Link Envelope Correlation in the Backscatter Channel

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***Abstract*— High-frequency backscatter radio systems operate in the dyadic backscatter channel, a pinhole channel whose envelope probability density function and bit-error-rate performance are strongly affected by link envelope correlation – the envelope correlation between the forward and backscatter links of the dyadic backscatter channel. This paper shows that link envelope correlation is most detrimental for backscatter radio systems using co-located reader transmitter and receiver antennas and a single RF transponder antenna. It is shown that using separate reader antennas and multiple RF transponder antennas will decrease link envelope correlation effects and a near maximum bit-error-rate can be achieved with link envelope correlation less than 0.6.**

***Index Terms*— Radio frequency identification, probability, rayleigh channels, multipath channels, RFID, correlation, pinhole channel.**

I. Introduction

SINCE the concept of modulating backscatter for communication was first proposed by Stockman in 1948 [1], work on backscatter radio has crescendoed to a virtual research and development explosion in the 21st century [2]. This excitement has been driven by many compelling backscatter radio applications such as radio frequency identification (RFID), passive sensors, and passive data storage. Such systems operate in the backscatter channel and suffer from small-scale fading that has *radically* different statistical properties than conventional one-way channels, which result in deeper fades [3]. Therefore, in order to affect a much needed improvement in radio frequency transponder (RF tag) range and reliability, small-scale fading in the backscatter channel must be mitigated. To this end, Ingram, *et al.*, [4] was the first to propose using multiple reader transmitter, reader receiver, and RF tag antennas for transmit diversity and spatial multiplexing. Kim, *et al.*, [3] reported the first measured cumulative density function (CDF) of the backscatter channel consisting of a single reader transmitter, reader receiver, and RF tag antenna. Kim, *et al.*, found that the measured CDF closely matched that of the product of two independent Rician distributions.

The *dyadic backscatter channel*, a pinhole channel [5] composed of a forward and backscatter link was presented by Griffin, *et al.*, [6] along with analytic expressions for two special cases of the channel probability density function (PDF) – i.e., the PDF for both independent and fully correlated forward and backscatter links. This paper will study the correlation between the envelopes of the forward and backscatter links, *link envelope correlation*, and show its effects on the channel PDF and bit-error-rate (BER). For context, Section II will briefly describe the dyadic backscatter channel and analytic PDFs reported previously [6].

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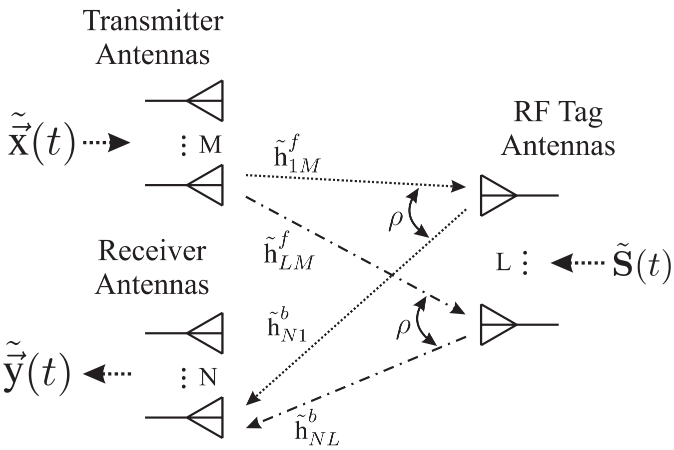


Fig. 1. The general $M \times L \times N$ dyadic backscatter channel with M reader transmitter antennas, L RF tag antennas, and N reader receiver antennas. The path from the m^{th} transmitter antenna to the n^{th} receiver antenna is shown where $\tilde{h}^f_{lm}$ and $\tilde{h}^b_{nl}$ are elements of the forward and backscatter link matrices, $\tilde{\mathbf{H}}^f$ and $\tilde{\mathbf{H}}^b$, respectively. Elements of the forward and backscatter link matrices that terminate or originate on a common RF tag antenna have link correlation, ρ; otherwise, the matrix elements are independent.

