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QUARK CONFINEMENT
AND THE
PROPERTIES OF HADRONS.

BY

MICHAEL HOWARD McCALL. M.A.

A thesis presented for the degree of
Doctor of Philosophy
at the University of Durham.
March 1980.

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PREFACE

The work presented in this thesis was carried out between October 1977 and March 1980 in the Department of Mathematics at the University of Durham under the supervision of Professor E.J.Squires.

The material herein has not been previously submitted for any degree in this or any other university. No claim of originality is made for chapter one. The work described in chapters two and three was undertaken jointly by the author and Professor Squires. Much of the work of chapter three has been published in reference 1. Chapters four and five contain work based on published papers by the author, respectively references 2 and 3.

I would like to express my thanks to Professor Squires for his help, guidance, and encouragement, and to Mrs J.McGough for her careful typing of this thesis. I am grateful to the Science Research Council for providing a grant.

ABSTRACT

This thesis considers the effects of quark confinement on the properties of hadrons. We begin with a brief introductory chapter on quark models. In chapter two a simple free quark model is used to calculate the structure functions for deep inelastic lepton nucleon scattering. Quark confinement is then taken into account by giving the quarks a distribution in energy-momentum. Various sum rules are derived, and a comparison is made with other work.

In chapter three we consider the bubble sea, the existence of which is a consequence of quark confinement. A formula for its contribution to the structure function is derived, evaluated, and compared with an estimate of the sea from experimental data. In chapter four a comparison is made between the spin and flavour dependences of the bubble sea and of the gluon-induced sea. It is suggested that it may be possible to deduce which of these is the dominant contribution to the sea from measurements of antiquark asymmetries.

In chapter five the magnetic moments of the octet baryons are discussed. It is found that they are best explained in the M.I.T. Bag Model of quark confinement, by taking a larger variation in the radius of the bag, when a light quark in the baryon is replaced by a strange quark, than was thought previously. We conclude with the comment that a flexible bag is needed both to make a reasonable prediction of the bubble sea, and to explain to a fair degree of accuracy the experimental measurements of the octet baryon magnetic moments.

In the postscript it is shown that serious difficulties are encountered in explaining the magnetic moments of leptons in composite models.

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CHAPTER 1

INTRODUCTION - QUARK MODELS1.1 Quarks and partons

In 1964, Gell-Mann⁴ and Zweig⁵ independently postulated quarks as the basic constituents of hadrons. Their model in which baryons are composed of three quarks, and mesons of a quark and an antiquark, has had many theoretical successes; for example, it gave an understanding of the hadron spectrum, and led to the prediction - correct to within a few per cent - of $2/3$ for the ratio of the π -N total cross-section to the N-N total cross-section at high energy.

However, experimental searches for quarks proved unsuccessful, and so quarks were considered by many theorists merely as a tool for doing group theory calculations. This state of affairs lasted until the early 1970's, when deep inelastic scattering experiments on nucleons gave excellent indirect evidence for the existence of quarks.

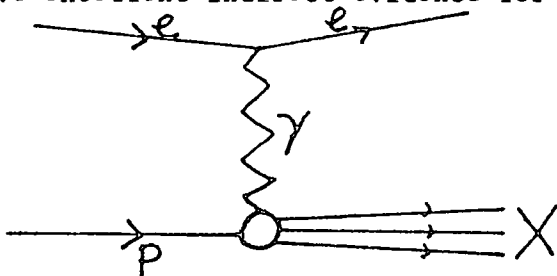


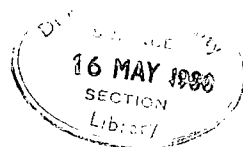
Figure 1.1 Deep Inelastic Scattering of electrons on protons $ep \rightarrow eX$

The cross section for this process is given by the formula⁶

$$\frac{d^2\sigma}{\Omega' dE'} = \frac{e^4 E'^2}{(2\pi)^2 q^4 M} \left(\frac{M}{q^0} F_2(x, q^2) \cos^2 \frac{\theta}{2} + 2F_1(x, q^2) \sin^2 \frac{\theta}{2} \right) \quad 1.1$$

where θ is the electron scattering angle, E' is the final electron energy, q is the photon 4-momentum (all in the proton rest frame), M is the proton mass, and the dimensionless scaling variable x is defined by

$$x = \frac{-q^2}{2Mq^0} \quad (0 \leq x \leq 1)$$



$F_1(x, q^2)$ and $F_2(x, q^2)$ are known as structure functions. They are found to be approximately q^2 independent, for q^2 above 2GeV^2 . This behaviour is known as approximate Bjorken scaling. In the limit of exact Bjorken scaling, the structure functions depend on the dimensionless variable x alone. This led Feynman⁷ to propose that the scattering took place off light pointlike objects inside the nucleon, which he named partons. Evidence has accumulated that the partons are, in fact, quarks. Deep Inelastic Scattering, and structure functions, are considered in much greater detail in Chapter 2.

In the naive parton model, the structure functions are taken as independent of q^2 . The Callan-Gross relation, $F_2 = 2xF_1$, is obeyed for spin $\frac{1}{2}$ partons. We write

$$F_2(x) = \sum_i e_i^2 x f_i(x). \quad 1.3$$

where the sum is over the different types of partons, $f_i(x)$ is the probability density function for a parton of type i (See Section 2.4).

Experimentally $F_2(x)$ tends to a constant, 0.32, as x tends to 0. This means that $f_i(x) \sim \frac{0.32}{x}$ as $x \rightarrow 0$, for some i , and thus that the number of partons rises as $x \rightarrow 0$. This is explained by describing a hadron as containing a 'sea' of quark-antiquark pairs as well as valence quarks. The sea distribution dominates at small x .

1.2 Quark confinement

The success of the parton model, and the continued lack of any observation of free quarks led to the formulation of the hypothesis of quark confinement, and, thence, to models incorporating quark confinement. The most successful of these is the M.I.T. Bag Model,^{6,8,9,10} in which quarks are confined within a bag of radius R by a bag surface pressure B .

Quantum Chromodynamics¹¹ is a possible theory of the strong interaction. It is a non-abelian gauge theory, based on an exact SU (3) symmetry of the quarks. This symmetry is described by saying that quarks exist in three different 'colours', usually chosen to be red, green and blue. Quarks couple to the intermediate vector bosons of the theory, called gluons, which also couple to themselves. Hadrons have no overall 'colour'.

The theory of quantum chromodynamics is not completely solved. There is no proof that it predicts colour confinement, though it is generally assumed to do so. It is worth remarking that recently fractional charges of $\pm \frac{1}{3}e$ have been measured on niobium spheres¹². A possible explanation of this is the presence of free quarks. DeRujula, Giles and Jaffe¹³ spontaneously break QCD by giving the gluons a small mass μ . This allows free quarks and gluons to exist with a large mass of $O\left(\frac{1}{\mu}\right)$. Whether quarks are, in fact, approximately or absolutely confined within hadrons, should have little effect on the calculation of hadronic properties. Thus it is eminently reasonable to use a model, such as the M.I.T. Bag Model, in which they are absolutely confined.

Structure functions cannot be directly calculated in QCD - only their variation with q^2 can be. This is consistent with the experimentally observed deviations from Bjorken scaling. However, structure functions can be calculated in the cavity approximation to the M.I.T. Bag Model,^{14, 15, 16} ignoring strong interaction effects such as gluon emission. There are 3 diagrams to consider (See Figure 1.2)

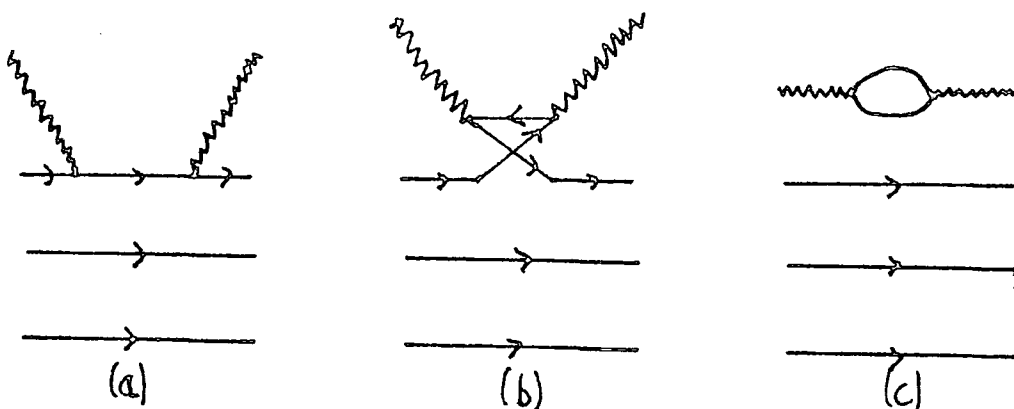


Figure 1.2. Diagrams contributing to the structure function.

Figure 1.2(a) corresponds to scattering off a valence quark. The bubble diagram of Figure 1.2(c) contributes to the sea of $q\bar{q}$ pairs observed in hadrons. The Z-graph of Figure 1.2(b) must be subtracted to remove the exclusion principle violating part of the bubble diagram. So $F = F_a - F_b + F_c$.

QCD also gives us a mechanism for generating the sea of $q\bar{q}$ pairs (see Figure 1.3). We call this the gluon induced sea, in comparison with the bubble sea of Figure 1.2(c).

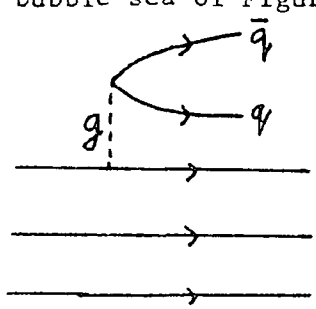


Figure 1.3 The Gluon-Induced Sea.

CHAPTER 2

FREE QUARK MODEL.2.1 Cross-section calculation

In the sections 2.1 - 2.3 we use a simple free quark model to calculate the structure functions. We assume that a nucleon is composed of three free massless valence quarks, and consider electron scattering off a quark, as shown in figure 2.1.

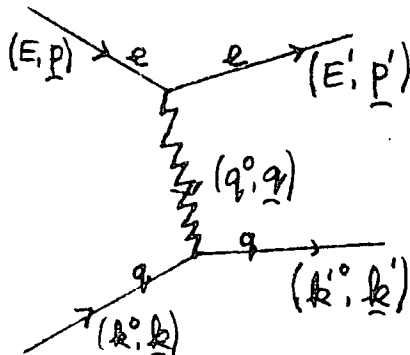


Figure 2.1 Electron-Quark Scattering

The differential cross section is given by

$$d\sigma = \int \frac{d^3 p'}{(2\pi)^6} \frac{d^3 k'}{(2\pi)^6} \frac{(2\pi)^4 \delta^{(4)}(p' - p + k' - k)}{2E 2E' 2k^0 2k'^0} |M|^2 \quad 2.1$$

where M is the matrix element for $e q \rightarrow e q$ scattering.

Using $k'^0 = |\underline{k}'| = |\underline{k} + \underline{q}|$ we have

$$d\sigma = \int \frac{d^3 p'}{16(2\pi)^2} \frac{\delta(q^0 + k^0 - |\underline{k} + \underline{q}|)}{E E' k^0 k'^0} |M|^2 \quad 2.2$$

We write $d^3 p' = p'^2 d|\underline{p}'| d\Omega$, and taking $|\underline{p}'| = E'$ (we put $m_e = 0$ throughout), find that

$$\frac{d^2\sigma}{d\Omega' dE'} = \frac{1}{16(2\pi)^2} \frac{E'}{E k^0 k'^0} |M|^2 \delta(q^0 + k^0 - |\underline{k} + \underline{q}|) \quad 2.3$$

Now, assuming the spherical symmetry of the proton in its rest

frame, we average over angles; we integrate over $z = \cos(\widehat{\underline{k}, \underline{q}})$ and normalise by dividing by $\int_{-1}^1 dz = 2$.

$$\begin{aligned} \text{We use } & \int dz \delta\left[q^0 + k^0 - \left(k^0^2 + |\underline{q}|^2 + 2k^0|\underline{q}|z\right)^{\frac{1}{2}}\right] \\ &= \frac{\left(k^0^2 + |\underline{q}|^2 + 2k^0|\underline{q}|z\right)^{\frac{1}{2}}}{k^0|\underline{q}|} \end{aligned}$$

2.4

$$= \frac{k^0 + q^0}{k^0|\underline{q}|}$$

to obtain

$$\frac{d^2\sigma}{d\mathcal{L}'dE'} = \frac{1}{32(2\pi)^2} \frac{E'}{Ek^0|\underline{q}|} |M|^2$$

2.5

Using Quantum Electrodynamics, and assuming point Dirac quarks, we find that

$$|M|^2 = \frac{8e^4 e_q^2}{q^4} \left((k \cdot p)(k' \cdot p') + (k \cdot p')(k' \cdot p) \right)$$

2.6

where e_q is the quark charge in units of e , the proton charge.

We write this in terms of the variables, s , t , and u , defined by

$$s = (k + p)^2 = (k' + p')^2 = 2k \cdot p = 2k' \cdot p'$$

$$u = (k - p')^2 = (k' - p)^2 = -2k \cdot p' = 2k' \cdot p$$

2.7

$$t = (k - k')^2 = (p - p')^2 = q^2$$

Using $(s + t + u) = \sum_i m_i^2 = 0$, we have

$$\begin{aligned} |M|^2 &= \frac{2e^4 e_q^2}{q^4} \left(\overline{s^2} + \overline{(s+t)^2} \right) \\ &= \frac{2e^4 e_q^2}{q^4} \left(2\overline{s^2} + 2\overline{s}q^2 + q^4 \right) \end{aligned}$$

2.8

where $\overline{}$ denotes an average over the azimuthal angle.

In the proton rest frame, in which \underline{k} is distributed spherically symmetrically, we write

$$\begin{aligned} p &= (E, E \cos \Theta, E \sin \Theta, 0) \\ q &= (q^0, q^1, 0, 0) \\ k &= (k^0, k^0 \cos \alpha, k^0 \sin \alpha \cos \beta, k^0 \sin \alpha \sin \beta) \end{aligned} \quad 2.9$$

The mass-shell conditions $p'^2 = 0, k'^2 = 0$, give us respectively

$$\cos \Theta = \frac{2Eq^0 - q^2}{2Eq^1} \quad 2.10$$

$$\text{and } \cos \alpha = \frac{2k^0 q^0 + q^2}{2k^0 q^1}$$

Now from (2.9) we find

$$k \cdot p = k^0 E (1 - \cos \alpha \cos \Theta - \sin \alpha \cos \beta \sin \Theta) \quad 2.11$$

We substitute in the values of α and Θ from (2.10), and average over β . The spherical symmetry of the proton gives $\overline{\cos \beta} = 0, \overline{\cos^2 \beta} = \frac{1}{2}$.

