

Independent Phase Belt Controlled Phase Pole Modulated Induction Machine for Integrated Starter Alternator Application

Saikat Subhra Ghosh*, Timothy J. Flack and Teng Long
Department of Engineering - Division B, Electrical Engineering
University of Cambridge, Cambridge CB3 0FA
*Email: ssg39@cam.ac.uk

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Abstract—Integrated Starter Alternators for micro and mild hybrid vehicles require wide torque-speed characteristics. This paper reports a Phase Pole Modulated (PPM) Induction Machine (IM) to achieve wide torque-speed characteristics by implementing different pole pair combinations in a single machine. A novel concept of independent phase belt control technique has been reported in detail, which allows higher flexibility than the phase pole modulated machines reported in previous literature. In this design study a 12 leg inverter has been used to achieve 2/4/6/8/10 pole configurations of a single machine. This design reports the possibility of removing the constraint of the magnetic space phase angle difference between two consecutive phase belts of a machine to be a submultiple of π or even 2π , which in turn allows fractional slots/pole for an induction machine. The proposed design has been validated using two-dimensional finite element analysis (2D FEA). Further, analytic expressions for equivalent circuit parameters of this proposed machine have been reported as well, which have been verified against FEA results.

I. INTRODUCTION

In accordance with the increasing level of pollution and concern over depleting oil storage, rising levels of regulatory restriction has resulted in a market for more fuel efficient, low emission automobiles. Electric and hybrid electric vehicles seem to provide a solution to this challenge, with a price for additional weight and cost of the electric power-train components. Further, due to the optimized size and weight of the electrical system, cost and fuel efficiency, micro and mild hybrid architecture have gained a lot of popularity amongst all other type of hybrids [1], which is expected to continue into the near future.

In conventional automobiles a starter motor (usually a DC motor) [1], is used to start the internal combustion engine (ICE), operating at low-speed high torque intermittently (only to start the engine, with a maximum duration of 20-30s). Further, there is a separate electric machine (usually a Lundell Alternator) [1], to generate electricity (wide speed range operation, 2000-12000 RPM), which powers up the vehicle power network and charge the battery. The Integrated Starter Alternator (ISA) [2] makes the first step towards micro or mild hybrid automobiles. The ISA is used in different modes of operation, such as high torque low speed for cranking, and high-speed low torque for alternator mode in cruising.

Further, the intermediate range of torque and speed will allow other hybrid functionality such as regenerative braking and torque boost. These different applications of a single machine demand a very wide torque and speed range of operation in both motoring and generating modes [2].

Different types of electric machines [3], induction motor (IM), interior and surface permanent magnet motors (IPM / SPM), Lundell alternator, switched reluctance motor (SRM) [3], have been discussed as potential candidates, with each of them having some advantages and disadvantages. A literature study shows, the robustness of the machine, fluctuating cost of rare earth material, ease of control, makes the Induction Machine as the most suitable choice, which is also being used by all major commercial manufacturers of hybrid and electric vehicles.

Although it has been shown extensively in the previous literature, significant flexibility is obtained using a power electronic converter to drive an induction machine, it can be further argued that the pole-pair number is a single most controlling factor of torque-speed characteristics of the induction machine [4]. Several ideas were developed to use mechanical switches to change the pole numbers of the machine before the advent of power electronics. Detailed research on 2:1 pole changing induction machine is reported in previous literature [4]. Further, a 3:1 pole changing technique was also reported. Power electronics allowed implementing these pole changes electronically from the inverter side. A 2:1 electronic pole changing configuration using power electronics is reported in [5]. It can be noted electronic pole changing allows higher flexibility on torque-speed characteristics. Further, it should be also emphasized, electronic pole changing allows changeover of winding configuration, without de-energizing the machine. Although the above mentioned techniques allow changing the pole configuration of the machine, in the order of multiples, such as 2:1, 3:1 etc., whereas phase amplitude modulation (PAM) allows pole changing in fractional order. It can be noted, a specific set of pole reconfiguration (set of two different pole numbers) can be obtained only, from a specific winding connection using PAM. However, for PAM, the number of phases remains the same in both the configuration. Phase pole modulation (PPM) allows further flexibility to change the number of phases as well. In [6] PPM was introduced by

