

20 **Question 1.1.** *Is it true that*

$$\lim_{n \rightarrow \infty} \frac{\hat{r}(P_n)}{n} = \infty \quad \text{and} \quad \lim_{n \rightarrow \infty} \frac{\hat{r}(P_n)}{n^2} = 0?$$

21 Answering Erdős' question, Beck [3] proved that the size-Ramsey number of paths is linear,
 22 i.e., $\hat{r}(P_n) = O(n)$, by means of a probabilistic construction. Alon and Chung [2] provided an
 23 explicit construction of a graph G with $O(n)$ edges such that $G \rightarrow P_n$. Recently, Dudek and
 24 Prałat [10] gave a simple alternative proof for this result (see also [21]). More generally, Friedman
 25 and Pippenger [14] proved that the size-Ramsey number of bounded-degree trees is linear (see
 26 also [8, 15, 17]) and it is shown in [16] that cycles also have linear size-Ramsey numbers.

27 A question posed by Beck [4] asked whether $\hat{r}(G)$ is linear for all graphs G with bounded
 28 maximum degree. This was negatively answered by Rödl and Szemerédi, who showed that there
 29 exists an n -vertex graph H and maximum degree 3 such that $\hat{r}(H) = \Omega(n \log^{1/60} n)$. The current
 30 best upper bound for bounded-degree graphs is proved in [19], where it is shown that for every Δ
 31 there is a constant c such that for any graph H with n vertices and maximum degree Δ :

$$\hat{r}(H) \leq cn^{2-1/\Delta} \log^{1/\Delta} n.$$

32 For further results on size-Ramsey numbers the reader is referred to [5, 18, 25].

33 Given an n -vertex graph H and an integer $k \geq 2$, the k th power H^k of H is the graph with
 34 vertex set $V(H)$ and all edges $\{u, v\}$ such that the distance between u and v in H is at most k .
 35 Answering a question of Conlon [6] we prove that all powers of paths have linear size-Ramsey
 36 numbers. The following theorem is our main result.

37 **Theorem 1.2.** *For any integer $k \geq 2$,*

$$\hat{r}(P_n^k) = O(n). \tag{1.3}$$

38 Since $C_n^k \subseteq P_n^{2k}$, the next corollary follows directly from Theorem 1.2.

39 **Corollary 1.4.** *For any integer $k \geq 2$,*

$$\hat{r}(C_n^k) = O(n). \tag{1.5}$$

40 Throughout the paper we use big O notation with respect to $n \rightarrow \infty$, where the implicit
 41 constants may depend on other parameters. For a path P , we write $|P|$ for the number of vertices
 42 in P . For simplicity, we omit floor and ceiling signs when they are not essential.

43 The paper is structured as follows. In Section 2 we introduce some preliminary definitions and
 44 give an outline of the proof. The proof of Theorem 1.2 is given in Section 3. In Section 4, we
 45 mention some related open problems.

46 §2. OUTLINE OF THE PROOF

47 To prove Theorem 1.2, we will show that there exists a graph G with $O(n)$ edges such
 48 that $G \rightarrow P_n^k$.

49 To construct G we begin by taking a pseudo-random graph H with bounded degree. The
 50 existence of such an H will be proved in Lemma 3.1. Given H^k , we then take a *complete blow-up*,
 51 defined as follows.

52 **Definition 2.1.** Given a graph H and a positive integer t , the *complete- t -blow-up* of H , denoted
 53 H_t is the graph obtained by replacing each vertex v of H by a complete graph with $r(K_t)$ vertices,
 54 the *cluster* $C(v)$, and by adding, for every $\{u, v\} \in E(H)$, every edge between $C(u)$ and $C(v)$.

55 Note that we replace each vertex with a clique on $r(K_t)$ vertices rather than t vertices as might
 56 have been expected.

57 The following immediate fact states that complete blow-ups of powers of bounded-degree graphs
 58 have a linear number of edges. This makes them valid candidates for showing $\hat{r}(P_n^k) = O(n)$.

59 **Fact 2.2.** *Let k, t, a and b be positive constants. If H is a graph with $|V(H)| = an$ and $\Delta(H) \leq b$,*
 60 *then $|E(H_t^k)| = O(n)$.*

61 The heart of the proof is to show that, given any 2-colouring of the edges of H_t^k , we can find a
 62 monochromatic copy of P_n . To do this we will use the fact that H satisfies a particular property
 63 (Lemma 3.2). We shall also make use of the following result.

