text{if } 0 < \rho^k \leq 0.25 \\ \Delta^k & \text{if } 0.25 < \rho^k \leq 0.75 \\ \min(2\Delta^k, \Delta^{\max}) & \text{if } 0.75 < \rho^k \end{cases} \quad (14)$$

where $\alpha_1 \in (0, 1)$, $\alpha_2 \in (0, 1)$ and Δ^{\max} are constants. This is a heuristic, and values for these parameters should be determined on a case-by-case basis in the context of the OPF problem (2).

Remark 1: If $\rho^k < 0$, then the iteration is considered as a failure. In such a case, the new point x^k is rejected, and the TR radius Δ^k for the next iteration is reduced to α_1 times its present value, and the TR-LP(x^{k-1}, Δ^k) is solved again. If $\rho^k \geq 0$, then the new point $x^k = x^{k-1} + d^k$ is accepted and the algorithm proceeds to the next step with the updated TR radius Δ^{k+1} .

We now summarize the TR-SLP algorithm via the pseudo-code in Algorithm 1, utilizing the aforementioned trust-region linear program (TR-LP), feasibility restoration (NLP-FR), and step acceptance/rejection (StepQuality) ingredients. It may be noted that the output (f^k, x^k) of the TR-SLP algorithm which satisfies the *convergence* conditions given in Section 3.4, is a local optimal solution of the OPF problem (2).

Algorithm 1:Trust-Region Sequential Linear Program (TR-SLP)

Input : $f, h, c, x^0, \alpha_1, \alpha_2, \Delta^1, \Delta^{\max}, K$ **Output**: f^k, x^k

```
1 while convergence not satisfied do
    /* solve trust-region linear program */
2    $(f^k, d^k, \lambda^k) \leftarrow \text{solve TR-LP}(x^{k-1}, \Delta^k)$ 
3   if  $\text{TR-LP}(x^{k-1}, \Delta^k)$  is not feasible then
    /* feasibility restoration phase */
4      $d^k \leftarrow \text{solve NLP-FR}$ 
5   end
    /* step quality determination phase */
6    $(\Delta^{k+1}, \rho^k) \leftarrow \text{StepQuality}(f, \nabla f, x^{k-1}, d^k, \alpha_1, \alpha_2, \Delta^{\max})$ 
7   if  $\rho^k < 0$  then
    /* reject step */
8      $x^k \leftarrow x^{k-1}$ 
9   else
    /* accept step */
10     $x^k \leftarrow x^{k-1} + d^k$ 
11     $k = k + 1$ 
12  end
13 end
```

3.4. Discussion on Convergence

We first give the necessary Karush-Kuhn-Tucker (KKT) conditions adopted from [34] for TR-SLP described in Algorithm 1. The TR-SLP stops when the following conditions are satisfied.

$$(a) \quad \|d^k\|_\infty \leq \epsilon_d$$

$$(b) \quad \|h(x^k)\|_\infty \leq \epsilon, \max\{c(x^k)\} \leq \epsilon$$

$$(c) \quad \max\{\|\nabla f(x^k) + \nabla c(x^k)\lambda_c^k + \nabla h(x^k)\lambda_h^k\|_\infty, \|c(x^k)\lambda_c^k\|_\infty\} < \epsilon_\lambda (1 + \|\lambda^k\|_2)$$

$$(d) \quad \lambda_c^k \geq 0$$

where ϵ_d , ϵ , and ϵ_λ are tolerances chosen for the step change, constraint satisfaction and KKT condition satisfaction respectively. Condition (a) implies that the step-size has reached the user-specified accuracy ϵ_d . Condition (b) implies that within the trust-region radius, the original nonlinear constraints h and c in the NLP (5) are satisfied to a user-specified accuracy of ϵ . Conditions (c) and (d) provide a measure of the closeness of the computed solution to a point satisfying the first-order optimality conditions for the NLP problem (5).

