FUZZY LINEAR PROGRAMMING IN INVESTMENTS

Nebojsa M. Ralevic[1], **Vladimir Dj. Djakovic**[2], **Goran B. Andjelic**[3], **Jelena S. Kiurski**[4]

[1]*University of Novi Sad, Faculty of Technical Sciences, Department of Fundamentals Sciences, Novi Sad, Serbia*
[2]*University of Novi Sad, Faculty of Technical Sciences, Department of Industrial Engineering and Management, Novi Sad, Serbia*
[3]*Educons University, Faculty of Business Economy, Sremska Kamenica, Serbia*
[4]*University Business Academy, Faculty of Economics and Engineering Management, Novi Sad, Serbia*
e-mail: nralevic@uns.ac.rs, v_djakovic@uns.ac.rs, goran.andjelic@educons.edu.rs, jelena.kiurski7@gmail.com

Abstract
The possibility of the investment performances assessment is especially important in the dynamic business conditions. Hence, the appropriate methodology enables an optimal investment decision. The main objective of the research is to test and analyse the possibility of fuzzy linear programming application in the investment processes with special attention to the investment performances. The methodology used in the research understands the methods of analyses and synthesis and also application of the fuzzy linear programming. The quantitative and qualitative results of the research imply that the application of fuzzy linear programming provide important information about the effects from the investment activities.

Keywords: fuzzy, fuzzy linear programming, investment performances, investments.

Introduction
The application of the contemporary engineering methods, techniques and tools present a necessity in the management processes, particularly in the investment processes. Namely, the investments processes are influenced by the dynamic nature of the investments, regarding its risk and return characteristics. The complexity of the investment environment induces the application of wide variety of mathematical methods in the function of formulating, implementation and evaluation the adequate investment strategy. In that way, investors are provided with the quality, scientific based information about the investments, determining different investment scenario effects, that is, outcomes from the investment activities. Solid, scientific based information is the key for acquiring and maintaining the competitive advantage.
When describing the fuzziness of certain system, it is important to observe it as an inherent characteristic, that is, phenomena originating from a fuzzy structure (Ralevic et al, 2011). Fuzzy linear programming presents an advantage while enabling the adequate investment decision making process. By its application it is possible to determine whether concrete investment provide the expected (requested) return from the investment activities. Specificities of the transitional markets, especially the market of the Republic of Serbia, require special attention while analysing the effects from the investment activities. Turbulent and dynamic business conditions understand the presence of high risk level and often even high level of uncertainty. Consequently, it is essential to gather high quality information about the planned investment and thus enabling the optimal investment decision.

Having in mind all mentioned above, the determination of the possibility and specificities of the fuzzy linear programming application on the transitional market of the Republic of Serbia present solid base for the academic and investment community, while recognizing its place, role and importance in the contemporary investment processes.

Methodology

The methodology used in the research comprises the application of the fuzzy linear programming on the data acquired from the Belgrade Stock Exchange (official internet site). The research sample present the Prime listing daily data of the following stocks:
- AERO - Aerodrom Nikola Tesla a.d., Beograd;
- NIIS - NIS a.d., Novi Sad.

The general type of fuzzy linear programming is formulated as follows:

$$\begin{aligned} &\max \sum_{j=1}^{n} \hat{c}_j \hat{x}_j \\ &\sum_{j=1}^{n} \hat{a}_{ij} \hat{x}_j \le \hat{b}_i, i = 1,\ldots,m \\ &\hat{x}_j \ge 0,\ j = 1,\ldots,n \end{aligned} \tag{1}$$

where $\hat{a}_{ij}, \hat{b}_i, \hat{c}_j$ are fuzzy numbers, and $\hat{x}_j$ are variables whose states are fuzzy numbers, the operation of addition and multiplication are operations of fuzzy arithmetic and $\le$ denotes the ordering of fuzzy numbers.

Special cases are considered:

$$\begin{aligned} &\max \sum_{j=1}^{n} c_j x_j \\ &\sum_{j=1}^{n} a_{ij} x_j \le \hat{b}_i, i = 1,\ldots,m \\ &x_j \ge 0,\ j = 1,\ldots,n \end{aligned} \tag{2}$$

Let fuzzy numbers $\hat{b}_i$ have the form

$$\hat{b}_1(x) = \begin{cases} 1, & x < b_i \\ \dfrac{b_i + d_i - x}{d_i}, & b_i \le x \le b_i + d_i, \\ 0, & x > b_i + d_i \end{cases} \tag{3}$$

Lower and upper of the optimal values are calculated, z_l and is z_u is obtained by solving the standard programming problems, respectively.

$$\begin{aligned} &\max \sum_{j=1}^{n} c_j x_j \\ &\sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1,\ldots,m \\ &x_j \ge 0,\ j = 1,\ldots,n \end{aligned} \tag{4}$$

$$\max \sum_{j=1}^{n} c_j x_j$$
$$\sum_{j=1}^{n} a_{ij} x_j \le b_i + d_i, i = 1,\dots,m \tag{5}$$
$$x_j \ge 0, \; j = 1,\dots,n$$

