

# Disentangling $\alpha$ and $\beta$ relaxation in orientationally disordered crystals with theory and experiments

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We use a microscopically motivated Generalized Langevin Equation (GLE) approach to link the vibrational density of states (VDOS) to the dielectric response of orientational glasses (OGs). The dielectric function calculated based on the GLE is compared with experimental data for the paradigmatic case of two OGs: Freon 112 and Freon 113, around and above  $T_g$ . The memory function is related to the integral of the VDOS times a spectral coupling function  $\gamma(\omega_p)$ , which tells the degree of dynamical coupling between molecular degrees of freedom at different eigenfrequencies. The comparative analysis of the two Freons reveals that the appearance of a secondary  $\beta$  relaxation in Freon 112 is due to cooperative dynamical coupling in the regime of mesoscopic motions caused by stronger anharmonicity (absent in Freon 113), and is associated with comparatively lower boson peak in the VDOS. The proposed framework brings together all the key aspects of glassy physics (VDOS with boson peak, dynamical heterogeneity, memory effects and dissipation, anharmonicity) into a single model.

## I. INTRODUCTION

Structural glass (SG) formers, which are usually obtained from supercooled liquids in which translational and orientational degrees of freedom are frozen below the glass transition temperature  $T_g$ , exhibit a complex response function on vibrational excitations [1–4]. When they undergo a rapid cooling to avoid crystallization, some anomalous physical properties emerge. For example, as temperature decreases, the relaxation time generally shows a stronger increase, faster than what is given by the Arrhenius law (super-Arrhenius behavior). For such cases, the temperature ( $T$ ) dependence of relaxation time ( $\tau$ ) is given through the empirical Vogel-Fulcher-Tammann (VFT) law [5, 6], or by physically-motivated double-exponential dependence of  $\tau$  on  $T$ , which includes the dependence on the steepness of interatomic repulsion and on thermal expansion via the more recent Krausser-Samwer-Zaccane (KSZ) relation [7]. To account for the deviation of the Arrhenius temperature dependence, i.e., for the faster increase of the relaxation time (or viscosity), the kinetic fragility index is defined as  $m = \frac{\partial \log \tau}{\partial (T_g/T)} \Big|_{T=T_g}$ , ranging between  $\approx 16$  (strong glasses) and 200 (fragile glasses) [10, 11].

In addition to the structural glasses, orientational glasses (OGs) can be obtained from orientationally disordered (OD) phases or plastic phases [20–22]. OD phases are high-symmetry crystal lattices in which weakly interacting molecules are orientationally disordered. On cooling, some OD phases exhibit the same features as structural (canonical) glass formers [21, 23–25]. With respect to the fragility index, OGs are usually strong [10] whereas for SGs a wide range of fragility values

are found, the most fragile being the cis- or trans-decahydronaphthalene ( $m \approx 147$ ) [26]. The most fragile OGs known to date contains Freon 112 ( $\text{CCl}_2\text{F}-\text{CCl}_2\text{F}$ , hereinafter F112) with  $m = 68$  [25]. On the other hand, for Freon 113 ( $\text{CCl}_2\text{F}-\text{CClF}_2$ , hereinafter F113), the kinetic fragility index was calculated to be  $m = 127$ , which is the highest so far reported for an OG [27].

Although mode-coupling theory provides a good description of dielectric relaxation of liquids for temperatures higher than the liquid crossover temperature  $T_c$  [9], the main relaxation mechanism by which supercooled liquids undergo a liquid-solid transition-like at or around  $T_g$  has remained elusive [8]. The  $\alpha$ -relaxation, typically associated with collective and strong cooperative motions of a large number of entities rearranging in a long-range correlated way, is related to the slowest decay of density correlations and is widely observed in dielectric and mechanical responses.

For supercooled liquids, the empirical Kohlrausch stretched-exponential function  $\sim \exp(-(t/\tau)^\beta)$  provides a good fit for the dielectric loss of the  $\alpha$ -relaxation, by taking the Fourier transform from time into frequency domain. Further, starting from the first principle assumption that the microscopic Hamiltonian can be modelled using a classical particle-bath coupling of the Caldeira-Leggett type, a simple and explicit relation between the dielectric relaxation function and the VDOS of SGs has been presented to provide a good interpretation of the  $\alpha$ -peak and stretched-exponential relaxation, through a memory function of friction [12].