We find, using $q^2 = -2Mq^0 x$,

$$\overline{S} = 2k \cdot p = \frac{-q^2}{q'^2} q^0 \left(E + Mx - k^0 + \frac{2Ek^0}{q^0} \right) \quad 2.12$$

$$\begin{aligned} \text{and } \overline{S^2} &= \overline{S}^2 + \frac{4E^2 k^{0^2} \sin^2 \alpha \sin^2 \Theta}{2} \\ &= \frac{q^4}{8q'^4} \left[8q^{0^2} \left(E^2 + 2E \left(Mx - k^0 + \frac{2Ek^0}{q^0} \right) \right. \right. \\ &\quad \left. \left. + \left(Mx - k^0 + \frac{2Ek^0}{q^0} \right)^2 \right. \right. \\ &\quad \left. \left. + (4EE' + q^2) (4k^{0^2} + 4k^0 q^0 + q^2) \right) \right] \quad 2.13 \end{aligned}$$

Substituting (2.12) and (2.13) into (2.8) gives

$$\begin{aligned}
|M^2 &= \frac{2e^4 e^2}{q'^4} \left[q^{\circ 4} + 2q^{\circ 3} \left(k^{\circ} + Mx + \frac{EE'}{q^{\circ}} \right) \right. \\
&\quad + q^{\circ 2} \left(3M^2 x^2 - 2Mxk^{\circ} + 2k^{\circ 2} + \frac{2EE'}{q^{\circ}} (6k^{\circ} - Mx) \right) \\
&\quad \left. + 2q^{\circ} \left(-Mxk^{\circ} + \frac{6EE' k^{\circ 2}}{q^{\circ}} \right) \right] \quad 2.14
\end{aligned}$$

and substituting (2.14) into (2.5) we have

$$\begin{aligned}
\frac{d^2 \sigma}{d\Omega' dE'} &= \frac{e^4 e^2}{16(2\pi)^2 q^4} \frac{E'}{Ek^{\circ 2} q'} \frac{q^4}{q'^4} q^{\circ 4} \left[1 + \frac{2}{q^{\circ}} \left(k^{\circ} + Mx + \frac{EE'}{q^{\circ}} \right) \right. \\
&\quad + \frac{1}{q^{\circ 2}} \left(3M^2 x^2 - 2Mxk^{\circ} + 2k^{\circ 2} + \frac{2EE' (6k^{\circ} - Mx)}{q^{\circ}} \right) \\
&\quad \left. + \frac{2}{q^{\circ 3}} \left(-Mxk^{\circ} + \frac{6EE' k^{\circ 2}}{q^{\circ}} \right) \right] \quad 2.15
\end{aligned}$$

We use $q^2 = -2Mq^{\circ} x = -4EE' \sin^2 \frac{\theta}{2}$, and $q' = q^{\circ} \left(1 + \frac{2Mx}{q^{\circ}} \right)^{\frac{1}{2}}$

to obtain

$$\begin{aligned}
\frac{d^2 \sigma}{d\Omega' dE'} &= \frac{e^4 e^2 E'^2}{(2\pi)^2 q^4} \frac{1}{2k^{\circ 2}} \left[Mx \sin^2 \frac{\theta}{2} \left(1 + \frac{(2k^{\circ} - 3Mx)}{q^{\circ}} + O\left(\frac{1}{q^{\circ 2}}\right) \right) \right. \\
&\quad \left. + \frac{M^2 x^2}{q^{\circ}} \left(1 + \frac{6(k^{\circ} - Mx)}{q^{\circ}} + O\left(\frac{1}{q^{\circ 2}}\right) \right) \right] \quad 2.16
\end{aligned}$$

We compare with the standard formulae, equation (1.1),

$$\text{i.e. } \frac{d^2 \sigma}{d\Omega' dE'} = \frac{e^4 E'^2}{(2\pi)^2 q^4} \left[\frac{F_2(x, q^{\circ})}{q^{\circ}} \cos^2 \frac{\theta}{2} + \frac{2F_1(x, q^{\circ})}{M} \sin^2 \frac{\theta}{2} \right] \quad 2.17$$

Writing $\cos^2 \frac{\theta}{2} = 1 - \sin^2 \frac{\theta}{2}$, and then equating the terms independent of θ in (2.16) and (2.17) gives us

$$F_2(x, q^0) = \frac{e_q^2 M^2 x^2}{2k^0{}^2} \left(1 + \frac{6(k^0 - Mx)}{q^0} + O\left(\frac{1}{q^0{}^2}\right) \right) \quad 2.18$$

Equating the coefficients of $\sin^2 \frac{\theta}{2}$ now yields

$$\frac{2F_1(x, q^0)}{M} - \frac{F_2(x, q^0)}{q^0} = \frac{e_q^2 Mx}{2k^0{}^2} \left(1 + \frac{(2k^0 - 3Mx)}{q^0} + O\left(\frac{1}{q^0{}^2}\right) \right) \quad 2.19$$

$$\text{So } F_1(x, q^0) = \frac{e_q^2 M^2 x}{4k^0{}^2} \left(1 + \frac{2(k^0 - Mx)}{q^0} + O\left(\frac{1}{q^0{}^2}\right) \right) \quad 2.20$$

and we find

$$2xF_1(x, q^0) = F_2(x, q^0) \left(1 - \frac{4(k^0 - Mx)}{q^0} + O\left(\frac{1}{q^0{}^2}\right) \right) \quad 2.21$$

Thus the Callan-Gross relation is obtained in the Bjorken limit, i.e. as q^0 tends to infinity.

2.2 Hadronic tensor calculation

The Hadronic Tensor for spin-independent processes is given by

$$W_{\mu\nu} = (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) W_1 + \frac{1}{M^2} (P_\mu - \frac{P \cdot q}{q^2} q_\mu) (P_\nu - \frac{P \cdot q}{q^2} q_\nu) W_2 \quad 2.22$$

It is symmetric. W_1 and W_2 are structure functions. P_μ is the proton 4-momentum. It equals $(M, \underline{0})$ in the rest frame. So $P \cdot q = Mq^0$.

Now the relation

$$\frac{W_{00}}{q^0 + 2Mx} = \frac{W_{01}}{q^1} = \frac{W_{11}}{q^0} = \frac{1}{2Mx} \left(-W_1 + \left(1 + \frac{q^0}{2Mx}\right) W_2 \right) \quad 2.23$$

holds exactly.

$$\text{Also } W_{22} = W_{33} = W_1 \quad 2.24$$

$$\text{and } W_{12} = W_{23} = W_{31} = 0$$

The modulus squared of the matrix element for quark lepton scattering can be written in the form

$$|M|^2 = \frac{e^4 e_q^2}{q^4} X_{\mu\nu} L^{\mu\nu} \quad 2.25$$

where $X_{\mu\nu}$ and $L^{\mu\nu}$ are, respectively, the contributions of the quark-photon vertex, and the electron-photon vertex.

$$\begin{aligned} X_{\mu\nu} &= \frac{1}{2} \text{Tr} \not{k}' \gamma_\mu \not{k} \gamma_\nu \\ &= 2(k'_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} (k \cdot k')) \end{aligned} \quad 2.26$$

$$\begin{aligned} \text{Similarly } L^{\mu\nu} &= \frac{1}{2} \text{Tr} \not{p}' \gamma^\mu \not{p} \gamma^\nu \\ &= 2(p'_\mu p'_\nu + p'_\mu p_\nu - g_{\mu\nu} (p \cdot p')) \end{aligned} \quad 2.27$$

Thus we obtain equation (2.6) by substituting (2.26) and (2.27) into (2.25).

Now $X_{\mu\nu}$ is directly proportional to $W_{\mu\nu}$. For electron-proton scattering the differential cross-section is given by

$$\frac{d^2\sigma}{dE' d\Omega'} = \frac{1}{4(2\pi)^2} \frac{E'}{E} |M|^2 \quad 2.28$$

$$\text{where } |M|^2 = \frac{e^4}{q^4} W_{\mu\nu} L^{\mu\nu}.$$

Substituting equations (2.22) and (2.27) into (2.28), and writing $MW_1 = F_1$, and $q^0 W_2 = F_2$, gives us equation (2.17). Comparing equations (2.5), (2.25) and (2.28), we find that

$$W_{\mu\nu} = \frac{e^2}{8k^0 \frac{q^2}{q}} X_{\mu\nu} \quad 2.29$$

A comparison of the components of these two tensors will give us formulae for W_1 and W_2 , which should be consistent with those obtained for F_1 and F_2 in the previous section. From the mass-shell condition on the final quark, $k'^2 = 0$, we have $q \cdot k = Mq^0$. Using this, and equations (2.9), (2.10) and (2.26), we find that

$$\frac{X_{00}}{q^{\circ}+2Mx} = \frac{X_{01}}{q^1} = \frac{X_{11}}{q^{\circ}} = \frac{2(2k^{\circ}-Mx) + \frac{4k^{\circ 2}}{q^{\circ}}}{\left(1 + \frac{2Mx}{q^{\circ}}\right)} \quad 2.30$$

is obeyed exactly. Comparing with (2.23) and using (2.29) gives us

$$\left(1 + \frac{q^{\circ}}{2Mx}\right) W_2 = W_1 + \frac{e^2 Mx [(2k^{\circ}-Mx)q^{\circ} + 2k^{\circ 2}]}{2k^{\circ 2} q^1 (q^{\circ} + 2Mx)} \quad 2.31$$

$$\begin{aligned} \text{Now } X_{22} &= 2Mq^{\circ}x + 2\left(k^{\circ 2} + q^{\circ}\left(k^{\circ} - \frac{Mx}{2}\right)\right) \\ &\quad \left(\frac{1+q^{\circ}}{2Mx}\right) \\ &= 2Mq^{\circ}x \left[1 + \frac{(2k^{\circ}-Mx)}{q^{\circ}} + O\left(\frac{1}{q^{\circ 2}}\right)\right] \end{aligned} \quad 2.32$$

Comparing with (2.24) and using (2.29) we have

$$\begin{aligned} W_1 &= \frac{e^2 Mx q^{\circ}}{4k^{\circ 2} q^1} \left[1 + \frac{(2k^{\circ}-Mx)}{q^{\circ}} + O\left(\frac{1}{q^{\circ 2}}\right)\right] \\ &= \frac{e^2 Mx}{4k^{\circ 2}} \left[1 + \frac{2(k^{\circ}-Mx)}{q^{\circ}} + O\left(\frac{1}{q^{\circ 2}}\right)\right] \end{aligned} \quad 2.33$$

As $F_1 = MW_1$, this is consistent with equation (2.20).

We substitute for W_1 in equation (2.31) and obtain

$$W_2 = \frac{e^2 M^2 x^2}{2k^{\circ 2} q^{\circ}} \left[1 + \frac{6(k^{\circ}-Mx)}{q^{\circ}} + O\left(\frac{1}{q^{\circ 2}}\right)\right] \quad 2.34$$

Writing $F_2 = q^{\circ}W_2$ yields equation (2.18). So the picture is entirely consistent.

A similar calculation with neutrinos replacing electrons gives us, in the Bjorken limit,

$$\frac{-}{+} F_3(x) = 2F_1(x) = \frac{F_2(x)}{x} = \frac{M^2 x}{k^{\circ 2}}, \quad 2.35$$

the $-$ sign being for scattering off a quark, and the $+$ sign being for scattering off an antiquark.

2.3 Spin-dependent structure functions

The spin-dependent structure functions G_1 and G_2 are defined from the hadronic tensor by^{15,17}

$$\begin{aligned} -\frac{i}{2} (W_{\mu\nu} - W_{\nu\mu}) &= W_{\mu\nu}^{[A]}(P, q, s) \\ &= \epsilon_{\mu\nu\alpha\beta} q^\alpha s^\beta M G_1 + \frac{1}{M} \epsilon_{\mu\nu\alpha\beta} q^\alpha [(P \cdot q) s^\beta - (s \cdot q) P^\beta] G_2 \end{aligned} \quad 2.36$$

Here s denotes the spin of the proton. Note that $s \cdot q = -s^1 q^1$.

We can calculate $X_{\mu\nu}$, which is related to $W_{\mu\nu}$ by equation (2.29).

$$X_{\mu\nu} = +\frac{1}{2} \text{Tr}(\not{k}' + m) \gamma_\mu (\not{k} + m) (1 - \gamma^5 \alpha) \gamma_\nu \quad 2.37$$

We have included the quark mass m for reasons which will soon become apparent. a^μ is the polarisation 4-vector. We find that

$$-\frac{i}{2} (X_{\mu\nu} - X_{\nu\mu}) = -2m \epsilon_{\alpha\mu\beta\nu} q^\alpha a^\beta \quad 2.38$$

We use¹⁸

$$\underline{m a} = \underline{m s} + \frac{\underline{k} (\underline{s} \cdot \underline{k})}{k^0 + m} \quad 2.39$$

$$m a^0 = \underline{k} \cdot \underline{s}$$

and obtain, averaging over the azimuthal angle β ,

$$-\frac{i}{2} (X_{02} - X_{20}) = -2 \epsilon_{1032} q^1 s^3 \frac{k_3^2}{k_0}$$

$$-\frac{i}{2} (X_{12} - X_{21}) = -2 \epsilon_{0132} q^0 s^3 \frac{k_3^2}{k_0} \quad 2.40$$

$$-\frac{i}{2} (X_{23} - X_{32}) = -2 \epsilon_{0213} s^1 k_1 \frac{(q^0 k^1 - q^1 k^0)}{k_0}$$

in the limit of $m \rightarrow 0$.

We compare with

$$\begin{aligned} -\frac{i}{2} (W_{02} - W_{20}) &= \epsilon_{0213} q^1 s^3 (MG_1 + q^0 G_2) \\ -\frac{i}{2} (W_{12} - W_{21}) &= \epsilon_{1203} q^0 s^3 (MG_1 + q^0 G_2) \\ -\frac{i}{2} (W_{23} - W_{32}) &= \epsilon_{2301} M s^1 q^0 (G_1 - 2xG_2) \end{aligned} \quad 2.41$$

This gives us the following pair of simultaneous equations for G_1 and G_2 .

$$MG_1 + q^0 G_2 = \frac{2e_q^2 k_3^2}{k^0} \left(\frac{1}{8k^0 q^1} \right) \quad 2.42$$

$$M(G_1 - 2xG_2) = \frac{2e_q^2 k_1 (q^0 k^1 - q^1 k^0)}{8q^0 k^0 q^1}$$

Solving these we find that, for massless quarks,

$$\begin{aligned} g_1(x, q^0) &= M^2 q^0 G_1(x, q^0) \\ &= \frac{e_q^2 M^2 x}{4k^0 q^3} \left[(Mx - k^0) + \frac{(9k^0 Mx - 2k^0 q^2 - 6M^2 x^2)}{q^0} + O\left(\frac{1}{q^0 q^2}\right) \right] \end{aligned} \quad 2.43$$

and

$$\begin{aligned} g_2(x, q^0) &= Mq^0 G_2(x, q^0) \\ &= \frac{e_q^2 M^2 x}{4k^0 q^3} \left[\left(2k^0 - \frac{3Mx}{2}\right) + \frac{(3k^0 - 12k^0 Mx + \frac{15}{2}M^2 x^2)}{q^0} + O\left(\frac{1}{q^0 q^2}\right) \right] \end{aligned} \quad 2.44$$

2.4 Energy-momentum distribution of quarks

Quarks are confined inside hadrons. From the uncertainty principle

it follows that they have a distribution in momentum. We take this into account by giving the initial quark an energy-momentum distribution, described by $P(k^0, k)$. The normalisation is such that

$$\iint k^2 P(k^0, k) dk^0 dk = 1 \quad 2.45$$

The final quark, which has been struck by the photon and will thus be fast, will be 'nearly' on mass-shell, i.e. k'^2 is $O(1)$. In the Bjorken limit this gives

$$k^0 - Mx = k \cos \theta \quad 2.46$$

$$\text{So } |k^0 - Mx| < k \quad 2.47$$

If the initial quark is on mass-shell, as in sections (2.1) to (2.3), (2.46) gives us $0 < x < \frac{2k^0}{M}$.

As we are now varying k , i.e. no longer imposing $k^0 = k$, we must rewrite equation (2.4) thus

$$\begin{aligned} \int dz \delta[q^0 + k^0 - (k^2 + |q|^2 + 2k|q|z)^{\frac{1}{2}}] &= \frac{(k^2 + |q|^2 + 2k|q|z)^{\frac{1}{2}}}{k|q|} \\ &= \frac{k^0 + q^0}{k|q|} \end{aligned} \quad 2.48$$

So equation (2.5) now reads

$$\frac{d^2\sigma}{d\Omega'dE'} = \frac{1}{32(2\pi)^2} \frac{E'}{E k^0 k |q|} |M|^2 \quad 2.49$$

and our formulae for the spin-independent structure functions for valence quarks are, in the Bjorken limit,

$$F_2^{eq}(x) = 2x F_1^{eq}(x) = e_q^2 \iint_{|k^0 - Mx| < k} dk^0 dk k^2 P(k^0, k) \frac{M^2 x^2}{2k^0 k} \quad 2.50$$

$$\begin{aligned} \text{and } F_2^{vq}(x) &= 2x F_1^{vq}(x) = \frac{1}{3} x F_3^{vq}(x) \\ &= 2 \iint dk^0 dk k^2 P(k^0, k) \frac{M^2 x^2}{2k^0 k} \end{aligned} \quad 2.51$$

(- for quarks, + for antiquarks).

The factor 2 outside the integral sign in (2.51) appears as a result of the weak current being of the form (V-A). The cross-section is thus of the form (V-A)(V-A). So there are two equal contributions to the structure functions F_1 and F_2 from the VV and the AA terms, and two equal contributions to F_3 from the VA and AV terms.

We now consider sum rules, and find that

$$\begin{aligned} \int_0^1 dx \frac{F_2^{eq}(x)}{e_q^2 x} &= \int_0^1 dx \iint_{|k^0 - Mx| < k} dk^0 dk k^2 P(k^0, k) \frac{M^2 x}{2k^0 k} \\ &= \iint dk^0 dk k^2 P(k^0, k) \int_{\frac{k^0 - k}{M}}^{\frac{k^0 + k}{M}} dx \frac{M^2 x}{2k^0 k} \\ &= 1. \end{aligned} \tag{2.52}$$

using equation (2.45).

$$\text{Thus } \frac{F_2^{eq}(x)}{e_q^2 x} = f(x). \tag{2.53}$$

where $f(x)$ is the quark probability density function, and we obtain

$$F_2^{eq}(x) = \sum_q e_q^2 x f(x). \tag{2.54}$$

as in the naive parton model (equation 1.3).