64 **Theorem 2.3** (Pokrovskiy [23, Theorem 1.7]). *Let $k \geq 1$. Suppose that the edges of K_n are*
 65 *coloured with red and blue. Then K_n can be covered by k vertex-disjoint blue paths and a*
 66 *vertex-disjoint red balanced complete $(k + 1)$ -partite graph.*

67 We remark that we do not need the full strength of this result, in the sense that we do not
 68 need the complete $(k + 1)$ -partite graph to be balanced; it suffices for us to know that the vertex
 69 classes are of comparable cardinality. Such a result can be derived easily by iterating Lemma 1.5
 70 in [23], for which Pokrovskiy gives a short and elegant proof (see also [22, Lemma 1.10]).

71 We shall also use the classical Kővári–T. Sós–Turán theorem [20], in the following simple form.

72 **Theorem 2.4.** *Let G be a balanced bipartite graph with t vertices in each vertex class. If G*
 73 *contains no $K_{s,s}$, then G has at most $4t^{2-1/s}$ edges.*

74 Let us now give a brief outline of how we find our monochromatic copy of P_n^k in a 2-edge
 75 coloured H_t^k . Suppose the edges of H_t^k have been coloured red and blue by an arbitrary colouring χ .
 76 Recall that H_t^k is obtained by blowing up H^k ; in particular, the vertices v of H^k become large
 77 complete graphs $C(v)$ in H_t^k . By the choice of parameters, Ramsey's theorem tells us that each
 78 such $C(v)$ contains a monochromatic copy $B(v)$ of K_t . We may assume without loss of generality
 79 that at least half of the $B(v)$ are blue.

80 Let F be the subgraph of H induced by the vertices v such that $B(v)$ is blue. We shall define
 81 an auxiliary edge-colouring χ' of F^k . By using Theorem 2.3 we shall be able to find either (i) a
 82 blue P_n in F^k under χ' or (ii) a P_n in F (not in F^k) with certain additional properties. The
 83 path in (ii) will be found applying Lemma 3.2 with the sets A_i being the vertex classes of a

84 red complete $(k + 1)$ -partite subgraph of F^k . This red complete $(k + 1)$ -partite subgraph of F^k
 85 will be found using Theorem 2.3, applied to a suitable red/blue coloured complete graph (we
 86 complete F^k with its auxiliary colouring χ' to a red/blue coloured complete graph by considering
 87 non-edges of F^k red).

88 In case (i), where we find a blue P_n in F^k under the colouring χ' , we shall be able to find a
 89 blue P_n^k in H_t^k . In case (ii), the properties of the path P_n found in F will ensure the existence
 90 of a red P_n^k in F^k . It will then be easy to find a red P_n^k in $F_t^k \subseteq H_t^k$. The idea of defining an
 91 auxiliary graph on monochromatic cliques as above was used in [1].

92 §3. PROOF OF THEOREM 1.2

93 Our first lemma guarantees the existence of bounded-degree graphs with the pseudo-randomness
 94 property we require.

95 **Lemma 3.1.** *For every positive constants ε and a , there is a constant b such that, for any large*
 96 *enough n , there is a graph H with $v(H) = an$ such that:*

- 97 (1) *For every pair of disjoint sets $S, T \subseteq V(H)$ with $|S|, |T| \geq \varepsilon n$, we have $|E_H(S, T)| > 0$.*
 98 (2) $\Delta(H) \leq b$.

99 *Proof.* Fix positive constants ε and a . Let $c = 4a/\varepsilon^2$ and $b = 4ac$ and consider a sufficiently
 100 large n . Let $G = G(2an, p)$ be the binomial random graph with $p = c/n$. By Chernoff's inequality,
 101 with high probability we have $|E(G)| < (4a^2c)n$. Moreover, with high probability G satisfies (1)
 102 (with $H = G$) by the following reason: Let X_G be the number of pairs of disjoint subsets of $V(G)$
 103 of size εn with no edges between them. Then, from the choice of c and using Markov's inequality,
 104 we have

$$\mathbb{P}[X_G \geq 1] \leq \mathbb{E}[X_G] \leq \binom{2an}{\varepsilon n}^2 \left(1 - \frac{c}{n}\right)^{(\varepsilon n)^2} < 2^{4an} \cdot e^{-c\varepsilon^2 n} = o(1).$$

