MANAGING AND OPTIMIZING DECENTRALIZED NETWORKS WITH RESOURCE SHARING

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MANAGING AND OPTIMIZING DECENTRALIZED NETWORKS WITH RESOURCE SHARING

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To my parents.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

SUMMARY

Resource sharing is a common collaborative strategy used in practice. It has the potential to create synergistic value and leads to higher system efficiency. However, realizing this synergistic value can be challenging given the prevalence of decentralization in practice, where individual operators manage resources based on their own benefits. Hence, optimizing a decentralized system requires understanding not only the optimal operational strategy in terms of the overall system efficiency, but also the implementation of the strategy through proper management of individual incentives. However, traditional network optimization approaches typically assume a centralized perspective. The classic game theory framework, on the other hand, addresses incentive issues of decentralized decision makers, but mainly takes a high-level, economic perspective that does not fully capture the operational complexity involved in optimizing systems with resource sharing.

The purpose of this thesis is to bridge this gap between practice and theory by studying the design of tools to manage and optimize the operations in decentralized systems with resource sharing using approaches that combine optimization and game theory. In particular, we focus on decentralized network systems and analyze two research streams in two application domains: (i) implementation of environmental legislation, and (ii) managing collaborative transportation systems. These applications are characterized by their decentralized multi-stakeholder nature where the conflicts and tension between the heterogeneous individual perspectives make system management very challenging. The main methodology used in this thesis is to adopt game theory models where individual decisions are endogenized as the solutions to

network optimization problems that reflect their incentives. Such an approach allows us to capture the connection between the operational features of the system (e.g., capacity configuration, network structure, synergy level from resource sharing) and the individual incentives thus the effectiveness of the management tools, which is the main research contribution of this thesis.

In the first research stream, we consider designing effective, efficient and practical implementation of electronic waste take-back legislation based on the widely-adopted Extended Producer Responsibility (EPR) concept that mandates the financial responsibility of post-use treatment of their products. Typical implementations of EPR are collective, and allocate the resulting operating cost to involved producers. In this thesis, we demonstrate the complexity of collective EPR implementation due to the tension among different stakeholder perspectives, based on a case analysis of the Washington implementation. We then perform analytical studies of the two prominent challenges identified in current implementations: (i) developing cost allocation mechanisms that induce the voluntary participation of all producers in a collective system, thus promoting implementation efficiency; and (ii) designing collective EPR so as to encourage environmentally-friendly product design, thus promoting implementation effectiveness. Specifically, we prescribe new cost allocation methods to address the first challenge, and demonstrate the practicality and economic impact of the results using implementation data from the state of Washington. We then analyze the tensions between design incentives, efficiency and the effectiveness of the cost allocation to induce voluntary participation under collective EPR implementation. We show there exists a tradeoff among the three dimensions, driven by the network effects inherent in a collective system. The main contribution of this research stream is to demonstrate how the implementation outcomes of an environmental policy are influenced by the way that the policy "filters" through operational-level factors, and to propose novel and implementation mechanisms to achieve efficient and effective

EPR implementation. Hence, our study has the potential to provide guidance for practice and influence policy-making.

In the second research stream, motivated by the practice of transportation alliances, we focus on a decentralized network setting where the individual entities make independent decisions regarding the routing of their own demand and the management of their own capacity, driven by their own benefits. We study the use of market-based exchange mechanisms to motivate and regulate capacity sharing so as to achieve the optimal overall routing efficiency in a general multicommodity network. We focus on the design of capacity pricing strategies in the presence of several practical operational complexities, including multiple ownership of the same capacity, uncertainty in network specifications, and information asymmetry between the central coordinator and individual operators. Our study in this research stream produces two sets of results. First, we demonstrate the impact of the underlying network structure on the effectiveness of using market-based exchange mechanisms to coordinate resource sharing and to allocate the resulting synergistic benefit, and characterize the network properties that matter. Second, we propose efficient and effective pricing policies and other mechanism design strategies to address different operational complexities. Specifically, we develop duality-based pricing algorithms, and evaluate different pricing strategies such as commodity-based price discrimination, which is shown to have an advantage in coordinating networks under uncertainty.

CHAPTER I

INTRODUCTION

Cooperation among companies is widely practiced today in different business sectors. Facing increasing pressure to improve profitability, companies are responding by seeking and implementing solutions that require strategic collaboration with external partners, because these solutions afford benefits that cannot be achieved alone. Among the various forms of cooperation, resource sharing is a common strategy widely used in practice. In the transportation industry, the members of carrier alliances collaborate by pooling fleets and integrating service networks in order to "reduce costs, increase asset utilization and improve overall service levels" [3]. For example, modern air cargo alliances began at a global scale in the late 1980's. [122] reports that global airline groupings collectively account for 63.6% world share of passenger traffic (Revenue Passenger Kilometres), 55.8% of passenger numbers and 58.4% of group revenues [109]. Other examples of resource sharing include bandwidth pooling among autonomous systems on the Internet to efficiently process and deliver individual end-to-end flow [128]. In environmental operations, many electronics producers collaborate in post-use collection and recycling of their products by participating in a common product take-back program where they pool return volume and share capacity [68, 162]. Such collective programs are believed to "offer the simplest, most straightforward, and most cost-effective approach" in electronic waste (e-waste) management [178], and have been adopted worldwide.

Resource sharing has the potential to create synergistic value and leads to higher system efficiency. It allows for the exploitation of scale economies from consolidating

demand for resources. It can also potentially increase the efficiency in resource allocation, thus generate scope economies, or what we call *network synergies*. However, realizing such synergistic value can be challenging given the prevalence of *decentralization* in practice, where individual operators are entitled to independent management of their own resources, driven by their own benefits. Unless completely aligned, these multiple individual preferences and objectives give rise to conflicts and tensions that potentially undermine the overall performance of the system, and even threaten the stability of cooperation. This problem can be observed in many real-life settings, two prominent examples being the following.

Example 1 (Implementation of environmental legislation)**.** Extended Producer Responsibility (EPR) is a policy tool that holds producers financially responsible for the post-use collection, recycling and disposal of their products. A well-functioning EPR implementation can (i) ensure the proper recycling of e-waste in the short term via adequate financing, and (ii) reduce the environmental impact of e-waste in the long-term via design incentives. However, EPR implementation is a complex and challenging process influenced by multiple environmental, economic, and operational factors, and the different perspectives of many stakeholders involved including the large number of producers, collection and recycling service providers, customers, legislative bodies, NGOs and practitioners. An evidence of such complexity is the following: At the present, EPR implementations are typically *collective* - a large collection and recycling network (CRN) handles multiple producers' products by sharing processing capacity; the total cost is then allocated to producers proportional to producer-specific metrics, such as their return shares. Such collective EPR implementations have been much questioned and criticized for failing to account for individual producers' benefits and incentives, which undermines their effectiveness and efficiency in executing legislative goals. In particular, producers are concerned about being over-charged under weight-based allocations, and thus try to break away from a collective program and establish

2

their own independent recycling operations. Such fragmentation can result in recycling operations that are inefficient due to the loss of synergies from resource sharing, and additional overhead cost such as in monitoring producers' compliance. The current collective EPR implementation is also reported to mute producers' incentives to design more recyclable products, thus failing to achieve the ultimate legislation goal of EPR.

Example 2 (Managing collaborative transportation systems)**.** Restructuring and reorganization of carrier alliances have been common in the transportation industry. Companies are constantly on the lookout for partners with high synergy potential. They may also leave an alliance when their individual benefit from the collaboration diminishes, in many cases, due to the misalignment of their own goals and interests with how the alliance is operated [3]. Such a trend has been observed in liner shipping (see Figure 2 in [3] which summarizes the changes in the member structure of the four major alliances in liner shipping). For airline alliances, Continental Airlines exited the alliance of SkyTeam in 2009 after a 5-year membership and joined the Star Alliance network [149]. Mexicana Airlines left the Star Alliance in 2004 [155] and chose to participate in the airline partnership oneworld in 2009 [119]. While the above changes involve companies switching between alliances, Aer Lingus quit oneworld in 2007 due to an intention to "concentrate on becoming a non-frill operation", and thus an alliance membership became "less relevant for the airline" [168]. Besides such tactical decisions regarding with whom to collaborate, the concrete operations of individual companies can also exert a significant impact on the overall efficiency of the combined transportation network. For example, one major operational decision in transportation is route selection. Inefficiency in transportation systems resulting from decentralized individual routing decisions, such as reducing the network throughput or increasing the total congestion or routing cost, has been observed in many contexts and discussed extensively in the literature (see [139] for an overview).

Due to these incentive issues, optimizing a decentralized system is a very complicated problem. It requires understanding not only what operational strategy is the most desirable in terms of the overall system efficiency, but also how to implement the strategy through proper management of individual incentives. The classic optimization framework provides a rich set of tools for the former problem; yet it typically assumes a centralized perspective thus does not address the latter one. The notion of mechanism design in game theory suggests a solution for the latter problem: The basic tenet is to align the individual interests with system efficiency by designing the operational rules, which are often termed as a *mechanism*, that motivate individuals' voluntary collaboration towards the best system performance. However, traditional game theoretic analysis mainly takes a high-level, economic perspective that does not fully capture the operational complexity involved in optimizing systems with resource sharing. For example, the individual preferences are often modeled as exogenous variables based on stylized assumptions.

The purpose of this thesis is to bridge this gap between practice and theory by studying the design of tools to manage and optimize the operations in decentralized systems with resource sharing using approaches that combine optimization and game theory. In particular, we focus on decentralized network systems that consist of capacity privately owned by different entities, and is not restricted to be used only by its owner but can be employed in the routing operations of others. Such *combined network* models are versatile in modeling many real-life systems including the service networks of carrier alliances, Internet and telecommunication networks, as well as the collection and recycling system of collective product take back programs. Moreover, the structural properties of networks has motivated an extensive network optimization literature that has generated insightful methodologies and results [5]. In addition, the interconnected nature of network models effectively captures the mutual

influence between individual entities, which is essential to understanding the potential negative externalities of their operations on system efficiency, and to designing effective coordination mechanisms.

A central research question addressed in this thesis is the connection between the operational features of the system (e.g., capacity configuration, network structure, synergy level from resource sharing) and the individual incentives and decisions, based on which the coordination mechanisms are designed. For this purpose, the main methodology used in this thesis is to adopt game theory models where individual decisions are endogenized as the solutions to network optimization problems that reflect their incentives. We use this approach to analyze the two practical problems mentioned in Example 1 and Example 2, i.e., implementation of environmental legislation, and managing collaborative transportation systems. Motivated by these problems, we particularly consider two types of individual decisions depending on the specific context concerned: (i) participation decisions in a combined network with resource sharing, (ii) the concrete tactical and operational decisions once they join a combined network, e.g., product design, capacity management and routing strategies. In analyzing the first type of decisions, we adopt a cooperative game theory perspective and study incentivizing voluntary individual participation in resource sharing via designing cost/benefit allocation mechanisms. We study the coordination of the second type of decisions mainly based on an equilibrium analysis using non-cooperative game models, and by prescribing concrete resource sharing rules such as market-driven exchange of capacity. In cases where the problem involves both types of decisions, we adopt a combination of the cooperative and non-cooperative angles, for example, using a biform game model, to capture the additional tension between the two decision-making processes. Incorporating optimization models into game theory frameworks enables us to gain insights into the impact of operational factors on the effectiveness of mechanisms in coordinating decentralized incentives, which is the

main research contribution of this thesis.

In the rest of this chapter, we first review related literature on managing and optimizing decentralized networks. We then provide an outline of the contents and the structure of the thesis, which summarizes the main results and contribution in each of the individual studies performed.

1.1 Literature Review

The problem of managing decentralized systems involving self-optimizing entities has long been recognized and discussed from various aspects in the literature. In this section we review some closely-related research areas. Papers that are more focused on and more relevant to specific issues and contexts will be reviewed in the corresponding sections in the subsequent chapters.

One major motivation of the study in this thesis is the potential inefficiency due to the decentralized decision-making of individuals who operate based on their own benefits. A stream of research in algorithmic game theory is focused on quantifying how much worse a decentralized system can be in terms of system efficiency compared to its optimal state under central control. The main approach is to evaluate the system efficiency of a decentralized system at its *equilibrium* state where no participant benefits from unilateral deviation. The concepts of the *price of anarchy* (PoA) introduced by [96] measures the ratio between the worst case system efficiency at equilibrium and the optimal efficiency level, and has been very widely used since. For example, [137] studies the PoA in networks where individuals selfishly choose the fastest route for themselves while the social goal is to minimize the total travel delay experienced by all participants. Notice that this setting is different from the combined network model with resource sharing studied in this thesis, as the road capacity is not owned individually. However, in some cases the notion of PoA can be "overly pessimistic" [106], especially when a best case Nash equilibrium can be guaranteed. In the light of

6

this idea, [7] introduces a variation of PoA, the *price of stability* (PoS) which characterizes the best-case system efficiency at equilibrium compared to the social optimum. Their study is based on a network design problem where selfish entities try to form a connected network and allocate the total cost among themselves based on centrally designed policies. Our study in this thesis shares a similar spirit in the sense that we also study designing operational rules of the decentralized network such that the PoS of the system is equal to 1, i.e., the social optimum is achieved as an equilibrium solution.

In this thesis, we focus on using mechanism design approaches to coordinate the individual incentives of a decentralized network so that its operations can be optimized. As mentioned before, the notion of mechanism design provides an important framework to study managing decentralized incentives so that a socially desirable solution is implemented without direct intervention. The field of *cooperative game theory* provides tools and concepts in designing *fair* cost or benefit allocation mechanisms to motivate individual participation, which is one of the major individual decisions we consider in this thesis. Some important fairness concepts include the core, the stable set, the nucleolus and the Shapley value. We refer to [184] for a thorough review of basic cost allocation methods and to [22] for a survey of cooperative games associated with operations research problems. Cooperative game analysis on combined networks with resource sharing is found in literature under the category of *network flow games*. In [90], a flow game is introduced based on the maximum-flow problems on single-commodity networks where each edge is uniquely owned. It is shown that the core of such a flow game is guaranteed to be non-empty. Its extensions include the *pseudo-flow games*[90], which involve public edges but may give rise to empty cores, and the *flow-based market games* [151], which are defined on networks that model market transaction with differentiated primary and secondary markets. The *multicommodity flow* (MCF) *game* is another generalization defined on networks

with multiple source-sink pairs. One such example is the MCF game adopted to model Internet routing with node capacity controlled by individual autonomous systems in [128]. [105] further shows that the core of such a MCF game is non-empty with either transferable utility or non-transferable utility, as well as when the utility function is nonlinear, i.e., displaying diminishing marginal value. Another MCF game formulation with edge resources shared is studied in [2] which is also proven to have a non-empty core.

It is worth mentioning that linear flow games are special cases of linear programming (or production) games defined based on linear programs (e.g., see [123, 141]), which is one of the first models studied in literature that combines optimization and game theory. In this thesis, we adopt concepts and methodologies from linear programming games and MCF games to analyze the individuals' resource sharing incentives in networks; we further analyze the impact of operational conditions of the network on such incentives. In literature, there also exists a set of papers that study other versions of network flow games defined based on combinatorial network optimization problems. These papers provide insight about the impact of the combinatorial structure of the underlying network problems on the individual incentives. Examples include the Assignment game [148, 60], the Traveling Salesman game (TSG) [158] and the Minimum Cost Spanning Tree game [62].

In this thesis, we also study how to implement a cost/payoff allocation in a resource sharing setting. We consider rewarding marginal resource contribution of individuals, as well as designing unit prices based on which side payments are made between the players according to their resource usage. There exists a large literature on how fair cost/payoff allocations can be implemented and achieved. One prominent method is the *Moulin mechanism* introduced in [110], which charges a player the marginal cost of involving him or her into a collaborative project or the utilization of a public facility. This mechanism is shown to yield strong equilibrium selections that

are coalition strategy-proof, i.e., all sub-coalitions being truthful in reporting their private information. A series of papers expands the idea and investigates the design of mechanisms that incentivize strategyproof cost sharing and are budget balanced or satisfy other properties. Sample papers include [111] that establishes a relationship between such allocation mechanisms and the Shapley value, and [108] that generalizes the Moulin mechanism and reports better performance under more general settings. Specific applications of the Moulin mechanism and its variations are also discussed in the context of network related problems, for example, [21] studies a network design problem with capacity requirements between any pair of nodes. In our study, we show the connection between a market-based resource exchange mechanism and core payoffs under certain pricing strategies.

Cooperative game theory essentially assumes a centralized perspective in managing the concrete operations once a coalition is formed. Such an assumption is not valid in many practical applications and specific operational mechanisms are needed. The goal is to design rules governing individual behavior such that the collective outcome of the individual actions result in the social choice. This idea has been explored in the literature on exchange economies and algorithmic game theory. The notion of *competitive prices* in an exchange economy is a classic example under which the social benefit is maximized and individual preferences are met; such a state is defined as the *competitive equilibrium* [147]. In this thesis, we study a capacity exchange mechanism; we show that by using pricing strategies based on the dual solution of the associated network problems, the competitive equilibrium can be achieved. In fact, among the mechanisms analyzed in literature, pricing mechanisms constitute a major category. For example, price mechanisms are extensively discussed to reduce congestion in road networks, where individuals internalize the negative impact of their behavior by paying a pre-designed road toll. [130] introduces the principle of *marginal cost pricing* of

edge utilization[1] to determine user taxes, under which the individual selfish routing leads to the minimum total delay over the network. [34] further studies the taxation mechanism towards the social goal defined as to minimize latency plus tax paid. [104] studies another form of the price mechanism based on rebates to reduce congestion in urban transportation networks. In the context of managing resource sharing in combined networks, [2] considers a revenue maximization problem, and introduces a capacity exchange mechanism under which users trade their own edge capacity with each other based on a set of centrally pre-designed unit exchange prices. In this thesis, we also consider designing capacity exchange prices, focusing on the impact of additional operational complexities such as demand uncertainty on the effectiveness of the mechanism. The effectiveness of pricing mechanisms is also studied in the context of revenue management in airline alliances, e.g., in [183]. Moreover, the implication of price mechanisms on coalition stability is also discussed in the literature. [37] shows that the competitive equilibrium resulting from the *competitive prices* is in the core of an exchange economy. In network settings, [2] shows that under the condition of unique ownership of edges, prices that induce the social optimal routing leads to an allocation of the total revenue in the core. However, most of the papers under this topic are based on economic models (see [150] as one of the pioneering examples in this area), and a systematic study of this problem in the context of managing the operations of decentralized networks is missing to the best of our knowledge.

From a methodological point of view, one goal of our study in this thesis is to propose mechanisms to perform optimization in a decentralized setting, thus to bridge the gap between the traditional optimization framework that assumes a centralized perspective, and the prevalence of decentralized systems in practice. In recent years, decentralized optimization has attracted much attention in the computing systems

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[1]The principle is in the similar spirit to the stand alone cost sharing in the Moulin mechanism but provides a pricing tool. Pricing mechanisms derived based on the stand alone concepts are developed in [77].

literature. One focus in this literature is to design distributed algorithms for coordination and resource sharing in networks and systems, taking into account the restriction in information availability for local agents to make decisions in large-scale computational systems. Common methodologies used to design such algorithms in distributed optimization include the sub-gradient methods (e.g., [167, 116]), decomposition methods (e.g., see [117] for an overview), and consensus-based methods (e.g., [166]). In [99], the authors take a different approach to designing local agents' objective functions such that a system-wide optimal solution is achieved under the Nash equilibrium solution. Other relevant papers in this area include [54], where the authors present a unifying framework in designing a distributed optimization environment called *optimization services*, [159] and [160], where the authors study the equilibrium state in communication networks in the presence of multiple congestion control protocols that respond to different pricing signals.

1.2   Thesis Outline and Contribution

We divide our study in this thesis into two research streams based on their application domains, i.e., implementation of environmental legislation, and managing collaborative transportation systems. These applications are characterized by their decentralized multi-stakeholder nature where the conflicts and tension between the heterogeneous individual perspectives make system management very challenging. In the rest of this subsection, we summarize the main issues analyzed and the contribution of the results in each research stream.

Efficient and Effective Implementation of Electronic Waste Take-back Legislation   In this research stream (Chapter 2), we study the problem of how to translate and operationalize the policy concept of Extended Producer Responsibility (EPR) into an efficient and effective working system, focusing on the collective form of implementation that has been adopted in the majority of EPR legislations.

We first analyze the tensions between stakeholder perspectives and identify the resulting challenges and opportunities that are present in a current EPR implementation in practice, by conducting a detailed case study of the Washington state E-cycle program. Due to the scope, scale and maturity of the Washington implementation for e-waste, the Washington case is instructive and the findings generate insights not only for Washington, but for other states and even other waste streams as well. This case study also opens up a rich set of research opportunities to understand some regulatory and system design choices in EPR implementation, e.g., the cost allocation mechanism (which is studied analytically in details in this stream), the mechanism to mandate and drive collection, public education campaigns, and contract structure with recyclers.

Among these practical challenges in implementing EPR, two major problems stand out: (i) the homogeneity of the simple proportional weight-based cost allocation methods employed currently, which can result in some producers being overcharged in a collective system and thus the emergence of fragmented collection and recycling networks that undermine efficiency and (ii) the implications of a collective system for motivating environmentally-friendly product design. In the following two essays, we perform a game-theoretic study of the use of mechanism design approaches to address these two pressing issues in by (i) developing cost allocation mechanisms that induce the voluntary participation of all producers in a collective system, thus promoting implementation efficiency; and (ii) designing collective EPR to provide design incentives, thus promoting implementation effectiveness.

In the first essay, we adopt a network cooperative game to capture operational complexity such as product heterogeneity and the network synergies from capacity sharing in a collective CRN, according to which effective cost allocation mechanisms are prescribed. Motivated by the importance of the practicality of the methods used in EPR implementations, the cost allocations we propose are presented as adjustments

to the widely-used return share method, and include the weighing of return shares based on processing costs and the rewarding of valuable capacity contributions to the collective system. Revolving around the key issue of developing practical and effective cost allocations, we also address several relevant research questions that are much debated in practice in the EPR context, for example, the implication of scale economies, the implementation of the shortfall fee concept that has been adopted in several EPR bills to complement the recycling capacity shortage of individual producers, and the value of performing source separation of the return volume. We validate our theoretical results using Washington state EPR implementation data and provide insights as to how these mechanisms can be implemented in practice and the added economic value to be obtained by their implementation.

In the second essay regarding the design incentives of EPR, we move one step further to study not only the implementation details of EPR, but also the legislation's impact on producers' own operational decisions. Hence, such a problem involves multiple stages of individual decision-making: The design choice of products and the participation decision into a collective system for post-use recycling. This motivates us to use a biform game model where the first stage is an equilibrium game on product design and the second stage is a cooperative network game. We analyze the tensions between design incentives, efficiency and the effectiveness of the cost allocation to induce voluntary participation under collective EPR implementation. We show there exists a tradeoff among the three dimensions, driven by the network effects inherent in a collective CRN. Based on this observation, we show that, contrary to conventional wisdom, a collective implementation can achieve superior design incentives relative to an individual system provided the proper cost allocation mechanism is implemented.

In all, the contribution of this research stream is to highlight the pressing challenges in achieving the potential of efficient and effective EPR implementation, and to propose novel and implementable mechanisms to address them. At the same time,

our results reveal the important roles of operational factors, such as product-capacity configuration and network structure of a CRN, and stakeholder incentives in determining the implementation outcome of a policy concept. Hence, given that the current understanding of EPR largely remains at a high level from an economic perspective, our results in this stream have the potential to inform the current debate on these issues and provide guidance for practitioners, producers and legislative bodies in e-waste management. These results have been used as policy input for the revision of the WEEE Directive in the UK [83].

Managing Decentralized Resource Sharing in Networks   In carrier alliances, the individual companies often select the shipping routes for their own demand as "full centralization is generally not an option given the technical and legal challenges associated with integrating the information systems of autonomous carriers" [79]. Hence, in this research stream, we consider a combined network setting where the players are entitled to independent decision making regarding the routing of their own demand and the management of their own capacity, driven by their own benefits. In this case, a major problem is that the resulting aggregate routing is not guaranteed to be efficient and it may even violate capacity limits and lead to overflow in the network. Hence, the main question addressed in this research stream is how to incentivize participants to adopt routing solutions that are aggregately efficient through imposing specially tailored operational guidelines. Taking into account the features of a combined network, one natural way to do this is to design the access rules according to which capacity can be used by individuals other than its owner. In this study, since market trade is one of the most common form of sharing private resources, we consider coordinating a combined network by designing a capacity exchange mechanism under which capacity is traded according to a set of centrally-designed unit prices. Via such a mechanism, a central authority of the system can

influence individual participants' profits and thus their selfish operations by choosing the appropriate exchange prices.

The contribution of this study is the analysis of the effectiveness of market-based mechanisms and propose capacity pricing strategies in the presence of several practical operational complexities, including multiple ownership of the same capacity, potential existence of free riders, uncertainty in network specifications, and information asymmetry between the central coordinator and individual operators. We address these issues respectively in three essays in Chapter 3 of this thesis. The main methodology we adopt is a non-cooperative game model based on a set of network optimization problems that represent individual operators' routing and capacity allocation decisions, and analyze the model using equilibrium analysis and inverse optimization techniques [6].

In the first essay, we begin by proposing a dual-based method to price capacity in a network such that in any network, not only a routing is achieved under which the system efficiency is maximized (which we call the *social optimal routing*), but also the stability of the collaboration is guaranteed as no players can strictly benefit by breaking away from the combined network and quitting sharing resource with others. In other words, with such dual prices, the capacity exchange mechanism also identifies a core allocation of the total system benefit achieved under the social optimal routing. We further show that in cases where there exist multiple owners of the capacity on one edge, a core allocation is not guaranteed under any pricing scheme that also induces a social optimal routing. This indicates a potential misalignment between the optimality and the stability of a decentralized network under the capacity exchange mechanism in the presence of multiple owners of the same resource. We show that such diseconomies of multiple ownership is mainly due to the asymmetry in the capacity levels among the multiple capacity owners on an edge relative to their commodity demand.

The second essay is motivated by the observation that dual-based prices can be zero in an over-capacitated network and thus are inappropriate as exchange prices in many practical applications. One of the biggest problems caused by zero prices is that this implies the existence of free riders who obtain positive shipping revenues using others' capacities at no cost. Such a phenomenon can cause perception of unfairness within the combined network, especially for the capacity owners, and thus discourages them from sharing capacity. We address this problem by studying the design of strictly positive capacity exchange prices to ensure the optimal operation within a combined network. We show that such prices do not generally exist due to the difference in individuals' perspectives towards the profitability of edge capacities over the network under strictly positive prices; such heterogeneity is caused by the different capacity ownership levels among the players and the existence of certain path structures. As a solution to this problem, we then propose a strictly positive pricing scheme that *partially* coordinates each user's individual routing towards the social optimal one. We show that under such partial coordination, a social optimal routing can always be achieved, but may not always be guaranteed to be an equilibrium. We further analyze this problem by identifying network conditions such that the social optimal routing can be maintained as an equilibrium under the pricing algorithm we propose; we also design an auxiliary capacity allocation mechanism under which an equilibrium is guaranteed at the social optimal routing given any combined network under partial coordination of the capacity exchange mechanism.

The central issue addressed in the third essay is how to design effective capacity exchange mechanism under the practical challenge of demand uncertainty, as in many practical cases, a pricing mechanism is designed and announced *ex-ante* before the demand is revealed. Hence, it is desirable that the capacity exchange mechanism is robust, i.e., it can effectively coordinate the network under all potential demand scenarios using a fixed set of exchange prices. This task is more challenging in practice,

as there often exists a certain level of information asymmetry between the mechanism designer and the players on demand. To address this problem, we perform the following two studies on the robustness of the capacity exchange mechanism under demand uncertainty. First, we characterize how network structure affects the robustness of the mechanism. Second, we investigate the algorithmic dimension of designing a robust capacity exchange mechanism in any given network. In particular, we propose a general pricing algorithm and quantify the routing performance under the resulting prices by providing bounds to the expected total revenue and the degree of potential capacity violation in the network. We also evaluate different pricing strategies such as commodity-based price discrimination, which are shown to have an advantage in coordinating networks under uncertainty.

CHAPTER II

EFFICIENT AND EFFECTIVE IMPLEMENTATION OF ELECTRONIC WASTE TAKE-BACK LEGISLATION

Extended Producer Responsibility (EPR) is a policy tool that holds producers responsible for the post-use collection, recycling and disposal of their products [100]. The basic concept is to promote environmental impact reduction at end of life by (i) making producers internalize the end-of-life costs of their products so as to incentivize the design of products that are more recyclable and have lower toxicity ; and (ii) to ensure there is sufficient and stable financing for running a collection and recycling system for post-use products [107].

EPR initiatives have rapidly diffused throughout the US in recent years. They target the waste streams of various products, including electronic waste (”e-waste”), mercury lights, carpets, packaging, paint, and pharmaceuticals [115]. Among those, e-waste stands out: Over the last decade, twenty five states have passed e-waste bills, and legislation is pending in several other states; the vast majority of these e-waste bills are based on the EPR principle [45]. The impetus is the potential impact of post-use electronics at home and abroad: Consumer electronics contain toxic materials that are harmful to both the environment and human health if not managed properly. Non-governmental organizations (NGOs) such as Californians against Waste argue that in the U.S., 70% of the toxic heavy metals found in landfills are estimated to come from e-waste [27]. Documented e-waste exports to developing countries, where they are handled in a way that is particularly harmful, have attracted attention [14]. A well-functioning EPR implementation, in conjunction with adequate environmental

regulation, can help alleviate these problems by (i) ensuring the proper recycling of e-waste in the short term (via adequate financing), and (ii) reducing the environmental impact of e-waste in the long-term (via design incentives).

The process to implement the EPR concept typically involves the following three stages. First, an appropriate policy instrument that embodies the EPR principles is identified, and a legislative framework is developed. Second, the legislation is translated into an EPR program. This involves designing a set of detailed operational rules, e.g., the specific mechanisms to finance the operations of the program and to monitor the legal compliance of each entity involved, within the parameters of the legislation. For example, many states in the US adopt a collective form of the EPR legislation, i.e., there exists a centrally-operated network that handles a mixture of products from multiple producers in an aggregate manner. Note that there is no definite boundary between these two stages: While some states adopt e-waste laws with high-level guidelines (e.g., in South Carolina [152]), there is also state legislation that already contains some operational details (e.g., in Washington [176]). The final stage is the execution of the EPR program into a working system in practice. This stage is characterized by numerous interactions among multiple stakeholders whose own managerial and operational strategies are affected by the EPR legislation, and thus each of whom has its unique perspective towards the program. These interactions greatly contribute to the shaping of the actual practice of e-waste collection and recycling (see [11] for a detailed discussion).

A prominent phenomenon that arises during the above transitions is that the decision made in each stage is influenced by various factors other than environmental concerns. In particular, the proper handling of e-waste is typically a costly operation, and this economic burden is shifted from local governments to the electronic industry under EPR. In addition, EPR typically allows end-users to return used electronics free of charge and requires advertising of take-back programs. This, in turn, typically

increases the collection volume, implying that the economic burden can be higher under EPR than in its absence. A mandated EPR program can also give rise to economic opportunities for businesses involved in e-waste collection and recycling. In addition, political factors such as the lobbying influence of stakeholders may play a significant role. Another crucial element in EPR implementation is the challenge of executing legislative objectives using practical and efficient methods, especially considering the existing infrastructure related to collection and recycling operations in the area. For example, in Washington state, transportation efficiency depends on the geographic location of the route. The biggest differentiation occurs between the so-called "west-of-the-mountains" and "east-of-the-mountains" areas, as the former contains the Seattle-Vancouver corridor where many trucking companies operate busy routes and thus can provide ample back-haul miles at cheap prices. This is an important factor when determining the location of collection and recycling facilities. Moreover, due to the multi-agent nature of an EPR program, the efficiency of its implementation is also greatly influenced by the heterogeneity in the perspectives and individual incentives among the entities involved, even within a single stakeholder group. These challenges often result in a gap between the EPR system in practice and what is intended by the EPR principle and/or EPR legislation [11].

The difficulty of achieving policy objectives in the EPR context has received some attention in the literature. For example, according to the environmental economics literature, mandated producer take-back policy may not be able to motivate producers to adopt product designs that are more environmentally friendly (e.g., [171]). Along similar lines, a number of papers (e.g., [126, 55, 25]) study policy instruments such as recycling subsidies, advance disposal fees, and command and control standards and point out the impact of certain externalities in determining the efficiency of these policy instruments. The environmental policy literature also recognizes the complexity in the implementation of policy tools. One seminal work is the study of public

policy implementation by Pressman and Wildavsky [133], which introduces a general framework to analyze factors in the implementation process that result in differences between the intended and the actual outcomes from the policies. This literature also highlights the additional complexity of the problem when environmental objectives are infused into the process (e.g., [118])

The recent operations management literature also recognizes the challenges with implementing EPR, and investigates the translation of EPR principles into working systems from an operations perspective. For example, Atasu and Van Wassenhove [11] provide a systematic overview of the operational issues in implementing e-waste take-back legislation. A set of other papers study specific outcomes of EPR implementation using analytical models. For example, [165] studies the impact of recycling competition, [86] investigates supply chain configuration decisions under product take-back mandates; [97], [173], and [73] study reverse logistics and network design; and [186], [131], [48], and [10] study product design implications of EPR. The industrial ecology literature has also provided evidence regarding the problem, pointing out the drawbacks and limitations of the current execution of EPR principles and suggesting conceptual solutions for improvement (e.g., [107, 101, 169, 163]).

The goal of this chapter is to contribute to the understanding of how the multiple, and sometimes conflicting, stakeholder perspectives and prevailing conditions (economic, geographic, etc.) in the implementation locality shape EPR "on the ground." Based on this understanding, we explore regulatory and system design choices to improve efficiency and effectiveness of EPR implementation. The chapter is organized as follows. In Section 2.1, we examine concrete activities at the operational front of a collection and recycling system, and probe the hidden tensions that have driven a specific system to its status quo. To this end, we conduct a detailed case study of the Washington state EPR implementation for electronic waste, based on which we provide an overview of various stakeholder perspectives and their implications for the

attainment of EPR policy objectives in practice. In Section 2.2 and 2.3, we take a deep dive and conduct analytical study into two specific problems identified in the gap between the current implementation outcomes of EPR and its legislative goals, i.e., how to (i) achieve operational efficiency via proper cost allocation design, and (ii) realize the design potential of EPR in implementation. We use a game theoretic approach based on a network model, and propose policy recommendations accordingly. Finally, in Section 2.4, we discuss some future research directions.

2.1 Implementing Extended Producer Responsibility Legislation: A Multi-Stakeholder Case Analysis

2.1.1 Specifics of the Washington EPR Legislation and the "E-Cycle" Recycling Program

In the Washington state, the Washington e-waste legislation [176] mandates free collection, transportation and recycling services to be provided for covered entities (any household, charity, school district, small business, local government in Washington state) for covered electronic products (CEPs) defined as TVs, monitors and computers (excluding peripherals). The collection and recycling system is financed by producers or manufacturers[1]. The Washington Materials Management and Financing authority (WMMFA) is established to put in place and run a "default" collection, transportation and recycling program (hereafter called the "standard plan"), and collect funds from the participating producers to finance the operational and administrative expenses incurred. The authority is governed by a board of directors comprised of representatives from participating producers. All producers must register with the Department of Ecology (hereafter referred to as "Ecology"' for short) and participate in either the standard plan or, alternatively, operate and finance their own "independent plan" if certain criteria are met (after approval of such plan by Ecology) in order

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[1]Please refer to [176] for the definition of a manufacturer, a collector, a transporter and a processor.

to sell covered electronic products in the state. Any plan (independent or standard) must provide collection service in every county, and every city of size greater than 10,000 (called "the convenience standard"), and implement and finance the sampling of brands processed in the plan for every program year. In addition, the standard plan is expected to try to come to a negotiated agreement with all collectors and processors that want to be in it. Each plan will be charged/paid for the deficit/surplus, if the e-waste processed by the plan within a program year, relative to the total weight processed by all plans, is below/above its return share, defined as the ratio of the participating producers' products returned to the total amount of electronic products returned by weight.

The electronic product recycling program [175] further defines detailed regulations with respect to the requirements for each entity involved in the system (see the section on "Stakeholder Roles and Perspectives" for details), as well as the specific procedures that Ecology will use to enforce these requirements and implement the legislation.

2.1.1.1 Implementation Overview.

To date, only the standard plan operates in Washington (no independent plans have been approved by Ecology) and is thus responsible for handling all returned covered electronic products in the state. A description of the material and financial flows associated with the standard plan is provided in Figure 1.

Product Flows Consumers (including households and small businesses) bring post-use CEPs to collection points, which are transported to processors (potentially after being taken to consolidation points). Material flows are handled by transporters (represented by solid lines in Figure 1.) The Authority determines to which processor each lot from each collection point will be transported so as to minimize the total cost it is charged. Transport is either in the form of self-transport by collectors or takes place on transport capacity purchased by the Authority (typically in the form
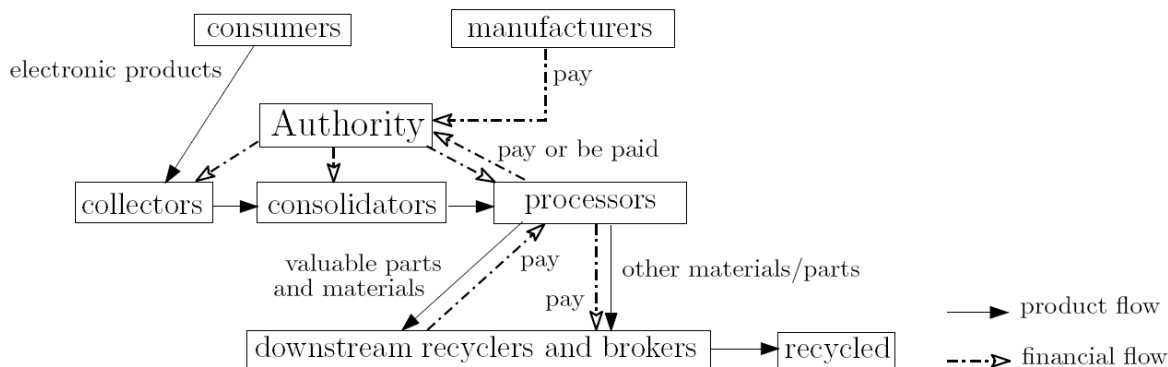
Figure 1: Product and financial flows of collective collection and recycling of electronics.

of backhaul miles). At the processors, the CEPs are dismantled into parts and/or shredded and sorted into different materials. Processors incur the operational costs of dismantling and/or shredding materials. Then the valuable parts and materials, such as computer processors and metals, are sold to downstream parties (smelters, brokers, refurbishers, etc.) to be reused or further processed, which creates a net value. The rest of the parts and materials, such as leaded glass, can be delivered to downstream brokers or facilities for further recycling and/or landfilling, both of which lead to a net cost that the processor needs to pay. For example, in the Washington case, some processors send the leaded glass to a Mexican facility that processes and sells the glass for new CRT TV production in India. Processors must follow environmental, health and safety standards (including those for the downstream brokers they interact with) as outlined by Ecology.