2.5 $\frac{G_A}{G_V}$ and sum rules for g_1 and g_2

In order to find $\frac{G_A}{G_V}$, the ratio of the axial vector coupling constant to the vector coupling constant, we need to calculate $\frac{\bar{\Psi}(\underline{\sigma} \cdot \hat{z})\Psi}{\bar{\Psi}\Psi}$, where Ψ is a solution of the Dirac equation for a quark mass m .

We find that

$$\bar{\Psi}(\underline{\sigma} \cdot \hat{z})\Psi = \bar{\Psi} \gamma^5 \gamma^3 \Psi = \frac{2}{3} (k^0 + 2m). \tag{2.55}$$

and $\bar{\Psi}\Psi = 2k^0$

This is the contribution of one quark only. Thus we need to multiply by 3, the number of valence quarks in the nucleon, and also, assuming SU (6) symmetry, by the SU (6) factor $\frac{5}{9}$.

So we obtain

$$\frac{G_A}{G_V} = \frac{5}{3} \left(\frac{1}{3} + \frac{2m}{3k^0} \right) \quad 2.56$$

Inserting the quark energy-momentum distribution $P(k^0, k)$ gives us the formula

$$\frac{G_A}{G_V} = \frac{5}{3} \left(\frac{1}{3} + \frac{2}{3} \iint dk^0 dk k^2 P(k^0, k) \frac{m}{k^0} \right) \quad 2.57$$

We can also insert $P(k^0, k)$ into the formulae (2.43) and (2.44) for g_1 and g_2 . We replace the e_q^2 in these formulae, which give the structure functions for electron scattering off one quark only, by the SU (6) factor. This factor takes into account the charges and spins of the quarks in the nucleon, assuming SU (6) symmetry. For the proton it is $\frac{5}{9}$, and for the neutron it is 0.

So, in the Bjorken limit, and now including terms in the quark mass, which are obtained by retaining terms in m as we go from equations (2.38) and (2.39) to (2.40), we have

$$g_1^P(x) = \frac{5}{9} \iint_{|k^0 - Mx| < k} \frac{dk^0 dk k^2 P(k^0, k)}{4k^0 k} M \frac{(m^2 + Mx m + M^2 x^2 - k^0 Mx)}{(k^0 + m)} \quad 2.58$$

$$\text{and } g_2^P(x) = \frac{5}{9} \iint_{|k^0 - Mx| < k} \frac{dk^0 dk k^2 P(k^0, k)}{8k^0 k} M \left[3k^0 - 2Mx - m - \frac{3(k^0 - Mx)^2}{k^0 + m} \right] \quad 2.59$$

We find that

$$\begin{aligned} \int_0^1 g_1^P(x) dx &= \frac{5}{9} \iint dk^0 dk k^2 P(k^0, k) \int_{\frac{k^0 - k}{M}}^{\frac{k^0 + k}{M}} \frac{M dx}{4k^0 k} \left(\frac{m^2 + Mx m + M^2 x^2 - k^0 Mx}{k^0 + m} \right) \\ &= \frac{5}{9} \iint dk^0 dk k^2 P(k^0, k) \left(\frac{1}{6} + \frac{m}{3k^0} \right). \end{aligned}$$

using $k_0^2 - k^2 = m^2$.

Comparing with equation (2.57) we obtain the Bjorken Sum Rule¹⁹

$$\int_0^1 g_1^P(x) dx = \frac{1}{6} \frac{G_A}{G_V} \quad 2.61$$

We also find $\int_0^1 g_2^P(x) dx = 0$, 2.62

as is required by the conservation of angular momentum^{7,15}.

2.6 Comparison with other work

Franklin²⁰ also calculates structure functions in the proton rest frame. However, he gives the initial quark a distribution in momentum only. He fits the proton structure function with a simple Gaussian momentum distribution down to $x \sim 0.1$, and fits the proton and neutron structure functions simultaneously, down to $x \sim 0.1-0.2$, with harmonic oscillator momentum distributions. The failure for $x < 0.1-0.2$ is because the sea quarks manifest themselves in this region.

Jaffe¹⁴ in the cavity approximation to the Bag Model takes the initial quarks as having fixed energy, and gives them a distribution in momentum. He obtains reasonable qualitative agreement with experiment.

We now compare in detail our formulae for the structure functions with those obtained equivalently from a light cone analysis and the operator product expansion by Barbieri, Ellis, Gaillard, and Ross²¹. These are for massless quarks,

$$F_1(x, q_0^2) = \frac{Mx F(\xi)}{\sqrt{1 + \frac{2Mx}{q_0^2}}} \left[1 + \frac{G(\xi)}{F(\xi)} \right] \quad 2.63$$

$$F_2(x, q_0^2) = \frac{2Mx^2 F(\xi)}{\left(1 + \frac{2Mx}{q_0^2}\right)^{3/2}} \left[1 + \frac{3G(\xi)}{F(\xi)} \right] \quad 2.64$$

ξ is the Nachtmann scaling variable defined for massless quarks by

$$\begin{aligned}\xi &= \frac{2x}{1 + \sqrt{1 + \frac{2Mx}{q^0}}} \\ &= x \left[1 - \frac{Mx}{2q^0} + O\left(\frac{1}{q^{02}}\right) \right]\end{aligned}\quad 2.65$$

$$G(\xi) = \frac{M}{q^0 \sqrt{1 + \frac{2Mx}{q^0}}} \int_{\xi}^1 d\xi' F(\xi') + \frac{M^2}{q^{02} \left(1 + \frac{2Mx}{q^0}\right)} \int_{\xi}^1 d\xi' \int_{\xi'}^1 d\xi'' F(\xi'').$$

2.66

Now G/F is $O\left(\frac{1}{q^0}\right)$. We now write formulae (2.63) and (2.64) to this order.

$$F_1(x, q^0) = Mx F\left(\frac{x}{q^0}\right) \left(1 - \frac{Mx}{q^0}\right) + \frac{M^2 x}{q^0} \int_{\xi}^1 d\xi' F(\xi') + O\left(\frac{1}{q^{02}}\right)\quad 2.67$$

$$F_2(x, q^0) = 2Mx^2 F\left(\frac{x}{q^0}\right) \left(1 - \frac{3Mx}{q^0}\right) + \frac{6M^2 x^2}{q^0} \int_{\xi}^1 d\xi' F(\xi') + O\left(\frac{1}{q^{02}}\right).\quad 2.68$$

We compare with formulae (2.18) and (2.20), now giving the quark a distribution in energy k^0 , described by $P(k^0)$, which is normalised so that

$$\int_0^{\infty} P(k^0) dk^0 = 1\quad 2.69$$

The quarks remain on mass-shell. The condition $k'^2 = 0$ gives us

$k^0 > \frac{M\xi}{2}$, working to $O\left(\frac{1}{q^0}\right)$. So (2.18) and (2.20) become

$$F_1(x, q^0) = e_q^2 \int_{\frac{M\xi}{2}}^{\infty} dk^0 P(k^0) \frac{M^2 x}{4k^{02}} \left(1 + \frac{2(k^0 - Mx)}{q^0}\right) + O\left(\frac{1}{q^{02}}\right)\quad 2.70$$

$$F_2(x, q^0) = e_q^2 \int_{\frac{M\xi}{2}}^{\infty} dk^0 P(k^0) \frac{M^2 x^2}{2k^{02}} \left(1 + \frac{6(k^0 - Mx)}{q^0}\right) + O\left(\frac{1}{q^{03}}\right)\quad 2.71$$

Comparing to 0th order we find that

$$F(\xi) = e_q^2 \int_{\frac{M\xi}{2}}^{\infty} dk^0 P(k^0) \frac{M}{4k^{02}} \quad 2.72$$

$$\begin{aligned} \text{so } \int_{\xi}^1 d\xi' F(\xi') &= e_q^2 \int_{\xi}^1 d\xi' \int_{\frac{M\xi'}{2}}^{\infty} dk^0 P(k^0) \frac{M}{4k^{02}} \\ &= e_q^2 \int_{\frac{M\xi}{2}}^{\infty} dk^0 \frac{P(k^0)}{4k^{02}} \int_{\xi}^{\frac{2k^0}{M}} M d\xi' \\ &= e_q^2 \int_{\frac{M\xi}{2}}^{\infty} dk^0 \frac{P(k^0)}{4k^{02}} (2k^0 - M\xi) \end{aligned} \quad 2.73$$

Substituting back in (2.67) and (2.68), and, bearing in mind that $\frac{\xi}{q^0} = \frac{x}{q^0} + O\left(\frac{1}{q^{02}}\right)$, we obtain equations (2.70) and (2.71). Thus these two approaches are equivalent to first order for the spin-averaged structure functions.

Bace and Scholz²² have obtained formulae for the spin-dependent structure functions, using the operator product expansion. They find that, for massless quarks,

$$2(g_1(x, q^0) + g_2(x, q^0)) = \frac{\xi F(\xi)}{\sqrt{1 + \frac{2Mx}{q^0}}} - \frac{M}{2q^0} \frac{\partial}{\partial x} \left[\frac{x \xi}{\sqrt{1 + \frac{2Mx}{q^0}}} (G(\xi) - K_1(\xi) - K_2(\xi)) \right] \quad 2.74$$

$$\begin{aligned} \text{and } 2g_2(x, q^0) &= \left(1 + x \frac{\partial}{\partial x} \right) \left[\frac{\xi}{\sqrt{1 + \frac{2Mx}{q^0}}} H(\xi) \right] \\ &\quad - \frac{M}{q^0} \left[\frac{\partial}{\partial x} + \frac{x}{2} \frac{\partial^2}{\partial x^2} \right] \left[\frac{x \xi}{\sqrt{1 + \frac{2Mx}{q^0}}} (G(\xi) - K_1(\xi) - K_2(\xi)) \right] \end{aligned} \quad 2.75$$

$$\text{where } G(y) = \int_y^1 \frac{dy'}{y'} \int_{y'}^1 dy'' F(y'')$$

$$K_1(y) = \int_y^1 dy' H(y') \quad 2.76$$

$$K_2(y) = \int_y^1 \frac{dy'}{y'} \int_{y'}^1 dy'' H(y'')$$

We obtain from equation (2.43) and (2.44), excluding the e_q^2 factor, and giving the quarks a distribution in energy, the following equations.

$$2g_1(x, q^0) + 2g_2(x, q^0) = \int_{\frac{Mx}{2}}^{\infty} dk^0 P(k^0) \frac{M^2 x}{4k^{03}} \left((2k^0 - Mx) + \frac{(2k^{02} - 6k^0 Mx + 3M^2 x^2)}{q^0} \right) \\ + \int_{\frac{Mx}{2}}^{\frac{Mx}{2}} dk^0 P(k^0) \frac{M^2 x}{4k^{03}} (2k^0 - Mx) + O\left(\frac{1}{q^{02}}\right)$$

2.77

We observe that $\int_{\frac{Mx}{2}}^{\frac{Mx}{2}} dk^0 \dots$ is $O\left(\frac{1}{q^{02}}\right)$.

$$2g_2(x, q^0) = \int_{\frac{Mx}{2}}^{\infty} dk^0 P(k^0) \frac{M^2 x}{4k^{03}} \left[(4k^0 - 3Mx) + \frac{6k^{02} - 24k^0 Mx + 15M^2 x^2}{q^0} \right] \\ + \int_{\frac{Mx}{2}}^{\frac{Mx}{2}} dk^0 P(k^0) \frac{M^2 x}{4k^{03}} (4k^0 - 3Mx) + O\left(\frac{1}{q^{02}}\right).$$

2.78

We find that

$$\int_{\frac{Mx}{2}}^{\frac{Mx}{2}} dk^0 P(k^0) \frac{M^2 x}{4k^{03}} (4k^0 - 3Mx) = -\frac{M^2 x}{2q^0} P\left(\frac{Mx}{2}\right) + O\left(\frac{1}{q^{02}}\right).$$

2.79

Writing (2.74) and (2.75) up to and including terms of $O\left(\frac{1}{q^0}\right)$, we have

$$2g_1(x, q^0) + 2g_2(x, q^0) = x \left(1 - \frac{3Mx}{2q^0}\right) \left(F(x) - \frac{Mx^2}{2q^0} F'(x) \right) \\ - \frac{M}{2q^0} \frac{d}{dx} \left[x^2 (G(x) - K_1(x) - K_2(x)) \right]$$

2.80

and $2g_2(x, q^0) = \left(1 + x \frac{d}{dx}\right) \left[x \left(1 - \frac{3Mx}{2q^0}\right) \left(H(x) - \frac{Mx^2}{2q^0} H'(x) \right) \right] \\ - \frac{M}{q^0} \left[\frac{d}{dx} + \frac{x}{2} \frac{d^2}{dx^2} \right] \left[x^2 (G(x) - K_1(x) - K_2(x)) \right].$

2.81

A comparison to 0th order gives us

$$F(x) = \int_{\frac{Mx}{2}}^{\infty} dk^0 P(k^0) \frac{M^2}{4k^{03}} (2k^0 - Mx) \quad 2.82$$

$$\text{and } \frac{d}{dx} (x^2 H(x)) = \int_{\frac{Mx}{2}}^{\infty} dk^0 P(k^0) \frac{M^2 x}{4k^{03}} (4k^0 - 3Mx). \quad 2.83$$

This yields

$$H(x) = F(x) \quad 2.84$$

and so from (2.76)

$$G(x) = K_2(x) \quad 2.85$$

We find that

$$K_1(x) = \int_{\frac{Mx}{2}}^{\infty} dk^0 P(k^0) \frac{M(2k^0 - Mx)^2}{8k^{03}} \quad 2.86$$

$$\text{and } \frac{M}{2q^0} \frac{d}{dx} (x^2 K_1(x)) = \int_{\frac{Mx}{2}}^{\infty} dk^0 P(k^0) \frac{M^2 x}{4k^{03} q^0} (2k^{02} - 3k^0 Mx + M^2 x^2). \quad 2.87$$

$$\text{and } -\frac{3Mx^2}{2q^0} - \frac{Mx^3}{2q^0} F'(x) = \int_{\frac{Mx}{2}}^{\infty} dk^0 P(k^0) \frac{M^2 x}{4k^{03} q^0} (2M^2 x^2 - 3k^0 Mx). \quad 2.88$$

Adding (2.87) and (2.88) together gives the scaling violation term in equation (2.77).

$$\begin{aligned} \text{Now } & \left(1 + x \frac{d}{dx}\right) \left(-\frac{3Mx^2}{2q^0} H(x) - \frac{Mx^3}{2q^0} H'(x)\right) \\ &= \int_{\frac{Mx}{2}}^{\infty} \frac{dk^0 P(k^0)}{4k^{03} q^0} \left[M^2 x (8M^2 x^2 - 9k^0 Mx)\right] - \frac{M^2 x}{2q^0} P\left(\frac{Mx}{2}\right) \end{aligned} \quad 2.89$$

and

$$\frac{M}{q^0} \left(\frac{d}{dx} + \frac{x}{2} \frac{d^2}{dx^2}\right) (x^2 K_1(x)) = \int_{\frac{Mx}{2}}^{\infty} \frac{dk^0 P(k^0)}{4k^{03} q^0} M^2 x (6k^{02} - 12k^0 Mx + 5M^2 x^2) \quad 2.90$$

Summing equations (2.89) and (2.90) gives a contribution of $O\left(\frac{1}{q^0}\right)$ to

$2g_2(x, q^0)$ of

$$\int_{\frac{Mx}{2}}^{\infty} \frac{dk^0 P(k^0)}{4k^{03} q^0} M^2 x (6k^{02} - 21k^0 Mx + 13M^2 x^2) - \frac{M^2 x}{2q^0} P\left(\frac{Mx}{2}\right).$$

2.91

This differs from the corresponding term in equation (2.78) by

$$\frac{M^3 x^2 (3k^0 - 2Mx)}{4k^{03} q^0}$$

2.92

Considering that the two methods give the same result for $(2g_1 + 2g_2)$, we find this discrepancy most surprising, and can find no explanation for it. Excepting this, the mass-shell quark model gives us formulae for the structure functions, which are equivalent to those obtained from the operator product expansion, at least up to $O\left(\frac{1}{q^0}\right)$.

2.7 The momentum sum rule

The fraction of momentum of a nucleon, held by one quark, is given by $\int_0^1 xf(x)dx$. Multiplying by 3, we obtain the momentum fraction of a nucleon, held by the valence quarks. The contribution of the sea quarks to this sum rule will be small, as the sea is large only for x small.

