



**Improved image-based deformation measurement for
geotechnical applications**

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Improved image-based deformation measurement for geotechnical applications

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1 **IMPROVED IMAGE-BASED DEFORMATION**
2 **MEASUREMENT FOR GEOTECHNICAL APPLICATIONS**

3 *S. A. Stanier, J. Blaber, W. A. Take and D. J. White*

4 **ABSTRACT**

5 This paper describes and benchmarks a new implementation of image-based deformation
6 measurement for geotechnical applications. The updated approach combines a range of
7 advances in image analysis algorithms and techniques best suited to geotechnical
8 applications. Performance benchmarking of the new approach has used a series of artificial
9 images subjected to prescribed spatially-varying displacement fields. An improvement by at
10 least a factor of ten in measurement precision is achieved relative to the most commonly used
11 particle image velocimetry (PIV) approach for all deformation modes, including rigid body
12 displacements, rotations and strains (compressive and shear). Lastly, an example analysis of a
13 centrifuge model test is used to demonstrate the capabilities of the new approach. The strain
14 field generated by penetration of a flat footing and an entrapped sand plug into an underlying
15 clay layer is computed and compared for both the current and updated algorithms. This
16 analysis demonstrates that the enhanced measurement precision improves the clarity of the
17 interpretation.

1 INTRODUCTION

2 Measurement of soil deformations in geotechnical models using Particle Image Velocimetry
3 (PIV) techniques (Adrian 1991) – also known as Digital Image Correlation (DIC) (Sutton et
4 al. 1983, Sutton et al. 2000) – has become routine experimental practice in many geotechnical
5 research laboratories over the past fifteen years (White et al. 2001, White et al. 2003,
6 Rechenmacher and Finno 2004, Iskander 2010, Hall 2012, Take 2015). The capabilities of the
7 algorithms and analysis techniques that underpin PIV/DIC have also significantly improved
8 over the same timeframe.

9 Figure 1(a) illustrates the typical setup of a geotechnical PIV/DIC analysis for a shallow
10 footing experiment. A wide range of deformation levels are present and accurate and precise
11 displacement data is desired in both the near- and far-field regions of the model. In a typical
12 PIV/DIC analysis, a Region of Interest (RoI) is first defined within the initial ('reference')
13 image of the model and populated with a mesh of subsets (or 'patches') of user-defined size.
14 The displacements of these subsets in subsequent ('target') images are found using one of the
15 approaches illustrated in Figure 1(b). Most freely available PIV/DIC software used for
16 geotechnical analyses use some form of cross-correlation to obtain integer pixel
17 displacements followed by sub-pixel interpolation of the correlation peak (e.g. GeoPIV
18 (White et al. 2003); MatPIV (Sveen and Cowen, 2004); PIVlab (Thielicke and Stamhuis,
19 2014) and OpenPIV (Taylor et al. 2010)). In these algorithms the subsets (or 'patches') are
20 generally not allowed to deform (so-called zero-order deformation).

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21 More sophisticated PIV/DIC algorithms exist, and are introduced later in this paper. They
22 incorporate higher-order subset shape functions (typically first-order, thus allowing
23 displacement gradients across the subsets), image intensity interpolation and deformation
24 parameter optimization, (e.g. Pan et al. 2006, 2012, Sutton et al. 2000). Some of these
25 advances have been incorporated in commercially-available PIV/DIC software (e.g. Vic-2D

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(Correlated Solutions, 2015); LaVision DaVis8 (LaVision, 2015); MI-2D (Match-ID (2015)) although these are not freely available to the academic community and the specific algorithms used are often proprietary. For applications in which small strains are of interest, such as geotechnical modeling and structural monitoring, photogrammetric corrections are often needed to obtain sufficiently accurate PIV/DIC results and these correction routines are not usually integrated within either freeware or commercial programs.

The purpose of this paper is to: (i) describe the advantages that the more sophisticated genera of PIV/DIC algorithms provide for geotechnical applications involving small and large deformations; and (ii) quantify these advantages via benchmark cases using a freely available non-commercial algorithm, that is well-suited to the analysis of geotechnical model test images.

The specific software used for the benchmark cases is referred to as GeoPIV-RG and is an update of the GeoPIV program (which represents the typical algorithms currently used in research, and is described by White et al. (2003)). A brief overview of the computational approach is first given. The comparison is then performed, using artificial 'soil-like' images subjected to various modes of deformation. Lastly, an example application is given that illustrates the impact this improvement in measurement precision can have on the interpretation of a classical geotechnical problem.

COMPUTATIONAL METHOD

Digital images captured during a geotechnical model test are usually analysed in sequence, starting with an initial 'reference' image. If the 'reference' image is retained as the initial image (the so-called 'leapfrog' scheme; see Figure 2(a)) then zero-order subsets can suffer a loss of correlation in regions experiencing large deformations (Figure 2(d)) due to a mismatch between the subset shape and the deformation being observed. Alternatively, if the 'reference' image is updated after every computation (the so-called 'sequential' scheme; see Figure 2(b))

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1 so as to minimize the distortion that would reduce the correlation in the 'target' images,
2 random walk errors are accumulated (White et al. 2003) because the overall displacement is
3 being found as the sum of many small displacements, each of which have an associated error
4 (Figure 2(d)). Random walk errors can become significant in regions of low deformation
5 when calculating strains from the derivatives of displacement.

6 The current version of GeoPIV uses a combination of these two schemes to minimise
7 accumulated random walk errors whilst maintaining tolerable correlations. The number of
8 increments to be performed using the 'leapfrog' scheme prior to updating the 'reference'
9 image (the so-called 'leapfrog' parameter) is manually defined by the user by trial and error
10 guided by the amount of deformation occurring between sequential images. Also, the 'search
11 zone' over which the correlation measure is computed for each subset (s_{zone}) is specified by
12 the user in GeoPIV. Unnecessarily large values of s_{zone} lead to computational inefficiency in
13 regions of images experiencing small displacements so small values are preferred. However,
14 the value specified must be larger than the maximum displacement expected to occur between
15 the 'reference' and 'target' images (see Figure 2(a,b)). Therefore, s_{zone} also cannot be
16 predetermined and requires further trial-and-error refinement to achieve the best balance
17 between accuracy and computational efficiency.

18 The new implementation presented in this paper avoids the need for trial-and-error refinement
19 of either the 'leapfrog' value or the 'search zone' parameter by following the process
20 illustrated in Figure 2(c) for each subset. The overarching framework controlling the
21 computation process is the Reliability-Guided (RG) method proposed by Pan (2009), as
22 implemented in Matlab by Blaber et al. (2015) (so the software is referred to as GeoPIV-RG).
23 Each 'reference' subset is allowed to deform using a shape (or warp) function describing first-
24 order deformations in conjunction with image intensity interpolation techniques to improve
25 the correlation between 'reference' and 'target' subsets via optimisation (Schreier and Sutton
26 2002). After an initial 'seed' subset has been analysed, subsequent computations are

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1 preconditioned using the results from the previously computed neighbouring subset that has
2 the highest correlation (the so-called 'preconditioned optimisation' scheme; see Figure 2(c)).
3 This approach leads to the definition of a 'search zone' being unnecessary whilst allowing the
4 effective 'search zone' to be the whole image if necessary should the preconditioning process
5 not yield a close match to the optimised solution for any particular subset. The 'reference'
6 image is updated when the correlation coefficient for either the seed or one of the subsets
7 contravenes user-defined thresholds, effectively optimising the 'leapfrog' parameter. The
8 first-order subset shape function (which allows for linear gradients of displacement across the
9 subset) leads to improved precision and reduced random walk errors (see Figure 2(d))
10 because correlation is better preserved allowing the 'reference' image to be updated less
11 frequently for image sequences experiencing low deformations. The overall programmatic
12 structure of the implementation is summarised by the flowchart in Figure 3.