105 Thus, there is a graph G with $|E(G)| < (4a^2c)n$ and $X_G = 0$.

106 Now let H be a subgraph of G obtained by iteratively removing a vertex of maximum degree
 107 until exactly an vertices remain. Then $\Delta(H) \leq b$, as otherwise, from the choice of b we would
 108 have deleted more than $b \cdot an > |E(G)|$ edges from G during the iteration, which contradicts
 109 property (1). Moreover, as H is an induced subgraph of G , (1) is maintained. This completes the
 110 proof of the lemma. \square

111 We now show that any graph satisfying the hypothesis of Lemma 3.1 and property (1) also
 112 satisfies an additional property.

113 **Lemma 3.2.** *For every integer $k \geq 1$ and every $\varepsilon > 0$ there exists $a_0 > 0$ such that the*
 114 *following holds for any $a \geq a_0$. Let H be a graph with an vertices such that for every pair*
 115 *of disjoint sets $S, T \subseteq V(H)$ with $|S|, |T| \geq \varepsilon n$ we have $|E_H(S, T)| > 0$. Then, for every*
 116 *family $A_1, \dots, A_{k+1} \subseteq V(H)$ of pairwise disjoint sets each of size at least εan , there is a path*
 117 $P_n = (x_1, \dots, x_n)$ *in H with $x_i \in A_j$ for all $1 \leq i \leq n$, where $j \equiv i \pmod{k+1}$.*

Algorithm 1:

Input : a graph H with $v(H) = an$ satisfying (1) and sets $A_i \subseteq V(H)$ ($1 \leq i \leq k+1$) with $A_i \cap A_j = \emptyset$ for all $i \neq j$ and $|A_i| \geq \varepsilon an$ for all i .

Output : a path $P_n = (x_1, \dots, x_n)$ in H with $x_i \in A_j$ for all i , where $j \equiv i \pmod{k+1}$.

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1 foreach  $1 \leq i \leq k+1$  do
2    $U_i \leftarrow A_i; \quad D_i \leftarrow \emptyset$ 
3 while  $|D_i| \leq |A_i|/2$  for all  $i$  do
4   pick  $x_1 \in U_1$  and let  $P = (x_1); \quad r \leftarrow 1; \quad U_1 \leftarrow U_1 \setminus \{x_1\}$ 
5   while  $1 \leq |P| < n$  do
6     //  $P = (x_1, \dots, x_r)$  with  $r \geq 1$ 
7     if  $\exists u \in U_{r+1}$  with  $\{x_r, u\} \in E(H)$  then
8        $x_{r+1} \leftarrow u; \quad U_{r+1} \leftarrow U_{r+1} \setminus \{u\}$ 
9        $P \leftarrow (x_1, \dots, x_r, x_{r+1}); \quad r \leftarrow r+1$ 
10    else
11       $D_r \leftarrow D_r \cup \{x_r\}$ 
12       $P \leftarrow (x_1, \dots, x_{r-1}); \quad r \leftarrow r-1$ 
13    if  $|P| = n$  then
14      return  $P$  // path has been found
15 STOP with failure // this will not happen

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118 To prove Lemma 3.2, we analyse a depth first search algorithm, adapting a proof idea in [5,
 119 Lemma 4.4]. More specifically, we run an algorithm (stated formally as Algorithm 1). Our
 120 algorithm receives as input a graph H with $v(H) = an$ satisfying property (1), and a family of
 121 pairwise disjoint sets $A_1, \dots, A_{k+1} \subseteq V(H)$ with $|A_i| \geq \varepsilon an$ for all i . The output of \mathcal{A} is a path
 122 $P_n = (x_1, \dots, x_n)$ in H with $x_i \in A_j$ for all i , where $j \equiv i \pmod{k+1}$.