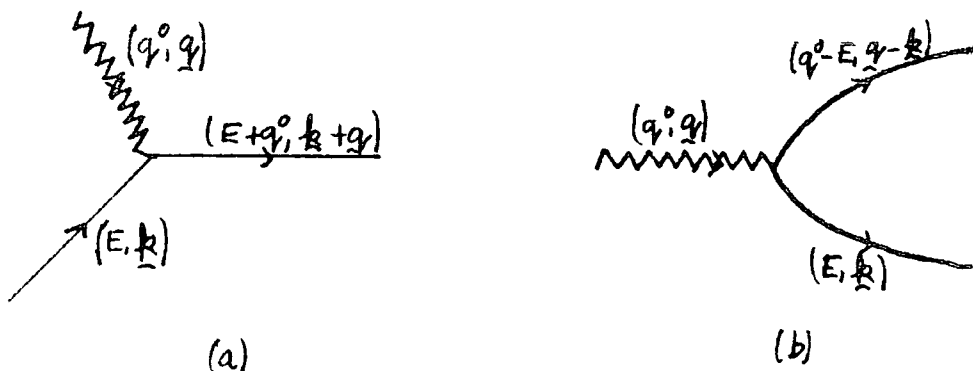
Using equations (2.50) and (2.53) we find that

$$3 \int_0^1 xf(x)dx = \int dk^0 dk k^2 P(k^0, k) \left(\frac{3k^0}{M} + \frac{k^2}{k^0 M} \right) \quad 2.93$$

For free massless quarks this equals $\frac{4k^0}{M}$ - which oversaturates the sum rule unless $k^0 \ll 0.235$ GeV.

In the cavity approximation to the bag model, this sum rule is saturated by the valence quarks alone. However experimentally the fraction of the nucleon momentum, held by quarks, is found to be about $\frac{1}{2}$. Gluons presumably account for the rest of the momentum.

CHAPTER 3

THE BUBBLE SEA.3.1 Quark distributions and structure functionsFigure 3.1(a) Valence Quark Scattering(b) The Bubble Diagram.

In this Chapter and the next, we write, for convenience, the quark energy as E , rather than as k^0 as in Chapter 2 (where E denoted the initial electron energy). We assume that the 'slow' initial valence quark in Figure 3.1(a) is in the ground state with fixed energy E , and thus has a distribution in momentum only. So the contribution of Figure 3.1(a) to the structure function is given in the Bjorken limit by (see Equation 2.50).

$$F_2^{eq}(x) = e_q^2 \int_{|E-Mx| < k} dk k^2 P(E, k) \frac{M^2 x^2}{2Ek} \quad 3.1$$

We define

$$V(x) = \int_{|E-Mx| < k} dk k^2 P(E, k) \frac{M^2 x^2}{2Ek} \quad 3.2$$

The difference between figures 3.1(a) and 3.1(b) is that the slow initial valence quark in (a) has 'swung round' to become an antiquark. Kinematically this means that $x \rightarrow -x$. Clearly, in the Bjorken limit, at least one of the quark and antiquark in the sea must be fast. Our

convention is that the lower quark (or antiquark) in the bubble is the slower. We split the bubble sea into a contribution, in which the lower quark, is in its ground state, and one in which it is in an excited state. These contributions may overlap, depending on our choice of the lower limit for the excited quark energy.

The ground state bubble contribution to the structure function is given by $aV(-x)$. The factor a , which is due to spin and colour, varies with the flavour of the quark in the ground state. If the quark is strange, charmed, etc, (or is an antiquark of any flavour), then for a proton (or neutron) $a = 6$. However for a proton we find $a = 4$ for $q = u$, and for $q = d$, $a = 5$, as certain states are not allowed by the exclusion principle. This is equivalent to subtracting off the z -graph of Figure 1.2(b).

We write the excited state bubble contribution to the structure function as $\Delta(x)$. It is evaluated in the next section. We now obtain formulae for various structure functions in terms of these contributions. We ignore the strange and charmed seas.

We find, using (2.54), that

$$F_2^{ep}(x) = \frac{4}{9} (xu_{VAL}(x) + 2\Delta(x) + xu_{SEA}^{GS}(x) + x\bar{u}_{SEA}^{GS}(x)) \\ + \frac{1}{9} (xd_{VAL}(x) + 2\Delta(x) + xd_{SEA}^{GS}(x) + x\bar{d}_{SEA}^{GS}(x)) \quad 3.3$$

Assuming that the u and d valence quark distributions are the same, this gives

$$F_2^{ep}(x) = \frac{4}{9} (2V(x) + 2\Delta(x) + 10V(-x)) + \frac{1}{9} (V(x) + 2\Delta(x) + 11V(-x)) \quad 3.4$$

Similarly we have

$$F_2^{en}(x) = \frac{4}{9} (V(x) + 2\Delta(x) + 11V(-x)) + \frac{1}{9} (2V(x) + 2\Delta(x) + 10V(-x)) \quad 3.5$$

So we find that

$$F_2^{ep}(x) - F_2^{en}(x) = \frac{1}{3}V(x) - \frac{1}{3}V(-x) \quad 3.6$$

We now consider charged-current neutrino structure functions.

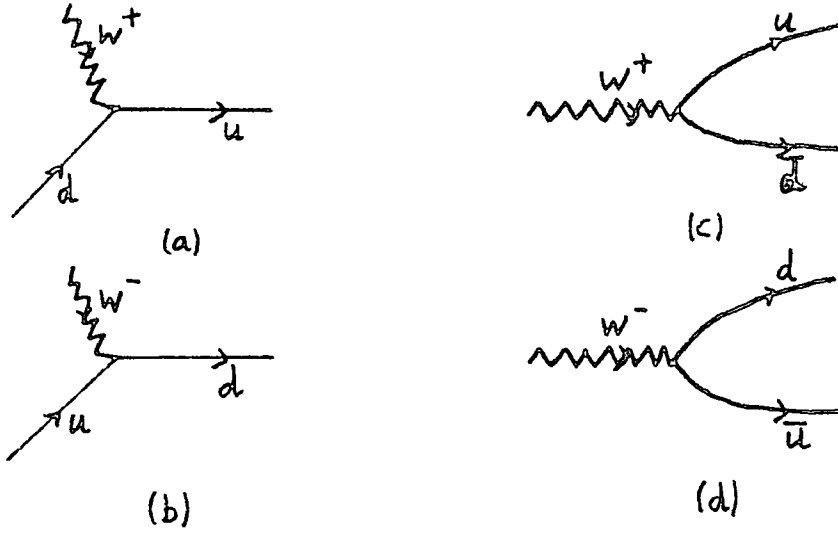


Figure 3.2(a) and (b). Valence Quark Scattering with W^+ as probe.
 (c) and (d). Bubble Diagrams produced by W^+ .

Considering the diagrams in Figure 3.2, we obtain

$$\begin{aligned} F_1^{\text{VP}}(\mathbf{x}) &= d_{\text{VAL}}(\mathbf{x}) + u_{\text{SEA}}(\mathbf{x}) + \bar{d}_{\text{SEA}}(\mathbf{x}) \\ &= \frac{1}{\mathbf{x}}(V(\mathbf{x}) + 2\Delta(\mathbf{x}) + 10V(-\mathbf{x})) \end{aligned} \quad 3.7$$

$$\begin{aligned} F_1^{\bar{\text{VP}}}(\mathbf{x}) &= u_{\text{VAL}}(\mathbf{x}) + d_{\text{SEA}}(\mathbf{x}) + \bar{u}_{\text{SEA}}(\mathbf{x}) \\ &= \frac{1}{\mathbf{x}}(2V(\mathbf{x}) + 2\Delta(\mathbf{x}) + 11V(-\mathbf{x})) \end{aligned} \quad 3.8$$

$$\begin{aligned} \frac{1}{2} F_3^{\text{VP}}(\mathbf{x}) &= -d_{\text{VAL}}(\mathbf{x}) + u_{\text{SEA}}(\mathbf{x}) + \bar{d}_{\text{SEA}}(\mathbf{x}) \\ &= \frac{1}{\mathbf{x}}(-V(\mathbf{x}) + 2\Delta(\mathbf{x}) + 10V(-\mathbf{x})) \end{aligned} \quad 3.9$$

$$\begin{aligned} \text{and } \frac{1}{2} F_3^{\bar{\text{VP}}}(\mathbf{x}) &= -u_{\text{VAL}}(\mathbf{x}) + d_{\text{SEA}}(\mathbf{x}) + \bar{u}_{\text{SEA}}(\mathbf{x}) \\ &= \frac{1}{\mathbf{x}}(-2V(\mathbf{x}) + 2\Delta(\mathbf{x}) + 11V(-\mathbf{x})) \end{aligned} \quad 3.10$$

Thus we obtain

$$\frac{1}{2}(F_1^{\text{VP}}(\mathbf{x}) - F_1^{\bar{\text{VP}}}(\mathbf{x})) + \frac{1}{4}(F_3^{\text{VP}}(\mathbf{x}) - F_3^{\bar{\text{VP}}}(\mathbf{x})) = \frac{-V(-\mathbf{x})}{\mathbf{x}} \quad 3.11$$

In the naive parton model

$$x^{-1}(F_2^{ep}(x) - F_2^{en}(x)) = \frac{1}{3}(u(x) + \bar{u}(x) - d(x) - \bar{d}(x)) \quad 3.12^*$$

$$\text{and } \frac{1}{2}(F_1^{vp}(x) - F_1^{\bar{v}p}(x)) + \frac{1}{4}(F_3^{vp}(x) - F_3^{\bar{v}p}(x)) = \bar{u}(x) - \bar{d}(x) \quad 3.13$$

These are consistent with equations (3.6) and (3.11) for

$$u(x) - d(x) = \frac{V(x)}{x} \quad 3.14$$

$$\text{and } \bar{u}(x) - \bar{d}(x) = \frac{-V(-x)}{x} \quad 3.15$$

3.2 Formula for $\Delta(x)$

Let us first suppose that the quark and antiquark in the bubble are both on mass shell. Then the condition

$$(q^0 - E)^2 - (q - \underline{k})^2 = m^2 \quad 3.16$$

yields in the Bjorken limit

$$k \cos \theta = E + Mx \quad 3.17$$

This only allows contributions in the unphysical $x < 0$ region. However, off mass-shell effects will ensure a contribution in the small positive x region.

We effectively take the 'fast' quark to be on mass shell, and describe the deviation of the slower quark from mass shell by the distribution $P(E, k)$. We have to sum over all allowed energy states of this quark. We do this by replacing the sum by an integral, $\int dE \rho(E)$, where $\rho(E)$ is the density of quark states with energy E . So we obtain the following formula for $\Delta(x)$ in the Bjorken limit, (c.f equation 3.2)

* $u(x)$ means the distribution of the u quark inside a proton, etc., isospin symmetry gives $u^p(x) = d^n(x)$, $u^n(x) = d^p(x)$, etc.

$$\Delta(x) = 6 \int_{E_1}^{\infty} dE \rho(E) \int_{E+Mx}^{\infty} dk k^2 P(E,k) \frac{M^2 x^2}{2Ek} \quad 3.18$$

E_1 , the lower limit of integration over E , should be greater than the quark ground state energy. There is a factor 6 outside the integral, as there are 3 colour and 2 spin states of the quark.

For ease of integration we take $k^2 P(E,k)$ to be an exponential distribution:

$$k^2 P(E,k) = \frac{\mu R}{2} \exp(-\mu R |k - (E^2 - m^2)^{\frac{1}{2}}|) \quad 3.19$$

where R is the effective radius of the confining region, and m is the quark mass. In the normalisation we have ignored a term $\frac{1}{2} \exp[-\mu R (E^2 - m^2)^{\frac{1}{2}}]$ compared with unity. The error is unimportant.

The average momentum associated with (3.19) is given by

$$R^2 \overline{[k - (E^2 - m^2)^{\frac{1}{2}}]^2} = \frac{2}{\mu^2} \quad 3.20$$

The uncertainty principle requires $\mu^2 \leq 2$.

The effective radius need not be the proton radius. Indeed, we expect it to increase with E , so that faster quarks will have less momentum uncertainty, i.e. will be closer to the mass shell. So we write

$$R(E) = c E^n \quad 3.21$$

where n will be determined in the next section. The constant of proportionality c is then fixed from knowledge of the proton radius and the quark energy inside the proton.

The density of states factor is given by

$$\rho(E) = 4\pi \left[\frac{V}{(2\pi)^3} \right] Ek \quad 3.22$$

where V is the volume of the proton. We take this as

$$V = \frac{4}{3} \pi R_0^2 R(E) \quad 3.23$$

where R_0 is the proton radius. This allows for an increase in effective radius in the longitudinal direction.

3.3 Evaluation of the formula

We now perform the integration over k in equation (3.18) to obtain

$$\Delta(x) = \frac{M^2 x^2 R_0^2 c}{\pi} \int_{E_1}^{\infty} dE E^n \exp[-\mu c E^n (E + Mx - (E^2 - m^2)^{\frac{1}{2}})] \quad 3.24$$

For $n = 0$, there is a linear divergence. Bell, Davis and Rafelski²³ use the cavity approximation to the M.I.T. Bag Model to calculate the sea contribution to the structure function, and also obtain a linear divergence. Hughes²⁴ remarks that the fixed radius bag does not provide an adequate description of deep inelastic scattering at small x . This indicates that a flexible bag is needed.

Note that we can write for large E in equation (3.19)

$$\begin{aligned} k - (E^2 - m^2)^{\frac{1}{2}} &\approx \frac{(k^2 - E^2)}{k + E} \\ &\approx \frac{(k^2 - E^2)}{2E} \end{aligned} \quad 3.25$$

So if R is independent of E , i.e. $n = 0$, the effective mass spread increases linearly with E . In calculations of the valence distribution in the simple quark model^{14, 16, 20, 21, 24, 25} it has been assumed that the effective mass of the struck quark after the collision remains finite in the scaling limit, thereby implicitly requiring $n \gg 1$. On the other hand, if we take $n > 1$, the integral in (3.24) converges for all x , in such a way that $F_2(x) \sim x^2$, as x tends to zero, unless $m = 0$, when $F_2(x) \sim x^{1 - \frac{1}{n}}$ as $x \rightarrow 0$.

So we are uniquely led to require $n = 1$, which leads to $F_2(x)$ constant as $x \rightarrow 0^{**}$, in agreement with experiment. Thus the confining

** Baacke²⁶ formally shows that the parton distribution for a static fermion bag is logarithmically divergent even for finite (non-confining) and smooth potentials. We do not obtain such a divergence; however, we are not using a formal bag model.

radius of the proton increases linearly with quark energy. This is compatible with the idea of a linear potential, used in charmonium models.

In order to evaluate the integral in equation (3.24) we make the approximation

$$(E^2 - m^2)^{\frac{1}{2}} \approx E - \frac{m^2}{2E} \quad 3.26$$

This is obviously valid for all x for $m \ll E_1$.

Now as $x \rightarrow 0$, $\frac{\Delta(x)}{x^2}$ diverges. We split the integral over E thus;

$$\frac{\Delta(x)}{x^2} \sim \int_{E_1}^{E_T} dE \dots + \int_{E_T}^{\infty} dE \dots \quad 3.27$$

Clearly the divergence occurs in the integral from E_T to ∞ ; this is true for any choice of E_T . So, for sufficiently small x , the approximation (3.26) is valid for $m \ll E_T$, i.e. for all m .

Now substituting (3.26) into (3.24) and integrating, we obtain

$$\Delta(x) = \frac{R_0^2}{\pi\mu^2c} (1 + \mu c M x E_1) \exp(-\mu c M x E_1) \exp(-\frac{1}{2}\mu c m^2) \quad 3.28$$

3.4 Comparison with data

Barger and Phillips²⁷ obtained values for the sea distribution of non-strange quarks at several values of x from fits to $\mu\bar{\mu}$ production. The data are taken at values of Q^2 ranging from 47 to 131 GeV². Ignoring the relatively small effect of scaling violations, we compare with our formula for $\Delta(x)$, equation (3.28) in Figure 3.3. We fit the data with parameters $\mu c M E_1 = 16$, $\frac{R_0^2}{\mu^2c} = 1$. The u and d quarks are taken to be massless, although, in fact, giving them a mass of up to 150 MeV will have little effect on the results..