Miller et al in [4]. To have a balanced machine operation the following equation must be satisfied for PPM

$$Q_s = 2pMq \quad (1)$$

Where Q_s is the number of stator slots, p is the pole pair number, M is the number of phases/pole and q is the phase belt number or number of stator slots/pole/phase. As for a given stator winding, q is fixed, p and M can be chosen arbitrarily satisfying Eqn. 1. It has been further argued in this paper compared to conventional winding connection, toroidally wound induction machine allows maximum flexibility in reconfiguration, when PPM is being used. This paper discusses a 9 leg inverter-fed 4 pole, 9 phase / 12 pole 3 phase machine. It is reported in [4], that much of literature on PPM induction machine is for very few variation of configuration, such as 4 pole / 12 pole and 6 pole / 18 pole. This study reports a generalized analysis of PPM machine. However, in line with initial work on PPM machines [6], this study also concludes that toroidal winding allows maximum flexibility for utilization of this type of machine.

However, so far in all these PPM IM [4], [6] all of the phase belts are not independently controlled, as half of the phase belts are wound to provide the electrical return path for the rest. In this paper, we have investigated controlling each of the phase belts independently while connecting the other end of all phase belts in a star point, which is only possible with a toroidal winding. This scheme allows more flexibility to achieve higher pole count.

A novel concept of independent phase belt control technique has been reported in this paper, which allows higher flexibility than the phase pole modulated machines reported in previous literature. In this design study, a 12 leg voltage source inverter has been used to achieve 2/4/6/8/10 pole configuration of a single machine. This design reports the possibility of removing the constraint of the magnetic space phase angle difference between two consecutive phase belts of a machine to be a submultiple of π or even 2π , which in turn allows fractional slots/pole for an induction machine at the expense of added harmonic content in the air-gap flux. It has been further shown that, due to the presence of sub-harmonics, although a 10 pole configuration is possible, that would lead to lower torque production. This has been by FEA as well.

As we are controlling the machine in independent phase-belt fashion, instead of conventional radial winding like structure, conventional analysis of induction machine with the complete winding (with go and return of a coil) cannot be used. Hence a rigorous mathematical approach has been developed to calculate the equivalent circuit parameters of the machine under the proposed excitation scheme, and the expressions are obtained for generalized pole count. It has been also shown, the developed analytical expressions can be used to obtain the equivalent circuit parameters for radial winding like structure, as a special case. The proposed design has been further validated using 2D FEA as well. The FEA has been further used to find the equivalent circuit parameters of the designed

machine under different pole configuration and compared against the same obtained from the analytical calculation as reported. Finally, the obtained circuit parameters have been used to obtain the machine torque-speed characteristics for different phase pole configurations.

II. MACHINE STRUCTURE AND ANALYSIS

Based on the literature survey on reconfigurable induction machine drives, particularly PPM strategy, a novel stator winding structure is proposed here. It is discussed in the previous research on PPM induction machine [4], [6], the toroidal winding machine provides maximum utilization of flexibility of PPM. Although toroidal machines have a poor utilisation of winding copper, and higher winding resistance compared to conventional radially wound machines, they allow flexibility in controlling each phase belt separately, as well controlling the number of turns per phase to accommodate required voltage and current rating.

This type of connectivity allows considerable flexibility of reconfiguring the machine in different pole pairs. In general the following equation has to be satisfied:

$$Q_s = 2pMq \quad (2)$$

Where Q_s is the number of stator slots, p is the pole pair, M is the number of phases/pole and q is the phase belt or the number of stator slots/inverter leg. As for the given machine, q is predetermined, for a given pole pair, M has to be decided from this equation. Applying this equation for 2, 4, and 6 pole configuration, number of phases equal to 6, 3, and 2 respectively are obtained. However, a further generalised equation can be used to obtain the phase difference between the legs, given as below:

$$Phase \text{ (in degree)} = \frac{p \times 360}{m} \quad (3)$$

Where, l_n is the total number of phase belts, given as below:

$$m = \frac{Q_s}{q} \quad (4)$$

It can be noted the number of the legs will decide possible phase-pole combinations as well as the number of switches in the inverter. With a large l_n many phase-pole combinations will be achievable at the expense of a large number of switches. For this work, we have considered a 12 leg inverter to have 12 phases.