To establish the convergence of TR-SLP (Algorithm 1) consider the following. TR-SLP solves a trust-region linear approximation (*i.e.* TR-LP (6)) of the original NLP (5) at each iteration k . If TR-LP is feasible, it will compute d^k (step 2). Subsequently, based on the previous iterates x^{k-1} and d^k , the actual-to-predicted cost ratio ρ^k and the new trust-region radius Δ^{k+1} are determined (step 6). Then, based on ρ^k , as the TR-SLP progresses, Δ^{k+1} shrinks, ensuring the tightness of the linear approximation TR-LP (cf. Figure 4). This leads the successive iterates x^k to converge to a local solution of NLP (5), satisfying the conditions (a)–(d).

4. Numerical Experiments and Discussion

In this section, we report numerical results with the proposed TR-SLP algorithm for OPF problem (2). The TR-SLP is analyzed on a benchmark test

252 suite consisting of IEEE (14, 30, 57, 118 and 300) systems, and Polish 2383wp,
 253 2746wop and 3012wp systems (the number refers to the number of buses in the
 254 respective test case) available in [35]. We note that [6] is a recent computational
 255 study on the AC-OPF problem for the aforementioned systems. It reports the
 256 OPF (2) results using general purpose optimization solvers like IPOPT [28] and
 257 KNITRO [29], and a penalty reformulations based successive linear program-
 258 ming (SLP) method. In [6], two types of OPF problems are solved for each
 259 system, *viz.* without line flow limits (baseline case), and with line flow lim-
 260 its (thermally constrained case). We obtain the line flow limits data for this
 261 work from [6, Table II] . Further, [6, Tables IV, V] report the results obtained
 262 using their proposed SLP, NLP solvers IPOPT and KNITRO for the various
 263 benchmark IEEE and Polish systems. **These results have been obtained with**
 264 **constraint satisfaction up to 0.001 tolerance.** We shall use these results in our
 265 study to compare the performance of our TR-SLP in terms of optimality and
 266 computational time.

267 Convergence and optimality of the solution of SLP algorithms depend on the
 268 selected initial point (cf. [6]). In this study, we consider two different initializa-
 269 tion strategies for TR-SLP, *viz.* flat start and DC warm start, to demonstrate
 270 the variation of performance with respect to the starting point. In the flat
 271 start, we assume unit voltage phasors and half-max outputs for all generation.
 272 The DC warm start is constructed with the solution obtained from the DC-
 273 OPF problem combined with unit voltage magnitudes and half-max reactive
 274 power generation. We conduct three case studies to showcase the performance
 275 of the TR-SLP. Firstly, the computational time of the TR-SLP with the afore-
 276 mentioned test cases is compared against that of KNITRO, IPOPT and SLP.
 277 Secondly, we study the optimality of the solution obtained using the TR-SLP
 278 for the same test cases against the KNITRO solver run in a multi-start mode
 279 (henceforth referred as KNITRO-MS). We note that KNITRO run in a multi-
 280 start mode results in improved local optimal solutions [29]. Finally, we tighten
 281 the tolerances (*i.e.* $\epsilon_d, \epsilon, \epsilon_\lambda$ in TR-SLP) and study the relative improvement
 282 in the optimality of the solution obtained using the TR-SLP for the same test

cases, and note the trade-off against computation time.

The TR-SLP algorithm is implemented in MATLAB and the optimization problems are formulated based on the MATPOWER library [35]. The LP subproblems are solved using CPLEX 12.6 [36] and feasibility restoration subproblems using the MATLAB *fmincon* solver based on the *interior-point method*. All experiments are carried out on a desktop PC with an Intel®Core i7-5500U 4 core CPU processor running at 2.40GHz with 8GB RAM. Based on our experience with the numerical experiments reported in this work, the parameters of TR-SLP are chosen as follows: $\alpha_1 = 0.1$, $\alpha_2 = 0.25$, $\Delta^{(0)} = 0.4$, $\Delta^{\max} = 2$, $K = 30$, $\epsilon_d = 0.1$, $\epsilon = \epsilon_\lambda = 0.01$. Note that, in all our numerical studies, the original nonlinear constraints (2b)–(2e) are satisfied up to the same accuracy employed by [6] (*i.e.* 0.001 or lesser). This is specifically demonstrated for two large Polish (2746, 3012) systems in columns A and B of Table 4.