The direct processor level is the "boundary" to which the oversight of Ecology and the financial responsibility of the Authority extend. The extent to which materials are recycled downstream does not need to be documented for plans that choose to adhere to the "minimum" performance standards. However, processors voluntarily adopting the "preferred" performance standards must send materials and parts to downstream vendors that certify that they do not export the e-waste to developing

countries that do not accept such waste.

Financial Flows The authority pays each collector, consolidator, and transporter based on a unit rate per weight that they handle; the main cost of these entities is operational in nature. Processors incur operational costs to dismantle/shred products and separate materials, and may either incur a cost or make money on each material/part stream. Hence, in implementation, the Authority pays processors (by weight) for products that incur a net cost, but obtains a reduction on its invoices for products that generate a net value. The Authority's total operational and administrative cost then gets allocated to producers whose products are sold in the state.

2.1.1.2 Washington Implementation in 2009.

Here we provide some implementation details in the first year of the E-cycle program in Washington (2009), mainly based on the annual report by Ecology [174]. We mention that the Washington implementation serves not only as a practical motivation of this research, but also as a main test bed of our analytical study in the subsequent sections.

In 2009, 38,509,563 pounds of CEPs were collected in the Washington state, of which TVs, monitors and computers accounted for 58%, 32% and 10% respectively. This corresponded to products from 137 different product brands from 87 producers with return shares varying from 0.0001% to 7.9%. The return rate among the counties ranged from 0.4 to 9.6 lbs. per capita, and King County alone, where Seattle is located, achieved a return volume of almost 15 million pounds, or about 38.5% of the total volume.

There were 244 collection points registered with Ecology in 2009, whose concentration widely varied from county to county because the population densities of counties in Washington vary (Figure 2(a)). Overall, the "west-of-the-mountains" area had a
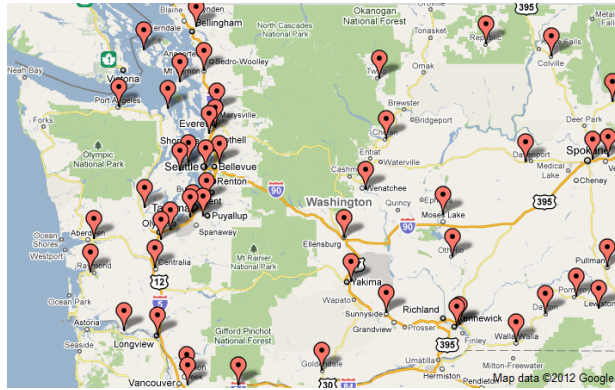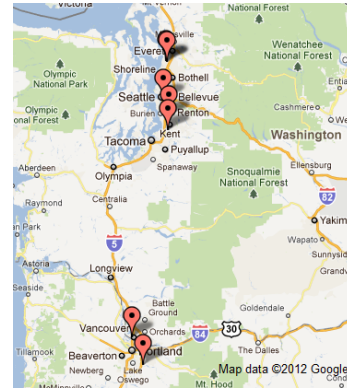
(a) Collection point
(b) Processors

Figure 2: Locations of (a) a representative sample of collection points (50 out of 244) and (b) all the in-state processors involved in the Washington E-Cycle program in 2009. Note that there are two processors near Seattle that are very close to each other and overlap in (b).

denser network of collection points compared to the "east-of-the- mountains" area. In particular, 15 counties, most of which are located in the "east-of-the-mountains" area, had only one collection point (which is mandated by the convenience standard), while King county (in the "west-of-the-mountains" area) had 58. The convenience standard assured that 38% of the 207 cities in Washington were covered and reached approximately 90% of the population (based on 2010 Census data).

Eight processors were involved in the E-cycle program in 2009; yet the majority of the total return volume (approximately 99%) was processed at the six in-state processors (Figure 2(b)). It can be observed that these processors are all located along the Seattle-Vancouver corridor with convenient and ample transportation capacity. Among these processors, there are large-volume high-tech processors that handled more than 60% of the total volume, and also small local businesses with mainly manual dismantling operations.

The average handling cost (including the cost of collection, transportation, processing and administrative expenses) was 24 cents/lb. in 2009. The rates paid to collectors ranged widely, depending on the location of the collection point and its

business scale. In particular, in counties with low population density, the collection points that were established largely due to the convenience standard mandated by the law and collected low volumes were typically compensated at a higher rate. The transportation rates depend on the location of the routes: As described in the introduction, backhaul capacity can often be utilized very cheaply to serve the collection points in the Seattle-Vancouver corridor to the west of the mountain where the processors are also located, while a slightly higher price is needed to transport the return volume from east to west. The processing costs are largely influenced by the product characteristics since products may require different recycling techniques and procedures, and/or generate parts and materials with different profitability levels. The biggest distinction exists between TVs/monitors and computers: TVs/monitors are expensive to recycle due to the hazardous materials contained in them, such as the leaded glass, while computers often lead to a positive net recycling profit as their components and materials have high reuse value. Hence, the E-cycle program pays processors for TVs/monitors and is effectively compensated by the processors for computers. The specific processing cost can also depend on the operational efficiency of the specific processors: Processors with large-scale and automated operations incur lower processing cost; in addition, those that can perform advanced processing operations besides basic dismantling of products, for example, shredding, material separation and even computer refurbishing, are able to achieve better product recovery and obtain more recycling profit, which can be reflected in the processing rates they quote to the E-cycle program. Similarly, processors that have in-house transport capacity can quote a combined and more advantageous rate to the Authority than those without such capacity.

2.1.2 Stakeholder Roles and Perspectives

As described in the previous section, the implementation of the WA program is the result of the joint participation of various parties including producers, collectors, processors, etc. Therefore, the perspectives of these stakeholders greatly influence and even shape the current practice of the E-cycle system. In particular, there exists misalignment of preferences among different stakeholders regarding various EPR policy implementation options. The attempt by both WMMFA and the Department of Ecology to balance these varying perspectives influences the final implementation structure. In this section, we illustrate this by providing examples of how such perspectives have been reflected in the Washington state implementation. We refer the readers to [67] for a detailed discussion of the individual perspectives of each stakeholder group.

First of all, the product recovery process directly monitored by the E-cycle program in Washington consists of four stages: collection, consolidation, transportation, and processing. In the collection stage, the Washington legislation departs from other states in adopting the convenience standard to ensure comprehensive coverage, reaching 90% of the population. This approach gives strong consideration to the consumers who highly value the convenience of the service, and also local governments who are concerned about the equity of the program across the state, as well as the economic and environmental benefits in their areas. While producers may be concerned about the cost of extensive coverage, and indeed small collectors are paid a higher collection rate as discussed earlier, the volume collected at these locations is rather limited as well so that the additional cost impact of the convenience standard does not appear to be onerous in this state.

The economic concerns of local governments are also reflected in the stipulation that the Authority give preference to processors operating in the state, creating an opportunity for small scale processors to be involved in the program. This can help

bolster the local economy by providing new employment opportunities. At the same time, focusing on local e-waste operations, especially in processing, may forgo the economies of scale advantage obtained by using only large processors and the use of state-of-the-art recycling technologies that exist outside the state. The consequence of balancing local economic development and cost efficiency concerns in Washington state is the presence of a mixture of high-volume, established facilities with partially automated equipment that are assigned a large percentage of the total return volume, and a set of facilities (some new) that are characterized by low volumes and manual operations.

The current scope of EPR legislation does not include peripherals (e.g., keyboards and mice), and yet, a steady stream of peripherals (typically associated with computers) is brought to collection sites by consumers. These returns are either not accepted, or accepted but handled outside the E-cycle program, with landfilling being the primary outlet. A recently proposed bill amendment would add some peripherals to the electronic products covered. Clearly, expanding the scope to include peripherals would be convenient for consumers and beneficial for collectors (and the environment), but primarily represent a cost burden for the program.

One prominent feature of e-waste recovery is that it is a multi-stage process, where different recovery methods such as parts reuse and product refurbishing can be used according to the product condition. The current Washington system, like many others, focuses primarily on recycling. The main concern from the perspective of the standard plan is that it would be much more complicated to manage and coordinate different recovery operations, especially considering the fact that different cost/revenue structures are to be observed from reuse and refurbishing compared to recycling. From a producer perspective, reuse and refurbishing can indeed be a desirable option. This, however, is the case only if each individual producer was to refurbish its own products and the market valuation of reused and refurbished products

would result in positive margins for the producers. If third parties (i.e., processors registered in the state) were to reuse or refurbish, this would effectively imply the creation of a strong secondary market that would cannibalize producers' new product sales. Moreover, reuse and refurbishing require a different skill set and expertise than new product manufacturing. Manufacturers who do not possess this capability are likely to prefer shredding, as it is cheap and keeps cannibalization at bay. Inclusion of reuse and refurbishing in the e-waste program appears to benefit consumers and third party remanufacturers the most. In particular, low-cost refurbished products can attract low-budget consumers to purchase products (albeit used) that they could not otherwise. For third parties possessing the skill to refurbish, an inclusion of reuse/refurbishing targets, similar to those that are considered for the recent revision of the European Waste Electrical and Electronics Equipment (WEEE) Directive [50] would imply a bigger revenue stream (i.e., refurbishing on behalf of producers who do not possess the skill) as well. The Washington state implementation appears to be maintaining its emphasis on recycling although some refurbishing is already taking place.

The EPR legislation in Washington allows producers to set up and operate their own independent plans, considering the perspectives of some producers who have established their own collection network and recycling facilities, and the economic advantage of using available resources. However, no independent plans have been approved in Washington to date, the main reason being the challenge of developing independent collection networks that meet the convenience standard. This is partially due to large scale collectors' concerns related to the operational difficulty and the revenue uncertainty inherent in managing capacity sharing among different plans, especially when no clear rules have been established regarding this issue. From the standard plan's perspective, the existence of independent plans may also imply an efficiency or bargaining power loss as well. Hence, the status quo with an efficient

standard plan benefits both the authority (and by extension, the producers) and the large scale collectors in the state.

A central implementation design issue for a working collective EPR program (such as the standard plan in Washington State) is to provide stable financing such that the program continues to run efficiently and benefits all stakeholders. Currently, this appears to be one of the major issues for producers associated with the program. Indeed, as described above, the discussions between TV and computer producers have led to the use of dynamically evolving combinations of return share and market share to determine each producer's cost allocation. These changes have not allayed fairness concerns, however, as some producers continue to benefit from a market share based cost allocation, while others prefer a return share based cost allocation, and others prefer to build their individual systems. The fact that the current cost allocation model in Washington (within the standard plan) uses a dynamically changing combination of return share and market share is an outcome of the need to balance these perspectives.

2.1.3   Challenges and Opportunities in EPR Implementation

The Washington EPR program is one of the most comprehensive working EPR implementations in the US, having initially enlisted more than 240 collection points (this number has risen to close to 300 recently) and several recyclers (including some new entrants), and collected approximately 6 lbs. per capita in 2009 and 2010. This volume may grow, especially if the scope of the covered electronics is expanded (considering its European counterpart that reached 17.6 lbs. per capita [51] over 11 product categories). Some of the future challenges and opportunities are discussed below. In particular, we focus on those associated with the two major issues of the financing mechanisms used and the product design implications under the current collective implementation, which will be studied analytically in Section 2.2 and 2.3,

31

respectively. We also provide a brief discussion of other challenges, regarding which we refer the readers to [67] for details.

2.1.3.1   *Fair Cost Allocation and Collective Efficiency.*

Since the EPR legislation essentially shifts the cost burden of e-waste management to producers, it is crucial to find a way to settle potential fairness concerns among producers regarding their fair cost shares within the standard plan or any independent plan; otherwise, the long-term viability of any plan will be at risk. The current weighing method between return share and market share does not seem to be effective and may not necessarily reflect fair cost shares of individual producers. Hence, finding new approaches for fair cost sharing remains as one of the most critical (if not the most) challenges of collective e-waste systems. In §2.2, we study this issue and propose adjustments to the return share method by (i) compensating producers for bringing in additional capacities based on rates that reflect the operational efficiency of these capacities; (ii) adjusting the return share of producers to reflect the use of critical resources to process their products; and (iii) using a cost-weighted return share to reward producers for having products with low processing cost and/or high recycling revenues. We provide theoretical and empirical analysis which indicates that these adjustments can be effective in better reflecting the differentials in cost burden among producers in the standard plan and thus improve the fairness of the allocation. Another approach in [107] proposes a novel cost allocation mechanism considering the requirements of the WEEE Directive regarding future and historical waste electrical and electronics equipment. It should be noted, however, that the feasibility of these approaches would significantly depend on the support of producers in the state, who may have varying preferences.

Another issue that is closely related to the cost allocation problem is the potential independent plans operated by the producers, which is allowed by the Washington

legislation (and also many of the current EPR laws.) Such actions have been indeed taken place in practice, especially given the producers' concerns regarding their cost allocation under the return/market share method: In 2009, two producer groups have submitted proposals of independent plans to Ecology but was declined on the ground that they were insufficiently developed; yet these efforts are continued. Hence, in the long term, the program should be able to handle the co-existence of independent plans along with the currently operational standard plan. Achieving this requires an unambiguous definition of program rules regarding the individual responsibility of each plan, especially when multiple plans share collection, transportation, and processing capacities. When independent plans become operational, they may bring in additional capacities that are located out-of-state and that may be more efficient. This raises the question of how to control, harmonize and utilize these capacities to achieve a higher operational efficiency of the entire system, while still promoting local economic development. We also note that such an integration and harmonization issue has long been a major concern in the EU because of the need to coordinate the legislation in different countries [177]. A similar problem has already emerged in the US at the state level and can be expected to become a significant challenge to effective EPR implementation that calls for a national solution [115]. In §2.2, we also propose an approach to incentivize producers to contribute private capacities to a collective system, thus promoting scope economies in EPR implementation.

2.1.3.2 Design Incentives.

Since the introduction of the EPR concept, it has been argued that it is not simply about diverting waste away from landfills, but more about providing incentives to producers to design more environmentally friendly products ([101], [11], [107]). Thus, an essential element to be considered in designing e-waste regulation is the type of design incentives that it provides to manufacturers. Under this issue, problems to

be addressed include free-rider prevention, reuse/refurbish incentives, and toxicity reduction. It is clear that simple volume-based cost allocations (such as return or market share heuristics) that are essentially targeted at managing the allocation in an effortless manner, are not going to provide these incentives. At best, they can result in reduced consumption (through increased prices to cover for end-of-life expenses), reduced weight or reduced product size (called miniaturization in practice) in order to reduce end-of-life costs. If the real goal of e-waste regulation is to achieve design incentives, there is the opportunity to tailor the implementation to exploit design improvements (e.g. by source separation and routing of e-waste to the appropriate processors) and to reflect their actual cost to each manufacturer (e.g., product or recycling fee differentiation with respect to product toxicity). In §2.3, we discuss how to do so via cost allocation design based on an analytical study of the design implication of the current collective EPR implementation.

2.1.3.3 Other Challenges.

1. Better Reflecting EPR Goals into E-waste Legislation and Implementation

   - **Reuse and refurbishing**: Reuse and refurbishing are clearly essential components of EPR with a number of advantages from both an environmental and economic perspective. However, incorporating reuse and refurbishing raises implementation issues such as (i) whether a newer device will have sufficiently improved environmental performance that it outweighs the benefits of waste diversion [69], (ii) the complications that can arise in managing and accounting for the contributions of reuse and refurbishing operations in a traditional take-back setting, (iii) how to handle exporting of e-waste (and is indeed observed in practice) under the guise of reuse ([161], [14]). These factors need to be considered carefully before attempting to incorporate reuse and refurbishing targets into EPR laws. Nevertheless, there is an important opportunity to observe the

implications of such requirements as the recent revision of the European WEEE Directive [50] considers the inclusion of reuse and refurbishing operations in the scope of the directive. The European experience could provide valuable policy input as to the impact and feasibility of such targets.

2. Building A More Comprehensive E-waste Recycling Program

- **Product scope:** A bill amendment to add peripherals to the electronic products covered under the Washington program was proposed in 2011. The expansion of covered products indicates a level of maturity and acceptance of the program. However, expanding product scope also brings operational challenges and has the potential to exacerbate complications in achieving a fair cost allocation and strong design incentives in a collective implementation, as different products have different characteristics.

- **Downstream material flows:** One of the implicit objectives of e-waste legislation is to avoid e-waste exports to undesirable parties. However, products are not fully recycled at the processors and there are many more steps in the entire product recovery process beyond the current scope of such programs. Yet tracing all e-waste to its ultimate destination would be onerous if not impossible at a processor or even Ecology level. The piecemeal nature of e-waste laws in the US makes this even more difficult. Hence, it is impossible to know the ultimate destination and usage of the e-waste, which is a serious problem as it can defeat the environmental goal of an EPR program. In particular, at present, toxic trade into developing countries is still a prominent phenomenon in the downstream recycling business despite various influential anti-toxic-trade campaigns worldwide [14]. Clearly, federal e-waste legislation can help close within-country loopholes. Absent this, building a national clearing house for data on material exports by the largest recyclers can be an effective information-based

tool. In addition, specifying the level of post-disassembly material separation and processing can be effective because some of the greatest abuses happen if non-working whole units are exported.

3. Towards an Effective and Efficient EPR Implementation

- **Local economic development:** Maintaining the balance between program efficiency and local resource utilization has been a consideration from the earliest stages of the EPR debate in Washington. The embedded trade-offs between local economic development and the cost efficiency resulting from the use of state-of-the-art recycling technologies and also the economies of scale advantage remains an open question to be investigated for a well-balanced choice.

- **Education and outreach:** Washington drives collection by consumer education, relying on producers, retailers and local governments to expend effort to do so. Hence, it is crucial for the viability of the program to design and implement efficient consumer education campaigns to achieve extensive diffusion of the EPR concepts, especially in the presence of multiple plans in parallel.

- **Volume uncertainty:** There was (and still is) a lot of uncertainty about how much e-waste would be collected under the program. Moreover, given the amount of unused electronics that are expected to have accumulated in households (if not small businesses), and the diffusion dynamics of information about the program, it can be expected that the composition of the return volume will also change over time. It is necessary to understand the implication of such a trend for the operations in the existing collection and recycling system and to design implementation strategies accordingly. Meanwhile, it is also important to develop forecast mechanisms for the specific changes in the demand volume and distribution over the region.

- **Long-term contracts:** A barrier to recycling technology investment by processors is the non-contractual nature of the relationship between the Authority and recyclers. To overcome it, a first step would be to evaluate what type of long-term contract (if any) would be most effective at both maintaining a competitive environment and incentivizing investment under collection volume and mix uncertainty.

2.1.4 Further Remarks

We would like to mention that our multi-stakeholder analysis of EPR implementation in this section uncovers a strong relationship between some of the issues identified and the characteristics of the electronics industry. In fact, we find that the electronics industry combines some features that complicate the development of a comprehensive and efficient EPR system. We summarize these features as follows. First, due to the rapid technological obsolescence of electronics, many products are still in working condition when they are replaced. This creates a need for multiple forms of product recovery to extract the most value out of post-use products, including reuse, refurbishing and different levels of recycling. This not only complicates the operations of an EPR system, but also leads to more stakeholders with different perspectives being involved. Second, the potential residual value, combined with the toxicity of the product, creates export concerns that are difficult to manage within the scope of EPR legislation. Third, there is high product heterogeneity even within a small range of electronics. For example, TVs and computers are very different in terms of weight, recovery cost/revenue, market share evolution, etc. Hence, weight-based cost allocation purely based on market or return share may not be sufficient to reflect the true cost burden of each producer, and designing fair cost allocation mechanisms becomes a challenge. Fourth, different manufacturers have made different levels of progress with respect to engaging in product recovery. Some had years of experience

before the E-cycle program was launched and had established mature infrastructure of their own, while some were new to the EPR concept. This contributes to different attitudes towards the state legislation, and brings about the provision that allows the establishment of independent plans. Some of these features are specific to electronics and will not carry over to other product categories. For example, few of the products for which EPR legislation is diffusing in the US (mercury lights, carpets, packaging, paint, and pharmaceuticals) lend themselves to reuse or refurbishing. At the same time, the heterogeneity in producer perspectives as well as some of the other fundamental tensions discussed herein are expected to persist.

2.2 Efficient Implementation of Collective Extended Producer Responsibility Legislation

2.2.1 Introduction

Our analysis of the Washington EPR program in §2.1 indicates that designing a proper financing mechanism is of crucial importance to the implementation efficiency of EPR. It is also a very challenging task considering the different stakeholder perspectives and the heterogeneous nature of the collection and recycling system in the Washington state. Indeed, the cost allocation problem is a major issue in EPR implementation that has been intensively discussed in practice. In this section, we study this problem based on a general analytical model.

As we have mentioned in the previous section, the proper handling of e-waste is typically costly, hence EPR introduces a significant economic burden on the electronics industry, the main stakeholder group affected by EPR. The desire to minimize this cost burden has resulted in the prevalence of collective implementations, which are believed to "offer the simplest, most straightforward, and most cost-effective approach" [178]. In a typical collective system, such as the one in the Washington state, a system operator (state- or producer-run) manages a large scale collection and recycling network (CRN) with many origins (waste collection points) and destinations

(processors). The CRN collects, transports and processes a mixture of electronic waste consisting of many different product types originally sold by producers participating in that collective system. Such implementations not only allow for the exploitation of scale economies from consolidating waste volumes, but also capitalize on the synergies that arise from integrating and sharing available collection, transportation and recycling capacities, which we call the *network synergies*. Moreover, a collective implementation can reduce compliance monitoring costs. It can also encourage competition among service providers (collectors, transporters and processors) by employing competitive contracting [102, 178], further increasing cost efficiency.

An important consideration in operating a collective system is the allocation of the total cost among participating producers, as it directly impacts producers' willingness to stay in that collective system. Typically, the system operator allocates the total cost to participating producers by return share or market share, i.e., in proportion to their shares (by weight) in the total e-waste volume returned or sold. Such simple weight-based allocation mechanisms do not differentiate between producers even if they impose heterogeneous costs on the system. Thus, producers whose products incur lower transportation or processing costs can find that they are charged more in a collective system than their potential stand alone cost. For example, at a time when a cell phone could be recycled at a profit of about $0.50 [56], cell phone producers in Europe were charged 0.03 Euros (approximately $0.05) by collective recycling systems [47]. Another drawback of return/market share is that they do not take into account the heterogeneous contributions of different producers to network synergies. A simple example illustrates this.

Consider two producers A and B who have products π_A and π_B, respectively, with return volumes of two and one. They have individual processing resources r_A and r_B, respectively, with capacity equal to their own return volumes. Let c_j^i denote the cost of processing product $i = \pi_A, \pi_B$ on capacity $j = r_A, r_B$. Suppose $c_{r_A}^{\pi_A} =$

$1, c_{r_B}^{\pi_A} = \$2, c_{r_A}^{\pi_B} = \$2$, and $c_{r_B}^{\pi_B} = \$4$. In this case, if these producers independently operate (i.e., using only their own individual resources), they incur a cost of $2 and $4, respectively, for a total of $6. If they form a collective system, routing each product to the other producer's facility to exploit the network synergy yields a total cost of $5. However, under cost allocation by return share, producer A would pay $3.3 and would be worse off relative to its independent operating cost, despite contributing efficient processing capacity at r_A. Given the existence of producer owned or contracted capacities in practice (e.g., HP's recycling facility in California [80] and backhaul miles available to producers through their delivery networks), the situation demonstrated is an important practical concern.

These problems associated with weight-based proportional allocations have resulted in significant producer concern. In the E.U., some producers have left or stated their interest in leaving existing collective systems and establishing their own networks [146, 84]. Similar action is taking place in the U.S. as well. In Washington, as mentioned in §2.1, two independent system proposals were filed in 2009 by two separate producer groups who believed their stand-alone costs would be lower than their cost allocation under the collective system run by the Washington Materials Management and Financing Authority (WMMFA). While these proposals were declined on compliance grounds, such efforts are continuing [43] and the emergence of independent systems operating in parallel with the default collective system is a real possibility. In Oregon, three producer groups have already chosen not to participate in the state's default collective system.

System fragmentation, unless explicitly barred[2], can result in recycling operations that are economically inefficient. Yet cost efficiency is a key concern from both legislative and producer perspectives. For example, in the E.U., a key consideration

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[2]Collective implementations of EPR legislation increasingly allow the defection of individual producers, with a few exceptions (e.g., in Belgium).

in WEEE legislation is cost efficiency [81]. Similarly, recycling systems that are directly operated by producers, such as the European Recycling Platform, aim to ensure cost-effective implementation of the WEEE Directive [49]. In the Washington state implementation in the U.S., WMMFA, which is a producer board-directed state authority, aims to operate "in the most cost-effective manner" [181].

Hence, motivating the voluntary participation of producers in collective implementations is an important concern and requires alternative approaches to cost allocation. What these approaches should be has elicited much debate among producers and policy makers [38, 83] and has led to specific suggestions. For instance, [107] interpreted cost allocation in collective systems as an accounting problem and proposed ad hoc methods to improve on return share. Similarly, in Washington, WMMFA experimented with different heuristics that constitute adjustments to return share. The WEEE advisory board of the UK Department of Business Innovation and Skills is also in the process of analyzing three possible modifications to return share [83].

Despite these attempts, finding an effective cost allocation method remains an open question. One fundamental problem is that the existing approaches are unable to address the network synergies in collective systems. A key principle in the current discussions is to find mechanisms that "relate to the actual costs of dealing with producers' own products at end of their life" [83]. However, what constitutes one's "actual" cost is undefined in a collective system because the value generated from network synergies is determined by combining all volumes and capacities. An allocation that fails to capture such effects can be ineffective in inducing voluntary participation of producers even in a simple CRN. In the above two-producer example, if each producer were allocated exactly the amount incurred processing its products in the collective system, then A would incur a cost of $3. This is higher than A's stand-alone cost of $2 and would motivate A to break away.

Hence, the goal of this section is to develop an effective tool for system operators

to implement EPR in a way that motivates the voluntary participation of producers. To this end, we adopt a network model of the operations of the collective system to capture network effects, and focus on the incentives that drive producers' defection from that system either individually or with others. A natural benchmark to evaluate such incentives is the *stand-alone* cost that a producer or a sub-coalition of producers can achieve. Hence, a desirable property of an allocation that induces voluntary participation is to charge each producer or sub-coalition no more than its stand-alone cost. We call this requirement *group incentive compatibility*. This notion, together with the property that all cost is allocated, is the concept of the *core* in cooperative game theory, which is one of the fundamental allocations widely studied in the literature [185]. The core notion is particularly appealing in our problem setting as it addresses the fragmentation issue that has plagued many implementations to date.

We note that proportional return/market share models remain widely used in practice (despite their ineffectiveness in motivating producers' participation) because of their intuitive nature and simplicity. Thus, we focus on identifying group incentive compatible mechanisms that can be presented as improvements to the return share model[3] in any CRN. In particular, we look for improvements based on measures that capture the cost difference of products and producers' heterogeneous contribution to network synergies. As our analysis will show, the core contains allocations that have these properties.

We study the cost allocation design problem using a cooperative network flow game with shared collection, transportation and processing capacities. We show that the cost allocation by return share is typically not group incentive compatible when

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[3]We do not take market share as the basis for cost allocation as it does not take into account return share differentials between producers, and is considered to be more arbitrary than return share [84].

there is cost and capacity heterogeneity in the system except under restrictive conditions regarding the relationship between the two factors. We demonstrate that two types of adjustments to return share can greatly reduce or eliminate its *incentive compatibility gap* (the cost increase experienced by producers in the collective system compared to their stand-alone costs): (i) making capacity-based side payments to producers who provide access to resources that can handle (collect, transport and process) e-waste more cost effectively; and (ii) adjusting/weighing return shares to reflect the processing cost differentials between products or the use of critical resources (i.e., highly-utilized, low-cost resources) in the network. These adjustments provide succinct and intuitive ways of capturing network synergies while remaining true to the return share concept.

In addition, we address three important issues. (i) Economies of scale is one of the main reasons why producers and policy makers argue for collective systems. An analysis of the implications of economies of scale, however, uncovers a surprisingly negative side-effect: Under proportional cost allocations that are not group incentive compatible (e.g., return share), scale economies may increase the incentive compatibility gap and introduce a stronger incentive for producers to break away. In particular, this can happen when products that generate revenues from processing are handled in the same network as those that impose costs. (ii) Some EPR bills mandate a penalty for producers who operate their own collection and recycling networks but do not fulfill their obligation. We show how the adjustments we propose can be implemented to operationalize this concept while ensuring a group incentive compatible allocation. (iii) An implementation requirement for the proposed adjustments is information on the operational costs and volumes of each producer's e-waste flows, which can be obtained by separating waste at the collection stage. While costly, the value that can be obtained from doing so (i.e., inducing industry-wide participation in the collective system to maximize cost efficiency) can justify the associated costs. Identifying

conditions under which the cost efficiency benefits of such information are realized is another practical contribution of our work.

To show how our results would translate into practice, we develop a sample network and a representative cost structure based on the Washington state implementation. A set of numerical experiments illustrate the concepts developed in this section and yield insights regarding their implementation. We observe that the incentive compatibility gap of the allocation by return share, and the associated efficiency loss from fragmentation, can be substantial. Economies of scale is shown to reduce the incentive compatibility gap of return share (without guaranteeing a core allocation), while greatly accentuating efficiency loss from fragmentation. We demonstrate that the economic implications of legislative targets are strongly influenced by network synergies.

In sum, our study in this section provides value at three levels: It identifies the operational-level factors that cause return share to be an ineffective mechanism in ensuring the voluntary participation of producers, and resolves the problem by providing simple implementable adjustments. It provides system operators new directions in how to manage cost allocation in collective systems and guidance to producers as to what to lobby for vis-a-vis the legislature. Finally, it informs policy makers and producer responsibility organizations about the implications of key legislative choices in EPR bills.

2.2.2 Contribution to the Literature

A stream of research in the environmental economics literature studies the economics of regulated collection and recycling of post-consumer products [125, 126, 55, 127, 172, 25, 26, 170, 171]. These papers use stylized economic models to identify the optimal form of environmental legislation that maximizes social welfare. The main finding in this stream of literature is that a deposit-refund policy maximizes social welfare. Yet,

44

the practice of e-waste legislation around the globe has converged to EPR implementations with mandated collection and recycling level targets imposed on producers. The recent operations management literature recognizes that implementing EPR is essentially an operational problem, and investigates how the principles of EPR can be effectively translated into working systems [164, 131, 12], but using stylized models of operational decisions that do not capture network synergies. Our work contributes to this growing literature by explicitly capturing the network synergies in a CRN, without which group incentive compatible cost allocations cannot be designed.

Another relevant stream in the operations literature [52, 88, 182, 20, 103, 140, 154] focuses on designing a reverse logistics network when product returns are valuable. [114] studies the design of e-waste networks, considering a competitive market for e-waste recycling. However, these papers do not investigate the efficient design and operation of collective recycling networks within the context of environmental regulation. We contribute to this literature by (i) explicitly modeling practical issues in designing and implementing recycling networks when this activity is costly and carried out because of EPR legislation, and (ii) identifying cost allocation mechanisms that guarantee voluntary participation of producers and thus efficient collective EPR implementations.

Cooperative game theory has been widely applied in operations management literature; one prominent application is to analyze profit sharing in supply chain alliances with resource sharing (see [112] for a survey). However, most of these papers focus on traditional supply chain problems and do not discuss product recovery issues. One paper that studies recycling activities is [162]. The authors investigate the structure of coalitions that would emerge given an exogenous unit recycling cost that is charged to each member of a coalition and that depends on the size and product diversity of the coalition; this cost structure defines an implicit cost allocation. In our approach,

we endogenously determine a coalition's total cost by solving a network flow problem which captures the cost heterogeneity and the network synergies, and focus on developing cost allocations that promote participation in the collective system.

From a methodological perspective, we also contribute to the literature on cooperative network games [91, 92, 60, 39, 123, 142]. A classical result in this area is that a dual-based cost allocation is guaranteed to be in the core. Ease of implementation, on the other hand, favors the use of simple proportional allocations. This is the case in the EPR context, where return share is prevalent. Thus, an important practical consideration for the cost allocation problem under collective EPR is identifying a set of core allocations that can be presented as derived from the return share concept. In this work, we show that adjustments to the return share model can achieve a core cost allocations under very general conditions. More importantly, we characterize the adjustments that matter, i.e., weighing return shares by cost burden and critical resource usage.

Collective EPR implementations often involve using both producers' independent capacities and those contracted by the system operator. By studying such a system as a cooperative network game, we also introduce a new perspective on the use of a mixture of player-owned and exogenous resources in collaboration. Existing analyses of such games (e.g., the pseudo-flow game introduced by [91] and the simple flow game studied in [134]) assume that the exogenous capacity is a public resource that is available free of charge, and show that the core of such games can be empty under certain conditions. [59] analyzes an extended linear production game where anyone may purchase the exogenous capacity in bundles, and gives bundle prices to guarantee the existence of the core. We extend this stream of research by studying a hybrid model in §2.2.4.5, where the exogenous capacity is operator-contracted and thus accessible at no additional cost to the operator-run grand coalition, while independent

sub-coalitions are allowed access to an arbitrary fraction of it for an additional (non-member) fee. Such user differentiation reflects the policy in some EPR bills under which capacity shortfalls in independent CRNs can be complemented by operator-contracted capacity at a surcharge. We derive lower bounds for the unit access fees under which the core must exist.

The organization of the rest of §2.2 is as follows: In §2.2.3, we introduce the notation, define the CRN, and develop the cooperative game model. §2.2.4 provides an analysis of the model and identifies group incentive compatible cost allocation methods under different operating environments. §2.2.7.4 illustrates how these results can be implemented in practice using data from Washington state and highlights the practical value of the results. §2.2.6 concludes with a summary of the results and discussion.

2.2.3   Model Description

In this section, we first introduce the notation and structure of the *collection and recycling network* (CRN) that represents the practice of e-waste collection and recycling. We then compute the minimum total operating cost that can be achieved in a CRN. Finally, we formulate a cooperative game model based on which we mathematically define the notion of a group incentive compatible cost allocation that will guarantee the attainment of this minimum cost.

2.2.3.1   Network Model Formulation.

A typical collection and recycling system of e-waste consists of four components: collection, consolidation, transportation and processing (dismantling, shredding, and/or recycling). We model the operations of such a system as a multicommodity network (Figure 3), called the collection and recycling network (CRN), where a variety of post-use products from different producers are handled.
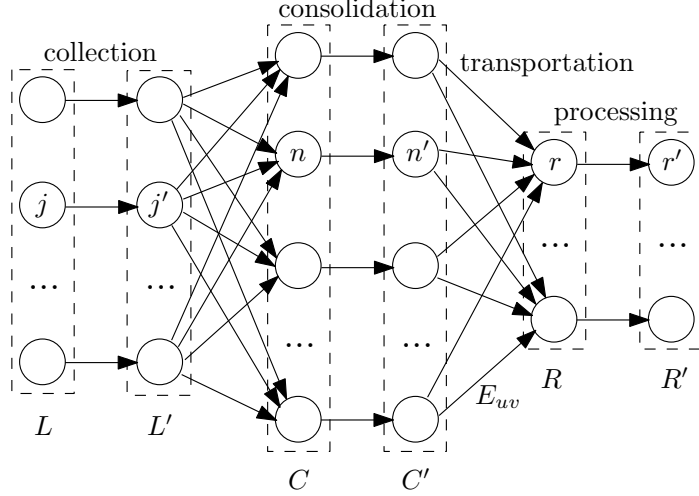
Figure 3: The structure of a generic collection and recycling network.

The Network To formulate a CRN, we construct three sets of nodes: $L = \{j : j \text{ is a collection point}\}$, $C = \{n : n \text{ is a consolidator}\}$, and $R = \{r : r \text{ is a processor}\}$. We denote the set of all edges as E. To represent capacity restrictions at each entity as edge capacities, we duplicate node sets L, C and R by generating $L' = \{j' : j \text{ is a collection point}\}$, $C' = \{n' : n \text{ is a consolidator}\}$ and $R' = \{r' : r \text{ is a processor}\}$, and link each original node with its counterpart in the duplicated sets. The capacities at collector j, consolidator n and processor r are then modeled as edge capacities on edges (j, j'), (n, n') and (r, r'), respectively. Transportation capacities on edges between any pair of nodes (u, v) in L' and C, and in C' and R are modeled by a set of parallel edges E_{uv}, where each edge corresponds to a different transporter.

Products, Costs, and Legal Requirement We denote a producer by i and the set of producers by M. Every producer i makes the set of products Π^i; these sets are mutually exclusive with each other. The set of all products is denoted by $\Pi = \bigcup_i \Pi^i$. On each edge e, the corresponding entity incurs an *operational cost* c_e^π to collect, transport or process a unit of product π. Moreover, each processor r also pays a *downstream cost* σ_r^π for or obtains a *downstream revenue* ρ_r^π (which is modeled as a *negative* cost) from sending the parts and materials extracted from a

48

unit of processed product π to downstream vendors, brokers and/or recyclers. For example, TVs typically incur downstream processing costs, while computers generate processing revenues. Hence, the *net processing cost* to r to process a unit of product π is $\hat{c}_r^\pi = c_{(r,r')}^\pi + \sigma_r^\pi + \rho_r^\pi$. In our main model, we assume that the unit costs do not depend on the volumes handled, i.e., there are no scale economies. We relax this assumption in §2.2.4.4 and show how our results can be extended under scale economies.