The fuzzy sets of optimal values G fuzzy subset of R^n is defined by

$$G(\mathrm{x}) = \begin{cases} 1, & \mathrm{cx} < z_u \\ \dfrac{\mathrm{c} \cdot \mathrm{x} - z_l}{z_u - z_l}, & z_l \le \mathrm{cx} \le z_u \,, \\ 0, & \mathrm{cx} > z_l \end{cases} \tag{6}$$

The problem becomes the classical problem

$$\max \lambda$$
$$\lambda(z_u - z_l) - \mathrm{c} \cdot \mathrm{x} \le -z_l$$
$$\lambda d_i + \sum_{j=1}^{n} a_{ij} x_j \le b_i + d_i, i = 1,\dots,m \tag{7}$$
$$\lambda, x_j \ge 0, \; j = 1,\dots,n$$

The above problem is a problem of finding $\mathbf{x} \in R^n$ such that

$$((\bigcap_{i-1}^{m} D_i) \bigcap G)(\mathrm{x}) \tag{8}$$

(where $D_i(\mathrm{x}) = \hat{b}_i(\sum_{j=1}^{n} a_{ij} x_j)$) reaches the maximum value.

Results and discussion

In order to overcome shortcomings in stock trading due to the uncertainty of the stock prices i.e. their volatility, in the simplified model considered, usually solved using linear programming (LP), fuzzy linear programming (FLP) is applied. Coefficients and unknowns which are crisp values in LP, in FLP are fuzzy numbers.

Such problem, as it is to be seen, is reduced to solving multiple linear programs, usually by classic simplex method using software, except in simple cases where the graphic method can also be applied.

The practical application of fuzzy linear programming is considered on AERO and NIIS stocks. Let the stock market price of AERO 1350 RSD and NIIS 680 RSD. Investment capital available for the investment activities is in the following range: 1 mil RSD to 1,2 mil RSD. Expected earnings for AERO is 5 RSD and for NIIS is 7 RSD.

The limitation of the investment is the following:

- Trade volume: 1000 to 1200;
- Investment capital: 1 mil RSD to 1,2 mil RSD.

The problem is to solve the next fuzzy FLP in the described way.

$$\max z = 5x_1 + 7x_2$$
$$x_1 + x_2 \le \hat{b}_1$$
$$1350x_1 + 680x_2 \le \hat{b}_2$$
$$x_1, x_2 \ge 0$$

$$\hat{b}_1(x) = \begin{cases} 1, & x < 1000 \\ \dfrac{1200 - x}{100}, & 1000 \le x \le 1200, \\ 0, & x > 1200 \end{cases}$$

$$\hat{b}_2(x) = \begin{cases} 1, & x < 1000000 \\ \dfrac{1200000 - x}{200000}, & 1000000 \le x \le 1200000, \\ 0, & x > 1200000 \end{cases}$$

Conclusion

Research results point to the adequacy of the fuzzy linear programming application on the transitional market of the Republic of Serbia. It is important to adequately determine the specificities of the investment focusing on the effects from the investment activities. Thus, the investment limitations, particularly trade volume and investment capital, affect the application performance of the fuzzy linear programming.

Further research in the subject field understand the continuous application of the fuzzy linear programming while focusing to the required return and investors risk preference while recognizing the dynamic conditions of the environment.

Acknowledgements

The authors acknowledge the financial support of the Ministry of Education, Science and Technological Development of the Republic of Serbia, within the Project No. TR34014.

References

[1] N.M. Ralević, V.Dj. Djaković, G.B. Andjelić, I.M. Kovačević, J.S. Kiurski, L.Lj. Čomić, Fuzzy Prediction Based on Regression Models: Evidence from the Belgrade Stock Exchange – Prime Market and Graphic Industry, SISY 2011, IEEE 9th International Symposium on Intelligent Systems and Informatics, September 8-10, 2011, Obuda University, Budapest, Hungary; Subotica Tech, Serbia; University of Novi Sad; Subotica, Serbia, pp. 151-156.

[2] D.F. Li, S.P. Wan, Fuzzy linear programming approach to multiattribute decision making with multiple types of attribute values and incomplete weight information, Appl. Soft. Comput. 13(11), 2013, pp. 4333-4348.

[3] D. Dubois, H. Prade, Fuzzy Sets and Systems: Theory and Applications, Academic Press, New York, 1980.

[4] H. Tanaka, H. Ichihashi, K. Asai, A formulation of fuzzy linear programming problem based on comparison of fuzzy numbers, Control Cybern. 3(3), 1991, pp. 185-194.

[5] R.C. Wang, T.F. Liang, Application of fuzzy multi-objective linear programming to aggregate production planning, Comput formulation Ind Eng. 46(1), 2004, pp. 17-41.