

PROJECT ADMINISTRATION DATA SHEET

ORIGINAL REVISION NO. _____

Project No. A-3195 DATE 3/22/82

Project Director: ~~Dr. D. J. Kozakoff~~ Mr. Joe M. Newton School/Lab EML/RSD

Sponsor: U. S. Army Missile Command; Redstone Arsenal, AL 35898

Type Agreement: Delivery Order No. 0042 under No. DAAH01-81-D-A003

Award Period: From 3/12/82 To 9/30/82 (Performance) _____ (Reports) _____

Sponsor Amount: \$30,173.90 Contracted through: _____

Cost Sharing: None GTRI/GPX _____

Title: Radome Computer Analysis Methodology RDF-43

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Defense Priority Rating: DO-A2 under DMS Reg. 1 Security Classification: unclassified

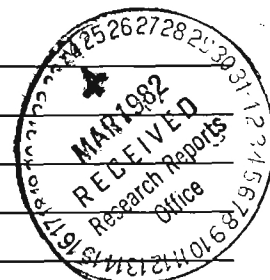
RESTRICTIONS

See Attached government Supplemental Information Sheet for Additional Requirements.

Travel: Foreign travel must have prior approval – Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of \$500 or 125% of approved proposal budget category.

Equipment: Title vests with government; except that items costing less than \$1,000 vests with GIT if prior approval to purchase is obtained from the Contracting Officer.

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FORM OCA 4:781

SPONSORED PROJECT TERMINATION SHEET

2-
SR-321

Date 9/30/82

Project Title: Radome Computer Analysis Methodology RDF-43

Project No: A-3195

Project Director: Dr. D. J. Kozakoff

Sponsor: US Army Missile Command; Redstone Arsenal, AL

Effective Termination Date: 9/30/82

Clearance of Accounting Charges: 11/30/82

Grant/Contract Closeout Actions Remaining:

- Final Invoice ~~and Closing Documents~~
- Final Fiscal Report
- Final Report of Inventions
- Govt. Property Inventory & Related Certificate
- Classified Material Certificate
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Assigned to: EML (School/Laboratory)

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Technical Report #1
and
Cost and Performance Report #1

Report Period
1 March 1982 through 31 March 1982

RADOME COMPUTER ANALYSIS METHODOLOGY

D. J. Kozakoff

Contract No. DAAH01-81-D-A003
Delivery Order 0042
EES Project A-3195

Effective Date: 3/12/82
Expiration Date: 9/30/82

Prepared for

U.S. Army Missile Command
Redstone Arsenal, AL 35898
Attn: DRSMI-RDF

Prepared by

Engineering Experiment Station
Georgia Institute of Technology
Atlanta, GA 30332

WORK PERFORMED IN THIS REPORTING PERIOD

This project was initiated in this reporting period. The only work accomplished were telecons and planning sessions with the contract technical monitor.

PROBLEMS ENCOUNTERED IN THIS REPORTING PERIOD

None

WORK PLANNED FOR THE NEXT REPORTING PERIOD

A project kick-off meeting in Atlanta will be held to quantify the goals and plan of attack in accomplishing this study.

Cost Information

The following charges have been incurred against the contract during period 12 March through 31 March, 1982

	<u>Expended</u>	<u>Encumbered</u>
Personal Services (PS)	\$240.75	
Materials and Supplies	0	
Travel	0	-NONE-
Overhead (@ 55% of PS)	147.76	
Retirement (@ 11.59% of PS)	<u>27.90</u>	
TOTAL	\$416.41	

The breakdown of personal services is as follows:

	<u>Dollars</u>	<u>Approximate Man Hours</u>
Principal Research Scientists/Engineers	0	0
Senior Research Scientists/Engineers	240.75	11
Research Scientists II/Engineers II	0	0
Research Scientists I/Engineers I	0	0
Technicians/Draftsmen	0	0
Students	0	0
Secretarial/Clerical/Other	0	0
TOTAL	<u>\$240.75</u>	<u>11</u>

The current financial status of the contract is as follows:

	<u>Budget As Proposed</u>	<u>Expended</u>	<u>Free Balance</u>
Personal Services (PS)	\$11,620.22	\$240.75	\$11,379.47
Materials and Supplies	600.00	0	600.00
Travel and Shipping	900.00	0	900.00
Computer	5,000.00	0	5,000.00
Overhead	10,706.90	147.76	10,559.14
Retirement	<u>1,346.78</u>	<u>27.90</u>	<u>1,318.88</u>
Encumbered	\$30,173.90	\$416.41	\$29,757.49

FUNDING

Based on present full funding, the funding and equivalent man hours are sufficient to complete the task. Approximately 1.4% of the proposed task has been completed.

Technical Report #2
and
Cost and Performance Report #2

Report Period

1 April through 31 July 1982

RADOME COMPUTER ANALYSIS METHODOLOGY

D. J. Kozakoff

Contract No. DAAH01-81-D-A003
Delivery Order 0042
EES Project A-3195

Effective Date: 3/12/82
Expiration Date: 9/30/82

Prepared for

U.S. Army Missile Command
Redstone Arsenal, AL 35898
Attn: DRSMI-RDF

Prepared by

Engineering Experiment Station
Georgia Institute of Technology
Atlanta, Georgia 30332

WORK PERFORMED IN THIS REPORTING PERIOD

Complete radome modeling was accomplished to assess the affects of backwall and bulkhead reflections and antenna scattered energy. In addition, the flat panel wall transmission subroutine was scrutinized via comparison of measured and predicted data. The results show the predictions are larger insertion phase values than measured at MICOM.

Various trips were made to interface with the MICOM Technical Contract Monitor.

PROBLEMS ENCOUNTERED IN THIS REPORTING PERIOD

None.

WORK PLANNED FOR NEXT REPORTING PERIOD

The final written report should be completed and a rough draft will be delivered to MICOM for review.

Cost Information

The following charges have been incurred against the contract during period July 1 - July 31, 1982

	<u>Expended</u>	<u>Encumbered</u>
Personal Services (PS)	1,077.09	-0-
Materials and Supplies	4.50	-0-
Travel	202.10	-0-
Overhead (@ 73% of PS)	757.87	-0-
Retirement (@ 11.11% of PS)	190.50	-0-
TOTAL	<u>2,232.06</u>	<u>-0-</u>

The breakdown of personal services is as follows:

	<u>Dollars</u>	<u>Approximate Man Hours</u>
Principal Research Scientists/Engineers		
Senior Research Scientists/Engineers	882.75	39
Research Scientists II/Engineers II		
Research Scientists I/Engineers I		
Technicians/Draftsmen		
Students	194.34	28
Secretarial/Clerical/Other		
TOTAL	<u>1,077.09</u>	<u>67</u>

The current financial status of the contract is as follows:

	<u>Budget As Proposed</u>	<u>Expended</u>	<u>Free Balance</u>
Personal Services (PS)	11,620.22	13,084.89	(1,464.67)
Materials and Supplies	600.00	176.48	432.52
Travel and Shipping	900.00	514.03	385.97
Computer	5,000.00	267.54	4,732.46
Overhead	10,706.90	8,344.50	2,367.40
Fringe Benefits	1,346.78	1,356.58	(9.80)
Encumbered	-0-	-0-	-0-
FUNDING	<u>30,173.90</u>	<u>23,744.02</u>	<u>6,429.88</u>

Based on present full funding, the funding and equivalent man hours are sufficient to complete the task. Approximately 79% of the proposed task has been completed.

FINAL TECHNICAL REPORT

Project A-3195

RADOME COMPUTER ANALYSIS METHODOLOGY

September 1982

D. J. Kozakoff

and

D. Bagwell

Contract No. DAAH01-81-D-A003

Delivery Order 0042

Prepared for

U. S. ARMY MISSILE COMMAND
Redstone Arsenal, AL 35898
Attn: DRSMI-RDF

Prepared by

Engineering Experiment Station
Georgia Institute of Technology
Atlanta, Georgia 30332

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) RADOME COMPUTER ANALYSIS METHODOLOGY		5. TYPE OF REPORT & PERIOD COVERED Final Report 3/12 - 9/30/82
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) D. J. Kozakoff D. Bagwell		8. CONTRACT OR GRANT NUMBER(s) DAAH01-81-D-A003 Delivery Order 0042
9. PERFORMING ORGANIZATION NAME AND ADDRESS Engineering Experiment Station Georgia Institute of Technology Atlanta, Georgia 30332		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Missile Command Redstone Arsenal, Alabama 35898 Project Monitor: K. Letson		12. REPORT DATE September 1982
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the U. S. Army Missile Command.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Radome Analysis, Computer Methodology, Boresight Errors.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This document contains an assessment of the accuracy of a three-dimensional backward ray trace computer code when various parameters are factored into the mathematics. This includes backwall and bulkhead reflections. Predictions are compared to measured data for a test case.		

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1.0 INTRODUCTION

This document summarizes a study with an objective of quantifying prediction errors in various computerized radome analyses techniques. In an earlier study [1], the various analysis methods including forward and backward ray trace, surface integration and plane wave spectra were investigated. It was determined that for antennas greater than approximately five wavelengths in diameter, the three-dimensional backward ray trace offered potential for superior accuracy while maintaining reasonable operating cost relative to the surface integration or plane wave spectra approaches which could cost up to \$10 per data point in computer cost (i.e. one look angle and one frequency).

Of the computer code survey completed in the first study, the three most viable which were selected for study were the Georgia Tech three-dimensional backward ray trace, and RADEP3 codes and the Auburn University code. All these are backward ray trace formulations; a comparative summary of modeling features are given in Table 1-1. The tradeoffs in analysis method performance results appear in Table 1-2.

In this study, the preferred Georgia Tech code was modified in detail to model additional error contributors which were believed to be the major error sources between theoretical predictions and actual results. In addition, the wall transmissions subroutine was exercised for an example and compared with actual measurements (performed as part of this program) to determine the validity of the theory. Finally, the program was exercised for a particular radome problem and the results compared to actual measurements to determine if prediction improvement could be obtained with the modeling modifications.

Table 1-1

COMPARISON OF SELECTED COMPUTER CODES

Program	Bulkhead Reflections	Backwall Reflections	Radome Tip Modeling	Radome Wall Taper	Arbitrary Circular Polarization	Circumferential Wall Variations	Antenna Scattering	Multilayer Walls
Georgia Tech	No	No	Yes	Yes	Yes	Yes	No	Yes
Auburn University	No	Yes	Yes	No	No	No	No	Yes
RADEP3	No	No	Yes	Yes	No	No	No	Yes

Table 1-2

TRADEOFF OF ANALYSIS METHODS

	Operating Cost	BSE Accuracy	Sidelobe Predictor Accuracy	Machine Size Required
Ray Trace Transmit	Low	Good	Poor	Small
Ray Trace Receive	Low	Good	Good	Small
PWS	High	Good	Good	Large
Surface Integration	Extremely High	Excellent	Good	Very Large

2.0 OVERVIEW OF SELECTED RADOME ANALYSIS METHOD

A flow chart of the ray trace program used herein is shown in Figure 2-1. The program interactively asks the user to input the necessary parameters to describe the (tangent ogive) radome, the antenna, and the incoming wave polarization. The program is currently preselected to the principal planes ($EI = 0$ and $AZ = 0$) from 0 to 30 degrees in increments of 1 degrees. The antenna sample spacing is selected by the user. A ray is then traced outward from each sample point in the direction of the incoming wavefront. The intersection of each ray with the radome is then found by a modified regula falsi root solving method. The normal to the tangent ogive at the intersection is found and used along with the direction of propagation to define the plane of incidence. The electric field is then broken into components perpendicular and parallel to the plane of incidence. The transmission and IPD are then calculated for both of the above cases, the radome wall being assumed to be locally flat at the intercept point. The subroutine that calculates the affect of the wall takes into account multi-layer sandwiches and multiple reflections within the radome wall (see Section 5).

The electric field is then reflected in terms of the original azimuth and elevation directions for numerical integration (summation) at the antenna aperture. Monopulse sum and difference illuminations are used in sequence to allow computation of standard monopulse error voltages and to thus derive a measure of boresight error in both the elevation and azimuth channels. The ray trace technique is illustrated in Figure 2-2.

The specific subroutines that compute the affects of bulkhead, backwall and internal antenna reflections are described in detail in subsequent sections of this report.

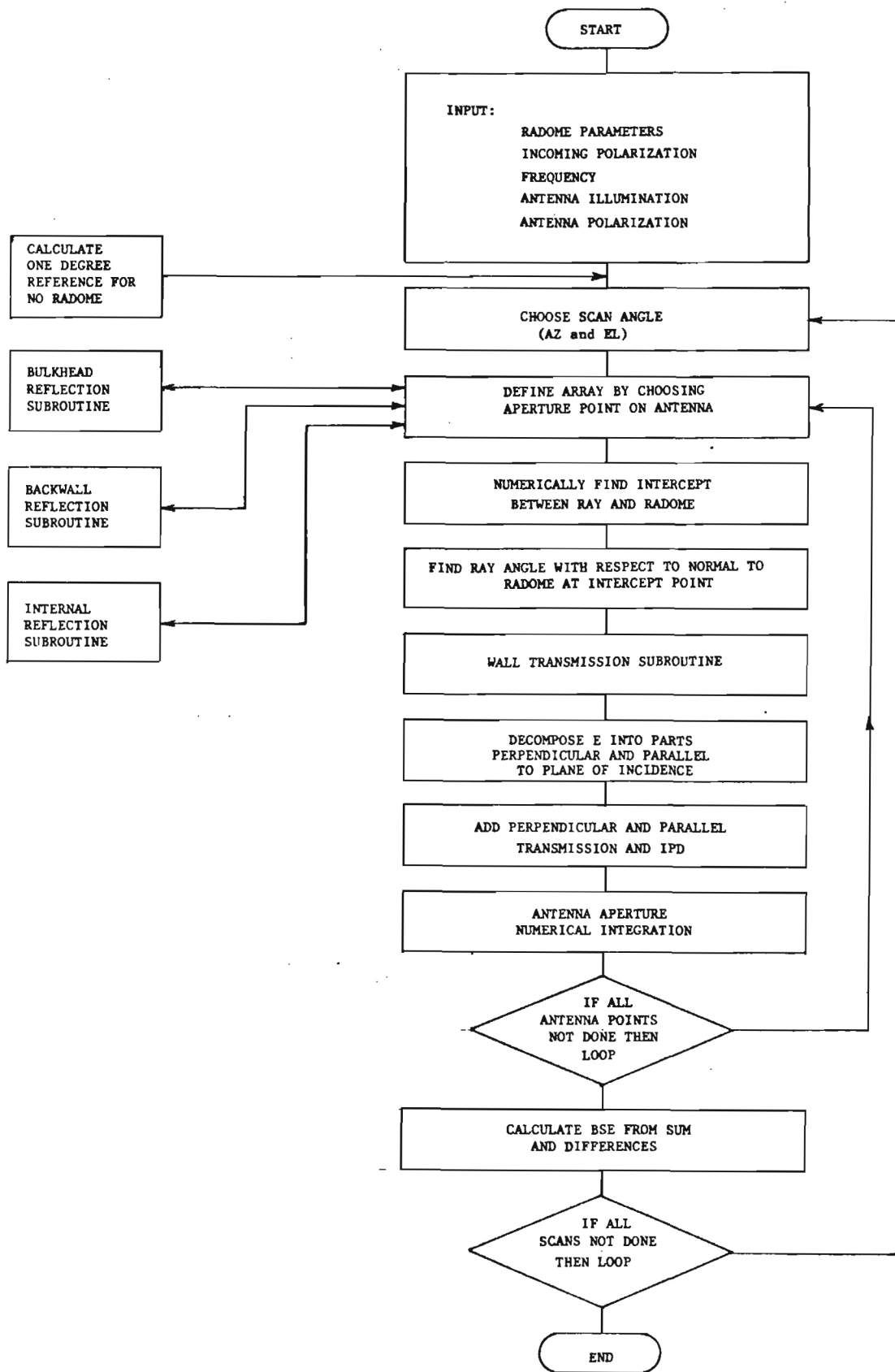


Figure 2-1. Program Flow Chart

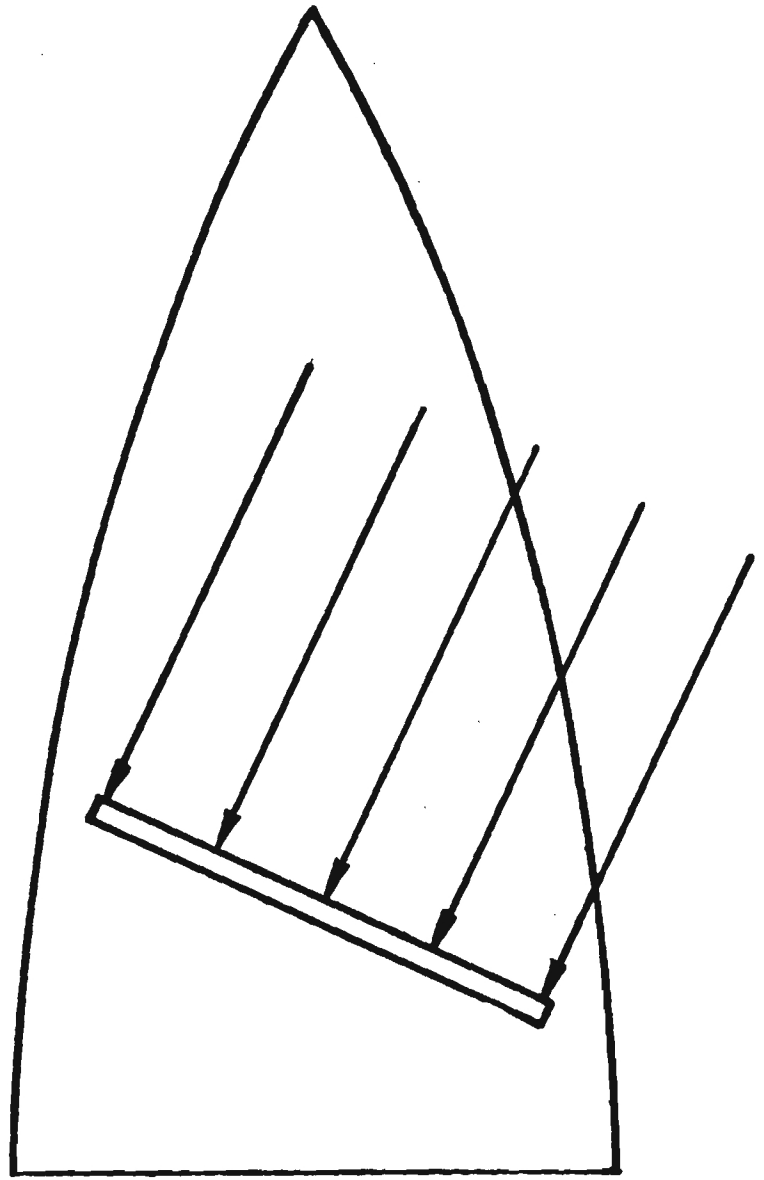


Figure 2-2. Backward Ray Trace Method.