We use d_j^π to denote the waste volume (by weight) of product π at collection point j, i.e., the amount of product π brought to j by the consumers. These waste volumes determine a producer's legal obligations given a mandated recycling requirement, modeled by the parameter τ, which is the minimum fraction of the return volume of each product that should be recycled during the entire collection and recycling process (including all downstream stages). Since landfilling mainly occurs after the processing/downstream recycling stage, and is outside the boundary of the CRN that is relevant for cost allocation purposes, we capture the influence of the stringency of such a requirement by modeling $c_{(r,r')}^\pi$, σ_r^π and ρ_r^π, and thus the net processing cost $\hat{c}_r^\pi$ at each processor r, as functions of τ, i.e., $\hat{c}_r^\pi(\tau) = c_{(r,r')}^\pi(\tau) + \sigma_r^\pi(\tau) + \rho_r^\pi(\tau)$. Such a relationship is derived from the fact that the stringency of the mandated recycling requirement influences the processing operations as well as where the resulting parts and materials are sent. For example, when the mandated recycling requirement is stringent, the processors should not only refrain from direct landfilling, but also contract with downstream recyclers that are able to thoroughly recycle the lead and the glass obtained from CRT TVs/monitors, both of which increase the processing costs.

Capacities Producers can privately own or contract for collection, transportation and processing capacities, called their *independent capacity*. We denote the CRN formed with the independent capacity of producer i by N^i, where each edge e has

k_e^i amount of independent capacity available. There is a system operator whose role is to establish a collective CRN that includes the independent capacities of those producers who are willing to join it and that collects and processes their e-waste, as well as that of producers with no independent capacity. The operator can also contract for additional capacity, which we call *operator-contracted capacity*, up to a limit of K_e^p on edge e, so as to supplement the independent capacity brought by the producers who join the collective CRN. We assume that independent capacities are reported truthfully to the operator; in practice, verification of capacities is feasible through a certification process (e.g., in Washington the Department of Ecology certifies collectors, transporters and processors.)

Minimum Total System Cost Let N denote the network obtained by pooling all independent and operator-contracted capacity together. Because it includes all available resources, the total cost to collect and process all the producers' products will be minimized on N. Given a certain stringency level τ, this minimum total cost can be computed by a minimum cost flow problem defined on N, which we call the *centralized problem* (C).

$$(C): \quad \min \quad Z(f) = \sum_{e \in E \setminus \{(r,r')\}} \sum_{\pi \in \Pi} c_e^\pi \cdot f_e^\pi + \sum_{r \in R} \sum_{\pi \in \Pi} \hat{c}_r^\pi(\tau) \cdot f_{(r,r')}^\pi \tag{1}$$

$$s.t \quad \sum_{e=(u,v) \in E} f_e^\pi - \sum_{e=(v,w) \in E} f_e^\pi = 0 \quad \forall v \in V \setminus \{L, R'\}, \forall \pi \in \Pi \quad [v_v^\pi] \tag{2}$$

$$f_{(j,j')}^\pi = d_j^\pi \quad \forall \pi \in \Pi, \forall j \in L \quad [\beta_j^\pi] \tag{3}$$

$$\sum_{\pi \in \Pi} f_e^\pi \leq \sum_{i \in M} k_e^i + K_e^p \quad \forall e \in E \quad [\alpha_e] \tag{4}$$

$$nonnegativity \quad constraints. \tag{5}$$

In the above program, (2) represents flow conservation constraints at every node except for the source and terminal nodes, (3) guarantees that all collected units are processed, and (4) represents the capacity constraint on every edge in N. The variable listed beside each constraint is its corresponding dual variable. The objective

function (1) minimizes the total cost incurred by the flow in the entire network N under the mandated recycling requirement τ. The optimal solution to (C) is denoted by f^*, which we call the *socially optimal routing*. Note that the implementation of f^* requires operator-contracted capacity to be used on edge e at the level of $k_e^p \doteq \max\{0, \sum_{\pi \in \Pi}(f^*)_e^\pi - \sum_{i \in M} k_e^i\}$ $\forall e \in E$. Thus, an equivalent formulation of (C) can be obtained by replacing K_e^p with k_e^p.

In conclusion, $Z(f^*)$ represents the optimal system cost that can be achieved given τ. However, to attain $Z(f^*)$ requires the formation of the collective network N by pooling all independent and operator-contracted capacity so that f^* can be implemented by the system operator. This is not guaranteed to happen. The problem is that individual producers or sub-coalitions can be better off by not joining the centrally-operated system if the cost allocated to them under the centrally-operated system is above their stand-alone costs. Thus, modeling the perspectives of individual producers is an important part of the problem, which is discussed next.

2.2.3.2   A Cooperative Game on CRNs.

In this section, we formulate a cooperative game on the CRNs, called the *collection and recycling flow* (CRF) *game*. In this game, individual producers are assumed to have two options: (i) join the centrally-operated CRN or (ii) form and operate an independent CRN individually or in a sub-coalition.

In option (i), it is the system operator that oversees and coordinates the e-waste flow of the participating producers using the independent capacities of those producers and additional operator-contracted capacity, if needed. Specifically, independent and operator-contracted capacity availabilities and unit prices are collected by the system operator. The return volumes are observed, routed optimally by the operator, and processed. The operator is billed by the collectors, transporters and processors based on the volumes they handled, at the unit prices previously communicated. Note that

in this model, the unit cost of using independent capacity is assumed to be unchanged when integrated into the centrally-operated CRN under the same mandated recycling requirement τ. The participating producers pay the portion of the total cost in this CRN that is allocated to them by the operator according to the cost allocation mechanism in force. Such an amount is often determined ex-post in practice. For example, under the return share mechanism, both the total cost and producers' return shares are observed/calculated after the products are processed, which are then used to determine the amounts to be billed to producers.

In option (ii), producers process their products in a sub-coalition, using only the independent capacities belonging to that sub-coalition. We call the network associated with such a coalition an *independent CRN*. Capacity within an independent CRN is restricted to the usage of the corresponding sub-coalition members. Moreover, in this main model, we assume that sub-coalitions cannot access operator-contracted capacity; this assumption is relaxed in §2.2.4.5, where we allow sub-coalitions operating their independent CRNs to pay a fee to use operator-contracted capacity.

When all producers choose to join the centrally-operated CRN, the *centrally-operated grand coalition* is formed. If not, a *fragmented system* is created, which consists of the centrally-operated CRN and independent CRNs of defecting sub-coalitions. Mathematically, we define the value of the centrally-operated grand coalition, $v(M)$, to be the minimum total system cost on network N (that includes the operator-contracted capacities), i.e., $Z(f^*)$. The value of sub-coalition $S \subsetneq M$ in a CRF game, $v(S)$, is defined as the minimum total cost achievable on the corresponding independent CRN, and can be computed by a program (C^S) that can be interpreted as the centralized problem within S. (C^S) differs from (C) in that (i) the product set is restricted to $\Pi^S = \bigcup_{i \in S} \Pi^i$, and (ii) sub-coalitions can only use their independent

capacities, i.e., constraint (4) is replaced with

$$\sum_{\pi \in \Pi^S} f_e^\pi \leq \sum_{i \in S} k_e^i \quad \forall e \in E.$$ (6)

Based on this CRF game model, we define a cost allocation by $x = \{x^i, \forall i \in M\}$ such that $\sum_{i \in M} x^i = v(M)$, i.e., all cost is allocated. In the context of our problem, the central question in allocating the costs is to prevent defection of producers (i.e., having them choose option (i)). The notion of the *core* of the CRF game [57] provides a good solution to this problem: An allocation x provides incentives for the participation of all producers in the collective system if $\sum_{i \in S} x^i \leq v(S) \ \forall S \subsetneq M$, i.e., no sub-coalition of producers is allocated a higher cost within the centrally-operated grand coalition compared to operating their independent CRNs. In this thesis, we refer to such cost allocations as being *group incentive compatible*.

2.2.4 Designing Group Incentive Compatible Cost Allocations

We first analyze the cost allocation by return share and identify its shortcomings. Based on this analysis, we construct an allocation mechanism that is obtained by making adjustments to return share and is guaranteed to be a core allocation of the CRF game. This mechanism is called *cost-corrected return share with capacity rewards* (§2.2.4.2). We then identify an alternative mechanism (*return share with capacity rewards*) that does not require cost adjustments while guaranteeing group incentive compatibility under mild conditions (§2.2.4.3). We conclude with a discussion and analysis of practical concerns: economies of scale (§2.2.4.4) and non-member access fees (§2.2.4.5).

2.2.4.1 Cost Allocation by Return Share.

A producer's *return share* is defined as the ratio of the producer's products returned to the total amount of electronic products returned by weight [42]. For notational simplicity, we denote the return volume belonging to sub-coalition S as

$R^S \doteq \sum_{j \in L} \sum_{\pi \in \Pi^S} d_j^\pi$, and let $R \doteq \sum_{i \in M} R^i$ be the total volume of products returned. We assume $R^i > 0 \ \forall i \in M$. The cost allocation by *return share*, x_r, is defined such that the cost allocated to producer i is computed as

$$(x^i)_r \doteq v(M) \cdot \frac{R^i}{R}. \tag{7}$$

Note that $\bar{v}^M \doteq \frac{v(M)}{R}$ can be interpreted as a flat rate charge equal to the average cost within the centrally-operated grand coalition. Let $\bar{v}^S \doteq \frac{v(S)}{R^S}$ be the average cost within a sub-coalition $S \subsetneq M$ operating an independent CRN. We can evaluate the maximum cost increase experienced by a sub-coalition $S \subseteq M$ compared with its stand-alone cost $v(S)$ under the allocation by return share. We call this measure the *incentive compatibility gap* of the allocation and denote it by $\mathcal{G}(x_r)$, where

$$\mathcal{G}(x_r) \doteq \max_{S \subseteq M} \{ \sum_{i \in S} (x^i)_r - v(S) \} = \max_{S \subseteq M} \{ R^S \cdot (\bar{v}^M - \bar{v}^S) \}. \tag{8}$$

The last term in (8) indicates that the incentive compatibility gap can be interpreted as the maximum increase in the average unit cost that a sub-coalition S will experience when joining the grand coalition multiplied by its return volume over all $S \subseteq M$. The change in the average unit cost for S, $\bar{v}^M - \bar{v}^S$, is influenced by network synergies. Intuitively, a sub-coalition of producers who make cheaper-to-recycle electronics and has established an efficient collection and recycling infrastructure with sufficient capacity tends to have a smaller average unit cost when operating its independent CRN, and thus is more likely to suffer a cost increase from joining the grand coalition when cost is allocated by return share. This intuition is substantiated by our following analysis on the network conditions for the allocation by return share to be in the core of the CRF game. First, we present a general sufficient condition in Proposition 1.

Proposition 1. *Given a CRN, x_r is in the core of the CRF game if* (i) *for any edge $e \in E$ and any processor $r \in R$, the operational cost and net processing cost of all products are identical, i.e., $c_e^\pi = c_e$ and $\hat{c}_r^\pi = \hat{c}_r \ \forall \pi \in \Pi$;* (ii) *for each product π, its*
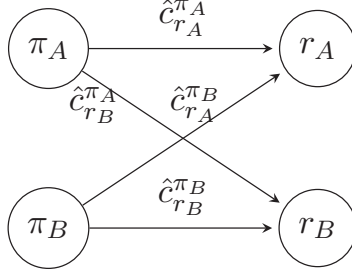
Figure 4: An equivalent network structure of CRN_2.

return share is the same at all collection points and is equal to its return share in the entire CRN, i.e. $\frac{d_j^\pi}{\sum_{\pi \in \Pi} d_j^\pi}$ is equal $\forall j \in L$; and (iii) *the socially optimal routing f^* can be implemented without operator-contracted capacity, i.e., $\sum_{\pi \in \Pi} f_e^{*\pi} < \sum_{i \in M} k_e^i$ $\forall e \in E$.*

Proof. All proofs are presented in Appendix 2.2.7.1. □

Under the sufficient conditions presented in Proposition 1, the CRN is entirely homogeneous and essentially uncapacitated with respect to independent capacities. These sufficient conditions are very restrictive and generally not satisfied in practice. Hence, further analysis is needed regarding the stringency of the necessary conditions to ensure group incentive compatibility of return share. Since developing insightful necessary conditions is very hard in a general network setting, in this analysis we consider a special case of the general CRN, denoted by CRN_2 (Figure 4), and focus on the impact of the heterogeneity in processing costs and capacity.

In CRN_2, there are two producers A and B who have products π_A and π_B with return volume d^{π_A} and d^{π_B}, respectively. They have independent processing resources r_A and r_B, respectively, with capacities $k_{r_A} \geq d^{\pi_A}$ and $k_{r_B} \geq d^{\pi_B}$. Without loss of generality, let r_A be more efficient than r_B such that both products are processed more cheaply at r_A, i.e., $\hat{c}_{r_A}^{\pi_A} < \hat{c}_{r_B}^{\pi_A}$ and $\hat{c}_{r_A}^{\pi_B} < \hat{c}_{r_B}^{\pi_B}$. Proposition 2 characterizes the necessary and sufficient conditions for the allocation by return share to be in the core.

Proposition 2. *Consider CRN_2 as defined above. When $\hat{c}_{r_A}^{\pi_A} \leq \hat{c}_{r_A}^{\pi_B}$, x_r is in the*

core of the CRF game if and only if (i) there is sufficient capacity at processor r_A to process both products, i.e., $k_{r_A} \geq d^{\pi_A} + d^{\pi_B}$, and (ii) the unit processing costs of both products at processor r_A are identical, i.e., $\hat{c}_{r_A}^{\pi_A} = \hat{c}_{r_A}^{\pi_B}$. When $\hat{c}_{r_A}^{\pi_A} > \hat{c}_{r_A}^{\pi_B}$, there exist two constants $\underline{\Delta}$ and $\bar{\Delta}$ such that x_r is in the core of the CRF game if and only if the difference between the unit processing costs of product π_A and π_B at processor r_A satisfies $\underline{\Delta} \leq \hat{c}_{r_A}^{\pi_A} - \hat{c}_{r_A}^{\pi_B} \leq \bar{\Delta}$.

Proposition 2 indicates that even under a two-producer setting, return share is not group incentive compatible unless under very restrictive conditions. When $\hat{c}_{r_A}^{\pi_A} \leq \hat{c}_{r_A}^{\pi_B}$, producer A has both the more efficient capacity and the product that is cheaper to recycle. Thus, in order to ensure his participation in the collective system under return share, the average cost in the collective system should equal $\hat{c}_{r_A}^{\pi_A}$. This is essentially equivalent to all products being recycled at the same cost under the optimal routing, which requires a homogeneous and uncapacitated CRN. This condition becomes less stringent in the case where $\hat{c}_{r_A}^{\pi_A} > \hat{c}_{r_A}^{\pi_B}$, i.e., where the producers are complementary as they either contribute a cheaper-to-recycle product or an efficient processor to the collective system. In this case, a certain degree of cost heterogeneity between the products is allowed given it is confined to a certain range $[\underline{\Delta}, \bar{\Delta}]$ that essentially reflects a balance between the cost burden and the capacity contributions of the two producers (see Appendix 2.2.7.1 for the detailed formulae of $\underline{\Delta}$ and $\bar{\Delta}$).

The above observations provide insights regarding the factors that drive the incentive compatibility gap of the allocation by return share in general, namely, cost heterogeneity and independent capacity contribution. Accounting for those factors helps us identify the right ingredients for designing a cost allocation model based on return share that is in the core, as illustrated in the next section.

2.2.4.2 A Cost Allocation in the Core: Cost-corrected Return Share with Capacity Rewards.

The adjustments we propose to cost allocation by return share account for the differences among producers regarding their cost burdens and their differences in capacity usage versus contribution. First, recall that $v^{*\pi}$, $\beta_j^{*\pi}$ and α_e^* denote the dual optimal solutions of the centralized problem (C) with respect to the constraints (2) - (4). Hence, we can interpret the term $\beta_j^{*\pi}$ as the marginal cost to process one additional unit of product π returned to collection point j in the centrally-operated grand coalition, which captures the network synergies under a cost-minimization objective due to the requirement that all e-waste returned is to be processed. We weigh the return volumes of products at each collection point by their marginal costs to obtain a *cost-corrected return share* for each producer i, denoted by μ^i:

$$\mu^i = \frac{\sum_{j \in L} \sum_{\pi \in \Pi^i} d_j^\pi \beta_j^{*\pi}}{\sum_{j \in L} \sum_{\pi \in \Pi} d_j^\pi \beta_j^{*\pi}} . \tag{9}$$

Second, let p_e denote the unit reward price on edge e. Then producer i receives a total capacity reward equal to $\sum_{e \in E} p_e k_e^i$, i.e., producers are compensated for their independent capacity contributions to the grand coalition according to a set of unit reward prices on the edges of the CRN. These monetary rewards increase the total cost to be allocated to $v(M) + \sum_{i \in M} \sum_{e \in E} p_e k_e^i$.

Consider the following allocation, denoted by x_μ^p:

$$(x^i)_\mu^p = \left[v(M) + \sum_{e \in E} p_e \sum_{i \in M} k_e^i \right] \cdot \mu^i - \sum_{e \in E} p_e k_e^i \quad \forall i \in M. \tag{10}$$

We call this allocation method *cost-corrected return share with capacity rewards* and prove that it guarantees group incentive compatibility.

Theorem 1. *Given any CRN, $\exists$ capacity reward prices $p_e \geq 0$ $\forall e \in E$ such that x_μ^p is in the core of the CRF game.*

Theorem 1 is a strong result showing that simple but powerful adjustments to return share can guarantee a core allocation. The proof proceeds as follows: When

$\mu^i \geq 0 \ \forall i \in M$ (i.e., all producers exert a nonnegative cost burden on the system), it can be shown that if the capacity reward price p_e is set equal to $|\alpha_e^*| \ \forall e \in E$, x_μ^p resides in the set of dual-based allocations, and therefore, according to Theorem 2 in [58], must be contained in the core of the CRF game. The situation becomes more complicated when there exists a large heterogeneity in product costs and some producer i makes a net revenue contribution to the system (i.e., $\mu^i < 0$). In that case, the above capacity reward prices $p_e = |\alpha_e^*|$ cannot guarantee a core allocation if operator-contracted capacity exists, but we show constructively that a set of prices based on $\{|\alpha_e^*|\}$ can be found that results in an allocation equivalent to a dual-based allocation.

The practical value of this mechanism is that it can be presented as an allocation based on the return share notion with adjustments for operational costs and independent capacity contributions. To illustrate how Theorem 1 would be implemented, consider the simple example presented in §1, for which the optimal dual solution is $\beta_{\pi_1}^* = 2, \beta_{\pi_2}^* = 3, \alpha_{r_1}^* = -1$, and $\alpha_{r_2}^* = 0$. Hence, we calculate that $\mu^A = \frac{2 \cdot 2}{2 \cdot 2 + 1 \cdot 3} = \frac{4}{7}$ and $\mu^B = \frac{3}{7}$, and set capacity rewards $p_{r_1} = 1$ and $p_{r_2} = 0$ (only capacity at r_1, i.e., only producer A, is rewarded). By formula (10), the total cost $v(M) = \$5$ should be allocated such that $(x^A)_\mu^p = \frac{4}{7}(5 + 1 \cdot 2) - 1 \cdot 2 = 2$ and $(x^B)_\mu^p = \frac{3}{7} \cdot (5 + 1 \cdot 2) = 3$, which is clearly a core allocation.

2.2.4.3   An Alternative Cost Allocation Model.

In this subsection, we exploit the underlying network structure to develop an alternative model that can generate a core allocation by only focusing on capacity rewards. Consider the following cost allocation model (*return share with capacity rewards, x_r^p*), which uses simple return shares instead of cost-corrected return shares compared to formula (10):

$$(x^i)_r^p \doteq \left[v(M) + \sum_{i \in M} \sum_{e \in E} p_e k_e^i \right] \cdot \frac{R^i}{R} - \sum_{e \in E} p_e k_e^i \quad \forall i \in M . \tag{11}$$

58

It is easy to see that when the capacity reward prices p_e on all edges are set to zero, this model would be equivalent to the return share model. Hence, to what degree capacity rewards can reduce the incentive compatibility gap of the allocation by return share and whether this model can guarantee a core allocation depends on how the $\{p_e\}$ are chosen. For example, in the simple example of §1, we can set $p_{r_1} = 2, p_{r_2} = 0$ and thus obtain the core allocation $(x^A)_r^p = \frac{2}{2+1} \cdot (5 + 2 \cdot 2) - 2 \cdot 2 = \$2$ and $(x^A)_r^p = \frac{1}{2+1} \cdot (5 + 2 \cdot 2) = \$3$. However, it is not straightforward whether a core allocation can be achieved in general.

To explore this issue further, let $\bar{k}_e^S = \frac{\sum_{i \in S} k_e^i}{R^S}$ denote the ratio of the independent capacity availability on edge e to the total return volume of producers in S, i.e., the *normalized independent capacity* of S. We can then define the incentive compatibility gap of x_r^p given reward prices $\{p_e\}$ similarly as that for the allocation by return share, and obtain

$$\mathcal{G}(x_r^p) \doteq \max_{S \subseteq M} \{\sum_{i \in S} (x^i)_r^p - v(S)\} = \max_{S \subseteq M} \{R^S \cdot [\bar{v}^M - \bar{v}^S - \sum_{e \in E} p_e(\bar{k}_e^S - \bar{k}_e^M)]\}. \quad (12)$$

The last term in (12) allows us to observe that the effectiveness of capacity rewards largely depends on the value of $\bar{k}_e^S - \bar{k}_e^M$, which measures the difference between the normalized independent capacity in S and in the grand coalition M. A critical observation is that *only* sub-coalitions with $\bar{k}_e^S > \bar{k}_e^M$ on at least one edge can potentially benefit from such a capacity reward. Hence, intuitively, we can expect the capacity rewarding mechanism to be effective in reducing the incentive compatibility gap of return share when the average independent capacity availability in the grand coalition is low. In order to analyze this situation further, we assume for the rest of section 2.2.4.3 that the collective independent capacity contracted by all producers is insufficient to process the total volume of products returned, necessitating additional operator-contracted capacity. We analyze the factors limiting the maximum throughput in the network $\bigcup_{i \in M} N^i$ (i.e., the grand coalition network without operator-contracted capacity) as follows.

We first transform the CRN into a capacitated single-commodity network by adding an artificial origin node o that is linked to each of the collection points j via a fictitious edge (o, j) with a capacity equal to $\sum_\pi d_j^\pi$, the total return volume at j. We also add an artificial destination node d and connect every node $r' \in R'$ to d with an infinite capacity edge. Let the resulting network be called N_o^d. Then the maximum flow on N_o^d is equivalent to the maximum throughput of $\bigcup_{i \in M} N^i$. A *cut* in N_o^d is a set of nodes containing the origin o but not the sink d and the corresponding *cut set* is defined by the set of edges that cross the cut. Let $\bar{C}$ be the minimum capacity $o - d$ cut in N_o^d and let $E(\bar{C})$ be its cut set. The max-flow-min-cut theorem [5] indicates that the maximum flow passing from o to d in N_o^d equals the total capacity of $E(\bar{C})$. Let the capacity of $E(\bar{C})$ be $K_{E(\bar{C})} \doteq \sum_{e \in E(\bar{C}) \backslash \{(o,j), j \in L\}} \sum_{i \in M} k_e^i + \sum_{j:(o,j) \in E(\bar{C})} \sum_\pi d_j^\pi$. Hence, $K_{E(\bar{C})} < R$ indicates an inadequate level of independent capacity in N_o^d.

Theorem 2. *Assume the minimum cut $\bar{C}$ is unique. If $K_{E(\bar{C})} < R$ and $E(\bar{C}) \cap \{(o, j), j \in L\} = \emptyset$, indicating that the throughput of $\bigcup_{i \in M} N^i$ is not restricted by the return volumes at the collection points, then $\underline{\mathcal{G}}(x_r^p) = 0$, i.e., $\exists$ capacity reward prices $p_e \geq 0$ $\forall e \in E$ such that return share with capacity rewards generates an allocation in the core of the CRF game.*

The intuition behind this result is the following: The condition $K_{E(\bar{C})} < R$ implies an inadequate independent capacity availability within the grand coalition M on the edges in the minimum cut set. Hence, for any sub-coalition S that has sufficient independent capacity to operate its own independent CRN, $\bar{k}_e^S > \bar{k}_e^M$ on at least one edge in $E(\bar{C})$. Since we assume $E(\bar{C}) \cap \{(o, j), j \in L\} = \emptyset$, this edge must be in the original CRN and we can associate a capacity reward with it. According to our analysis to formula (12), this fact guarantees that the capacity rewarding mechanism potentially can benefit all sub-coalitions that are likely to break away. In other words, there exists a set of nonnegative reward prices $\{p_e\}$ that can adjust return share to be incentive compatible. Note that Theorem 2 can also be extended to cases where

the min-cut is not unique (see Appendix 2.2.7.2).

Also note that under the conditions in Theorem 2 and assuming the minimum cut $\bar{C}$ is unique, an additional unit returned to any collection point j cannot increase the maximum throughput of $\bigcup_{i \in M} N^i$ since $E(\bar{C}) \cap \{(o,j), j \in L\} = \emptyset$. This indicates that the *volume burdens* of products on $\bigcup_{i \in M} N^i$ are not differentiated. Such homogeneity is essential to the group incentive compatibility of return share with capacity rewards, as Theorem 2 may not hold when it is violated. In that case, we can show that the group incentive compatibility of the allocation can still be guaranteed without a cost correction; instead, we design a volume adjustment to producers' return shares based on the relative volume burdens of their products. The details are presented in Appendix 2.2.7.3.

We would like to mention that the practical value of the above findings is that under certain circumstances, the widely-adopted return share calculation can be retained and group incentive compatibility be guaranteed by a simple capacity rewarding mechanism, without resorting to cost-based adjustments. In other words, cost heterogeneity among different producers' products can be reflected in the cost allocation mechanism through simple capacity rewards.

2.2.4.4   Extensions: Economies of Scale.

In the rest of §2.2.4, we extend our results in two practically important directions: incorporating economies of scale and non-member access fees.

Economies of scale is one of the frequently mentioned advantages (and the reason for the popularity of) collective implementations of EPR. To gain insight into the impact of economies of scale on the group incentive compatibility of cost allocations, we consider a model where a global discount (increment) factor that is a function of the total return volume handled in the network is applied to the unit operational or downstream cost (unit downstream revenue) over the entire CRN; this ensures

tractability while capturing the essence of economies of scale. The discount and the increment factors are modeled respectively as a decreasing function $\eta \in (0, 1]$ and an increasing function $\zeta \in [1, \infty)$ in the total return volume. In particular, we denote the factors associated with a sub-coalition S by $\eta^S \doteq \eta(R^S)$ and $\zeta^S \doteq \zeta(R^S)$ respectively. We let the operational cost of product π on each edge e be $\eta^S \cdot c_e^\pi$ when S operates independently. The downstream cost/revenue that processor r pays for/obtains from downstream recycling changes to $\eta^S \cdot \sigma_r^\pi$ and $\zeta^S \cdot \rho_r^\pi$ respectively. Thus, the net processing cost becomes $\eta^S \cdot c_{(r,r')}^\pi + \eta^S \cdot \sigma_r^\pi + \zeta^S \cdot \rho_r^\pi$. By replacing the cost vector in the objective function in the program (C^S) introduced in §2.2.3 by the unit costs defined above, we obtain a new program $(C^{S(\eta,\zeta)})$ for each coalition $S \subseteq M$, and we denote its optimal objective function value by $v^{(\eta,\zeta)}(S)$. We define the core of the CRF game under scale economies and the incentive compatibility gap of an arbitrary allocation in this setting accordingly.

We calculate the three cost allocations $x_r^{(\eta,\zeta)}$, $x_r^{p(\eta,\zeta)}$ and $x_\mu^{p(\eta,\zeta)}$ by replacing $v(M)$ by $v^{(\eta,\zeta)}(M)$ in formulas (7) and (11), and by adjusting both $v(M)$ and μ^i in (10) based on $(C^{M(\eta,\zeta)})$. Let $v^r(M) = \sum_{r \in R} \sum_{\pi \in \Pi} \rho_r^\pi \cdot f_{(r,r')}^{*\pi}$, where f^* is the socially optimal routing, denote the total processing revenue obtained within the grand coalition under no scale economies. Define $v^r(S)$ in the same way for each sub-coalition $S \subsetneq M$ with adequate independent capacity to fulfill the recycling obligation of its members (i.e., $v(S) < \infty$).

Proposition 3. *Given scale economies parameters η and ζ that decrease and increase respectively with respect to the total return volume in the CRN,*

1. *$x_\mu^{p(\eta,\zeta)}$ is in the core of the CRF game under scale economies;*

2. *In the case where $\forall \pi \in \Pi$, $\rho_r^\pi = \rho^\pi \ \forall r \in R$, if $\forall S \subsetneq M$ such that $v(S) < \infty$, $\zeta^M \cdot |\frac{v^r(M)}{R}| \geq \zeta^S \cdot |\frac{v^r(S)}{R^S}|$ holds, i.e., the average processing revenue obtained within S is no higher than that within the grand coalition under scale economies, then the*

62

incentive compatibility gaps $\mathcal{G}(x_r^{(\eta,\zeta)})$ and $\min_{p_e \geq 0} \mathcal{G}(x_r^{p(\eta,\zeta)})$ satisfy $\mathcal{G}(x_r^{(\eta,\zeta)}) \leq$

$\eta^M \cdot \mathcal{G}(x_r)$ and $\min_{p_e \geq 0} \mathcal{G}(x_r^{p(\eta,\zeta)}) \leq \eta^M \cdot \min_{p_e \geq 0} \mathcal{G}(x_r^p)$.

Thus, according to the first result of Proposition 3, cost-corrected return share with capacity rewards continues to guarantee an allocation in the core of the CRF game under scale economies. Now consider the models of return share and return share with capacity rewards. When the conditions in the second result of Proposition 3 are met, economies of scale can reduce the incentive compatibility gap of both models by at least a fraction of $(1 - \eta^M)$. This is because all producers can enjoy a percentage cost saving of $(1 - \eta^M)$ within the centrally-operated grand coalition under the return-share based cost allocations, which is no less than their economies of scale benefits under smaller sub-coalitions in a fragmented system in this case. Otherwise, the effect of scale economies becomes more complex and scale can even lead to an increased incentive compatibility gap. We analyze this effect by studying CRN_2 defined in §2.2.4.1 under return share, and focusing on the case where $\exists$ a sub-coalition S such that $\zeta^M \cdot |\frac{v^r(M)}{R}| < \zeta^S \cdot |\frac{v^r(S)}{R^S}|$. In particular, we assume that π_A creates a positive unit processing revenue ρ and zero downstream cost at both processors, while π_B has no processing revenue but exerts a downstream cost $\sigma_{r_A}^{\pi_B}$ and $\sigma_{r_B}^{\pi_B}$ at both processors. We denote this network as CRN_2^ρ. Proposition 4 provides sufficient conditions under which $\mathcal{G}(x_r^{(\eta,\zeta)}) > \mathcal{G}(x_r)$.

Proposition 4. *Consider a CRN_2^ρ. Assume that the increment factors satisfy $\frac{\zeta^A - 1}{\zeta^M - 1} >$ $\frac{d_{\pi_A}}{d_{\pi_A} + d_{\pi_B}}$. There exists a constant $\bar{\rho}$ such that $\mathcal{G}(x_r^{(\eta,\zeta)}) > \mathcal{G}(x_r)$ if the unit processing revenue $\rho \geq \bar{\rho}$.*

Intuitively, the condition $\frac{\zeta^A - 1}{\zeta^M - 1} > \frac{d_{\pi_A}}{d_{\pi_A} + d_{\pi_B}}$ indicates that producer A incurs a bigger loss in average revenue from participating in the collective system after scale economies are factored in. In other words, the revenue component of the incentive compatibility gap is increased by scale economies. When the unit revenue ρ is large

enough, such an increase dominates the reduction in the cost component of the gap due to scale economies; the threshold $\bar{\rho}$ reflects the relative magnitude of the two effects. This intuition can be used to explain the potential effect of scale economies to increase the incentive compatibility gap in general CRNs: When $\zeta^M \cdot |\frac{v^r(M)}{R}| < \zeta^S \cdot |\frac{v^r(S)}{R^S}|$, the sub-coalition S experiences a loss in average recycling revenue participating in the grand coalition, which contributes to the incentive compatibility gap of the allocation by return share. Since such a revenue component of the incentive compatibility gap is affected differently by scale economies (through ζ) than that derived from cost (through η), a larger incentive compatibility gap may occur when scale economies are factored in. Note that such a situation can only occur when processing revenues associated with different products/processors vary widely. The practical implication of these observations is that in the presence of revenue heterogeneity among products or processors, economies of scale may not be as effective in reducing the incentive compatibility gap as has been advocated (Shao and Lee 2009), particularly if a cost allocation mechanism that is not group incentive compatible, such as return share, is used. In other words, the scale advantage of collective systems may be undermined by the prevalent return share model.

2.2.4.5   Extensions: Non-member Access Fees.

Some EPR bills are designed with flexibility provisions so that a capacity shortfall in an independent CRN can be complemented by the system operator using available operator-contracted capacity at a surcharge. For example, in Washington, a sub-coalition operating an independent CRN will be charged a unit shortfall fee by the system operator for the amount that it fails to process compared to its mandated share of the total return volume collected. Such a unit fee often covers the operational and downstream cost to handle the missing part of the sub-coalition's obligation within the centrally-operated CRN plus a surcharge. This policy is essentially equivalent to

allowing sub-coalitions that operate independent CRNs to use operator-contracted capacity for supplementing their independent capacities at a surcharge. We call these surcharges *non-member access fees*. We show that when non-member access fees are incorporated into the CRF game, the allocation by cost-corrected return share with capacity rewards remains in the core under properly designed non-member access fees.

Let the unit non-member access fee for operator-contracted capacity on edge e be ϕ_e. We construct a CRF game with $\{\phi_e\}$ by modifying the program (C^S) in §2.2.3.2 that computes the value of a sub-coalition $S \subsetneq M$ as follows: (i) add the term $\sum_{e \in E} \phi_e \cdot \max\{0; \sum_{\pi \in \Pi^S} f_e^\pi - \sum_{i \in S} k_e^i\}$ to the objective function to account for the total amount of non-member access fees paid for operator-contracted capacity; and (ii) increase the right-hand side of the capacity constraint on each edge e by the amount of operator-contracted capacity, i.e., k_e^p. Define the optimal value of this modified program as the value of sub-coalition S with non-member access fees $\{\phi_e\}$, denoted by $v_\phi(S)$. One important feature of this CRF game under non-member access fee is the user differentiation of the operator-contracted capacity: Producers need to pay to use the operator-contracted capacity only if they defect from the collective system. Hence, this game essentially combines features of existing cooperative games in literature that assume for all players and coalitions, the exogenous capacity not owned by players is available either for free (e.g., the pseudo-flow game by [91]) or at a price (e.g., the extended linear production game by [59]).

In the following, we analyze the choices of $\{\phi_e\}$ such that cost-corrected return share with capacity rewards continues to guarantee an allocation in the core. To build intuition, we first observe that when $\phi_e = \infty \ \forall e \in N$, sub-coalitions operating independent CRNs in a fragmented system will only use their own independent capacities. In this case, the CRF game with non-member access fees is equivalent to the original one described in §2.2.3.2 and thus x_μ^p must be in the core by Theorem 1.

65

The opposite extreme case occurs when $\phi_e = 0 \; \forall e \in N$, resulting in a special case of the pseudo-flow game, which is not guaranteed to have a non-empty core [91]. Our analysis finds a threshold for $\{\phi_e\}$ in closed-form expressions, such that all values of ϕ_e above this threshold guarantee that x_μ^p resides in the core of the CRF game with non-member access fees. Moreover, the threshold characterizes how the impact of incorporating non-member access fees into the game model is related to the cost heterogeneity in the CRN and producers' capacity ownership conditions. In presenting the result, we use e^0 to denote an edge in the CRN where independent capacity exists, i.e., $\sum_{i \in M} k_{e^0}^i > 0$.

Theorem 3. *Given any CRN, $\exists$ capacity reward prices $p_e \geq 0 \; \forall e \in E$ such that x_μ^p is guaranteed to be in the core of the CRF game under non-member access fees $\{\phi_e\}$ if*

1. *$\phi_e \geq \max_{i \in M}\{(1 - \mu^i)\} \cdot |\alpha_e^*| \; \forall e \in E$ when $\mu^i \geq 0 \; \forall i \in M$.*

2. *$\phi_e \geq \left[\min_{e^0 \in E: \sum_{i \in M} k_{e^0}^i > 0} \max_{i \in M}\{(1 - \frac{k_{e^0}^i}{\sum_{i \in N} k_{e^0}^i})\} \right] \cdot |\alpha_e^*| \; \forall e \in E$ otherwise.*

Theorem 3 implies that non-member fees can be effectively utilized to induce participation in a collective system. Notice that the lower bounds given as the right-hand-side in the above formulas will be smaller if the cost-corrected return shares (μ^i's) or the percentage independent capacity ownerships ($\frac{k_{e^0}^i}{\sum_{i \in N} k_{e^0}^i}$'s) are similar among the producers. In fact, they attain their lowest value if μ^i or $\frac{k_{e^0}^i}{\sum_{i \in N} k_{e^0}^i}$ equals $\frac{1}{n} \; \forall i \in N$. The practical implication of this observation is that it requires lower non-member access fees to guarantee a core allocation under the model of cost-corrected return share with capacity rewards if there is a higher level of homogeneity in cost/revenue among products and in the independent capacity ownerships among producers.

2.2.5 Implications for Practice

This section uses the Washington state implementation - one of the first U.S. implementations and the best documented one - as a test bed to investigate the practicality, economic value added and implications of the results developed in the previous section. The information used includes 2009 public e-waste data from Washington state [145, 180, 41] and input from stakeholder interviews with collectors, processors, transporters and NGOs (conducted by the authors in May 2011).