13 **Seed Computation**

14 The computation process begins at a selected subset (circular in shape in this instance) from
15 part of the RoI that experiences minimal deformation, meaning that the correlation between
16 the 'reference' and 'target' subset will be high and thus the chances of an incorrectly
17 computed seed occurring will be low. This point is used as a 'seed' from which the RG
18 computation process propagates. The displacement of this subset is computed following the
19 procedure outlined in Figure 4(a). Initially, the displacement of the seed subset is estimated to
20 the nearest integer pixel value using Normalised Cross Correlation (NCC) (Lewis, 1995).
21 Next the subset is allowed to deform using a subset shape function (p) that describes a
22 superposition of the first order subset deformation modes illustrated in Figure 5 (Pan et al.
23 2006). The Inverse Compositional Gauss-Newton (IC-GN) method described by Pan et al.
24 (2013) is used, in combination with bi-quintic B-spline interpolation of the deformed subset
25 pixel intensities (e.g. Cheng et al. 2002) to adjust the subset shape function until the
26 correlation between the 'reference' and 'target' subsets is optimised. The exit criterion for the

1 optimisation is a user-defined maximum magnitude for the norm of the subset shape function
2 difference vector ($|\Delta p|_{max}$) between successive computations (typically 1×10^{-5}) and a
3 maximum number of iterations (max_{iter}) per subset (typically 50). The zero-normalised cross-
4 correlation coefficient (CC_{ZNCC}) is used to indicate the degree of match where
5 values of 1, 0 and -1 indicate perfect, zero and inverse correlations respectively (Pan et al.
6 2010). The seed computation is deemed successful if the CC_{ZNCC} is greater than a user-defined
7 limit, $CC_{ZNCC-seed-tol}$ (typically 0.9).

8 **Reliability-Guided Computations**

9 Assuming the 'seed' computation was successful, a high-density grid of subsets is processed
10 using the RG framework outlined in Figure 4(b). Firstly the CC_{ZNCC} of the four subsets
11 surrounding the seed are estimated using the displacement and deformation parameters for the
12 seed subset as an initial estimate (Zhou and Chen, 2012). These four subsets are then placed
13 in a queue of descending CC_{ZNCC} from which the subset with the optimal correlation
14 coefficient is selected first. IC-GN and bi-quintic B-spline interpolation is once again used to
15 optimise the deformation parameters for this subset, then it is removed from the queue. If not
16 already in the queue, the CC_{ZNCC} is computed for the neighboring subsets and those subsets
17 are added to the queue. This process repeats, calculating the displacement and deformation of
18 all of the subsets across the entire RoI. The advantage of the RG framework over the usual
19 processing of subsets across successive rows is three-fold: firstly, the NCC, which is
20 computationally expensive to determine, is only computed for the seed subset and covers the
21 entire region of interest. Secondly, the subsets with higher correlation coefficient are
22 processed first, and the optimised deformation parameters used to precondition the IC-GN
23 optimisation of the neighbouring subsets. Thirdly, as a result of this approach, the need to
24 specify the expected maximum displacement within the displacement field is eliminated.

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1 Reference Image Updating

2 Compared to conventional PIV/DIC applications, geotechnical model testing can involve
3 tracking of a larger range of deformations and highly circuitous displacement paths. For
4 example soil often flows around penetrometers and deeply buried foundations. Unlike fluid
5 mechanics studies, where constitutive relations are not a focus, geotechnical research is
6 concerned with both instantaneous flow fields and also the strain path histories, stress-strain
7 behaviour and the associated constitutive relations. Additional measures are therefore
8 required to handle the resulting changes in subset appearance, for example due to grain
9 rearrangement, because these lead to a reduction in the subset correlation and cause erroneous
10 displacements to be estimated (known as 'wild' vectors). To counter this degradation of
11 correlation the 'reference' image can be periodically updated. In earlier versions of GeoPIV
12 the updating interval was specified manually by the user and refined through trial and error
13 (White et al. 2003). An automatic 'reference' image-updating scheme, similar to that
14 proposed by Pan et al. (2012), is used in GeoPIV-RG. After completion of the RG
15 computations for each target image the CC_{ZNCC} for each subset is compared to a second,
16 slightly relaxed, user-defined limit (typically 0.75) denoted $CC_{ZNCC-min-tol}$. Using a relaxed
17 tolerance on the minimum permissible full field CC_{ZNCC} allows large deformations to occur in
18 certain regions of the model prior to 'reference' image updating. If the CC_{ZNCC} for any subset
19 is less than $CC_{ZNCC-min-tol}$ then the 'reference' image is updated to the target image from the
20 last successful increment, otherwise the current 'reference' image is carried forward.

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21 The RG method is programmed to compute the displacement of regularly gridded subsets.
22 Consequently, an interpolation scheme is required to compute the displacement of the user-
23 defined subset locations from the regularly gridded RG output. Bi-cubic spline interpolation
24 of the output from the RG process achieves this. To safeguard the precision of the
25 measurements during the interpolation process, the subset spacing used in the RG process is
26 reduced relative to that given in the user-defined mesh. For the benchmarking analyses

1 presented in this paper halving the grid spacing was sufficient to preserve accuracy of the
 2 measurements. Due to the computational efficiency of the preconditioned IC-GN optimisation
 3 process, the computational cost of this four-fold increase in RG computation grid density is
 4 minimal. The 'reference' image updating procedure ensures that the updating interval is
 5 always optimised. The user can control the frequency of 'reference' image updating indirectly
 6 by varying the correlation coefficient tolerances, $CC_{ZNCC-seed-tol}$ and $CC_{ZNCC-min-tol}$, with stricter
 7 values resulting in more frequent 'reference' image updating.

8 PERFORMANCE COMPARISON

9 Methodology

10 The performance of GeoPIV-RG is compared to GeoPIV (described by White et al. 2003 and
 11 White et al. 2005) which is widely used in geotechnical research and is typical of the many
 12 freely available zero-order algorithms (e.g. MatPIV, PIVlab and OpenPIV). Therefore the
 13 following benchmarking analyses are generally indicative of the improvements in
 14 measurement precision that can be attained by incorporating advances in PIV/DIC including
 15 first-order subset shape functions, image intensity interpolation, deformation parameter
 16 optimisation and automatic 'reference' image updating schemes similar to those described
 17 earlier.