II. The Dyadic Backscatter Channel and PDF

A. Dyadic Backscatter Channel

The $M\times L\times N$ dyadic backscatter channel, the most general backscatter channel, consists of M transmitter, L RF tag, and N receiver antennas, shown in Fig. 1. The received, complex baseband signal is:

$$\vec{\tilde{y}}(t) = \frac{1}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \tilde{\mathbf{H}}^b(\tau_b;t)\tilde{\mathbf{S}}(t)\tilde{\mathbf{H}}^f(\tau_f;t) \times \vec{\tilde{x}}(t-\tau_b-\tau_f)d\tau_b d\tau_f + \vec{\tilde{n}}(t) \quad (1)$$

where $\vec{\tilde{y}}(t)$ is an $N\times 1$ vector of received complex baseband signals, $\tilde{\mathbf{H}}^b(\tau_b;t)$ is the $N\times L$ complex, baseband channel impulse response matrix of the backscatter link, and $\tilde{\mathbf{H}}^f(\tau_f;t)$ is the $L\times M$ complex, baseband channel impulse response matrix of the forward link [7]. $\tilde{\mathbf{S}}(t)$ is a narrowband $L\times L$ matrix that describes the time-varying modulation that an RF tag places on the radio signals absorbed and scattered by the L RF tag antennas. This matrix is equivalent to the scattering matrix commonly used in RF design. Furthermore, $\vec{\tilde{x}}(t)$ is an $M\times 1$ vector of signals transmitted from the reader transmitter antennas and $\vec{\tilde{n}}(t)$ is an $N\times 1$ matrix of noise components.

B. Dyadic Backscatter Channel PDF

In this paper, propagation in the forward and backscatter links is assumed to experience uncorrelated Rayleigh fading, an extreme, limiting case of the Rician fading distribution found in a typical line-of-sight (LOS) backscatter channel. Consequently, each element of $\tilde{\mathbf{H}}^f$ and $\tilde{\mathbf{H}}^b$ is an independent, identically distributed (i.i.d.), zero-mean, complex Gaussian random variable. The variance of the ij^{th} element of each link, $\tilde{h}_{ij}^f$ and $\tilde{h}_{ij}^b$, is σ_f^2 and σ_b^2, respectively. The signal received at the n^{th} reader receiver antenna, $\tilde{y}_n(t)$, is proportional to the sum of ML complex Gaussian products, L of which are independent. The characteristic function of the signal envelope received at the n^{th} reader receiver antenna through the general $M \times L \times N$ dyadic backscatter channel is [6]

$$\Phi(\nu;\rho) = \left[\frac{\sigma_b^4\sigma_f^4 M^2}{16}\frac{\gamma^4}{\left(1-|\rho|^2\right)^2} \times \left(\nu^2 + \frac{4\left(|\rho|-1\right)^2}{\sigma_b^2\sigma_f^2 M\gamma^2}\right)\left(\nu^2 + \frac{4\left(|\rho|+1\right)^2}{\sigma_b^2\sigma_f^2 M\gamma^2}\right)\right]^{-L/2}, \quad (2)$$

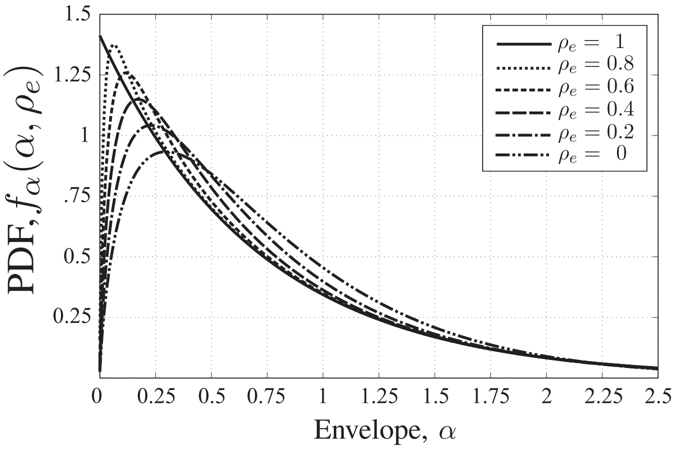
where $\gamma = 1-\rho^2$, ν is the index of the characteristic function, and ρ is the normalized link correlation coefficient which describes the correlation between $\text{Re}\{\tilde{h}_{Lm}^f\}$ and $\text{Re}\{\tilde{h}_{nL}^b\}$ and between $\text{Im}\{\tilde{h}_{Lm}^f\}$ and $\text{Im}\{\tilde{h}_{nL}^b\}$ ($-1 \le \rho \le 1$, $\text{Re}(\tilde{z})$ and $\text{Im}(\tilde{z})$ are the real and imaginary parts of the complex scalar, $\tilde{z}$). It is assumed that the correlation between $\text{Re}\{\tilde{h}_{ij}^f\}$ and $\text{Im}\{\tilde{h}_{ij}^f\}$ and between $\text{Re}\{\tilde{h}_{ij}^b\}$ and $\text{Im}\{\tilde{h}_{ij}^b\}$ is zero. The PDF of (2) can be found using the inverse Hankel transform which, for most values of ρ, requires numerical techniques. However, analytic expressions of the envelope PDF have been derived [6] for $\rho = 0$ and $\rho = 1$ which are useful upper and lower bounds of link correlation. These PDFs are

$$f_\alpha(\alpha,\rho) = \alpha^{L/\beta}\left(\frac{2}{\beta\sqrt{M}\sigma_b\sigma_f}\right)^{1+L/\beta} \times \frac{2^{1-L/\beta}}{\Gamma\left(L/\beta\right)}\mathrm{K}_\nu\left(\frac{2\alpha}{\beta\sqrt{M}\sigma_b\sigma_f}\right), \quad (3)$$