2.2.5.1 Washington State EPR Implementation Description.

In the state of Washington, the 2006 e-waste bill [176] mandated the formation of the Washington Materials Management and Financing Authority (WMMFA) whose job is to create a state-wide, collective "standard plan" to process the allowable e-waste (TVs, monitors, computers, and laptops) brought to its collection points by the consumers. Please refer to Section 2.1.1 for the details regarding the legislation and the operations of the "standard plan". In particular, the bill also allowed producers to opt out of the standard plan (subject to Department of Ecology approval) and operate their own collection and recycling networks. As mentioned, two such independent plans were filed in 2009 by two producers groups who believed their stand-alone costs would be lower than their cost allocation under the standard plan. Although these plans were rejected on the grounds that they were not sufficiently developed, they are expected to be resubmitted [43], as the adjustments made by the Authority to the return share model based on market share have not fully addressed their concerns. Consequently, a fragmented state-wide recycling system in the state of Washington is a real possibility, and our proposed cost allocation mechanism can serve to resolve these issues as follows: The system operator (WMMFA) would communicate the cost allocation mechanism to be used. Since independent capacities are subject to verification prior to approval of the plan by the Department of Ecology, the WMMFA

has access to this information. After returns are observed, the WMMFA would incorporate this independent capacity information in calculating the optimal routing of e-waste flows, determine the cost-corrections and the reward prices based on this routing, and in turn, the cost to be allocated to each producer. Ex-post, producers would be able to verify that their allocation is superior to what they would have achieved on their own, giving them the incentive to stay with the WMMFA-operated collective system.

To demonstrate how our proposed solution can be implemented in the Washington example and its potential economic implications, we construct a highly representative version of the Washington state collection and recycling network, including a sample of fifty collection points, eight consolidation points, eight processors, and seventeen producers who produce two product categories (TV/monitors and computers) with a waste volume equal to the entire 2009 volume in Washington. In the set of processors, we include the six in-state processors involved in the WA program, among which two of them (denoted by r_1 and r_2) are high-tech processors and the rest (denoted by $r_5 - r_8$) are low-tech manual facilities, plus two out-of-state high-tech processors (denoted by r_3 and r_4) associated with potential independent plans (discussed below). The distinction in our example between "high-tech" and "low-tech" reflects the operational differences among processors in practice, and results in cost heterogeneity in the CRN. The detailed construction of the sample CRN is provided in Appendix 2.2.7.4.

The waste volume is taken equal to the entire 2009 volume. We distinguish between two product categories - TVs/monitors ($\approx$27M lbs and 70% by volume) and computers ($\approx$12M lbs and 30% by volume), as the products' processing costs and post-use values primarily depend on this distinction at present. Specifically, TVs/monitors are expensive to process due to the hazardous materials contained in them, while computers generate revenues for processors as their components and/or materials have high reuse value. The net processing cost structures for each product are reproduced

Table 1: Representative net processing cost structure (cents/lb) by facility and product as a function of τ, the mandated recycling requirement. The negative numbers indicate revenues.

	High-tech processor			Low-tech processor $(r_5 - r_8)$
	Local $(r_1 - r_2)$	Out-of-state TV/monitor-specialized (r_3)	Out-of-state IT-specialized (r_4)	
TVs/monitors	$5 + 9\tau$	$5 + 5\tau$	$5 + 11\tau$	$7 + 13\tau$
Computers	-10	-7	-20	-6

in Table 1 for convenience, which are disguised estimates but structurally representative (see Appendix 2.2.7.4 for the construction of these costs and that of the collection, consolidation and transportation costs).

Motivated by the existing independent recycling capacity of producers around the WA region, we model two producers as having access to independent CRNs. In particular, a group of TV producers has contracted with a processor in Oregon that has advanced TV/monitor recycling technology in order to fulfill its recycling obligations in that state, and has filed one of the two independent plans in Washington. An IT producer has established a nationwide collection and recycling system including its own processing facility in Northern California, and is expected to apply for an independent plan in WA. In light of this information, and based on these producers' return volumes by category, we consider a TV producer A with 5.5M return volume, and a computer producer B with 2.1M lbs (25%) monitor volume and 6.4M lbs (75%) computer volume. We assume that A and B have access to 6M lbs and 9M lbs of independent recycling capacities at two out-of-state high-tech processors that are specialized in recycling TVs/monitors (r_3) and IT products such as computers (r_4), respectively. In addition, we model that the Authority can contract with in-state processors r_1, r_2, and each of $r_5 - r_8$ for up to 10M, 5M and 6M lbs of capacity, respectively. Collection and transportation are assumed to be uncapacitated for simplicity of exposition. In experiments not reported here, we observed similar network

effects as discussed below when such capacities were included.

2.2.5.2 Impact Assessment: Fairness Gap and Efficiency Loss.

In the following, the "centrally-operated grand coalition" refers to all producers participating in the standard plan, whereas the "fragmented system" refers to the one where producers A and/or B do not. In either case, the "system cost" includes the cost generated by the collection, transportation and processing of all the producers' e-waste. The "efficiency loss" refers to the increase in the system cost after the grand coalition disintegrates into a fragmented system.

In Washington, the incentive compatibility gap of the cost allocation by return share, and the associated efficiency loss from fragmentation, are substantial. The solid line in Figure 5(a) reports the incentive compatibility gap of the cost allocation by return share (x_r) when τ varies from 0 to 1, assuming no scale economies. Return share is clearly not group incentive compatible and at worst (when $\tau = 1$), producer B is allocated about \$0.6M more within the centrally-operated grand coalition, which is almost 60% more than what it could achieve on its own. This is because the cost allocated to producer B according to its return share reflects neither the value of B's independent capacity in reducing the total operating cost within the grand coalition, nor the much smaller cost burden (even a positive revenue) associated with computers, which are dominant in B's waste stream. Thus, B has an incentive to defect. While A alone would not defect, if B defects, A also finds it preferable to do so provided the A-B subcoalition allocates cost in a group incentive compatible way (i.e., adopts a core allocation within themselves). Figure 5(b) plots the absolute efficiency loss that would result. This loss is between 4.5-6.5% of the system cost of the grand coalition, due to the reduction in network synergies in a fragmented system. It represents a lower bound on the real efficiency loss as it is calculated under no economies of scale.

Remark 1. *The economic implications of legislative targets are strongly influenced*
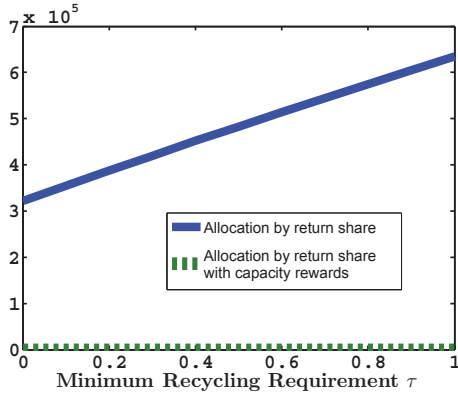
(a) Fairness gaps (in dollars) of cost allocation by return share and by return share with capacity rewards. In this figure, producer B experiences the highest cost increase under the return share model, which defines the incentive compatibility gap of the allocation.
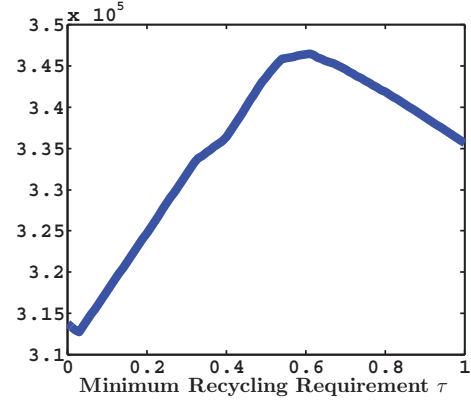
(b) Efficiency loss (in dollars) when the A-B sub-coalition does not join the centrally-operated plan.

Figure 5: Fairness gaps and efficiency loss under return share based allocations as a function of τ, the mandated recycling requirement, under no scale economies.

by product heterogeneity and network effects.

A revealing property of Figure 5(b) is that the efficiency loss is non-monotone. This phenomenon derives from the change in cost heterogeneity in the CRN and the consequent network effects as the recycling requirement becomes more stringent. Specifically, there is a change in the relative cost saving from rerouting product flow from a local low-tech processor to a high-tech one between TVs/monitors and computers when τ reaches 0.5. Hence, in the fragmented system, it is optimal to process the computer volume of producers other than A and B at the local high-tech (low-tech) processors when τ is below (above) 0.5. Therefore, how e-waste flows are rerouted relative to this baseline when a centrally-operated grand coalition is formed changes with τ, yielding a non-monotone efficiency loss function. Another way to see this is to note that network effects depend not only on the absolute magnitude of costs, but also on their relative values. Hence, while one expects the efficiency loss to increase as τ rises, the reverse may be observed. This discussion highlights the strong influence of network effects in determining the implications of legislative choices.

71

Remark 2. *This case is an instance where allocation by return share with capacity rewards generates a core allocation.*

This is observed by referring to the dashed line in Figure 5(a), and is a demonstration of our general theoretical discussion regarding the sufficient conditions to guarantee core allocations under this model in §2.2.4.3.

We now turn to the effect of scale economies. We model the discount factor η^S of a coalition S as a convex quadratic decreasing function of R^S (the parameters of the function are calculated based on input from WMMFA; see Appendix 2.2.7.4 for details); the increment ζ is assumed to be 1 for any return volume. To highlight the effects of scale economies, we fix $\tau = 1$ and let the return volumes of producers A and B vary from 0.5 to 1 times their base values, while the amount of their independent capacity remains constant. Figure 6 yields the following observation:

(a) Fairness gaps (in dollars) of the allocation by return share.

(b) Efficiency loss (in dollars) when the A-B sub-coalition does not join the centrally-operated plan.
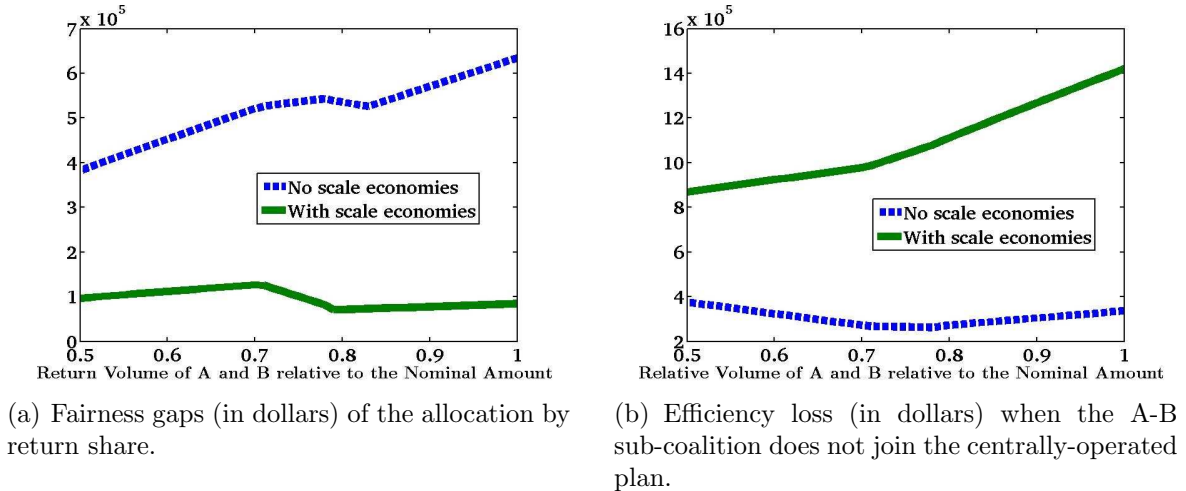
Figure 6: Comparison of efficiency loss and incentive compatibility gaps with/without scale economies as a function of the return volume of A and B relative to the nominal value. The capacity levels remain unchanged.

Remark 3. *The presence of economies of scale reduces the incentive compatibility gap of the allocation by return share (yet cannot guarantee a core allocation), and can greatly accentuate the efficiency loss from fragmentation.*

Note that proponents of collective systems tend to fall back on the economies of scale argument when faced with criticisms regarding the issues such as over-charging and potential system fragmentation. Our analysis underlines that economies of scale may not be sufficient to ensure group incentive compatibility of return share and thus voluntary participation of all producers in a collective system. The resulting efficiency loss from fragmentation can reach about \$1.42M (equivalent to a 20% increase in system cost) when $\tau = 1$, which is more than *four* times that without scale economies (about \$0.35M). We conclude that scale makes it all the more important to resolve incentive compatibility issues as it multiplies the efficiency loss that results from fragmentation.

Collectively, these observations highlight the significant incentive compatibility gaps and potential efficiency losses that can be incurred in practical implementations, and the difficulty in predicting the sensitivity of these quantities with respect to legislative choices and system characteristics. They also underline why finding a group incentive compatible cost allocation is so important from an economic perspective.

2.2.5.3   *The Mechanics of Adjustments to Return Share.*

Return share and cost-corrected return share with capacity rewards generate a core allocation in the Washington example as discussed. Here, we demonstrate how these two proposed adjustments to return share work.
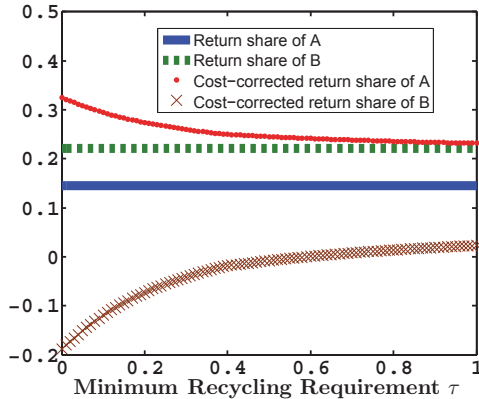
Figure 7: Return shares and cost-corrected return shares of producers A and B.

Figure 7 shows that the cost correction adjusts A's return share upward and B's downwards. This is because producer A makes TVs, while producer B's return volume consists mainly of computers (75% of the volume), which generate recycling revenue and exert a much smaller cost burden on the system than TVs. Note that the cost-corrected return share of producer B can even be negative, as the recycling revenue from computers can dominate the costs of the smaller volume of TVs. As the mandated recycling requirement τ increases, the cost correction becomes smaller. This is because an increasing recycling requirement raises the processing cost of TVs/monitors, which reduces the significance of the revenue contribution of B and decreases the relative cost burden exerted by A.

(a) Unit capacity reward price at the TV/monitor-specialized processor where A has independent capacity

(b) Unit capacity reward price at the IT-specialized processor where B has independent capacity
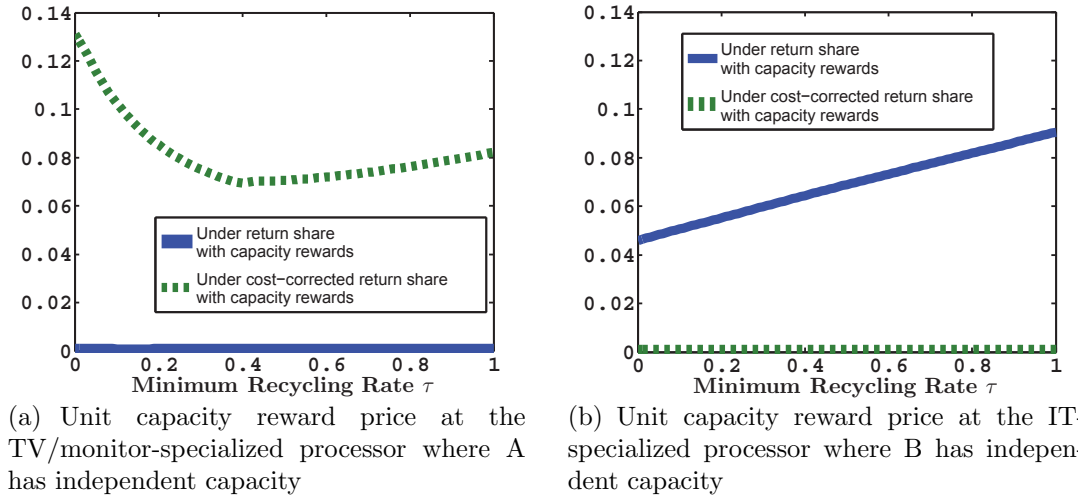
Figure 8: The unit capacity reward prices (in dollars) which lead to a cost allocation in the core using the minimum total capacity reward under each model of the mechanisms of return share with capacity rewards and cost-corrected return share with capacity rewards.

Remark 4. *To ensure a core allocation, the capacity reward prices should be tailored to whether a cost correction is implemented.*

Figure 8(a) and (b) plot the unit reward prices for the independent capacity

contributed by A and B (at the TV/monitor-specialized and the IT-specialized processors, respectively) that guarantee a core allocation under the two proposed mechanisms[4]. Worth noting is the significant interaction between capacity reward prices and cost corrections: With a cost correction, only the TV/monitor specialized capacity brought in by producer A commands a capacity reward; without a cost correction, only producer B gets a capacity reward for the IT specialized capacity it brings in. The underlying reason for the two contrasting cases is the different roles cost correction can play: A is penalized by the cost correction and thus may end up with too high a cost without a capacity reward, while B is rewarded and the reward from the cost correction is enough to account for both its capacity contribution and the processing revenues of its products. These observations highlight the nuances in the implementation of the proposed mechanisms.

Finally, we evaluate the non-member access fees to be charged at the local processors (where capacity is centrally contracted) that ensure a core allocation under the cost-corrected return share model with capacity rewards. The lower bounds of such fees are positive at the two local high-tech processors where the capacity is fully-used under the optimal routing; they vary from 2.3-6.1 cents/lb for r_2 and 0.3-3.9 cents/lb for r_3 as the minimum recycling requirement τ varies from 0 to 1. Adding this to the 24 cents/lb average processing cost in the Washington case, this is equivalent to charging no less than around a 26-30 cents/lb shortfall fee to producers operating independent plans, depending on τ. We conclude that the 50 cents/lb shortfall fee charged in Washington [44] is above this lower bound and thus helps to motivate producers' participation in the collective system.

___

[4]Since there exist multiple such prices under both mechanisms, we focus on those under which the total capacity reward to the A-B sub-coalition is the smallest.

Implementing the optimal routing and calculating the proposed cost allocations requires counting and separation (by product type and producer) at collection points. In practice, e-waste is often routed to the processors without separation and return shares are calculated by sampling. Thus, the CRN gains by avoiding the separation cost, but it loses from not routing the e-waste optimally through the CRN. Moreover, a group incentive compatible cost allocation as proposed here cannot be implemented.

In this section, we investigate the value of source separation in the Washington example. To this end, we develop a "myopic routing" policy inspired by the current practice in Washington (as opposed to the "optimal routing") where the e-waste is shipped to the processors without separation based on the transportation cost on each edge so as to minimize the total consolidation and transportation cost subject to capacity constraints at each processor. Sampling is carried out at each processor so as to achieve a desired accuracy level, based on which the total processing cost is calculated (please refer to Appendix 2.2.7.4 for the detailed specification of the myopic policy). We incorporate new elements that influence the operating cost under the myopic routing policy in the Washington example. Specifically, we include a separation cost and a sampling cost. We model product heterogeneity as the fraction p of the TVs in the total volume; a p value close to 0.5 means a higher heterogeneity level.

Remark 5. *The myopic routing has an advantage over the optimal one in terms of the total cost only under a high cost difference between separation and sampling, and a low level of product heterogeneity. However, such an advantage may not be realized as the allocation by return share is generally not group incentive compatible under the myopic routing.*

Figure 5(a) compares the centralized CRN's cost under the optimal routing with

(a) Efficiency difference

(b) Fairness gap under the myopic policy
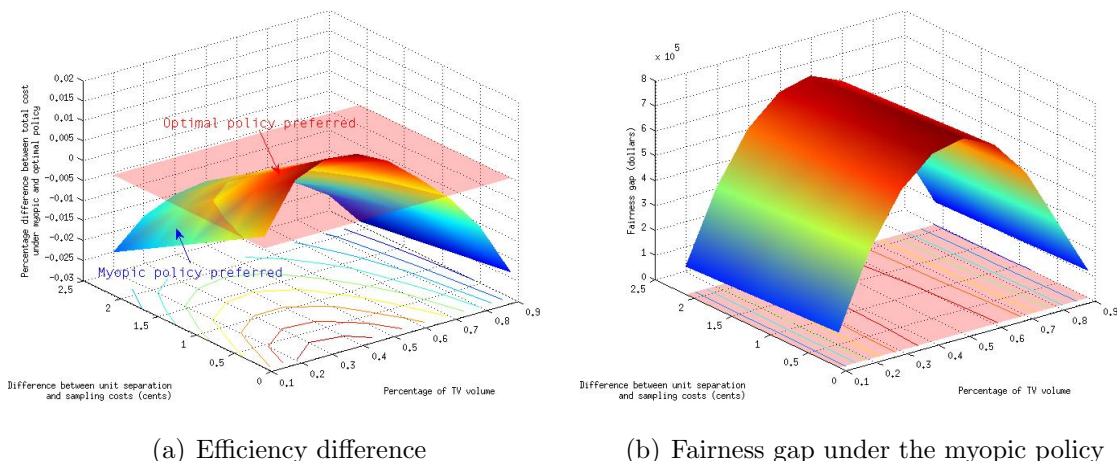
Figure 9: Percentage difference in the centralized system's total cost (including separation or sampling cost) between the myopic policy and the optimal policy, and the incentive compatibility gap under the myopic policy.

source separation and the myopic routing. A big difference between the cost of separation and sampling makes it relatively too expensive to implement the optimal routing. Moreover, a lower level of product heterogeneity requires a smaller sample size to achieve the desired sampling precision and thus further reduces the cost of the myopic policy. Note that the Washington instance is essentially a special case of the numerical example used in this section with $p=70\%$ (see Appendix 2.2.7.4 for details), and for this particular instance, the myopic policy dominates the optimal one. This implies that from the WMMFA perspective, whose objective is to minimize the overall system cost, the myopic policy will be preferred. An important caveat is in order, however: The centralized cost of the myopic policy can only be achieved if return share produces a core allocation, since return share is the only cost allocation that can be implemented with sampling at the processors. We find return share to be unfair for the entire range of the difference between the unit separation cost and sampling cost values (Figure 5(b)). Therefore, although the myopic routing with return share may seem attractive because it saves on separation cost, it will continue to raise over-charging concerns that may culminate in a fragmented system in Washington.

A critical driver of the status quo, where separation is economically not preferable, is that the current composition of e-waste flows is dominated by mainly CRT TVs. This results in a lower sample size requirement and a lower total sampling cost. Presumably, the e-waste composition will change in the near future, and will consist of a variety of different TV sets (e.g., fewer CRTs and more LCDs with Hg backlights, LCDs without Hg backlights and LEDs) and more IT products. The increased product heterogeneity would require larger sample sizes, and more variation in the value and processing requirements would increase the attractiveness of source separation. Indeed, our analysis suggests that the optimal policy with source separation would be highly justified as long as the per unit separation cost is modest (not more than 2 cents higher than the unit sampling cost at $p = 50\%$). With technological improvements yielding higher RFID read rates and lower tag costs [76], this appears to be feasible in the near future. In other words, our analysis suggests that centralized systems such as the WMMFA should seriously consider the potential from separation at source, both in terms of overall cost efficiency and a group incentive compatible cost allocation.

2.2.6 Conclusions

In this study, we contribute to the ongoing debate regarding how to implement EPR legislation in an effective and efficient manner. The choice in practice is often framed as one between an efficient collective system where producers share a lower total cost, and an individual system, where a producer is only responsible for its own cost, but is unable to benefit from network synergies. We propose an alternative paradigm that is capable of resolving this dilemma. The resolution we propose is based on the observation that the active debate is not necessarily critical of collective systems per se, but rather of the concerns regarding the prevalent proportional cost allocation methods used in these systems. Accordingly, we focus on identifying group incentive

compatible adjustments to the return share method, which is prevalent in practice due to its simplicity. We first show that the cost allocation by return share is generally not group incentive compatible due to its inability to account for processing cost heterogeneity among products or for network synergies that arise in the CRN. Then, we show that these shortcomings can be alleviated by simple adjustments such as correcting return shares to account for differences in processing costs, and rewarding independent capacities according to the value they bring to the collective CRN.

These results can influence the practice of EPR, as already evidenced by their adoption as policy input for the revision of the WEEE Directive in the UK [83]. They can also help different stakeholders shape their EPR implementation strategies. For example, producers lobbying for bills and regulations that reflect their cost burdens more accurately can focus their efforts on promoting these two easy-to-communicate concepts. Similarly, state or producer-operated systems who aim to achieve scale economies by drawing as many producers as possible to their system can implement these concepts. As we show, scale by itself is not a guarantor of stability for collective systems, and can even exacerbate their incentive compatibility gap, so these notions continue to be valuable for any size organization. Finally, collective systems who wish to institute flexibility provisions can institute non-member access fees developed here and maintain group incentive compatibility in cost allocation.

The producer dynamics in the state of Washington provide the opportunity to apply the concepts we develop and to assess their practical significance. To capture the producer behavior observed in Washington, we consider a setting where two producers are proposing to set up independent CRNs. We illustrate how the proposed cost allocation models can be implemented in this state. Our findings suggest that these cost allocation models, by guaranteeing group incentive compatibility, can retain these producers in the state-wide collective system and result in efficiency improvement of 5 - 20% for the state of Washington, which translates to $0.45M - $1.8M of

opportunity cost. With a simple population-based projection of this scenario-based analysis to the United States, this cost efficiency improvement would amount to approximately $22M - $90M for the electronics industry. Note also that this projection is based on the 5.78 lbs/capita collection rate in Washington, which is much lower than its European counterpart that reaches 17.6 lbs/capita because of its broader scope [51]. If similar collection volumes are attained (e.g. via scope expansion) in the US, the predicted efficiency improvement by a group incentive compatible cost allocation can go up to $67M - $274M for the electronics industry. Moreover, if fragmentation results in fewer processors being involved in each component of the fragmented system, processor competition may soften. In this case, our calculation of the efficiency loss would provide a conservative estimate of the true loss. More importantly, projecting the sales volume in Washington to the sales volume in the United States, we find that the collective system with cost allocation by return share could charge producers A and B up to $30M more than their actual cost burdens. While this is a case-based analysis, it underlines the economic potential of achieving collective system implementation and group incentive compatible cost allocation. The analysis also highlights the sensitivity of the outcomes with respect to legislative choices and network characteristics.

Implementing group incentive compatible cost allocations requires source separation, which can be a costly activity. Nevertheless, our analysis of the WA state data suggests that collective systems should seriously consider finding efficient ways of source separation, especially in the face of high product heterogeneity. This appears to be economically feasible: the Japanese implementations of EPR [163] show that separation can be achieved at low cost by a simple barcode technology.

While our focus in this section is cost efficiency given an existing set of products, an important policy goal of EPR is to make producers internalize the end-of-life burden of their products and encourage them to design better products. In the next

section, we incorporate such a design incentive dimension of EPR implementation into the analysis. We investigate whether the cost allocations we develop are effective in providing design incentives, and explore the interactions between individual design incentives, efficiency and group incentive compatibility of the mechanism under collective EPR.

2.2.7 Appendix

2.2.7.1 Proofs

Proof to Proposition 1. According to [58], the following *dual-based cost allocation* is guaranteed to be in the core of the CRF game.

$$(x^i)^\nu_d = \sum_{j\in L} \sum_{\pi\in\Pi^i} \beta_j^{*\pi} d_j^\pi + \sum_{e\in E} \alpha_e^* k_e^i + \sum_{e\in E} \alpha_e^* \nu_e^i \quad \forall i \in M, \tag{13}$$

where $\nu_e^i \geq 0 \ \forall i \in M \ \forall e \in E$, and $\sum_{i\in M} \nu_e^i = k_e^p \ \forall e \in E$.

According to conditions (i) and (iii) in Proposition 1, $\beta_j^{*\pi}$ are identical for all π given any $j \in L$ and $\alpha_e^* = 0 \ \forall e \in E$. Hence

$$(x^i)_r \doteq \frac{v(M)}{R} \cdot R^i = \frac{\sum_{j\in L} \beta_j^* \cdot [\sum_{\pi\in\Pi} d_j^\pi]}{R} \cdot R^i = \sum_{j\in L} \beta_j^* \cdot [\sum_{\pi\in\Pi^i} d_j^\pi]. \tag{14}$$

The last equality holds due to condition (ii) that for each product π, $\frac{d_j^\pi}{\sum_{\pi\in\Pi} d_j^\pi}$ is identical for all collection points j. Hence, if we denote $l_\pi \doteq \frac{d_j^\pi}{\sum_{\pi\in\Pi} d_j^\pi} \ \forall \pi \in \Pi$, then $\frac{R^i}{R} = \sum_{\pi\in\Pi^i} l_\pi$ and thus

$$\frac{\sum_{\pi\in\Pi} d_j^\pi}{R} \cdot R^i = \sum_{\pi\in\Pi} d_j^\pi \cdot \sum_{\pi\in\Pi^i} l_\pi = \sum_{\pi\in\Pi^i} (\sum_{\pi\in\Pi} d_j^\pi) \cdot l_\pi = \sum_{\pi\in\Pi^i} (\sum_{\pi\in\Pi} d_j^\pi) \cdot \frac{d_j^\pi}{\sum_{\pi\in\Pi} d_j^\pi} = \sum_{\pi\in\Pi^i} d_j^\pi \tag{15}$$

Hence, x_r is equivalent to the dual-based allocation defined in (13). □

Proof of Proposition 2. We first prove the result when $\hat{c}_{r_A}^{\pi_A} \leq \hat{c}_{r_A}^{\pi_B}$. The average cost incurred when A operates alone is $\bar{v}^A = \hat{c}_{r_A}^{\pi_A}$. Hence by equation (7), if x_r is in the core, the minimum average cost within the grand coalition $M = \{A, B\}$ must satisfy $\bar{v}^M \leq \bar{v}^A = \hat{c}_{r_A}^{\pi_A}$. Furthermore, when $\hat{c}_{r_A}^{\pi_A} \leq \hat{c}_{r_A}^{\pi_B}$, $\hat{c}_{r_A}^{\pi_A}$ is the smallest unit processing

cost on CRN$_2$, thus $\hat{c}_{r_A}^{\pi_A} \leq \bar{v}^M$. Therefore, we conclude that x_r being a core allocation indicates that $\bar{v}^M = \hat{c}_{r_A}^{\pi_A}$, which requires that all products are processed at r_A and $\hat{c}_{r_A}^{\pi_A} = \hat{c}_{r_A}^{\pi_B}$. On the other hand, when there is sufficient capacity at r_A to process all products and $\hat{c}_{r_A}^{\pi_A} = \hat{c}_{r_A}^{\pi_B}$, $\bar{v}^M = \bar{v}^A = \bar{v}^B$, and x_r must be in the core according to equation (7).

When $\hat{c}_{r_A}^{\pi_A} > \hat{c}_{r_A}^{\pi_B}$, we consider four scenarios and calculate the minimum average cost $\bar{v}^M$ respectively. We first define the following notation that represents the unit processing cost difference at the two processors for each product: $\delta^{\pi_i} \doteq \hat{c}_{r_B}^{\pi_i} - \hat{c}_{r_A}^{\pi_i}$ $i = A, B$.

scenario 1 where there are sufficient capacity at processor r_A, i.e., $k_{r_A} \geq d^{\pi_A} + d^{\pi_B}$.

scenario 2 where $k_{r_A} < d^{\pi_A} + d^{\pi_B}$ and $\delta^{\pi_A} \geq \delta^{\pi_B}$.

scenario 3 where $d^{\pi_B} \leq k_{r_A} < d^{\pi_A} + d^{\pi_B}$ and $\delta^{\pi_A} < \delta^{\pi_B}$.

scenario 4 where $k_{r_A} < d^{\pi_B}$ and $\delta^{\pi_A} < \delta^{\pi_B}$.

The minimum average cost within the grand coalition in each scenario is calculated as below.

$$
\bar{v}^M = \frac{1}{d^{\pi_A} + d^{\pi_B}} \cdot
\begin{cases}
[\hat{c}_{r_A}^{\pi_A} \cdot d^{\pi_A} + \hat{c}_{r_A}^{\pi_B} \cdot d^{\pi_B}] & \text{scenario 1} \\[2mm]
[\hat{c}_{r_A}^{\pi_A} \cdot d^{\pi_A} + \hat{c}_{r_A}^{\pi_B} \cdot (k_{r_A} - d^{\pi_A}) + \hat{c}_{r_B}^{\pi_B} \cdot (d^{\pi_B} - k_{r_A} + d^{\pi_A})] & \text{scenario 2} \\[2mm]
[\hat{c}_{r_A}^{\pi_B} \cdot d^{\pi_B} + \hat{c}_{r_A}^{\pi_A} \cdot (k_{r_A} - d^{\pi_B}) + \hat{c}_{r_B}^{\pi_A} \cdot (d^{\pi_A} - k_{r_A} + d^{\pi_B})] & \text{scenario 3} \\[2mm]
[\hat{c}_{r_A}^{\pi_B} \cdot k_{r_A} + \hat{c}_{r_B}^{\pi_B} \cdot (d^{\pi_B} - k_{r_A}) + \hat{c}_{r_B}^{\pi_A} \cdot d^{\pi_A}] & \text{scenario 4}
\end{cases}
$$

$$(16)$$

By equation (7), the allocation by return share is in the core of the CRF game if and only if $\bar{v}^M \leq \bar{v}^A = \hat{c}_{r_A}^{\pi_A}$ and $\bar{v}^M \leq \bar{v}^B = \hat{c}_{r_B}^{\pi_B}$. By solving these two inequalities in each scenario with $\bar{v}^M$ replaced by the corresponding formula in (16), we obtain the following sufficient and necessary condition for the allocation by return share to be

in the core.

$$
\hat{c}_{r_A}^{\pi A} - \hat{c}_{r_A}^{\pi B} \in
\begin{cases}
[0, \frac{d^{\pi A} + d^{\pi B}}{d^{\pi A}} \cdot \delta^{\pi B}] & \text{scenario 1} \\[2mm]
[\frac{d^{\pi A} + d^{\pi B} - k_{r_A}}{d^{\pi B}} \cdot \delta^{\pi B}, \frac{k_{r_A}}{d^{\pi A}} \cdot \delta^{\pi B}] & \text{scenario 2} \\[2mm]
[\frac{d^{\pi A} + d^{\pi B} - k_{r_A}}{d^{\pi B}} \cdot \delta^{\pi A}, \frac{1}{d^{\pi A}} \cdot ((d^{\pi A} + d^{\pi B}) \cdot \delta^{\pi B} - (d^{\pi A} + d^{\pi B} - k_{r_A}) \cdot \delta^{\pi A})] & \text{scenario 3} \\[2mm]
[\frac{d^{\pi A} + d^{\pi B} - k_{r_A}}{d^{\pi B}} \cdot \delta^{\pi B}, \frac{1}{d^{\pi A}} \cdot ((d^{\pi A} + k_{r_A}) \cdot \delta^{\pi B} - d^{\pi A} \cdot \delta^{\pi A})] & \text{scenario 4}
\end{cases}
\tag{17}
$$

Hence, we can prove the proposition when $\hat{c}_{r_A}^{\pi A} > \hat{c}_{r_A}^{\pi B}$ by defining $\underline{\Delta}$ and $\bar{\Delta}$ as the lower and upper bounds specified in (17) in each scenario. $\qquad\square$

Proof of Theorem 1. We discuss two cases to prove this theorem. Note that in our problem, the dual variables β_j^π are unrestricted while $\alpha_e \leq 0 \ \forall e \in E$. In addition, since we assume sufficient public capacity is available, the centralized problem (C) is feasible and obviously lower bounded (e.g., by zero). Thus strong duality must hold for (C).

Case 1 When $\mu^i \geq 0 \ \forall i \in M$, we set $p_e = |\alpha_e^*| \ \forall e \in E$. Since $v(M) = \sum_{j \in L} \sum_{\pi \in \Pi} \beta_j^{*\pi} d_j^\pi + \sum_{e \in E} \alpha_e^* \cdot (\sum_{i \in M} k_e^i + k_e^p)$ due to strong duality for (C), we conclude that $v(M) + \sum_{e \in E} p_e \sum_{i \in M} k_e^i = \sum_{j \in L} \sum_{\pi \in \Pi} \beta_j^{*\pi} d_j^\pi + \sum_{e \in E} \alpha_e^* k_e^p$. Thus by formula (10),

$$
\begin{aligned}
(x^i)_\mu^p &= \left[\sum_{j \in L} \sum_{\pi \in \Pi} \beta_j^{*\pi} d_j^\pi + \sum_{e \in E} \alpha_e^* k_e^p \right] \cdot \frac{\sum_{j \in L} \sum_{\pi \in \Pi^i} \beta_j^{*\pi} d_j^\pi}{\sum_{j \in L} \sum_{\pi \in \Pi} \beta_j^{*\pi} d_j^\pi} + \sum_{e \in E} \alpha_e^* k_e^i \\
&= \sum_{j \in L} \sum_{\pi \in \Pi^i} \beta_j^{*\pi} d_j^\pi + \sum_{e \in E} \alpha_e^* k_e^i + \sum_{e \in E} \alpha_e^* \mu^i k_e^p.
\end{aligned}
\tag{18}
$$

Hence, x_μ^p is equivalent to the cost allocation x_d^ν defined in (13) if we set $\nu_e^i = \mu^i k_e^p$ $\forall e \in E \ \forall i \in M$. With $\mu^i \geq 0 \ \forall i \in M$ in this case, $\nu_e^i \geq 0 \ \forall e \in E \ \forall i \in M$. Hence, this cost allocation is a dual-based cost allocation, and thus is in the core of the CRF game.