An extra shoulder or wing in addition to  $\alpha$ -relaxation also decorates the imaginary part of the dielectric response, which is referred to as the  $\beta$ -relaxation, or as Johari-Goldstein or secondary relaxation. As discov-

ered by Johari and Goldstein [13] in glasses of rigid molecules and as described by the coupling model [14], the secondary relaxation involves the motion of the entire molecule. Knowing the underlying mechanism of  $\beta$ -relaxation, is of great importance for understanding many crucial unresolved issues in glassy physics and materials science and consequently for a wide potential application in technologies, ranging from glass transitions, deformation mechanisms, to diffusion and the breakdown of the Stokes-Einstein relations, physical ageing, as well as the conductivity of ionic liquids and the stability of glassy pharmaceuticals and biomaterials. Yet, the nature and mechanism of the  $\beta$ -relaxations are still not clear [15–18].

Previous study on thermal conductivities of Freon 112 and Freon 113 reveals the existence of quasilocalized low-energy vibrational modes (soft harmonic oscillators as described through the soft potential model [19]) at energy lower than the values of the maximum of the boson peak compared with other OGs, which results in an increase of VDOS [23]. It was thought that the high values of kinetic ( $m = 127$ ) fragility of F113 is produced by strong orientational correlations, which is evidenced by low values of the stretching exponent in Kohlrausch stretched-exponential function close to  $T_g$ , where only  $\alpha$ -relaxation is observed with no sign of the  $\beta$ -relaxation. On the contrary, in dielectric spectra of F112,  $\beta$ -relaxation emerges as temperature decreases to  $T_g$  and becomes evident below  $T_g$ .

The above experimental facts are the origin of our interest in applying a microscopic theoretical model to plastic crystals. In particular, freons F112 and F113 are chemically and molecularly similar compounds displaying glassy states (they both belong to the series  $C_2X_{(6-n)}Y_n$ , with X, Y=Cl, F, Br, and  $n = 0, \dots, 6.$ ), but with completely different dynamics and relaxation. This provides a unique opportunity to explain, from a microscopic point of view, the physical origin of secondary  $\beta$  relaxation.

We therefore develop a modified theoretical model in the spirit of Ref. [12], to account for both  $\alpha$  and  $\beta$  relaxation and we apply it to OG states of freons F112 and F113. From the analysis of experimental data, it is evident that: i) the proposed generalized Langevin equation theory successfully describes both  $\alpha$  and  $\beta$  relaxation process in the dielectric response, by using the experimentally measured VDOS as input; ii) the model provides a new insight into the dynamical origin of the secondary relaxation; iii) the model also clarifies, for the first time, which eigenmodes dynamically couple with the secondary relaxation process. This framework represents a new microscopic modelling of the glassy relaxation in orientationally disordered crystals for which no theoretical model was available so far.

## II. THEORY

Focusing on a tagged particle (e.g. a molecular sub-unit carrying a partial charge which reorients under the electric field), it is possible to describe its motion under the applied field using a particle-bath Hamiltonian of the Caldeira-Leggett type, in the classical dynamics regime [12]. The particle's Hamiltonian is bi-linearly coupled to a bath of harmonic oscillators which represent all other molecular degrees of freedom in the system [28]. Any complex system of oscillators can be reduced to a set of independent oscillators by performing a suitable normal mode decomposition. This allows us to identify the spectrum of eigenfrequencies of the system, i.e. the experimental VDOS, with the spectrum of the set of harmonic oscillators forming the bath.

### A. Particle-bath Hamiltonian and GLE

The particle-bath Hamiltonian under a uniform AC electric field, is given by [12]:  $H = H_P + H_B$  where  $H_P = P^2/2m + V(Q) - q_e Q E_0 \sin(\omega t)$  is the Hamiltonian of the tagged particle with the external electric field ( $q_e$  is the charge carried by the particle),  $H_B = \frac{1}{2} \sum_{\alpha=1}^N \left[ \frac{P_{\alpha}^2}{m_{\alpha}} + m_{\alpha} \omega_{\alpha}^2 \left( X_{\alpha} - \frac{F_{\alpha}(Q)}{\omega_{\alpha}^2} \right)^2 \right]$  is the Hamiltonian of the bath of harmonic oscillators that are coupled to the tagged particle [28]. Two parts in  $H_B$  are of physical interest: The first part is the ordinary harmonic oscillator; the second is the coupling term between the tagged particle position  $Q$  and the bath oscillator position  $X_{\alpha}$ . The coupling function is taken to be linear in the displacement of the particle,  $F_{\alpha}(Q) = c_{\alpha} Q$ , where  $c_{\alpha}$  is known as the strength of coupling between the tagged atom and the  $\alpha$ -th bath oscillator. Hence, there is a spectrum of coupling constants  $c_{\alpha}$  by which each particle interacts with all other molecular degrees of freedom in the system. This spectrum of coupling strengths will play a major role in the subsequent analysis. The equation of motion for the tagged particle can then be derived straightforwardly, which leads to the following GLE

