

Some Thoughts on the Nature of Reality

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Summary

The following few pages contain few thoughts on reality in general, Kantian view of it, whether mathematics captures it and why reality is an interconnected phenomenon. This is not a comprehensive essay on reality but some personal views that may be useful in understanding our grasp of reality.

What does my reality means?

On one wintry night I reversed my car out of a parking space and was about to move forward. Then I noticed another car reversing out of the parking space next to where I had parked. I froze in time and was wishing my car would not be hit. But the little thud happened. Later when I recalled the incident, I only could think of a rear of a black car softly bumping the front bumper bar of my car. But the photos showed a dark maroon coloured car instead of a black car. The image my mind built was taking into account the darkness of the surrounds. My perceptions are not always the reality. The world I perceive can be coloured by many things including the ambient light.

Unless we are unconscious or asleep, we live in the world of our minds rather than in our bodies. With every passing second, our existence becomes one of the mind rather than one of the body. Our life before this exact second is a series of memories, at times, jumbled up. Aptly, as Plato wrote about teachings of Heraclitus, “nothing ever is, everything is becoming” (Russell, 1961). In a book review I wrote previously I talked about three types of human existence based on memories, namely biographical, autobiographical and historical¹. Cartesian “Cogito ergo sum” becomes more complicated. Descartes asks “How could one deny that these hands and that my whole body exist?” (Cahn, 1990). Do I have a continuous existence, even in the constant flux, as I am a ‘thinking being’? Am I the ‘thinking being’ as perceived by myself and as remembered by others? The person who remembered the colour of the car as black in my previous anecdote is the same person who experienced the accident as my memories link me back to the accident. But my memory failed me and I am now months older. Can a memory of a past dream get spliced with my other memories and create a new reality? Thus, the reality of our existence possibly is subjective at best and Descartes’ body in front of the fireplace feeling his hands is simply a continuum of momentary proprioception and thought. Our biographical existence together with our autobiographical and historical existence make us feel real. Basically, our existence or reality in general is not an isolated phenomenon but a whole consisting of collection of frames of reference².

¹ Review of Sapiens: A Brief History of Humankind by Yuval Noah Harari: https://www.amazon.com/gp/customer-reviews/R3GS9OC9PGBM9K?ref=pf_vv_at_pdctrvw_srp or <http://thelureofnoma.com/posts-are-here/2016/03/imagined-orders-chains-of-memories-and-biographical-existence-a-brief-history-of-humankind-by-yuval-noah-harrari-harvill-secker-london-2014/>

² See the review of Visual Intelligence; How We Create What We See by Donald D. Hoffman: <https://hcommons.org/deposits/item/hc:15519/>

Can we cease the reality?

When I read Plato's allegory in the book VII of Republic about the prisoner's in the cave, the epistemological side of the above scenario also comes to mind. The prisoners, it is very likely, couldn't distinguish the shadow of man from a shadow of a statue of a man. Do we really know the world we live in? Perhaps we do as much as Plato's prisoners did about the cave that they were entrapped in. Whatever we perceive is whatever flows through the doors of knowledge including mind into our beings defined by our bodies. Do we call all these perceptions reality? That is the question of philosophy humans grapple with since and perhaps, long before Plato's time.

As pragmatic creatures, we humans have a very little regard to the nature of reality. For us, in our mundane existence, it is more important to be aware of our surrounding rather than be more precise about it. We use what I call "the rule of thumb" logic (Arachige, 2009) in the world which we experience. I don't measure the distance between my car driven on the right-hand side and the car on the left lane when passing one another at 100 kilometres an hour. This miraculous sense has been built into us by Nature and we use it not only for the physical world but also for the mental activities. The best examples of such mental activities come from Theory of Mind. We cannot exactly know what the other party has in the mind. But we, engaging in a rule of thumb deduction, when the other person looks at us, sense a threat or friendly gesture. Oftentimes, flight or fight responses are proactive rather than reactive.