3.0 DISCUSSION OF ANALYSIS ERRORS

3.1 General

The extent that the sidewall and bulkhead reflections and antenna scattered energy can affect boresight error calculations is highly dependent upon the properties of the radome and the antenna that it encloses. A radome can be theoretically designed so that reflections have negligible effects on the boresight error, but in practice, the reflections usually are large enough to influence the boresight errors.

The effect of bulkhead reflections depend mainly on the backlobe properties of the antenna. The magnitude of rays reflecting off the rear bulkhead plate are approximately the same as those directly striking the antenna. If the antenna is located close to the bulkhead then the area on the bulkhead for rays to reflect and strike the antenna is restricted and the extent of bulkhead reflections decrease.

Generally, bulkhead reflections will not have much affect until the angle of incidence of the incoming rays (with respect to the radome axis) become larger. Our antenna model assumes that the rear pattern is a mirror image of the forward pattern, decreased in magnitude by a user specified number of decibels (generally 20 dB). This should reasonably be a good model for a variety of antennas. In actual applications, positioners and electronic gear may block some of the reflections from reaching the antenna. As the incidence angle of the incoming rays gets much greater than 45 degrees, the bulkhead reflections cannot hit the antenna since they become parallel to the plane of the antenna.

The reflections off the radome side walls is another parameter that can affect boresight errors. The reflectance of a design wall may only be one percent for the electric field (.01% for the power) for a well-designed radome. In practice, erosion and ablation may change the thickness and heating may change the electrical properties of the wall. Reflectances may then get very large, potentially even being larger than fifty percent. The sidewall reflections strike only a small segment of the antenna. As radome wall reflectances become large, the boresight errors may become very large as a result of this concentration of reflected energy. The size and position of the antenna also has an effect on sidewall reflections.

A limiting incidence angle exists, below which no sidewall reflections can occur (see Section 3.2). As the size of the antenna approaches that of the radome immediately surrounding it, this limiting angle goes to 0. As the size of an antenna increases, the effects of sidewall reflections will increase regardless of incidence angle (as long as the incidence angle is larger than the limiting angle).

The fineness ratio of the radome also has an effect on sidewall reflections, since the position and angle of the radome wall at the reflection point determines where the reflected rays can strike the antenna. A large fineness ratio should lead to increased sidewall reflections, as should moving the antenna towards the base of the radome. Sidewall effects would be expected to increase if the antenna were not perpendicular to the incident radiation.

The effects of antenna scattered energy on boresight error calculations are highly dependent upon the reflectance of the radome walls. Some of these rays may reflect off the radome wall twice before striking the antenna, enhancing the need for high reflectance to get measureable effects. The properties of the antenna control how much of the scattered energy is absorbed. A large antenna increases the probability that a reflecting ray can hit the antenna. The antenna's illumination function determines how much of the incident rays energy is scattered and the distribution of this scattered energy. A low fineness ratio for the radome should allow more rays to hit the antenna with fewer sidewall reflections. Antenna scattered energy should not be as important in typical radomes as the other types of reflections, because the reflected energy is distributed fairly evenly over the antenna aperture, reducing its affect on boresight error calculations.

3.2 Sidewall Reflections

Part of the energy incident on an antenna enclosed in a radome consists of rays that transmit through the radome, hits another position of the radome where some of the energy is reflected back into the antenna (or misses the antenna and is reflected again). The rays that reflect off the radome wall only once before striking the antenna are the only ones that are considered herein. Higher order reflections can be ignored for typical radomes since the amount of energy reflected at each contact with the radome wall becomes insignificant.

The amount of energy that is reflected off a radome wall is dependent on the angle of the incoming rays. To develop the analyses we define the antenna reference plane to be the plane of the antenna when the antenna is looking down the axis of the radome illustrated in Figure 3-1. At some incidence angle part of the incident wave that would normally hit the antenna reference plane outside of the radome is unable to hit this plane because it has already been reflected by the radome. The radome casts a "shadow" on the antenna reference plane.

The energy absent from this shadow region is that which will be reflected off the radome wall. A limiting angle of incidence with respect to the radome axis exists, below this limit no shadow will be cast and no energy can be reflected. This angle is found to be

$$\theta_{\text{lim}} = \tan^{-1} \left(\frac{D'/2}{L-\Delta} \right) \quad (3-1)$$

An array of sample points is set up inside any shadow that is cast. The sample point spacing is selected according to the sample point spacing used on the antenna in the standard ray-tracing method.

A ray incident on the shadow intercepts the radome in two locations. These locations are found numerically using the same techniques as the standard ray-trace. The energy striking the antenna must be transmitted at the first intercept and reflected at the second. The effects on the magnitude and phase of the incoming ray by the transmission and reflection are calculated using the model described in

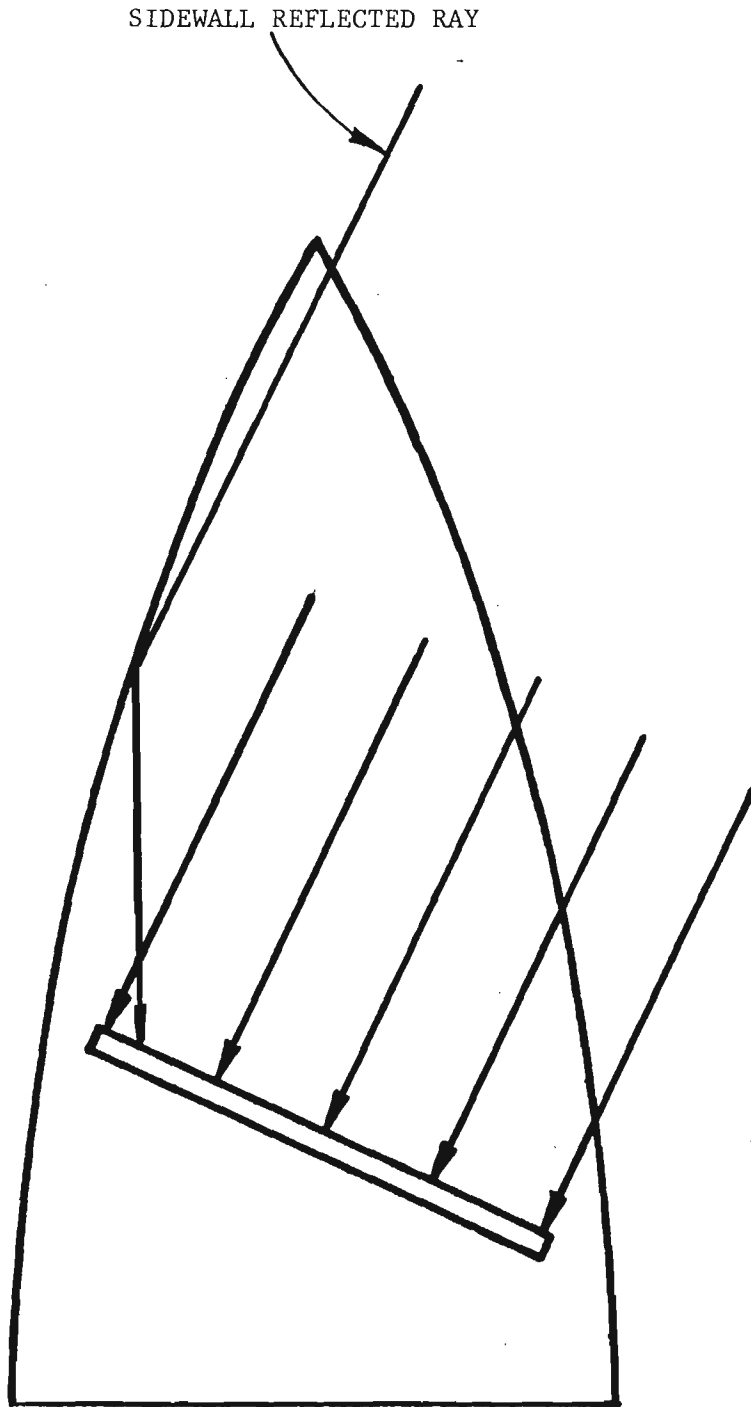


Figure 3-1. Sidewall Reflection Component Added to Ray Trace.

Section 5. The direction vector C of the reflected ray is:

$$C = K - (K \cdot N) N \quad (3-2)$$

The coordinates of the point where the reflected ray intersects the antenna aperture must be found for all reflected rays that will hit the antenna. The equation for the rotated antenna's aperture plane is obtained by inverting the rotation matrix that rotated the antenna to its position and solving for the points $X'=0$ which are contained in the aperture plane. (See Appendix C for details). The intersection point can now be found by solving the equations for the antenna plane with those describing the vector C.

A phase shift is added to the electric field at the antenna due to the longer path that the reflected ray must take with respect to a reference ray coming directly in. The magnitude of the electric field absorbed by the antenna is decreased for fields not in the plane (direction vector normal to the plane) of the antenna. After these two effects on the electric field are accounted for, the electric field is added to the computer model of the monopulse network in the same method as direct incidence rays.

3.3 Bulkhead Reflections

Many missiles have a bulkhead plate between the radome of the missile body. If the bulkhead is not absorber treated, then incoming electromagnetic rays can reflect off this plate and strike the backside of the antenna; these rays are capable of causing boresight errors.

The analysis treats the reflections off the bulkhead plate as if it were flat, perfectly conducting, and located at the origin of the radome coordinates. This should be a fairly accurate model for many missiles.

The antenna is modeled the same as in the standard ray-trace, i.e., the same sample point definitions are used. However, the rays incident on the back of the antenna are reduced in gain. The user corrects for this in the program by inputting the rear antenna gain factor. This factor can also include any losses which come from imperfect reflection off of the bulkhead.

Mathematics

The rays are traced from the antenna sample points to the plane of the bulkhead separation plate along the vector $(-K_x, K_y, K_z)$ illustrated in Figure 3-2. (Direction vector for incident rays is K_x, K_y, K_z . The vector $(-K_x, K_y, K_z)$ was chosen because it will become the direction vector \bar{K} after reflection off of the plate. Any rays that pierce the side of the radome before hitting the bulkhead plate are discarded since they are not reflected by the bulkhead. The rays are then traced along the direction vector \bar{K} to the plane of the antenna. The intersection between the plane and ray is found by inverting the rotation matrix as described in Section 3.2. If this ray hits the antenna itself then it is discarded because rays directly incident on the antenna have already been treated in the standard ray-trace. The rays are then ray traced from the antenna plane through the radome wall along the direction \bar{K} in the same manner as the standard ray-trace.

The remaining rays may be thought of as being incident at the radome, reflecting off the bulkhead, then striking the antenna. The magnitude of the electric field is adjusted to account for non-normal incidence on the antenna as described in Section 3.2. The phase of the

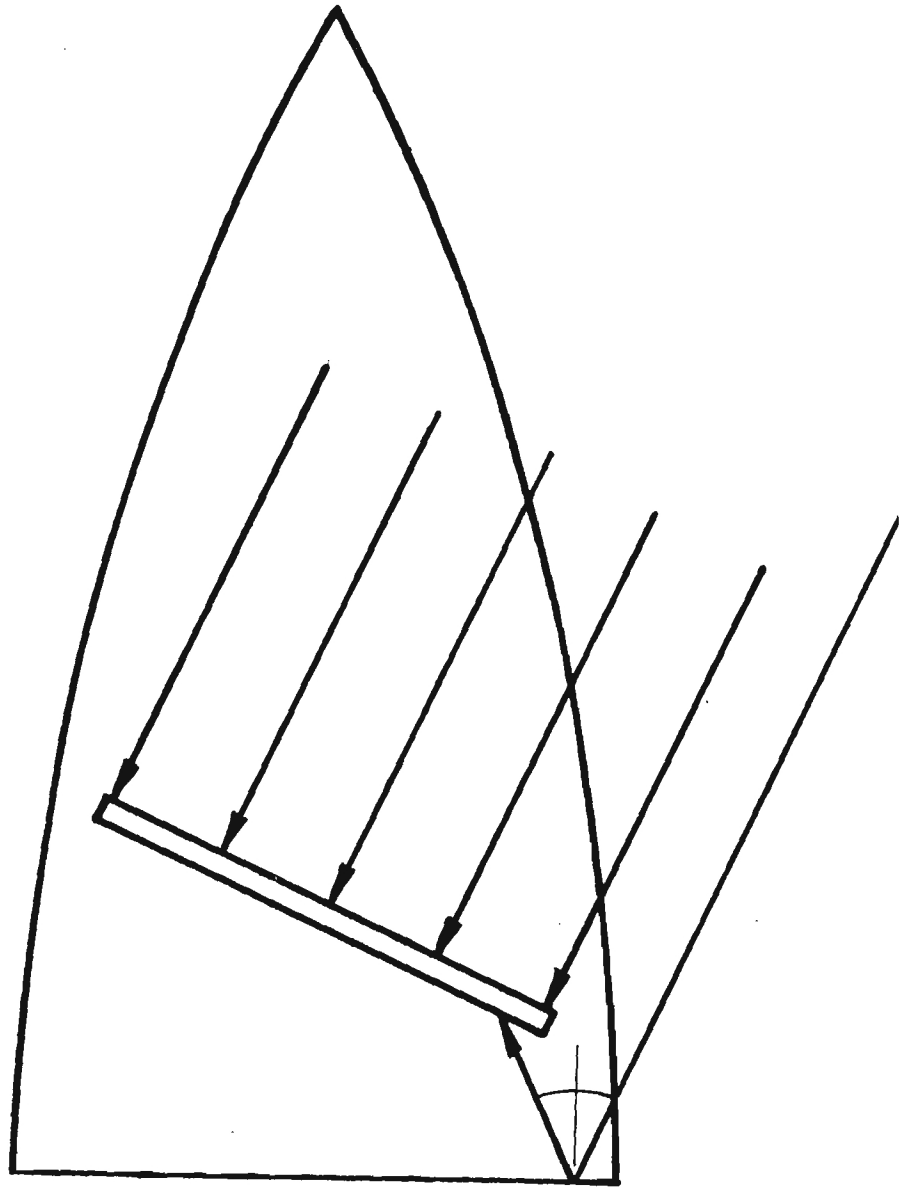


Figure 3-2. Bulkhead Reflection Added to Backward Ray Trace.

electric field is adjusted to allow for the additional path from the antenna to the bulkhead to the antenna plane. The electric field is then summed to the monopulse model at the sample point on the antenna.

3.4 Antenna Scattered Energy

Not all of the energy striking an antenna is absorbed by the antenna. The energy absorbed by an antenna is dependent on the antenna's illumination function. An approximation to the energy reflected by the antenna is the inverse of the illumination functions.

The antenna scattered energy can bounce off the radome wall one or more times before striking the antenna. At each reflection most of the energy is transmitted through the radome wall if the radome is well designed. Our model takes into account one and two reflections off the radome, ignoring rays that miss the antenna after 2 radome wall reflections.

Mathematics

The standard ray-tracing method is used to trace the rays through the radome wall to the antenna, including use of the same sample points. At the antenna the rays are multiplied by the illumination function inverse being evaluated at the sample point. The antenna is assumed to be flat, because of this the rays reflecting off the antenna will intersect the radome wall at the same place that the ray initially entered the radome (see Figure 3-3). At the reflection point the direction of the ray is changed and the magnitude and phase of the electric field are changed using the same techniques as the sidewall reflections described in Section 3.2. The ray is then tested to see if it will intersect the antenna, go through the base of the radome, or strike the radome wall in another location.

If the ray intersects the antenna the intersection point is found, the electric fields magnitude and phase are adjusted, and the electric field is summed to the monopulse model, all of these being done in the same manner as the sidewall reflections of Section 3.2. If the ray passes through the base of the radome without intersecting the antenna

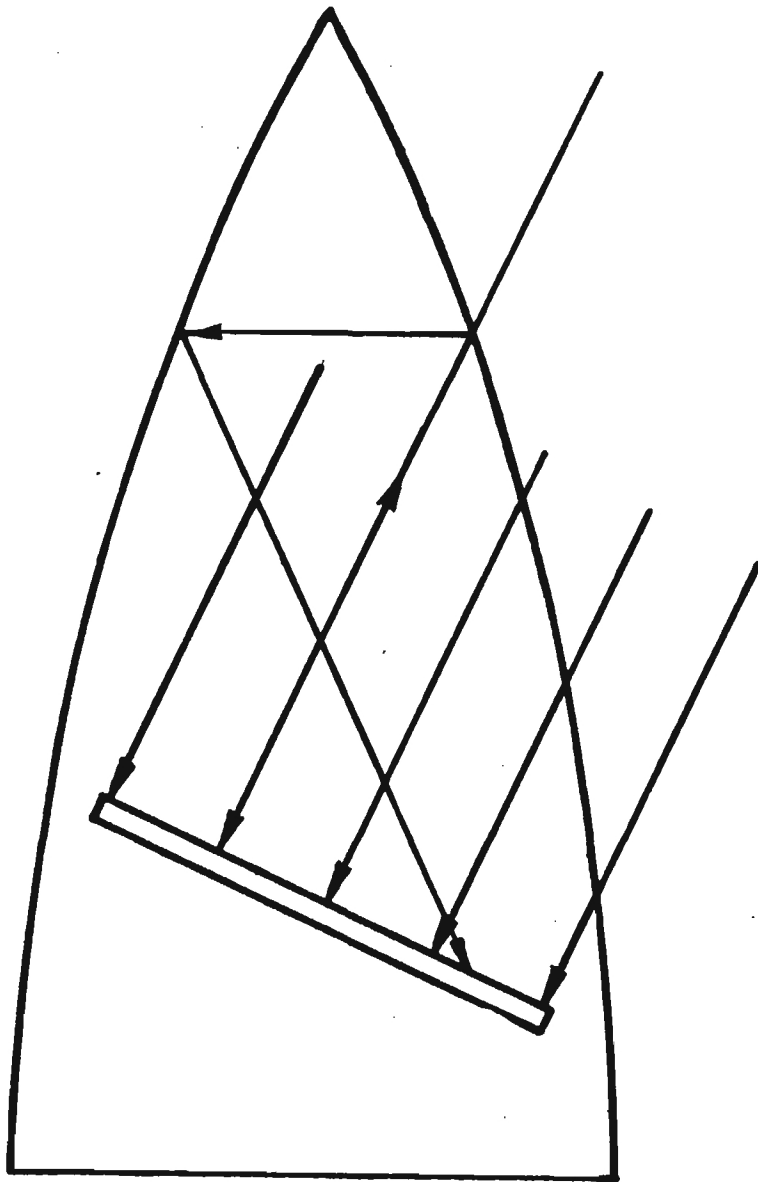


Figure 3-3. Antenna Scattered Energy Added to Backward Ray Trace.

then the ray is discarded. If the ray strikes the radome wall the intersection point is found using numerical techniques and the new direction of the ray and effects of reflection on the ray are calculated as above. The ray is then tested to see if it strikes the antenna. If it does strike the antenna it is summed, if not the ray is dropped.