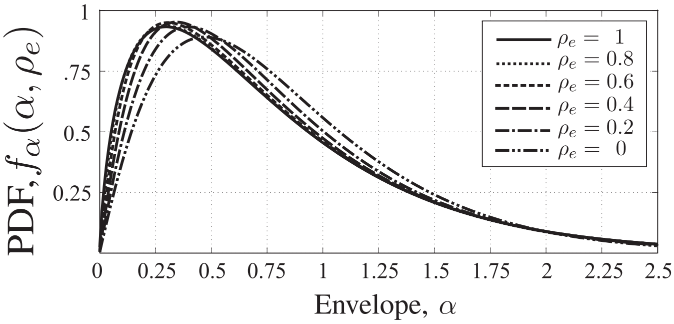
where α is the channel envelope, $\Gamma(\cdot)$ is the gamma function, $\mathrm{K}_\nu(\cdot)$ is a modified bessel function of the second kind with order $\nu = 1 - L/\beta$, and $\beta = \rho + 1$. It should be noted that, while (2) holds for any value of M, L, or ρ, care must be taken in the choice of ρ so that the correlation matrix formed by the entries of $\tilde{\mathbf{H}}^f$ and $\tilde{\mathbf{H}}^b$ is positive semi-definite [8]. The range of ρ is a function of M and, for the $1 \times L \times 1$ channel considered in this paper, the range $0 \le \rho \le 1$ is permissible. It can be shown that $|\rho|^2 \approx \rho_e$ where ρ_e denotes the link envelope correlation and the argument of ρ is usually assumed to be zero [9], [10]. In the remainder of this paper, results will be presented in terms of ρ_e.

III. Link Envelope Correlation

In a conventional one-way channel, spatial fading correlation will hinder communication and limit available diversity gains. In a pinhole channel, an additional source of fading correlation, link envelope correlation, can have the same effect by coupling fading in the forward and backscatter links – even if fading in each link is uncorrelated. Previous work on realistic pinhole channels has focused on situations in which the two links of the pinhole channel are likely dissimilar (e.g. outdoor propagation [11] or amplify-and-forward channels [12]) justifying the assumption of independent links. In many backscatter radio systems, however, reader transmitter and receiver antennas may be closely spaced or even co-located giving rise to potentially high link envelope correlation.



(a) Normalized $1 \times 1 \times 1$ dyadic backscatter channel PDF



(b) Normalized $1 \times 2 \times 1$ dyadic backscatter channel PDF

Fig. 2. The PDF of the signal received at the n^{th} reader receiver antenna for (a) the $1 \times 1 \times 1$ and (b) $1 \times 2 \times 1$ channels with different ρ_e values. The random variables that correspond with these PDFs have been normalized to unit power (i.e., $\mathcal{E}\{\alpha^2\} = 1$ where $\mathcal{E}\{\cdot\}$ denotes the ensemble average).

A. The Effect of Link Envelope Correlation on the PDF

Fig. 2(a) and Fig. 2(b) show the PDF of the signal received at the n^{th} reader receiver antenna as a function of ρ_e. The random variables that correspond with these PDFs have been normalized to unit power (i.e., $\mathcal{E}\{\alpha^2\} = 1$ where $\mathcal{E}\{\cdot\}$ denotes the ensemble average). In both figures, the probability of a fade decreases with ρ_e. The most significant fading improvement is for the $1\times1\times1$ channel, shown in Fig. 2(a), in which the PDF transitions from an exponential distribution to that of a product Rayleigh. Though fading in the $1\times2\times1$ channel is significantly less than the $1 \times 1 \times 1$ channel, reducing ρ_e in this channel results in smaller PDF improvements. This is because multiple RF tag antennas provide additional statistically independent pinholes through which signals may propagate [6]. In fact, the number of pinholes available in the channel determines the shape of the PDF. Analysis of (3) shows that this is a general result; when normalized to equal power, the $1 \times L \times 1$ channel