Toroidal machines, as reported in [4], [6], connects certain phase belts together, which reduces the number of inverter legs, at the expense of reducing the number of pole configurations achievable. In [4], [7], a 9 leg inverter has been used to achieve 2/6 pole configurations. However, at the expense of 3 more inverter legs (total 12 leg inverter) we have shown here up to 5 different pole configurations of the same machine can be achieved. The phase angle for the different pole configurations (2,4,6 8 and 10) can be obtained using

Eqn. 3 and 4. The machine winding schematic is shown in Fig. 1.

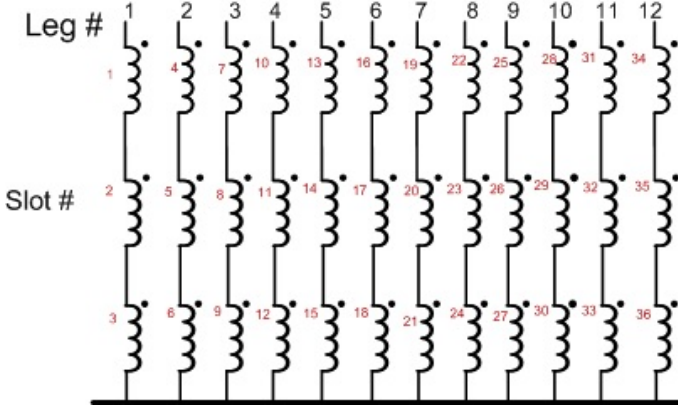


Fig. 1. Winding diagram of the proposed 12 Leg PPM toroidal winding

It can be noted, for the 8 pole configuration, the phase angle difference between two phase belts is not a submultiple of π , which has not been investigated before. In conventional radially wound m phase machine as well as previously reported PPM IM [4], [6], as the return paths of different phase belts are connected electrically, the achievable magnetic space phase angle difference between two consecutive phase belts are a submultiple of π . This constraint limits the possible pole configurations achievable. Mathematically the magnetic space phase angle difference is given by $\phi = \frac{\pi}{M}$, and M being an integer ph is a submultiple of π . Under the pole configuration $p = 5$, we explore even more freedom with respect to the magnetic space phase angle difference, by making it to be $\frac{5\pi}{6}$ which is not a submultiple of 2π .

This novelty can be expressed as an implementation of fractional slots/pole, as the slots per pole is given as 4.5 for 8 pole and 3.6 for 10 pole configurations respectively, which has never been reported in previous literature. It has been discussed in the later section this fractional slots per pole will give rise to unequal flux density under different poles, however, the skewed rotor can be used to address this problem. With this fractional slots/pole the PPM scheme allows unprecedented flexibility to widen torque-speed characteristics of a machine.

III. FINITE ELEMENT ANALYSIS

For initial evaluation of the proposed PPM IM configuration under different Phase-Pole combination, an FEA has been done. The details of the FEA package used, along with step by step methodology for extraction of equivalent circuit parameters are reported below.

A. Finite Element Package

To implement finite element simulation, a two-dimensional finite element software FEMM [8] has been used. The software allows defining the geometry of the system, and material property of different part of the problem along with boundary condition. Further, the electrical circuit connection (series or

parallel) can be defined, to set up excitation. The electrical current can be defined as an a.c. quantity (phasor representation for different phases) along with the frequency of the problem.

To study the proposed PPM induction machine, current at different stator slot is defined, with different phase belt current as given in Eqn 3. The software, calculates rotor current to meet the boundary condition, which is set as $\vec{A} = 0$ at the stator boundary, where \vec{A} denotes the magnetic vector potential given as below

$$\vec{B} = \nabla \times \vec{A} \quad (5)$$

Where \vec{B} is the flux density. The FEA package yields flux density, current density and magnetic vector potential over the geometry. It also gives a direct output for circuit properties, such as voltage drop, resistive loss, and flux linkage. The package further allows using the non-linear $B - H$ curve to model saturation in the magnetic material.