4.1. Case Study 1: Computational Time Comparison

In this study, TR-SLP is executed with a flat start strategy for OPF problem (2) and is compared in Table 1 with different solution approaches for several test cases. It should be noted that in [6], four different initialization strategies, *viz.* flat start, DC warm start, AC warm start and uniform cold start are used and the best solver time recorded for each test case is reported (see Table 1; KNITRO, IPOPT, and SLP).

It can be observed that for all IEEE systems, the CPU times of KNITRO and IPOPT are almost the same. Comparatively in Polish systems, KNITRO is noted to be slower. SLP is found to be the slowest for all IEEE systems. However, its performance is observed to improve for Polish systems with respect to KNITRO and IPOPT. We observed TR-SLP to be fastest among all solution approaches in many test cases (except IEEE 300 and Polish 2383 thermally constrained cases, where KNITRO and IPOPT are slightly faster). The TR-SLP reports comparatively the best CPU time for the largest Polish 3012 system, approximately 6, 2 and 3 times faster than KNITRO, IPOPT, and SLP, respectively, for the baseline case and approximately 5, 1.5 and 2 times faster

Table 1: The Computational Time Comparison (in Seconds) among All Solvers/Algorithms. TR-SLP is Executed under a Flat Start Strategy.

Test Case	Computational Time (s)							
	Baseline Case				Thermally Constrained Case			
	KNITRO	IPOPT	SLP	TR-SLP	KNITRO	IPOPT	SLP	TR-SLP
IEEE 14	0.12	0.16	0.24	0.06	0.12	0.19	0.19	0.1
IEEE 30	0.19	0.2	0.84	0.07	0.19	0.19	0.58	0.08
IEEE 57	0.32	0.38	0.76	0.14	0.32	0.35	0.79	0.17
IEEE 118	0.84	1.18	2.53	0.94	1.23	1.18	2.86	1.01
IEEE 300	2.27	2.63	4.88	2.17	1.96	2.22	5.23	3.61
Polish 2383	38.65	88.47	37.05	15.94	43.63	26.32	36.42	28.09
Polish 2746	42.43	20.67	51.71	8.97	82.02	39.66	43.78	12.85
Polish 3012	96.51	33.15	49.73	16.40	99.53	35.56	46.34	20.07

313 than KNITRO, IPOPT, and SLP, respectively, for the thermally constrained
 314 case.

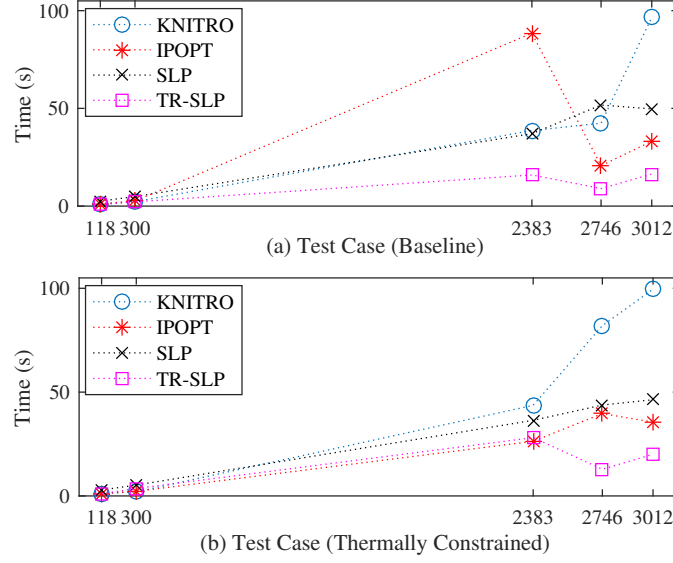


Figure 1: The computational scalability comparison among all solvers/algorithms for various IEEE (118, 300) and Polish (2383, 2746, 3012) systems. TR-SLP is simulated under the flat start strategy.

314
 315 Figure 1 reports the computational time growth among all solution ap-
 316 proaches for both the baseline and thermally constrained cases. It can be seen
 317 that both SLP and TR-SLP give almost linear increase in time against the
 318 test case size. This demonstrates to the better scalability of these LP based
 319 approaches in case of large-size optimization problems.