123 As it runs, the algorithm builds a path $P = (x_1, \dots, x_r)$ with $x_i \in A_j$ for all i and j with $j \equiv i$
 124 $\pmod{k+1}$. Furthermore, it maintains sets U_j and $D_j \subseteq A_j$ for all j , with the property that U_j ,
 125 D_j , and $V(P) \cap A_j$ form a partition of A_j for every j . The cardinality of the sets U_j decrease as
 126 the algorithm runs, while the D_j increase. As the algorithm runs, we have $r = |P| < n$ and it
 127 searches for an edge $\{x_r, u\} \in E(H)$ where u belongs to the set U_{r+1} of *unused* vertices in A_{r+1} .
 128 If such a vertex $u \in U_{r+1}$ is found, then P is made one vertex longer by adding u to it. If there is
 129 no such vertex u , then x_r is declared a *dead end* and it is put into D_r . Moreover, the path P is
 130 shortened by one vertex; it becomes $P = (x_1, \dots, x_{r-1})$. Our algorithm iterates this procedure.
 131 If we find a path P with n vertices this way, then we are done.

132 We now analyse Algorithm 1.

133 *Proof of Lemma 3.2.* We will prove that Algorithm 1 returns a path P on line 13 as desired,
 134 instead of terminating with failure on line 14.

135 Fix an integer $k \geq 1$ and $\varepsilon > 0$. Let

$$a_0 = 2 + \frac{4}{\varepsilon(k+1)}, \quad (3.3)$$

136 fix $a \geq a_0$ and let n be sufficiently large. Let H be a graph with an vertices satisfying property (1),
 137 i.e., for every pair of disjoint sets $S, T \subseteq V(H)$ with $|S|, |T| \geq \varepsilon n$ we have $|E_H(S, T)| > 0$. Let
 138 $A_1, \dots, A_{k+1} \subseteq V(H)$ be a family of pairwise disjoint sets each of size at least εan .

139 First recall that U_i, D_i , and $V(P) \cap A_i$ form a partition of A_i for every i . Since the path P is
 140 always empty on line 4, at this point we have $|U_1| \geq |A_1| - |D_1| \geq |A_1|/2 > 0$. Then, line 4 is
 141 always executed successfully.

142 Suppose now that \mathcal{A} stops with failure on line 14. Then, for some i , say $i = r$, the set $D_i = D_r$
 143 became larger than $|A_r|/2 \geq \varepsilon an/2 \geq \varepsilon n$. Furthermore, we have $|P| < n$ and $|D_{r+1}| \leq |A_{r+1}|/2$
 144 (indices modulo $k+1$) and hence,

$$|U_{r+1}| \geq |A_{r+1}| - |D_{r+1}| - |V(P) \cap A_{r+1}| \geq \frac{1}{2}|A_{r+1}| - \left\lceil \frac{n}{k+1} \right\rceil \geq \frac{1}{2}\varepsilon an - \frac{2n}{k+1} > \varepsilon n.$$

145 Note that this is the only place where the exact value of a_0 is used. Applying property (1)
 146 to the pair (D_r, U_{r+1}) , we see that there is an edge $\{x, u\} \in E(H)$ with $x \in D_r$ and $u \in U_{r+1}$.
 147 Consider the moment in which x was put into D_r . This happened on line 10, when P had x as
 148 its foremost vertex and \mathcal{A} was trying to extend P further into U_{r+1} . At this point, because of
 149 the edge $\{x, u\} \in E(H)$, we must have had $u \notin U_{r+1}$ (see line 6). Since the set U_{r+1} decreases
 150 as \mathcal{A} runs, this is a contradiction and hence \mathcal{A} does not terminate on line 14.

151 Since $\sum_{1 \leq i \leq k+1} (|D_i| - |U_i|)$ increases as Algorithm 1 runs, we know the algorithm terminates.
 152 Therefore, we conclude that it returns a suitable path P as claimed. \square

153 We are now ready to complete the proof of Theorem 1.2.

154 *Proof of Theorem 1.2.* Fix $k \geq 1$ and let $\varepsilon = 1/3(k+1)$. Let a_0 be the constant given by an
 155 application of Lemma 3.2 with parameters k and ε . Set $a = \max\{6k, a_0\}$ and let b be given by
 156 Lemma 3.1 for this choice of a . Moreover, let H be a graph with $|V(H)| = an$ and $\Delta(H) \leq b$ be
 157 as in Lemma 3.1. Finally, put $t = (64k)^{2k}$ and $s = 2k$.