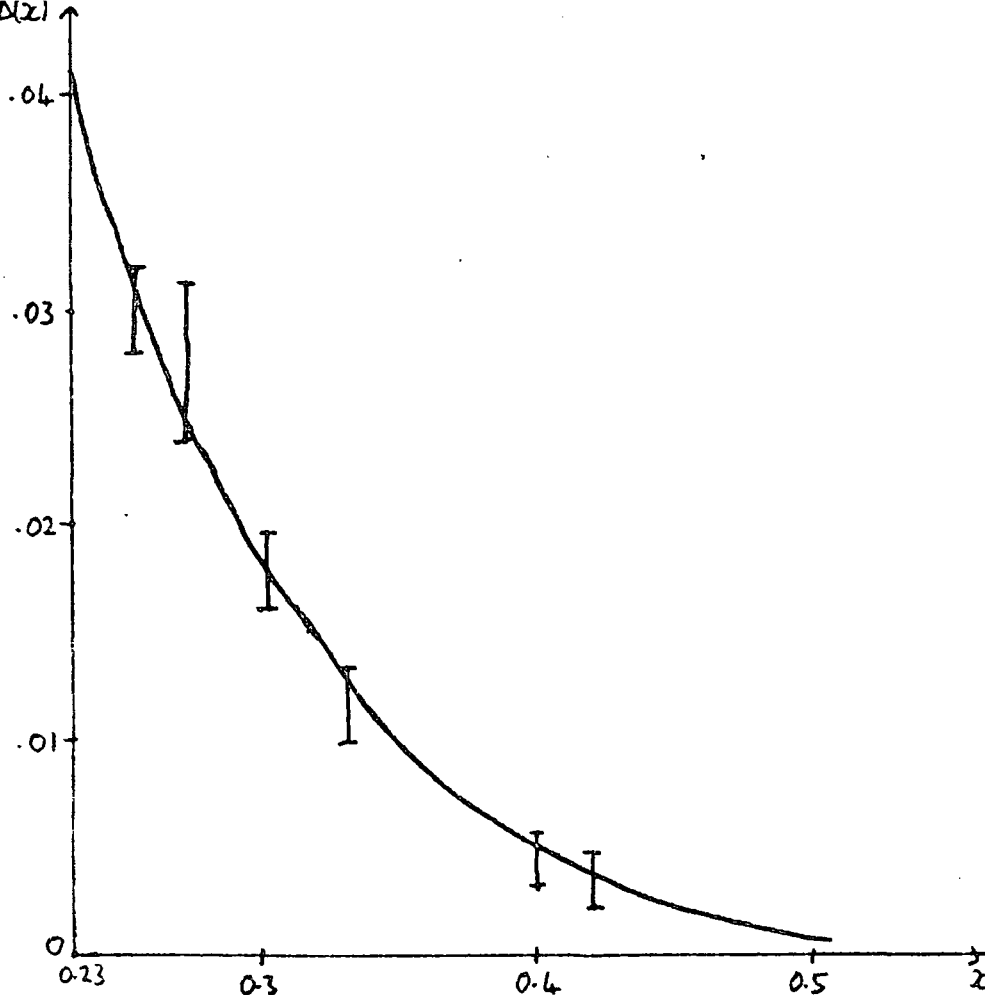


Figure 3.3 Sea of Light Quarks, and comparison with the values extracted from the data by Barger and Phillips²⁷.

At $x = 1$, $\Delta(x) \sim 6e^{-16}$, which is tiny. However, $\Delta(x)$ should tend to 0, as x tends to 1. The reason it does not is that the model is not translationally invariant. The condition $x \leq 1$ arises because the invariant mass squared of the final particle in electron proton scattering (see Figure 1.1) must be greater than or equal to M^2 . The case $(P + q)^2 = M^2$, which yields $x = 1$, corresponds to elastic scattering.

In the uncertainty principle limit of $\mu^2 = 2$, taking $R_0 = 6 \text{ GeV}^{-1}$, we require c to be 18 GeV^{-2} , and $E_1 = 0.6 \text{ GeV}$ - a value in between the energy of the ground state and the first excited state of the proton in the MIT Bag Model. So it is a reasonable choice as the lower limit of our continuum approximation. Putting R equal to the proton radius, R_0 , in equation (3.21)(with $n = 1$), we find $E = \frac{1}{3} \text{ GeV}$, which is probably a fair estimate of the ground state quark energy in the proton.

It is unlikely that we are in fact close to the uncertainty principle limit, as the quark and antiquark produced are in excited states. If we take $\mu^2 = 1$, we find $E_1 \approx 0.5 \text{ GeV}$, again taking a proton radius of 6 GeV^{-1} . It is clear that, however we choose to 'fiddle' with the parameters, reasonable agreement is obtained with data.

The factor $e^{-\frac{1}{2}\mu\text{cm}^2}$ in equation (3.28) shows that the heavier quark seas are suppressed for all x . Taking the values of the parameters from our fit with $\mu^2 = 2$, we find that the suppression factor is given by

$$\exp[-9\sqrt{2}(m_q^2 - m_{ud}^2)] \quad 3.29$$

For a strange quark with mass 0.3 GeV and massless u, d quarks, this factor is about $\frac{1}{3}$. This compares well with the experimental value²⁸ of

$$\frac{\xi_s}{\xi} = \frac{\int x s(x) dx}{\int x d(x) dx} = 0.35 \quad 3.30$$

for s and d in the sea.

Of course, our suppression factor is highly dependent on the values chosen for the quark masses. For example, if we take $m_s = 0.4 \text{ GeV}$, and $m_{u,d} = 0.14 \text{ GeV}$, we obtain a suppression factor of about $\frac{1}{6}$.

Tittel²⁹ gives details of measurements of quark and antiquark distributions. He defines

$$\begin{aligned} q(x) &= u(x) + d(x) + s(x) \\ Q &= \int x q(x) dx \\ \bar{S} &= \int x \bar{s}(x) dx, \text{ etc} \end{aligned} \quad 3.31$$

Averaging the various experimental results, he obtains

$$\frac{\bar{Q}}{Q+\bar{Q}} = 0.13 \pm 0.02 \quad 3.32$$

$$\text{and } \frac{\bar{S}}{Q + \bar{Q}} = 0.02 \pm 0.01 \quad 3.33$$

and concludes that

$$\frac{\bar{S}}{Q + \bar{Q}} \leq \frac{1}{2} \frac{1}{2} \left(\frac{\bar{U} + \bar{D}}{Q + \bar{Q}} \right) \quad 3.34$$

For details of the CDHS results, see reference 30.

The mean charge of a quark in the sea at high energy has been measured as 0.13 ± 0.03 ³¹. If the sea were SU(3) symmetric, n would be 0, and were it SU(2) symmetric, n would be $\frac{1}{6}$. This indicates that the strange sea is fairly small compared to the sea of light quarks.

Taking the charmed quark mass as 1.5 GeV, equation (3.29) gives us a suppression factor for the charmed sea of less than e^{-28} ! So there is little chance of its ever being observed. This is in agreement with the currently accepted view that the charmed sea does not play a significant role in the observed ψ and ψ' production. Other heavier quark seas, viz. the b sea, will be even more suppressed.

3.5 $\langle k_T^2 \rangle$

The expected value of k_T^2 for a quark in the sea is given in our model by

$$\langle k_T^2 \rangle_{\Delta(x)} = 6 \int_{E_1}^{\infty} \frac{dE}{E} \rho(E) \int_{E+Mx}^{\infty} dk k^2 p(E, k) \frac{M^2 x^2}{2k} k_T^2 \quad 3.35$$

$$\begin{aligned} \text{Now } k_T^2 &= k^2 (1 - \cos^2 \theta) \\ &= k^2 - (E + Mx)^2 \end{aligned} \quad 3.36$$

using equation (3.17).

Proceeding, as in section 3.2, we obtain, for massless u and d quarks,

$$\Delta(x) \langle k_T^2 \rangle = \frac{2M^2 x^2 R_0^2}{\pi} \int_{E_1}^{\infty} dE \left(\frac{E + Mx}{\mu} + \frac{1}{\mu^2 CE} \right) e^{-\mu c E M x} \quad 3.37$$

We approximate by replacing $\frac{1}{E}$ in the last term of the bracket by its upper limit $\frac{1}{E_1}$. This leads to

$$\Delta(x) \langle k_T^2 \rangle \approx \frac{R_0^2}{\pi \mu^2 c} \frac{2}{\mu c} (1 + \mu c E_1 M x) \left(1 + \frac{M x}{E_1}\right) \exp(-\mu c E_1 M x) \quad 3.38$$

$$\Rightarrow \langle k_T^2 \rangle \approx \frac{2}{\mu c} \left(1 + \frac{M x}{E_1}\right) \quad 3.39$$

using equation (3.28).

So using the parameters from the fit with $\mu^2 = 2$ in section (3.4), we have

$$\langle k_T^2 \rangle \approx 0.08 + 0.11x \quad 3.40$$

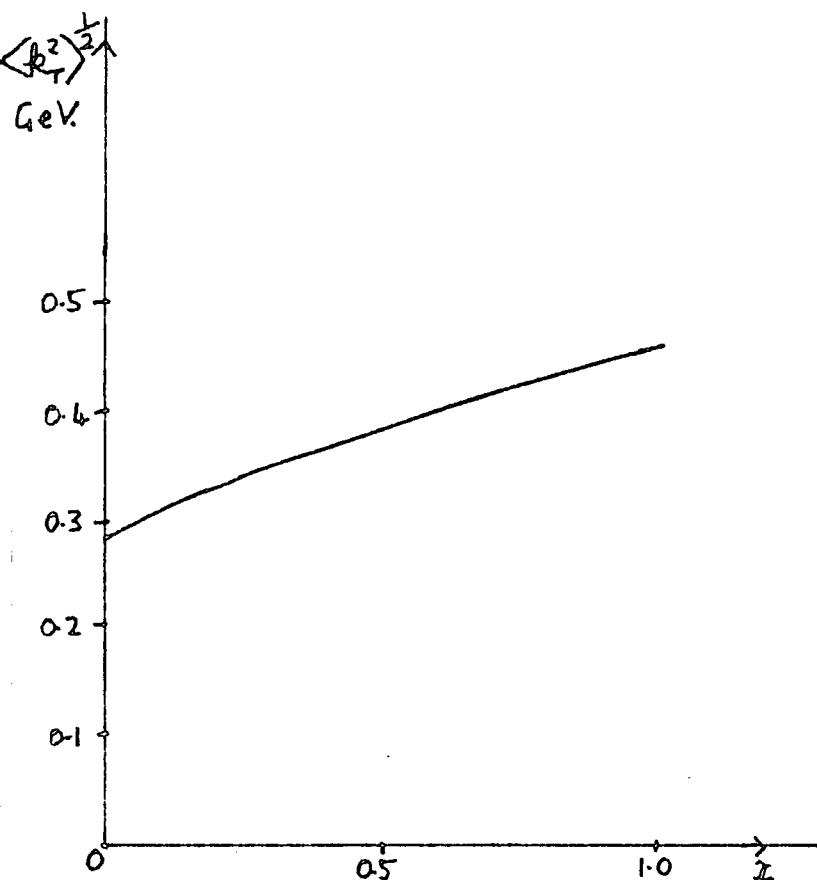


Figure 3.4 Average transverse momentum $\langle k_T^2 \rangle$ as a function of x .

The experimental value of $\langle k_T \rangle$ at $x = 0$ is about 0.5 GeV^{32} . This gives a value for $\langle k_T^2 \rangle$ at $x = 0$ of $\frac{0.5}{\sqrt{2}} \approx 0.35 \text{ GeV}$. $\langle k_T \rangle$ rises with x^{32} at a rate faster than is predicted by our model.

3.6 Approach to the scaling limit

We show in Figure 3.5 the approach to the scaling limit for massless quarks obtained by replacing the upper limit of the E integration in equation (3.18) by $\frac{q^0}{2}$. We have not included the kinematic non-scaling terms of order $\frac{k^0}{q^0}$ and $\frac{Mx}{q^0}$, calculated in sections 2.1 and 2.2.

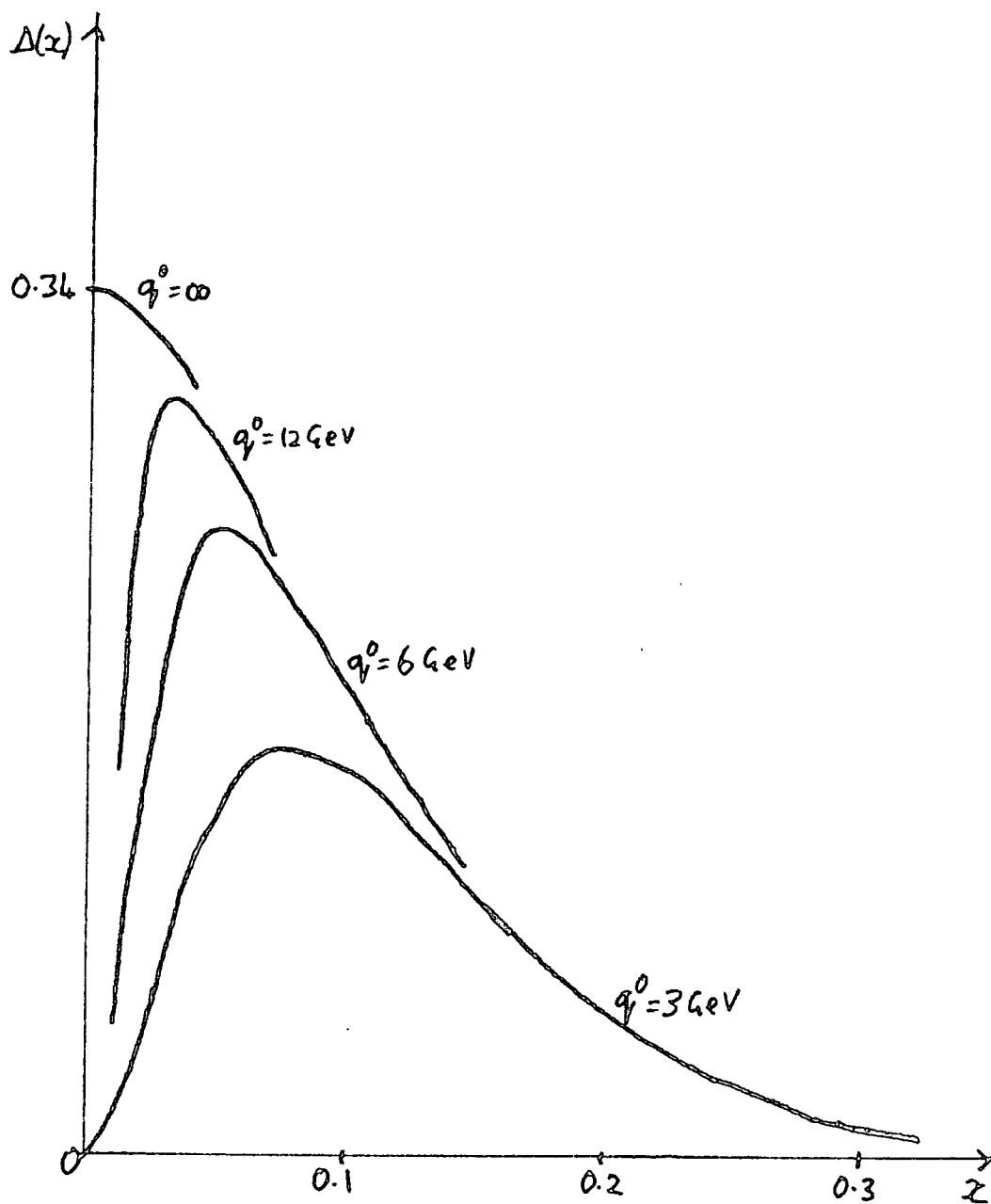


Figure 3.5 Showing the non-uniform approach to scaling.

This illustrates the non-uniformity of the limit as $x \rightarrow 0$, and should be observed in quark and antiquark distributions. However, the effect is not important in the region where data are available (see Figure 3.3).

3.7 The pion sea

Equation (3.28) can be used to calculate the quark sea in any hadron. The variable parameters are M , R_0 and E_1 . We now attempt to estimate the pion sea. The mass of the pion is just under 0.14 GeV. The pion is an enigmatic particle. Its properties are not well predicted by the M.I.T. Bag Model¹⁰. So it is difficult to know what values to take for R_0 and E_1 .

First we put $R_0 = 5 \text{ GeV}^{-1}$, and use the Bag Model relation^{9, 10} that the energy of a massless quark in its lowest mode multiplied by the radius of the hadron equals 2.04, to estimate E_1 as 0.4 GeV. We obtain $C = 12.5 \text{ GeV}^{-2}$ from $R_0 = cE_1$, and for $\mu = \sqrt{2}$, we have

$$\Delta^\pi(x) \approx \frac{1}{\pi}(1+x)e^{-x} \quad 3.41$$

Squires³³ suggests that $R_\pi \sim 1.7R_p$. For $R_\pi = 10 \text{ GeV}^{-1}$, $E_1 = 0.2 \text{ GeV}$, and $\mu = \sqrt{2}$, we find

$$\Delta^\pi(x) \approx \frac{1}{\pi}(1+2x)e^{-2x} \quad 3.42.$$

See Figure 3.6.