18 Artificial images that represent geomaterials are preferred for such benchmarking as they can
 19 be subjected to precisely prescribed deformations and are unaffected by camera-induced lens
 20 distortions and camera-target movements (Lee et al. 2012). The images were generated in
 21 Matlab by randomly projecting thousands of white dots onto a black background to sub-pixel
 22 positional resolution. Each white dot is defined by a Gaussian brightness peak centered at a
 23 specified location. In this way the location of the dot can be precisely controlled allowing
 24 smooth spatially-varying displacement fields to be prescribed. The 'reference' artificial image
 25 used in all of the artificial benchmarking analyses presented herein is shown in Figure 6(a)

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1 with the 1681 subset locations marked by yellow crosses in a centrally located zone, of $400 \times$
2 400 pixels. This subset population is sufficiently large to generate statistically valid
3 measurements of the error in the image-based displacement measurements. For GeoPIV the
4 side length of the square subsets, L_s , was 45 pixels, while for GeoPIV-RG the subset
5 diameter, D_s , was taken as 50 pixels giving comparable total pixels per subset (within $\sim 3\%$).

6 The theoretical and measured displacements at the 1681 subset locations were compared for
7 four deformation modes: (i) rigid body translation, (ii) rigid body rotation, (iii) vertical strain
8 and (iv) pure shear strain as illustrated in Figure 6 (b-e). For rigid body translation,
9 displacement was applied in 0.025 pixel increments up to a maximum of 1 pixel. For the
10 rotation, vertical strain and pure shear strain analyses the deformation magnitude imposed
11 was increased over 100 logarithmically spaced intervals up to a maximum of 90° of rotation
12 and 50% strain. The total deformation applied was chosen such that 'reference' image
13 updating was periodically required so the efficacy of the full computational scheme has been
14 validated. For the analyses performed using the current version of GeoPIV the 'reference'
15 image was updated manually as infrequently as possible to minimise the summation of
16 random walk errors.

17 The precision error in the displacement measurements is quantified by the standard error, ρ_{px} ,
18 defined as the standard deviation of the difference between the theoretical and calculated
19 subset displacement over the 1681 subsets. It is shown later that this error increases as the
20 deformation of the subset increases. To convert these standard errors to a measure of the
21 precision with which deformations can be determined, the measurement errors (δ) are defined
22 as the error in the measured deformation, for a given level of that deformation mode. An
23 estimate of the random error δ for each mode (rotation, vertical strain and pure shear strain)
24 can then be defined as:

$$1 \quad \delta_{\theta} = \tan^{-1} \left(\frac{\sqrt{2} \rho_{px}}{L} \right) \quad 1$$

$$2 \quad \delta_{\varepsilon_y} = \frac{\sqrt{2} \rho_{px}}{L} \quad 2$$

$$3 \quad \delta_{\varepsilon_{xy}} = \frac{\sqrt{2} \rho_{px}}{L} \quad 3$$

4 where θ , ε_y and ε_{xy} , denote rotation in degrees, vertical strain and pure shear strain respectively
 5 and L is a nominal gauge length in pixels. Each random error δ is estimated for $L = 25$, 250
 6 and 2500 pixels, which cover the range typically relevant. When viewing a geotechnical
 7 model test, the varying deformation throughout the image is of interest, so the relevant gauge
 8 length is comparable to the spacing of the subsets, i.e. of the order of 25 pixels. Alternatively,
 9 when viewing a geotechnical element test, at a stage when the deformation is uniform, the
 10 gauge length might be significantly larger, i.e. of the order of 2500 pixels. The δ estimates,
 11 combined with a tolerable measurement error, allow the required subset spacing to be
 12 identified. Alternatively, they show the image scale (e.g. pixels/mm) required in a model test
 13 to detect a specific level of deformation.

14 **Results**

15 *Rigid body translation*

16 The standard errors, ρ_{px} , in Figure 7(a) are ~ 0.005 and ~ 0.0008 pixels for GeoPIV and
 17 GeoPIV-RG respectively, indicating a modest improvement in precision for sub-pixel
 18 displacement measurement for the new methods. Meanwhile, Figure 7(b) presents the mean
 19 bias error, μ_{bias} (Schreier et al. 2000), which is the mean discrepancy between the actual and
 20 measured displacement, for the sub-pixel rigid body translation analysis. For GeoPIV, which
 21 uses NCC to obtain the integer pixel displacements prior to sub-pixel interpolation using bi-
 22 cubic splines, a significant periodic variation in μ_{bias} is evident for non-integer or non-half-

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1 integer displacements. This behaviour is consistent with that reported by Amiot et al. (2013)
2 for PIV/DIC software incorporating bi-cubic interpolation. In contrast, the bi-quintic B-spline
3 interpolation process used in the IC-GN optimisation of GeoPIV-RG suffers from mean bias
4 errors that never exceed 0.0005 pixels. This is comparable to the performance reported by Lee
5 et al. (2012) for NCC with bi-quintic B-spline sub-pixel interpolation and consistent with the
6 best performing PIV/DIC software (w.r.t. mean bias errors) reported by Amiot et al. (2013)
7 that also incorporated bi-quintic b-spline interpolation. Minimising bias errors is particularly
8 important if strains are to be derived from the derivatives of displacement fields. Periodic bias
9 can lead to erroneous localizations in strain fields if periodic bias errors are evident.

10 *Rigid body rotation*

11 Figure 8(a) shows the evolution of ρ_{px} with rigid body rotation. GeoPIV accumulates
12 significant errors with increasing rotation because the ability of NCC to accurately track
13 subset displacements progressively degrades with rotation (Dutton et al. 2014), causing
14 accumulating drift in the displacements. The iterative subset deformation optimisation
15 performed by GeoPIV-RG mitigates the degradation of correlation, resulting in a
16 comparatively small precision error of $\rho_{px} < 1/1000^{\text{th}}$ of a pixel, irrespective of the rotation
17 magnitude, whilst the mean errors are always less than 1×10^{-4} pixels. As a result, the
18 rotation error, δ_{θ} , is approximately constant for rotation magnitudes greater than 1° (Figure
19 8(b)).

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20 The divergence in performance between GeoPIV and GeoPIV-RG observed in Figure 8 has a
21 profound effect on the abilities of the respective algorithms to measure strain fields because
22 the magnitude of the error is random and not linked to the magnitude of the displacement of
23 the subset. To illustrate this an additional analysis was performed using the same analysis
24 parameters and artificial images for a horizontal row of subsets spaced at 1-pixel intervals at
25 increasing distance from the origin of rotation (L_o) up to a maximum of 200-pixels. For these
26 subsets and for all rotation increments, the rotation angle is the same but the displacement

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1 magnitude increases proportionally with the distance from the origin of rotation, as illustrated
 2 in Figure 9(a). The resultant error magnitudes (δ_r) were calculated for rotation angles of 0.0°,
 3 0.5°, 1.0°, 1.5° and 2.0° and are plotted with respect to the distance from the origin of rotation
 4 in Figure 9(b) and (d) for GeoPIV and GeoPIV-RG respectively.

5 From these results it is clear that the error magnitudes and directions are random and
 6 unrelated to the distance from the origin of rotation. However, the error magnitudes are
 7 clearly linked to the rotation angle as the error magnitude tends to increase with rotation
 8 angle, as is confirmed for GeoPIV in Figure 9(c) and GeoPIV-RG Figure 9(e) where the
 9 evolution of errors are presented with respect to the rotation angle imposed for subsets located
 10 at sections A–A, B–B and C–C.