4.0 COMPARISON OF THEORETICAL AND MEASURED RADOME DATA

4.1 Radome Description

The radome posed for theoretical analysis was a tangent ogive having the basic geometry defined in Figure 4-1; for this geometry the values of the various wall parameters are:

$$D = 13.46 \text{ in.}$$

$$L = 48.47 \text{ in.}$$

$$\Delta = 10.75 \text{ in.}$$

$$\delta = 0 \text{ in.}$$

$$d = 10.7 \text{ in.}$$

In addition, the monolithic wall was specified to have a dielectric constant of 5.0 and loss tangent of 0.005.

The wall thickness specified for the test radome was a sophisticated prescription summarized in Table 4-1. To model this in the program, a closed form expression for radome wall thickness was derived as:

$$\begin{aligned} \text{THK (inches)} &= (0.282 + 0.0064\theta^2) \\ &\cdot \text{COS} \left(\frac{(\text{DIST} - 34 - 4\theta)}{34 - 4\theta} \right) (0.5 + 0.108 \text{ abs} (\theta - 0.628)) \end{aligned} \quad (4-1)$$

Where

DIST = Station referenced to radome tip (inches)

θ = Radome circumferential angle from vertical
(radians)

The radome roll angle reference, antenna geometry and polarization are depicted in Figure 4-2. The theoretical radome thickness resulting from the use of equation (4-1) is tabulated in Table 4-2.

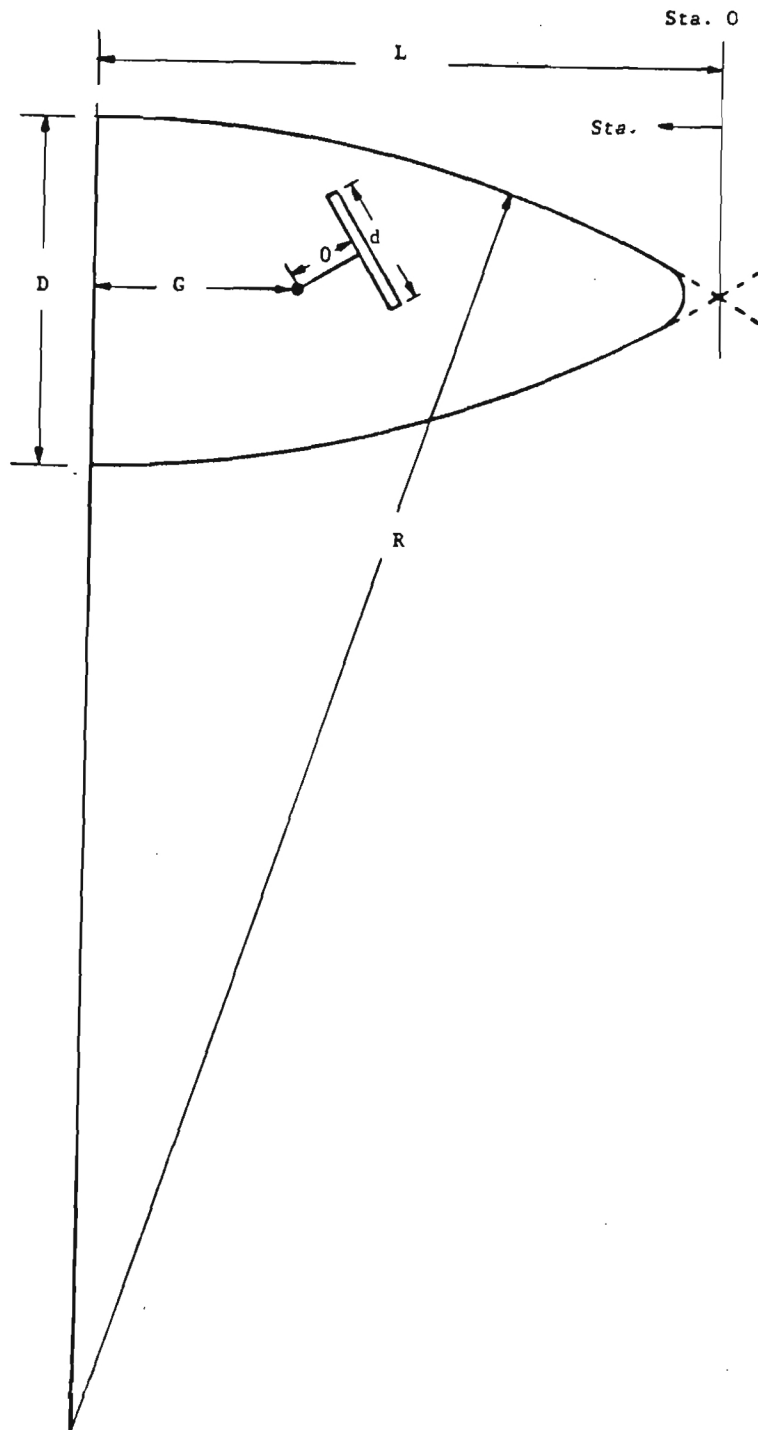


Figure 4-1. Geometry for Tangent Ogive Radome Performance Calculations.

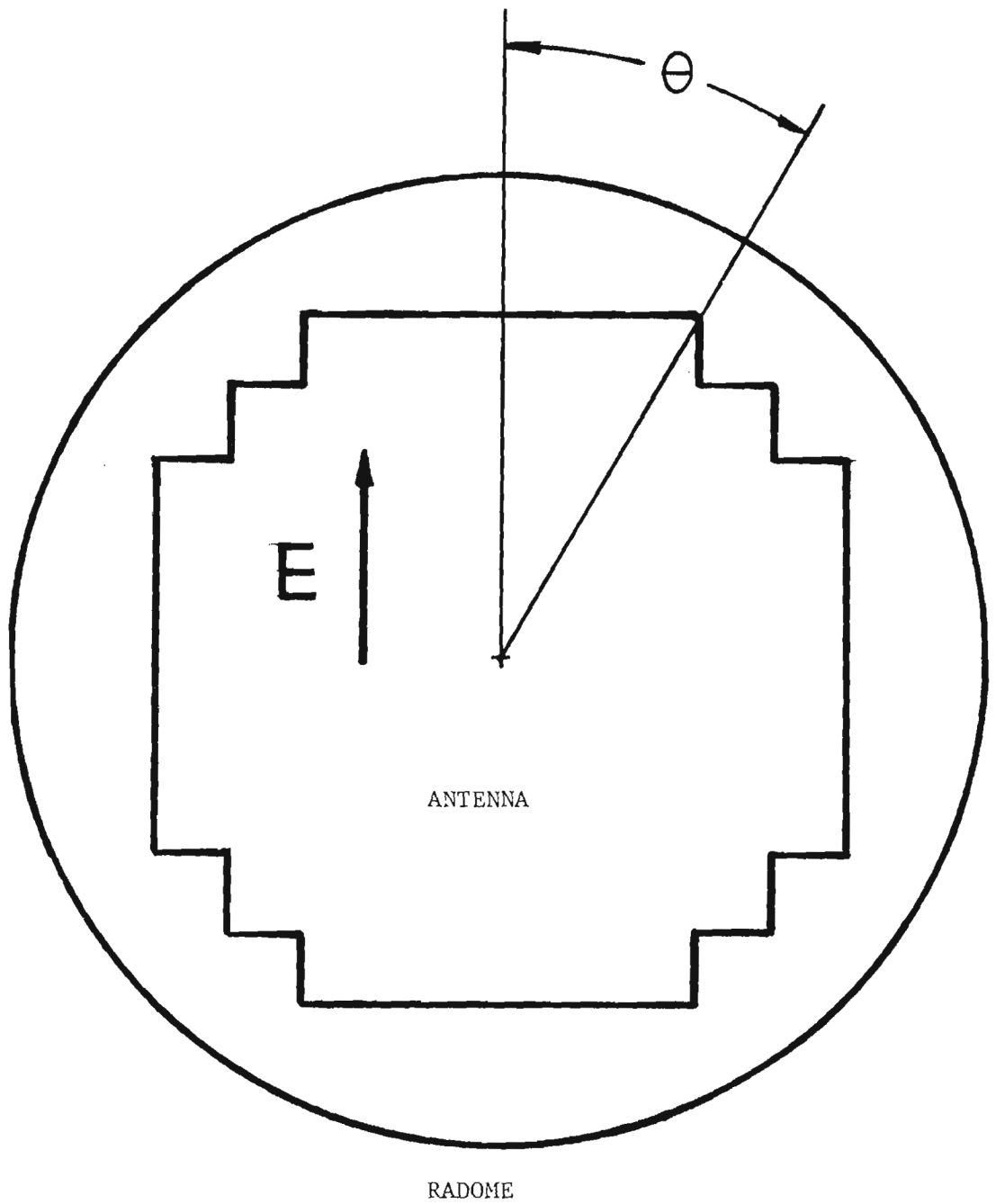


Figure 4-2. Antenna Orientation for Thickness Taper Prescription

TABLE 4-1
WALL THICKNESS VERSUS DISTANCE FROM BASE (DIST)

DIST	THETA				
	0	22.5	45	67.5	9
0	.238	.246	.250	.249	.248
2	.238	.246	.250	.249	.248
4	.247	.253	.273	.278	.279
6	.251	.259	.276	.281	.284
8	.255	.260	.277	.282	.285
10	.259	.270	.267	.282	.284
12	.258	.267	.259	.285	.287
14	.267	.259	.259	.262	.288
16	.271	.283	.261	.290	.276
18	.273	.273	.270	.291	.280
20	.274	.267	.280	.285	.290
22	.269	.269	.275	.291	.268
24	.265	.281	.263	.274	.274
26	.278	.271	.288	.266	.295
28	.283	.274	.281	.300	.297
30	.285	.281	.275	.281	.297
32	.281	.280	.278	.280	.291
34	.281	.276	.282	.281	.290
36	.281	.282	.279	.283	.283
38	.281	.281	.280	.284	.283
40	.281	.277	.280	.278	.279
42	.275	.275	.279	.276	.275
44	.272	.271	.279	.276	.275
46	.271	.267	.271	.284	.274

THETA (deg)

	0	22.5	45	67.5	90
Distance	0 0.2377466481	0.2448168637	0.2485762907	0.2465428489	0.2454710513
from	2 0.2426787305	0.2492868444	0.253171747	0.2522585635	0.252558735
Base	4 0.2473400924	0.2534950975	0.2574829753	0.2576061945	0.2591702031
(in)	6 0.251725534	0.2574372047	0.2615051354	0.2625779389	0.2652929893
	8 0.255830163	0.2611090271	0.2652337118	0.2671665422	0.2709155486
	10 0.2596494005	0.2645067098	0.2686645185	0.271365309	0.2760272794
	12 0.2631789859	0.2676266854	0.2717937039	0.2751681125	0.2806185431
	14 0.2664149819	0.2704656782	0.2746177548	0.2785694039	0.2846806827
	16 0.2693537785	0.2730207077	0.277133501	0.2815642203	0.2882060387
	18 0.2719920973	0.2752890911	0.2793381179	0.2841481917	0.2911879638
	20 0.2743269952	0.2772684471	0.2812291306	0.2863175477	0.2936208354
	22 0.2763558674	0.2789566974	0.282804416	0.2880691229	0.2955000663
	24 0.2780764507	0.2803520695	0.2840622057	0.2894003616	0.2968221129
	26 0.2794868257	0.2814530984	0.2850010875	0.2903093211	0.2975844825
	28 0.2805854189	0.2822586281	0.2856200074	0.2907946752	0.2977857375
	30 0.2813710049	0.282767813	0.2859182706	0.2908557157	0.2974254986
	32 0.2818427074	0.2829801183	0.2858955422	0.2904923535	0.2965044449
	34 0.282	0.2828953213	0.2855518477	0.2897051188	0.2950243132
	36 0.2818427074	0.2825135109	0.284887573	0.2884951604	0.2929878943
	38 0.2813710049	0.281835088	0.2839034638	0.2868642437	0.2903990281
	40 0.2805854189	0.2808607649	0.282600625	0.2848147485	0.287262596
	42 0.2794868257	0.2795915646	0.2809805192	0.2823496653	0.2835845119
	44 0.2780764507	0.2780288195	0.2790449653	0.279472591	0.2793717111
	46 0.2763558674	0.2761741704	0.2767961362	0.2761877238	0.2746321372

Table 4-2. Theoretical Wall Thickness Model.

4.2 Comparison of Measured and Predicted Data

The boresight errors for the azimuth and elevation principal plane scans are shown in Figures 4-3 and 4-4. Here the same general features are found in both the measured and the calculated curves; for both plus and minus azimuth values the boresight error curve approximates a sine curve for one half of a cycle. The azimuth BSE, curves differ by about 3 milliradians between the calculated and measured while the calculated and measured curves for the elevation scans agree within 2 milliradians over most of the scan.

Similar data, but with added -20 dB rear bulkhead reflections, appear in Figures 4-5 and 4-6. The affects of -15 dB rear bulkhead reflections appear in Figures 4-7 and 4-8.

These bulkhead reflection components noticeably made the computed curves approach the measured curve, especially in the elevation scan. The affects of antenna backwall reflections were then investigated in Figures 4-9 and 4-10. The computed curves come more closely to the measured curves, but the affect due to sidewall reflections in this case was at most .3 milliradians.

The errors including antenna scattered energy were also calculated but not plotted here since the maximum affect of the energy was about 0.1 milliradians.

BSE
AZIMUTH
(Milliradians)

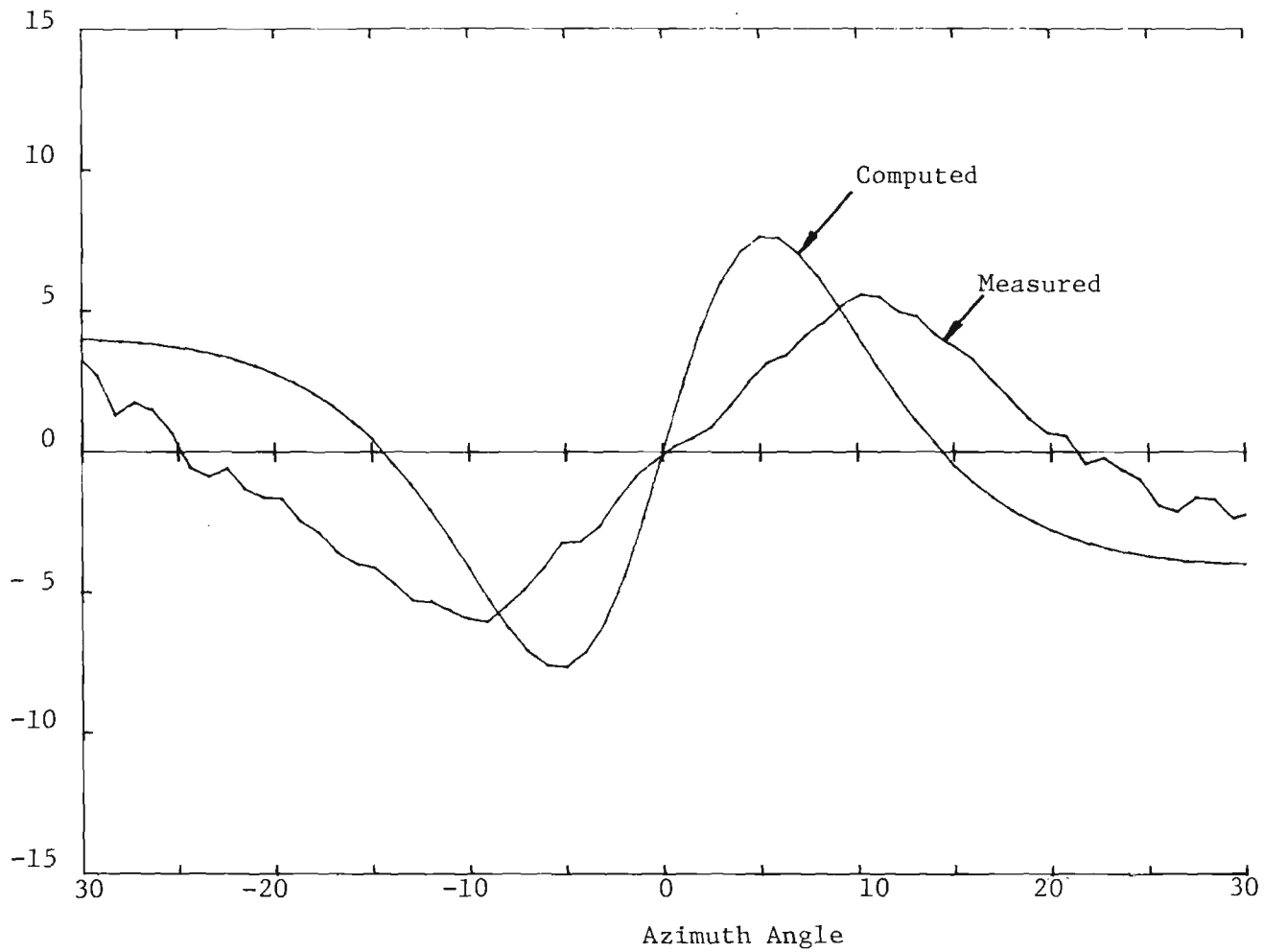
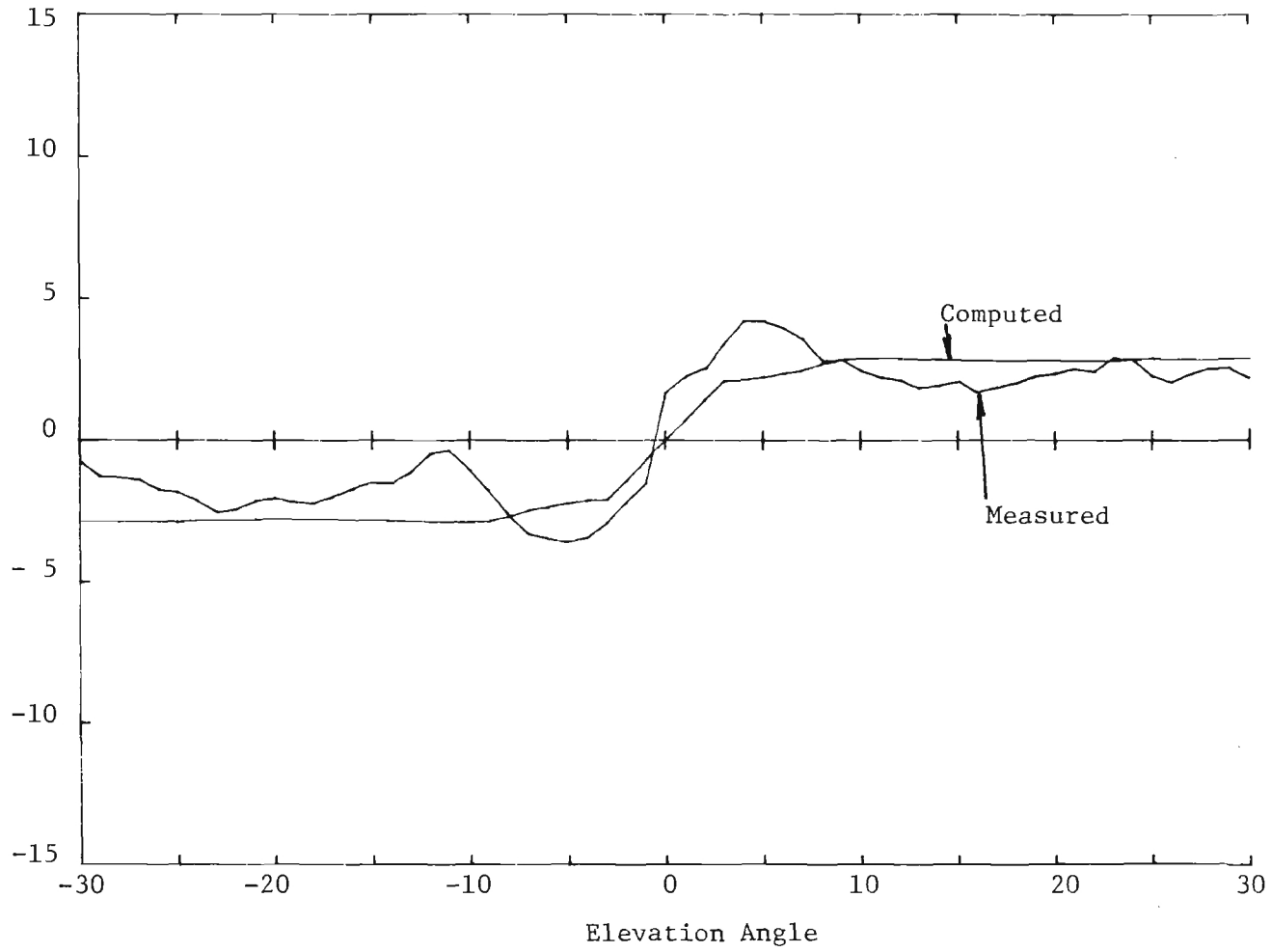