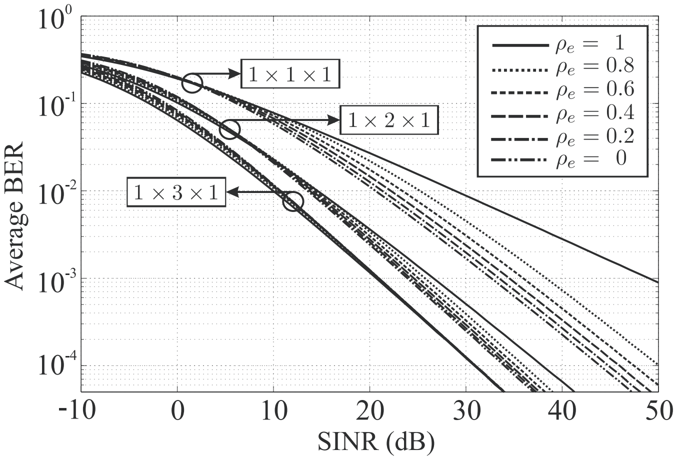


Fig. 3. Average BER plots for the $1\times1\times1$, $1\times2\times1$, and $1\times3\times1$ dyadic backscatter channels for several values of link envelope correlation, ρ_e. In these Monte Carlo simulations, Rayleigh fading forward and backscatter links, uncoded BPSK modulation, and noise and interference that is additive, white, and Gaussian were assumed. Each curve represents the average BER of the signal received at the n^{th} reader receiver antenna with no diversity combining. Each BER curve is plotted against the signal-to-noise plus interference ratio (SINR) at the n^{th} reader receiver antenna in the $1\times1\times1$ channel.

with independent links has the same PDF as the $1 \times 2L \times 1$ channel with fully correlated links. This may be seen in Fig. 2(a) and Fig. 2(b).

B. The Effect of Link Envelope Correlation on the BER

The changes in the dyadic backscatter channel PDF caused by ρ_e and multiple RF tag antennas are reflected in the average BER, shown in Fig. 3. For each dyadic backscatter channel (i.e., the $1 \times 1 \times 1$, $1 \times 2 \times 1$, and $1 \times 3 \times 1$ channels), the BER improves as ρ_e is lowered. However, even greater BER gains are available as antennas are added to each RF tag. The source of the BER improvements seen in Fig. 3 is the improved PDF shown in Fig. 2(a) and Fig. 2(b). Like the PDF of the dyadic backscatter channel, the BER plots for higher numbers of RF tag antennas are less sensitive to ρ_e. Note that for the $1\times1\times1$ and $1\times2\times1$ channels, the greatest increase in BER occurs between $\rho_e = 0.6$ and $\rho_e = 1$. If ρ_e is kept at or below approximately 0.6 in these channels, near maximum BER performance is achieved. This agrees with the generally accepted rule that envelope correlation between diversity branches in a one-way channel is acceptable below 0.5 to 0.7 [10]. The close relationship between the $1 \times L \times 1$ and $1 \times 2L \times 1$ channels noted in Section III-A is not evident in Fig. 3 because, in this figure, each channel has a different level of power. The power of each channel is dependent on the number of RF tag antennas, L.

C. Discussion

A high degree of link envelope correlation will occur when the dominant mechanism of wave propagation (i.e., non-line-of-sight propagation in this paper) and the angles of arrival/departure at the reader are similar. Since a high level of link envelope correlation implies that the propagation environment of the forward and backscatter links are similar, fully correlated links can *only* occur when the reader transmitter and receiver antennas are co-located and have the same antenna patterns. If the antennas are spatially separated and/or the antenna patterns are different, ρ_e will be reduced which allows the designer some control over the level of link envelope correlation. The separation distance and pattern required to reduce ρ_e to an acceptable level will vary depending upon the channel.

In this analysis, link envelope correlation can occur only between propagation paths that terminate or originate on the same RF tag antenna and it is assumed that ρ_e is equal for each set of paths (see Fig. 1). In an actual dyadic backscatter channel, antenna coupling, close spacing of RF tag antennas, and the scattering environment will likely cause unequal levels of correlation between all propagation paths. Such a channel, which will likely have correlated, Rician fading, may best be studied by measurements.

IV. CONCLUSION

Link envelope correlation will worsen both the PDF and BER in the dyadic backscatter channel, especially for backscatter radio systems with a single RF tag antenna and co-located reader receiver and transmitter antennas. If separate reader transmitter and receiver antennas are used in conjunction with multiple RF tag antennas, link envelope correlation will be significantly reduced and yield a near minimum BER for link envelope correlation lower than approximately 0.6.

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