11 For a basic PIV/DIC analysis where vector plots are used to illustrate soil flow mechanisms
 12 the poor performance of GeoPIV for rotation is not necessarily problematic as the large
 13 magnitude of the vectors within the soil flow mechanism will mask the errors induced by
 14 rotation. However, if strains are derived from the derivatives of the displacement components
 15 the errors become very significant. For example, for a strain element with length $L=25\text{px}$
 16 positioned either side of subset A–A, similarly to Equations 1-3, a generic estimate of the
 17 strain error (δ_ε) can be taken as follows:

$$18 \quad \delta_\varepsilon = \frac{2|\delta_r|}{L} \quad 4$$

19 For a rotation angle of 2°, the strain error for GeoPIV is of the order of ~3%, significantly
 20 limiting the ability to plot meaningful strain fields. On the other hand, the strain errors for
 21 GeoPIV-RG are less than ~0.03%, resulting in a two order of magnitude improvement in
 22 strain measurement resolution as a result of the additional degrees of freedom provided by the
 23 first order subset shape function. The impact of this improvement is significant for
 24 geotechnical research applications where more than simple instantaneous flow mechanisms

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1 are to be observed, such as when gross element distortions are to be monitored through large
2 rotations.

3 *Vertical and shear strain*

4 Figures 10(a) and 11(a) illustrate that measurement precision during deformation – either
5 through vertical strain or shear strain – is similar to rotation ($\rho_{px} \sim 1/1000^{\text{th}}$ pixel) until ~ 0.01
6 or 1% strain, beyond which the precision error rises approximately linearly with strain.

7 When converted to δ estimates (Figures 10(b) and 11(b)), these results allow the practicality
8 of detecting a given strain level to be assessed. For example, if a model test with a varying
9 deformation is being viewed, then a subset spacing of typically 25 pixels might apply. In this
10 case, if zones of the model have undergone strains of 1% or 10% (points M1 and M2 on
11 Figures 10b and 11b), the resulting strain errors would be $\sim 0.025\%$ and $\sim 0.25\%$ respectively
12 (equivalent to a signal-to-noise ratio, *SNR*, of ~ 40), which is likely to be adequate for
13 producing detailed and smooth deformation fields (e.g. 20 contours of 0.05% or 0.5% up to
14 the maximum of 1% or 10% respectively). Alternatively, if the application is an element test
15 with uniform deformation, so the gauge length is a larger proportion of the image width –
16 typically 2500 pixels in width – then at a strain of 0.1% (100 microstrain) the strain error is
17 $\sim 5 \times 10^{-7}$ (point E on Figures 10(b) and 11(b)) or 0.05 microstrain (equivalent to a signal-to-
18 noise ratio, *SNR*, of ~ 2000).

19 All of these artificial image analyses show that the new approach is at least an order of
20 magnitude more precise than the combination of NCC and bi-cubic spline interpolation
21 method employed by GeoPIV (White et al. 2003), for both small and large deformations.

22 EXAMPLE APPLICATION

23 To demonstrate the application of the new methodology, images from a model test performed
24 in the drum centrifuge at UWA are used. The test involved similar techniques to the work

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1 reported by Hu et al. (2014), investigating punch-through of a 30mm diameter flat footing (at
2 200g, so equivalent to 6m diameter at prototype scale) on a 20mm deep (4m prototype) sand
3 layer overlying clay. In the experiment ~550 images were recorded at a frequency of 5Hz
4 using the apparatus described by Stanier and White (2013). Artificial texture was applied to
5 the exposed face of the model at the optimal Artificial Seeding Ratio (*ASR*) following the
6 procedure proposed by Stanier and White (2013) to maximise the precision of the image-
7 based deformation measurements.

8 Analyses were performed on the series of images for the underlying clay layer only, using
9 both GeoPIV and GeoPIV-RG with the analysis settings summarised in Table 1. The time
10 taken by each of the algorithms to perform an analysis is dependent upon a number of factors,
11 including: the subset spacing, deformation magnitude, image texture quality and available
12 processing power. For this particular analysis GeoPIV-RG performed the computations in
13 ~20% of the time taken by GeoPIV. The total maximum shear strain ζ (i.e. $\Delta\varepsilon_1 - \Delta\varepsilon_2$ summed
14 through the deformation) was calculated from the displacement fields following the large
15 strain procedure of White and Bolton (2004).

16 Figure 12 presents the distributions of ζ after 1D of footing penetration. Significant noise is
17 evident in the results from GeoPIV from summed random walk errors and degradation of
18 correlation due to subset rotation and deformation. In contrast the analysis generated by
19 GeoPIV-RG has lower noise, as is evident in the regions experiencing small strains. Figure 13
20 shows a horizontal cross section through both analyses at an initial normalised depth, z/D of
21 1.5, presented in terms of both the normalised displacement magnitude and total maximum
22 shear strain. The first-order deformation algorithm of GeoPIV-RG results in smoother spatial
23 variation of both the displacements and strains across the model, compared to the stepped
24 cross-section resulting from the zero-order deformation algorithm of GeoPIV.

25 These enhancements result from the subset deformation optimisation capability of GeoPIV-
26 RG as it preserves correlation and precision in regions of large deformation. The 'reference'

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1 image also requires updating less frequently (see Table 1), which in turn minimises random
2 walk errors. These advances create more precise deformation measurements, which unlocks
3 additional potential applications. For example, more detailed verification of constitutive
4 models by extracting element-level responses within model tests, and the quantification of
5 geomaterial behavior at both smaller strains and higher levels of rotation and deformation
6 than was possible using previous image analysis methods.

7 CONCLUSIONS

8 ~~This paper has shown that recent advances in PIV/DIC algorithms coupled with~~
9 ~~photogrammetric correction routines allow improved deformation measurements~~ for
10 geotechnical applications. ~~The algorithms have been incorporated in an update of a~~
11 ~~commonly-used freeware PIV/DIC program. The prior version has been used as a benchmark~~
12 ~~representing the approaches commonly used in geotechnical physical modelling. The~~
13 ~~benchmarking used a~~ series of artificial soil-like images subjected to prescribed
14 displacements and deformations. The ~~advanced~~ algorithms are faster and more precise than
15 ~~the simpler zero-order PIV/DIC approach that is widely used and freely available to the~~
16 ~~research community~~. Rigid-body displacements can be detected to a precision of ~ 0.001
17 pixels. There is a modest reduction in precision when the tracked soil is deforming. The effect
18 of the gauge length (i.e. the separation of the measurement points) and the level of
19 deformation or rotation on the precision of deformation measurements is quantified. For
20 example, it is shown that when soil elements at close spacing experience rotation in a model
21 test, the new implementation is approximately two orders of magnitude more precise than the
22 existing approach, resulting in significantly less noise in strain fields derived from the
23 derivatives of the displacements. At the other end of ~~the~~ scale, in an element test in which
24 digital images are used to monitor the overall response of the sample (i.e. the gauge length is
25 significantly larger and taken here as 2500 pixels), the standard error is ~ 0.05 microstrain at a
26 strain of 0.1%.

Deleted: A new implementation of

Deleted: has been written and optimized

Deleted: This software, called GeoPIV-RG,

Deleted: benchmarked using

Deleted: new

Deleted: previously reported geotechnical PIV/DIC programs.

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1 An example analysis illustrates the value of the new approach by showing improved
2 measurement of deformations during punch-through of a flat footing on a sand-over-clay
3 stratigraphy. These results demonstrate the benefits of the enhanced measurement precision
4 provided by this software, which is freely available to the geotechnical research community.