Figure 4-3. Azimuth Scan BSE (no Added Reflections)

BSE
ELEVATION
(Milliradians)



24

Figure 4-4. Elevation Scan BSE (no Added Reflections)

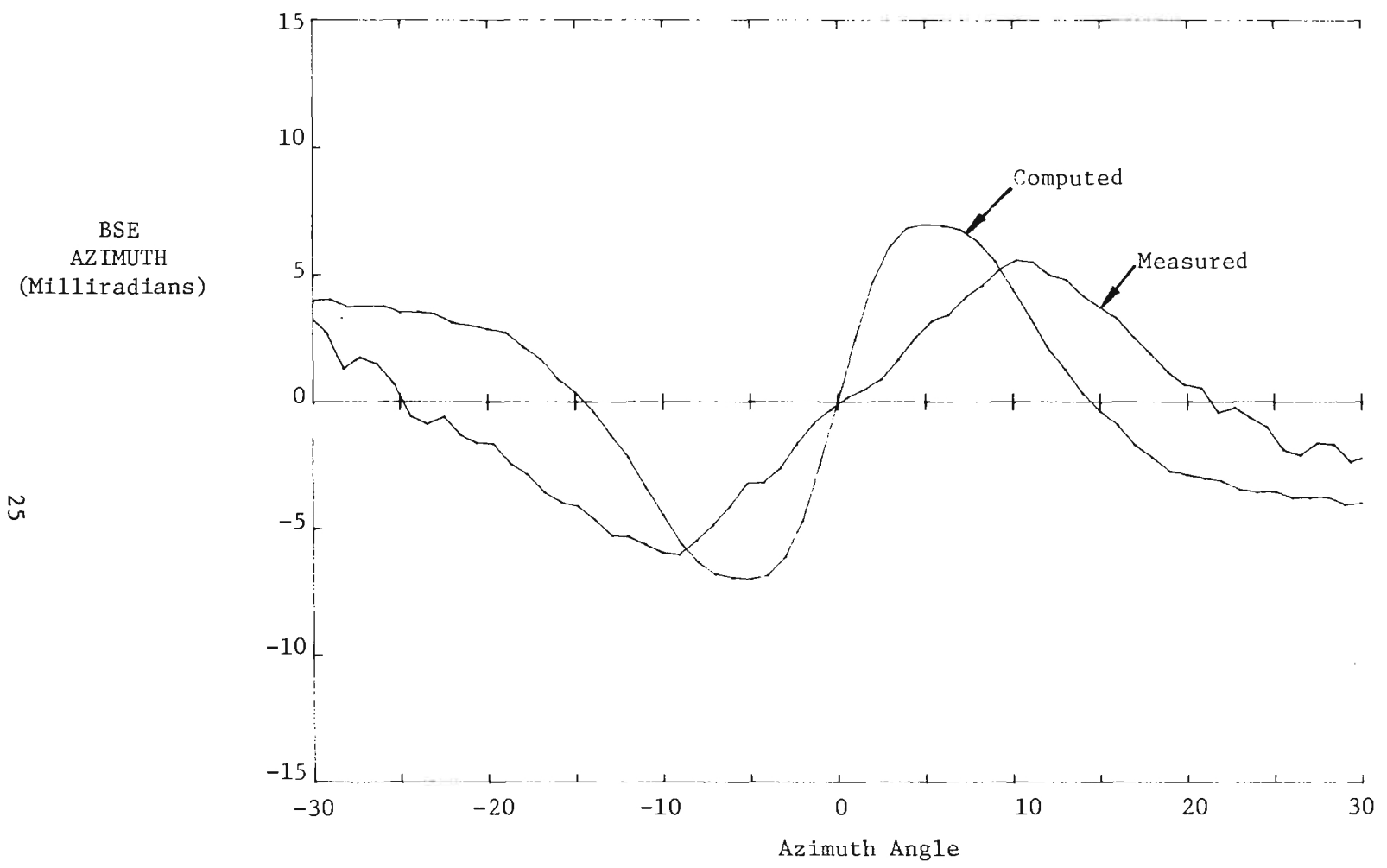


Figure 4-5. Azimuth Scan BSE with Bulkhead Reflection of -20 dB

BSE
ELEVATION
(Milliradians)

26

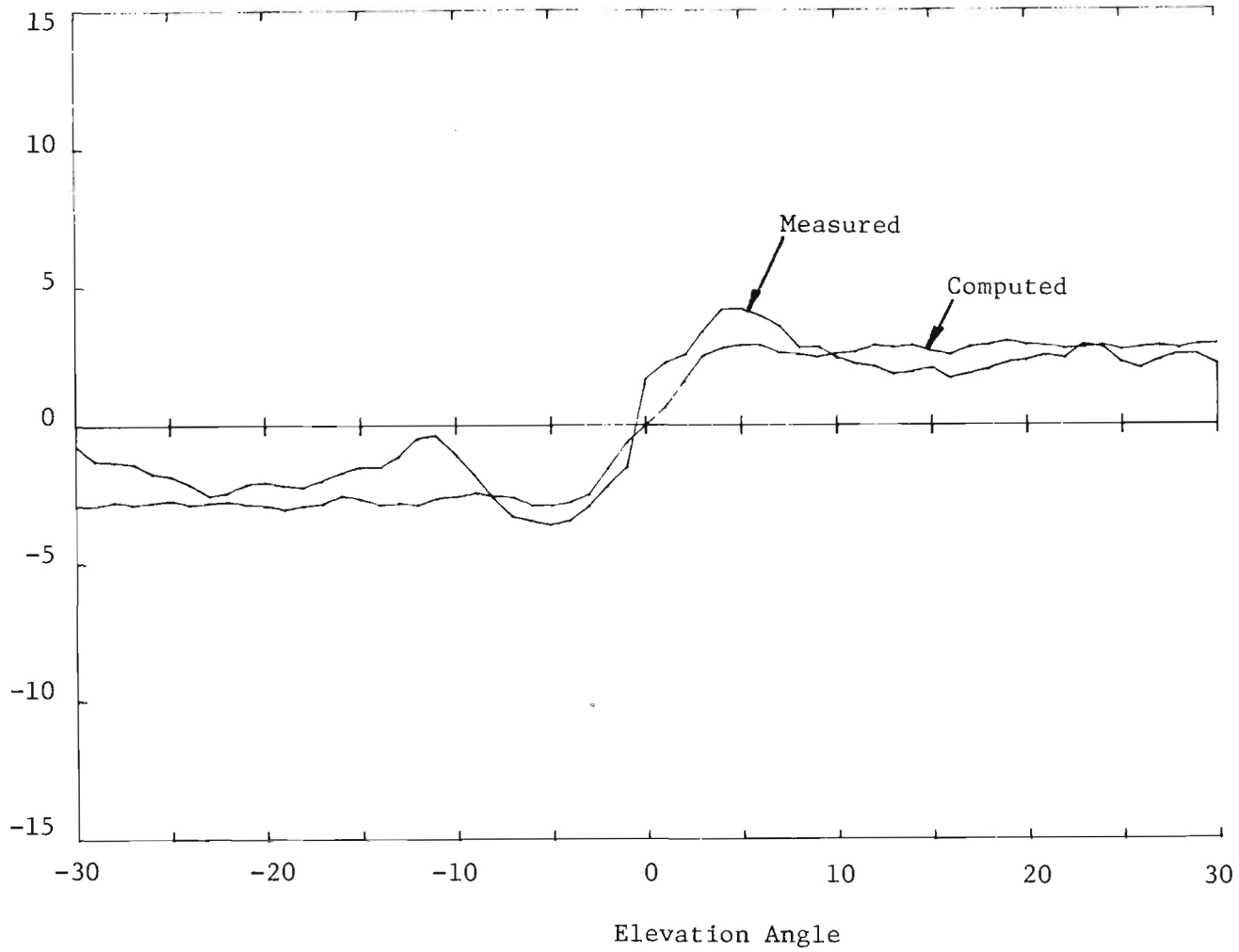


Figure 4-6. Elevation Scan BSE with Bulkhead Reflection of -20 dB

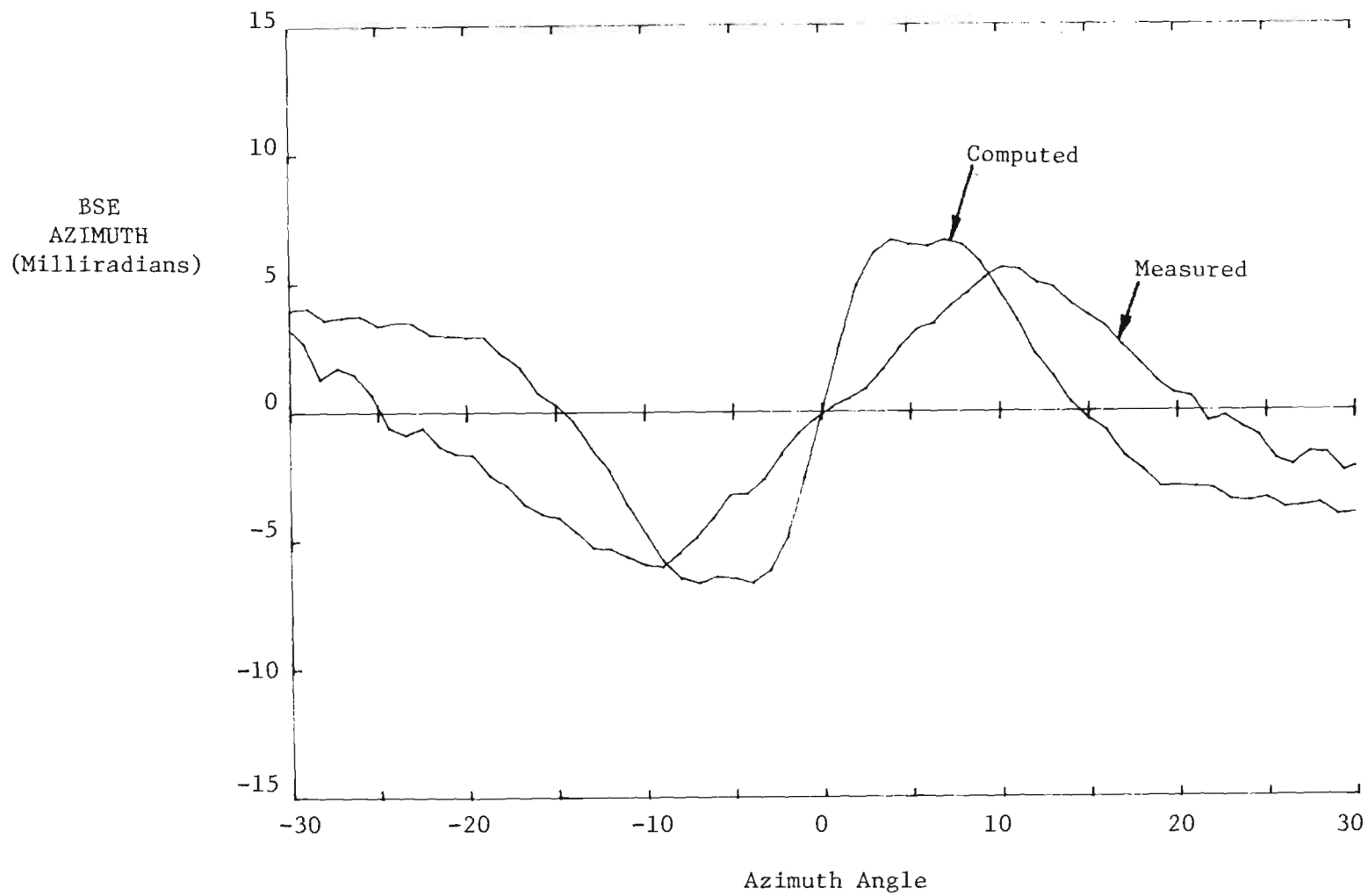


Figure 4-7. Azimuth Scan BSE with Bulkhead Reflection of -15 dB

BSE
ELEVATION
(Milliradians)