B. Circuit Parameters from FEA

As the modelled machine is a cage rotor induction machine the analysis is performed [9], using the above mentioned FEA package. The steps for obtaining equivalent circuit parameters are shown below:

- 1) The machine is excited with balanced 1A (RMS) current in all phases (with the phase angles from Eqn. 3) at two frequencies ω_1 and ω_2
- 2) The stator loss has been extracted from FEA.
- 3) Using stator current information, and stator loss, stator resistance (R_s) is obtained as below

$$P_{sloss} = m|I_s|^2 R_s \quad (6)$$

- 4) Stator phase voltage has been obtained from the FEA result. Stator voltage V_s is related to equivalent circuit parameters as below:

$$V_s = I_s(R_s + j\omega L_s + \frac{j\omega L_m(R_r + j\omega L_r)}{R_r + j\omega(L_m + L_r)}) \quad (7)$$

Where I_s and I_r denotes stator and rotor phase current, and R_s and R_r denotes stator and rotor resistances, and m is the number of phases. As the stator phase current excitation is set in FEA, this value is readily available. L_s, L_r and L_m denotes stator leakage, rotor leakage and stator to rotor mutual inductances respectively. Breaking down Eqn 7 in real and imaginary parts, and for two different known frequencies ω_1 and ω_2 the rest four unknown quantities, L_s, L_r, L_m and R_r (as R_s is already obtained by Eqn. 6) can be found. The equivalent circuit parameters for the possible pole configurations of the machine discussed, has been obtained by following the above steps are shown in table III in the results section, along with the same obtained by analytical closed form expressions.

IV. ANALYTICAL EXPRESSIONS

Although FEA analysis is capable of extracting equivalent parameters of the machine accurately, it consumes more processing time. As we intend to optimize the design for a particular application using an optimization algorithm, (the optimization of the design is not presented in this work) which needs to evaluate the system for a very large number of machine dimensions possible. Hence to reduce the processing time analytical closed-form expressions for the equivalent circuit parameters have been obtained in this section and validated against the same obtained from FEA in the previous section. Further, analytical expressions will provide a good theoretical understanding of electro-magnetics of the proposed PPM scheme.

A. Stator MMF

The proposed machine connects the consecutive slots of a phase belt in series, and all phases in star fashion. We can not use magnetics expressions for a conventional coil with go and return. However, it can be shown the stator excitation scheme is somewhat similar to the induced currents inside a squirrel cage induction machine, where each bar current is phase shifted by $\frac{2p\pi}{N_r}$, where p is the pole pair number, and N_r is the number of rotor bars. Hence a similar mathematical treatment reported for rotor cage current, reported in [10], has been used to develop the expression for the stator MMF, which has been used to find equivalent circuit parameters. Considering m phases and q slots per phase, the stator current density can be expressed as

$$\vec{K}_s(\phi, t) = \sum_{k=0}^{m-1} \sum_{l=0}^{q-1} \text{Re}\left(\frac{I}{R} e^{j(\omega t - k\frac{2\pi p}{m})} \delta\left(\phi - \frac{2\pi(qk+l)}{N_s}\right)\right) \quad (8)$$

Where I is the current in each stator phase, R is the radius of the machine. The unit impulse function $\delta(\cdot)$ allows modelling the stator MMF as a series of impulsive currents around the stator. For simplicity of the derivation, we have assumed all the slots of a phase belt in a single position, later the equivalent distribution factor for the phase belt is introduced. Assuming each phase belt in a single location, Eqn. 8 can be written as

$$\vec{K}_s(\phi, t) = \sum_{k=0}^{m-1} \text{Re}\left(\frac{I_{pb}}{R} e^{j(\omega t - k\frac{2\pi p}{m})} \delta\left(\phi - \frac{2\pi k}{m}\right)\right) \quad (9)$$