Case 2 When $\exists i \in M$ such that $\mu^i < 0$, we show that any set of prices $\{p_e\}$ that satisfy the following conditions give rise to an allocation x_μ^p in the core of the

CRF game.

$$
\begin{cases}
p_e \geq |\alpha_e^*| \quad \forall e \in E \quad (a) \\
\sum_{e \in E} p_e \cdot \sum_{i \in M} k_e^i = \sum_{e \in E} |\alpha_e^*| \cdot (\sum_{i \in M} k_e^i + k_e^p) \quad (b)
\end{cases}
\tag{19}
$$

To show this, consider an arbitrary sub-coalition $S \subset M$. Due to strong duality and condition (19b), $v(M) + \sum_{e \in E} p_e \sum_{i \in M} k_e^i = \sum_{j \in L} \sum_{\pi \in \Pi} \beta_j^{*\pi} d_j^\pi + \sum_{e \in E} \alpha_e^* \cdot (\sum_{i \in M} k_e^i + k_e^p) + \sum_{e \in E} |\alpha_e^*| \cdot (\sum_{i \in M} k_e^i + k_e^p) = \sum_{j \in L} \sum_{\pi \in \Pi} \beta_j^{*\pi} d_j^\pi$. Then by formula (10), the cost allocated to S satisfies

$$
\begin{aligned}
\sum_{i \in S} (x^i)_\mu^p &= \sum_{i \in S} \left\{ \left[\sum_{j \in L} \sum_{\pi \in \Pi} \beta_j^{*\pi} d_j^\pi \right] \cdot \frac{\sum_{j \in L} \sum_{\pi \in \Pi^i} \beta_j^{*\pi} d_j^\pi}{\sum_{j \in L} \sum_{\pi \in \Pi} \beta_j^{*\pi} d_j^\pi} - \sum_{e \in E} p_e k_e^i \right\} \\
&= \sum_{i \in S} \left\{ \sum_{j \in L} \sum_{\pi \in \Pi^i} \beta_j^{*\pi} d_j^\pi - \sum_{e \in E} p_e k_e^i \right\} \\
&\leq^1 \sum_{i \in S} \left\{ \sum_{j \in L} \sum_{\pi \in \Pi^i} \beta_j^{*\pi} d_j^\pi + \sum_{e \in E} \alpha_e^* k_e^i \right\} \leq^2 v(S)
\end{aligned}
\tag{20}
$$

The inequality $\leq^1$ is due to condition (19a). The second one $\leq^2$ is due to the fact that $[\beta^*, \alpha^*]$ is a feasible dual solution to the centralized problem within S, i.e., (C^S), and weak duality. Hence, by definition, the allocation x_μ^p is in the core of the CRF game.

We then show that there exist prices that satisfy condition (19). Pick one edge e^0 such that $\sum_{i \in M} k_{e^0}^i > 0$ (such an edge always exists otherwise no producer can operate alone). Define a set of reward prices such that

$$
p_{e^0} \doteq |\alpha_{e^0}^*| + \frac{\sum_{e \in E} |\alpha_e^*| \cdot k_e^p}{\sum_{i \in M} k_{e^0}^i}; \qquad p_e = |\alpha_e^*| \quad \forall e \neq e^0
\tag{21}
$$

It is easy to verify that such a set of prices satisfies both conditions specified in (19). In fact, when these prices are adopted, we can calculate that the cost allocation to producer i equals

$$
(x^i)_\mu^p = \sum_{j \in L} \sum_{\pi \in \Pi^i} \beta_j^{*\pi} d_j^\pi - \sum_{e \in E} p_e k_e^i = \sum_{j \in L} \sum_{\pi \in \Pi^i} \beta_j^{*\pi} d_j^\pi + \sum_{e \in E} \alpha_e^* k_e^i + \sum_{e \in E} \alpha_e^* k_e^p \cdot \frac{k_{e^0}^i}{\sum_{j \in M} k_{e^0}^j}
\tag{22}
$$

Hence the cost allocation x_μ^p is equivalent to a dual-based cost allocation x_d^ν defined in (13) if we set $\nu_e^i = \frac{k_{e0}^i}{\sum_{i \in M} k_{e0}^i} \cdot k_e^p \ \forall e \in E \ \forall i \in M$. $\qquad\qquad\qquad\qquad\square$

Proof of Theorem 2. To prove this theorem, we need the following lemma which calculates $\underline{\mathcal{G}}(x_r^p)$ as the optimal objective value of a linear program given any CRN.

Lemma 1. *Let $\Psi = \{S \subseteq M : v(S) < \infty\}$ be the set of sub-coalitions whose members have sufficient independent capacity to process their own return volumes in their individual CRNs. Then*

$$\underline{\mathcal{G}}(x_r^p) = \max \quad \sum_{S \in \Psi} [\bar{v}^M - \bar{v}^S] \cdot y^S \qquad (23)$$

$$s.t. \quad \sum_{S \in \Psi} \left[\bar{k}_e^S - \bar{k}_e^M \right] \cdot y^S \le 0 \quad \forall e \in E \qquad (24)$$

$$\sum_{S \in \Psi} \frac{1}{R^S} \cdot y^S \le 1 \qquad (25)$$

$$nonnegativity \quad constraints. \qquad (26)$$

Proof of Lemma 1. The problem of $\min_{p_e \ge 0} \mathcal{G}(x_r^p) = \min_{p_e \ge 0} \max_{S \subseteq M} \{R^S \cdot [\bar{v}^M - \bar{v}^S - \sum_{e \in E} p_e(\bar{k}_e^S - \bar{k}_e^M)]\}$ is equivalent to the following linear program:

$$\min \quad z \quad s.t. \quad z \ge R^S \cdot [\bar{v}^M - \bar{v}^S - \sum_{e \in E} p_e(\bar{k}_e^S - \bar{k}_e^M)] \quad \forall S \in \Psi \quad p_e \ge 0 \quad \forall e \in E. \quad (27)$$

We obtain the program (23)-(25) by taking the dual of the above problem (27). $\quad\square$

Continuing the proof of Theorem 2, since $E(\bar{C}) \cap \{(o, j), j \in L\} = \emptyset$, $K_{E(\bar{C})} = \sum_{e \in E(\bar{C})} \sum_{i \in M} k_e^i < R$. Hence, adding up the constraints (24) over the set $E(\bar{C})$, we obtain another constraint $\sum_{S \in \Psi} \left\{ \sum_{e \in E(\bar{C})} \left[\bar{k}_e^S - \bar{k}_e^M \right] \right\} \cdot y^S \le 0$. Since $\forall S \in \Psi$ such that $v(S) < \infty$, subset S has enough independent capacity to process its own return volume in its private CRN, and $\sum_{e \in E(\bar{C})} \sum_{i \in S} k_e^i \ge R^S$. Thus $\sum_{e \in E(\bar{C})} \left[\bar{k}_e^S - \bar{k}_e^M \right] = \frac{\sum_{e \in E(\bar{C})} \sum_{i \in S} k_e^i}{R^S} - \frac{\sum_{e \in E(\bar{C}) \setminus \{(F,d)\}} \sum_{i \in M} k_e^i}{R} > 0$, as the first (second) term is greater than or equal to (strictly less than) 1. Due to the nonnegativity constraints on y^S, we conclude that the only feasible solution to the program (23)-(25) is the zero vector and thus $\underline{\mathcal{G}}(x_r^p) = 0$ due to Lemma 1. $\qquad\qquad\qquad\qquad\square$

Proof of Proposition 3. We first prove that $\underline{\mathcal{G}}(x_\mu^{p(\eta,\zeta)}) = \underline{\mathcal{G}}(x_\mu^p) = 0$. Consider a CRN with unit cost (revenue) modified by the discount factors η^M (increment factor ζ^M) associated with the grand coalition M. From Theorem 1, we know that there exists a set of prices $\{p_e\}$ such that the allocation by cost-corrected return share with capacity rewards $x_\mu^{p(\eta,\zeta)}$ is inside the core of the CRF game defined based on the above CRN. In other words, if we denote the minimum total cost incurred within any sub-coalition S under the discount and increment factors (η^M, ζ^M) as $v(\eta^M, \zeta^M, S)$, then $\sum_{i \in S}(x^i)_\mu^{p(\eta,\zeta)} \leq v(\eta^M, \zeta^M, S)$. Recall that in the CRF game under scale economies, the stand-alone value of S, $v^{(\eta,\zeta)}(S)$, is defined with the unit cost (revenue) modified by the discount factors η^S (increment factor ζ^S). Since η and ζ decrease and increase respectively as the return volume in the CRN grows, for each sub-coalition $S \subsetneq M$, $\eta^M \leq \eta^S$ and $\zeta^M \geq \zeta^S$. Hence, we conclude that $v(\eta^M, \zeta^M, S) \leq v^{(\eta,\zeta)}(S)$ and thus $\sum_{i \in S}(x^i)_\mu^{p(\eta,\zeta)} \leq v(\eta^M, \zeta^M, S) \leq v^{(\eta,\zeta)}(S) \; \forall S \subsetneq M$. By definition the allocation $x_\mu^{p(\eta,\zeta)}$ remains to be in the core in the CRF game under scale economies.

Next we prove the second result. Since the unit processing revenue of each product π is identical among the processors, we know that under any flow within a coalition $S \subseteq M$, the total revenue obtained equals $\sum_{\pi \in \Pi^S} \rho^\pi \cdot \sum_{j \in L} d_j^\pi = v^r(S)$. Let $v^c(S) \doteq v(S) - v^r(S)$. Hence, $\forall S \subseteq M$, $v^{(\eta,\zeta)}(S) = \eta^S \cdot v^c(S) + \zeta^S \cdot v^r(S)$. Now consider an arbitrary sub-coalition S such that $v(S) < \infty$. Since $\eta^M \leq \eta^S$ and $v^c(S) \geq 0$, we obtain the following inequality.

$$
\begin{aligned}
\bar{v}^{(\eta,\zeta)M} - \bar{v}^{(\eta,\zeta)S} &= \left(\frac{\eta^M \cdot v^c(M)}{R} - \frac{\eta^S \cdot v^c(S)}{R^S}\right) + \left(\frac{\zeta^M \cdot v^r(M)}{R} - \frac{\zeta^S \cdot v^r(S)}{R^S}\right) \\
&\leq \eta^M \cdot \left(\frac{v^c(M)}{R} - \frac{v^c(S)}{R^S}\right) + \left(\frac{\zeta^M \cdot v^r(M)}{R} - \frac{\zeta^S \cdot v^r(S)}{R^S}\right) \quad (28)
\end{aligned}
$$

We further derive the following inequalities considering two cases respectively.

Case I: When $|\frac{v^r(M)}{R}| \geq |\frac{v^r(S)}{R^S}|$, then since $\zeta^M \geq \zeta^S$ and both $v^r(M)$ and $v^r(S)$ are

nonpositive, and $\eta^M \leq \zeta^M$,

$$\eta^M \cdot (\frac{v^c(M)}{R} - \frac{v^c(S)}{R^S}) + (\frac{\zeta^M \cdot v^r(M)}{R} - \frac{\zeta^S \cdot v^r(S)}{R^S})$$

$$\leq \eta^M \cdot (\frac{v^c(M)}{R} - \frac{v^c(S)}{R^S}) + \zeta^M \cdot (\frac{v^r(M)}{R} - \frac{v^r(S)}{R^S})$$

$$\leq \eta^M \cdot (\frac{v^c(M)}{R} - \frac{v^c(S)}{R^S} + \frac{v^r(M)}{R} - \frac{v^r(S)}{R^S}) = \eta^M(\bar{v}^M - \bar{v}^S) \qquad (29)$$

Case II: When $|\frac{v^r(M)}{R}| < |\frac{v^r(S)}{R^S}|$, then since $\zeta^M \cdot |\frac{v^r(M)}{R}| \geq \zeta^S |\frac{v^r(S)}{R^S}|$

$$\eta^M \cdot (\frac{v^c(M)}{R} - \frac{v^c(S)}{R^S}) + (\frac{\zeta^M \cdot v^r(M)}{R} - \frac{\zeta^S \cdot v^r(S)}{R^S}) \leq \eta^M \cdot (\frac{v^c(M)}{R} - \frac{v^c(S)}{R^S})$$

$$\leq \eta^M \cdot (\frac{v^c(M)}{R} - \frac{v^c(S)}{R^S} + \frac{v^r(M)}{R} - \frac{v^r(S)}{R^S}) = \eta^M(\bar{v}^M - \bar{v}^S) \qquad (30)$$

Hence, combining (28) with both (29) and (30), we conclude that $\bar{v}^{(\eta,\zeta)M} - \bar{v}^{(\eta,\zeta)S} \leq \eta^M \cdot (\bar{v}^M - \bar{v}^S)$. Hence, by formulas (8), we know that $\mathcal{G}(x_r^{(\eta,\zeta)})) = \max_{S \subseteq M}\{R^S \cdot (\bar{v}^{(\eta,\zeta)M} - \bar{v}^{(\eta,\zeta)S})\} \leq \eta^M \cdot \max_{S \subseteq M}\{R^S \cdot (\bar{v}^M - \bar{v}^S)\} = \eta^M \cdot \mathcal{G}(x_r)$. As for the model of return share with capacity rewards, recall that by Lemma 1, the smallest incentive compatibility gap under the model of return share with capacity rewards, $\underline{\mathcal{G}}(x_r^p)$, can be calculated as the optimal value of an linear program (23)-(26). Hence, if we denote the feasible region of the above linear program as $Y = \{\{y^S, S \subset \Psi\} : (24) - (26)\}$, then $\underline{\mathcal{G}}(x_r^{p(\eta,\zeta)}) = \max_Y \sum_{S \in \Psi}[\bar{v}^{(\eta,\zeta)M} - \bar{v}^{(\eta,\zeta)S}] \cdot y^S \leq \eta^M \cdot \max_Y \sum_{S \in \Psi}[\bar{v}^M - \bar{v}^S] \cdot y^S = \eta^M \cdot \underline{\mathcal{G}}(x_r^p)$. $\qquad \square$

Proof of Proposition 4. Let $\bar{v}^{cS} \doteq \frac{v^c(S)}{R^S}$ be the average of the cost component incurred within each sub-coalition S. Then given any CRN$_2^\rho$, we can calculate that $\bar{v}^M = \frac{v^c(M) + \rho \cdot d_{\pi_A}}{d_{\pi_A} + d_{\pi_B}} = \bar{v}^{cM} + \rho \cdot \frac{d_{\pi_A}}{d_{\pi_A} + d_{\pi_B}}$, $\bar{v}^A = \frac{v^c(A) + \rho \cdot d_{\pi_A}}{d_{\pi_A}} = \bar{v}^{cA} + \rho$, $\bar{v}^B = \bar{v}^{cB}$, $\bar{v}^{(\eta,\zeta)M} = \eta^M \cdot \bar{v}^{cM} + \zeta^M \cdot \rho \cdot \frac{d_{\pi_A}}{d_{\pi_A} + d_{\pi_B}}$, $\bar{v}^{(\eta,\zeta)A} = \eta^A \cdot \bar{v}^{cA} + \zeta^A \cdot \rho$. Under the condition of

$\frac{\zeta^A - 1}{\zeta^M - 1} > \frac{d_{\pi_A}}{d_{\pi_A} + d_{\pi_B}}$, we can show the following.

$$\text{If} \quad |\rho| \geq \bar{\rho}_1 \doteq \frac{\bar{v}^{cM}(1 - \eta^M) - \bar{v}^{cA}(1 - \eta^A)}{(\zeta^A - 1) - (\zeta^M - 1) \cdot \frac{d_{\pi_A}}{d_{\pi_A} + d_{\pi_B}}},$$

$$\text{then} \quad k_{\pi_A} \cdot (\bar{v}^{(\eta,\zeta)M} - \bar{v}^{(\eta,\zeta)A}) \geq k_{\pi_A} \cdot (\bar{v}^M - \bar{v}^A).$$

$$\text{If} \quad |\rho| \geq \bar{\rho}_2 \doteq \frac{\bar{v}^{cM}(\frac{d_{\pi_B}}{d_{\pi_A}} - \eta^M) + \bar{v}^{cA} \cdot \eta^A - \bar{v}^{cB} \cdot \frac{d_{\pi_B}}{d_{\pi_A}}}{\zeta^A - \zeta^M \cdot \frac{d_{\pi_A}}{d_{\pi_A} + d_{\pi_B}} + \frac{d_{\pi_B}}{d_{\pi_A} + d_{\pi_B}}},$$

$$\text{then} \quad k_{\pi_A} \cdot (\bar{v}^{(\eta,\zeta)M} - \bar{v}^{(\eta,\zeta)A}) \geq k_{\pi_B} \cdot (\bar{v}^M - \bar{v}^B).$$

$$\text{If} \quad |\rho| \geq \bar{\rho}_3 \doteq \frac{\bar{v}^{cA} \cdot \eta^A - \bar{v}^{cM} \cdot \eta^M}{\zeta^A - \zeta^M \cdot \frac{d_{\pi_A}}{d_{\pi_A} + d_{\pi_B}}}, \quad \text{then} \quad k_{\pi_A} \cdot (\bar{v}^{(\eta,\zeta)M} - \bar{v}^{(\eta,\zeta)A}) \geq 0. \tag{31}$$

Hence, we can conclude that when $|\rho| \geq \bar{\rho} \doteq \max\{\rho_1, \bar{\rho}_2, \bar{\rho}_3\}$, $\mathcal{G}(x_r^{(\eta,\zeta)})) = \max_{S \subseteq M}\{R^S \cdot (\bar{v}^{(\eta,\zeta)M} - \bar{v}^{(\eta,\zeta)S})\} \geq k_{\pi_A} \cdot (\bar{v}^{(\eta,\zeta)M} - \bar{v}^{(\eta,\zeta)A}) \geq \max_{S \subseteq M}\{R^S \cdot (\bar{v}^M - \bar{v}^S)\} = \mathcal{G}(x_r)$. $\square$

Proof of Theorem 3. To prove Theorem 3, we first present the following lemma that calculates the lower bound of non-member access fees $\{\phi_e\}$ under which a given dual-based allocation is guaranteed to be in the core of the CRF game $(M, v_\phi(S))$.

Lemma 2. *The dual-based allocation x_d^ν is in the core of the CRF game with non-member access fees $\{\phi_e\}$ if $\phi_e \geq \max_{i \in M}\{(1 - \frac{\nu_e^i}{k_e^p}) \cdot |\alpha_e^*|\}\ \forall e \in E$.*

Proof of Lemma 2. We first model the problem (C_ϕ^S) as the following linear program.

$$(C_\phi^S) \quad v_\phi(S) = \min \quad \sum_{e \in E \setminus \{(r,r')\}} \sum_{\pi \in \Pi^S} c_e^\pi f_e^\pi + \sum_{r \in R} \sum_{\pi \in \Pi} \hat{c}_r^\pi f_{(r,r')}^\pi + \sum_{e \in E} \phi_e \cdot (k_e^p)^S \tag{32}$$

$$s.t. \quad \sum_{e=(u,v) \in E} f_e^\pi - \sum_{e=(v,w) \in E} f_e^\pi = 0 \quad \forall v \in V \setminus \{L, R'\}, \forall \pi \in \Pi^S \quad [(v^S)_v^\pi] \tag{33}$$

$$f_{jj'}^\pi = d_j^\pi \quad \forall \pi \in \Pi^S, \forall j \in L \quad [(\beta^S)_j^\pi] \tag{34}$$

$$\sum_{\pi \in \Pi^S} f_e^\pi \leq \sum_{i \in S} k_e^i + k_e^p \quad \forall e \in E \quad [\alpha_e^S] \tag{35}$$

$$\sum_{\pi \in \Pi^S} f_e^\pi - (k_e^p)^S \leq \sum_{i \in S} k_e^i \quad \forall e \in E \quad [\omega_e^S] \tag{36}$$

$$\text{nonnegativity constraints}, \tag{37}$$

where $[v, \beta, \alpha, \omega, \sigma]$ are the dual variables associated with the constraints in (C_ϕ^S). Set $(v^S)_j^\pi = 0\ \forall j \in L$ and $(v^S)_{r'}^\pi = 0\ r' \in R'$. Define the set $E_j = \{(j, j'), \forall j \in L\}$,

and $E_r = \{(r, r'), \forall r \in R$. Then the dual to the above linear program is formulated as follows:

$$(D_\phi^S) \quad \max \quad \sum_{j \in L} \sum_{\pi \in \Pi^S} d_j^\pi (\beta^S)_j^\pi + \sum_{e \in E} \sum_{i \in S} k_e^i \cdot (\alpha_e^S + \omega_e^S) + \sum_{e \in E} k_e^p \cdot \alpha_e^S \quad (38)$$

$$s.t. \quad (v^S)_{j'}^\pi - (v^S)_j^\pi + \alpha_e^S + \omega_e^S + (\beta^S)_j^\pi \le c_e^\pi \quad \forall \pi \in \Pi^S, \forall e = (j, j'), \forall j \in L \quad (39)$$

$$(v^S)_v^\pi - (v^S)_u^\pi + \alpha_e^S + \omega_e^S \le c_e^\pi \quad \forall \pi \in \Pi^S, \forall e \in E \setminus (E_j \cup E_r) \quad (40)$$

$$(v^S)_v^\pi - (v^S)_u^\pi + \alpha_e^S + \omega_e^S \le \hat{c}_r^\pi \quad \forall \pi \in \Pi^S, \forall e = (r, r'), \forall r \in R \quad (41)$$

$$-\omega_e^S \le \phi_e, \quad \alpha_e^S \le 0, \quad \omega_e^S \le 0 \quad \forall e \in E. \quad (42)$$

We define a solution to the above program (D_ϕ^S) based on the optimal dual solutions to the centralized problem (C) $[v_v^{*\pi}, \alpha_e^*, \beta_j^{*\pi}]$.

$$(v^S)_v^\pi \doteq v_v^{*\pi} \quad \forall v \in V \setminus \{L, R'\}, \forall \pi \in \Pi^S \quad (43)$$

$$\alpha_e^S \doteq \frac{\sum_{i \in S} \nu_e^i}{k_e^p} \cdot \alpha_e^* \quad \omega_e^S \doteq (1 - \frac{\sum_{i \in S} \nu_e^i}{k_e^p}) \cdot \alpha_e^* \quad \forall e \in E \quad (44)$$

$$(\beta^S)_j^\pi \doteq \beta_j^{*\pi} \quad \forall \pi \in \Pi^S, \forall j \in L. \quad (45)$$

Due to the optimality of $[v_v^{*\pi}, \alpha_e^*, \beta_j^{*\pi}]$ with respect to the dual problem of (C), it is easy to check that the solution defined in (43)-(45) is feasible for (D_ϕ^S) under the condition given in the theorem. Hence by weak duality, we conclude that the objective value of (D_ϕ^S) for the above solution, which is exactly the cost allocated to sub-coalition S under the dual-based allocation x_d^ν, is no greater than $v_\phi(S) \, \forall S \subsetneq M$. Hence, the dual-based allocation x_d^ν lies in the core of the CRF game with non-member access fee ϕ_e. $\qquad \square$

Continuing the proof of Theorem 3, it is easy to see that it directly follows from Lemma 2, because according to the proof to Theorem 1, we can design prices $\{p_e\}$ such that x_μ^r is equivalent to a dual-based allocation. In particular, in Case 1 (i.e., when $\mu^i \ge 0 \, \forall i \in N$), according to (18), we can see that by adopting the prices $p_e = |\alpha_e^*| \, \forall e \in E$, a dual-based allocation is obtained with $\nu_e^i = \mu^i \cdot k_e^p \, \forall e \in E$

$\forall i \in M$. Hence, replacing ν_e^i in Lemma 2 by μ^i, we obtain the first bound in Theorem 3. Similarly, when there exists some producer i whose $\mu^i < 0$, we can adopt the prices given by (21), and according to the proof of Theorem 1 (Case 2) and obtain a dual-based allocation where $\nu_e^i = \frac{k_{e0}^i}{\sum_{i \in M} k_{e0}^i} \forall e \in E \; \forall i \in M$. Hence, by choosing e^0 to be the edge such that the value of $\max_{i \in M}\{(1 - \frac{k_e^i}{\sum_{i \in N} k_e^i})\}$ is minimized, we obtain the second bound given in Theorem 3. □

2.2.7.2 Generalization of Theorem 2 under Multiple Minimum Cuts $\bar{C}$

Given a CRN $\bigcup_{i \in M} N^i$, we transform it into a single commodity network N_o^d as described in §2.2.4.3. Assume multiple minimum cuts $\{\bar{C}_1, ..., \bar{C}_i, ..., \bar{C}_n\}$ exist in N_o^d. Define $\bar{L}_i = \{j \in L : (o, j) \notin E(\bar{C}_i)\}$. Then the set $\bigcap_{i=1}^n (L \setminus \bar{L}_i) = L \setminus \bigcup_{i=1}^n \bar{L}_i$ is comprised of collection points j such that the corresponding fictitious edge (o, j) is contained in the edge set of all mininum cuts. Such an edge set can be identified by the following algorithm (Algorithm 1).

ALGORITHM 1: Computing $\bigcap_{i=1}^n (L \setminus \bar{L}_i)$, the set of collection points j such that the corresponding fictitious edge (o, j) is contained in the edge set of all mininum cuts.

Input: A single commodity network N_o^d with multiple cuts.
Output: An edge set $\bigcap_{i=1}^n (L \setminus \bar{L}_i)$ as defined above
Let $L^* = \emptyset$ and $t = 0$.
while $L \neq \emptyset$ **do**

    1. Pick an edge $j^t \in L$ and let $L = L \setminus \{j^t\}$.

    2. Increase the return volume at the collection point j^t by one unit, i.e., let the capacity on the edge (o, j^t) in N_o^d be $\sum_\pi d_j^\pi + 1$.

    3. Solve the maximum flow problem on the new network. **if** *the maximum flow strictly increases* **then**
    $L^* = L^* \cup \{j^t\}$.

       else
    $L^* = L^*$.

       end

    4. Decrease the capacity on the edge (o, j^t) back to $\sum_\pi d_j^\pi$. Let $t = t + 1$.

end
Let $\bigcap_{i=1}^n (L \setminus \bar{L}_i) = L^*$.

We prove the validity of the above algorithm by the following Lemma.

Lemma 3. *Collection point $j \in \bigcap_{i=1}^{n}(L \setminus \bar{L}_i)$ if and only if the maximum flow of the network N_o^d strictly increases after an additional unit is collected at j.*

Proof to Lemma 2.2.7.2. "$\Rightarrow$": Assume the maximum flow does not increase after an additional unit is collected at j. Then according to the max-flow-min-cut theorem, there must exists a minimum cut that does not contain the edge (o, j) in the original N_o^d network, a contradiction to the definition of $\bigcap_{i=1}^{n}(L \setminus \bar{L}_i)$.

"$\Leftarrow$": Assume there exists a minimum cut in N_o^d that does not contain the edge (o, j). Then increasing the return volume at collection point j does not change the value of this cut, which implies that the maximum flow should not be increased when an additional unit is collected at j. $\square$

Note that the set $\bigcap_{i=1}^{n}(L \setminus \bar{L}_i)$ is the family of collection points where the products collected do not burden the collective CRN within the grand coalition. We show that there exists a minimum cut $\bar{C}$ on N_o^d such that $\{j \in L : (o, j) \in E(\bar{C})\} = \bigcap_{i=1}^{n}(L \setminus \bar{L}_i)$. Based on Lemma 2.2.7.2, given any collection point $j \notin \bigcap_{i=1}^{n}(L \setminus \bar{L}_i)$, no augmenting path exists in the network N_o^d after the capacity on the corresponding edge (o, j) is increased by 1. Since every path in N_o^d involves only one (o, j) edge, no augmenting path exists if the capacities on *all* edges (o, j) are increased by 1 for $j \notin \bigcap_{i=1}^{n}(L \setminus \bar{L}_i)$. Then increasing the capacities on all such edges by 1 will not change the maximum flow and any minimum cut $\bar{C}$ in the new N_o^d network is also a minimum cut in the original one. Hence $\bar{C}$ must satisfy $\{j \in L : (o, j) \in E(\bar{C})\} = \bigcap_{i=1}^{n}(L \setminus \bar{L}_i)$.

Theorem 2 hold if we consider such a minimum cut $\bar{C}$. Specifically, whenever $\bigcap_{i=1}^{n}(L \setminus \bar{L}_i) = \emptyset$, a minimum cut with no artificial edges (o, j) can be found and thus Theorem 2 holds.

Assume $E(\bar{C}) \cap \{(o,j), j \in L\} \neq \emptyset$, that is, at least one artificial edge (o,j) in N_o^d is in the cut set $E(\bar{C})$, indicating that increasing the volume returned to this collection point j by one unit will increase the maximum throughput in N_o^d when the min-cut is unique. Let $\bar{L} \doteq \{j : (o,j) \notin E(\bar{C})\}$, that is, $\bar{L}$ contains all collection points that collect more products than they can process through the network $\bigcup_{i \in M} N^i$. The volume burdens of products returned to the collection points in $\bar{L}$ and in $L \setminus \bar{L}$ are differentiated according to their capacity usage in the centrally-operated grand coalition. In particular, it can be shown that processing the products returned to the collection points in $L \setminus \bar{L}$ does not take up any capacity on the edges in $E(\bar{C}) \setminus \{(o,j), j \in L\}$. In other words, the independent capacity shortage in the grand coalition is caused entirely by the products collected at $\bar{L}$. Let $R_{\bar{L}}^i \doteq \sum_{j \in \bar{L}} \sum_{\pi \in \Pi^i} d_j^\pi$ be the return volume of producer i collected at $\bar{L}$, and $R_{\bar{L}} \doteq \sum_{i \in M} R_{\bar{L}}^i$; $R_{\bar{L}} > 0$ is guaranteed under the assumption of inadequate independent capacity in the centrally-operated grand coalition. We define the *volume-corrected return share* of producer i as $\frac{R_{\bar{L}}^i}{R_{\bar{L}}}$ and the cost allocation by *volume-corrected return share with capacity rewards* as x_{rVC}^p, where the allocation to the i-th producer is calculated as

$$(x^i)_{rVC}^p = \left[v(M) + \sum_{e \in E} p_e \sum_{i \in M} k_e^i \right] \cdot \frac{R_{\bar{L}}^i}{R_{\bar{L}}} - \sum_{e \in E} p_e k_e^i \quad \forall i \in M. \tag{46}$$

The above allocation is identical to (11) except for the proportions used to allocate the total cost, i.e., $\frac{R_{\bar{L}}^i}{R_{\bar{L}}}$, which is the same as $\frac{R^i}{R}$ in return share, but only applied to the volume returned to $\bar{L}$.

Theorem 4. *Assume the minimum cut $\bar{C}$ is unique. If $K_{E(\bar{C})} < R$ and $v(S) \geq 0$ $\forall S \subseteq M$, then $\underline{\mathcal{G}}(x_{rVC}^p) = 0$, i.e., $\exists$ capacity reward prices $p_e \geq 0$ $\forall e \in E$ such that the volume-corrected return share with capacity rewards generates an allocation in the core of the CRF game.*

Proof of Theorem 4. The proof of Theorem 4 is similar to that of Theorem 2 except for the following changes. Since $v(S) \geq 0 \; \forall S \subseteq M$, for the volume-corrected return share, it is sufficient to consider all sub-coalitions with $v(S) < \infty$ and $R_{\bar{L}}^S \doteq \sum_{i \in S} R_{\bar{L}}^i > 0$ to test for the core membership of the allocation x_{rVC}^p. Let Ψ_{VC} be the set of such sub-coalitions. Also define the volume-corrected average cost and average capacity availability within $S \subseteq M$ as $\bar{v}_{VC}^S = \frac{v(S)}{R_{\bar{L}}^S}$ and $\bar{k}_{eVC}^S = \frac{\sum_{i \in S} k_e^i}{R_{\bar{L}}^S}$. Hence the incentive compatibility gap of the allocation x_{rVC}^p can be calculated as $\max_{S \in \Psi_{VC}} \{ R_{\bar{L}}^S \cdot [\bar{v}_{VC}^M - \bar{v}_{VC}^S - \sum_{e \in E} p_e(\bar{k}_{eVC}^S - \bar{k}_{eVC}^M)] \}$. Modify Lemma 1 by replacing Ψ, $\bar{v}^S$ and $\bar{k}_e^S$ by their counterparts under the volume-corrected return share. Since $K_{E(\bar{C})} = \sum_{e \in E(\bar{C}) \backslash \{(o,j), j \in L\}} \sum_{i \in M} k_e^i + (R - R_{\bar{L}}) < R$ and $\sum_{e \in E(\bar{C}) \backslash \{(o,j), j \in L\}} \sum_{i \in S} k_e^i + (R^S - R_{\bar{L}}^S) \geq R^S$, we conclude that $\sum_{e \in E(\bar{C}) \backslash \{(o,j), j \in L\}} [\bar{k}_{eVC}^S - \bar{k}_{eVC}^M] > 0, \forall S \in \Psi_{VC}$. The theorem can then be derived by summing up the modified inequalities (24) over all edges in $E(\bar{C}) \backslash \{(o,j), j \in L\}$ and following the same argument in the proof of Theorem 2. Note that the result can also be extended to situations with multiple min-cuts based on similar arguments in Appendix 2.2.7.2, by defining the allocation by volume-corrected return share with capacity adjustment based on the collection points in $\bigcup_{i=1}^n \bar{L}_i$. $\qquad \square$

It can be observed that Theorem 2 is essentially a special case of Theorem 4 in which $R_{\bar{L}}^i = R^i \; \forall i \in M$. When such homogeneity no longer exists, Theorem 4 indicates that weighted corrections of producers' return shares that are proportional to the relative volume burden of their own products can adjust for the network synergies arising in a centrally-operated CRN with low independent capacity availability. Hence, when the total cost is allocated based on such proportions, the capacity reward mechanism remains effective in reducing the incentive compatibility gap of return share even when the gap is due to the cost differences among products. Note that this is only guaranteed when the return volume within any sub-coalition $S \subseteq M$

results in a nonnegative cost burden, i.e., $v(S) \geq 0$. The reason is that the mechanism essentially penalizes volume burden and rewards capacity contribution; yet when $\bar{L} \neq L$, it may be that the participation of some producer neither exerts any volume burden nor alleviates capacity shortage in the grand coalition, yet it reduces the total cost due to the high processing revenue obtained from its products. In this case, volume-corrected return share with capacity rewards is not sufficient to produce a group incentive compatible allocation.

Hence, we conclude that while the volume correction improves the group incentive compatibility of the allocation by return share with capacity rewards, the differentiation among products is derived based on their volume burdens, which does not explicitly and fully capture the network synergies arising from the heterogeneous operational and downstream costs in the CRN, in particular when processing revenue is involved. Furthermore, in cases with adequate independent capacity to process the entire return volume in the grand coalition, the volume-corrected return share becomes undefined as $\bar{R}_{\bar{L}} = 0$. In these situations, a cost-correction to the return shares of producers may be necessary to guarantee group incentive compatibility of the allocation.

2.2.7.4 Construction of the Numerical Example in §2.2.7.4

Facilities We build a sample CRN based on the state of Washington's implementation of its EPR program in 2009. First, 50 collection points are chosen from the 244 that were registered with the WMMFA [180]. The sample contains at least one collection point for every county and is generated according to Table 2. In counties where more than one collection point is to be chosen, we select one in each of the largest cities by population. In case no collection point is registered in such a city, we pick the one that is the nearest to it. The resulting sample is displayed in Table 4.

We assume that there is a consolidator in each of the 8 counties with more than

Table 2: The number of collection points chosen in each county.

Number of collection points registered	Number of collection points chosen
1-5	1
6-19	2
≥ 20	3

one collection point. Each consolidator is in/near the biggest city by population of the corresponding county and is identical to the collection point chosen in that city. The consolidators are assumed to only handle the return volume within their respective counties. In counties with only one collection point, we assume the return volume is directly transported from the collection point to the processors.

Table 3: The list of processors in the sample CRN.

Processor	Facility location	Operation type
r_1	Seattle	In-state high-tech
r_2	Vancouver	In-state high-tech
r_3	Clackamas, OR	Out-of-state TV/monitor-specialized high-tech
r_4	Roseville, CA	Out-of-state IT-specialized high-tech
r_5	Mukilteo	Low-tech
r_6	Auburn	Low-tech
r_7	Lynnwood	Low-tech
r_8	Tukwila	Low-tech

WMMFA has contracted with 8 processors in 2009, 6 within Washington state, which are reported to have processed 98.85% of the total volume in 2009. Within these 6 processors, 2 have high-tech facilities, while the remaining 4 operate mainly based on manual labor [180]. We incorporate these 6 processors into our example and label them as processor $r_1 - r_2$ (high-tech) and $r_5 - r_8$ (low-tech), plus two out-of-state processors (labeled r_3 and r_4) associated with two potential "independent plans" (see Table 3). The locations of the sampled collection points and processors in the example CRN are depicted in Figure 10. Note that all transportation distances between entities in the sample CRN are measured by the minimum traveling time in order to account for the differences in road conditions.

Table 4: The list of collection points in the sample CRN.

1	Experience Merchandise Thrift Store	2	CEP Recycle Asotin Co.
3	Clayton-Ward Company Kennewick	4	City of Chelan Recycle Center
5	Goodwill Port Angeles Store	6	Goodwill Vancouver Outlet
7	Goodwill Battleground Store	8	CEP Recycle Columbia Co.
9	Goodwill Longview Store	10	Goodwill East Wenatchee Store
11	Torboy Transfer Station	12	Tommy's Steel & Salvage
13	CEP Recycle Garfield Co.	14	CDSI Transfer & Recycling Center
15	Waste Connections Inc Aberdeen Sanitation	16	Oak Harbor Drop Box Station
17	Goodwill Port Townsend Store	18	RE-PC Seattle
19	Goodwill Bellevue Store	20	Goodwill Federal Way Store
21	Bremerton St. Vincent dePaul	22	Goodwill Bainbridge Island Don. Center
23	Goodwill Ellensburg Store	24	Regional Disposal Company - Goldendale Transfer
25	Goodwill Centralia Store	26	Lincoln County Transfer Station
27	Wilson Recycling LLC	28	Methow Recycles
29	Royal Heights Transfer Station	30	Deer Valley Transfer Station
31	Green PC Recycling	32	Goodwill Lakewood Store
33	Public Recycling Center - Canyon Rd	34	Consignment Treasures LLC
35	Appliance Recycling Connection	36	Stevenson Transfer Facility
37	St. Vincent dePaul Everett	38	Goodwill Marysville
39	E-Waste, LLC	40	Earthworks Recycling, Inc.
41	Jaco Environmental	42	Goodwill Colville Store
43	MIDWAY RECOVERY INC.	44	Goodwill Lacey Store
45	Stanley's Sanitary Service	46	Walla Walla Recycling
47	Safe And Easy Recycling Bellingham	48	Pullman Disposal Shop
49	Yakima Waste Systems Yakima	50	Sunnyside Christian Thrift Shop

Figure 10: Locations of the collection points and processors (except r_4 located in CA) considered in sample CRN. Note that recyclers r_5 and r_7 are very close to each other and overlap in figure (b).