158 Let H_t^k be a complete- t -blow-up of H^k , as in Definition 2.1, and let $\chi: E(H_t^k) \rightarrow \{\text{red}, \text{blue}\}$ be
 159 an edge-colouring of H_t^k . We shall show that H_t^k contains a monochromatic copy of P_n^k under χ .
 160 By the definition of H_t^k , any cluster $C(v)$ contains a monochromatic copy $B(v)$ of K_t . Without
 161 loss of generality, the set $W := \{v \in V(H): B(v) \text{ is blue}\}$ has cardinality at least $v(H)/2$. Let
 162 $F := H[W]$ be the subgraph of H induced by W , and let F' be the subgraph of $F_t^k \subseteq H_t^k$ induced
 163 by $\bigcup_{w \in W} V(B(w))$.

164 Given the above colouring χ , we define a colouring χ' of F^k as follows. An edge $\{u, v\} \in E(F^k)$
 165 is coloured *blue* if the bipartite subgraph $F'[V(B(u)), V(B(v))]$ of F' naturally induced by the
 166 sets $V(B(u))$ and $V(B(v))$ contains a blue $K_{s,s}$. Otherwise $\{u, v\}$ is coloured *red*.

167 **Claim 3.4.** *Any 2-colouring of $E(F^k)$ has either a blue P_n or a red P_n^k .*

168 *Proof.* We apply Theorem 2.3 to F^k , where if an edge is not present in F^k , then we consider it
 169 to be in the red colour class. If F^k contains a blue copy of P_n , then we are done. Hence we may
 170 assume F^k contains a balanced, complete $(k+1)$ -partite graph K with parts A_1, \dots, A_{k+1} on at
 171 least $v(F^k) - kn \geq an/2 - kn$ vertices, with no blue edges between any two parts. As $a \geq 6k$,
 172 each one of these parts has size at least

$$\frac{1}{k+1} \left(\frac{1}{2}a - k \right) n \geq \varepsilon an. \quad (3.5)$$

173 By Lemma 3.2 applied to the collection of sets of vertices A_1, \dots, A_{k+1} of $F \subseteq H$ (specifically
 174 F and not F^k), we see that $F[V(K)]$ contains a path with n vertices such that any consecutive
 175 $k+1$ vertices are in distinct parts of K . Therefore $F^k[V(K)]$ contains a copy of P_n^k in which
 176 every pair of adjacent vertices are in distinct parts of K . By the definition of K , such a copy is
 177 red. \square

178 By Claim 3.4, F^k contains a blue copy of P_n or a red copy of P_n^k under the edge-colouring χ' .
 179 Thus, we can split our proof into these two cases.

180 **Case 1.** First suppose F^k contains a blue copy (x_1, \dots, x_n) of P_n . Then, for every $1 \leq i \leq n-1$,
 181 the bipartite graph $F'[V(B(x_i)), V(B(x_{i+1}))]$ contains a blue copy of $K_{s,s}$, with, say, vertex classes
 182 $X_i \subseteq V(B(x_i))$ and $Y_{i+1} \subseteq V(B(x_{i+1}))$. As $|X_i| = |Y_i| = s = 2k$ for all $2 \leq i \leq n-1$, we can
 183 find sets $X'_i \subseteq X_i$ and $Y'_i \subseteq Y_i$ such that $|X'_i| = |Y'_i| = k$ and $X'_i \cap Y'_i = \emptyset$ for all $2 \leq j \leq n-1$.
 184 Let $X'_1 = X_1$ and $Y'_n = Y_n$.

185 We now show that the set $U := \bigcup_{i=1}^{n-1} X'_i \cup \bigcup_{i=2}^n Y'_i$ provides us with a blue copy of P_{2kn}^k
 186 in $F' \subseteq H_t^k$. Note first that $|U| = 2k + 2k(n-2) + 2k = 2kn$. Let u_1, \dots, u_{2kn} be an ordering
 187 of U such that, for each i , every vertex in X'_i comes before any vertex in Y'_{i+1} and after every
 188 vertex in Y'_i . By the definition of the sets X'_i and Y'_i and the construction of $F' \subseteq F_t^k \subseteq H_t^k$, each
 189 vertex u_j is adjacent in blue to $\{u_{j'} \in U : 1 \leq |j - j'| \leq k\}$. Thus, U contains a blue copy of P_{2nk}^k ,
 190 as claimed.