Where I_{pb} represents equivalent current for the whole phase belt of q slots. Taking the Fourier transform [10, Eqn. 4.86-91], it can be shown the n th space harmonic amplitude of the flux is given as

$$K_{s,n} = \frac{I_{pb}}{2\pi R} \sum_{k=0}^{m-1} e^{j(n-1)2\pi kp/m} \quad (10)$$

It can be further shown, the sum is zero except where $\frac{(n-1)p}{m}$ is an integer, in which case the sum equals to m . It can be argued, most important harmonics will be for $n = 1, \frac{m}{p} \pm 1$, as they produce largest magnetic fields. Each of these surface

current harmonics will produce a magnetic field, amplitude of which is given by

$$B_{s,n} = \frac{\mu_0}{2\pi} \frac{m I_{pb}}{pg} \quad (11)$$

where g is the air gap length of the machine. So far the derivation has been done, without considering the distribution of the winding over the phase belt, and stator turn number. This can be accounted, by the following equation

$$I_{pb} = k_d n_s I \quad (12)$$

where k_d is the distribution factor for phase belt q for n th harmonic, given as

$$= \text{Re}\left(\left[\frac{1 - e^{-jp\lambda qn}}{1 - e^{-jp\lambda n}}\right] e^{-jp\lambda \frac{q}{2}}\right) \text{ if } q \text{ is even} \quad (13)$$

$$\text{Re}\left(\left[\frac{1 - e^{-jp\lambda qn}}{1 - e^{-jp\lambda n}}\right] e^{-jp\lambda \frac{q+1}{2}}\right) \text{ if } q \text{ is odd}$$

Combining the above equations, the n th harmonic component of stator MMF can be given as

$$B_{s,n} = \frac{\mu_0}{2\pi} I \frac{k_d m n_s}{n p g} \quad (14)$$

B. Stator Self Inductance (L_{ss})

Using energy balance approach [10], it can be written

$$\frac{m}{4} L I^2 = \frac{L g R}{2} \int_0^{2\pi} H(\phi, t) \cdot B(\phi, t) d\alpha \quad (15)$$

Further, assuming linear relation ($B = \mu_0 H$) and using expression from Eqn. 14 in 15, and simplifying, stator mutual inductance can be obtained as

$$L_{ss} = \frac{\mu_0 L R}{g} \frac{(k_d n_s)^2}{2\pi p^2} \left(\frac{m}{p}\right) \quad (16)$$

It can be verified by putting $m = 6p$, for a conventional 3 phase machine, the stator self inductance will be given as

$$L_{ss} = 2 * \frac{\mu_0 L R}{g} \frac{(k_d n_s)^2}{2\pi p^2} \left(\frac{6p}{p}\right) = \frac{3}{2} \frac{4}{\pi} \frac{\mu_0 L R}{g} \frac{(k_d n_s)^2}{p^2}$$

The expression is multiplied by 2, as in conventional radial winding, each stator phase consists of both go and return of independent phase belts.

Based on these expression, and following a similar mathematical approach, as reported in [10], and considering major harmonics for $n = \frac{m}{p} \pm 1$ the expression for the equivalent circuit parameters of rotor leakage inductance and resistance have been obtained. The expression for stator leakage inductance has been obtained considering several components, such as slot leakage, zigzag leakage, along with stator dominant harmonic leakages given at $n = \frac{m}{p} \pm 1$, and the expressions from [10] have been used. Due to the similarity of the expressions, and space constraint in this paper, these expressions are not presented here.

C. Dominant Stator Harmonics

The major harmonics present in the stator, are tabulated in I. It should be noted in the fractional phase belt configuration, where the phase belt angle is not a sub-multiple of π , e.g 8 pole, the even harmonics are present. Further, in case of 10 pole, the phase belt angle not being a sub-multiple leads to the presence of sub-harmonics.

TABLE I
DOMINANT STATOR HARMONICS AT DIFFERENT POLE CONFIGURATION

Pole Pair	Dominant Harmonics	
p	$n = \frac{m}{p} - 1$	$n = \frac{m}{p} - 1$
1	11	13
2	5	7
3	3	5
4	2	4
5	0.2	2.2

V. RESULTS

A particular machine geometry is needed to implement FEA, and also to benchmark the proposed design against reported literature, the same machine dimensions are used as [4] given in Table II.