$$\ddot{q} = -V'(q) - \int_{-\infty}^t \nu(t') \frac{dq}{dt'} dt' + q_e E_0 \sin(\omega t). \quad (1)$$

where the non-Markovian friction or memory kernel  $\nu(t)$  is given by:

$$\nu(t) = \sum_{\alpha} \frac{c_{\alpha}^2 m_{\alpha}}{\omega_{\alpha}^2 m} \cos(\omega_{\alpha} t). \quad (2)$$

Note we have converted into rescaled coordinates for standard normal-mode analysis:  $q = Q\sqrt{m}$ . This means  $V(Q)$  and  $V(q)$  are basically different functions. We have also redefined  $q_e = e/\sqrt{m}$  as the (partial) reduced charge in rescaled coordinates. Then we can let the spectrum be continuous and  $c_{\alpha}$  be a function of  $\omega_p$  which leads to

the following expression for the friction kernel:

$$\nu(t) = \int_0^\infty d\omega_p D(\omega_p) \frac{\gamma(\omega_p)^2}{\omega_p^2} \cos(\omega_p t), \quad (3)$$

where  $\gamma(\omega_p)$  is the continuous spectrum of coupling constants, i.e. the continuous version of the discrete set  $\{c_\alpha\}$  averaged over all tagged particles. For any given (well behaved) VDOS function  $D(\omega_p)$ , the existence of a well-behaved function  $\gamma(\omega_p)$  that satisfies Eq. (3) is guaranteed by the fact that we can always decompose  $\nu(t)$  into a basis of  $\{\cos(\omega_p t)\}$  functions, by taking a cosine transform. The inverse cosine transform in turn gives the spectrum of coupling constants  $\gamma(\omega_p)$  as a function of the memory kernel:

$$\gamma^2(\omega_p) = \frac{2\omega_p^2}{\pi D(\omega_p)} \int_0^\infty \nu(t) \cos(\omega_p t) dt. \quad (4)$$

This coupling function contains information on how strongly the particle's motion is coupled to the motion of other particles in a mode with vibrational frequency  $\omega_p$ . This is an important information, because it tells us about the degree of long-range anharmonic couplings in the motion of the molecules.

## B. Memory function modelling

Looking at Eq. (3), it is evident that the particle-bath Hamiltonian does not provide any prescription to the form of the memory function  $\nu(t)$ , which can take any form depending on the values of the coefficients  $c_\alpha$  [28]. Hence, a shortcoming of particle-bath models is that the functional form of  $\nu(t)$  cannot be derived a priori for a given system, because, while the VDOS is certainly an easily accessible quantity from simulations of a physical system, the spectrum of coupling constants  $\{c_\alpha\}$  is basically a phenomenological parameter.

However, for a supercooled liquid, the time-dependent friction, which is dominated by slow collective dynamics, has been famously derived within kinetic theory (Boltzmann equation) using a mode-coupling type approximation by Sjoegren and Sjoelander [29], and is given by the following elegant expression:

$$\nu(t) = \frac{\rho k_B T}{6\pi^2 m} \int_0^\infty dk k^4 F_s(k, t) [c(k)]^2 F(k, t) \quad (5)$$

where  $k$  is the wavevector,  $c(k)$  is the direct correlation function of liquid-state theory,  $F_s(k, t)$  is the self-part of the intermediate scattering function (ISF)  $F(k, t)$  [29]. All of these quantities are functions of the wave-vector  $k$ . Clearly, the integral over  $k$  leaves a time-dependence of  $\nu(t)$  which is controlled by the product  $F_s(k, t)F(k, t)$ . For a chemically homogeneous system,  $F_s(k, t)S(k, t) \sim F(k, t)^2$ , especially in the long-time regime.