320 4.2. Case Study 2: Relative Optimality Comparison

321 In this study, TR-SLP is executed with the flat start strategy as well as
 322 with the DC warm start strategy for the OPF problem (2). As pointed out in
 323 [6, Section V], multi-start mode increases the probability of finding better local
 324 solutions for KNITRO-MS. Hence, we assume KNITRO-MS solutions as one
 325 of the best known solution sets for our benchmarking. Therefore in terms of

Table 3: The Relative Quality in the Optimality of TR-SLP Solution in Comparison to KNITRO-MS. TR-SLP is Executed under the DC Warm Start Strategy.

Test Case	Baseline Case		η	Thermally Constrained Case		
	Optimality (\$/h)			Optimality (\$/h)		
	KNITRO-MS	TR-SLP		KNITRO-MS	TR-SLP	
IEEE 14	8,091	8,088	0.03%	9,294	9,270	0.26%
IEEE 30	575	575	0%	582	583	-0.51%
IEEE 57	41,817	41,769	0.15%	41,978	41,961	0.04%
IEEE 118	129,903	129,895	0%	135,189	135,017	0.13%
IEEE 300	720,149	720,414	-0.03%	726,794	726,192	0.08%
Polish 2383	1,858,447	1,857,929	0.03%	1,863,627	1,862,377	0.07%
Polish 2746	1,185,115	1,208,277	-1.91%	1,185,507	1,208,281	-1.80%
Polish 3012	2,581,020	2,581,810	-0.03%	2,597,386	2,583,008	0.6%

326 optimality, we compare the quality of the solution obtained by TR-SLP against
 327 the KNITRO-MS solution. For Table 2 and Table 3, we define the following
 328 performance metric:

$$\eta = \frac{\text{KNITRO-MS} - \text{TR-SLP}}{\text{KNITRO-MS}} \times 100\% \quad (15)$$

329 where η indicates the relative improvement (if η is positive) or deterioration (if
 330 η is negative) in optimality of the solution obtained by TR-SLP with respect to
 331 the best known KNITRO-MS solution.

332 Table 2 reports the optimal solution obtained using KNITRO-MS and TR-
 333 SLP (with the flat start strategy) for the baseline and thermally constrained
 334 cases. In both cases, it can be observed that for half of the IEEE and Polish
 335 systems, TR-SLP results in slightly improved optimality with respect to the
 336 KNITRO-MS (ranging 0.03 % to 0.17 %). Performance of TR-SLP for the
 337 Polish 2746 system is observed to be slightly suboptimal for both the baseline
 338 and thermally constrained cases. However, KNITRO-MS is computationally
 339 slower than TR-SLP due to the multi-start feature, and represents a trade-off
 340 for using multi-start in practical OPF applications.

341 Table 3 reports the optimal solution obtained using KNITRO-MS and TR-
 342 SLP with DC warm start strategy for the baseline and thermally constrained
 343 cases. The convergence and optimality of the solution of TR-SLP depends upon
 344 the initial point. As such we noted TR-SLP with the DC warm start strategy
 345 locates slightly different optimal values compared to those of TR-SLP with the
 346 flat start strategy. Hence, in order to compare the relative effectiveness between
 347 the flat start and DC warm start strategies, we analyze the number of iterations
 348 taken under each strategy to converge to the final solution. Figure 2 depicts
 349 the TR-SLP iterations of the IEEE (118, 300) and Polish (2383, 2746, 3012)
 350 systems for both the baseline and thermally constrained cases. We observe that
 351 the DC warm start strategy speeds up the convergence compared to the flat start
 352 strategy, except for the Polish 2383 system. However, both strategies converge
 353 within the maximum number of iterations set for TR-SLP, *i.e.* $K = 30$.