191 **Case 2.** Now suppose F^k contains a red copy P of P_n^k . That is, F^k contains a set of vertices
 192 $\{x_1, \dots, x_n\}$ such that x_i is adjacent in red to all x_j with $1 \leq |j - i| \leq k$. We shall show that,
 193 for each $1 \leq i \leq n$, we can pick a vertex $y_i \in V(B(x_i))$ so that y_1, \dots, y_n define a red copy of P_n^k
 194 in $F' \subseteq F_t^k \subseteq H_t^k$. We do this by applying the local lemma [13, p. 616] (a greedy strategy also
 195 works).

196 We have to show that it is possible to pick the y_i ($1 \leq i \leq n$) in such a way that $\{y_i, y_j\}$ is a
 197 red edge in F' for every i and j with $1 \leq |i - j| \leq k$. Let us choose $y_i \in V(B(x_i))$ ($1 \leq i \leq n$)
 198 uniformly and independently at random. Let $e = \{x_i, x_j\}$ be an edge in $P \subseteq F^k$. We know that e
 199 is red. Let A_e be the event that $\{y_i, y_j\}$ is a blue edge in F' . Since the edge e is red, we know
 200 that the bipartite graph $F'[V(B(x_i)), V(B(x_j))]$ contains no blue $K_{s,s}$. Theorem 2.4 then tells
 201 us that $\mathbb{P}[A_e] \leq 4t^{-1/s}$.

202 The events A_e are not independent, but we can define a dependency graph D for the collection
 203 of events A_e ($e \in E(P)$) by adding an edge between A_e and A_f if and only if $e \cap f \neq \emptyset$.
 204 Then $\Delta(D) \leq 4k$. Given that

$$4\Delta\mathbb{P}[A_e] \leq 64kt^{-1/s} = 1 \quad (3.6)$$

205 for all e , the Local Lemma tells us that $\mathbb{P}[\bigcap_{e \in E(P)} \bar{A}_e] > 0$, and hence a simultaneous choice of
 206 the y_i ($1 \leq i \leq n$) as required is possible. This completes the proof of Theorem 1.2. \square

207 Throughout our proof we have used probabilistic methods to show the existence of G . We now
 208 briefly discuss how our proof could be made constructive. For instance, it suffices to take for H a
 209 suitable (n, d, λ) -graph as in Alon and Chung [2], namely, it is enough to have $\lambda = O(\sqrt{d})$ and d
 210 large enough with respect to k and $1/\varepsilon$.

211 §4. OPEN QUESTIONS

212 We make no attempts to optimise the constant given by our argument, so the following question
 213 is of interest.

214 **Question 4.1.** *For any integer $k \geq 2$, what is $\limsup_{n \rightarrow \infty} \hat{r}(P_n^k)/n$?*

215 It is also interesting to consider what happens when more than two colours are at play. For
 216 $q \in \mathbb{N}$, let $\hat{r}_q(H)$ denote the q -colour size-Ramsey number of H , that is, the smallest number of
 217 edges in a graph that is q -Ramsey for H .

218 **Conjecture 4.2.** *For any $q, k \in \mathbb{N}$ we have $\hat{r}_q(P_n^k) = O(n)$.*

219 It is conceivable that in hypergraphs the size-Ramsey number (defined analogously as for
 220 graphs) of tight paths may be linear. Let $H_n^{(k)}$ denote the tight path of uniformity k on n vertices;
 221 that is, $V(H_n^{(k)}) = [n]$ and $E(H_n^{(k)}) = \{\{1, \dots, k\}, \{2, \dots, k+1\}, \dots, \{n-k+1, \dots, n\}\}$. The
 222 following question appears as Question 2.9 in [9].

223 **Question 4.3.** *For any $k \in \mathbb{N}$, do we have $\hat{r}(H_n^{(k)}) = O(n)$?*

224 Finally, we note that for fixed k , our main result implies the linearity of the size Ramsey
 225 number for the grid graphs $G_{k,n}$, the cartesian product of the paths P_k and P_n . Indeed our main
 226 result implies the linearity of the size Ramsey number for any sequence of graphs with bounded
 227 bandwidth. For the d -dimensional grid graph G_n^d , obtained by taking the cartesian product of d
 228 copies of P_n , we raise the following question.

229 **Question 4.4.** *For any integer $d \geq 2$, is $\hat{r}(G_n^d) = O(n^d)$?*

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236

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