From theory and simulations, we know that in supercooled liquids  $F(k, t) \sim \exp(-(t/\tau)^b)$  for some  $\tau$  and  $b$ ,

when only  $\alpha$ -relaxation is present. When both  $\alpha$  and  $\beta$  relaxation are present, the ISF has a two-step decay (one for  $\alpha$  and one for  $\beta$ ) [2]. It is easy to check that a two-step decay of the ISF within Eq. (5) is perfectly compatible with a memory function  $\nu(t)$  given by a sum of two stretched-exponential terms.

While the elegant relation by Sjoegren and Sjoelander Eq. (5) relies on mode-coupling type assumptions which may be questionable below  $T_g$ , we also point out that a more physically meaningful justification comes from its ability to generate an intermediate scattering function (ISF) with a two-step decay for Freon 112 if  $\nu(t)$  is a sum of two stretched-exponentials, which is able to describe dielectric data (see below). This qualitative behavior for the ISF with a two-step decay has been demonstrated for the Freon 112 system in simulations, e.g. Ref. [30], and also in experiments [31]. Hence, despite the fact that the Sjoegren and Sjoelander relation relies on assumptions of mode-coupling type, the relationship between our memory function and the intermediate scattering function is physically meaningful and supported by data in the literature.

Hence, in light of the above discussion, we will take the following phenomenological expression for our memory function

$$\nu(t) = \nu_0 \sum_i e^{-(t/\tau_i)^{b_i}}, \quad (6)$$

where  $\tau_i$  is a characteristic time-scale, with  $i = 1$  for pure  $\alpha$  relaxation and  $i = 1, 2$  for co-existing  $\alpha$  and  $\beta$  relaxation.  $\nu_0$  is a constant pre-factor.

## C. Dielectric response and link with the VDOS

Following the same steps as those described in Ref.[12], we obtain the complex dielectric function

$$\epsilon^*(\omega) = 1 - A \int_0^{\omega_D} \frac{D(\omega_p)}{\omega^2 - i\omega\tilde{\nu}(\omega) - \omega_p^2} d\omega_p \quad (7)$$

where  $A$  is an arbitrary positive rescaling constant,  $\omega_D$  is the Debye cut-off frequency (i.e. the highest eigenfrequency in the VDOS spectrum), and tilde over  $\nu$  denotes Fourier transformation. As one can easily verify, if  $D(\omega_p)$  were given by a Dirac delta, one would recover the standard simple-exponential (Debye) relaxation.

The VDOS is an important key input to the theoretical framework. The essential experimental VDOS were measured by means of inelastic neutron scattering using the direct spectrometer MARI of the ISIS facility (UK) and are shown in Fig. 1. The VDOS of F113 clearly exhibits a much more significant excess of low-frequency (boson-peak) modes, with respect to F112, in the range 2 – 5meV.

For F113, we use only one stretched-exponential term in the memory  $\nu(t)$ , hence  $i = 1$  in Eq. (6). For F112, instead,  $\nu(t)$  is the sum of two terms ( $i = 1, 2$  in Eq. (6)),

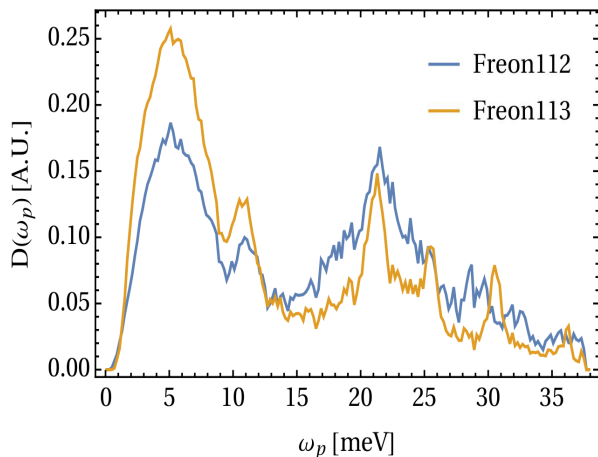


FIG. 1: Experimental vibrational density of states (VDOS) for Freon 112 (blue) and Freon 113 (yellow). The data for Freon 112 were published in Ref. [24], while the data for Freon 113 were taken from Ref. [27].

both of which are stretched-exponential. The first term represents mainly the  $\alpha$  process although it also affects the  $\beta$  relaxation (hence the two are coupled, as one can anticipate in the spirit of the Ngai coupling model [17]). The second term describes only  $\beta$  relaxation. Thus, the time-scale of  $\beta$  relaxation is not identically equal to the time-scale of the second stretched-exponential parameter, which is  $\tau_2$ . This amounts to the fact that  $\beta$  relaxation is a process which is cooperative (hence coupled to the  $\alpha$ ) and at the same time quasi-localized.