Philosophers of mathematics argue that mathematics provides a glimpse into reality. As Penrose (p 12-23: Penrose, 2005) points out, the reality in this sense is about the objectivity of the mathematical truths. When we say that Pythagorean Theorem is always true for rectangular triangles, it is an objective truth irrespective of other externalities. We know we only can perceive different models of Platonic forms such as specific triangles and squares. Plato believed in the existence of these forms in an idealised world beyond physical existence. In other words, as an example, it is not a particular triangle, but the notion of a triangle; a pure, somewhat mystical existence beyond the physical word. Pythagorean Theorem is not specific to a particular rectangular triangle as it is true for Platonic rectangular triangle. Does this Platonic world of mathematics exist? Is there some reality beyond physical existence?

Reality and Quantifiable Existence

I am neither a philosopher nor a mathematician. Thus, following discussion will be more an expression of an opinion than an analysis. To discuss the reality, let us start with the following assertion of Immanuel Kant about principle of anticipation of perceptions:

"In all appearances the real, which is an object of the sensation, has intensive quantity, that is has a degree³." According to Kant, the real should cause sensation and should have magnitude. Pain or heat or light, all internal representations, have quantities not in intuition but in sensation. The quantity of quality is degree in sensation not in intuition. The appearances occupying a part of space or time, that can be intuited a priori, fall under the concept of quantity. In other words, only those appearances as objects of sensations or intuition quantifiable in degrees are real⁴. He says "what is

³ Kant, I The Critique of Pure Reason, Translated by J. M. D. Meiklejohn, Project Gutenberg Ebook#4280

⁴ P.964, Cahn (1990). This is my interpretation of what Kant wrote.

real in all appearance has a degree⁵. The degree here can be represented by numbers. Kant believes that pure mathematics can only have objective reality because it refers to objects of sense. However, all mathematical concepts are constructs of intuition and the concept of numbers, according to Kant, is based on the successive addition of units in time⁶. The last points make the numbers a construct of intuition as space and time arise from intuition. In other words, numbers themselves should not tell us about things-in-themselves or their appearances.

Russell following mathematicians such as Cantor and Dedekind rejected considering numbers as referring to quantities due to the existence of irrational numbers that cannot be linked to physical quantities. Then there is projective geometry using cross ratios that cannot be quantified. He also makes a distinction between quantity and magnitude and for him, a magnitude that is particularised in space-time is called a quantity; thus, two quantities of same kind resulting from particularization of same magnitudes are equal (p167: Russell, 1937). Weights of one pound of two kinds are equal in relation to the magnitude. Two kinds of materials whose weights were thus measured are quantities. In other words, the quantity is the physical existence while the magnitude is about some measurable quality of it. As Russell points out Kantian view makes reality synonymous with existence. An intensive quantity fills space or time. The reality is about the objects of sensations with such intensive magnitudes. I believe this argument is something Kant has been unfairly criticised for. I believe Kant is telling us that the sensations we feel about any object are caused by and proportional to the magnitudes of the object. The extensive magnitudes of appearance such as size and shape alone don't make a sensation real. Kantian view says that we cannot know the thing-in-itself. We only can know its appearance.

Despite mathematical objections to Kantian view that the reality should be quantifiable and should be attached to magnitudes it is very intuitive. The objection Russell (p: 177; Russell, 1937) expressed as two equal quantities of same kind have more existence than one is somewhat unfair assessment of Kant's view. Two equal quantities obviously lead to a larger intensity of perception that tells us about the appearance in space time of two quantities.

Thus, in Kantian worldview, numbers seem to be not-so-real as pointed out elsewhere in this article. Numerals represent symbols that were agreed upon by humans as a result of convention. They help us reason. Their reality only arises in the context of quantities and magnitudes.

Mathematics and Reality

Let us now investigate the reality of the numbers a little deeper. We know that the world of mathematics is extremely dependent on numbers. Numbers started as counting aides. I remember reading the way Veddass of ancient Ceylon kept track of the coconuts they plucked. They pair each coconut they have with a stick and keep the matching number of sticks securely. When the time comes, they pair the sticks from the bundle with the remaining coconuts and confirm whether they still have all. Thus, in a pragmatic world, counting numbers are only meaningful when they can be matched to an existing object. The concept of number itself is in the Platonic world. As we know the

⁵ P.963, Cahn (1990).