TABLE II
DETAILS OF THE MACHINE USED FOR FEA AND ANALYTICAL STUDY OF PPM INDUCTION MACHINE

Dimension	Value(in mm)
Stator Outer Dia	203.5
Stator Inner Dia	114.1
Rotor Outer Dia	113.3
Stator Slot Number	36
Rotor Slot Number	28
Stator Slot width (air gap side)	4.95
Stator Slot width (yoke side)	7.92
Stator Slot Depth	17.5
Rotor Slot width (air gap side)	5.15
Rotor Slot width (shaft side)	3.0
Rotor Slot Depth	11.15

The machine described in Table II, has been studied under 5 different pole -phase configuration. The flux density plots, with contours, are shown in the Fig. 2, for different possible pole configurations. For the FEA analysis, excitation frequency is kept at 2.5 Hz , which equates to 5% slip, considering 50Hz supply frequency. Exciting the system at slip frequency, in the static model, is equivalent to analysing the system in rotor frame of reference.

Following the method discussed in the previous sections, the machine parameters are calculated. Also using the analytical expressions developed in the previous section, same parameters have been calculated. Both the results have been tabulated in Table.III , for comparison. As the FEA simulation is done in two-dimension, hence the end ring terms in the analytical result are neglected to emulate similar condition. Table. III shows that the results obtained from FEA are closely

matching with analytical results, which proves the validity of the expressions obtained. As the equivalent circuit model used for FEA analysis is for fundamental frequency only, due to increase in harmonics (lower order harmonics for higher pole pair configuration) leads to a greater mismatch for 8 pole and 10 pole configuration.

A. Steady state characteristics

Applying the FEA, and analytical expressions described before the per phase equivalent circuit model is obtained for the machine. The steady state torque-speed characteristics, assuming a 42V DC bus voltage has been shown in Fig. 3 for these different configurations. It should be noted these curves corresponds to maximum torque available from the machine. It can be further noted, as indicated by analytical expression, due to the presence of sub-harmonics, the available torque for 10 pole configuration is lower than 8 pole configuration. Hence although this is a theoretically possible pole configuration, practically it will not be used.

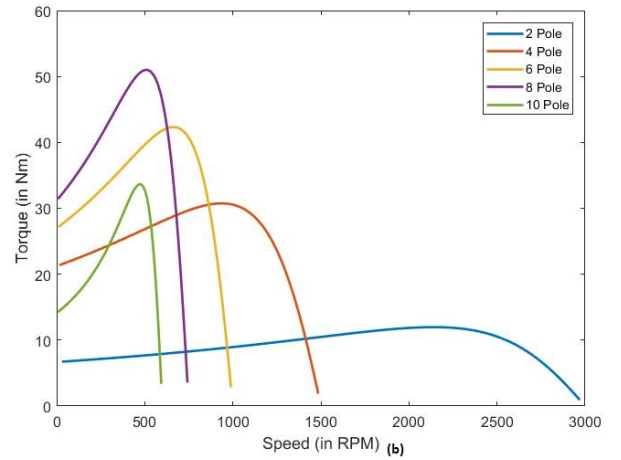


Fig. 3. Torque-speed characteristics of the machine under different Pole configuration at 50 Hz

VI. CONCLUSIONS

In this paper, a novel PPM IM concept is discussed in detail. The proposed machine, shows the capability of more flexibility in terms of achievable pole configuration, hence leading to wider torque-speed characteristics. The proposed concept is validated using FEA, as well as analytical method. The derived analytical expression shows a good match with FEA results. Further, these analytical expressions are useful for machine design, and design optimization. A hardware prototype is currently under development, and experimental results will be presented in future work.