5 ACKNOWLEDGEMENTS AND SUPPLEMENTARY DATA

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10 in Offshore Foundations and the Shell EMI Chair in Offshore Engineering (held by the fourth
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13 Centre at Queen's-RMC. The software code and example images relevant to this paper can be
14 downloaded from www.geopivrg.com.

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NOMENCLATURE

ASR	artificial seeding ratio
CC_{ZNCC}	zero normalised cross-correlation correlation coefficient
$CC_{ZNCC-min-tol}$	full field correlation coefficient tolerance
$CC_{ZNCC-seed-tol}$	seed correlation coefficient tolerance
d	displacement
δ_ε	strain error
$\delta_{\varepsilon_{xy}}$	shear strain error
δ_{ε_y}	vertical strain error
δ_r	resultant error
δ_θ	rotation error
D	diameter
D_s	diameter of GeoPIV-RG subset
ε_1	major principal strain
ε_2	minor principal strain
ε_{xy}	pure shear strain
ε_y	vertical strain
L	gauge length
L_s	length of GeoPIV subset
max_{iter}	maximum number of iterations per subset
<u>NCC</u>	<u>normalised cross correlation</u>
p	subset deformation shape function
$ \Delta p _{max}$	maximum norm of the shape function difference vector
px	pixel
ρ	undrained shear strength gradient

Deleted: Hz → frequency

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ρ_{px}	standard error of displacement measurement in pixels
r	radius
<u>RoI</u>	<u>region of interest</u>
s	subset spacing
SNR	signal-to-noise-ratio
s_{zone}	search zone parameter
u	horizontal displacement
v	vertical displacement
x	horizontal position
x_f	final horizontal position
x_i	initial horizontal position
y	vertical position
y_f	final vertical position
y_i	initial vertical position
z	depth
<u>$ZNCC$</u>	<u>zero normalised cross correlation</u>
ζ	total maximum shear strain

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FIGURES

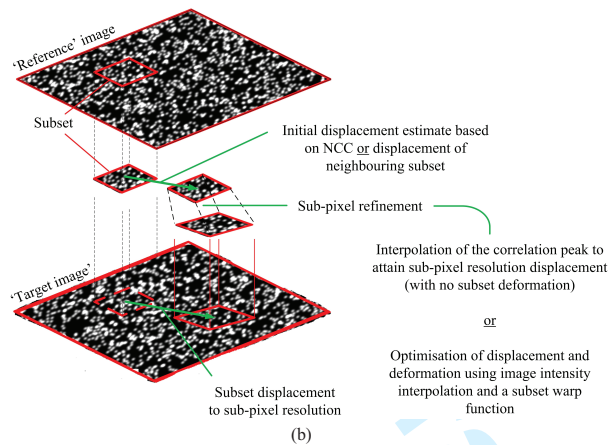
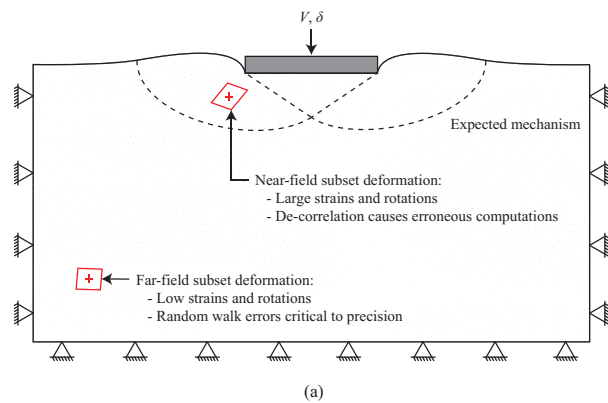
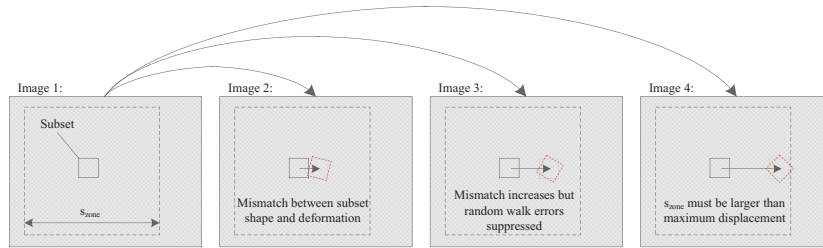


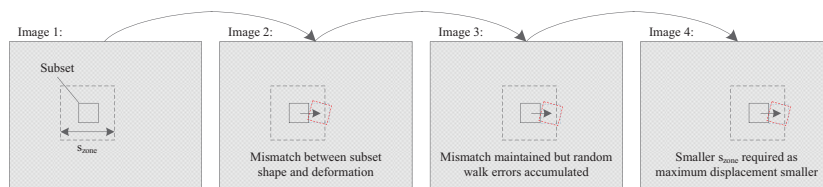
Figure 1: PIV/DIC analysis overview: (a) typical PIV/DIC scenario with associated causes of error, (b) general overview of the PIV/DIC method.

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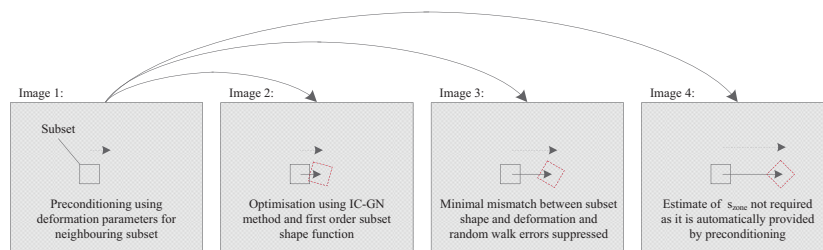
(a) GeoPIV 'leapfrog' scheme with zero-order subset shape function:



(b) GeoPIV 'sequential' scheme with zero-order subset shape function:



(c) GeoPIV-RG 'preconditioned optimisation' scheme with first-order subset shape function:



Key:
 □ - Subset location in 'reference' image
 □ - Computed subset location in 'target' image
 □ - Actual subset location in 'target' image
 □ - Mismatch between subset shape function and deformation
 → - Neighbouring computation used in preconditioning
 → - Computed displacement

(d) Schematics of anticipated performance for subset correlation (LHS) and random walk errors (RHS):

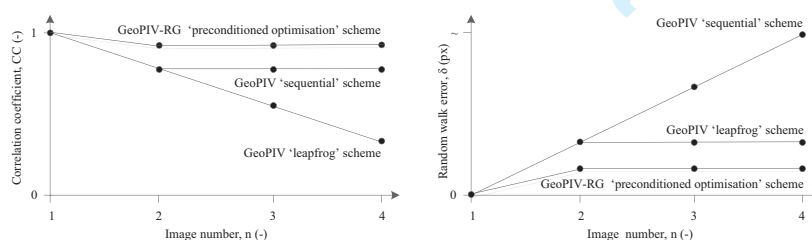


Figure 2: Overview of (a) 'leapfrog', (b) 'sequential', and (c) 'preconditioned optimisation' computation schemes alongside (d) schematic plots of the expected evolution of correlation coefficient and random walk errors.