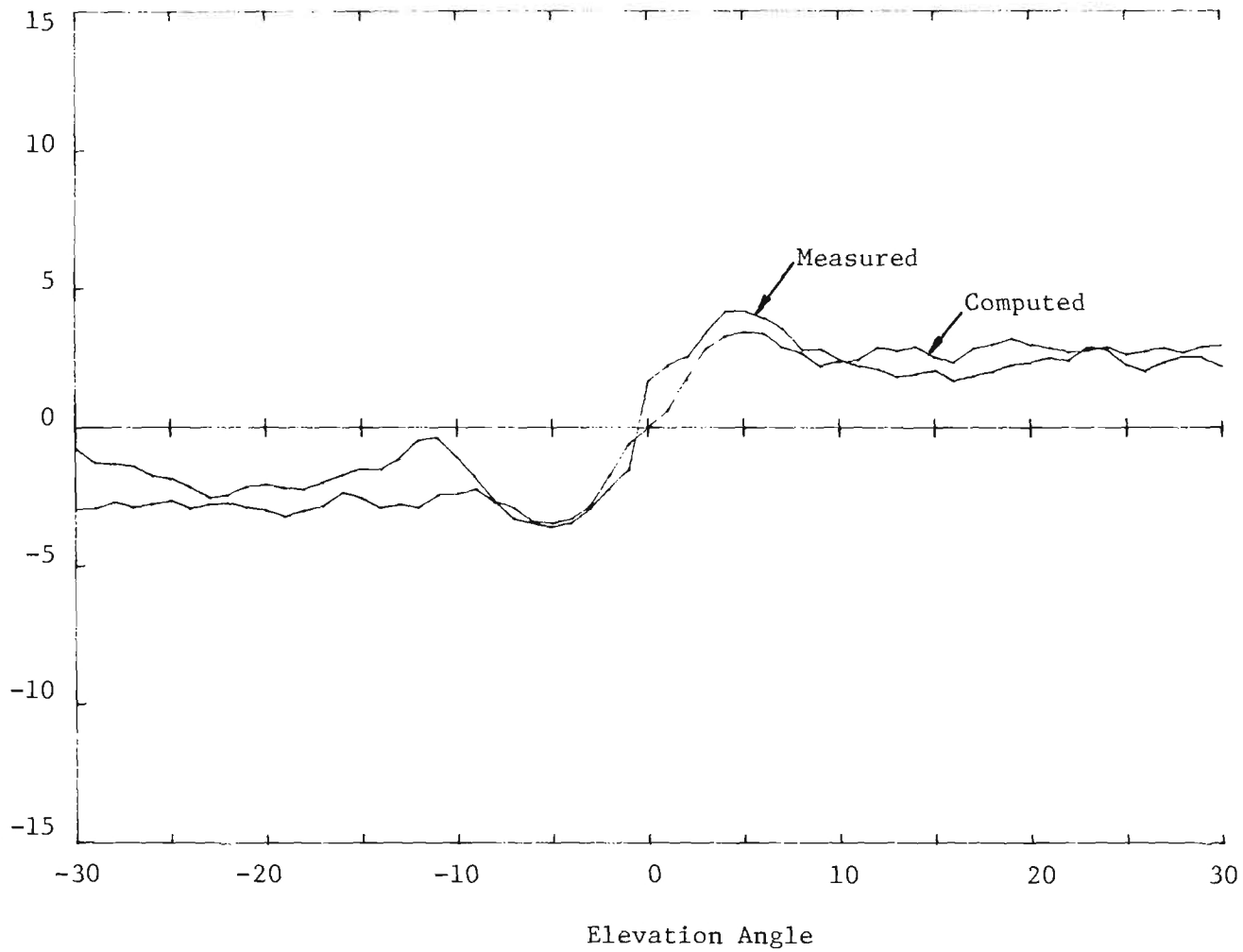


Figure 4-8. Elevation Scan BSE with Bulkhead Reflection of -15 dB

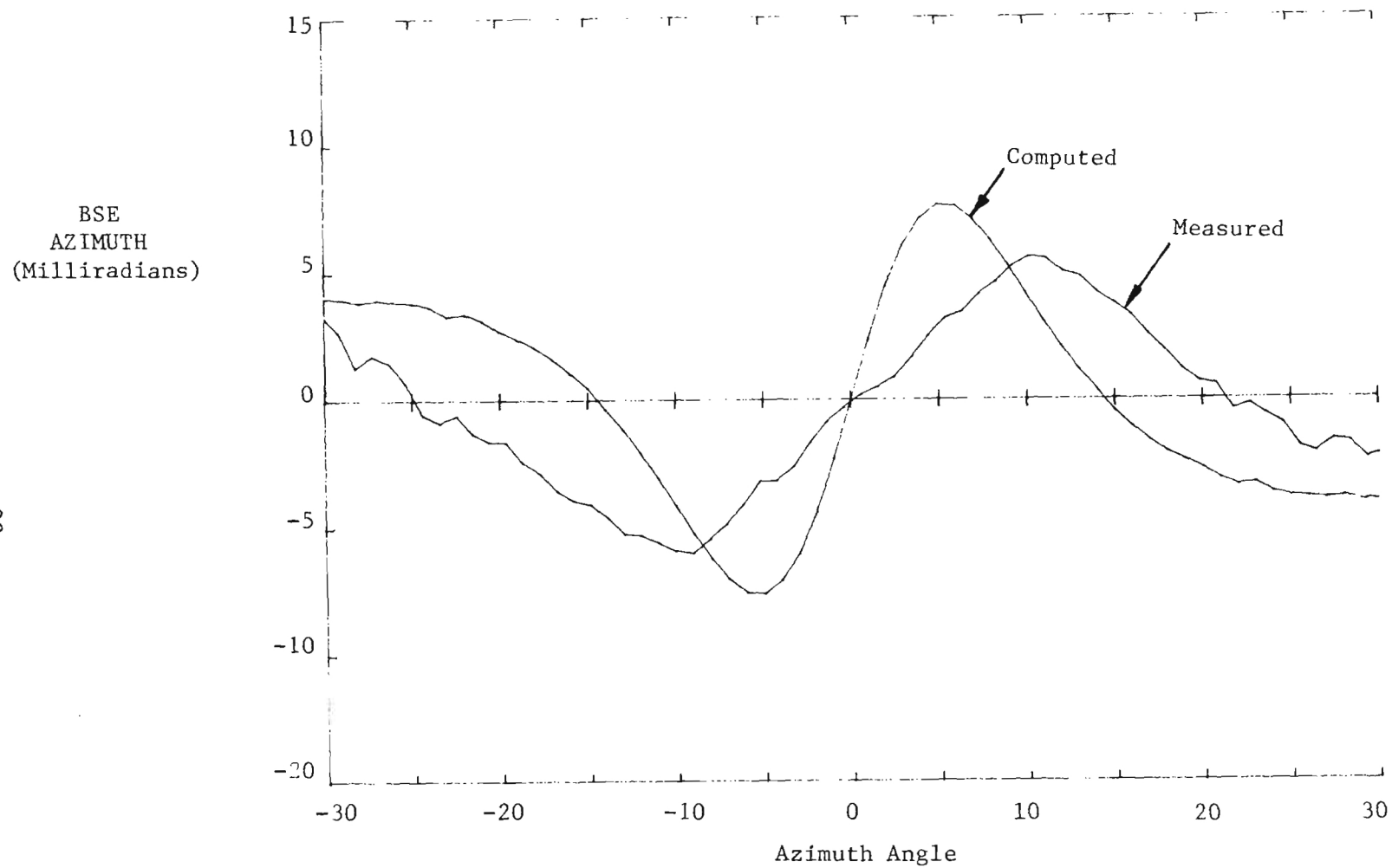


Figure 4-9. Azimuth Scan BSE with Sidewall Reflection Included

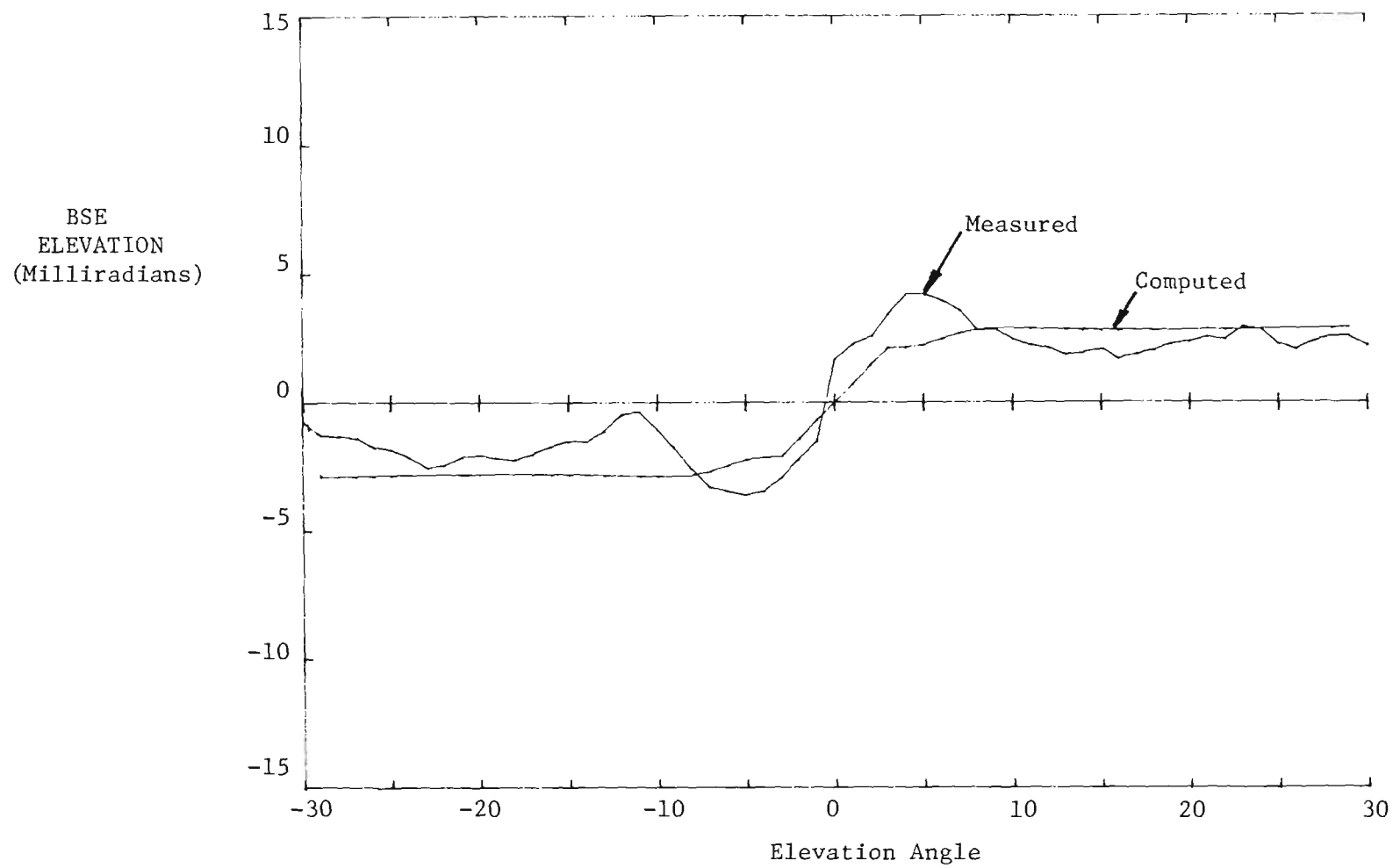


Figure 4-10. Elevation Scan BSE with Sidewall Reflection Included

5.0 Evaluation of Wall Transmission Model

5.1 General

An evaluation of possible sources of computational errors suggested that the wall transmission subroutine used in the computer analysis be considered. Here, the theoretical approach is a matrix solution developed by Collin [1] who formulated expressions based on reflection and transmission components at each boundary of a multilayer dielectric media.

In the domain of geometrical optics, radome walls are approximated as being locally flat and infinite in extent. The method is based on this approximation, and is not valid for thick, highly curved walls. An incident electromagnetic wave is decomposed into components with the electric field vector perpendicular and parallel to the plane of incidence, that plane being defined as containing both the local normal to the surface and the direction vector of propagation. Arbitrary incidence angles, electrical properties, layer thickness, and number of layers can be handled. The method assumes that the dielectric and magnetic properties must be homogeneous and isotropic within individual layers.

A modification can be made for anisotropic materials [4], but added computer time cannot be justified when the anisotropy is not large. Multiple reflections between the layer boundaries are analyzed by complex matrix multiplications, one matrix being needed for each boundary. The mathematics for setting up the matrices is based on solving a boundary value problem at each interface.

Applicable Equations

In terms of the layer geometry, and following the development in references [1] and [2], the applicable matrix solution is:

$$\begin{bmatrix} c_1 \\ \vdots \\ b_1 \end{bmatrix} = \prod_{i=1}^N \left\{ \frac{1}{T_i} \begin{vmatrix} e^{j\phi_i d_i} & R_i e^{-j\phi_i d_i} \\ R_i e^{j\phi_i d_i} & e^{-j\phi_i d_i} \end{vmatrix} \right\} \cdot \left\{ \frac{1}{T_{N+1}} \begin{vmatrix} 1 & R_{N+1} \\ R_{N+1} & 1 \end{vmatrix} \right\} \begin{bmatrix} c_{N+1} \\ \vdots \\ b_{N+1} \end{bmatrix} \quad (1)$$

where $N =$ number of layers,

$d_i =$ i th layer thickness (inches) $T_i = 1 - R_i$, and

$$\phi_i = k_o (\epsilon_i' - \sin^2 \theta)^{1/2} \quad (2)$$

$$R_i = \frac{Z_i - Z_{i-1}}{Z_i + Z_{i-1}} \quad (3)$$

Note: $Z_0 = Z_{N+1} = 1$

$$Z_i = \frac{\cos \theta}{(\epsilon_i - \sin^2 \theta)^{1/2}} \quad (\text{TE or perpendicular polarization}) \quad (4)$$

$$= \frac{(\epsilon_i - \sin^2 \theta)^{1/2}}{\epsilon_i \cos \theta} \quad (\text{TM or parallel polarization}) \quad (5)$$

Further refinement is made via equation (2) modification:

$$\phi_i' = k_o \sqrt{\epsilon_i - \sin^2 \theta} \quad (6)$$

$$\begin{aligned} \phi_i' &= k_o \sqrt{(\epsilon_i' - \sin^2 \theta) - j(\epsilon_i'')} = \\ &= k_o \sqrt{\epsilon_i' - \sin^2 \theta} \left\{ 1 - \frac{j\epsilon_i' \tan \delta_i}{(\epsilon_i' - \sin^2 \theta)} \right\}^{1/2} \quad (7) \end{aligned}$$

$$\phi_i' \approx k_o \sqrt{\epsilon_i' - \sin^2 \theta} \left\{ 1 - \frac{j\epsilon_i' \tan \delta_i}{2(\epsilon_i' - \sin^2 \theta)} \right\} \quad (8)$$

Where in terms of the relative dielectric constant and loss tangent:

$$\epsilon_i = \epsilon_i' - j\epsilon_i'' = \epsilon_i' (1 - j \tan \delta_i) \quad (9)$$

$$\text{Defining, } \alpha_i = \frac{k_o \epsilon_i' \tan \delta_i}{2(\epsilon_i' - \sin^2 \theta)^{1/2}} = \frac{K_o^2 \tan \delta_i}{2\phi_i} \quad (10)$$

Then (7) becomes,

$$\begin{aligned} \phi_i' &= k_o \sqrt{\epsilon_i' - \sin^2 \theta} - j\alpha_i \\ &= \phi_i - j\alpha_i \end{aligned} \quad (11)$$

From which (1) can be expressed:

$$\begin{bmatrix} c_1 \\ b_1 \end{bmatrix} = \frac{1}{T_1 T_2 T_3 T_4 T_5} \prod_{i=1}^4 \begin{vmatrix} e^{-j\phi_i' d_i + \alpha_i d_i} & R_i e^{+j\phi_i' d_i - \alpha_i d_i} \\ R_i e^{-j\phi_i' d_i + \alpha_i d_i} & e^{+j\phi_i' d_i - \alpha_i d_i} \end{vmatrix} \quad (12)$$

$$\begin{vmatrix} 1 & R_5 \\ R_5 & 1 \end{vmatrix} \begin{bmatrix} c_6 \\ b_6 \end{bmatrix}$$

or,

$$\begin{bmatrix} c_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} c_{N+2} \\ b_{N+2} \end{bmatrix} \quad (13)$$

The voltage transmission coefficient is

$$|T| = 20 \log (c_6/c_1) = -20 \log |A_{11}| \quad (14)$$

The voltage reflection coefficient is

$$|R| = 20 \log (b_1/c_1) = 20 \log |A_{21}/A_{11}| \quad (15)$$

The insertion phase is defined as the difference in electrical thickness between the panel and that of free space over the same thickness as the panel [3].

C_1 is incident on the wall and C_{N+2} is transmitted then

$$C_{INC} = A_{11} C_{TRAN} \quad (16)$$

$$C_{TRAN} = \frac{C_{INC}}{A_{11}} \quad (17)$$

$$\text{angle } (C_{TRAN}) = \text{angle } (C_{INC}) - \text{angle } (A_{11}) \quad (18)$$

$$\text{IPD} = -\text{angle } (A_{11}) - \frac{360 d(\text{total})}{\lambda} \cos \theta \quad (\text{degrees}) \quad (19)$$

A phase delay also is incurred on the reflecting ray, b_1 .

$$\begin{aligned}
B_{\text{refl}} &= A_{21} C_{\text{TRAN}} \\
&= \frac{A_{21}}{A_{11}} C_{\text{INC}}
\end{aligned}
\tag{20}$$

$$\text{angle } (B_{\text{refl}}) = \text{angle } (C_{\text{INC}}) + \text{Angle } (A_{21}) - \text{Angle } (A_{11})
\tag{21}$$

Limitations

This method computes the transmission coefficient and insertion phase delay for a plane wave incident at angle θ on a N-layer dielectric sheet with free space on either side; relative permeability of all layers is assumed unity.

There are no restrictions on the range of any of the variables except loss tangent. Here, the approximation made is seen in equation (7). This approximation generally restricts accuracy for loss tangents greater than about 0.10.

5.2 Comparison of Measured and Predicted Data

To assess program accuracy, a sheet of 0.375-inch plexiglass (polymethyl methacrylate) was evaluated. The dielectric properties at X-band published in the technical literature indicate a value of $\epsilon = 2.59$ and $\tan \delta = 0.0067$. These values were input to the wall program to compute the parallel and perpendicular transmission coefficient and insertion phase delay (IPD).

Utilizing the measurements facility at MICOM in conjunction with the Georgia Tech radome measurements instrumentation system, actual data were obtained on a sheet approximately four-feet square. The range of incidence angles measured was limited by the radome positioner to angles less than 60 degrees.

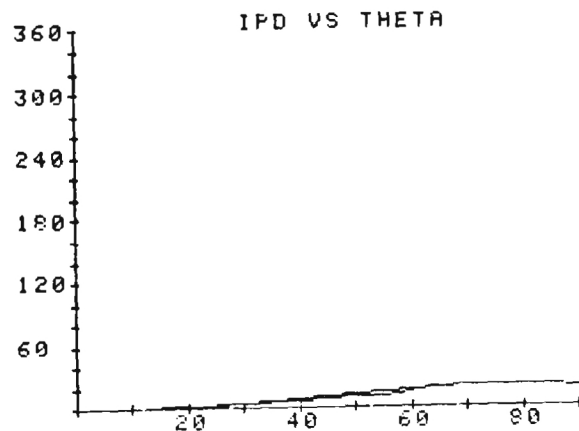
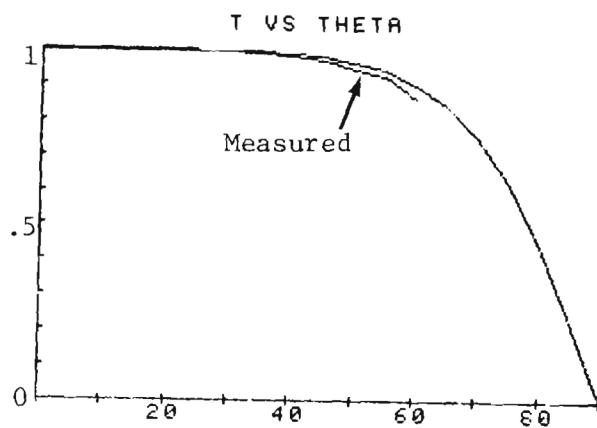
The measured data did not include any reference measurements for the free space or "no sheet" case. The measured transmission was set to 1 and the measured IPD was set to 0 for an incidence angle of 0 degrees. The computed IPD was also set to 0 for 0° incidence angle for comparison

with the measured. (These offsets are not important for radome analysis since they effect all rays entering the radome equally.)

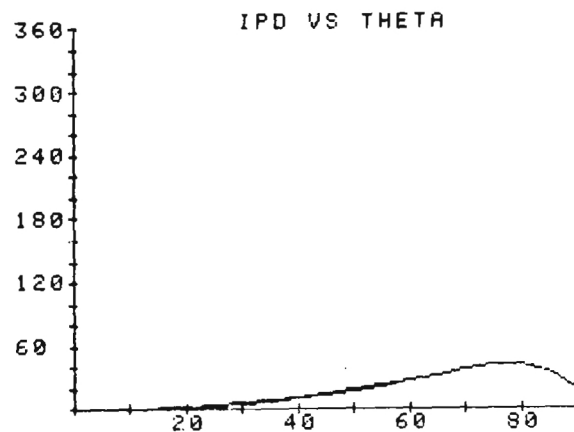
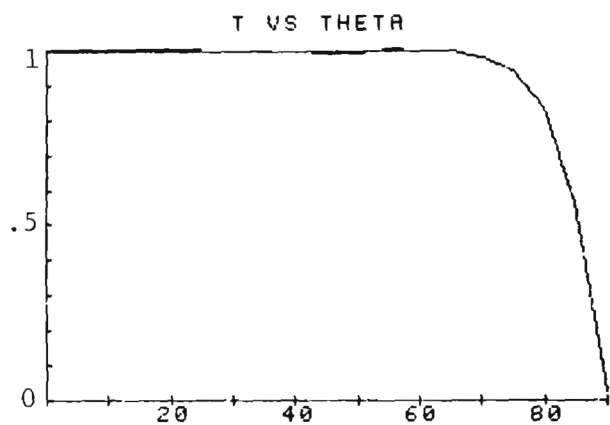
The measured and computed values for the transmission and IPD are shown in Figures 5-1 and 5-2. The parallel polarization case had the most difference between measured and computed values for both transmission and IPD coefficients. At 60° the measured IPD differs from the computed by approximately 2 degrees (or 15%) and the measured transmission differs from the computed by approximately -0.05 (or 6%).