(a) Collection point

(b) Processors
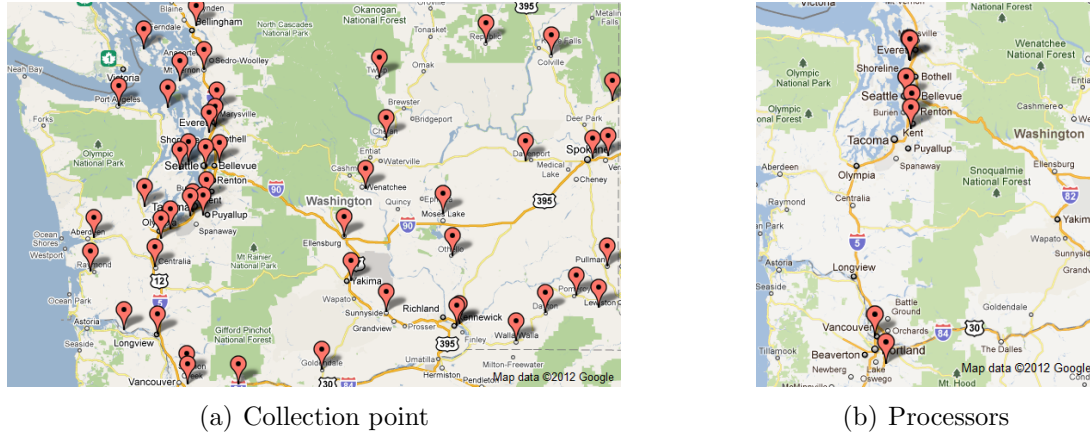
Products, Return Volumes and Capacity The EPR bill in Washington covers TVs, computers, laptops and monitors [180]. Based on their processing costs, the products are basically classified into CRT-TV/monitor (which is reported to account for 98.5% of the total TV/monitor return volume in WA), LCD-TV/monitor, desktops, laptops and computers, because these are the only cost drivers with the current processing technology. In particular, TVs/monitors contain hazardous materials and thus are costly to recycle under certain environmental standards, while the parts and materials used in computers have high reuse value and usually generate a revenue in recycling. Hence, we distinguish in the example two product types, TVs/monitors and computers. A total volume of 38,509,563 lbs of products, among which about 30% are computers and 70% are TVs/monitors, were collected in Washington in 2009 in the form of 137 different product brands from 87 producers with return shares varying from 0.001% to 7.9% [41]. In constructing the sample CRN, we uniformly choose a set of 19 products (labeled from π_1 to π_{19}), manufactured by 17 producers (labeled from m_1 to m_{17}) from the pool to capture the heterogeneity of the actual return shares in the Washington implementation. The products are also chosen in

97

order to reflect the 7:3 proportion of TVs/monitors vs computers in the total volume. We then calculate the volume of each of the 19 products in the example proportional to their relative return shares based on the actual collection volume of 38,509,563 lbs (Table 5).

In order to distribute the collective return volume of the 19 products shown in Table 5 among individual collection points, we first calculate the total amount collected in each of the collection points in the sample CRN. Specifically, if only one collection point is chosen in a county, then we assume that the entire volume within this particular county is returned to this collection point. Otherwise, the county volumes are proportionally allocated among the sampled collection points based on the corresponding city populations. Based on this data, we calculate each collection point's share of the total volume by $\lambda_j = \frac{\text{collective volume returned to collection point } j}{38,509,563}$. Then, the volume of each product π at each collection point j is calculated based on a homogeneous distribution of the product's total volume among the collection points that follows $\{\lambda_j, j = 1, 2..., 50\}$, i.e., $d_j^\pi = \lambda_j \cdot$ return volume of π shown in Table 5.

Cost Structure on the CRN and Economies of Scale Model   The unit costs in the sample CRN are disguised but structurally representative of costs in WA that are reported as aggregate averages within each stage of the CRN for a product type. All unit prices are in cents per weight (lb) except for the transportation cost, which is in cents per pound hour (lb×hr).

We assume no administrative cost in the sample CRN, as this cost is negligible in Washington. For each product type, the unit price for collection and consolidation is assumed to be identical at each such site (10 cents/lb), while different processing cost structures are quoted by different processors (see Table 6 for details). The unit downstream recycling cost (revenue) used in this example is a weighted average over all parts and materials according to their proportions by weight inside one unit of the

Table 5: List of sampled producers and their return volume and product type.

Producer	Product(s)	Volume (lbs)	Product type	Producer	Product(s)	Volume (lbs)	Product type
m_1	π_1	4,016	TV/monitor	m_2	π_2	6,718	TV/monitor
m_3	π_3	10,193	Computer	m_4	π_4	14,672	TV/monitor
m_5	π_5	25,329	TV/monitor	m_6	π_6	34,055	TV/monitor
m_7	π_7	67,724	TV/monitor	m_8	π_8	115,216	TV/monitor
m_9	π_9	140,468	TV/monitor	m_{10}	π_{10}	185,334	TV/monitor
m_{11}	π_{11}	231,127	TV/monitor	m_{12}	π_{13}	361,015	TV/monitor
	π_{12}	1,995,040		m_{13}	π_{14}	436,461	TV/monitor
m_{14} (A)	π_{16}	5,565,725	TV/monitor		π_{15}	1,097,793	
m_{15}	π_{17}	6,965,767	TV/monitor (25%) / Computer (75%)	m_{16} (B)	π_{18}	8,510,292	TV/monitor (25%) / Computer (75%)
m_{17}	π_{19}	12,742,619	TV/monitor				

Table 6: Representative full processing cost structure (cents/lb) by facility and product. τ is the mandated recycling requirement. The negative numbers indicate revenues. The landfilling cost is 3 cents/lb.

| | | High-tech processor | | | Low-tech processor |
		Local $(r_1 - r_2)$	Out-of-state TV/monitor-specialized (r_3)	Out-of-state IT-specialized (r_4)	Low-tech processor $(r_5 - r_8)$
TVs / monitors	Operational cost	2	2	2	4
	Downstream recycling cost	12	8	14	16
	Net processing cost	$5 + 9\tau$	$5 + 5\tau$	$5 + 11\tau$	$7 + 13\tau$
Computers	Operational cost	12	10	15	4
	Downstream cost (revenue)	-22	-17	-35	-10
	Net processing cost	-10	-7	-20	-6

corresponding product. Hence, given the mandated recycling requirement τ, the net processing cost is calculated as a linear approximation as follows: For TVs/monitors, the net processing cost equals *operational cost* $+ \tau \cdot$ *downstream recycling cost* $+ (1 - \tau) \cdot$ *landfilling cost*, while that for computers is simply *operational cost* $+$ *downstream revenue*, because 100% recycling will be implemented for computers regardless of τ due to the potential processing revenues.

The transportation rates are reported to be based on the geographic location of the route. Specifically, all processors in WA are located along the north-south corridor between Seattle and Vancouver (which is referred to as the "west-of-the-mountain-area"). Many common carriers operate busy routes along this north-south corridor and thus can provide cheap back-haul miles for the collection points within the area. In contrast, the transportation services for collection points located "east of the mountain" are more expensive. Hence, in our example, we use two different rates for the collection points sampled in these two areas (0.8 and 2 cents/lb×hr). The calculation is done based on a 2 cents/lb average and the assumption that the return volume at each collection point is distributed to the 8 processors according to the aggregate percentages reported by [180], and that the cost in the "west-of-the-mountain-area" is about 50% cheaper than that in the east.

As for our modeling of economies of scale in the example, our interviews indicated that a 20% cost reduction can be expected in Washington when the total volume doubles. According to this information, we calculate a decreasing quadratic function $\eta(x)$ such that $\eta(38,509,563) = 1$ and $\eta(38,509,563 * 2) = 0.8$, which produces the function $0.011146 \cdot (\frac{1}{38,509,563} \cdot x - 10.47214)^2$.

The Myopic Policy in Studying the Value of Source Separation In this section, we develop the myopic policy used for the numerical analysis to study the value of source separation of products at the collection points. The assumption behind

the myopic policy is that the e-waste volumes at collection centers are not separated by brand. In other words, the distribution of individual producers' e-waste volumes are not known at the point of collection. Rather, the total volume at each collection point j, $d_j = \sum_{\pi \in \Pi} d_j^\pi$ is the only available volume information. Because separating each producer's products at collection can be costly for high return volumes and a heterogeneous mixture of products, the myopic policy estimates the mixture of individual producers' return shares by sampling at the processors. This set-up is similar to many practical settings. Sampling typically takes place at processors, because return volumes from different collection points are consolidated there, which increases the statistical significance of the sampling procedure. This myopic policy can be cost effective if sampling costs in practice are significantly smaller than separation costs. Because sampling requires the handling of only a fraction of the waste volume, the return share calculation can be achieved at a very low cost under this policy.

To highlight the above effects, for the rest of the discussion, we assume an identical unit processing cost among all products on every edge of the CRN except at the processors (which is exactly the case in the sample CRN used in the numerical study). Because the myopic policy does not assume that the actual mixture of TVs/monitors and computers at collection points is readily available, the network flow problem cannot be solved optimally. Rather, the system operator needs to transport the e-waste volumes to the processors using a suboptimal procedure. We model this myopic procedure as follows: First the e-waste is routed to the processors to minimize the total *collection, consolidation and transportation* cost, which can be solved by the

following program.

$$(C): \quad \min \quad Z_1(y) = \sum_{e \in E \setminus \{(r,r')\}} \sum_{\pi \in \Pi} c_e \cdot y_e \tag{47}$$

$$s.t \quad \sum_{e=(u,v) \in E} y_e - \sum_{e=(v,w) \in E} y_e = 0 \quad \forall v \in V \setminus \{L, R'\} \tag{48}$$

$$y_{(j,j')} = d_j \quad \forall j \in L \tag{49}$$

$$y_e \leq \sum_{i \in M} k_e^i + K_e^p \quad \forall e \in E \tag{50}$$

$$nonnegativity \ \ constraints. \tag{51}$$

Let y^* denote the optimal solution to the above program. Next, we model the sampling procedure at the processors to determine the composition of the arriving volumes, i.e., the amount of each product π (TVs/monitors or computers), at processor r. Assume that the volume of TVs/monitors at a collection point j is generated from a binomial distribution with parameters (d_j, p), and the products are transported to processors in a uniform mix of the two product types. Let $d_r(y^*) \doteq \sum_{n' \in C'} y^*_{(n',r)}$ The required size of the product sample at each processor r is calculated by (52) (adapted from [33]), which estimates the actual volume mixture of TVs/monitors and computers with a 1% precision rate and a 99% confidence level.

$$Samplesize_r = \frac{Z^2_{1-0.01/2} \cdot p*(1-p)}{0.012} \cdot \left[1 + \frac{1}{d_r(y^*)} \cdot \left(\frac{Z^2_{1-0.01/2} \cdot p*(1-p)}{0.012} - 1 \right) \right]^{-1} \tag{52}$$

Based on the composition information of the arriving volume under the myopic routing y^* at each processor r, the processing cost at r can be calculated, which we denote as $Z_2(y^*)$. Hence, the total cost incurred under the myopic routing policy is $Z_1(y^*) + Z_2(y^*) + Samplesize \times c_{samp}$ where c_{samp} is the unit sampling cost. Our numerical analysis compares this cost with $Z(f^*) + \sum_{\pi \in \Pi, j \in J} d_j^\pi \cdot c_{sep}$, where c_{sep} is the unit separation cost, based on the sample CRN constructed. We also analyze the potential group incentive compatibility of the cost allocations under the myopic policy. We show that they can result in an inefficient fragmented system under the

assumption that producers who defect from the centrally-operated CRN need to separate their own products to operate their own private CRNs. Both the cost difference and the incentive compatibility gap are plotted as a function of product heterogeneity (modeled by the percentage of TV/monitor volume, p) and $c_{sep} - c_{samp}$. Note that we also assume that as p changes, the return volume and the independent capacity of producer A and B varies in the way such that their shares of the total TV and IT volume, as well as their normalized independent capacity availability (i.e., capacity/volume), remain constant and identical to those values in the nominal sample CRN based on the Washington instance.

2.3 Product Design or Cost Efficiency?: A Network Perspective on Extended Producer Responsibility

2.3.1 Introduction

Since its introduction in the early 1990's, EPR has been regarded at its heart a policy strategy to create incentives for more environmentally-friendly design. By internalizing the end-of-life treatment cost of their products, producers are expected to have the incentive to reduce this cost by designing more recyclable products. The question of how to realize the design potential of EPR in implementation has been discussed intensively in practice. In particular, there exist growing concerns that the current collective implementations may mute design incentives, as the weight-based cost allocations used, such as return or market share, do not differentiate between the unit costs that producers are charged for different products. Such homogeneous cost allocations do not penalize or reward producers in line with their product designs. For example, one producer's bad design and thus an expensive recycling cost of his products at end-of-life is expected to be absorbed jointly by all producers within the collective system; similarly, the cost saving resulting from the design improvement of a product will be uniformly distributed in the collective system instead of being fully

reflected in the cost allocated to the corresponding producer [84]. Indeed, some producers have indicated that they lose design incentives after participating in a collective system, as the costs they are allocated and need to internalize are not necessarily related to their own products [10]. These concerns have resulted in strong advocacy for Individual Producer Responsibility (IPR) in practice as a solution to restore design incentives in EPR implementation, a principle that requires each producer to only bear cost for *his own products* even within a collective recycling system [101, 84]. In particular, it is commonly agreed that an individual system where every producer only processes his own products incentivizes the most recyclable product design, as it involves no cost sharing and thus no free riding [10, 131, 48]. Nevertheless, it is also believed that the same level of design incentives can be achieved in a collective system by adopting an appropriate cost allocation mechanism that implements the tenet of IPR, i.e., assigning each producer the end-of-life treatment cost of his own products [143, 83, 38].

The operational nature of collective EPR implementations raises the following questions and motivates us to re-evaluate the above well-established statements. First, under EPR, the cost to collect and recycle a product is influenced by not only the recyclability of the product design but also the operational efficiency of the processing capacities. Hence, operational factors including the capacity configuration and utilization within a collection and recycling system can have an impact on the design incentives that the system can provide. In particular, a collective system with resource sharing may have different design implications compared to an individual system even if the concept of IPR is implemented via the cost allocation used. Second, how to operationalize IPR in a collective system is not a straightforward issue, either. The main reason is that sharing capacitated resources leads to synergistic benefits (called the *network synergies*) under collective implementation due to more efficient product routing. Since the value derived from such synergies is determined

by combining all product volumes and capacities, it is difficult to isolate the portion of the total cost in a collective system that is solely related to one producer's products. This gives rise to the following questions: (i) What is a reasonable way to implement IPR in a collective system taking into account these operational factors? (ii) How is the resulting design outcome in the collective system compared to that of an individual system where producers indeed only bear cost for their own products?

In this section, we aim to address the above problems by studying the impact of collective recycling operations on the design implications of collective EPR, using the collection and recycling network model introduced in §2.2. In order to highlight the design incentive dimension of collective EPR, there are several notable differences in the model used in this study compared to the one in §2.2. First, the processing stage of the post-use treatment of electronics is the most relevant stage to producers' design choices as the unit processing costs are directly influenced by the recyclability of the products. Hence, in this section, we focus on the processing operations in a CRN by studying a transportation model, where a set of different products with varying volumes is transported to and recycled at a set of processors that can be different in their capacity and processing efficiency levels. Second, the producers' design incentives are influenced by the recycling costs that they would expect to internalize at the end of life of their products. Hence, in this study, we consider a model that involves two stages of decisions under collective EPR: (i) the producers' product design choices in the first stage, (ii) the operations of a collective CRN in the second stage. The goal of this study is to understand the interactions between these two stages. A major complexity to do this under collective EPR is the mutual dependence among producers due to their network synergies in the collective CRN. Specifically, a producer's design choice determines the processing costs of his products, thus potentially influencing the operations and the cost allocation in the collective CRN, which ultimately impact all producers' design incentives. This motivates us to

105

adopt a non-cooperative game model of producers' design decisions in the first stage. We define a collection of design choices of all producers (called a *design profile*) to be a Nash equilibrium if no unilateral deviation of any producer is beneficial for himself. Such an equilibrium design profile represents a stable outcome of the producers' interactions under a collective implementation.

One important policy choice that influences the design incentives generated by collective EPR is the cost allocation mechanism used in the second stage. This has been recognized by the existing discussion in practice, which suggests the IPR principle as a crucial property of the cost allocation to eliminate the negative impact of joint responsibility in cost sharing. However, as we mentioned before, implementing IPR is not straightforward given the network setting of the CRN. In §2.2, we conduct a detailed analysis of this issue from the perspective of producers' participation incentives into the collective system, and propose a cost allocation method called the cost-corrected return share with capacity rewards. This is a cost allocation mechanism based on two adjustments to the return share model that capture the two differentiating attributes of producers' cost burdens on a collective system: (i) heterogeneity in post-use treatment cost structures, (ii) contribution of individual capacity with different processing efficiency levels. We show that this mechanism reflects the cost related to each producer's own products in the sense that it is guaranteed to be *group incentive compatible* (Theorem 1), i.e., no producer group is allocated a higher cost compared to its stand-alone cost to process the products of its members independently. In other words, it is a *core* allocation of the corresponding cooperative CRF game.

In this study, we adopt a similar approach and model the collective recycling operations in the second stage as a cooperative CRF game based on the transportation model. Hence, to summarize, we study a two-stage *biform game* model in this section involving both non-cooperative and cooperative elements (Figure 11). We
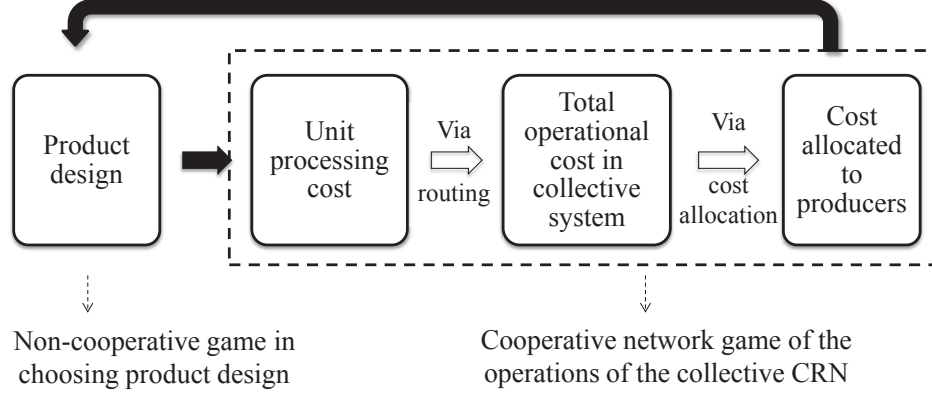
Figure 11: A two-stage biform game model to analyze the design implication of collective EPR implementation.

assume that the allocation by cost-corrected return share with capacity rewards is implemented, and thus all producers' voluntary participation in the collective system is guaranteed in the second stage. In analysis, we focus on studying the equilibrium design profile in the first stage under the influence of this core allocation. In particular, we evaluate the products' recyclability levels under such an equilibrium with those that producers are incentivized to adopt in an individual system where everyone processes his own product independently. We call a system providing *superior* design incentives for a producer if it incentivizes more recyclable design of his products. We also mention that to use a core allocation in the cooperative part of a biform game is a common approach in analyzing such models in the literature, for example, in the original biform game proposed by [23] and in its applications, such as in supply chain management (e.g., [9]).

Our main observation in this study is that under the allocation by cost-corrected return share with capacity rewards, the design implication of collective EPR strongly depends on the level of potential network synergy in the collective CRN, which is measured by, for each producer, the availability of more efficient capacity owned by

107

others that he can potentially benefit from in the collective system. Specifically, our analysis of a two-producer case indicates that when the network synergy level is high enough, a collective system provides inferior design incentives than an individual one for every producer. We show that this is mainly because the high network synergy already significantly reduces the recycling cost allocated to each producer, such that the cost reduction potential by improving the product design becomes quite limited. However, in cases where the synergy level is low, the opposite result can be observed, i.e., a collective system provides superior design incentives than an individual one for every producer. This is due to the network setting of the CRN where the products' marginal costs matter in designing group incentive compatible cost allocations. In the low synergy case, although the total processing cost decreases due to the network synergy, the marginal costs of both producers' products go up, thus creating incentives for more recyclable design. We show that both of these two results can be extended to the general case with any number of producers. The situations in between, which we call the medium synergy case, is shown to be more complex even with two producers; in particular, the marginal value of capacity exerts an additional impact such that a pure Nash equilibrium may not exist. However, by studying mixed-strategy equilibria, we show that the collective system provides inferior incentives for the two producers in the medium synergy case. How the result will generalize when more producers are included is an immediate open question to be addressed.

In order to study the design issue under the current EPR implementations that typically use weight-based proportional allocations, we further analyze the equilibrium design profile when the allocation by return share is used in the second stage of the biform game model. Our analysis discovers a surprising result: In the low- and medium-synergy case, a collective system can provide superior design incentives compared to an individual system even under return share, if the capacity heterogeneity in the collective CRN is high enough in terms of processing efficiency. This result

underlines the significant influence of the operational factors, which can dominate the negative economic impact derived from joint responsibility under such a homogeneous cost sharing mechanism by return share.

To summarize, our analysis in this section demonstrates the importance of taking into account the operational factors of a collective system in order to understand the design implication of collective EPR implementations, especially the network synergy from a more efficient routing due to capacity sharing. In particular, the network synergy gives rise to a potential tradeoff between cost efficiency and design incentives under collective EPR. At the same time, operational details associated with product routing, such as the marginal costs of products, can also exert a big impact, as it is demonstrated in the low synergy case. Based on these results, we can characterize the operational conditions regarding the network structure and the capacity configuration of a collection and recycling system, under which a collective implementation may excel or be strictly dominated in creating design incentives compared to an individual system. This observation challenges the economic perspective generally held in practice, which suggests that an individual system always provides the best design incentives, yet the same incentives can also be achieved in a collective system via proper cost allocation mechanisms.

The study in this section contributes to the existing discussion on realizing the design potential of EPR, where the operational factors, especially the network operations under capacity sharing, are generally not captured. For example, in the environmental economics literature, a set of environmental policy instruments, such as producer take-back mandates and combined fee/subsidy approaches, are analyzed for their design-for-environment (DfE) potential (e.g., see a review in [171]); the study is conducted from an economic perspective and does not address implementation issues in translating these policies into working systems. The current operations literature on achieving the design potential of EPR focuses on stylized analysis of the

implications of implementation issues, such as cost sharing in collection and recycling, information about return volumes, competition in the product market, new product introduction frequency, and supply chain coordination (e.g., [10, 186, 48, 131, 156]). The main contribution of our study to this literature is to explicitly model the network operations in a collective system, and thus to capture the impact of the synergistic effect from capacity sharing.

From a methodological point of view, the original biform game model proposed by [23] considers the setting where the strategy set of individual players in the first stage is finite, while in our problem, each producer is assumed to have a continuous choice of product design. Biform models have been widely adopted to study multi-stage decision-making processes that involve both competitive and collaborative elements, most prominently in the literature on supply chain management (e.g., [65, 31, 9]). The paper whose approach is the closest to ours is [9]. The paper studies an inventory management problem where retailers make independent inventory decisions and then, after the demand is realized, collaborate in transshipment. The authors identify core allocations in the second-stage cooperative game, under which the first-stage equilibrium of retailers' inventory decisions is further analyzed. Note that in this formulation, the transshipment problem is modeled as a simple linear program whose optimal profit can be fully captured by a closed-form differentiable function. Although using a similar approach, we provide a more comprehensive discussion of the impact of network effects. Specifically, the interactions between the second-stage cost allocation and the first-stage design decisions are critically based on the pattern of the optimal routing in the CRN, e.g., the set of processors where a product is processed. We study this problem by analyzing the combinatorial structure of the optimal routing under three network configurations of the CRN, based on which we characterize the equilibrium design profile.

2.3.2 Model Description

In the following, we first introduce the notation and the processing cost structure of the products considered in this study in §2.3.2.1. We then define the biform game model in §2.3.2.2.

2.3.2.1 Notation and Processing Cost Structure.

We denote a producer by i and the set of producers by M. Since our goal is to compare producers' design incentives in a collective system versus his operating alone in an individual system, we assume that each producer has sufficient *independent capacity* to process his own product. Moreover, as mentioned in the introduction, we focus on the processing stage of the post-use treatment of electronics in order to highlight the end-of-life impact of product designs. Hence, in this study, we consider the following setting: Each producer i has one product, denoted by $\pi(i)$, with return volume $d^{\pi(i)}$, and has independent capacity at its own processor, denoted by $r(i)$; the amount of capacity at $r(i)$ is denoted by $k_{r(i)}$ and satisfies $k_{r(i)} \geq d^{\pi(i)} \ \forall i \in M$. We also denote the set of all products and processors as $\Pi = \{\pi(i)\}$ and $R = \{r(i)\}$, respectively. In the analysis, we will also sometimes drop (i) and refer to a generic product and processor as π and r, respectively.

In this study, we focus on products that incur net unit processing costs[5], denoted by c_r^{π} for each product π processed at each facility r. In order to model the attributes that influence this unit cost, we first model c_r^{π} as the sum of two parts $\bar{c}^{\pi} + \underline{c}_r^{\pi}$. The first component $\bar{c}^{\pi}$ represents a base cost to process a unit of π and is determined by the universal characteristics of the product category. For example, the cost required to dismantle any TV might have a unit base cost. The second component $\underline{c}_r^{\pi}$, for example, the cost to process the leaded glass of a CRT TV, depends on the product

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[5]The framework can also be used to study scenarios with products that generate recycling revenues.

design of π and the processing efficiency at r, which we model as follows. Let r_0 be a fixed *standard* processor whose efficiency level is taken as a benchmark. The unit cost $\underline{c}_{r_0}^\pi$ incurred at r_0 reflects the end-of-life impact of the product design of π. We capture such a relationship by defining the *recyclability level* of π, denoted by κ^π, to be equal to the cost $\underline{c}_{r_0}^\pi$. Intuitively, the smaller the κ^π is, the more recyclable the product design is. For discussion convenience, we allow κ^π to be any number in $(0, \infty)$. For a given processor r, a natural measure of its processing efficiency level is the ratio between the unit costs incurred at r and at the standard processor r_0 for each product π, i.e., $\frac{c_r^\pi}{\underline{c}_{r_0}^\pi}$. In this study, we focus on a homogeneous case where for each processor r, this ratio $\frac{c_r^\pi}{\underline{c}_{r_0}^\pi}$ is identical for all products. Hence, we assign to each processor r a parameter $\tau_r \doteq \frac{c_r^\pi}{\underline{c}_{r_0}^\pi}$ to represent its efficiency level. Similarly, a smaller τ_r value indicates more efficient capacity at r. To summarize, we model the unit cost to process a product π at a processor r as a function of the product's recyclability level κ^π and the processors' efficiency level τ_r in the following way:

$$c_r^\pi = \bar{c}^\pi + \kappa^\pi \cdot \tau_r \quad \forall \pi, \forall r \tag{53}$$

2.3.2.2 A Biform Game Model.

In this subsection, we first introduce the transportation network model of the processing operations in the collective system in the second stage, as well as the corresponding cooperative network game and the allocation by cost-corrected return share with capacity rewards. We then describe the non-cooperative game model of producers' product design decisions in the first stage, based on which the notion of an equilibrium design profile is defined.

Second Stage Model: Collective Processing Operations We begin the description of the biform game model with collective recycling operations in the second stage. Specifically, we model the processing stage in a collective system as a transportation problem where the returned products are assigned to different processors.

$$\Pi = \pi(i) \left\{ \right\} R = r(i)$$
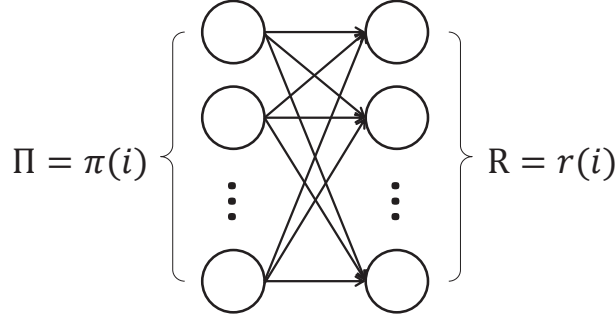
Figure 12: A Transportation CRN that represents the processing stage in the collection and recycling procedure.

We mention that a similar transportation model is used in [89] to study the assignment of electronics components to different reuse options. The collective transportation CRN structure is shown in figure 12, where the two tiers of nodes represent the set of products Π and the set of processors R, respectively. There exists an edge between every pair of product π and processor r, and the unit cost on that edge is modeled as the unit processing cost c_r^π. In this study, we focus on the case where the transportation network only involves independent capacity of producers. Note that to process the entire return volume on such a network is feasible since $k_{r(i)} \geq d^{\pi(i)}$ $\forall i \in M$. The goal in managing the product routing on the transportation network is to minimize the total cost incurred, which can be computed by the following *centralized transportation problem* (C_t) (In this section, we use an additional subscript t to differentiate some of the notations associated with the transportation network from their counterparts used in the previous section.)

$$(C_t): \quad \min \quad Z_t(f_t) = \sum_{\pi \in \Pi} \sum_{r \in R} c_r^\pi \cdot f_{tr}^\pi \tag{54}$$

$$s.t. \quad \sum_{r \in R} f_{tr}^\pi = d^\pi \quad \forall \pi \in \Pi \quad [\beta^\pi] \tag{55}$$

$$\sum_{\pi \in \Pi} f_{tr}^\pi \leq k_r \quad \forall r \in R \quad [\alpha_r] \tag{56}$$

$$nonnegativity \;\; constraints. \tag{57}$$

In the above program, constraint (55) makes sure that all return volume is processed. Constraint (56) guarantees that the capacity at each processor is not exceeded. The variables β^π and α_r denote the dual variables associated with the corresponding constraints. Let f_t^* denote the social optimal routing to the above centralized transportation problem. The minimum total processing cost $Z_t(f_t^*)$ is then the total amount to be allocated among the producers. In order to model this cost allocation problem, we setup a cooperative CRF_t game in a similar way as in §2.2.3. Specifically, we define the value of the grand coalition $v_t(M) \doteq Z_t(f^*)$. For each sub-coalition of producers $S \subsetneq M$, its incentive to participate in a collective system is captured by the difference between its allocated cost in the collective system and its stand-alone cost $v_t(S)$, which is defined as the minimum total cost to process the return volume of the coalition members using their independent capacity. Such a cost can be calculated by a similar program as (C_t), where the product set Π and the processor set R are replaced by those within the sub-coalition S, i.e., $\Pi^S = \{\pi(i), i \in S\}$ and $R^S = \{r(i), i \in S\}$.

We show that the model of cost-corrected return share with capacity rewards gives rise to a core allocation of the above CRF_t game. Specifically, let $[\beta^{*\pi}, \alpha_r^*]$ be the optimal dual solution of the problem (C_t). In the transportation setting defined above, the allocation by cost-corrected return share with capacity rewards, denoted by $x_{t\mu}^p$, can be calculated as follows.

$$(x^i)_{t\mu}^p = \left[v_t(M) + \sum_{r \in R} p_r k_r \right] \cdot \mu_t^i - p_{r(i)} k_{r(i)} \quad \forall i \in M. \tag{58}$$

where p_r is the unit price rewarded to the independent capacity at processor r, and the cost-corrected return share μ_t^i is calculated as

$$\mu_t^i = \frac{d^{\pi(i)} \beta^{*\pi(i)}}{\sum_{\pi \in \Pi} d^\pi \beta^{*\pi}}. \tag{59}$$

It is easy to derive the following result as a corollary to Theorem 1.

114

Corollary 1. *Given any transportation CRN, the allocation $x_{t\mu}^p$ is guaranteed to be in the core of the CRF_t game if $p_r = |\alpha_r^*| \ \forall r \in R$.*

For the rest of this section, we denote by $x_{t\mu}^\alpha$ the cost allocation by cost-corrected return share with the capacity reward price at each processor equal to the marginal value of the capacity there. Corollary 1 indicates that under the allocation $x_{t\mu}^\alpha$, each sub-coalition S incurs a total cost within the grand coalition that is no higher than its stand-alone cost, i.e., $\sum_{i \in S}(x^i)_{t\mu}^\alpha \le v_t(S) \ \forall S \subset M$. Hence, the allocation guarantees the voluntary participation of all producers in the collective system and thus the formation of a grand coalition. In the rest of this study, we will assume that this allocation is implemented in the second stage unless stated otherwise.

First Stage Model: Product Design Decisions Now we move up to the first stage and consider how to incorporate the producers' product design choices into the model. First, for each producer i, we model the recyclability level $\kappa^{\pi(i)}$ of his product $\pi(i)$ as i's decision variable in designing the product. We call a collection of design choices of all producers a *design profile*, and denote it by $\mathcal{K} = \{\kappa^{\pi(i)}, \forall i \in M\}$.

Next we analyze how producers' design decisions in the first stage interact with the CRF_t game in the second stage. As modeled in (53), a design profile $\mathcal{K}$ affects the unit processing costs of the products and thus has an impact on the total processing cost $Z_t(f^*)$ (i.e., the value of the grand coalition $v_t(M)$) via the transportation problem (54) - (57). Hence, $\mathcal{K}$ will potentially influence the cost allocated to each producer. To capture this relation, we regard the cost allocation mechanism $x_{t\mu}^\alpha$ as a vector function that maps a design profile $\mathcal{K}$ to a vector in $R^{|M|}$ whose elements correspond to the costs allocated to all producers. In order to highlight such a functional relationship, in the remaining analysis of this section, we will also write the allocation as $x_{t\mu}^\alpha(\mathcal{K})$ and the cost allocated to each producer i as $(x^i)_{t\mu}^\alpha(\mathcal{K})$. Note that such an allocated cost determines a producer's design incentive in the collective system. In particular,

115

the producer would like to balance between the allocated post-use recycling cost and the investment required to achieve a design improvement. Hence, we first model the investment cost for each producer i as a decreasing convex function of the recyclability level of his own product, denoted as $Q^i(\kappa^{\pi(i)})$; this function can be different among the producers. Intuitively, the smaller $\kappa^{\pi(i)}$ is, the more recyclable the product is, which requires a higher investment, as well as a larger marginal cost for further improvement. Furthermore, for analysis convenience, we make the following smoothness assumption of Q^i, which is referred to in the rest of this section as the *property $\star$*.

Definition 1. *Q^i is defined to have the property $\star$ if (i) it is continuously differentiable; (ii) its first order derivative $Q^{i\prime}$ is strictly monotone; and (iii) $Q^{i\prime}$ satisfies $\lim_{\kappa^{\pi(i)} \to 0} Q^{i\prime}(\kappa^{\pi(i)}) = -\infty$ and $\lim_{\kappa^{\pi(i)} \to \infty} Q^{i\prime}(\kappa^{\pi(i)}) = 0$.*

Clearly, a function $Q^{i\prime}$ that satisfies the property $\star$ is invertible on $(0, \infty)$ and we denote its inverse function by $(Q^{i\prime})^{-1}$. Then we model that each producer i chooses the recyclability level of his product $\kappa^{\pi(i)}$ in the first stage in order to minimize his total cost, which is calculated as the summation of the investment cost and the allocated recycling cost as follows.

$$Cost^i(\mathcal{K}) \doteq Q^i(\kappa^{\pi(i)}) + (x^i)^\alpha_{t\mu}(\mathcal{K}). \tag{60}$$

Clearly, since each producer's total cost is a function of $\mathcal{K}$ that includes the design of all products, there exists mutual impact between producers' design decisions. Hence, we propose the following notion of an *equilibrium design profile*. For each producer i, let $\mathcal{K}^{-i} \doteq \{\kappa^{\pi(j)}, \forall j \neq i, j \in M\}$ denote the set of design decisions of other producers.

Definition 2. *A design profile is defined to be a Nash equilibrium design profile, denoted by $\mathcal{K}_{ne} = \{\kappa^{\pi(i)}_{ne}, \forall i \in M\}$, if $\forall i \in M$, producer i achieves the smallest total cost by adopting the design $\kappa^{\pi(i)}_{ne}$, given that others all adopt the designs in $\mathcal{K}^{-i}_{ne}$. Mathematically,*

$$Cost^i(\mathcal{K}_{ne}) \doteq Cost^i(\kappa^{\pi(i)}_{ne}, \mathcal{K}^{-i}_{ne}) \leq Cost^i(\kappa^{\pi(i)}, \mathcal{K}^{-i}_{ne}) \quad \forall \kappa^{\pi(i)} \tag{61}$$

Intuitively, an equilibrium design profile represents a stable design outcome of a collective implementation from which no unilateral deviation is beneficial for any producer. In this study, we focus on evaluating the products' recyclability levels under an equilibrium $\mathcal{K}_{ne}$.

We would like to make the following additional remark about the above model. Notice that in order to analyze the influence of the post-use recycling cost to producers' first-stage design choices, the model implicitly assumes that the producers have access to deterministic estimates of their future return volumes, as well as to information regarding the capacity availability in the collective CRN. This is relevant in the case where the CRN operates based on stable capacities, and the EPR program is mature enough such that most products sold will be returned for recycling, hence the return volume can be estimated from the sales volume with a certain degree of precision. Moreover, the return volumes are also assumed to be exogenous parameters in this model. We mention that in practice, return volumes are typically dependent on sales volumes, which are influenced by various factors in the product market, for example, the competition among the producers in the same product category. Notice that these factors can be in turn influenced by the producers' product design choices especially with environmentally-sensitive customers. Yet in this study, our focus is to analyze the impact of an EPR regulation and the collective form of its implementation on the design strategies of producers, assuming given product market conditions such that the sales volumes and thus the return volumes can be estimated. We mention that the influence of product market competition on the design implication of collective EPR has been addressed in the literature, for example, in [10], using stylized models that do not capture network effects. Hence, we take this problem, as well as the impact derived from the intrinsic uncertainty in return volumes, as important and interesting future research directions in this research stream.

2.3.3 Analysis of Equilibrium Design Profiles under Collective Implementation

In this section, we formally analyze the equilibrium design profile under a collective implementation. This is generally a complex problem given the network operations with capacity sharing in a collective CRN. The main challenge is that the cost allocation depends on the social optimal routing, which can change under different design profiles and thus is generally difficult to be fully captured by a closed-form formula. Hence, in the following, we first analyze the biform game model with two producers and discuss the intuition and insights from the observations (§2.3.3.1). Based on this analysis, we further study the general situation with any number of producers in §2.3.3.2. Finally, in §2.3.3.3, we study the equilibrium design profile if the widely-used allocation by return share is adopted in the second stage.