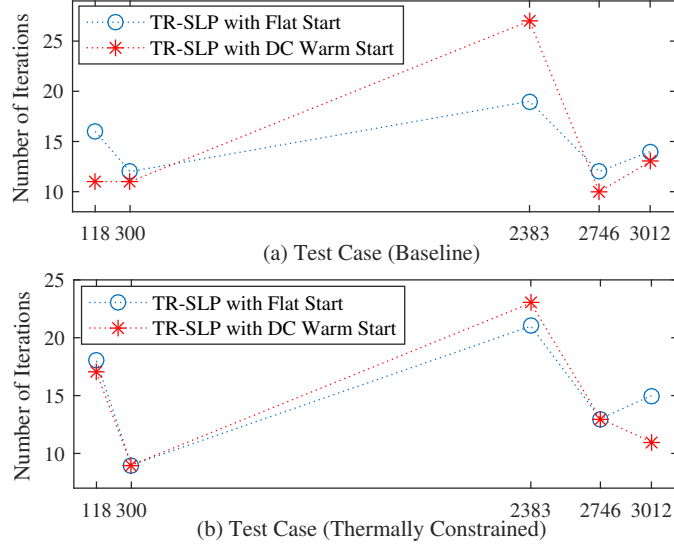


Figure 2: The relative comparison in iterations for the convergence of TR-SLP under flat start and DC warm start strategies for various IEEE (118, 300) and Polish (2383, 2746, 3012) systems.

4.3. Case Study 3: Discussion on Trust-Region Activation

In this study, we discuss the two main components in TR-SLP, *viz* the trust-region linear program (TR-LP) and feasibility restoration (NLP-FR). Specifically, we report the findings about the exact number of occurrences of TR-LP and NLP-FR in the execution of TR-SLP. We also study the effects of activation of a trust-region band on the execution of TR-SLP. The obtained results are reported for both the baseline and thermally constrained cases under the flat start strategy.

It can be seen in Figure 3 that NLP-FR is activated at least once in all the test cases, irrespective of the baseline or thermally constrained case. The NLP-FR was activated only once in all the test cases, except for the IEEE 118-bus system in the thermally constrained case, in which it was activated twice. Therefore, in all these cases the TR-SLP convergence is mainly governed by the TR-LP component. Furthermore, the relative propagation of the maximum

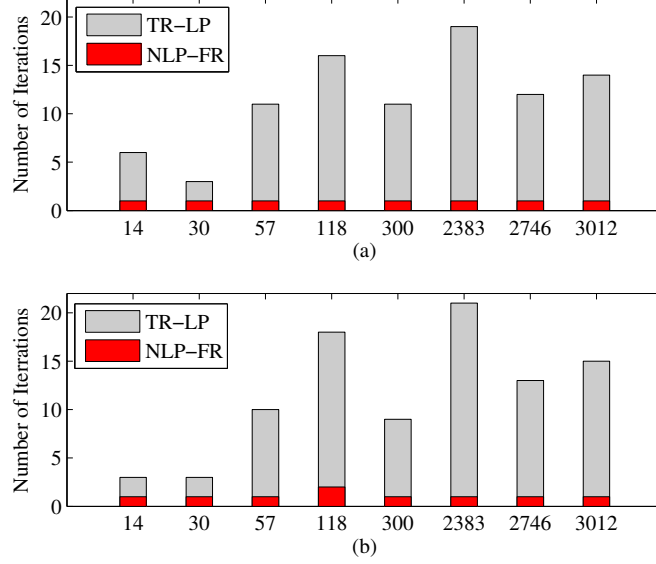


Figure 3: The relative comparison of NLP-FR and TR-LP executions for (a) baseline case and (b) thermally constrained case under flat start strategy for various IEEE (118, 300) and Polish (2383, 2746, 3012) systems.

step-size (d^k) along with a trust-region band ($[-\Delta^k, \Delta^k]$) is illustrated in Figure 4 for the Polish 2746 and 3012-bus systems. We note that NLP-FR has been first executed in the four simulations. Then, the trust-region initially expands due to insufficient closeness of the TR-LP to the original NLP. However, as the TR-SLP progresses the trust-region becomes smaller, converging to the maximum step-size solution within the specified solution tolerance band (ϵ_d). Overall, from Figure 4 it can be concluded that the trust-region assists convergence of the linear approximation neatly to the final solution.