So we expect the pion sea distribution to be of similar magnitude to the nucleon sea distribution near $x = 0$, but to be very much flatter. This flatness occurs predominantly because the mass of the pion is much smaller than that of the nucleon. $\Delta^\pi(x)$ is still fairly large at $x = 1$ for both values of R , shown in figure 3.6, - unlike $\Delta^p(x)$, which is tiny at $x = 1$. The reason for this is that recoil effects,

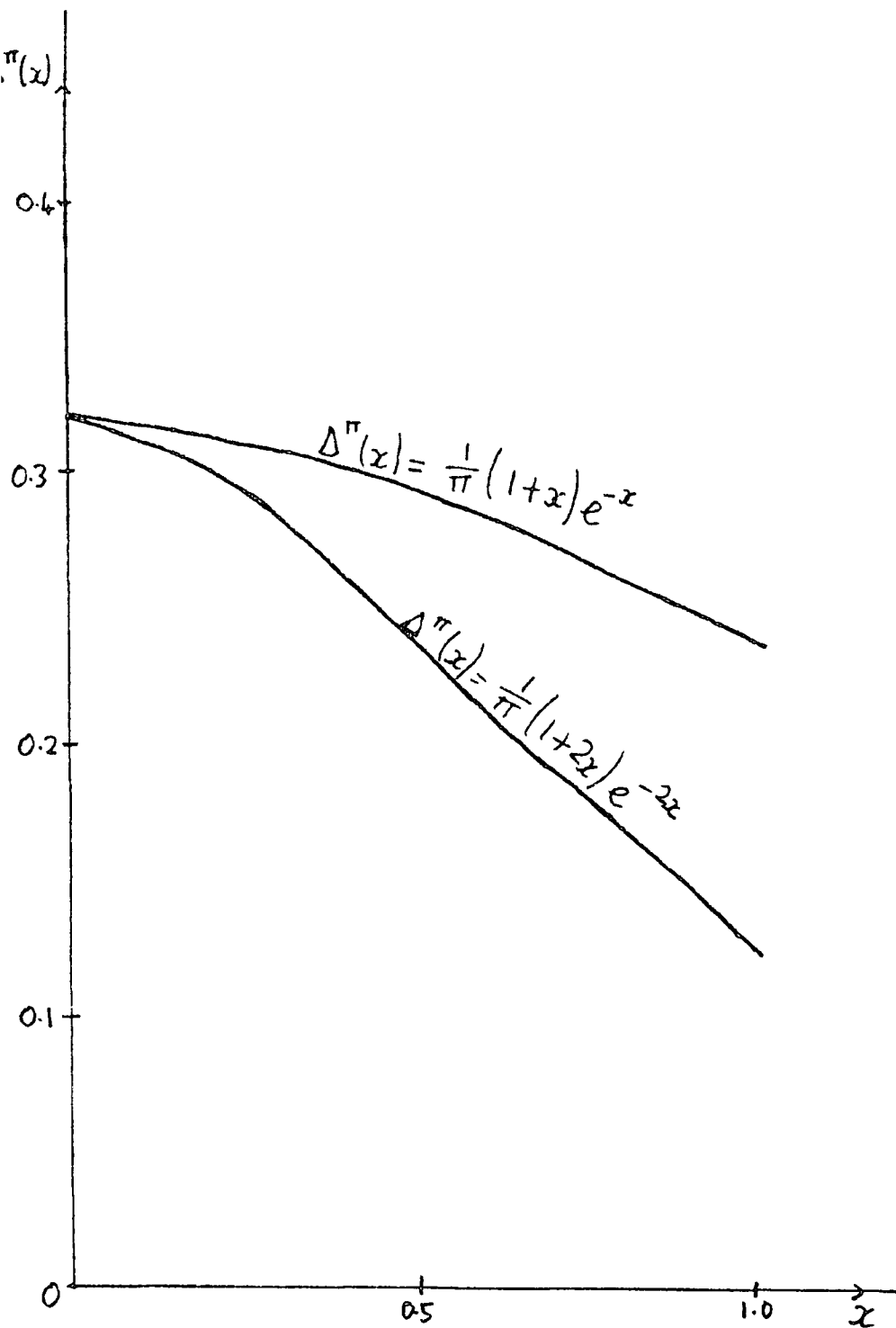


Figure 3.6 Pion Sea Predictions ignoring recoil effects.

which we have ignored, are much more important for the light pion than for the proton. In reference 33, recoil effects are taken into account by replacing x by $\log(1-x)^{-1}$. Making this substitution in our formula for $\Delta(x)$, equation 3.28, results in $\Delta(x)$ tending to 0, as x tends to 1. See Figure 3.7 for predictions of $\Delta^\pi(x)$ for which this

substitution has been made in equations (3.41) and (3.42). The pion structure function has been measured (see the figure in reference 33), but has not been separated into valence and sea contributions.

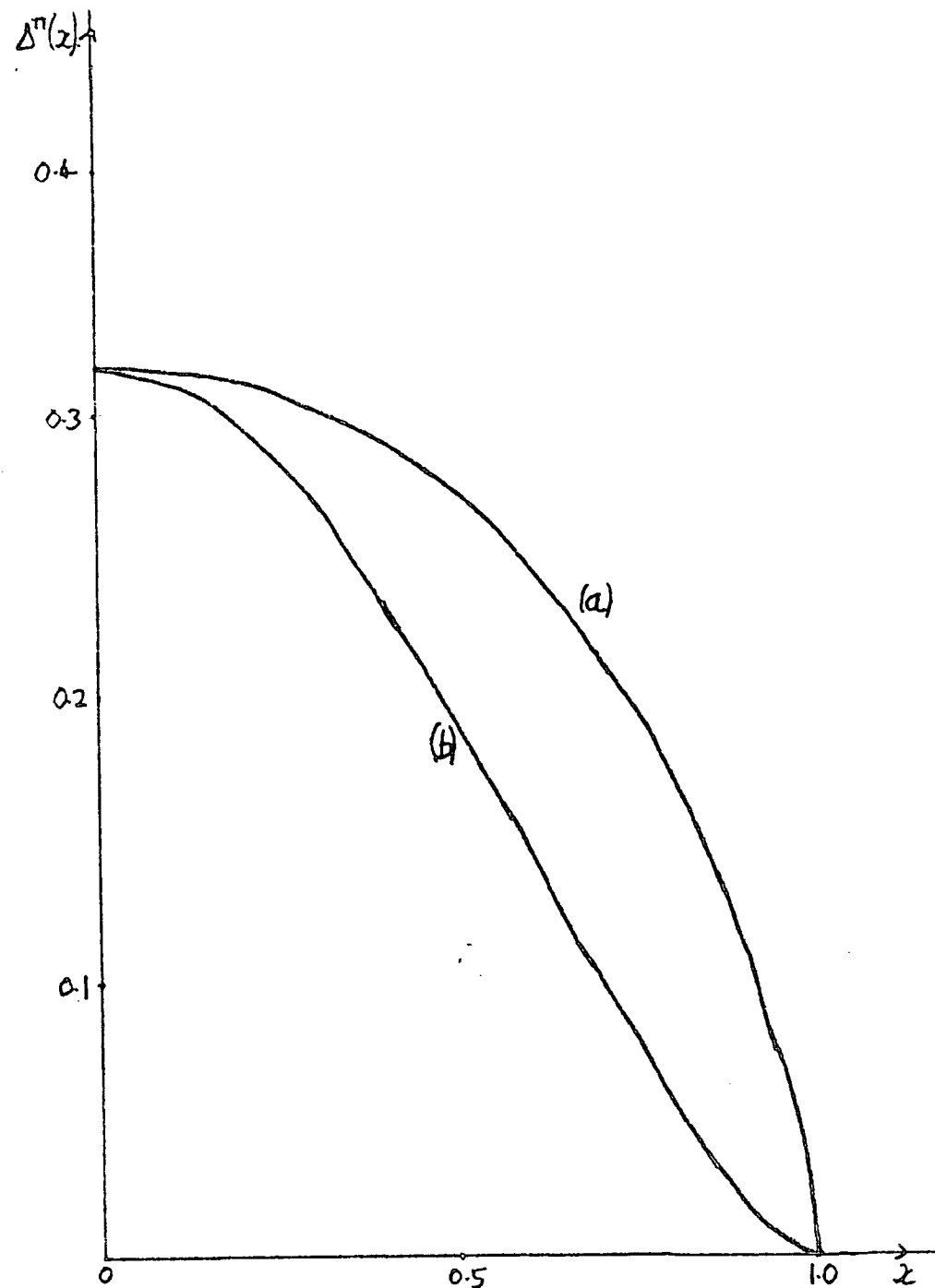


Figure 3.7 Pion Sea Predictions including recoil effects.

$$(a) \quad \Delta^\pi(x) = \frac{1}{\pi} (1 - \log(1-x)) (1-x)$$

$$(b) \quad \Delta^\pi(x) = \frac{1}{\pi} (1 - 2 \log(1-x)) (1-x)^2$$

CHAPTER 4

FLAVOUR AND SPIN EFFECTS IN THE SEA4.1 Flavour dependence of the bubble sea

The only dependence of the excited state bubble contribution to the sea, $\Delta(x)$, on the spin and flavour of the quarks in the sea arises from the quark mass, which leads to the suppression of the strange and charmed seas. As the quark and antiquark in the sea are both in excited states, there is no exclusion principle effect. So the flavour dependence of the light quark bubble seas arises solely from the ground state term.

The ground state contribution depends on $V(-x)$ (See section 3.1).

$$V(-x) = \int_{|E+Mx| < k} dk k^2 P(E, k) \frac{M^2 x^2}{2Ek} \quad 4.1$$

To evaluate this integral we choose

$$k P(E, k) = \frac{\mu R_0}{2E} \exp(-\mu R_0 |k - E|) \quad 4.2$$

for massless quarks. In the normalisation we have ignored a term

$\frac{e^{-\mu R_0 E}}{2\mu R_0 E}$ in comparison with unity. The error involved in this can safely be neglected.

Substituting (4.2) in (4.1) gives us

$$V(-x) = \frac{M^2 x^2}{4E^2} \exp(-\mu R_0 Mx) \quad 4.3$$

Taking $\mu = \sqrt{2}$, $R_0 = 6 \text{ GeV}^{-1}$, $E = \frac{1}{3} \text{ GeV}$, we estimate that

$$V(-x) \approx 2x^2 e^{-8x} \quad 4.4$$

See Figure 4.1.

We observe that the ground state contribution to the sea is zero at $x = 0$. Its contribution, relative to that of the excited states, rises as x increases from 0 to 1. However, where the sea is at its

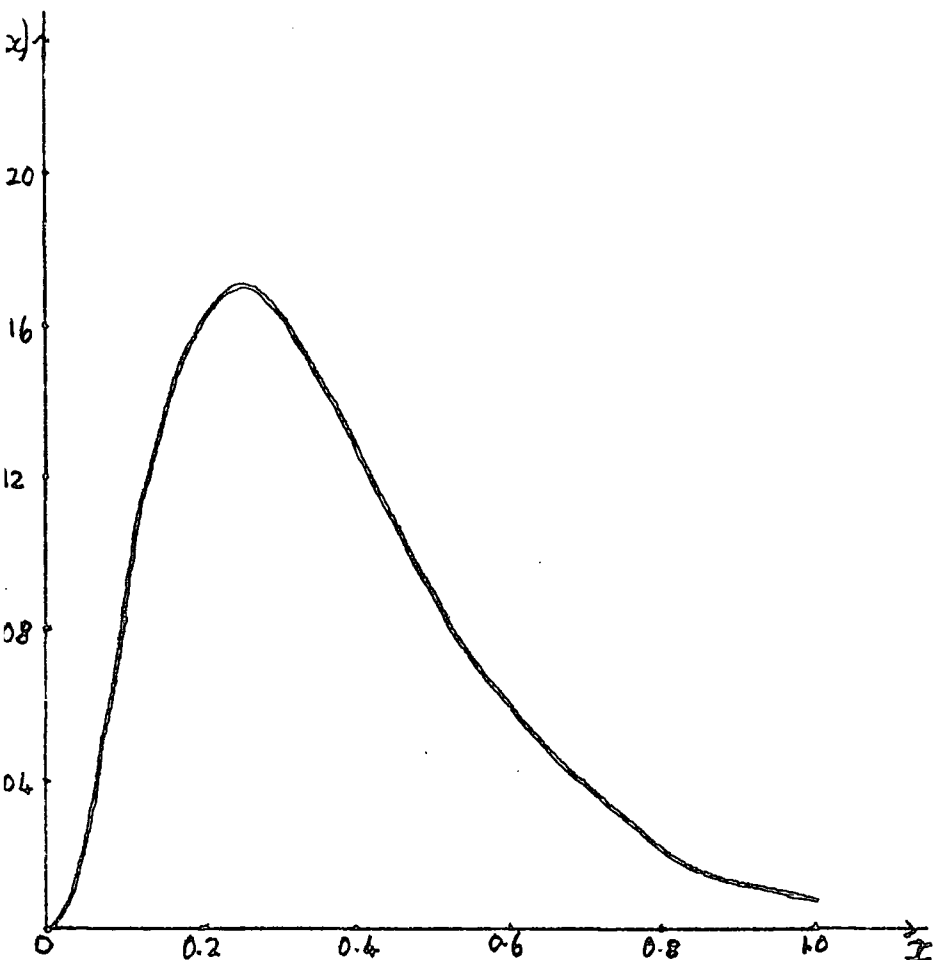


Figure 4.1 Plot of $V(-x)$ against x .

largest in the region $x < 0.1$, the ground state contribution to it is small. $V(-x)$ should tend to zero, as x tends to 1. It does not, as the model is not translationally invariant.

Now Feynman and Field³⁴ have estimated from the experimental data³⁵ that

$$\int_0^1 (F_2^{\text{ep}}(x) - F_2^{\text{en}}(x)) \frac{dx}{x} \approx 0.27 \quad 4.5$$

From equation (3.6), we obtain

$$\begin{aligned} \int_0^1 (F_2^{\text{ep}}(x) - F_2^{\text{en}}(x)) \frac{dx}{x} &= \frac{1}{3} \int_0^1 (V(x) - V(-x)) \frac{dx}{x} \\ &= \frac{1}{3} - \frac{1}{3} \int_0^1 \frac{V(-x)}{x} dx \end{aligned} \quad 4.6$$

We find that

$$\int_0^1 \frac{V(-x)}{x} dx = \frac{1}{(2E\mu R_0)^2} [(1 - e^{-\mu R_0 M}(1 + \mu R_0 M))] \quad 4.7$$

$$\approx \frac{1}{(2E\mu R_0)^2}$$

neglecting the $e^{-\mu R_0 M}$ term, as $\mu R_0 M \approx 8$. With $\mu = \sqrt{2}$, $R_0 = 6\text{GeV}^{-1}$, $E = \frac{1}{3}\text{GeV}$, this integral takes the value of approximately 0.03. Were we to take $\mu = 1$, this would double to 0.06. In order to agree with the experimental estimate we need a value of 0.19 for $\int_0^1 \frac{V(-x)}{x} dx$.

It does not appear that the bubble sea can entirely account for this. However, we must bear in mind that we have only roughly estimated this integral, and that equation (4.5) is also only an estimate. For a discussion of the contribution of the gluon induced sea to the difference between the \bar{u} and \bar{d} distributions in the proton, see section 4.3

4.2 Spin dependence of the bubble sea

The spin dependence of the bubble sea also arises solely from exclusion principle effects in the quark ground state term. We find, assuming SU(6) symmetry, that the spin and colour factor a (which appears in section 3.1) for a spin up proton, equals $\frac{4}{3}$ for $q = u\uparrow$, equals $\frac{8}{3}$ for $q = u\downarrow$ and $d\uparrow$, and equals $\frac{7}{3}$ for $q = d\downarrow$. For any antiquark in the ground state $a = 3$, provided that the corresponding quark is in an excited state.

The quark asymmetry $\alpha(x)$ is defined as

$$\alpha(x) = \frac{q\uparrow(x) - q\downarrow(x)}{q\uparrow(x) + q\downarrow(x)} \quad 4.8$$

Now in electron-electron scattering the spin non flip term dominates over the spin flip term (If we take $m_e = 0$, the latter is zero). This

means that in the dominant case the virtual photon has spin $|1, 0\rangle$.