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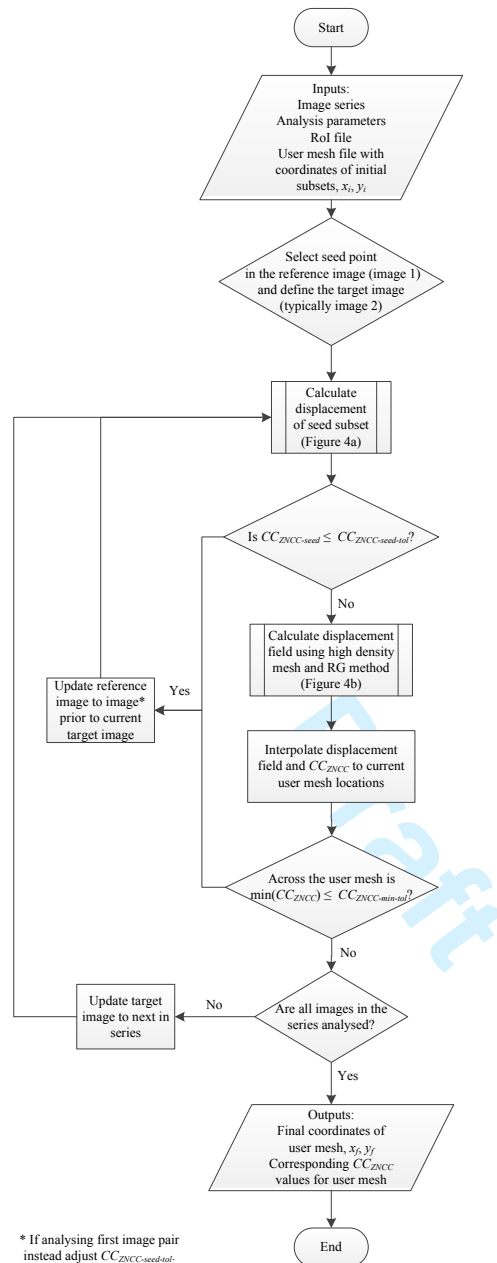


Figure 3: Flowchart for GeoPIV-RG computations.

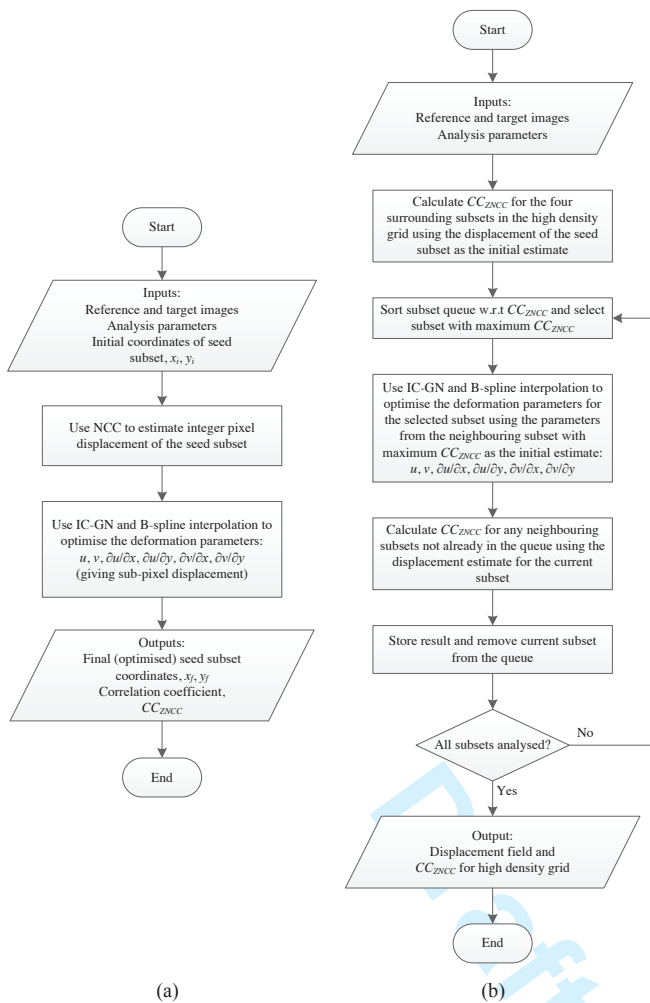


Figure 4: Flowcharts for seed subset (a) and RG subset computation sub-routines (b).

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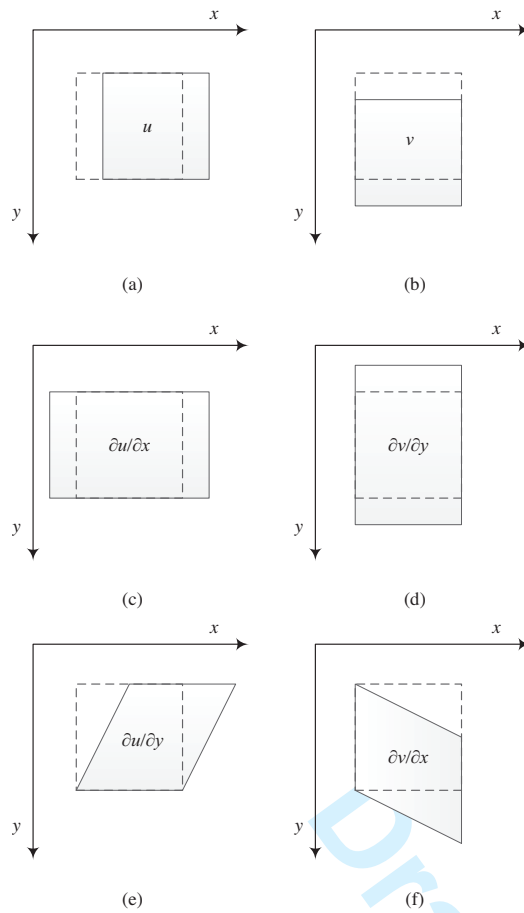


Figure 5: Subset deformation modes considered by the first-order shape function p : (a, b) displacements and (c-f) displacement gradients. Note: subsets are represented as squares for clarity but subset shape is arbitrary.

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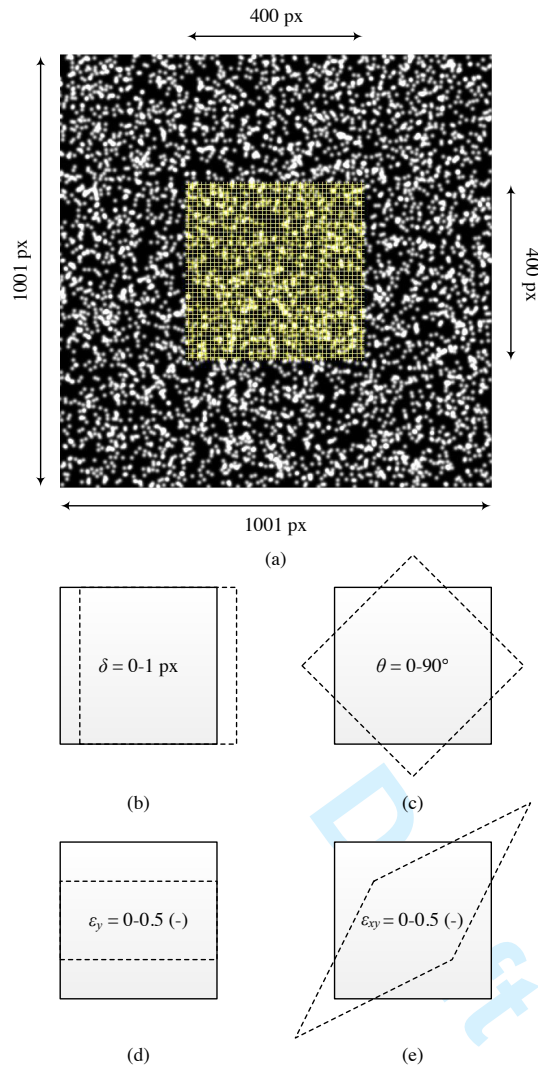


Figure 6: Artificial images and imposed deformations: (a) example image and subset locations (b) rigid body translation (c) rigid body rotation (d) vertical strain (e) pure shear strain.