4.4. Case Study 4: Relative Efficiency with Tightening the Tolerances

In this study, we further tighten the tolerances in TR-SLP and evaluate its impact on the quality of the OPF solution. For brevity, we restrict ourselves to the large test cases (Polish 2746, 3012) and TR-SLP is executed under the DC

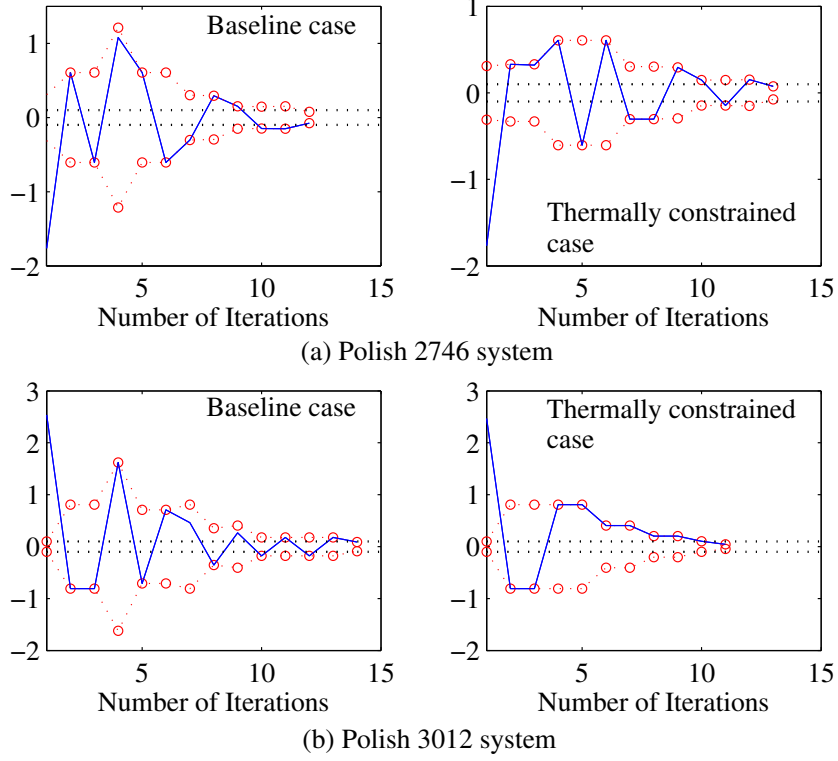


Figure 4: The relative propagation for the maximum step-size $\|d^k\|_\infty$ (solid line), trust-region band $[-\Delta^k, \Delta^k]$ (dotted o line), and solution tolerance band $[-\epsilon_d, \epsilon_d]$ (dotted line) under flat start strategy for Polish (2746, 3012) systems.

380 warm start strategy. Initially, the tolerances are set to $\epsilon_d = 0.1$, $\epsilon = \epsilon_\lambda = 0.01$
 381 (column A in Table 4), and are then tightened to $\epsilon_d = 0.01$, $\epsilon = \epsilon_\lambda = 0.001$
 382 (column B in Table 4). The maximum number of iterations K is set to 100 in
 383 this case study. The results of the two experiments are showcased in Table 4. In
 384 comparison, the constraints are satisfied more accurately in the second experi-
 385 ment (as shown in column B) which is expected due to the tolerance tightening.
 386 However, we observed that this does not benefit the TR-SLP algorithm in a
 387 significant manner. For instance, the optimality improves only by very small
 388 amount relative to that of the first experiment (shown in column A); which is

less than 0.001% for Polish 2746 and approximately 0.0018% for Polish 3012 test systems. On the other hand, the total TR-SLP iterations and the corresponding computational time required to reach the desired optimality are increased by approximately 2 to 7 times. Therefore, this experiment shows that the original tolerances used in TR-SLP are good enough to reach the optimal solution while satisfying constraints for the test cases considered in the study.

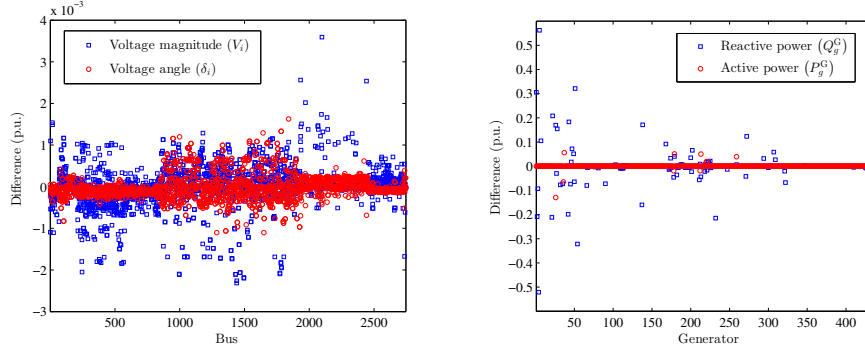
Table 4: Computational Results with Tightening the Tolerances ($\epsilon_d, \epsilon, \epsilon_\lambda$) in TR-SLP under the DC Warm Start Strategy.