In terms of physical meaning,  $\tau_1$  represents the time-scale of  $\alpha$ -relaxation, and the stretching exponent is related to the distribution of escape times from larger metastable basins in the glassy energy landscape. This is because stretched-exponential form arises from the integral average of simple exponential decays weighted by a distribution of time-scales, the broader the distribution the lower the resulting stretching-exponent [33]. Similarly, the second stretched-exponential required to describe  $\beta$ -relaxation is possibly related to the distribution of smaller wells within the same meta-basin.

### III. COMPARISON WITH EXPERIMENTAL DATA OF DIELECTRIC LOSS

Fitting parameters for F112 and F113 at different temperatures are listed in Table I & II and resulting fittings of dielectric loss are displayed in Fig. 2.

For the fitting procedure, we have assumed that  $D(\omega_p)$  and the overall scaling for the height of curve,  $A$ , are  $T$ -independent.

Temperature	91K	115K	131K
$b_1$	0.45	0.625	0.7
$\tau_1$ (seconds)	0.558	$3.12 \cdot 10^{-7}$	$6.99 \cdot 10^{-9}$
$b_2$	0.2	0.56	
$\tau_2$ (seconds)	$1.55 \cdot 10^{-2}$	$5.48 \cdot 10^{-8}$	
$\nu_0$	$4 \cdot 10^6$	$3.9 \cdot 10^6$	$6.3 \cdot 10^6$

TABLE I: Parameters of the memory function for Freon 112.

Temperature	72K	74K	76K
$b$	0.26	0.3	0.35
$\tau$ (seconds)	7.133	0.326	$2.38 \cdot 10^{-2}$
$\nu_0$	$8.28 \cdot 10^6$	$7.28 \cdot 10^6$	$6.28 \cdot 10^6$

TABLE II: Parameters of the memory function for Freon 113.

### IV. PHYSICAL MECHANISM OF SECONDARY RELAXATION

To physically understand the difference between F112 and F113, their dynamical coupling parameters (Eq. (4)) have been analysed (see Fig. 3). In general, the coupling spectrum decays from the highest Debye cut-off frequency of short-range high-frequency in-cage motions, down to the low eigenfrequency part where the coupling goes up with decreasing  $\omega_p$  towards zero, due to phonons or phonon-like excitations, which are collective and long-wavelength and hence result in a larger value of  $\gamma$ .

There is a substantial difference between F112 and F113, especially in the middle part of the coupling spectrum where F112 shows much stronger coupling, which corresponds to medium-range correlated motions. This means that motions are strongly coupled also in the intermediate eigenfrequency domain, where modes are typically quasi-localized, which corresponds to mesoscopic string-like motions [32] typically associated with  $\beta$ -relaxation [34]. In addition, the F113 spectrum is overall comparatively much lower in that energy regime, which clearly indicates that, for F113, the intermediate part of the coupling spectrum, i.e. the one of mesoscopic and string-like motions, is scarcely populated and one has a steep decay from the short-range high-frequency in-cage motions to the long-wavelength phonon-like excitations, with not much in between in the mesoscopic range. Hence in F113, the anharmonicity is much less prominent and intermediate excitations are not important. This origin of the secondary relaxation aligns with the simulation results of Refs. [35, 36] which point at the cooperative, though localized or quasi-localized, nature of secondary relaxation.