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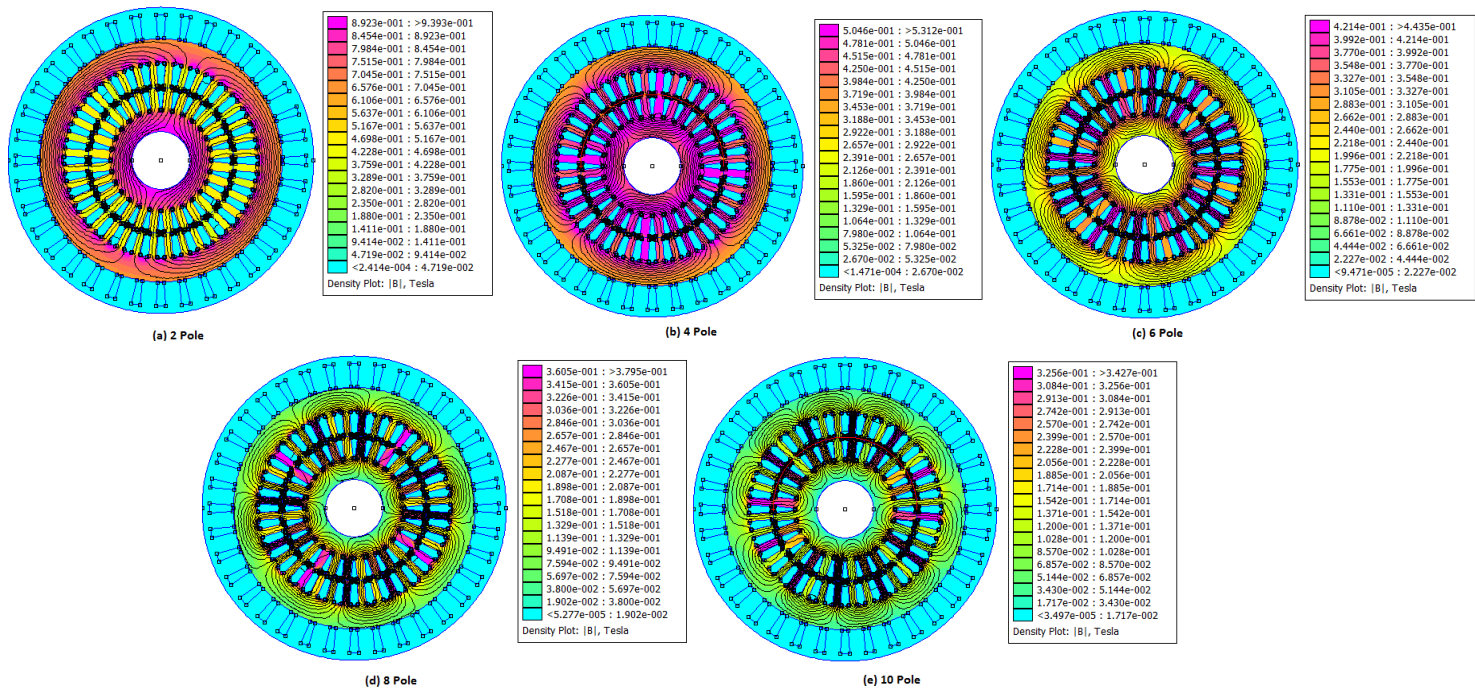


Fig. 2. FEA of the proposed machine under different phase pole configuration of the machine under different pole configuration

TABLE III
EQUIVALENT CIRCUIT PARAMETERS FROM FEA AND ANALYTICAL METHOD

Pole No.	R_s (in Ω)		R_r (in Ω)		L_s (in mH)		L_r (in mH)		L_m (in mH)	
	Calc	FEA	Calc	FEA	Calc	FEA	Calc	FEA	Calc	FEA
2	0.0484	0.0484	0.1158	0.1127	0.87	1.03	0.18	0.23	49.3	49.9
4	0.0484	0.0484	0.1075	0.1004	0.89	0.82	0.09	0.07	12.3	10.8
6	0.0484	0.0484	0.0969	0.0821	0.90	0.78	0.09	0.10	5.5	4.2
8	0.0484	0.0484	0.0825	0.0626	0.91	0.77	0.09	0.05	3.1	1.9
10	0.0484	0.0484	0.0620	0.0453	0.91	0.85	0.28	0.22	2.0	1.0

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