The measured data was actually an average of three trials. The IPD and transmission coefficients did vary from trial to trial, but the average would seem to have a low standard deviation as evidenced by the smoothness of the measured curves. The computed curves depend on the thickness and electrical properties of the plexiglass, both of which may have moderate tolerances in the commercial sheet used.

The deviation between measured and computed coefficients is large enough to effect radome boresight errors, particularly for large angles of incident (high fineness ratio radomes). It is the opinion of the authors that some of the discrepancy between measured and computed data is due to measurement errors. While it is difficult to quantify the measurement error component magnitude, it does suggest that if computed radome data is to be compared with actual radome data measured in the same facility, one should factor the measurement data into the data evaluation. Secondly, the measured data herein suggests a theoretical discrepancy of the mathematical model for large angles-of-incidence which can only be resolved via a more exhaustive perfection of the WALL transmission formulation.

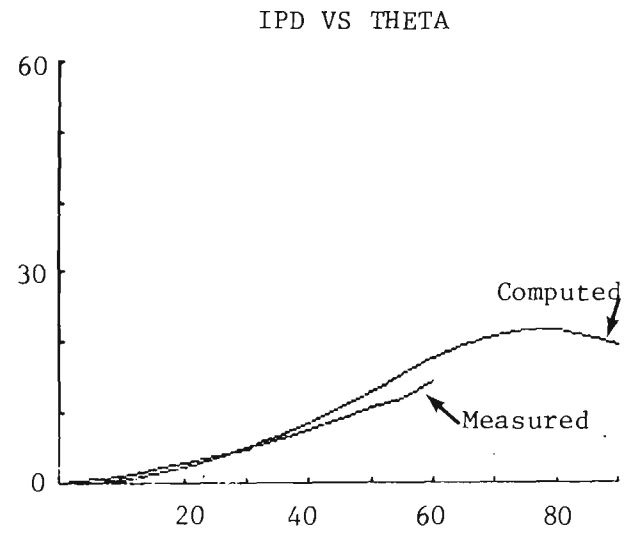
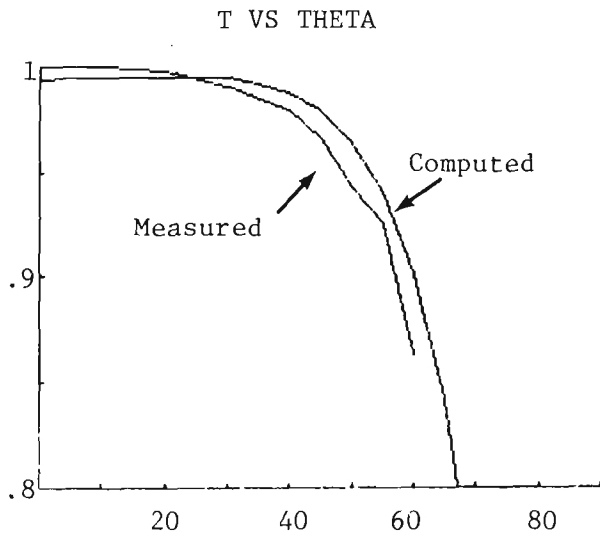


(a) Parallel Polarization

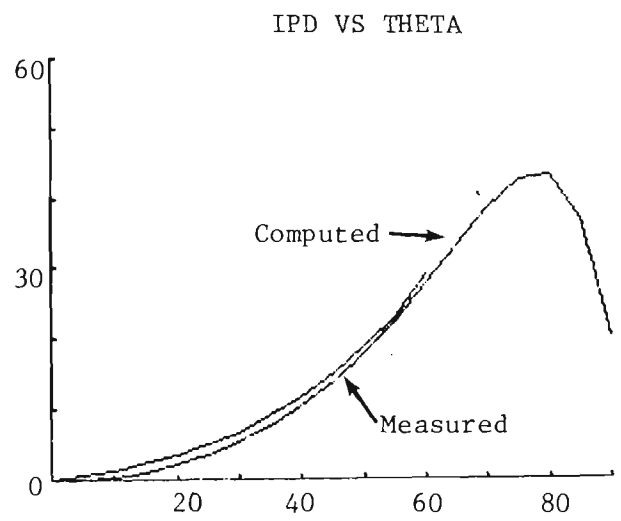
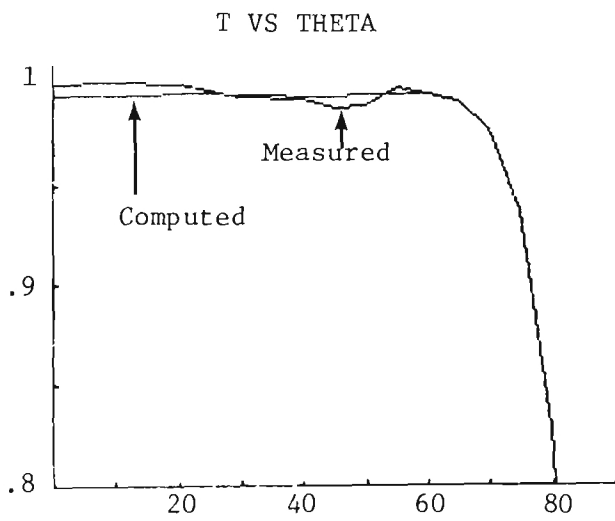


(b) Perpendicular Polarization

Figure 5-1 Comparison of Measured to Calculated Transmission and IPD Values



(a) Parallel Polarization



(b) Perpendicular Polarization

Figure 5-2 Close Up of Comparison of Measured to Calculated Transmission and IPD Values

6.0 REFERENCES

1. R. E. Collin, "Field Theory of Guided Waves," McGraw-Hill Book Company, New York, 1960.
2. C. H. Krueger, Jr., "A Computer Program for Determining the Reflection and Transmission Properties of Plane Impedance Boundaries," Report AFATL-TR-67-191, Research and Technology Division, Wright-Patterson AFB, Ohio, September, 1967.
3. H. Jasik, "Antenna Engineering Handbook," McGraw Hill Book Company, New York, 1961.
4. D. J. White and D. J. Banks, "Plane Wave Transmission and Reflection for Anisotropic Sheets of Radome Materials," Proceedings of the Sixteenth Symposium on Electromagnetic Windows, Atlanta, Georgia, June 9-11, 1982.
5. D. J. Kozakoff, G. K. Huddleston and M. West, "Missile Radome Performance Assessment", Final Report on Project A-2939, prepared for U. S. Army Missile Command under Contract DAAH01-81-D-A003, Delivery Order 0015, Georgia Tech, Atlanta, Georgia, December 1981.

APPENDIX A
PROGRAM LISTING

```

1      PROGRAM RAD(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE7)
2      REAL L,LTAN(5),IPD0,IPD1,THK(5),ER(5)
3      COMMON/A1/NLAY,ER,LTAN
4      COMMON/A2/APAZ,APEL,ILLUM,FREQ,THETA,IPD0,IPD1,RPD0,RPD1
5      COMMON/AA2/TRANO,TRAN1,BKDB,CPHI,SPHI,THK(5)
6      COMMON/A3/NRCT1,NARA1,NBLK1,NSAR,USUM,VDAZ,VEL
7      COMMON/A4/AZS,ELS,AZ,EL,PI,CONV,RSQ,B,THAZ,THEL,EEL,EAZ,L,TIPL
8      COMMON/A5/NF,NFIR,D2,ARAD,ADIS,NREF,NARA,NRCT,NANT,NBLK,DBLK
9      COMMON/AA5/PY,PZ,DEL
10     PI=3.1415927
11     CONV=PI/180.
12     AZS=0.
13     ELS=0.
14     NANT=0
15     PRINT*,"RAY(1) OR ANTENNA PATTERN(2)"
16     READ(5,*)NF
17     PRINT*,"ENTER:LENGTH AND DIAMETER OF RADOME(IN.)"
18     READ(5,*)L,D
19     D2=D/2.
20     B=(L*L-D2*D2)/D
21     R=B+D2
22     RSQ=R*R
23     PRINT*,"ENTER ANTENNA DIAMETER"
24     READ(5,*)ADIA
25     ARAD=ADIA/2.
26     PRINT*,"ENTER:APERTURE ILLUMINATION FUNCTION"
27     PRINT*,"      1=UNIFORM ILLUMINATION  "
28     PRINT*,"      2=COSINE ILLUMINATION  "
29     PRINT*,"      3=TABLE  "
30     READ(5,*)ILLUM
31     PRINT*,"ANTENNA NPOL.  AZ,EL"
32     READ(5,*)APAZ,APEL
33     PRINT*,"ENTER:RADOME TIP DIAMETER"
34     READ(5,*)TIPD
35     TIPL=L-SQRT(RSQ-(B+TIPD/2.)*(B+TIPD/2.))
36     PRINT*,"ENTER # OF PTS PER SIDE (EVEN)"
37     READ(5,*)NSAR
38     IF(ILLUM.EQ.1) GO TO 7B
39     PRINT*,"INCLUDE A-R-A REFL. Y(1) N(0)?"
40     READ(5,*)NARA
41     NARA1=NARA
42 7B   PRINT*,"INCLUDE WALL REFL. Y(1) N(0)?"
43     READ(5,*)NRCT
44     NRCT1=NRCT
45     PRINT*,"INCLUDE BLKHEAD REFL. Y(1) N(0)?"
46     READ(5,*)NBLK
47     NBLK1=NBLK
48     IF(NBLK.EQ.0) GO TO 80
49     PRINT*,"DB'S DOWN FOR BLKHEAD"
50     READ(5,*)BKDB
51     BKDB=10.**(BKDB/20.)
52 80   PRINT*,"DISTANCE TO ANTENNA"
53     READ(5,*)DEL
54     PRINT*,"EAZ,EEL"
55     READ(5,*)EAZ,EEL
56     PRINT*,"THAZ,THEL"
57     READ(5,*)THAZ,THEL
58     THAZ=THAZ*CONV
59     THEL=THEL*CONV

```

```

60      PRINT*, "ENTER FREQ (GHZ)"
61      READ(5,*)FREQ
62      NLAY=1
63      PRINT*, "ENTER: DIELECTRIC CONST., LOSS TANGENT"
64      READ(5,*)ER(1),LTAN(1)
65      IF(NF.EQ.2)GO TO 109
66      NREF=1
67      AZS=1.
68      ELS=0.
69      CALL ARRAY
70      VAZ=VDAZ/VSUM
71      AZS=0.
72      ELS=1.
73      CALL ARRAY
74      VEL=VDEL/VSUM
75      NREF=0
76      ELS=0.
77 C      EL SCANNER
78      AZ=0.
79      DO 24 I=1,30
80      EL=1.*I
81      PRINT*, "EL", EL
82      CALL ARRAY
83      VT=17.45*(VDEL/VSUM)/VEL
84      PRINT*, "BSE", VT
85 24     CONTINUE
86      PRINT*, " "
87      EL=0.
88      DO 25 I=1,30
89      AZ=1.*I
90      PRINT*, "AZ", AZ
91      CALL ARRAY
92      VT=17.45*(VDAZ/VSUM)/VAZ
93      PRINT*, "BSE", VT
94 25     CONTINUE
95      STOP
96 C      ANTENNA PATTERNS
97 109    PRINT*, "ENTER DIR LOOKED AZ,EL"
98      READ(5,*)AZ,EL
99      PRINT*, "ENTER LOWER LIMIT,UPPER LIMIT,INC"
100     PRINT*, "FOR AZ SCAN"
101     READ(5,*)LLAZ,LUAZ,IAZ
102     PRINT*, "FOR EL SCAN"
103     READ(5,*)LLEL,LUEL,IEL
104     NREF=0
105     CALL ARRAY
106     RSUM=VSUM
107     NANT=1
108     I1=LUAZ+IAZ-LLAZ
109     I2=LUEL+IEL-LLEL
110     DO 106 I=1,I1,IAZ
111     AZS=(I-1+LLAZ)*1.
112     DO 106 IE=1,I2,IEL
113     ELS=(IE-1+LLEL)*1.
114     CALL ARRAY
115     H3=20.*ALOG10(VSUM/RSUM)
116     H1=AZ+AZS
117     H2=EL+ELS
118 106   PRINT*, H1, H2, H3
119     STOP

```

```

120      END
121      SUBROUTINE ARRAY
122      COMMON/A2/APAZ, APEL, ILLUM, FREQ, THETA, IPD0, IPD1, RPD0, RPD1
123      COMMON/AA2/TRANO, TRAN1, BKDB, CPHI, SPHI, THK(5)
124      COMMON/A3/NRCT1, NARA1, NBLK1, NSAR, VSUM, VDAZ, VIDEL
125      COMMON/A4/AZS, ELS, AZ, EL, PI, CONV, RSQ, B, THAZ, THEL, EEL, EAZ, L, TIPL
126      COMMON/A5/NF, NFIR, D2, ARAD, ADIS, NREF, NARA, NRCT, NANT, NBLK, DBLK
127      COMMON/AAS/PY, PZ, DEL
128      COMMON/A6/VRSUM, VISUM, VRDEL, VIDEL, VRDAZ, VIDAZ, BKPS
129      COMMON/C1/POS(3), K(3), CAN, SAN, RM, PSN(3), S, C(3), NSKP
130      COMMON/C2/CAZ, SAZ, CEL, SEL
131      COMMON/AR/RAZ, IAZ, REL, IEL, E91, P91, E92, P92, N(3), P7
132      COMMON/F1/T, COSF
133      REAL IAZ, IEL, I1, I2, N(3), K, L
134      NRCT=0
135      VRSUM=0.
136      VISUM=0.
137      VRDEL=0.
138      VIDEL=0.
139      VRDAZ=0.
140      VIDAZ=0.
141      NFIR=1
142      DO 26 N1=1, NSAR
143      DO 26 N2=1, NSAR
144      NARA=0
145      NBLK=0
146      NSKP=0
147      FY=(ARAD/NSAR)*(2*N2-NSAR-1)
148      PZ=(ARAD/NSAR)*(2*N1-NSAR-1)
149      ADIS=SQRT(FY*FY+PZ*PZ)
150      IF(ADIS.GT.ARAD) GO TO 26
151      CALL RAY
152      IF(NREF.EQ.1) GO TO 26
153      IF(POS(2)**2+POS(3)**2.LT.D2*D2*.01) GO TO 107
154      IF(NARA1.EQ.0) GO TO 107
155      NARA=NARA1
156      PY1=PY
157      PZ1=PZ
158      E91=E91*(1.-TRAN0)*(1.-COSF)
159      P91=P91+RPD0
160      E92=E92*(1.-TRAN1)*(1.-COSF)
161      P92=P92+RPD1
162      R1=COS(P91)*E91
163      I1=SIN(P91)*E91
164      R2=COS(P92)*E92
165      I2=SIN(P92)*E92
166      RAZ=CPHI*R1+SPHI*R2
167      IAZ=CPHI*I1+SPHI*I2
168      REL=CPHI*R2-SPHI*R1
169      IEL=CPHI*I2-SPHI*I1
170      P7=T*.531976*FREQ
171      CALL BNC(K, N)
172      T=-POS(1)/C(1)
173      T=(POS(2)+T*C(2))**2+(POS(3)+T*C(3))**2
174      IF(T.LT.D2*D2) GO TO 107
175      CALL MRF(.01*D2, 2.*D2, C)
176      P7=P7+T*.531976*FREQ
177      Z11=K(1)
178      Z12=K(2)
179      Z13=K(3)

```

```

180      K(1)=C(1)
181      K(2)=C(2)
182      K(3)=C(3)
183      CALL RAY
184      K(1)=Z11
185      K(2)=Z12
186      K(3)=Z13
187      PY=PY1
188      PZ=PZ1
189      NARA=0
190      NBLK=NBLK1
191  107  IF(NBLK.EQ.0.OR.K(1).EQ.1.) GO TO 26
192      POS(1)=-CAZ*SEL*PY+SAZ*PZ+DEL
193      POS(2)=CEL*PY
194      POS(3)=SEL*SAZ*PY+CAZ*PZ
195      S1=POS(1)/K(1)
196      FOS(1)=0.
197      FOS(2)=POS(2)+S1*K(2)
198      FOS(3)=POS(3)+S1*K(3)
199      IF(SQRT(FOS(2)*FOS(2)+FOS(3)*FOS(3)).GT.D2) GO TO 26
200      C(1)=K(1)
201      C(2)=K(2)
202      C(3)=K(3)
203      NSKP=2
204      CALL RAY
205      DBLK=SQRT((PSN(1)-DEL)*(PSN(1)-DEL)+PSN(2)*PSN(2)+PSN(3)*PSN(3))
206      IF(DBLK.LT.ARAD)GO TO 26
207      BKPS=(S1+S)*FREQ*.531976
208      NSKP=0
209      CALL RAY
210  26  CONTINUE
211      NSKP=0
212      NBLK=0
213      NRCT=NRCT1
214      IF(NRCT.EQ.0.OR.NREF.EQ.1) GO TO 40
215      RM=K(1)/SQRT(K(3)*K(3)+K(2)*K(2))
216      TEMP=(L-DEL)/D2
217      IF(RM.GT.TEMP) GO TO 40
218      RADM=(L-DEL)/RM
219      ANG=SIGN(PI/2.,-K(2))
220      IF(K(3).NE.0.) ANG=ATAN2(-K(2),-K(3))
221      NY=INT(D2*NSAR/(2.*ARAD)+.25)*2
222      NZ=INT(RADM*NSAR/(4.*ARAD)+.25)*2
223      IF(NZ.EQ.0) GO TO 40
224      IF(NY.LT.2) NY=2
225      IF(NZ.LT.2) NZ=2
226      DO 30 N1=1,NY
227      DO 30 N2=1,NZ
228      NRCT=1
229      PY=(2*N1-NY-1)*D2/NY
230      FZ=(2*N2-1)*RADM/(2.*NZ)
231      ADIS=SQRT(PY*PY+PZ*PZ)
232      IF(ADIS.LT.D2) GO TO 30
233      DIS=(RADM-PZ)*RM
234      IF(ABS(PY).GT.SQRT(RSQ-(L-DIS)*(L-DIS))-B) GO TO 30
235      CAN=COS(ANG)
236      SAN=SIN(ANG)
237      POS(1)=DEL
238      POS(3)=CAN*PZ-SAN*PY
239      POS(2)=SAN*PZ+CAN*PY