⁶ P.949, Cahn (1990).

cardinality of a number is defined using elements of a set. This is similar to the number of sticks in the Vedda's bundle. However, this is where the connection between the reality and the numbers ends. Mathematicians, however, try to make the numbers meaningful in themselves. Perhaps, Pythagoras who proclaimed that all things are numbers started this. Even though numbers and mathematics can relate to a higher truth, that Platonic world doesn't necessarily relate to the reality as our senses perceive it.

Let us assume that our Veddas are like some Hottentot tribes whose vocabulary didn't have names for numbers larger than three (p 3: Gamow, 1961). Any number larger than three was referred to as 'many'. Let us also assume that one bright fellow counted one coconut, two coconuts, and three coconuts and gave it to the chief in three separate baskets. This bright fellow also paired sticks with each coconut and placed them in three bundles. This makes the set of three bundles:

$\{1,2,3\}$

If we use Von Neuman's definition of ordinals the above can be written as $\{\{\}, \{\{\}\}, \{\{\{\}\}\}, \{\{\{\{\}\}\}\}\}$. Let us ignore empty set and use counting number notation. Then we can write $\{1, \{1,2\}, \{1,2,3\}\}$ to denote the ordinality of each stick in each bundle or subset in set theoretic terminology. It is noteworthy to remember if we ignore 0, we can get rid of the Burali Fortis's paradox⁷. If, ignoring mathematical niceties, we use our sticks definition and use cardinality of elements within all subsets, we have 6 sticks making a set of cardinality of six. For a moment let us say we have extended the above series to all the natural numbers with cardinalities ending in the largest finite number. The elements of the set S, now are all the bundles of sticks with a cardinality equal to each counting number. Let us also say that the conceivable largest ordinality of the subsets is Ω that is the last number before the countable infinity. Then the above set S is $\{1,2,3,\dots, \Omega\}$. The greatest ordinality of the number of sticks in a new set, S^* assuming each stick in all the bundles in S have been separately identified and made an element in the new set is $\Omega ((1+ \Omega)/2)$ and definitely greater than the cardinality of countable infinity of the original set. Thus, in a very mundane sense as I am not attempting a mathematical proof, the countable infinity is an arbitrary definition that can be exceeded in rather a simple manner. The largest number that existed in Archimedes time was myriad or 10,000 (p. 6: Gamow,1961). Archimedes introduced a myriad myriad for 100,000,000. Georg Cantor built similar structure using transfinite numbers.

In a physical sense, we can be sceptical about the objective existence of a power set that Cantor used to argue for transfinite numbers. As the power set is a set of all possible subsets of a set, its connection to reality will be lost. This is somewhat similar to previous case. Simply a power set is not a physical concept since we lose the correspondence to our stick world. The largest number of physically meaningful subsets that can be made out of a set is dependent on each element of the original set. For the above example that used sticks the largest cardinality of all elements of such subsets, in a physically realistic sense, is confined to a number less than or equal to $\Omega ((1+ \Omega)/2)$. This is not true for the power set arising from all countable numbers.

Let us look at Cantor's diagonality argument. Let us assume there is a sequence of binary digits of length α . Then, one can construct 2^α sequences of variations of the original sequence. Can we now say that Cantor's diagonality argument can be interpreted as a proof of non-existence of an infinity?

⁷ Russell, B in the introduction to the second edition of The Principals of Mathematics

With this argument we learn by contradiction that if α is substituted by \aleph_0 , the lowest order infinity of rational numbers, then we only can find an injection between the sequences and the infinite set of countable numbers. If we add the digit 1 in a diagonal manner (i.e. add 1 to first digit of first sequence, second digit of second sequence and so on) to every sequence we constructed, we can construct a new sequence not in the first \aleph_0 sequences. Thus, there are larger cardinalities than \aleph_0 . Instead of arguing there are higher cardinalities than \aleph_0 , can we interpret this as \aleph_0 , as a limiting number, does not exist? In other words, as long as there is some physical reality that is countable, the countability doesn't end. Thus, in my view, the notion of infinity is a figment of imagination to symbolise the artificial limit imposed by human reasoning.