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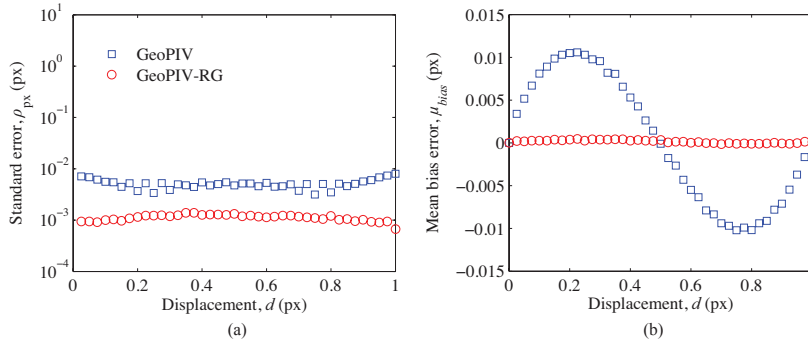
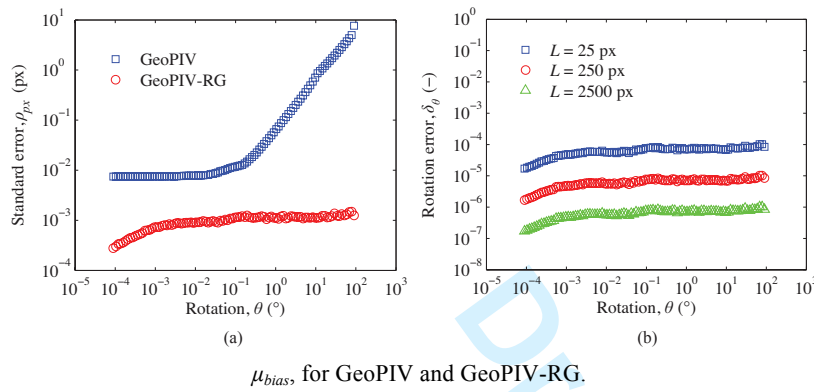


Figure 7: Rigid-body translation performance: (a) standard error, ρ_{px} , and (b) mean bias error,



μ_{bias} for GeoPIV and GeoPIV-RG.

Figure 8: Rotation performance: (a) standard error for GeoPIV and GeoPIV-RG (b) effect of gauge length on rotation error, δ_θ , for GeoPIV-RG.

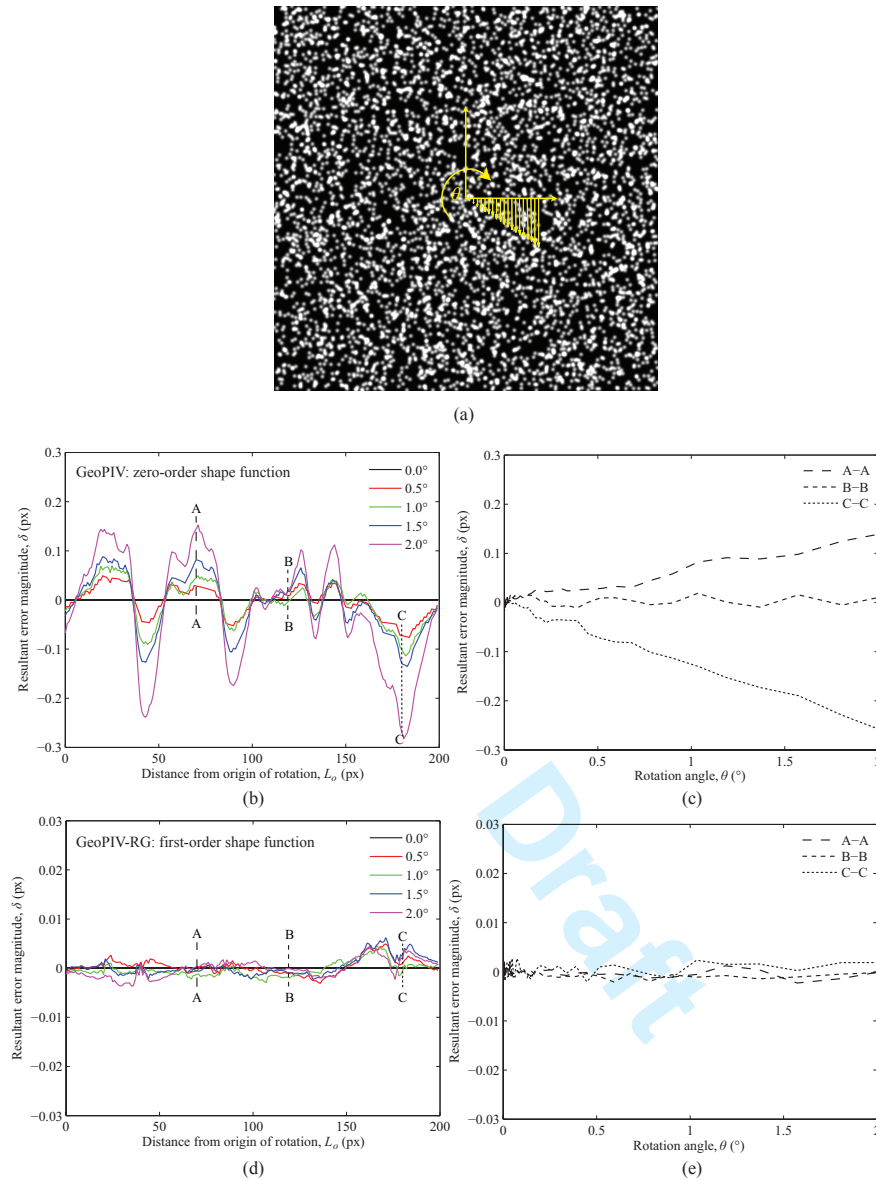


Figure 9: Randomness of rotation performance: (a) artificial image and a row of subset displacements (magnitudes amplified) illustrating the rotation imposed; (b,c) resultant error, δ_r , as a function of distance from the origin of rotation and rotation angle for GeoPIV; and (d,e) GeoPIV-RG. Note: Vertical scales are different between (a,b) and (c,d) for clarity.