```

```

240      CALL RAY
241 30    CONTINUE
242 40    CONTINUE
243 C     BSE FINDER
244      VSUM=SQRT(VRSUM*VRSUM+VISUM*VISUM)
245      IF(NF.EQ.2) RETURN
246      IF(NREF.EQ.1) GO TO 27
247      ANSUM=ATAN2(VISUM,VRSUM)
248      ANDAZ=ATAN2(VIDAZ,VRDAZ)
249      ANDEL=ATAN2(VIDEL,VRDEL)
250 27    VDAZ=SQRT(VRDAZ*VRDAZ+VIDAZ*VIDAZ)*SIGN(1.,SIN(ANDAZ-ANSUM))
251      VDEL=SQRT(VRDEL*VRDEL+VIDEL*VIDEL)*SIGN(1.,SIN(ANDEL-ANSUM))
252      RETURN
253      END
254      SUBROUTINE RAY
255      REAL K(3),N(3),T2(3),IAZ,IEL,I1,I2,I3,IPD0,IPD1,L,PINT(3)
256      COMMON/A2/APAZ,APEL,ILLUM,FREQ,THETA,IPD0,IPD1,RPD0,RPD1
257      COMMON/A4/AZS,ELS,AZ,EL,PI,CONV,RSQ,B,THAZ,THEL,EEL,EAZ,L,TIPL
258      COMMON/AA2/TRAN0,TRAN1,BKDB,CPHI,SPHI,THK(5)
259      COMMON/A5/NF,NFIR,D2,ARAD,ADIS,NREF,NARA,NRCT,NANT,NBLK,DBLK
260      COMMON/AAS/PY,PZ,DEL
261      COMMON/A6/VRSUM,VISUM,VRDEL,VIDEL,VRDAZ,VIDAZ,BKPS
262      COMMON/B1/NLST,AR(4),AI(4),BR(4),BI(4),CR(4),CI(4)
263      COMMON/C1/POS(3),K(3),CAN,SAN,RM,PSN(3),S,C(3),NSKP
264      COMMON/C2/CAZ,SAZ,CEL,SEL
265      COMMON/AR/RAZ,IAZ,REL,IEL,E91,P91,E92,P92,N(3),P7
266      COMMON/F1/T,COSF
267      IF(NARA.EQ.0) P7=0.
268      IF(NSKP.EQ.2) GO TO 990
269      IF(NARA.EQ.1) GO TO 63
270      IF(NBLK.EQ.1) P7=P7+BKPS
271      IF(NRCT.EQ.1.OR.NBLK.EQ.1) GO TO 59
272      IF(NF.NE.2.AND.NREF.EQ.0) GO TO 18
273      CAZ=COS(AZS*CONV)
274      SAZ=SIN(AZS*CONV)
275      CELS=COS(ELS*CONV)
276      SELS=SIN(ELS*CONV)
277      PSH1=SAZ*PY-CAZ*SELS*PY
278      PSH2=CELS*PY
279      PSH3=SELS*SAZ*PY+CAZ*PZ
280      ALP=ACOS((ABS(PSH2*PY+PSH3*PZ)/(ADIS*ADIS)-1E-8))
281      P7=SIGN(SIN(ALP)*ADIS*FREQ*.531976,PSH1)
282 18    IF(NREF.EQ.1) GO TO 1000
283      IF(NFIR.EQ.0) GO TO 19
284      NFIR=0
285      H=(AZ+AZS)*CONV
286      CAZ=COS(H)
287      SAZ=SIN(H)
288      H1=(EL+ELS)*CONV
289      CEL=COS(H1)
290      SEL=SIN(H1)
291      IF(NANT.EQ.1) GO TO 19
292      K(1)=CAZ*CEL
293      K(2)=SEL
294      K(3)=-SAZ*CEL
295 19    POS(1)=-CAZ*SEL*PY+SAZ*PZ+DEL
296      POS(2)=CEL*PY
297      POS(3)=SEL*SAZ*PY+CAZ*PZ
298 C     MOD REGULA FALSI
299 59    IF(NRCT.EQ.1) GO TO 62

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300      CALL MRF(0.,L+ARAD,K)
301      GO TO 63
302 62    CDIS=SQRT(PZ*PZ+RM*RM*PZ*PZ)
303      CALL MRF(CDIS,2.*CDIS,K)
304 63    POS(1)=POS(1)+T*K(1)
305      IF(POS(1).GT.L-TIPL) RETURN
306      POS(2)=POS(2)+T*K(2)
307      POS(3)=POS(3)+T*K(3)
308      N(1)=POS(1)/SQRT(RSQ-POS(1)*POS(1))
309      U=SQRT(1+N(1)*N(1))
310      N(1)=N(1)/U
311      U=U*SQRT(POS(2)*POS(2)+POS(3)*POS(3))
312      N(2)=POS(2)/U
313      N(3)=POS(3)/U
314      T2(1)=K(2)*N(3)-K(3)*N(2)
315      T2(2)=K(3)*N(1)-K(1)*N(3)
316      T2(3)=K(1)*N(2)-K(2)*N(1)
317      U=SQRT(T2(1)*T2(1)+T2(2)*T2(2)+T2(3)*T2(3))
318      T2(1)=T2(1)/U
319      T2(2)=T2(2)/U
320      T2(3)=T2(3)/U
321      THETA=ASIN(ABS(U))
322      PHI=SIGN(ACOS(T2(1)*SAZ+T2(3)*CAZ),T2(2))
323      CPHI=COS(PHI)
324      SPHI=SIN(PHI)
325      IF(NRCT.EQ.2.OR.NARA.EQ.1) GO TO 23
326      RAZ=COS(THAZ)*EAZ
327      IAZ=SIN(THAZ)*EAZ
328      REL=COS(THEL)*EEL
329      IEL=SIN(THEL)*EEL
330 23    R3=CPHI*RAZ+SPHI*REL
331      I3=CPHI*IAZ+SPHI*IEL
332      E91=SQRT(R3*R3+I3*I3)
333      P91=ATAN2(I3,R3)
334      R3=CPHI*REL-SPHI*RAZ
335      I3=CPHI*IEL-SPHI*IAZ
336      E92=SQRT(R3*R3+I3*I3)
337      P92=ATAN2(I3,R3)
338 C    ADD EFFECTS OF RADOME
339      IF(NRCT.NE.2.AND.NARA.NE.1) GO TO 999
340      CALL THIC(POS,THK)
341      CALL WALL
342      E91=E91*(1.-TRAN0)/K(1)
343      P91=P91+RPD0
344      E92=E92*(1.-TRAN1)/K(1)
345      P92=P92+RPD1
346      CALL BNC(K,N)
347 990   YM=SEL/(CEL*CAZ)
348      ZM=SAZ/CAZ
349      S=DEL+POS(3)*ZM-POS(2)*YM-POS(1)
350      S=S/(C(1)+C(2)*YM-C(3)*ZM)
351      PSN(1)=POS(1)+S*C(1)
352      PSN(2)=POS(2)+S*C(2)
353      PSN(3)=POS(3)+S*C(3)
354      IF(NSKF.EQ.2) RETURN
355      IF(NARA.EQ.1) GO TO 996
356      D1=PINT(1)-POS(1)
357      D4=PINT(2)-POS(2)
358      D3=PINT(3)-POS(3)
359      PHA=SQRT(D1*D1+D4*D4+D3*D3)+S

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360 996 PY=SEL*SAZ*PSN(3)+CEL*PSN(2)-SEL*CAZ*(PSN(1)-DEL)
361 PZ=SAZ*(PSN(1)-DEL)+CAZ*PSN(3)
362 ADIS=SQRT(PY*PY+PZ*PZ)
363 IF(ADIS.GT.ARAD) RETURN
364 IF(NARA.EQ.1) GO TO 997
365 DIS=ABS((PINT(1)-PSN(1))*K(1)+(PINT(2)-PSN(2))*K(2)
366 *+(PINT(3)-PSN(3))*K(3))
367 PHA=(PHA-DIS)*.531976*FREQ
368 P7=P7+PHA
369 GO TO 998
370 997 P7=P7+S
371 GO TO 998
372 999 CALL THIC(POS,THK)
373 CALL WALL
374 E91=E91*TRANO
375 P91=P91+IPDO
376 E92=E92*TRAN1
377 P92=P92+IPD1
378 998 SPHI=-SPHI
379 R1=COS(P91+P7)*E91
380 I1=SIN(P91+P7)*E91
381 R2=COS(P92+P7)*E92
382 I2=SIN(P92+P7)*E92
383 IF(NRCT.NE.1) GO TO 1000
384 RAZ=CPHI*R1+SPHI*R2
385 IAZ=CPHI*I1+SPHI*I2
386 REL=CPHI*R2-SPHI*R1
387 IEL=CPHI*I2-SPHI*I1
388 NRCT=2
389 - PINT(1)=POS(1)
390 PINT(2)=POS(2)
391 PINT(3)=POS(3)
392 POS(1)=DEL
393 POS(2)=SAN*PZ+CAN*PY
394 POS(3)=CAN*PZ-SAN*PY
395 CALL MRF(0.,CDIS,K)
396 GO TO 63
397 1000 COSF=1.
398 IF(NBLK.EQ.1) COSF=COSF*BKDB
399 SINFEL=SIGN(1.,PY)
400 SINFAZ=SIGN(1.,PZ)
401 IF(ILLUM.NE.2) GO TO 50
402 COSF=COS(1.24507*ADIS/ARAD)*COSF
403 SINFEL=SIN(PI*PY/ARAD)
404 SINFAZ=SIN(PI*PZ/ARAD)
405 50 AAZ=1.
406 AEL=1.
407 IF(NRCT.EQ.0.AND.NARA.EQ.0) GO TO 51
408 ZM=SQRT(C(1)*C(1)+C(3)*C(3))*SQRT(1-K(2)*K(2))
409 YM=SQRT(C(1)*C(1)+C(2)*C(2))*SQRT(1-K(3)*K(3))
410 AAZ=(C(1)*K(1)+C(3)*K(3))/ZM
411 AEL=(C(1)*K(1)+C(2)*K(2))/YM
412 51 IF(NBLK.EQ.0) GO TO 55
413 AAZ=COS(2.*ACOS(K(1)/SQRT(1.-K(2)*K(2))))
414 AEL=COS(2.*ACOS(K(1)/SQRT(1.-K(3)*K(3))))
415 55 IF(NF.NE.2) GO TO 56
416 AAZ=AAZ*CAZS*CAZS*CELS*CELS/SQRT(1.-SELS*SELS)
417 AEL=AEL*CAZS*CAZS*CELS*CELS/SQRT(1.-SAZS*SAZS*CELS*CELS)
418 56 APAZ1=APAZ*AAZ
419 APEL1=APEL*AEL

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420      IF(NREF.EQ.0) GO TO 52
421      RSS=(EAZ*COS(THAZ+P7)*APAZ1+EEL*COS(THEL+P7)*APEL1)*COSF
422      ESS=(EAZ*SIN(THAZ+P7)*APAZ1+EEL*SIN(THEL+P7)*APEL1)*COSF
423      GO TO 54
424  52    RSS=((CPHI*R1+SPHI*R2)*APAZ1+(CPHI*R2-SPHI*R1)*APEL1)*COSF
425      ESS=((CPHI*I1+SPHI*I2)*APAZ1+(CPHI*I2-SPHI*I1)*APEL1)*COSF
426  54    VRSUM=VRSUM+RSS
427      VISUM=VISUM+ESS
428      VRDEL=VRDEL+RSS*SINFEL
429      VIDEL=VIDEL+ESS*SINFEL
430      VRDAZ=VRDAZ+RSS*SINFAZ
431      VIDAZ=VIDAZ+ESS*SINFAZ
432      RETURN
433      END
434      SUBROUTINE MRF(A,B1,K2)
435      REAL K1(3),K2(3)
436      COMMON/F1/T,COSF
437      COMMON/F2/K1
438      K1(3)=K2(3)
439      K1(2)=K2(2)
440      K1(1)=K2(1)
441      F=FH(A)
442      G=FH(B1)
443      W=A
444      F2=F
445      DO 20 N3=1,5
446      W1=(G*A-F*B1)/(G-F)
447      F0=F2
448      F2=FH(W1)
449      F1=F2
450      IF(A.NE.W1) F1=FH(A)
451      IF(SIGN(1.,F2).EQ.SIGN(1.,F1)) GO TO 21
452      B1=W1
453      G=F2
454      IF(SIGN(1.,F2).EQ.SIGN(1.,F0)) F=F/2.
455      GO TO 20
456  21    A=W1
457      F=F2
458      IF(SIGN(1.,F2).EQ.SIGN(1.,F0)) G=G/2.
459  20    W=W1
460      T=(G*A-F*B1)/(G-F)
461      RETURN
462      END
463      FUNCTION FH(T1)
464      REAL K,K1(3),L
465      COMMON/A4/AZS,ELS,AZ,EL,PI,CONV,RSQ,B,THAZ,THEL,EEL,EAZ,L,TIFL
466      COMMON/C1/POS(3),K(3),CAN,SAN,RM,PSN(3),S,C(3),NSKP
467      COMMON/F2/K1
468      X=POS(1)+T1*K1(1)
469      Y=POS(2)+T1*K1(2)
470      Z=POS(3)+T1*K1(3)
471      FH=SQRT(RSQ-X*X)-B-SQRT(Y*Y+Z*Z)
472      RETURN
473      END
474      SUBROUTINE BNC(VK,UN)
475      DIMENSION VK(3),UN(3)
476      COMMON/C1/POS(3),K(3),CAN,SAN,RM,PSN(3),S,C(3),NSKP
477      DOT=2.*(UN(1)*VK(1)+UN(2)*VK(2)+UN(3)*VK(3))
478      C(1)=VK(1)-DOT*VN(1)
479      C(2)=VK(2)-DOT*VN(2)

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480      C(3)=VK(3)-DOT*VN(3)
481      RETURN
482      END
483      SUBROUTINE THIC(POS,THK)
484      DIMENSION POS(3),THK(5)
485      THE=ABS(ATAN(POS(3)/POS(2)))
486      THK(1)=34,-4.*THE
487      THK(1)=(POS(1)-THK(1))/THK(1)
488      THK(1)=COS(THK(1)*(.5+ABS((THE-.628)*.108)))
489      THK(1)=THK(1)*(.282+.010*THE)*2.54
490      RETURN
491      END
492      SUBROUTINE WALL
493      REAL IPD,IPD0,IPD1,KO,LTAN(5),Z(5),U1(5),U2(5),R(5)
494      COMMON/B1/NLST,AR(4),AI(4),BR(4),BI(4),CR(4),CI(4)
495      COMMON/A1/NLAY,ER(5),LTAN(5)
496      COMMON/A2/APAZ,APEL,ILLUM,FREQ,THETA,IPD0,IPD1,RPD0,RPD1
497      COMMON/AA2/TRAN0,TRAN1,BKDB,CPhi,SPHI,THK(5)
498      NPOL=0
499      CTH=COS(THETA)
500      STH=SIN(THETA)**2
501      KO=0.2094395*FREQ
502      MLAY=NLAY+1
503      11  NLST=0
504          DO 10 I=1,MLAY
505             IF(I.NE.(MLAY)) GO TO 12
506             Z(I)=1.
507             GO TO 13
508      12  U1(I)=SQRT(ER(I)-STH)
509          Z(I)=CTH/U1(I)
510          U2(I)=THK(I)*KO*LTAN(I)/(2.*U1(I))
511          U1(I)=U1(I)*KO
512          IF(NPOL.EQ.1) Z(I)=1./(ER(I)*Z(I))
513          IF(I.NE.1) GO TO 13
514          R(1)=(Z(1)-1.)/(Z(1)+1.)
515          GO TO 10
516      13  R(I)=(Z(I)-Z(I-1))/(Z(I)+Z(I-1))
517      10  CONTINUE
518          AR(1)=EXP(U2(1))
519          AR(4)=1./AR(1)
520          AR(2)=R(1)*AR(4)
521          AR(3)=R(1)*AR(1)
522          AI(1)=U1(1)*THK(1)
523          AI(2)=-AI(1)
524          AI(3)=AI(1)
525          AI(4)=AI(2)
526          DO 14 J=2,NLAY
527             IF(NLAY.EQ.1) GO TO 40
528             BR(1)=EXP(U2(J))
529             BR(4)=1./BR(1)
530             BR(2)=R(J)*BR(4)
531             BR(3)=R(J)*BR(1)
532             BI(1)=U1(J)*THK(J)
533             BI(2)=-BI(1)
534             BI(3)=BI(1)
535             BI(4)=BI(2)
536          CALL MULT
537      14  CONTINUE
538      40  CONTINUE
539          BR(1)=1.

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540      BR(4)=1.
541      BR(2)=R(MLAY)
542      BR(3)=BR(2)
543      BI(1)=0.
544      BI(2)=0.
545      BI(3)=0.
546      BI(4)=0.
547      NLST=1
548      CALL MULT
549      TRAN=1.
550      DO 15 K=1,MLAY
551 15     TRAN=TRAN*(1.-R(K))
552      TRAN=TRAN/CR(1)
553      SUM=0.
554      DO 16 L=1,NLAY
555 16     SUM=SUM+THK(L)
556      IPD=CI(1)-CTH*KO*SUM
557      IF(NPOL.EQ.0) GO TO 17
558      TRAN1=TRAN
559      IPD1=IPD
560      RPD1=CI(1)-CI(3)
561      RETURN
562 17     TRAN0=TRAN
563      IPD0=IPD
564      RPD0=CI(1)-CI(3)
565      NPOL=1
566      GO TO 11
567      END
568      SUBROUTINE MULT
569      COMMON/B1/NLST,AR(4),AI(4),BR(4),BI(4),CR(4),CI(4)
570  C     COMPLEX MATRIX MULTIPLICATION
571      DO 32 N1=1,3,2
572      DO 32 N2=1,2
573      IF(NLST.EQ.1.AND.N1-N2.EQ.1) RETURN
574      R1=AR(N1)*BR(N2)
575      R2=AR(N1+1)*BR(N2+2)
576      E1=AI(N1)+BI(N2)
577      E2=AI(N1+1)+BI(N2+2)
578      R3=R1*COS(E1)+R2*COS(E2)
579      E3=R1*SIN(E1)+R2*SIN(E2)
580      NSET=N2+N1-1
581      CR(NSET)=SQRT(R3*R3+E3*E3)
582 32     CI(NSET)=ATAN2(E3,R3)
583      DO 33 I=1,4
584      AR(I)=CR(I)
585 33     AI(I)=CI(I)
586      RETURN
587      END

```

APPENDIX B.