This also gives insights into the difference in the form of the memory function used for the fittings of the two Freons. Upon focusing on the integration in Eq. (4): the integral of  $\nu(t)$  from 0 to  $\infty$  increases from high  $\omega_p$  (short-range and fast vibration) to low  $\omega_p$  (long-range and slow vibration), since for slow collective vibration there is clearly much more extended friction due to con-

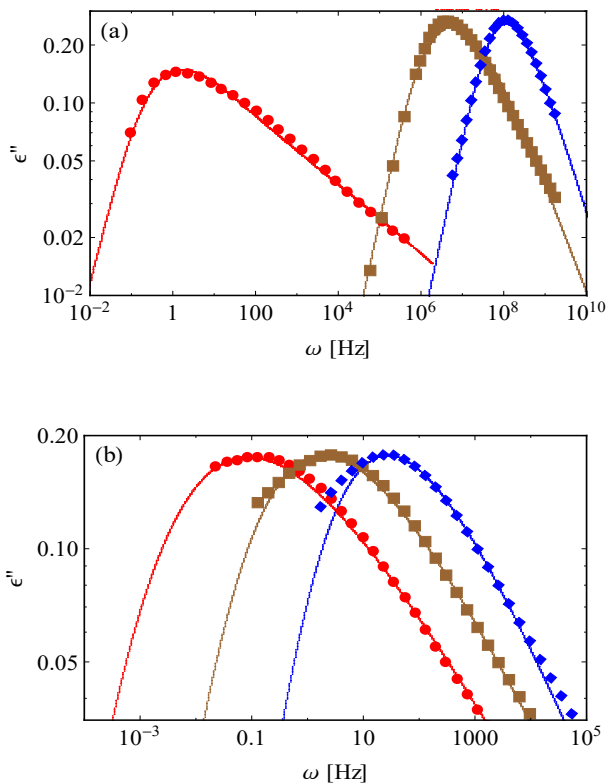


FIG. 2: Fitting of experimental data using the proposed theoretical model for Freon 112 (a) at 91 K (red circles), 115 K (brown squares) and 131 K (blue diamonds) and for Freon 113 (b) at 72 K (red circles), 74 K (brown squares) and 76 K (blue diamonds). Solid lines are the theoretical model presented here. A rescaling constant was used to adjust the height of the curves since the data are in arbitrary units. Experimental data for Freon 112 were taken from Ref. [25], while data for Freon 113 were taken from Ref. [27].

tact between many particles all moving at the same time. Thus, the integral factor definitely contributes to the coupling being overall stronger for F112 than for F113. However, also the boson peak contributes to the coupling of F112 being larger than that of F113 (via the VDOS in the denominator of Eq. (4)) in the specific frequency range that corresponds to the boson peak). The boson peak maximum (in  $D(\omega)/\omega^2$ , not shown) for both materials is of the order of 2 – 3 meV, i.e.,  $\approx 0.5 - 0.7$  THz, which corresponds virtually to the lowest minimum in the coupling function (see Fig. 4) where, in addition, the minimum value is much lower for F113 (with larger boson peak) than for F112. That means that in such region not only we have larger dynamical coupling for F112 due to stronger medium-range correlations/anahrmnicity, in general, but also for the additional effect of boson peak (soft weakly-coupled modes, see Fig. 1) being smaller for F112 in that regime of vibrations.

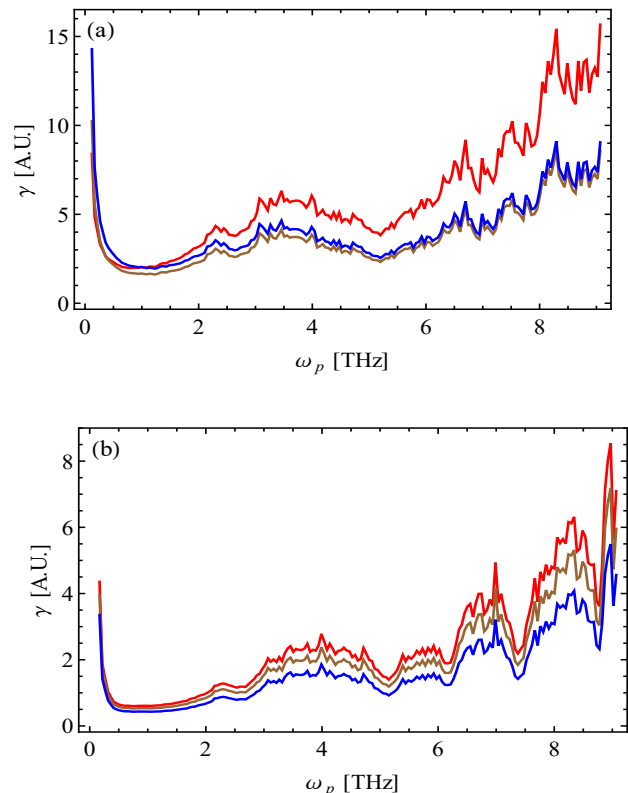


FIG. 3: Spectrum of coupling constants of Freon 112 (a) and Freon 113 (b) as a function of the vibrational eigenfrequency computed according to Eq. (4) using the phenomenological memory functions  $\nu(t)$  used in the fitting of dielectric response in Fig. 2, with same colour setting for the different temperatures.