The reality is something we can introduce to our calculations or we can simply play with the numbers. In the world of frames of reference neither zero nor infinity can exist. Similarly, all numbers are not real and only exist in the human psyche as Platonic realities. In other words, they only have a symbolic meaning tied to an order of magnitude. We don't use a musical scale for counting even though each note represents a quantifiable frequency. The reason is that the mind is based on efficiency. We adopted Indo-Arabic numerals in place of Roman numerals because of the efficiency it provided. This allowed many eminent mathematicians to build empires in their imagination. Pure mathematics is pure imagination. What this means is that we cannot see an existential frame of reference in a number. An existential frame of reference should be observer independent and a number is not. Number needs a human mind versed in numerical concepts to see its existence. Thus, numbers need human consciousness for their existence and to make sense. This will be further discussed in the following section.

Frames of Reference and Reality

In my review of "visual intelligence"⁸ it was mentioned that there exist two types of frames of reference, namely, Existential and Relational. We can see that a thing to exist it should be subject to both these frames of reference. Let us define the following:

Any existence that can be grasped by our senses to be real should have at least one magnitude (a degree) that is independent of the observer- Existential frame of reference.

More than one observer, if circumstances allow, should be able to perceive the same existence irrespective of the magnitudes each of them perceive – Relational frame of reference.

Space, time and many other quantifiable properties of an existence should be described by the magnitudes of various relevant dimensions.

The observer is the person who is the subject of the sensation. The first definition excludes any mental sensation such as pain, pleasure or fear that, though intrinsically quantifiable, depend on the subject. Schopenhauer also says that feeling is not rational knowledge of reason that is communicable. Reason for humans is synonymous with speech that comes from the language (p:37: Schopenhauer, 1969). Feelings are thus considered differently from the rational or abstract knowledge of reason as they are in the concept of feeling, their own family of heterogeneous things

⁸ See the review of Visual Intelligence; How We Create What We See by Donald D. Hoffman: <https://hcommons.org/deposits/item/hc:15519/>

that cannot be described in the abstract. I cannot express feelings to another person in a magnitude that is common to both of us. Thus, they are not associated with an existential frame of reference as they only can exist within a living organism of some complexity. The reality of these mental states should be described using the substrate they arise from. Some comparisons between these mental states are possible not because they themselves are compatible but because the substrates they are residing on are compatible and are possible of reasoning. Even though heat or light can be objectively measured as physical quantities both are sensations devoid of form or shape. This can yet be described by an existential frame of reference as they are not dependent on the observer.

Russell (p:687: Russell 1961) points out that Kant believes that the eyes and the mouth exist as things in themselves. In the frames of reference view of the world, the person as a whole exists as defined by his or her co-ordinate system. For a non-physical existence like localised light or heat it may be a description of some localised field.

As numeric zero and infinity can only exist in human mind, they don't have independent existence in the real world. We cannot place a stick and say it's zero as zero is just a numeral, a man-made symbol, to represent unbounded nothingness. If a Vedda counts 3 coconuts, he or she wouldn't count empty space in-between. However, Empty set is simply like an empty vessel and not nothingness (p.28: Stewart, 1981). Here we can refer back to our previous discussion about Von Neuman's definition using empty set. The meaning of zero vanishes when the human is taken out of the picture. The same can be said about infinity. As Robert Weiner's famous hotel with infinite number of rooms full of infinite number of guests can still accommodate another infinite number of guests, $\infty + \infty = \infty$. The same is true for 0, and is understandable as nothingness added to nothingness is still the nothingness.

Cantor denoted the above concept as $\aleph_0 + \aleph_0 = \aleph_0$. If this is about quantifiable world, we know from the Newtonian concept of inertial mass that states that mass of two objects taken together is the sum of their individual inertial masses (p.9: Reynolds, 1965). The first law of thermodynamics states that the energy is conserved in an isolated system. Thus, the infinity violates the basic laws of physics and cannot be represented by an existential frame of reference or quantified using primary or secondary dimensions. If we are dealing with a numeral, say 1, we are dealing with one unit of some measure. Thus, the infinity, unlike zero, is a symbol for "too many to count". A Hottentot in the old days would have written, if the person had known simple algebra, $4+4 = 8$ as many + many = many. Similarly, in modern terms, $\infty + \infty = \infty$ means 'too many to count' but not uncountable. Thus, we have justifiable doubts about the reality of zero, infinity and numbers in general. We found them to be human constructs. Numbers occupy a place between feelings and quantities such as heat or light. Unlike feelings, numerals, however, convey the same magnitude to all people familiar with them.