As mentioned before, in order to evaluate the design implication of collective implementations, we take the product designs that the producers will choose if operating independently in an individual system as the benchmark. Hence, we first characterize such a benchmark as follows. For every producer i, we can calculate that the stand-alone processing cost that i incurs in an individual system is $d^{\pi(i)} \cdot c_{r(i)}^{\pi(i)} = d^{\pi(i)} \cdot [\kappa^{\pi(i)} \cdot \tau_{r(i)} + \bar{c}^{\pi(i)}]$. The producer's design incentive operating independently is then captured by the value of the recyclability variable $\kappa^{\pi(i)}$ that minimizes the total cost function $d^{\pi(i)} \cdot [\kappa^{\pi(i)} \cdot \tau_{r(i)} + \bar{c}^{\pi(i)}] + Q^i(\kappa^{\pi(i)})$. Since Q^i is a convex function, it is easy to see that such a minimum, which we denote as $\kappa_{ind}^{*\pi(i)}$, must satisfy $d^{\pi(i)} \cdot \tau_{r(i)} + Q^{i\prime}(\kappa_{ind}^{*\pi(i)}) = 0$, i.e., the first-order derivative of the total cost function equals zero. Hence, we can solve that

$$\kappa_{ind}^{*\pi(i)} = (Q^{i\prime})^{-1}(-d^{\pi(i)} \cdot \tau_{r(i)}) \tag{62}$$

We call $\kappa_{ind}^{*\pi(i)}$ producer i's *individual optimal design*.

118

Consider two producers 1 and 2. Without loss of generality, assume that producer 1's independent capacity is more efficient, i.e., $\tau_{r(1)} < \tau_{r(2)}$. In this case, a collective system can benefit from the network synergy when producer 2's product $\pi(2)$ is processed at $r(1)$ at a lower unit cost. Hence, we evaluate the potential synergy level in the collective CRN by the availability of the efficient capacity $k_{r(i)}$ relative to the return volume, in particular $d^{\pi(2)}$, i.e., the return volume of producer 2's product. Specifically, we consider three scenarios in this subsection, in each of which the main observations are summarized in Table 2.

Table 7: Summary of observations in the two-producer case.

	Low-synergy	Medium-synergy	High-synergy
Definition	$k_{r(1)} \leq d^{\pi(2)}$	$d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$	$d^{\pi(1)} + d^{\pi(2)} \leq k_{r(1)}$
Existence of a equilibrium design profiles	Guaranteed	Not guaranteed (mixed-strategy equilibrium is further analyzed)	Guaranteed
Type of system providing superior design incentives	Collective	Individual	Individual

Low-synergy Scenario Our approach to study the equilibrium design profile of the biform game under collective EPR is based on characterizing each producer's best design choice in the first stage given the other producer's decision. In other words, for each producer, we first solve his *best design response function* (best response function for short in the rest of this section) to the other producer's design decision under collective implementation. For discussion convenience, we denote such functions as $\kappa_{clt}^{*\pi(1)}$ and $\kappa_{clt}^{*\pi(2)}$.

We begin by analyzing producer 1's best design choice, assuming that producer 2 has adopted a recyclability level $\hat{\kappa}^{\pi(2)}$ for his own product $\pi(2)$. Depending on producer 1's design decision, we can solve the social optimal routing (Figure 13) and

(a) When $\kappa^{\pi(1)} < \hat{\kappa}^{\pi(2)}$ (b) When $\kappa^{\pi(1)} \geq \hat{\kappa}^{\pi(2)}$
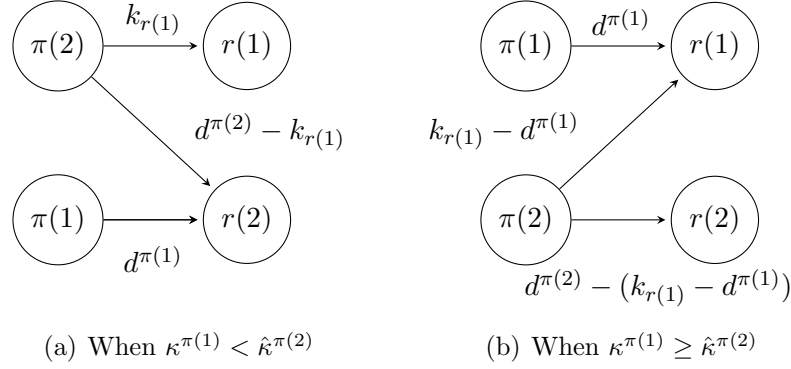
Figure 13: Social optimal routing in the low-synergy scenario.

calculate the recycling cost allocated to producer 1 under cost-corrected return share with capacity rewards accordingly.

$$
(x^1)^{\alpha}_{t\mu}(\kappa^{\pi(1)}) = \begin{cases} d^{\pi(1)} \cdot \left[\kappa^{\pi(1)} \cdot \tau_{r(2)} + \bar{c}^{\pi(1)}\right] - k_{r(1)} \cdot \hat{\kappa}^{\pi(2)} \cdot \left(\tau_{r(2)} - \tau_{r(1)}\right) & \text{if } \kappa^{\pi(1)} < \hat{\kappa}^{\pi(2)} \\ \\ d^{\pi(1)} \cdot \left[\kappa^{\pi(1)} \cdot \tau_{r(1)} + \bar{c}^{\pi(1)} + \hat{\kappa}^{\pi(2)} \cdot \left(\tau_{r(2)} - \tau_{r(1)}\right)\right] - k_{r(1)} \cdot \hat{\kappa}^{\pi(2)} \cdot \left(\tau_{r(2)} - \tau_{r(1)}\right) \\ \\ \hspace{8cm} \text{if } \kappa^{\pi(1)} \geq \hat{\kappa}^{\pi(2)} \end{cases}
$$

$$(63)$$

The total cost of producer 1 is then calculated as $Cost^1(\kappa^{\pi(1)}) = Q^1(\kappa^{\pi(1)}) + (x^1)^{\alpha}_{t\mu}(\kappa^{\pi(1)})$. Since $(x^1)^{\alpha}_{t\mu}(\kappa^{\pi(1)})$ is piecewise linear and $Q^1(\kappa^{\pi(1)})$ is convex, we can conclude that given producer 2's design choice $\hat{\kappa}^{\pi(2)}$, $Cost^1(\kappa^{\pi(1)})$ is a piecewise convex function. Under the behavioral model that producer 1 aims at minimizing his total cost, we can obtain his best design response to producer 2's design choice $\hat{\kappa}^{\pi(2)}$ by solving the global minimum of the function $Cost^1$. In the next proposition, we present a characterization of producer 1's best response function $\kappa^{*\pi(1)}_{clt}$ in the low-synergy scenario.

Proposition 5. *In the low-synergy scenario, given any convex decreasing function Q^1 that satisfies the property $\star$, the best response function of producer 1, $\kappa^{*\pi(1)}_{clt}$, is a*

120

(a) When $\hat{\kappa}^{\pi(2)} < l_2$

(b) When $l_2 \leq \hat{\kappa}^{\pi(2)} \leq \kappa_0^{\pi(2)}$

(c) When $\kappa_0^{\pi(2)} < \hat{\kappa}^{\pi(2)} \leq l_1$

(d) When $\hat{\kappa}^{\pi(2)} > l_1$
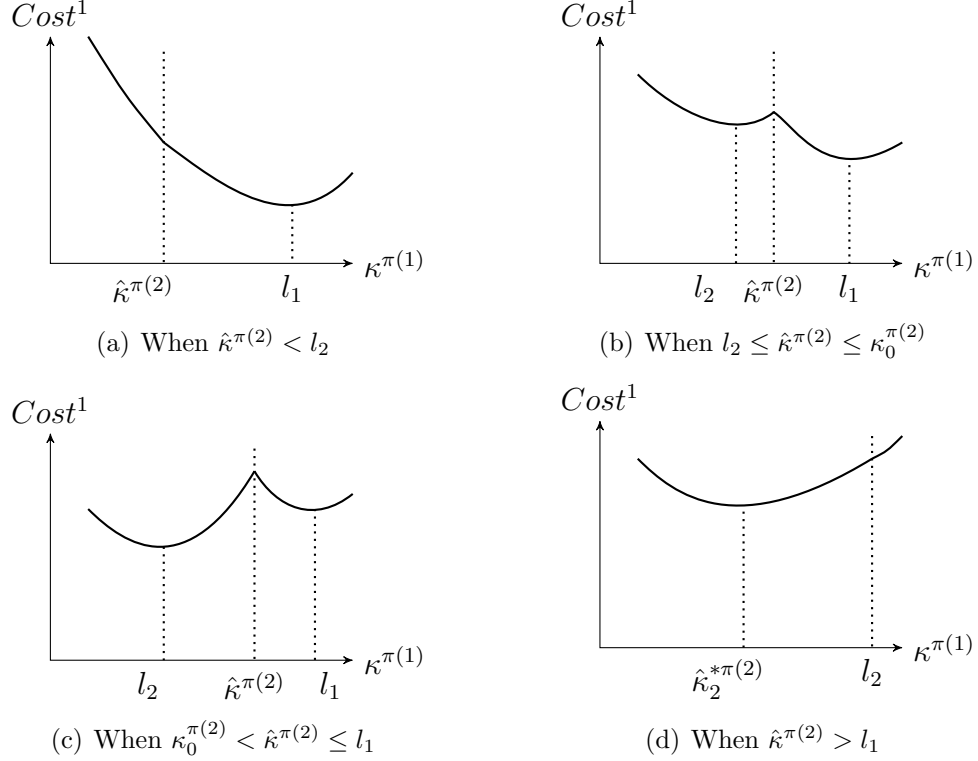
Figure 14: Four situations of the graph of the function $Cost^1$ in the low-synergy scenario.

step function that satisfies

$$
\kappa_{clt}^{*\pi(1)} = \begin{cases} l_1 \doteq (Q^{1\prime})^{-1}(-d^{\pi(1)} \cdot \tau_{r(1)}) & \forall \hat{\kappa}^{\pi(2)} \leq \kappa_0^{\pi(2)} \\ l_2 \doteq (Q^{1\prime})^{-1}(-d^{\pi(1)} \cdot \tau_{r(2)}) & \forall \hat{\kappa}^{\pi(2)} > \kappa_0^{\pi(2)} \end{cases} \tag{64}
$$

where $\kappa_0^{\pi(2)} \in (l_2, l_1)$ is a constant defined based on Q^1.

To prove the above result, first notice that the cost allocation to producer 1, $(x^1)_{t\mu}^{\alpha}$, is a *concave* piecewise linear function. Hence, we can show that, given any convex decreasing function Q^1 that satisfies the property $\star$, there are four situations of the graph of the total cost function $Cost_1 = Q^1 + (x^1)_{t\mu}^{\alpha}$ (Figure 14; please refer to Lemma 7 in Appendix 2.3.6 for details.)

Now we analyze the best design response of producer 2 to producer 1's design choice in the low-synergy case. It turns out that under the allocation by cost-corrected return share with capacity rewards, the cost allocated to producer 2 is not influenced

by producer 1's design choice, and can be calculated as a linear function of his own design variable $\kappa^{\pi(2)}$ as follows.

$$(x^2)^{\alpha}_{t\mu}(\kappa^{\pi(2)}) = d^{\pi(2)} \cdot \left[\kappa^{\pi(2)} \cdot \tau_{r(2)} + \bar{c}^{\pi(2)}\right] \tag{65}$$

Hence, it is easy to see that in order to minimize his own total cost $Cost^2(\kappa^{\pi(2)}) = Q^2(\kappa^{\pi(2)}) + (x^2)^{\alpha}_{t\mu}(\kappa^{\pi(2)})$, producer 2's best design strategy is to adopt the following recyclability level

$$q_1 \doteq (Q^{2\prime})^{-1}\left(-d^{\pi(2)} \cdot \tau_{r(2)}\right). \tag{66}$$

In other words, producer 2's best response function $\kappa^{*\pi(2)}_{clt}$ is a constant function equal to q_1.

By definition, an equilibrium design profile is the intersection of both producers' best response functions (Figure 15). Hence, based on the above analysis, we conclude the following theorem.
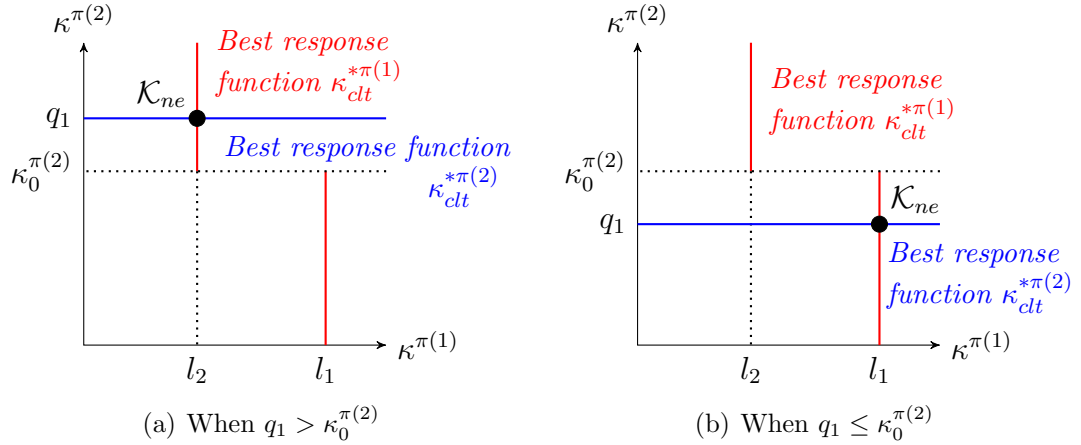
(a) When $q_1 > \kappa_0^{\pi(2)}$ (b) When $q_1 \leq \kappa_0^{\pi(2)}$

Figure 15: The Equilibrium design profile under collective implementation in the low-synergy scenario.

Theorem 5. *In the low-synergy scenario, given any convex decreasing functions Q^1 and Q^2 that satisfy the property $\star$, there exists at least one equilibrium design profile $\mathcal{K}_{ne}$. Moreover, for both producers, the recyclability levels of their product designs under any $\mathcal{K}_{ne}$ are no worse than their own individual optimal designs, i.e., $\kappa_{ne}^{\pi(i)} \leq \kappa_{ind}^{*\pi(i)}$ for both $i = 1, 2$.*

We point out that the equilibrium design profile in the low-synergy scenario is unique when $\kappa_0^{\pi(2)} \neq q_1$. In particular, when $q_1 > \kappa_0^{\pi(2)}$, collective implementation incentivizes a strictly more recyclable design of product $\pi(1)$ (Figure 15(a); see Appendix 2.3.6 for details.) The intuition underlying the theorem is the following. When the allocation mechanism of the cost-corrected return share with capacity rewards is used, both producers' design incentives depend on the sensitivity of their marginal costs in the collective CRN to the changes in their product designs. In particular, formula (64) and (66) indicate that in the low-synergy scenario, the faster the rate of change in the marginal cost of a product is, a more recyclable product design will be adopted by the corresponding producer, due to the convexity of the investment cost. Note that such sensitivity is determined by the efficiency level of the facility where the last unit of the product is processed. In the low-synergy scenario, we can observe from Figure 13 that in the collective CRN, although the optimal total processing cost is reduced compared to that of an individual system due to the benefits from resource sharing, the last unit of either product is processed using capacity no more efficient than the independent capacity of the corresponding producer. This results in superior or equal design incentives to each producer under collective implementation in the low-synergy scenario compared to their operating independently in an individual system.

Next, in order to highlight the impact of network synergy, we analyze the other extreme case, the high-synergy scenario, where there exists sufficient capacity at the efficient processor $r(1)$ to process the entire volume. We obtain the opposite result in that situation, i.e., collective implementation offers inferior design incentives compared to an individual system.

High-synergy Scenario The high-synergy scenario is defined such that there is sufficient efficient capacity to process the entire volume, i.e., $d^{\pi(1)} + d^{\pi(2)} \leq k_{r(1)}$.

It is easy to see that in this case, it is socially optimal to process all products at the efficient processor $r(1)$. Hence, we can calculate the allocation by cost-corrected return share with capacity rewards to each producer as follows.

$$(x^1)^\alpha_{t\mu}(\kappa^{\pi(1)}) = d^{\pi(1)} \cdot [\kappa^{\pi(1)} \cdot \tau_{r(1)} + \bar{c}^{\pi(1)}]$$
$$(x^2)^\alpha_{t\mu}(\kappa^{\pi(2)}) = d^{\pi(2)} \cdot [\kappa^{\pi(2)} \cdot \tau_{r(1)} + \bar{c}^{\pi(2)}] \tag{67}$$

Notice that in this scenario, the cost allocations and thus the design decisions of individual producers are completely independent of each other. Thus, we can conclude the following result.

Proposition 6. *In the high-synergy scenario, given any convex decreasing functions Q^1 and Q^2 that satisfy the property $\star$, there exists a unique equilibrium design profile $\mathcal{K}_{ne}$ such that*

$$\kappa^{\pi(i)}_{ne} = (Q^{i\prime})^{-1}(-d^{\pi(i)} \cdot \tau_{r(1)}) \geq \kappa^{*\pi(i)}_{ind} \quad i = 1, 2 \tag{68}$$

In other words, for both producers, the recyclability levels of their product designs under $\mathcal{K}_{ne}$ are no better than their own individual optimal designs.

Proposition 6 demonstrates the potential tradeoff between cost efficiency and design incentives in a collective CRN. Specifically, when there exists a substantial level of network synergy in the collective CRN, not only the total processing cost is significantly reduced compared to that of an individual system, but also the marginal cost of each product is becomes smaller or remains unchanged, as the entire return volume is processed at the efficient facility $r(1)$. This leads to inferior design incentives under collective implementation.

A common feature of both the low-synergy and the high-synergy scenarios is that only the marginal costs of their products play a role in determining the producers' design incentives in the collective CRN. Next, we show that when there exists a medium level of network synergy in the collective CRN, the marginal value of the producers' independent capacity can also exert an impact through the capacity reward

(a) When $\kappa^{\pi(1)} < \kappa^{\pi(2)}$ (b) When $\kappa^{\pi(1)} \geq \kappa^{\pi(2)}$
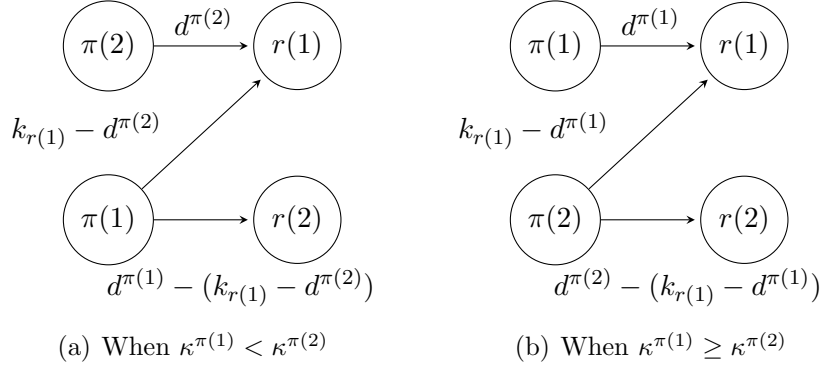
Figure 16: Social optimal routing in the medium-synergy scenario.

mechanism adopted under the cost allocation $x_{t\mu}^\alpha$. Such an impact adds significant complexity to the problem. For example, it may undermine the existence of an equilibrium design profile.

Medium-synergy Scenario We adopt a similar approach to analyze the medium-synergy scenario, starting with characterizing the social optimal routing in the collective CRN. Specifically, since the efficient capacity at $r(1)$ satisfies $\max\{d^{\pi(1)}, d^{\pi(2)}\} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$, the social optimal routing is to route the full return volume of the product that is more difficult to recycle (i.e., the one with the higher κ parameter) to $r(1)$, use the remaining capacity at $r(1)$ to process the other product, and finally route the rest of the return volume to $r(2)$ (Figure 16). Hence, we can calculate each producer's cost allocation by cost-corrected return share with capacity rewards within the collective CRN as follows.

- Producer 1's cost allocation $(x^1)_{t\mu}^\alpha$ given producer 2's design choice $\hat{\kappa}^{\pi(2)}$:

$$(x^1)_{t\mu}^\alpha(\kappa^{\pi(1)}) = \begin{cases} d^{\pi(1)} \cdot [\kappa^{\pi(1)} \cdot \tau_{r(2)} + \bar{c}^{\pi(1)}] - k_{r(1)} \cdot \kappa^{\pi(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) & \text{if } \kappa^{\pi(1)} < \hat{\kappa}^{\pi(2)} \\ d^{\pi(1)} \cdot [\kappa^{\pi(1)} \cdot \tau_{r(1)} + \bar{c}^{\pi(1)} + \hat{\kappa}^{\pi(2)} \cdot (\tau_{r(2)} - \tau_{r(1)})] - k_{r(1)} \cdot \hat{\kappa}^{\pi(2)} \cdot (\tau_{r(2)} - \tau_{r(1)}) \\ \hspace{10cm} \text{if } \kappa^{\pi(1)} \geq \hat{\kappa}^{\pi(2)} \end{cases}$$
$$\tag{69}$$

- Producer 2's cost allocation $(x^2)_{t\mu}^\alpha$ given producer 1's design choice $\hat{\kappa}^{\pi(1)}$:

125

$$(x^2)^\alpha_{t\mu}(\kappa^{\pi(2)}) = \begin{cases} d^{\pi(2)} \cdot \left[\kappa^{\pi(2)} \cdot \tau_{r(2)} + \bar{c}^{\pi(2)} \right] & \text{if } \kappa^{\pi(2)} < \hat{\kappa}^{\pi(1)} \\ \\ d^{\pi(2)} \cdot \left[\kappa^{\pi(2)} \cdot \tau_{r(1)} + \bar{c}^{\pi(2)} + \hat{\kappa}^{\pi(1)} \cdot \left(\tau_{r(2)} - \tau_{r(1)} \right) \right] & \text{if } \kappa^{\pi(2)} \geq \hat{\kappa}^{\pi(1)} \end{cases}$$

$$(70)$$

From the above formulas, we observe that the cost allocation of producer 2, $(x^2)^\alpha_{t\mu}$, is a *concave* function of his design choice $\kappa^{\pi(2)}$. We can conclude a similar result as in Proposition 5, i.e., the best response function of producer 2 is a step function with two constant pieces.

Proposition 7. *In the medium-synergy scenario, given any convex decreasing function Q^2 that satisfies the property $\star$, the best response function of producer 2, $\kappa^{*\pi(2)}_{clt}$, is a step function that satisfies*

$$\kappa^{*\pi(2)}_{clt} = \begin{cases} q_2 \doteq (Q^{2\prime})^{-1}(-d^{\pi(2)} \cdot \tau_{r(1)}) & \forall \hat{\kappa}^{\pi(1)} \leq \kappa^{\pi(1)}_0 \\ \\ q_1 \doteq (Q^{2\prime})^{-1}(-d^{\pi(2)} \cdot \tau_{r(2)}) & \forall \hat{\kappa}^{\pi(1)} > \kappa^{\pi(1)}_0 \end{cases}$$

$$(71)$$

where $\kappa^{\pi(1)}_0 \in (q_1, q_2)$ is a constant defined based on Q^2.

However, it turns out that producer 1's best response function is not a step function in the medium-synergy scenario. Intuitively, according to formula (69), we can observe that improving the design of the product $\pi(1)$ may decrease the marginal value of producer 1's own independent capacity, thus can potentially result in a smaller capacity reward in his cost allocation. Because of this, producer 1's cost allocation $(x^1)^\alpha_{t\mu}$ turns out to be a *convex* function of his design variable $\kappa^{\pi(1)}$ in the medium-synergy scenario, as the slopes of the two linear pieces of $(x^1)^\alpha_{t\mu}$ satisfy

$$d^{\pi(1)} \cdot \tau_{r(2)} - k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) = d^{\pi(1)} \cdot \tau_{r(1)} - (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(2)} - \tau_{r(1)}) \leq d^{\pi(1)} \cdot \tau_{r(1)}.$$

In this case, the graph of the total cost of producer 1 $Cost^1$ (Figure 17) becomes different from that in the low-synergy scenario, which leads to a different best response function of producer 1.

126
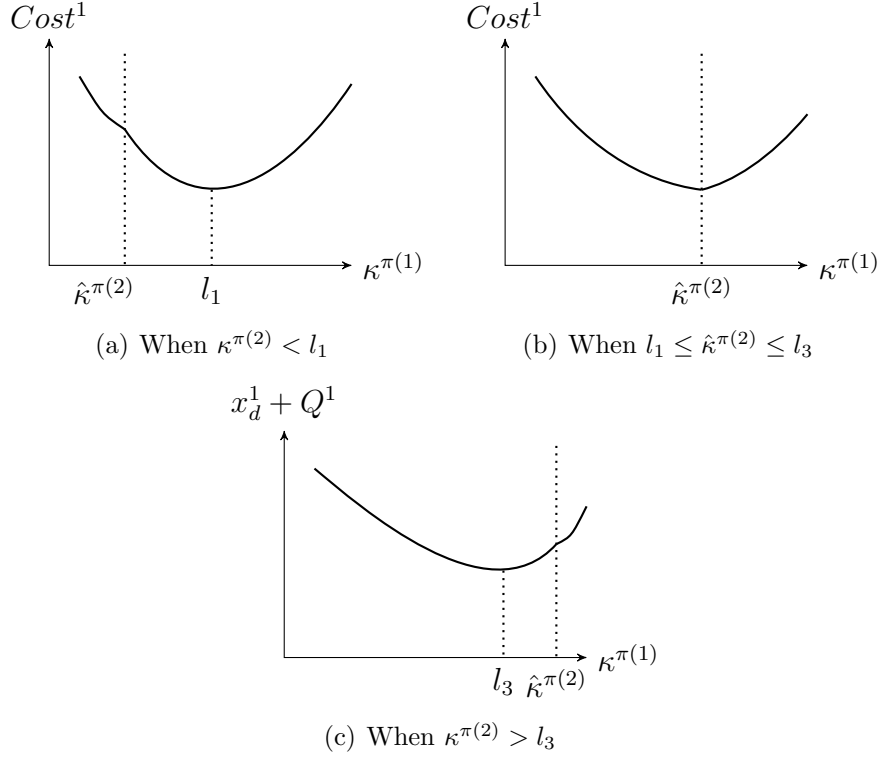
Figure 17: Three situations of the graph of the function $Cost^1$ in the medium-synergy scenario.

Proposition 8. *In the medium-synergy scenario, given any convex decreasing function Q^1 that satisfies the property $\star$, the best response function of producer 1, $\kappa_{clt}^{*\pi(1)}$, is continuous piecewise linear and satisfies*

$$\kappa_{clt}^{*\pi(1)} = \begin{cases} l_1 \doteq (Q^{1\prime})^{-1}(-d^{\pi(1)} \cdot \tau_{r(1)}) & \forall \hat{\kappa}^{\pi(2)} < l_1 \\ \hat{\kappa}^{\pi(2)} & \forall l_1 \leq \hat{\kappa}^{\pi(2)} \leq l_3 \\ l_3 \doteq (Q^{1\prime})^{-1}(-d^{\pi(1)} \cdot \tau_{r(2)} + k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)})) & \forall \hat{\kappa}^{\pi(2)} > l_3 \end{cases} \tag{72}$$

Based on Proposition 7 and 8, we plot the best response functions $\kappa_{clt}^{*\pi(1)}$ and $\kappa_{clt}^{*\pi(2)}$ in Figure 18; the intersection of the two functions represents the equilibrium design profile. In panel (a) and (b) of Figure 18, there must exist an equilibrium design profile; the equilibrium is also unique if $\kappa_0^{\pi(1)} \neq l_1$ and l_3. However, notice that this is no longer the case in Figure 18(c): The two best response functions do not intersect, indicating that no equilibrium design profile exists when $l_1 < \kappa_0^{\pi(1)} < l_3$.

(a) When $l_1 \geq \kappa_0^{\pi(1)}$

(b) When $l_3 \leq \kappa_0^{\pi(1)}$

(c) When $l_1 < \kappa_0^{\pi(1)} < l_3$. No equilibrium design profile exists in this case, yet a mixed-strategy equilibrium design profile must exist.
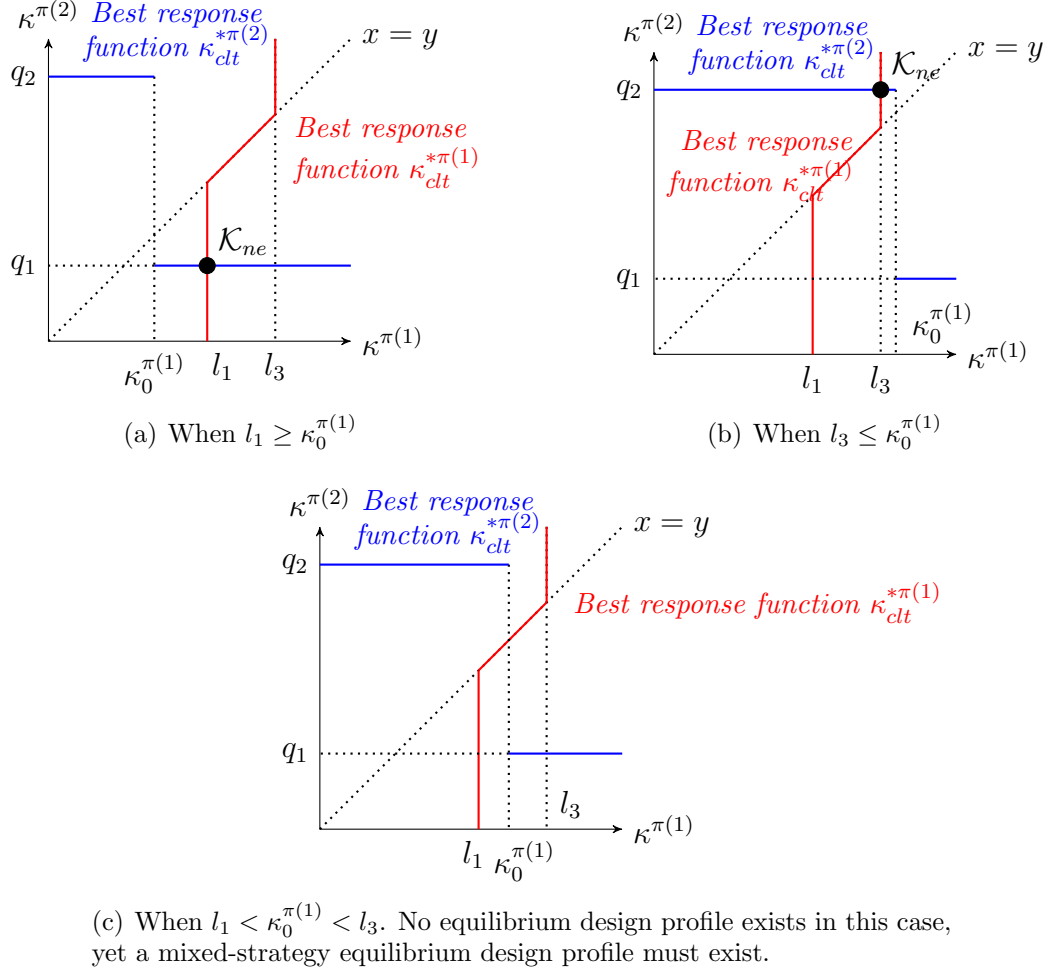
Figure 18: The Equilibrium design profile under collective implementation in the medium-synergy scenario.

In this case, we further analyze the *mixed-strategy equilibrium design profile* of the biform game. The idea is to allow each producer i to randomize among product design choices based on a chosen probability distribution f^i over his design variable $\kappa^{\pi(i)}$. Mathematically, we denote a mixed-strategy design profile as the collection of such probability distributions $\mathcal{K}_m = \{f^i, \forall i \in M\}$. We let $E_{\mathcal{K}_m}[Cost^i]$ denote the corresponding *expected* total cost of producer i.

Definition 3. *A mixed strategy design profile is called a mixed-strategy Nash equilibrium design profile, denoted by $\mathcal{K}_{mne}$, if $\forall i \in M$,*

$$E_{\mathcal{K}_{mne}}[Cost^i] \leq E_{\{f^i, \mathcal{K}_{mne}^{-i}\}}[Cost^i] \quad \forall f^i, \tag{73}$$

128

where $\mathcal{K}_{mne}^{-i} \doteq \{f^j \in \mathcal{K}_{mne} : j \neq i\}$.

We conclude the following theorem regarding the design implication of collective implementation in the medium-synergy scenario.

Theorem 6. *In the medium-synergy scenario, consider any convex decreasing functions Q^1 and Q^2 that satisfy the property $\star$.*

1. *If $\kappa_0^{\pi(1)} \in (0, l_1] \cup [l_3, \infty)$, there exists at least one equilibrium design profile $\mathcal{K}_{ne}$. Moreover, for both producers, the recyclability levels of their product designs under any $\mathcal{K}_{ne}$ are no better than their own individual optimal designs, i.e., $\kappa_{ne}^{\pi(i)} \geq \kappa_{ind}^{*\pi(i)}$ for both $i = 1, 2$.*

2. *If $\kappa_0^{\pi(1)} \in (l_1, l_3)$, there does not exist an equilibrium design profile. However, there must exist a mixed-strategy equilibrium design profile $\mathcal{K}_{mne} = \{f^1, f^2\}$, under which for both $i = 1, 2$, $\kappa^{\pi(i)} \geq \kappa_{ind}^{*\pi(i)} \ \forall \kappa^{\pi(i)}$ such that the probability $f^i(\kappa^{\pi(i)}) > 0$.*

We conclude that in the medium-synergy scenario, the network synergy decreases the marginal cost of producer 2's product thus reduces his design incentives, which is another example of the tradeoff between cost efficiency and design incentives under collective implementation. Meanwhile, for producer 1, the situation becomes more complicated: On one hand, the marginal cost of his product $\pi(1)$ may increase when producer 1 participates in a collective CRN, and thus can potentially motivate producer 1 to adopt a better design choice. However, on the other hand, such incentives are undermined by the concern that a better recyclability level of $\pi(1)$ may reduce the marginal value of producer 1's own independent capacity at $r(1)$ and thus his capacity reward. This result further underlines the complexity of the impact of the operational details in capacitated systems with shared resources.

In this subsection, we study the design implication of collective EPR with any number of producers under the allocation by cost-corrected return share with capacity rewards, based on the insights obtained from the two-producer case. We focus on the two extreme cases with a low and a high level of network synergy, respectively. In particular, we show that both Theorem 5 and Proposition 6 can be extended to the general case, i.e., a collective implementation with low/high network synergy leads to superior/inferior design incentives compared to an individual system. This analysis indicates the significance of the impact of the operations in a collective CRN in a general setting. As for the medium-synergy scenario, based on our analysis in the two-producer case, we can expect that the marginal value of capacity can exert an impact and thus the situation can be much more complex with a general number of producers involved; we leave this problem for future research.

In the following discussion, without loss of generality and similar to our assumption in the two-producer case, we assume that the relative efficiency levels among the processors are determined based on the order of producers' indices. Specifically, $r(i)$ is more efficient than $r(j)$ (i.e., $\tau_{r(i)} \leq \tau_{r(j)}$) if $i < j$.

Low-synergy Scenario   Recall that in the two-producer case, the low-synergy scenario represents the situation where the return volume of producer 2 is larger than the capacity at the efficient processor $r(1)$. A natural way to generalize this idea to a $m-$producer setting is assume that, for each producer, his own return volume is larger than the total capacity that is more efficient than his independent capacity. Mathematically, we define the low-synergy scenario with m producers as such that $d^{\pi(i)} > \sum_{j<i} k_{r(j)} \ \forall i \in M$. Intuitively, the motivation behind such a definition is that a collective CRN can benefit from network synergy when a product $\pi(i)$ is routed to a more efficient processor than $r(i)$. When $d^{\pi(i)} > \sum_{j<i} k_{r(j)} \ \forall i \in M$, no producer

can have his return volume entirely processed using the capacity more efficient than his own, indicating a limited level of potential network synergy. We show that in this scenario, collective implementation provides superior design incentives compared to an individual system.

Theorem 7. *Consider any set of producers $M = \{1, 2, ..., m\}$ and any convex decreasing functions $\{Q^i, \forall i \in M\}$ that satisfy the property $\star$. In the low-synergy scenario where $d^{\pi(i)} > \sum_{j<i} k_{r(j)} \ \forall i \in M$, there exists at least one equilibrium design profile $\mathcal{K}_{ne}$. Moreover, for all producers, the recyclability levels of their product designs under any $\mathcal{K}_{ne}$ are no worse than their own individual optimal designs, i.e., $\kappa_{ne}^{\pi(i)} \leq \kappa_{ind}^{*\pi(i)}$ $\forall i \in M$.*

We prove the above theorem in a constructive way by providing an algorithm to compute an equilibrium design profile. To do this, we first identify some structural properties of the social optimal routing in this low-synergy scenario, which lead to a full characterization of the best response function of each producer; we then design the algorithm based on these observations. We find the proof itself insightful in characterizing the key factors that influence producers' design incentives under collective EPR given a CRN structure with deficient efficient capacity. Hence, we highlight some key steps and observations from the proof in this subsection. The technical details are provided in Appendix 2.3.6.

We first propose the following greedy routing algorithm (Algorithm 2), which can be shown to compute a social optimal routing in any transportation CRN as defined in §2.3.2. For discussion convenience, we introduce the algorithm for nondegenerate cases where producers' recyclability levels and processors' efficiency levels are different. In general cases, the algorithm also applies with certain rules to break ties.

Lemma 4. *Algorithm 2 computes a social optimal routing on any transportation CRN as defined in §2.3.2.*

131

ALGORITHM 2: A greedy routing algorithm in a general transportation CRN

Input: A transportation CRN with a product set Π and a processors set R. For each product $\pi \in \Pi$, the return volume d^π and the recyclability level of its design κ^π are given. For each processor $r \in R$, the capacity k_r and its efficiency level τ_r are given.

Output: A routing $\{f_r^\pi\}$ of the products to the processors

Let $\Pi_0 = \Pi$ and $R_0 = R$. Let $f_r^\pi = 0 \; \forall r \in R \; \pi \in \Pi$. Let $t = 0$.

while $\Pi_t \neq \emptyset$ **do**

    1. Let $\bar{\pi} \doteq \arg\max_{\pi \in \Pi_t} \kappa^\pi$ be product with the worst recyclability level in Π_t. Let $\bar{r} \doteq \arg\max_{r \in R_t} \tau_r$ be the most efficient processor in R_t.