Similarly, in deep inelastic scattering, the dominant term will arise when the virtual photon has spin $|1, 0\rangle$. Thus the quark and antiquark in the bubble sea, produced by such a photon, will have opposite spins.

So the antiquark asymmetry

$$\bar{\alpha}(x) \stackrel{!}{=} -\alpha(x) \quad 4.9$$

for the bubble sea.

We find that

$$\bar{u}_\uparrow(x) - \bar{u}_\downarrow(x) = \frac{\frac{4}{3}V(-x)}{x} \quad 4.10$$

$$\text{and} \quad \bar{d}_\uparrow(x) - \bar{d}_\downarrow(x) = -\frac{1}{3} \frac{V(-x)}{x}$$

So were the ground state the dominant contribution to the sea, $\bar{\alpha}_u(x) = \frac{1}{3}$, and $\bar{\alpha}_d(x) = -\frac{1}{15}$. On the other hand, were the ground state contribution negligible, the antiquark asymmetry would be zero.

To make a realistic estimate we use

$$\bar{q}_\uparrow(x) + \bar{q}_\downarrow(x) = \bar{q}(x) = \frac{\Delta(x)}{x} \quad 4.11$$

where the lower limit of the integration in equation (3.24) is now taken to be E , the ground state energy. Thus we hope to include the ground state contribution to the antiquark sea in this integral. We obtain

$$\begin{aligned} \bar{\alpha}_u(x) = -4 \bar{\alpha}_d(x) &= \frac{4}{3} \frac{V(-x)}{\Delta(x)} \quad 4.12 \\ &= \frac{4}{3} h(x), \text{ say.} \end{aligned}$$

Now $\bar{\alpha}_u(x)$ must be less than or equal to $\frac{1}{3}$. This implies that $0 \leq h(x) \leq \frac{1}{4}$ for all x between 0 and 1.

Using equations (3.28) and (4.3) we find that

$$h(x) = \frac{M^2 x^2 \pi \mu^2 c}{4E^2 R_0^2} (1 + \mu c M x E)^{-1} \exp(-\mu M x (R_0 - cE)) \quad 4.13$$

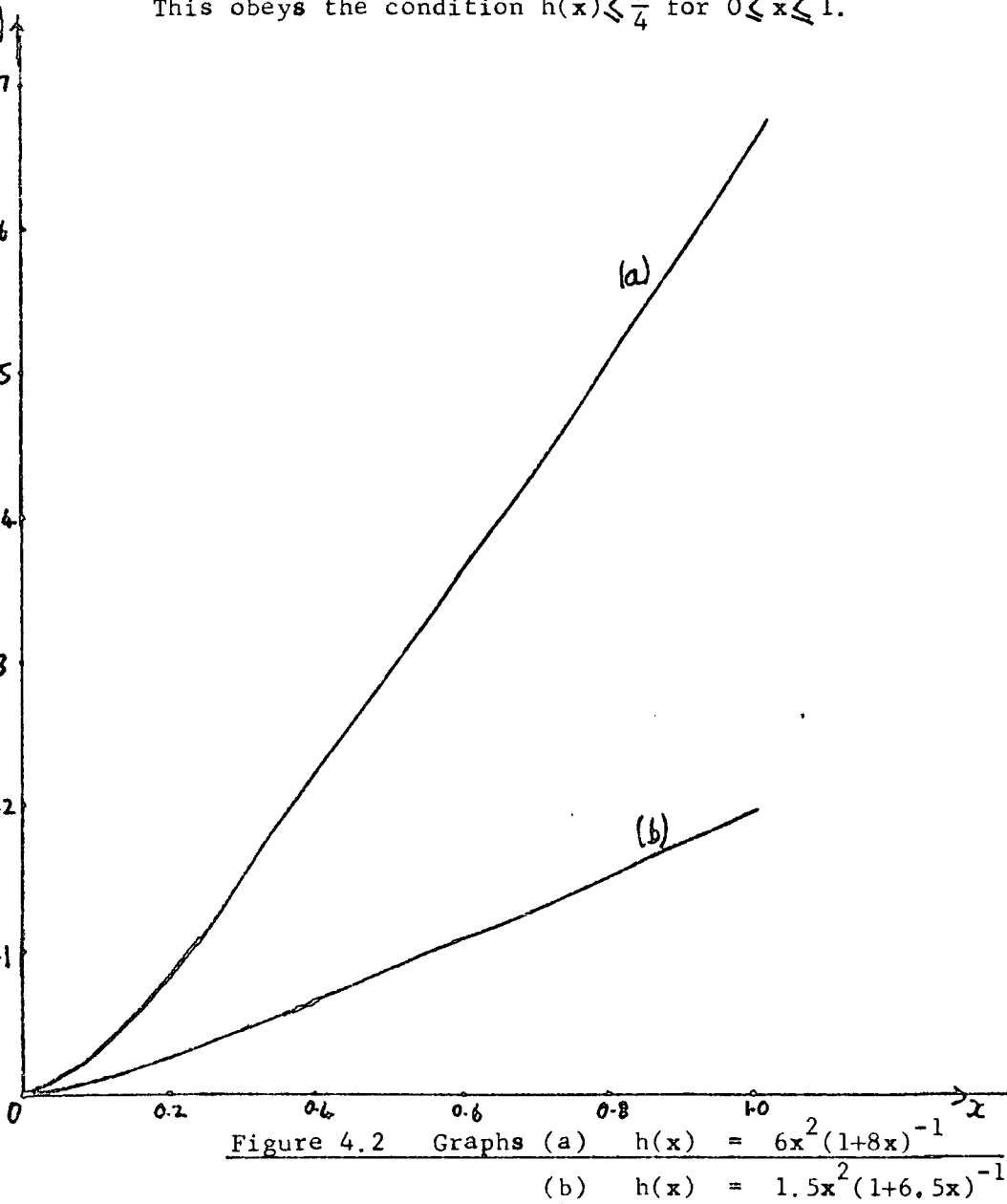
Putting $R_0 = cE$, which gives $c = 18\text{GeV}^{-2}$ for $R = 6\text{GeV}^{-1}$, and $E = \frac{1}{3}\text{GeV}$, and $\mu^2 = 2$, we obtain

$$h(x) \approx 6x^2(1+8x)^{-1} \quad 4.14$$

Now this is less than $\frac{1}{4}$, only for $x < 0.43$. This discrepancy occurs because our estimate of $V(-x)$ (see equation 4.4 and figure 4.1) does not die off as fast as it should as $x \rightarrow 1$, as observed in the previous section. However, if we 'fiddle' the parameters by choosing $R_0 = 7\text{GeV}^{-1}$, $E = 0.4\text{GeV}$, thus giving $c = 17.5\text{GeV}^{-2}$, and $\mu = 1$, we obtain

$$h(x) = 1.5x^2(1+6.5x)^{-1} \quad 4.15$$

This obeys the condition $h(x) \leq \frac{1}{4}$ for $0 \leq x \leq 1$.



So we observe how a comparatively small change in the parameters has a considerable effect on the function $h(x)$, and stress that our other estimates are likewise parameter dependent.

4.3 Comparison between flavour and spin effects in the gluon induced and bubble seas

Ross and Sachrajda³⁶ have calculated the difference between the \bar{u} and \bar{d} distributions in the gluon induced sea of the proton. They find that $\bar{u}(x) < \bar{d}(x)$, but that the difference is not large enough to explain the value of 0.27 for the integral in equation (4.5). They suggest that a significant non-perturbative effect is required to explain this discrepancy. The bubble diagram gives rise to such an effect. Adding these two contributions, and taking into account the uncertainty in both the theoretical and experimental estimates gives us a result not inconsistent with experiment.

It has been shown that the gluon sea is polarised^{37,38}. Close and Sivers³⁷ input a simply parametrized valence quark distribution, and calculate, using the method of Altarelli and Parisi³⁹, the gluon distributions, and thence the antiquark distributions inside a Δ particle. From this, working in the SU(6) limit, where spin-spin forces are neglected, they estimate the antiquark asymmetry for a proton (see their figure (2(b))). Note that $\bar{\alpha}_u(x) = \bar{\alpha}_d(x)$, and that $\bar{\alpha}(0) = 0$. This ignores exclusion principle effects. Taking them into account, we find that the relation $\bar{\alpha}_u(x) = \bar{\alpha}_d(x)$ holds only approximately, and that both are non-zero and different, although small, at $x = 0$.

Davis and Lambourne³⁸ estimate the spin asymmetries, using the cavity approximation to the MIT Bag Model. They find that $\alpha_u(x) = \alpha_d(x)$, and that $\bar{\alpha}_u(x) = \bar{\alpha}_d(x)$. They also calculate $\alpha(x)$ and $\bar{\alpha}(x)$ for s and c quarks. They find (see their figure 2) that $\alpha(x) > 0$, and $\bar{\alpha}(x) < 0$ for all flavours. Neither is near zero at $x = 0$. Thus their results

differ much from those of Close and Sivers³⁷. This is presumably because they are using bag model wave-functions. Their approach is similar to that of Donoghue and Golowich⁴⁰, one of whose predictions is that $\bar{u}(x) > \bar{d}(x)$ - which seems to be inconsistent with experiment.

The antiquark asymmetries should be measurable in polarised Drell-Yan production, as is discussed in reference 37. Separate measurements of $\bar{\alpha}_u(x)$ and $\bar{\alpha}_d(x)$ would serve as a good guide as to which if either, of the bubble and gluon induced contributions to the sea is dominant. If it were the bubble, we would find $\bar{\alpha}_u(x) \approx -4\bar{\alpha}_d(x)$. Were it the gluon induced sea, $\bar{\alpha}_u(x)$ would approximately equal $\bar{\alpha}_d(x)$.

CHAPTER 5

MAGNETIC MOMENTS OF THE OCTET BARYONS5.1 Magnetic moments in the naive quark model

The agreement between theoretical predictions of hadron magnetic moments and experiment is an important test for quark models. One of the initial successes of the non-relativistic SU(6) model was its prediction of -1.5 for the ratio of the proton and neutron magnetic moments, compared with the experimental value of -1.46.

The basic assumption required to predict the magnetic moments is that the magnetic moment of a hadron is the sum of the magnetic moments of its valence quarks, i.e.

$$\mu_H = \sum_q \mu_q = \sum_q e_q \lambda_q \langle \sigma_q \rangle \quad 5.1$$

where e_q is the quark charge in units of e , λ_q is the absolute value of the magnetic moment of the quark q divided by e_q ;

$$\lambda_q = \frac{|\mu_q|}{|e_q|} \quad 5.2$$

and $\frac{1}{2} \sigma_q$ is the quark spin operator. Taking the expectation values of σ_q^3 for the various baryons from the SU(6) model gives us⁴¹

$$\begin{aligned} \mu_p &= \frac{8}{9} \lambda_u + \frac{1}{9} \lambda_d \\ \mu_n &= \frac{2}{9} \lambda_u - \frac{4}{9} \lambda_d \\ \mu_\Lambda &= -\frac{1}{3} \lambda_s \\ \mu_{\Sigma^+} &= \frac{8}{9} \lambda_u + \frac{1}{9} \lambda_s \\ \mu_{\Sigma^-} &= -\frac{4}{9} \lambda_d + \frac{1}{9} \lambda_s \\ \mu_{\Xi^0} &= -\frac{2}{9} \lambda_u - \frac{4}{9} \lambda_s \\ \mu_{\Xi^-} &= \frac{1}{9} \lambda_d - \frac{4}{9} \lambda_s \\ \mu_{\Sigma^0} &= \frac{4}{9} \lambda_u - \frac{2}{9} \lambda_d + \frac{1}{9} \lambda_s \\ \mu_{\Sigma^0-\Lambda^0} &= -\frac{2}{3\sqrt{3}} \lambda_u - \frac{1}{3\sqrt{3}} \lambda_d \end{aligned} \quad 5.3$$

$\mu_{\Sigma^0-\Lambda^0}$ is the $\Sigma^0 \rightarrow \Lambda^0 \gamma$ transition moment. Only μ_{Σ^0} has not been measured.

If we neglect SU(3) breaking effects, we have $\lambda_u = \lambda_d = \lambda_s$. This yields the aforementioned prediction for the ratio of μ_p to μ_n , but gives poor predictions for the hyperon moments. In the non-relativistic constituent quark model, which takes account of SU(3) breaking, λ_q is given by the formula for Dirac moments for pointlike particles of mass m_q thus;

$$\lambda_q = \frac{\hbar}{2m_q c} \quad 5.4$$

It is independent of the hadron in which the quark is confined. The experimental values of $2M_p \mu_p = 2.793^{42}$, $2M_p \mu_n = -1.913^{42}$, and $2M_p \mu_{\Lambda} = -0.6138 \pm 0.0047^{43}$, determine the magnetic moments of the three quarks. It is found⁴¹ that $2M_p \lambda_u = 2.778$, $2M_p \lambda_d = 2.913$, and $2M_p \lambda_s = 1.84 \pm 1\%$. Now, averaging λ_u and λ_d , equation (5.4) yields $m_u = m_d = 0.331 \text{ GeV}$, and $m_s = 0.510 \text{ GeV}$. These values are in good agreement with the masses obtained in non-relativistic constituent quark models^{44,45}; so the picture is consistent.

The predictions for the other octet baryon moments are

$$\begin{aligned} 2M_p \mu_{\Sigma^+} &= 2.68 && (2.33 \pm 0.13)^{46} \\ 2M_p \mu_{\Sigma^-} &= -1.09 && (-1.48 \pm 0.37)^{42} \\ 2M_p \mu_{\Sigma^0} &= -1.44 && (-1.20 \pm 0.06)^{47} \\ 2M_p \mu_{\Xi^-} &= -0.495 && (-1.85 \pm 0.75)^{42} \\ 2M_p \mu_{\Xi^0-\Lambda^0} &= -1.63 && (-1.82 \pm 0.18 / 0.25)^{48} \end{aligned} \quad 5.5$$

where the experimental results are given in brackets. The unmeasured Σ^0 moment should be the average of the Σ^+ and Σ^- moments (i.e. 0.80).

We note that the ξ^+ and ξ^0 moments differ respectively by 2.7 and 4 standard deviations from the predictions. The experimental value for the ξ^- result differs by nearly 2 standard deviations, but the error here is very large. We obtain from (5.3) that

$$2M_p(\mu_{\xi^-} - \mu_{\xi^0}) = \frac{2M_p\lambda_u}{3} \approx 0.9 \quad 5.6$$

It would be surprising if this were not approximately correct. An accurate measurement of μ_{ξ^-} would be very interesting.

5.2 Magnetic moments in the M.I.T. Bag Model

In order to calculate the magnetic moment of a relativistic quark, we use the formula

$$\mu = \frac{1}{2} \int (\mathbf{r} \wedge \mathbf{j}) d^3x \quad 5.7$$

We work in momentum space, so $\mathbf{r} = i\nabla$. The current operator \mathbf{j} equals $e_q \psi \boldsymbol{\gamma} \psi$ for spin $\frac{1}{2}$ quarks, where ψ is a plane-wave solution of the relativistic Dirac equation. We find, working in units where $\hbar = c = 1$, that

$$\mu_q = \frac{e_q \langle \sigma \rangle}{2\omega} \quad 5.8$$

where ω is the quark energy. Thus we see, comparing equations (5.1) and (5.4) with equation (5.8) that the quark mass in non-relativistic models is replaced by the quark energy in a simple relativistic model.

Now the quark energy varies from hadron to hadron, with the result that the assumption that λ_q is independent of the hadron, in which the quark is confined, is no longer correct. We work in the M.I.T. Bag Model, which is relativistic, and confines quarks within hadrons. The effects of quark confinement were ignored in the previous section, and in the calculation* of equation (5.8). Previous work^{9,10,49,50,51,52} on magnetic moments has been carried out using this model, but this was

* Such effects can be taken into account by inserting an energy-momentum distribution into the equation, as was done in section 2.4 for structure functions.

before the new measurements of the Λ^0 , Σ^+ , and Ξ^0 moments were made.

The quark energy is given in the bag model by

$$\omega(mR) = \frac{1}{R} (x^2(mR) + m^2 R^2)^{\frac{1}{2}} \quad 5.9$$

where $x(mR)$ is the eigenfrequency of the lowest quark mode with mass m in a spherical cavity of radius R (see figure 2 of reference 10).

If gluon effects are ignored, the quark magnetic moment is given by the formula¹⁰

$$\lambda_q = \frac{R}{6} \left(\frac{4\alpha + 2\lambda - 3}{2\alpha(\alpha-1) + \lambda} \right) \quad 5.10$$

where R is the hadron radius, $\lambda = mR$, and $\alpha = R\omega(mR)$. Putting $m = 0$ in equation (5.10), and using $\alpha(0) = 2.04$, we find that $\lambda_q = \frac{0.41}{\omega}$.