As far as temperature effects on the coupling strength, we must point out first that, due to the fragility difference between the two freons, the temperature range in which fittings were performed are noticeably different. For F113 ( $T_g = 71$  K) experimental dielectric functions are available at the highest reduced temperature of  $T_r = 76/71 = 1.07$ , whereas for F112 ( $T_g = 90$  K) the highest value is around  $T_r = 131/90 = 1.46$ . Bearing this in mind, it can be noticed that upon increasing temperature, the “going up” tail at decreasing  $\omega_p$  towards zero becomes smaller, which means less phonon-like modes. In general, absolute coupling values shift down (lower coupling) with the increase of temperature, as expected, and the decay of correlated motions from high  $\omega_p$  to low  $\omega_p$  becomes also somewhat steeper with increasing  $T$ .

## V. DISCUSSION AND CONCLUSIONS

The stronger coupling between collective and individual motions for F112 could be a physical explanation of why in the dielectric study of F112 [25] the authors

described so many problems to disclose  $\alpha$ - from  $\beta$ -relaxation. For F112 collective vibrations, medium-ranged and slow motions are much more important than for F113, in such a way that individual molecular motions ( $\beta$ -relaxation) should correlate, i.e. are much more coupled, with motions of surrounding molecules (collective motions associated with the  $\alpha$ -relaxation). And, even more, if slow vibrations are more important and more heterogeneous in F112, this should mean stronger coupling between collective and individual motions, so then, much more phonon scattering for F112 and, as a consequence, lower thermal conductivity for F112 than for F113, as it has been experimentally shown (see Fig. 5 in [23]). In addition, it should be emphasized that the higher thermal conductivity for F113, analysed in terms of the soft-potential model, was also attributed to the low coupling strength between sound waves and the soft quasi-localized modes. Moreover, the dynamical coupling function  $\gamma$  extends over a frequency range much broader than that of the boson peak, and thus the role of the boson peak is confined to a specific frequency range which is around the minimum in the coupling spectrum. The fact that boson peak is stronger for F113 leads to a lower coupling in that region and contributes to the already lower coupling of F113 compared to F112 in that region. Because the boson peak is associated with soft modes, which "break" the coherence of phonons (hence more phonon scattering), it leads to even lower coupling in the boson peak frequency range for F113.

In conclusion, we have presented a new approach which makes it possible to directly link the vibrational density of states of orientational glasses measured experimentally with the macroscopic dielectric response and the underlying heterogeneous dynamics. Furthermore, the model

effectively accounts also for the medium and long-range anharmonic coupling among molecular degrees of freedom and allows one to disentangle  $\alpha$  and  $\beta$  relaxation on the basis of the extent of dynamical coupling in different eigenfrequency sectors of the vibrational spectrum. The appearance of secondary  $\beta$  relaxation is associated with higher values of the dynamical coupling strength of correlated particle motions in the regime of mesoscopic quasi-localized motions (e.g. string-like motions, vortices, etc. [34]) and is also promoted by a lower excess of soft modes in the boson peak frequency range.

In our model, we require two forms of stretched exponentials in the memory function, hence two relaxation times, to fit both alpha and beta relaxations. The beta-relaxation process cannot be recovered with only one stretched exponential (i.e. with only one term in the memory function). One of the stretched exponentials dominates the alpha peak while the co-existing effect of two stretched exponential terms in the memory function gives rise to the secondary relaxation. In other words, the two terms of memory function both affect the secondary relaxation, whereas only one of them controls the alpha relaxation. This implies that there is indeed a deep microscopic dynamical coupling between the two relaxation processes, which has not been unveiled so far. In future work this framework will be used to provide more microscopic insights into the dynamical nature of this coupling and in the context of the Ngai coupling model [18].

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