Test case	Performance metrics for TR-SLP		Tolerance settings	
			A	B
Polish 2746	Optimality (\$/h)		1,208,281	1,208,279
	Iterations		13	89
	Computational time (s)		12.85	85.45
	Constraint satisfaction*	Max	2.07×10^{-4}	7.46×10^{-7}
		Mean	1.99×10^{-7}	1.09×10^{-9}
Polish 3012	Optimality (\$/h)		2,583,008	2,582,962
	Iterations		15	35
	Computational time (s)		20.07	51.29
	Constraint satisfaction*	Max	7.59×10^{-5}	7.86×10^{-7}
		Mean	6.79×10^{-8}	5.45×10^{-10}

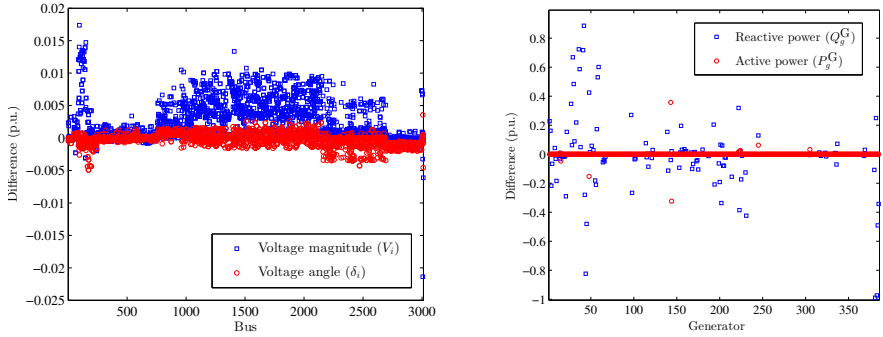
*indicates the accuracy up to which constraints (2b)–(2e) are satisfied.

In addition, let x_A be the solution of the experiment A and x_B be the solution of the experiment B. Figure 5 depicts the difference ($x_A - x_B$) in the final solutions of the two test systems in the two experiments. It can be observed that active power dispatch, which is related to the objective function, varies less than reactive power. Further, the voltage angle variation is also small. This may be due to the quasi-linear relationship of active power and voltage angles. However, the reactive power dispatch and voltage magnitudes are not included in the cost function, but only play a role in constraint satisfaction. Therefore,

we can observe slightly more variation in these two variables.



(a) Polish 2746 system.



(b) Polish 3012 system.

Figure 5: The difference between the bus voltages, voltage angles, active and reactive power values of the Polish 2746 system in (a) and Polish 3012 system in (b) under different tolerance settings (experiments A and B in Table 4). The vertical axis represents the magnitude difference ($x_A - x_B$) in p.u.

403

404 5. Conclusions

405 We reported the classical AC-OPF formulation and proposed the SLP-based
 406 approach to solve it. The linear models formulated in the SLP approach are
 407 valid only at the vicinity of the linearization point. Therefore, a trust-region
 408 that bounds variations in the decision variables was introduced to tighten the
 409 **SLP approximation**. This ensures the convergence of SLP approximation for the

OPF problem (referred to as TR-SLP in this work). In addition, we also proposed the feasibility restoration phase based on the original nonlinear constraints to quickly locate a feasible point when the **SLP approximation** is infeasible. Results show that our TR-SLP approach outperforms KNITRO, IPOPT and a recently reported SLP method based on penalty reformulations [6] in terms of computational time.

We also used two *generic starting point strategies* (flat and DC warm start) and the OPF results on IEEE and Polish systems demonstrated the capability of TR-SLP to locate good local optimal solutions. It was observed, with these two starting strategies, in some cases, TR-SLP converges to a better solution than KNITRO-MS. It would be interesting to develop and study good starting point strategies for TR-SLP in future.

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