COMPUTED DATA

The purpose of this Appendix is to document additional data which has been computed during the analysis method validation.

The data presented within the main text of this report has been run for a dielectric constant of 5.0, which is believed to be that of the measured radome article. An increase in dielectric constant to 5.1 resulted in the data shown in Figures B-1 and B-2. Here, there was poorer agreement between theoretical and measured compared to the data in the report.

Secondly, the variation of the distance of the antenna from the base of the radome were tried in an effort to get the crossing points in the measured and computed azimuth scans to match. An antenna to base distance of 17 inches made this agreement fairly good, as seen in Figures B-3 and B-4.

BSE
AZIMUTH
(Milliradians)

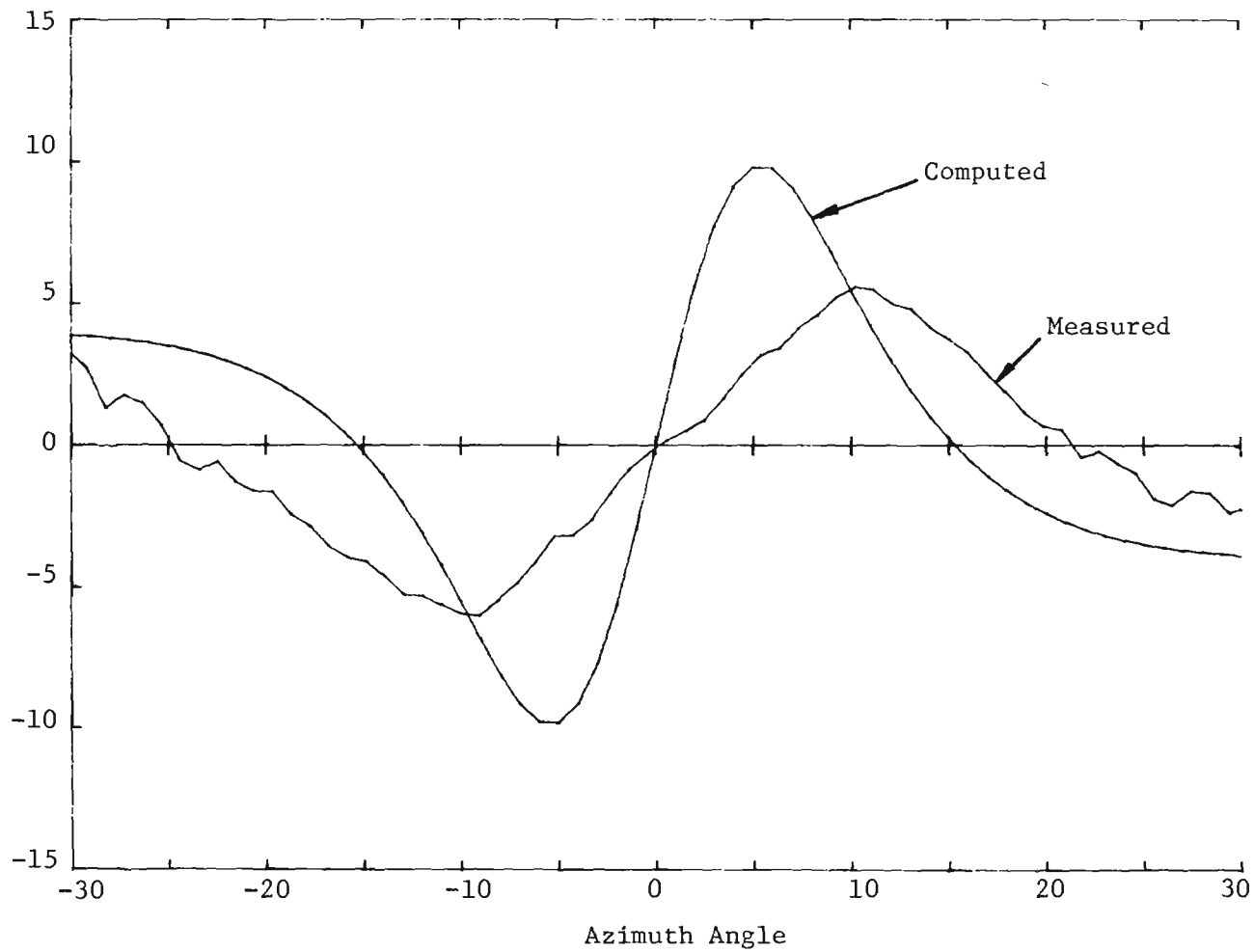


Figure B-1. Azimuth Scan with Dielectric Constant Changed from 5.0 to 5.1

BSE
ELEVATION
(Milliradians)

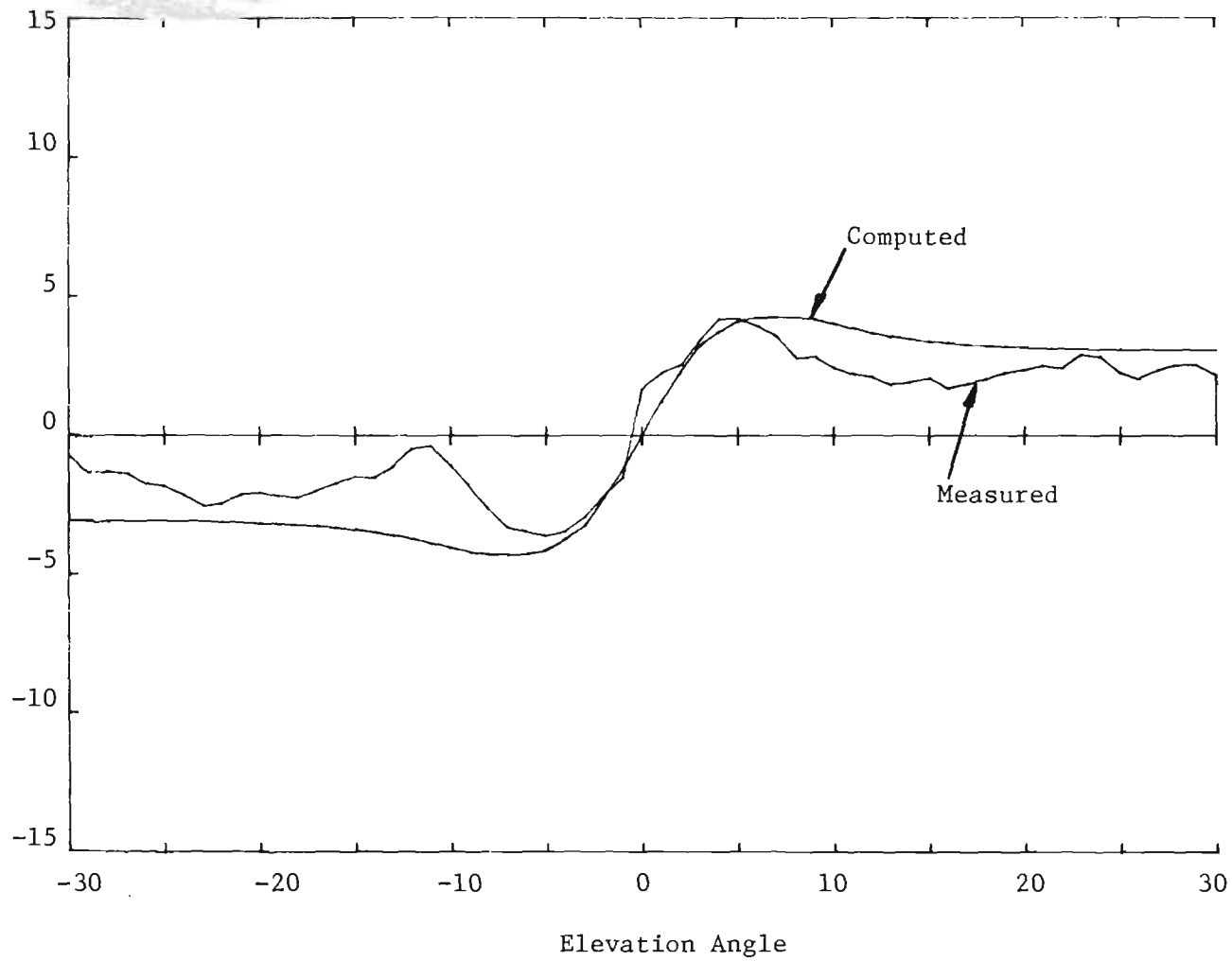


Figure B-2. Elevation Scan with Dielectric Constant Changed from 5.0 to 5.1

BSE
AZIMUTH
(Milliradians)

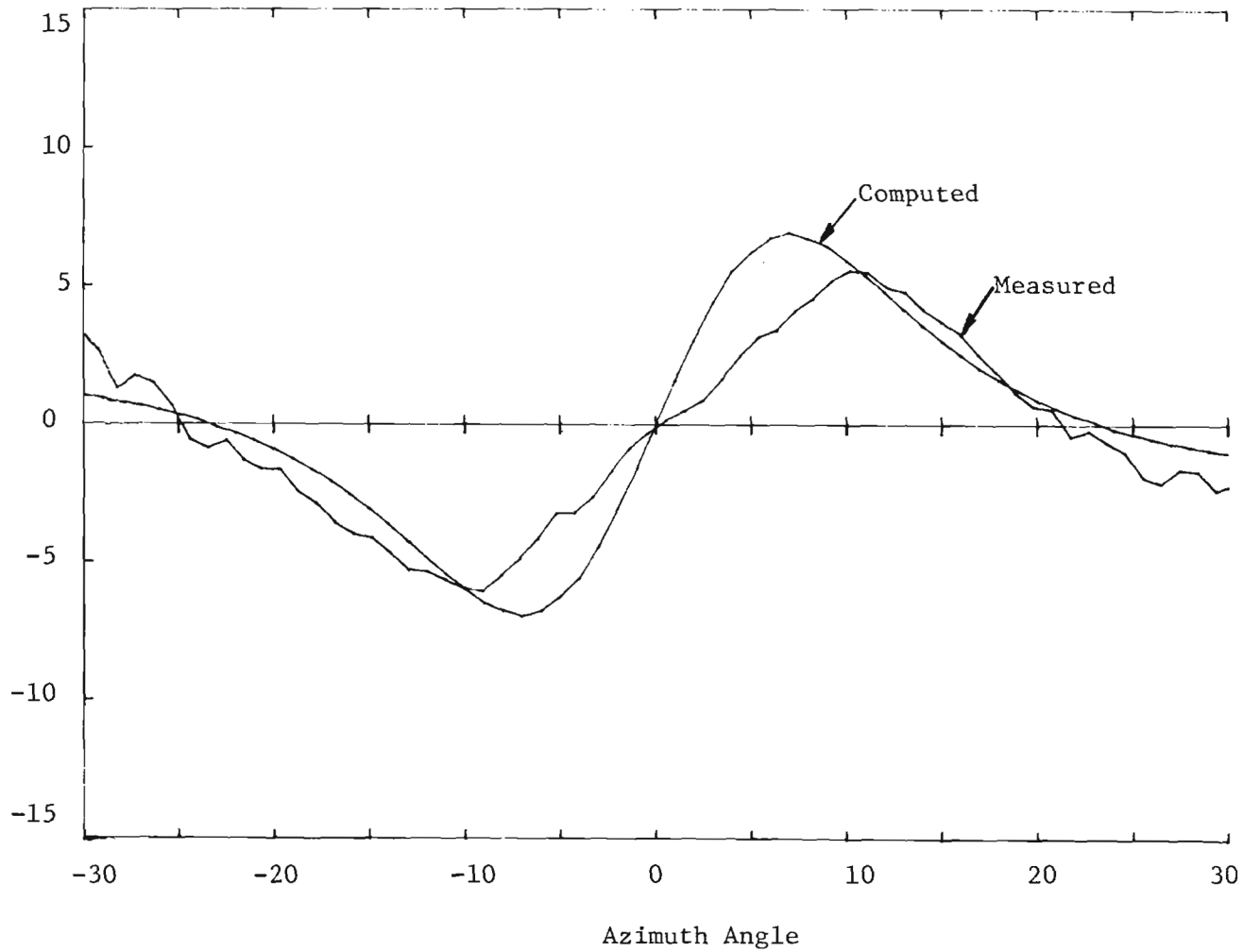


Figure B-3. Azimuth Scan with Antenna's Distance from Base Changed from 10.7 to 17 inches

BSE
ELEVATION
(Milliradians)

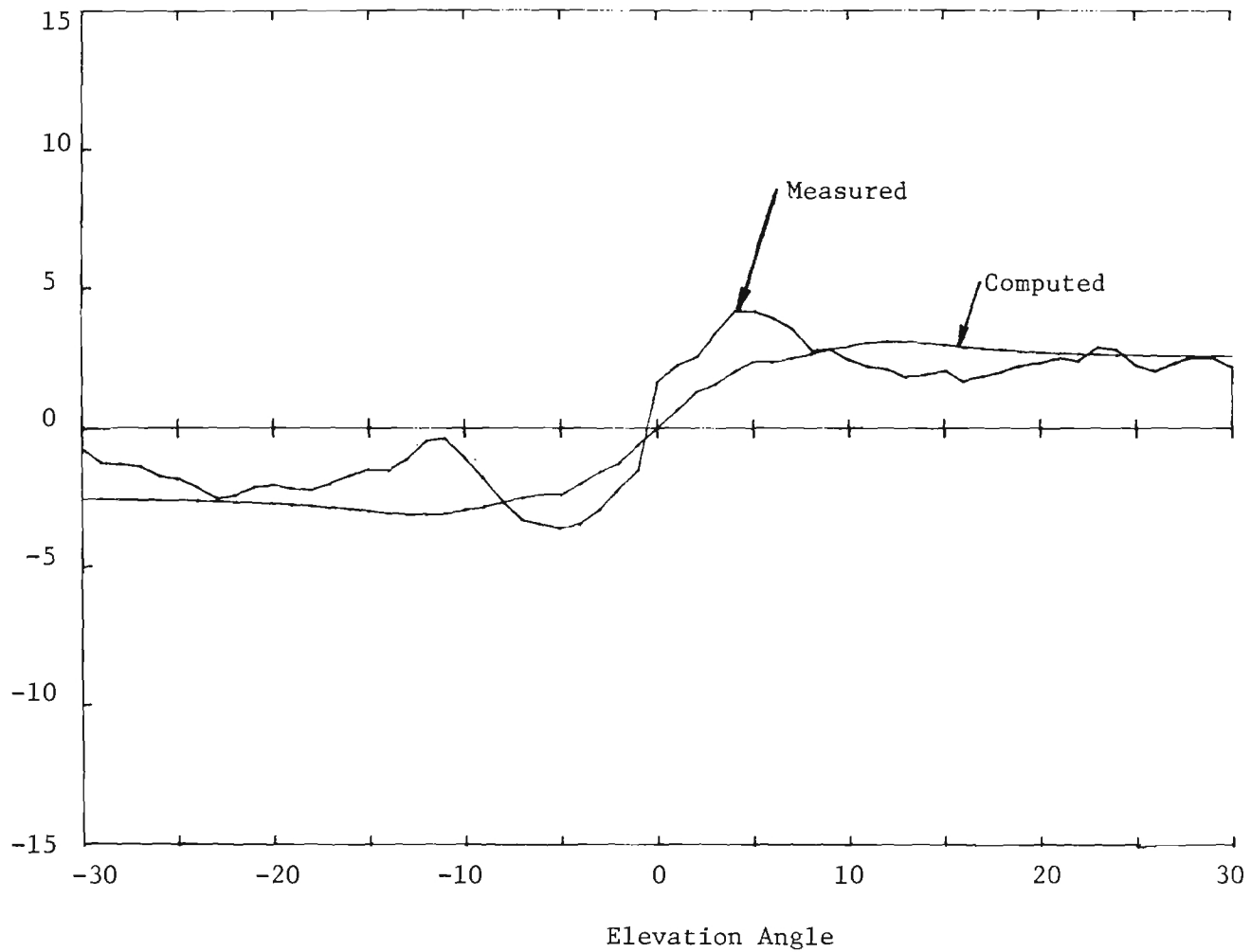


Figure B-4. Elevation Scan with Antenna's Distance from Base Changed from 10.7 to 17 inches.

APPENDIX C
ROTATIONAL MATRIX

The rotational matrix [M] is different for AZ/EL or EL/AZ gimbal configurations. Specifically, for an AZ/EL gimbal

$$[M] = \begin{bmatrix} \cos \theta_{EL} & \cos \theta_{AZ} & -\sin \theta_{EL} & \cos \theta_{EL} & \sin \theta_{AZ} \\ \sin \theta_{EL} & \sin \theta_{AZ} & \cos \theta_{EL} & \sin \theta_{EL} & \sin \theta_{AZ} \\ -\sin \theta_{AZ} & & 0 & \cos \theta_{AZ} & \end{bmatrix} \quad (A-1)$$

For an EL/AZ gimbal,

$$[M] = \begin{bmatrix} \cos \theta_{EL} & & -\cos \theta_{AZ} & \sin \theta_{EL} & \sin \theta_{AZ} \\ \sin \theta_{EL} & & \cos \theta_{EL} & 0 & \\ -\sin \theta_{AZ} & \cos \theta_{EL} & \sin \theta_{EL} & \sin \theta_{AZ} & \cos \theta_{AZ} \end{bmatrix} \quad (A-2)$$

A unit vector K at the origin in the direction θ is given in terms of the rotational matrices via

$$K = [M] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (A-3)$$

At a point y_p, z_p on the antenna face (see Figure A-1), a vector \bar{P}_1 from the origin to a point on the antenna (in terms of rotated coordinates) is:

$$P_1 = [M] \begin{bmatrix} 0 \\ y_p \\ z_p \end{bmatrix} \quad (A-4)$$

Finally, this is shifted by the distance (x = del) the antenna is offset into the radome.

$$P_2 = P_1 + \begin{bmatrix} del \\ 0 \\ 0 \end{bmatrix} \quad (A-5)$$