Kant discussing "How is Pure Mathematics Possible?" in *Prolegomena to Any Future Metaphysics* says that "everything which can be given to our senses (to the external senses in space and to the internal senses in time) is intuited by us as it appears to us, not as it is in itself" (p.950, Cahn, 1990). The above stated Kantian idea in the form of Transcendental idealism taken as two-world view, that is there exist two worlds that of the appearance and that of the things in themselves, is closer to the frames of reference view. The two-world view has more weight as Kant wrote "by [intuition] we can

know objects only as they appear to us (to our senses), not as they are in themselves⁹". He also says that the doubt of existence of things never came to his head¹⁰. Kant also had the view that laws of physics (Universal laws of nature) are what our understanding prescribe to nature but not what our understanding derives from nature. The laws of physics are what we prescribe to a phenomenon rather than what is in nature. When we discover behaviours that can be following such mathematically expressible regularities in nature, we simply ascribe these as laws to the nature¹¹. I must confess that I tend to agree with this view. Our reality is far from being discovered by mathematical manipulations. When we chance upon a natural phenomenon agreeing with our ideas mathematically expressed, we simply see the illusion that we discovered a law of nature. As Kant says the illusion like a dream is sensation devoid of experience. We cannot experience the gravitational law but we can only experience gravitational force. Gravitational law is something that we assume as Nature adhere to and thus, we prescribe to. However, scepticism, as proposed by Hume on causality is warranted here.

The frames of reference should not be an anthropic viewpoint. The frames of reference also rely on all measurable dimensions including space and time. According to Kant, space is "nothing but the form of all external appearances"¹². This can be interpreted to mean that space is just a 'perspective' in an artistic sense. Thus, Kant believes that the space is not a complete externality. As Schopenhauer, interpreting Kant, (p:120: Schopenhauer,1969) points out that time, space and causality are present in our consciousness not in thing-in-itself. Schopenhauer (p:51: Schopenhauer,1969) ties the concept of consciousness with the representation of thing-in-itself and thus, making the connection between Kantian appearance and the space and time. Kantian view challenges the independent existence of space and time. In a frame of reference view, space is an independent existence that can be measured. Time is dependent on the second law of thermodynamics and can be a dimension attached to frames. Every object can occupy some space at a given point in time and such space will be attached to the object itself. Kant seems to have a similar opinion on the space defined by an object when he says that all bodies together with the space in which they are, are mere representations existing in our thoughts. But the space between two objects has its own existence and with the arrow of time that space will be dynamically redefined.

Conclusion

In our previous discussion I believe we could show that reality has an objective aspect to it. Representations or appearances are only one aspect of it. This aspect is represented by the relational frame of reference. Apart from the appearances or representations, there exist reality represented by existential frames of reference. We can use mathematics and numbers to search for regularities in nature and thus, understand our world better. But they are unlikely to be anything more than imaginary constructs. Even though our memories and our perceptions may fail, there is some reality beyond the subjective that withstand these failures. We may not be able to exactly recall what we have in our memories. This is because our memories don't have an independent

⁹ P948, Cahn (1990) This shows, given the accuracy of translation, that Kant made the distinction between things that appear to us and that are in themselves.

¹⁰ P954, Cahn (1990)

¹¹ Interpretation of a statement in P970, Cahn (1990)

¹² P951, Cahn (1990)

existence. But as humans we only need what we are provided with by Nature. When our memories fail us, we may find other evidence to back up the past events. The human existence is not an isolated collection of parts but a whole that should be treated together. It is a mixture of existential frames and relational frames held together by dimensions such as space and time. My existence is interconnected with whole series of frames of reference. I exist together with all these existences and dependencies.

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