Comparing with equation (5.8) we see that confinement has the effect of reducing the quark magnetic moment. For a given quark, the variation of λ_q from hadron to hadron depends on the variation of the hadron radius. Our purpose is to see whether the effect of changing radius on the quark moments can be seen in the data.

We take $m_u = m_d = 0$ (small deviations from this will not greatly affect our results). The experimental value of μ_p then requires a proton radius of 7.35 GeV^{-1} . This is much larger than the value of 5 GeV^{-1} , obtained by Degrand et al¹⁰ from their fit to the proton mass, in which gluon effects are included. This is not consistent with their neglect of such effects in formula (5.10) for the quark magnetic moment. They also include the dubious zero-point energy term in their expression for the hadron masses. This term is not included in an earlier bag paper⁹, in which gluon exchange is also neglected. There, a value of $R_p = 6.9 \text{ GeV}^{-1}$ is obtained from a fit to the average mass of the $N-\Delta$ system. This leads to the result $2M_p \mu_p = 2.6$, to be compared with the 1.9 obtained in reference 10.

Donoghue and Johnson⁵³ introduce corrections to the bag model due

to momentum fluctuations of the quarks in the bag, and predict that $2M_p \mu_p$ equals 2.5. They include the same terms as Degrand et al¹⁰, but take somewhat different values for the parameters (see their table 2). They use a larger proton radius of 5.5 GeV^{-1} . This accounts for about one third of the increase in the magnetic moment from 1.9. The rest of the increase comes from the fluctuation correction.

Kobzarev et al⁵⁴ do not include the zero point energy term; instead they allow the bag constant B to take different values, B_2 for mesons, and B_3 for baryons. They find from a fit to the masses to the first order in the gluon coupling constant, that $R_p = 6.00 \text{ GeV}^{-1}$, leading to $2M_p \mu_p = 2.28$. If gluon corrections are included in the magnetic moment formula, this value is reduced to 1.95, but the ratio of μ_p to μ_n does not change. Thus it seems, in the very least, possible that gluon corrections have little effect on the ratios of the other baryon moments to μ_p . If this is so, they cannot explain the observed discrepancies from the predictions of naive quark models.

The conventional bag model predictions¹⁰ for the ratios of the other octet baryon magnetic moments to μ_p differ little from those of naive quark models, as the variation in radius over the baryon octet is found, from the fit to the masses, to be such that $\frac{R_\Lambda}{R_p} \approx 0.99$ (see reference 10. Table 3). Hackman et al⁵² choose the bag parameters so as to give the correct values for μ_p (and for several masses). They calculate $\frac{\mu}{\mu_p}$ for the octet baryons, and obtain ratios little different from those in reference 10 (see reference 52. Table IV).

5.3 The effect of the variation of baryon radius on magnetic moments in the Bag Model

In order to be able to form some idea as to how the bag radius should vary from hadron to hadron, we proceed as follows. Ignoring gluon

effects and excluding the zero-point energy term, we write the masses as^{9,49}

$$\begin{aligned}
 M_p(R) &= \frac{4}{3} \pi B R^3 + \frac{3x(0)}{R} \\
 M_\Lambda(R) &= M_\Sigma(R) = \frac{4}{3} \pi B R^3 + \frac{2x(0)}{R} + \left(\frac{x^2(mR)}{R^2} + m_s^2 \right)^{\frac{1}{2}} \\
 M_\Xi(R) &= \frac{4}{3} \pi B R^3 + \frac{x(0)}{R} + 2 \left(\frac{x^2(mR)}{R^2} + m_s^2 \right)^{\frac{1}{2}}
 \end{aligned} \tag{5.11}$$

Minimising the proton mass with respect to the radius gives

$$R_p = \left(\frac{3x(0)}{4\pi B} \right)^{\frac{1}{3}} \tag{5.12}$$

$$\text{So } M_p = \frac{4x(0)}{R_p} \tag{5.13}$$

We now write $R_\Lambda = R_p + \delta R$, where $\left| \frac{\delta R}{R_p} \right| \ll 1$,

and

$$M_\Lambda(R) = M_p(R) + \Delta(R) \tag{5.14}$$

$$\begin{aligned}
 \text{where } \Delta(R) &= \left(\frac{x^2(mR)}{R^2} + m_s^2 \right)^{\frac{1}{2}} - \frac{x(0)}{R} \\
 &\ll M_p(R).
 \end{aligned} \tag{5.15}$$

Working to first order, we find that

$$\dot{M}_\Lambda(R) = \ddot{M}(R_p) \delta R + \dot{\Delta}(R_p) \tag{5.16}$$

So for $M_\Lambda(R)$ to be a minimum,

$$\begin{aligned}
 \delta R &= \frac{-\dot{\Delta}(R_p)}{\ddot{M}(R_p)} \\
 &= \frac{-R_p}{12} \left[1 - \left(1 + \frac{m_s^2 R_p^2}{x^2} \right)^{-\frac{1}{2}} \right]
 \end{aligned} \tag{5.17}$$

ignoring the variation of x with mR . (The overall effect of this is small.) Putting $\frac{m_s R_p}{x} \sim 1$, we obtain $\frac{\delta R}{R_p} \sim -\frac{1}{40}$. So in this model,

replacing a massless u quark in a baryon by a massive strange quark

($m_s \sim 0.3$ GeV) reduces the radius by about $\frac{1}{40}$.

Taking $R_p = 7.35 \text{ GeV}^{-1}$, chosen to fit the experimental proton magnetic moment, and this ratio, we find $R_\Lambda = 7.17 \text{ GeV}^{-1}$, and $R_\Sigma = 6.99 \text{ GeV}^{-1}$. We obtain a value of 0.295 GeV for the strange quark mass by fitting the experimental value of μ_Λ^{43} . Then, assuming $R_\Lambda = R_\Sigma$, the other octet baryon moments can be predicted (see Table 5.1, column 2).

The effect of the variable radius is in the right direction to remove the principle discrepancies, noted in Section 5.1, but it is too small in magnitude. It is possible, by multiplying $\frac{\delta R}{R}$ by a factor of four, to fit all the magnetic moments within two standard deviations (see Table 5.1, column 3). Here we take $m_s = 0.252 \text{ GeV}$, $R_\Sigma = 5.98 \text{ GeV}^{-1}$, $R_\Lambda = 6.63 \text{ GeV}^{-1}$, and $R_p = 7.35 \text{ GeV}^{-1}$. Note that the relation μ_{Σ^0} equals $\frac{1}{2}(\mu_{\Sigma^+} + \mu_{\Sigma^-})$ still holds in the bag model. It is a consequence of isospin invariance, and the assumption that the three Σ particles have the same radius.

Baryon	$2M_p \mu \left(\frac{\delta R}{R_p} = -\frac{1}{40} \right)$		$2M_p \mu \left(\frac{\delta R}{R_p} = -\frac{1}{10} \right)$
p	2.79	-(input)-	2.79
n	-1.86		-1.86
Λ^0	-0.61	-(input)-	-0.61
Σ^+	2.62		2.44
Σ^-	-1.00		-0.92
Σ^0	-1.40		-1.28
Ξ^-	-0.51		-0.52
Ξ^0	0.81		0.76
$\Xi^0 - \Lambda^0$	-1.57		-1.46

Table 5.1 Predictions of Octet Baryon Magnetic Moments

Isgur and Karl⁵⁵ also fit the octet baryon magnetic moments within two standard deviations. They also take the magnetic moment of a given quark to decrease by 10%, when a u or d quark is replaced in the baryon

by an s quark. They do not work in the M.I.T. Bag Model. Their argument for the 10% decrease is somewhat 'ad hoc'. They note that the hadron radius decreases by 4% and 13% per additional s quark in harmonic and coulombic potential models respectively, and choose $\frac{\delta R}{R_p} = -\frac{1}{10}$ as they expect a combination of these two potentials. They quote the fact that in the bag model the quark magnetic moment is proportional to the bag radius to justify their taking a 10% decrease in the quark moment. They consider configuration mixing and isospin violating effects, and relativistic corrections to the additivity rule, in making their predictions. We have ignored such effects.

In the non-relativistic quark model somewhat better predictions for the magnetic moments than those given in equation (5.5) are obtained if the mass scale factor depends on the baryon mass^{56,57,58}. This means that the predictions of (5.5) must be reduced by a factor $\frac{M_p}{M_B}$. This effect is similar to ours, for we are reducing the baryon radius, and thus its magnetic moment, when the baryon mass is increased by replacing a light quark with a strange quark.

We now calculate the baryon masses, obtained by substituting the two sets of parameters for R_p , R_Λ , R_Σ , and m_s , into equations (5.11) and compare with the experimental masses⁴⁹ (see Table 5.2).

Baryon	Experimental Mass/MeV	Mass($\frac{\delta R}{R_p} = -\frac{1}{40}$)	Mass($\frac{\delta R}{R_p} = -\frac{1}{10}$)
N/ Δ	1180	1110	1110
N/ Σ / Λ	1300	1300	1278
Σ / Ξ *	1462	1485	1466

Table 5.2 Calculation of Baryon Masses

The predicted proton mass is 70 MeV less than the experimental value. This is a simple consequence of equation (5.13). We have chosen R_p so as to fit μ_p , and the value of the radius so obtained is

greater than that needed to fit M_p . In other words, it is impossible to fit both M_p and μ_p exactly in the bag model of reference 9. However, as the error is only about 6%, there is little cause for anxiety.

The predictions for M_Λ and M_Ξ are remarkably good. We note that there is little difference between the two sets of results, even though δR has changed by a factor of four. So increasing the difference between the octet baryon radii leads to a much better fit to the experimental magnetic moments, and has little effect on the predicted masses. More accurate measurements of the Σ^- and Ξ^- moments would be very helpful in determining if the picture is consistent, and, if so, what value of $\frac{\delta R}{R_p}$ best fits the data.

5.4 Quark mass, axial vector coupling constants, and charge radii in the bag model

Giving the u quark some mass, or increasing the mass of the s quark means that a larger value of the baryon radius is needed to fit the experimental magnetic moments. The axial vector coupling constant g_A also increases with m_u - from 1.09 for $m_u = 0$ to the experimental value of 1.25 for $m_u = 122$ MeV⁵⁹. See Table 1 of reference 49 for the variation of g_A , $2M_p \mu_p$ and $\langle r_p^2 \rangle^{\frac{1}{2}}$ with m_u . Note that the experimental N/ Δ mass average is fitted, and that the proton magnetic moment varies very little over the range of quark masses shown. A value of 44.1 MeV for m_u is chosen as giving the best results overall.

Jaffe⁶⁰ determines m_u , m_d and m_s as functions of scale (q^2) for all $q^2 < -\mu_0^2$, where $-\mu_0^2$ is the scale for baryon matrix elements of $\bar{q}q$ in the bag model. 'Constituent' and 'current' quark masses correspond to $m(q^2)$ at $q^2 = -\mu_0^2$, and as $q^2 \rightarrow -\infty$ respectively. $m(q^2)$ decreases with q^2 . He finds that the constituent quark masses are $\frac{1}{2}(m_u + m_d) = 43$ MeV, $(m_d - m_u) = 6.7$ MeV, and $m_s = 330$ MeV.

Donoghue and Johnson⁵³ take $\frac{1}{2}(m_u + m_d)$ to be 33 MeV to fit the pion mass in their improved bag model. They obtain a value of 1.27 for g_A including their fluctuation correction. However, they do not include gluon corrections in their formula for g_A , although they have taken gluon exchange into account in their calculation of the bag parameters. Kobzarev et al.⁵⁴ find that including gluon corrections reduces the value of g_A from 1.09 to 0.93. Reducing the above value of 1.27 by this ratio, we obtain $g_A = 1.08$. In general, there does seem to be some difficulty in incorporating gluons successfully into the M.I.T. Bag Model. We have not attempted to do so.

g_A can also be calculated for Λ , Σ and Ξ β -decays in the bag model^{10,50}. For a specific decay one adds up the contributions of the various quarks, as calculated from the formula given in table 1 of reference 10. The contribution of each quark is reduced from that in the SU(6) model by a factor, which increases slowly with mR - from 0.653 for $mR = 0$ to 0.707 for $mR = 1.41$ ¹⁰. Thus taking higher values for the baryon radii will lead to a slight increase in the predictions for $|g_A|$, e.g. from 0.87 to 0.895 for the decay $\Lambda^0 \rightarrow p e^- \bar{\nu}_e$, taking $m_s R_\Lambda = 2$. This increase is not significant. It does not, for example, explain the experimental result of $g_A = 0.435 \pm 0.035$ for the decay $\Sigma^- \rightarrow n e^- \bar{\nu}_e$, in comparison with bag model predictions of about 0.24⁵⁰.

The bag model formula for the charge radius (see table 1 of reference 10) predicts $\langle r_p^2 \rangle^{\frac{1}{2}} = 5.35 \text{ GeV}^{-1}$, for $R_p = 7.35 \text{ GeV}^{-1}$. The experimental value is $4.40 \pm 0.15 \text{ GeV}^{-1}$. This requires a value of R_p of about 6.2 GeV^{-1} . However, there is some doubt about the static bag model formula for the charge radius. This doubt arises because one actually measures a form factor, which is a non-static quantity. Donoghue and Johnson⁵³ overcome this difficulty by considering a moving bag as a wave packet with non zero net momentum (see the appendix of reference

53). They find, excluding gluon corrections, that $\langle r_p^2 \rangle^{\frac{1}{2}} = 4.10$
 GeV^{-1} .

5.5 A concluding comment

The work of this chapter suggests that the difference between the octet baryon radii in the M.I.T. Bag Model should be a good deal larger than previously thought. This indicates that a flexible bag is needed. We first formed this idea in Chapter 3, where, in evaluating the bubble contribution to the quark sea, we took the radius of the proton in the longitudinal direction to be proportional to the energy of the quark in the sea. So we conclude that a model which considers hadrons as bags with soft walls is closer to physical reality than one in which the bags have rigid boundaries.

POSTSCRIPTMAGNETIC MOMENTS OF LEPTONS IN COMPOSITE MODELS.

Recently there has been interest in composite models of quarks and leptons^{56,61,62}. An important test of such models is their ability to predict correctly the magnetic moments of observable particles. In this postscript we apply the ideas used in chapter 5 to this problem for leptons.

The magnetic moments of the electron and muon are excellently predicted on the assumption that they are pointlike particles obeying the Dirac equation. Furthermore it is known⁶¹ that the electron and muon show no structure down to distances of order 10^{-16} cm ($\sim 10^{-2}$ GeV⁻¹).

If a spin $\frac{1}{2}$ lepton is composed of three spin $\frac{1}{2}$ constituents, which, in the simplest case, have equal charges and magnetic moments, it is easy to see that⁵⁶

$$\mu_L = \mu_c \quad (P1)$$

and thus

$$\lambda_L = \frac{\mu_L}{e_L} = \frac{\mu_c}{3e_c} = \frac{1}{3} \lambda_c \quad (P2)$$

where L denotes the lepton, and c one of its components.

Now if the lepton is a Dirac particle, i.e. an electron or muon,

$$2M_L \lambda_L = 1 \quad (P3)$$

and so from (P2)

$$2M_L \lambda_c = 3. \quad (P4)$$

In the M.I.T. Bag Model λ_c is given by an analogous equation to that for λ_q - equation (5.10). Assuming that the lepton components are confined within leptons by a similar mechanism, we find that

$$\begin{aligned} 2M_L \lambda_c &\lesssim 0.4 M_L R_L \\ &\lesssim 4 \times 10^{-3} M_L \end{aligned} \quad (P5)$$

using $R_L \ll 10^{-2} \text{ GeV}^{-1}$. As $M_e \approx 5 \times 10^{-4} \text{ GeV}$, and $M_\mu \approx 10^{-1} \text{ GeV}$, this is totally inconsistent with equation (P4), the discrepancy being of the order of 10^6 for the electron. This is not a particular result of our using the M.I.T. Bag Model. For if we use a model for leptons and their components, analogous to the relativistic quark model, described in the first paragraph of section 5.2, we obtain

$$2M_L \lambda_c = \frac{M_L}{\omega_c} \ll M_L R_L \quad (\text{P6})$$

using the uncertainty principle. ω_c is the energy of a lepton component.

The discrepancy between equations P4 and P5 (or P6) is so large that it cannot be explained by changing the assumption that the lepton constituents have equal charges and magnetic moments. This contradiction arises because the known maximum values for the radii of the electron and muon are so small. As far as experiments can discern, they are pointlike particles. Any model, which tries to explain them as composite, will encounter serious difficulties.

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