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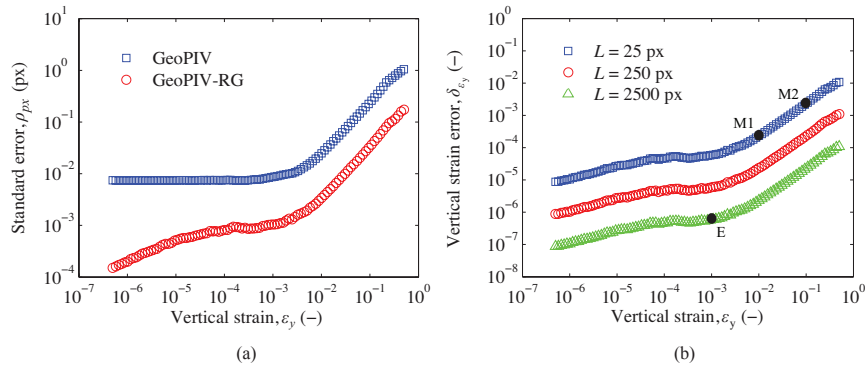


Figure 10: Vertical strain performance: (a) standard error for GeoPIV and GeoPIV-RG (b) effect of gauge length on vertical strain error, δ_{ϵ_y} , for GeoPIV-RG.

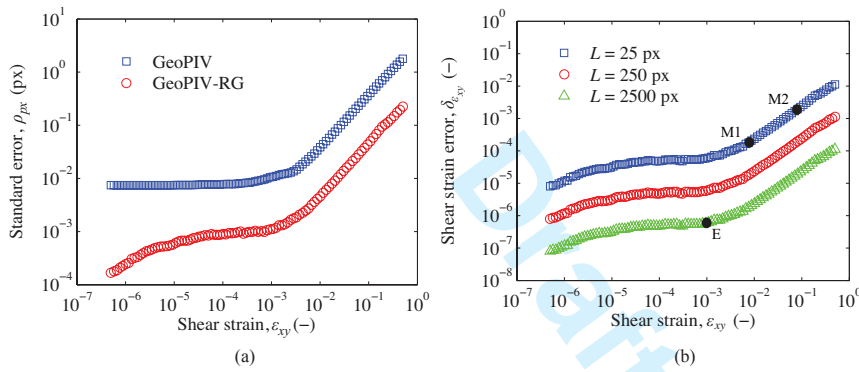


Figure 11: Shear strain performance: (a) standard error for GeoPIV and GeoPIV-RG (b) effect of gauge length on shear strain error, $\delta_{\epsilon_{xy}}$, for GeoPIV-RG.

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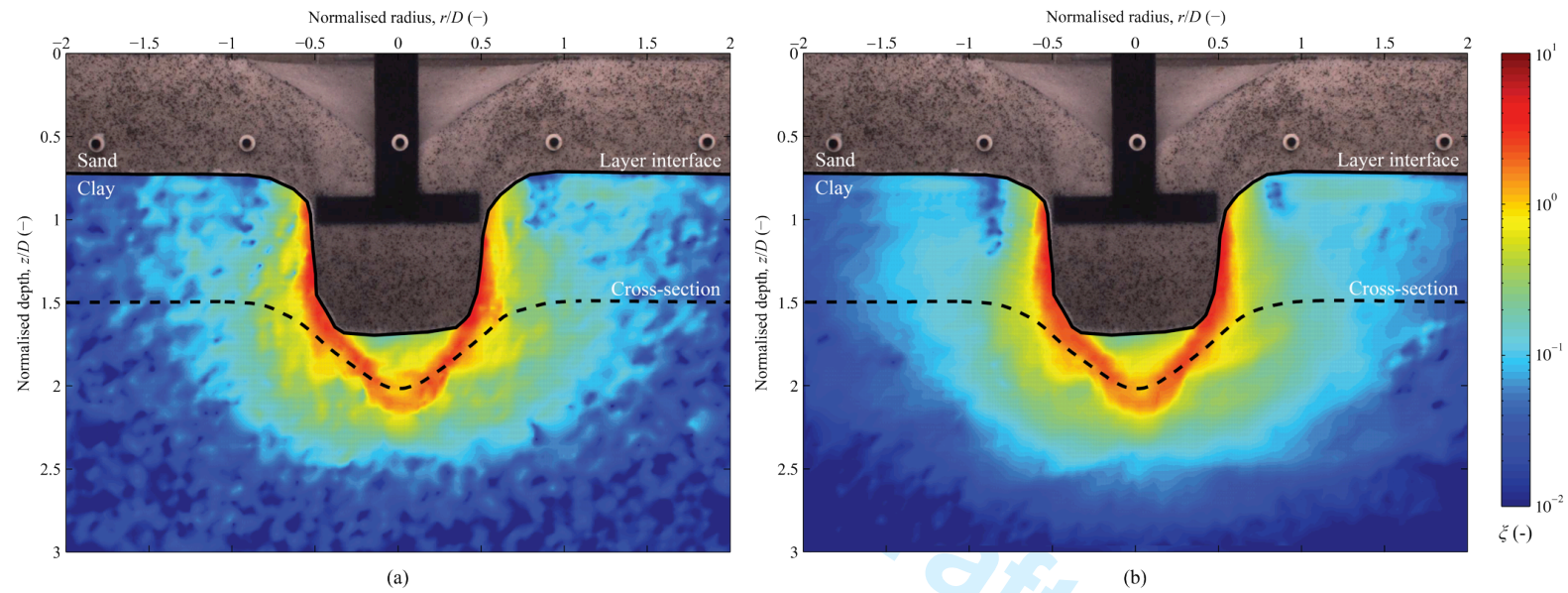


Figure 12: Example application: flat footing on sand-over-clay, total shear strain at $1D$ penetration using GeoPIV (a) and GeoPIV-RG (b).

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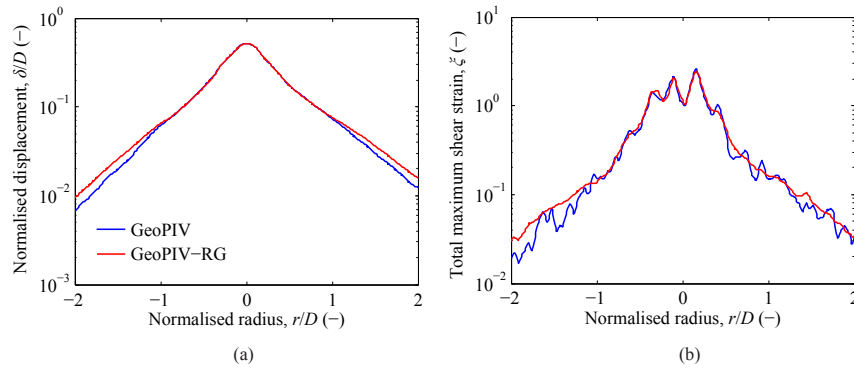


Figure 13: Normalised displacement, δ/D (a) and total maximum shear strain, ζ (b) along the cross-sections in Figures 12 (a, b) at an initial normalised depth, z/D of 1.5.

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TABLES

Table 1: Computation parameters for example analysis.

GeoPIV parameters	
L_s	45px ^a
s	25px
s_{zone}	15px
Ref. image updating interval	10
GeoPIV-RG parameters	
D_s	50px ^a
s	25px
max_{iter}	50
$ \Delta p _{max}$	1×10^{-5}
$CC_{ZNCC-seed-tol}$	0.9
$CC_{ZNCC-min-tol}$	0.75
Ref. image updating interval	~20

^a Subset sizes chosen to have equivalent area (within ~3%).

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