    2. Let $f_{\bar{r}}^{\bar{\pi}} = \min\{d^{\bar{\pi}}, k_{\bar{r}}\}$. Let $d^{\bar{\pi}} = d^{\bar{\pi}} - f_{\bar{r}}^{\bar{\pi}}$ and $k_{\bar{r}} = k_{\bar{r}} - f_{\bar{r}}^{\bar{\pi}}$.

    3. Let $\Pi_{t+1} \doteq \Pi_t \setminus \{\pi : d^\pi = 0\}$, and $R_{t+1} \doteq R_t \setminus \{r : k_r = 0\}$. Let $t = t + 1$.

end

Output $\{f_r^\pi\}$.

Intuitively, Algorithm 2 gives products that are more difficult to recycle the priority in using more efficient capacity. We prove the above lemma by duality theory. Based on this routing algorithm, we are able to explore the combinatorial structure of the social optimal routing, which leads to the following property of the cost allocation by cost-corrected return share with capacity rewards to each producer in the low-synergy scenario.

Lemma 5. *In the low-synergy scenario, for each producer i, the cost allocation to i under cost-corrected return share with capacity rewards, $(x^i)_{t\mu}^\alpha$, is a function of only the decision choices of the set of producers $\{j : j \geq i\}$.*

According to the above lemma, it can be concluded that in the low-synergy scenario, a producer i's best response function under collective implementation is only dependent on the design choices of producers with indices larger than i. This indicates that in this case, an equilibrium design profile can be computed in a sequential manner, which motivates us to design the following algorithm.

Lemma 6. *In the low-synergy scenario, a design profile is a Nash equilibrium if and only if it is a solution to Algorithm 3. Moreover, in the algorithm, for any producer*

ALGORITHM 3: An algorithm to compute a design profile.

Input: A set of producers M. For each producer i, the return volume $d^{\pi(i)}$, the capacity
$k_{r(i)}$ and its efficiency level $\tau_{r(i)}$, as well as the investment function Q^i, are given.

Output: A design profile $\mathcal{K}$

Let $\kappa^{\pi(m)} = (Q^{m\prime})^{-1}(-d^{\pi(m)} \cdot \tau_{r(m)})$. Let $t = m - 1$.

while $t > 0$ **do**

    Compute a global minimum of the function $Cost^t = Q^t + (x^t)^\alpha_{t\mu}$, with $\{\kappa^{\pi(j)}, j > t\}$ as
    parameters. Assign the value to the design variable $\kappa^{\pi(t)}$. Let $t = t - 1$.

end

Output $\mathcal{K} = \{\kappa^{\pi(i)}, \forall i \in M\}$.

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*i, given any convex decreasing functions Q^i that satisfies the property $\star$, the global
minimum of the total cost function $Cost^i = Q^i + (x^i)^\alpha_{t\mu}$ is equal to $(Q^{i\prime})^{-1}(d^{\pi(i)} \cdot \tau_{r(j)})$
for some $j \geq i$.*

Combining Lemma 4, Lemma 5 and Lemma 6, we conclude that under any equilibrium design profile in a collective system, the recyclability level of each product is no worse than that under the corresponding individual optimal design. This completes the proof to Theorem 7. To summarize, the key factor that leads to the superior design incentives under collective EPR in the low-synergy scenario is the potential increase in the marginal cost of each product under resource sharing, due to the deficiency in efficient capacity in the collective CRN.

High-synergy Scenario We define the high-synergy scenario with any number of producers as such that there exists sufficient capacity at the most efficient processor $r(1)$ to handle the entire return volume, i.e., $k_{r(1)} \geq \sum_{i=1}^m d^{\pi(i)}$. Hence, the social optimal routing is to route all products to $r(1)$. Applying the same argument used in the high-synergy scenario in the two-producer case, we conclude the following theorem.

Theorem 8. *Consider any set $M = \{1, 2, ..., m\}$ producers and any convex decreasing functions $\{Q^i, \forall i \in M\}$ that satisfy the property $\star$. In the high-synergy scenario where*

$k_{r(1)} \geq \sum_{i=1}^{m} d^{\pi(i)}$, *there exists a unique equilibrium design profile* $\mathcal{K}_{ne}$ *such that*

$$\kappa_{ne}^{\pi(i)} = (Q^{i\prime})^{-1}(-d^{\pi(i)} \cdot \tau_{r(1)}) \geq \kappa_{ind}^{*\pi(i)} \quad \forall i \in M \tag{74}$$

In other words, for all producers, the recyclability levels of their product designs under $\mathcal{K}_{ne}$ *are no better than their own individual optimal designs.*

2.3.3.3 The Design Implication of using the Allocation by Return Share in Collective Implementation.

We conclude our analytical study in this section of the design implication of collective EPR by studying the equilibrium design profile of the biform game assuming the recycling cost in the second stage is allocated proportional to producers' return shares. As mentioned in §2.2, such a return share model is widely-used in practice. However, it has been criticized by producers and practitioners for being the major factor to mute producers' design incentives under collective EPR implementation. Specifically, under such a homogeneous cost allocation, the externality of a producer's design choice is not completely reflected in his cost allocation, thus the producer's economic incentives to improve the design are undermined. Because of this, it is generally believed that an individual system always incentivizes product designs that are easier to recycle compared to a collective one where the allocation by return share is adopted. Surprisingly, our analysis indicates that while the common wisdom holds true under some circumstances, a collective system with return share may also provide superior design incentives compared to an individual one in certain cases.

We would like to mention that one additional complexity that arises from adopting the allocation by return share in our biform game model is that the grand coalition is not guaranteed to be sustained in the second stage, as the producers may be overcharged under such a homogeneous cost allocation and thus be incentivized to break away. In other words, the allocation by return share is generally not group incentive compatible (Proposition 1 and Proposition 2). The fragmentation of a collective

system can have a potential impact on producers' end-of-life recycling cost and thus their design incentives. In order to analyze such an impact, we need to study the coalition structure that producers form in the second stage under return share. This requires additional assumptions regarding the cost allocation used within each sub-coalition and an equilibrium analysis of producers' participation decisions in those sub-groups. In literature, such problems are analyzed based on farsighted, dynamic coalitional stability notions such as the largest consistent set, equilibrium process of coalition formation, subgame perfect consistent stability and etc. [32, 94, 64]. Applications of these models include supply chain management and product take-back economics [66, 113, 162]. In the context of collective EPR, it can be expected that the design implication of such fragmentation issues under return share can be a significant concern in the long run. However, the current debate focuses on the negative impact of joint responsibility in cost sharing within a collective recycling system due to the uniform nature of the allocation by return share. Hence, in this study, we analyze producers' design incentives in a collective system with return share, assuming there exists significant exit barrier for the producers to leave the collective system.

In this subsection, we consider the two-producer case and discuss the problem in the low-, medium- and high-synergy scenarios. In each scenario, we focus on characterizing the equilibrium design profile under a collective implementation where the cost allocation by return share to each producer i equals

$$x_r^i = \frac{d^{\pi(i)}}{d^{\pi(1)} + d^{\pi(2)}} \cdot v_t(M) \quad i = 1, 2 \,. \tag{75}$$

We then compare the recyclability level of each product under the equilibrium design profile with that under the corresponding individual optimal design. In the rest of this subsection, we provide an overview of our analysis and highlight the new observations. We refer the readers to Appendix 2.3.6 for technical details.
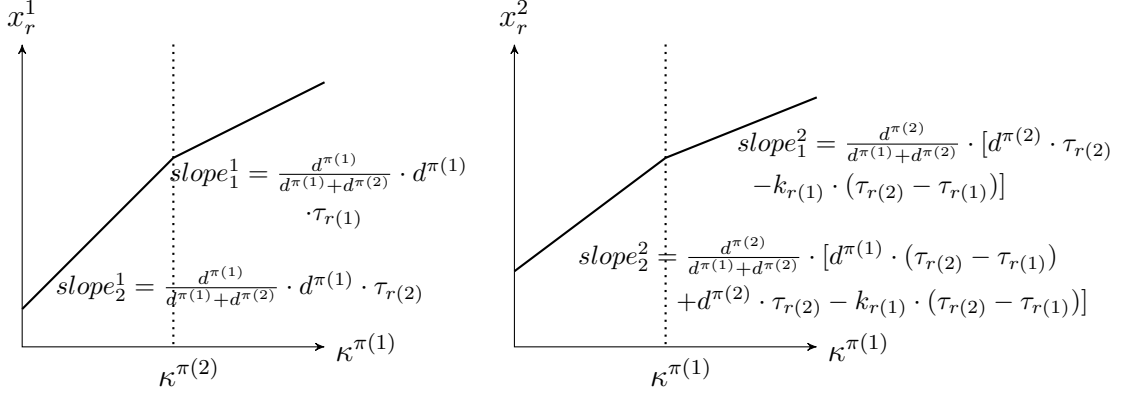
(a) Producer 1's allocation by return share

(b) Producer 2's allocation by return share

Figure 19: Producers' allocations by return share in the low-synergy scenario.

Low-synergy Scenario In this scenario, we can calculate that for each producer, the second-stage cost allocation by return share is a concave piecewise linear function of his own design variable (Figure 19). Hence, similar to our previous results in Proposition 5 and Proposition 7, it can be shown that the best response functions of both producers are step functions with two constant pieces. We show that the existence of an equilibrium design profile can be guaranteed (see Figure 20). We can also observe that, under return share, there exist multiple equilibrium design profiles in the situation depicted in Figure 20(d), i.e., the break point of producer 1's best response function lies between the two constant pieces of producer 2's best response function, and vice versa. This is different from the situation under cost-corrected return share with capacity rewards, where the equilibrium design profile is unique unless $\kappa_0^{\pi(2)} = q_1$ in the low-synergy scenario. Moreover, in cases with multiple equilibria, whether a producer experiences superior or inferior design incentive under a collective implementation with return share compared to an individual system depends on which equilibrium design profile is achieved.

Theorem 9. *Consider the two-producer case of the biform game where the allocation by return share x_r is adopted in the second stage. In the low-synergy scenario, given*

136

Figure 20: The Equilibrium design profile under collective implementation with allocation by return share in the low-synergy scenario.

any convex decreasing function Q^1 and Q^2 that satisfy the property $\star$, there exist at least one equilibrium design profile $\mathcal{K}_{ne}$. Moreover,

1. for producer 2, the recyclability level of his product design under any $\mathcal{K}_{ne}$ is strictly worse than his own individual optimal design, i.e., $\kappa_{ne}^{\pi(2)} > \kappa_{ind}^{*\pi(2)}$.

2. for producer 1,

   (a) the worst recyclability level of his product design under an $\mathcal{K}_{ne}$ is always strictly worse than his own individual optimal design, i.e., $\max_{\mathcal{K}_{ne}} \kappa_{ne}^{\pi(1)} > \kappa_{ind}^{*\pi(1)}$.

   (b) if there exists multiple equilibria, then when $\frac{\tau_{r(1)}}{\tau_{r(2)}} < \frac{d^{\pi(1)}}{d^{\pi(2)}+d^{\pi(1)}}$, the best recyclability level of his product design under an $\mathcal{K}_{ne}$ is strictly better than his own individual optimal design, i.e., $\min_{\mathcal{K}_{ne}} \kappa_{ne}^{\pi(1)} < \kappa_{ind}^{*\pi(1)}$.

Result 2(b) in Theorem 9 is a surprising and counter-intuitive result from an economic perspective that is generally held in practice. To see why this is the case, note that intuitively a small ratio $\frac{\tau_{r(1)}}{\tau_{r(2)}}$ indicates a large heterogeneity level in processing

efficiency among the processors in the collective CRN. In this case, the potential increase in the marginal cost of product $\pi(1)$ can be significant when its last unit is routed to processor $r(2)$ under the social optimal routing in the low-synergy scenario. Such a significant marginal cost increase can incentivize a better design choice of producer 1 in a collective system with return share even if the impact of a product's marginal cost is jointly absorbed by all producers under the homogeneous return share allocation. In this case, we conclude that the effect of the operational factors in a collective CRN (e.g., marginal cost) on producers' design incentives dominates the negative impact of joint responsibility in cost sharing.

Medium-synergy Scenario In the medium-synergy scenario, our analysis gives rise to very similar results as in the low-synergy scenario regarding the design incentives of both producers under collective EPR when the allocation by return share is used. Specifically, both producers' cost allocation by return share in the second stage are concave piecewise linear functions (Figure 21), which lead to best response functions that are step functions with two constant pieces. Hence, the existence of an equilibrium design profile is guaranteed yet it is likely that there are multiple equilibria in this system. We also derive the following result, which is very similar to Theorem 9. In particular, producer 1 may also experience superior design incentives in the medium-synergy scenario if there is a big enough difference between the efficiency level of the two processors.

Theorem 10. *Consider the two-producer case of the biform game where the allocation by return share x_r is adopted in the second 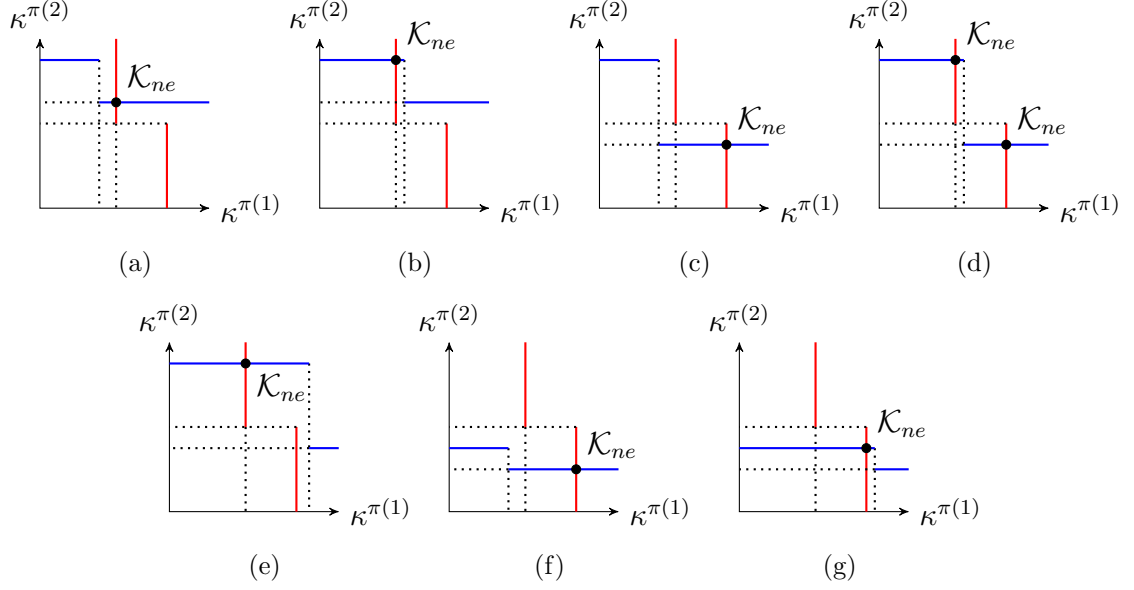stage. In the medium-synergy scenario, given any convex decreasing function Q^1 and Q^2 that satisfy the property $\star$, there exist at least one equilibrium design profile $\mathcal{K}_{ne}$. Moreover,*

1. *for producer 2, the recyclability level of his product design under any $\mathcal{K}_{ne}$ is strictly worse than his own individual optimal design, i.e., $\kappa_{ne}^{\pi(2)} > \kappa_{ind}^{*\pi(2)}$.*
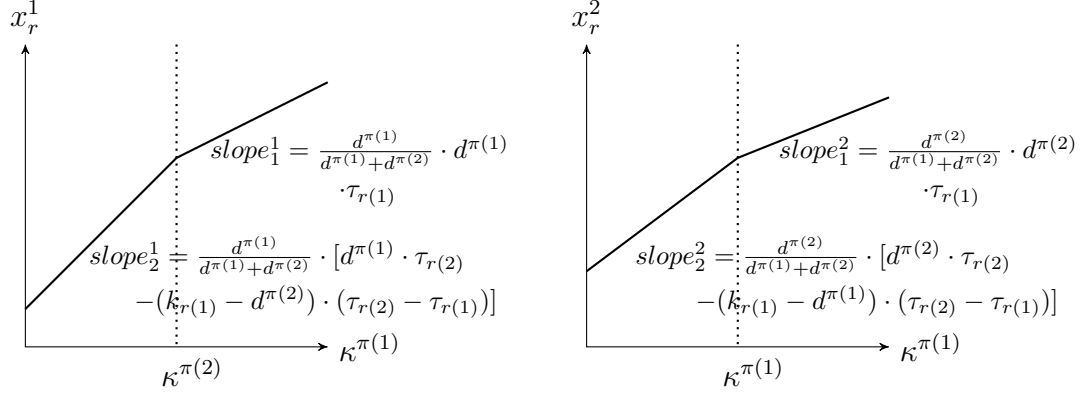
(a) Producer 1's allocation by return share   (b) Producer 2's allocation by return share

Figure 21: Producers' allocations by return share in the medium-synergy scenario

2. for producer 1,

   (a) the worst recyclability level of his product design under an $\mathcal{K}_{ne}$ is always strictly worse than his own individual optimal design, i.e., $\max_{\mathcal{K}_{ne}} \kappa_{ne}^{\pi(1)} > \kappa_{ind}^{*\pi(1)}$.

   (b) if there exists multiple equilibria, then when $\frac{\tau_{r(1)}}{\tau_{r(2)}} < \frac{d^{\pi(1)}+d^{\pi(2)}-k_{r(1)}}{d^{\pi(1)}+2d^{\pi(2)}-k_{r(1)}}$, the best recyclability level of his product design under an $\mathcal{K}_{ne}$ is strictly better than his own individual optimal design, i.e., $\min_{\mathcal{K}_{ne}} \kappa_{ne}^{\pi(1)} < \kappa_{ind}^{*\pi(1)}$.

High-synergy Scenario As we have mentioned before, the social optimal routing in the high-synergy scenario is to route the entire return volume to the efficient processor $r(1)$, due to the abundant capacity there. In this case, we can calculate the unique equilibrium design profile $\mathcal{K}_{ne}$ under collective implementation with return share as follows.

$$\kappa_{ne}^{\pi(1)} = (Q^{1\prime})^{-1}\left(-\frac{d^{\pi(1)}}{d^{\pi(1)} + d^{\pi(2)}} \cdot d^{\pi(1)} \cdot \tau_{r(1)}\right) > \kappa_{ind}^{*\pi(1)} \tag{76}$$

$$\kappa_{ne}^{\pi(2)} = (Q^{2\prime})^{-1}\left(-\frac{d^{\pi(2)}}{d^{\pi(1)} + d^{\pi(2)}} \cdot d^{\pi(2)} \cdot \tau_{r(1)}\right) > \kappa_{ind}^{*\pi(2)} \tag{77}$$

Hence, we conclude that in the high-synergy scenario, collective EPR with return share allocation leads to strictly inferior design incentives compared to an individual

139

system. We further show that in this case, for each producer, his equilibrium design under return share is strictly worse than that under cost-corrected return share with capacity rewards. This demonstrates that in the high-synergy scenario, both the operational factors (e.g., high-synergy level from resource sharing) and the impact of joint responsibility undermine producers' design incentives under collective implementations.

2.3.4 A Numerical Study

The analytical study above provides a general understanding towards the impact of the operational factors on the design implications of collective EPR. In particular, our results indicate the significant roles of the capacity configuration of a collective CRN and the network synergy derived from resource sharing. One prominent observation from the analysis is that collective implementation may lead to superior design incentives compared to an individual system if the efficient capacity availability is relatively low in the collective CRN. Moreover, the heterogeneity in capacity efficiency among the processors can also exert a substantial impact especially if the allocation by return share is used in the second stage of the biform game. Specifically, we provide analytical bounds to the value of $\frac{\tau_{r(1)}}{\tau_{r(2)}}$, under which collective implementation with return share may provide better design incentives. In this section, we perform a numerical study of the problem. The purpose of the numerical study is twofold: First, we validate our analytical results. Second, we study the joint impact of the availability of efficient capacity and the efficiency heterogeneity among the processors. In particular, our analytical results provide parameter ranges where a collective implementation leads to *strictly* superior/inferior design incentives compared to an individual system.

The numerical study is performed based on a collective CRN with two producers. Producer 1 has a return volume of $d^{\pi(1)} = 10M$ lbs and producer 2's return volume
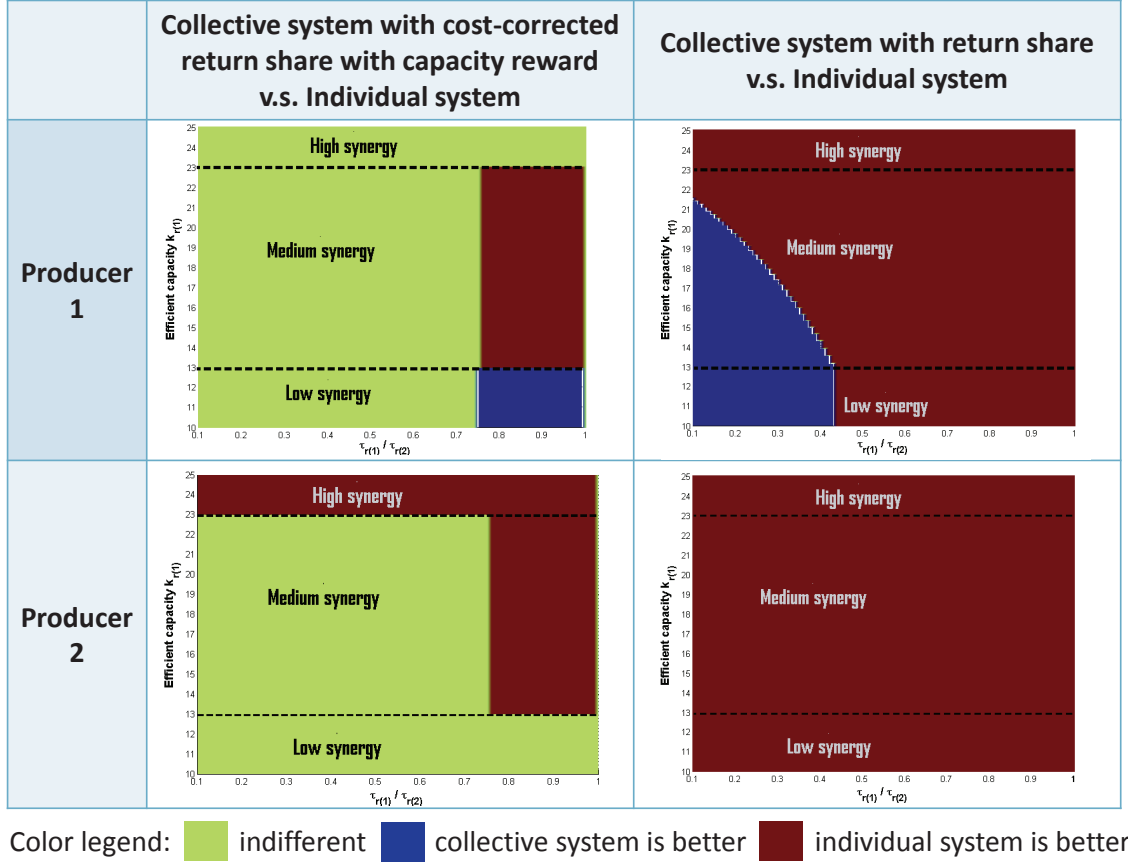
Figure 22: Producers' allocations by return share in the medium-synergy scenario

is $d^{\pi(2)} = 13\text{M lbs}$. The unit base cost of the products are assumed to be $\bar{c}^{\pi(1)} = 10$ cents/lb, and $\bar{c}^{\pi(2)} = 12$ cents/lb. The design investment function of the two producers are modeled as reciprocal function; specifically, $Q^1(\kappa^{\pi(1)}) = 200 \cdot \frac{1}{\kappa^{\pi(1)}}$ and $Q^2(\kappa^{\pi(2)}) = 300 \cdot \frac{1}{\kappa^{\pi(2)}}$. We take the processor $r(2)$ where producer 2 has independent capacity as the standard processor, i.e., $\tau_{r(2)} = 1$; the independent capacity there is assumed to be $k_{r(2)} = 20$ M lbs. Based on this instance, we consider a continuum of scenarios with different capacity availability at and efficiency level of the efficient processor $r(1)$. Specifically, we allow the efficiency level of $r(1)$ to vary in $[0.1, 1]$ and the capacity level there to vary in $[10, 25]$ million lbs.

Figure 22 compares the producers' individual optimal designs with the equilibrium design profiles under collective EPR with either the allocation by cost-corrected return

share with capacity rewards (column 1) or the allocation by return share (column 2). Note that the figure for producer 1 in column 2 is plotted based on his best equilibrium design under return share. Overall, the numerical results are consistent with our analytical results. Specifically, in the high-synergy case, an individual system indeed dominates collective implementation and motivates better design, as resource sharing with abundant efficient capacity in the collective system leads to high network synergy and thus the tradeoff between cost efficiency and design incentives. It is also the case for producer 2 for the entire parameter space. For producer 1, collective implementation has the potential to achieve superior design incentives for him in both the low-synergy and the medium-synergy scenarios, given that the choice of the allocation mechanism is made based on the specific configuration of the collective CRN.

One prominent observation from Figure 22 is that in this example, the degree of heterogeneity in capacity efficiency levels plays an important role in determining under which cost allocation mechanism collective EPR implementation can provide strictly superior design incentives compared to an individual system. We first observe that when the efficiency levels of the two producers are similar, i.e., when $\frac{\tau_{r(1)}}{\tau_{r(2)}}$ is greater than approximately 0.75, then under cost-corrected return share with capacity rewards, collective implementation can provide strictly superior design incentives to producer 1 in the low synergy case. However, also note that under the same range of $\frac{\tau_{r(1)}}{\tau_{r(2)}}$, doing so in the medium synergy case actually leads to strictly inferior design incentives to both producers, due to the additional impact from the marginal value of producer's independent capacity. This finding accentuates the complexity of the design implication of collective EPR implementation derived from the network operations in a collective system.

In contrast, we observe quite the opposite result if the allocation by return share is used. Specifically, under return share, producer 1 adopts a more recyclable product

142

design in the collective system compared to his individual optimal design in the low-synergy scenario when there is a big difference between the efficiency levels of the two processors, i.e., $\frac{\tau_{r(1)}}{\tau_{r(2)}}$ is small and is approximately in the range of $[0, 0.43]$. Notice that such a range gradually shrinks in the medium synergy case as the capacity at $r(1)$ increases. These numerical observations are consistent with our analytical results in Theorem 9 and Theorem 10: In the low-synergy case, Theorem 9 indicates that producer 1 may have higher design incentives under return share when $\frac{\tau_{r(1)}}{\tau_{r(2)}}$ is less than producer 1's return share $\frac{d^{\pi(1)}}{d^{\pi(1)}+d^{\pi(2)}}$, which can be calculated to be $\frac{10}{23} \approx 0.43$ in this example. In the medium-synergy case, Theorem 10 suggests that a collective system with return share may provide superior design incentives to producer 1 if $\frac{\tau_{r(1)}}{\tau_{r(2)}}$ is within $(0, \frac{d^{\pi(1)}+d^{\pi(2)}-k_{r(1)}}{d^{\pi(1)}+2d^{\pi(2)}-k_{r(1)}})$; such a range obviously becomes smaller with more capacity $k_{r(1)}$ in the system.

A natural question that arises when comparing the above two sets of observations is that why collective EPR does not provide strictly superior design incentives under small values of $\frac{\tau_{r(1)}}{\tau_{r(2)}}$ under cost-corrected return share with capacity rewards. The answer to that question is that the interaction and dynamics between the producers' decisions also play an important role under a collective implementation, which is captured by our equilibrium analysis of the biform game. Specifically, when $\frac{\tau_{r(1)}}{\tau_{r(2)}}$ is low, it is not an equilibrium for producer 1 to adopt a strictly more recyclable design under cost-corrected return share with capacity rewards, due to the influence of producer 2's design decision.

2.3.5 Conclusion

Providing design incentives has been regarded as one of the ultimate goals of Extended Producer Responsibility as a policy concept to treat post-use products. The current debate on how to realize such design potential in implementation essentially focuses

on finding effective ways to reflect the products' environmental impact in the post-use recycling cost that the corresponding producers need to internalize. An intuitive economic perspective suggests that one crucial element to achieve this is to prevent joint responsibility, i.e., to associate each producer's recycling cost only with his own products. This principle of *Individual Producer Responsibility* (IPR) has been widely supported in practice, and has steered the majority of the existing discussion on the design implication of EPR implementation. For example, it is based on this IPR principle that an individual system, where all producers operate independently, is generally regarded as being able to provide the best design incentives, as no cost sharing occurs thus joint responsibility is absolutely prevented. Moreover, under a collective implementation, although the recycling operations are done jointly, it is believed that the same design incentives can be achieved if the financing mechanism is designed to separate the cost for each participant.

In this section, we argue that IPR is not a sufficient and necessary condition for superior design incentives, due to the significant impact of operational-level factors, in particular, the capacity configuration of the CRN and the network synergy derived from resource sharing. The intuitive reason is that these factors can influence the post-use treatment cost in a collective system, thus potentially exerting an impact on producers' design incentives. In this section, we explore such an impact by adopting a network perspective towards the collective recycling operations, and studying a two-stage biform game model that captures the interaction between such operations and the producers' design decisions. Specifically, the first stage is modeled as a non-cooperative game where producers independently choose their product designs, while the second stage is a cooperative network game model of the recycling operations in a collective system at the end-of-life of the products.

Our first analysis assumes that the allocation by cost-corrected return share with capacity rewards is adopted in the second stage. Such an allocation is shown to be

group incentive compatible in §2.2, i.e., the costs allocated to producers are related to their own products in the sense that no sub-coalition of producers suffers a higher cost in the collective system compared to their operating independently. We mainly study three scenarios with different capacity configurations thus different levels of network synergy from capacity sharing. The analysis indicates that a producer's design incentive under collective implementation is strongly dependent on the synergy level in the collective system as well as operational details such as the marginal costs of his products. In particular, when abundant efficient capacity exists, we identify a tradeoff between cost efficiency and design incentives in a collective system. We also identify network conditions where collective implementation has the potential to provide superior design incentives compared to an individual system. Finally, we further analyze the design implication of using the allocation by return share in the collective system. The IPR principle suggests that the producers' design incentives should be significantly weakened under such a homogeneous cost allocation. However, we show that producers may be motivated to adopt a more recyclable design under return share compared to operating independently in an individual system when the capacity heterogeneity in the collective system is high in terms of processing efficiency. This result underlines the significance of the operational impact on the design implication of EPR. Hence, our study suggests that in order to realize the design potential of the EPR concept, it is important to make policy choices based on the operational features of the collection and recycling infrastructure in the area concerned.

Being our first study towards the operational impact on design implication of EPR, this section also motivates a rich set of issues to be further explored. For example, one immediate question is to study the medium-synergy scenario in the general case with any number of producers, from which we can expect to gain more general insights regarding the impact of the marginal value of capacity. Moreover,

as mentioned in the previous two sections, current collective implementations often involve capacities that are contracted by the system operator. The availability of such exogenous capacity changes the capacity configuration of the collective system and thus is expected to have an impact on the design incentives of the producers. Our initial study on small examples indicates that incorporating the operator-contracted capacity can increase the synergy level in a collective system and thus can potentially undermine producers' design incentives. Under what conditions this observation can be generalized is another interesting question to investigate. In addition, as we mention at the end of §2.3.2.2, the study in this section also motivates us to evaluate the impact of other operational factors such as the competition in the product market and the uncertainty in return volumes. Finally, we conclude from this study that the efficiency of capacities has a significant impact on the implementation outcomes of EPR. This motivates us to consider the question of whether and how an EPR regulation and its implementation may incentivize producers' investment in process improvement, such as adopting more advanced recycling technology. We discuss this issue in more details in the next section as a future research direction motivated from this research stream.

2.3.6   Appendix

Proof of Proposition 5. In order to prove this proposition, we first present the following lemma that characterizes the global minimum of the sum of a general convex decreasing function $F_c : (0, \infty) \to (0, \infty)$ and an increasing function $F_l : (0, \infty) \to (0, \infty)$ that consists of two linear pieces. Specifically, we defined F_l as

$$F_l(x) = \begin{cases} a_1 \cdot x + b_1 & x \in (0, c) \\ a_2 \cdot x + b_2 & x \in [c, \infty) \end{cases} \tag{78}$$

where $a_1 > 0$ and $a_2 > 0$. We assume that $a_1 \cdot c + b_1 = a_2 \cdot c + b_2$ to ensure continuity of the function F_l.

Lemma 7. *Given a convex decreasing function F_c and a two-step linear increasing function F_l as defined above. Assume that F_c satisfies the property $\star$.*

1. *If F_l is concave, i.e., $a_1 > a_2$, then the minimum of the function $F_c + F_l$ satisfies*

$$\arg\min F_c + F_l = \begin{cases} x_2^* \doteq (F_c')^{-1}(-a_2) & \text{if } c \leq c_0 \\ x_1^* \doteq (F_c')^{-1}(-a_1) & \text{if } c > c_0 \end{cases} \tag{79}$$

where c_0 is a constant contained in the interval (x_1^*, x_2^*).

2. *If F_l is convex, i.e., $a_2 < a_1$, then the minimum of the function $F_c + F_l$ satisfies*

$$\arg\min F_c + F_l = \begin{cases} x_2^* \doteq (F_c')^{-1}(-a_2) & \text{if } c < x_2^* \\ c & \text{if } c \in [x_2^*, x_1^*] \\ x_1^* \doteq (F_c')^{-1}(-a_1) & \text{if } c > x_1^* \end{cases} \tag{80}$$

Proof of Lemma 7. Consider the case where F_l is concave. Since $a_1 > a_2 > 0$ and F_c is a convex decreasing function that satisfies the property $\star$, it is easy to calculate that $(F_c')^{-1}(-a_1) < (F_c')^{-1}(-a_2)$. Denote $(F_c')^{-1}(-a_1)$ by x_1^* and $(F_c')^{-1}(-a_2)$ by x_2^*. We consider the following four situations. First, we define $c_0 \doteq \frac{F_c(x_1^*) + a_1 \cdot x_1^* - F_c(x_2^*) - a_2 \cdot x_2^*}{a_1 - a_2}$. Since x_1^* is the global minimum of the function $F_c + a_1 \cdot x$, we can calculate that

$$\frac{F_c(x_1^*) + a_1 \cdot x_1^* - F_c(x_2^*) - a_2 \cdot x_2^*}{a_1 - a_2} < \frac{F_c(x_2^*) + a_1 \cdot x_2^* - F_c(x_2^*) - a_2 \cdot x_2^*}{a_1 - a_2} = x_2^* \tag{81}$$

Similarly, since x_2^* is the global minimum of the function $F_c + a_2 \cdot x$, $c_0 > x_1^*$. Hence we conclude that $c_0 \in (x_1^*, x_2^*)$.

- When $c \leq x_1^*$. Then $\forall x < c$, $(F_c + F_l)' < 0$ thus $F_c + F_l$ is a decreasing function in the interval of $(0, c)$. When $x = c$, $F_c + F_l$ is continuous but not differentiable; however, we can calculate that both the left- and right-derivative at $x = c$ are nonpositive, indicating that $x = c$ is not a local minimum of the function. When $x > c$, $(F_c + F_l)' = 0$ when $x = x_2^*$. Hence, we conclude that in this case, $F_c + F_l$

147

decreases when $x < x_2^*$ and increases when $x > x_2^*$, thus the only local minimum and thus the global minimum of the function $F_c + F_l$ is x_2^*.

- When $x_1^* < c \leq c_0$. In this case, the function $(F_c + F_l)$ has two local minima at x_1^* and x_2^*. We can calculate that

$$(F_c + F_l)(x_1^*) - (F_c + F_l)(x_2^*) = a_1 \cdot x_1^* + b_1 + F_c(x_1^*) - a_2 \cdot x_2^* - b_2 - F_c(x_2^*)$$

$$=^1 a_1 \cdot (x_1^* - c) + F_c(x_1^*) - a_2 \cdot (x_2^* - c) - b_2 - F_c(x_2^*) \geq^2 0 \quad (82)$$

where the relation $=^1$ is derived from the assumption that $a_1 \cdot c + b_1 = a_2 \cdot c + b_2$, and $\geq^2$ is due to $c \leq c_0$. Hence, the global minimum of the function $F_c + F_l$ is x_2^*. We mention that both x_1^* and x_2^* are global minima when $c = c_0$.

- When $c_0 < c < x_2^*$, the situation is similar to the previous one where $x_1^* < c \leq c_0$, i.e., $(F_c + F_l)$ has two local minima at x_1^* and x_2^*. Using a similar argument, we calculate that in this case, $(F_c + F_l)(x_1^*) < (F_c + F_l)(x_2^*)$ since $c_0 < c$, thus the global minimum of the function $F_c + F_l$ is x_2^*.

- When $c \geq x_2^*$, we apply a similar argument as in the first case when $c \leq x_1^*$, and show that the function $F_c + F_l$ has only one local minimum at $x = x_2^*$. Hence, x_2^* is the global minimum of the function.

Combining the above four cases, we obtain formula (79).

Now we consider the other case where F_l is convex. Since $0 < a_1 < a_2$ and F_c is a convex decreasing function that satisfies the property $\star$, it is easy to calculate that $x_1^* \doteq (F_c')^{-1}(-a_1) > (F_c')^{-1}(-a_2) \doteq x_2^*$. We adopt a similar analysis as above. Specifically, when $c \leq x_2^*$ or $c \geq x_1^*$, the situation is similar as in the first and fourth situation in the concave case, i.e., the function $F_c + F_l$ has only one local minimum at $x = x_2^*$ or $x = x_1^*$, respectively, which is therefore the global optimum of the function. When $c \in (x_2^*, x_1^*)$, we can show that the function $F_c + F_l$ decreases in the interval $(0, c)$ and increases in (c, ∞). Since $F_c + F_l$ is continuous, we conclude that $x = c$ is

the only local minimum thus the global minimum of $F_c + F_l$. Therefore, formula (80) follows. $\qquad\square$

Continuing the proof of Proposition 5, according to (63), the cost allocated to producer 1 based on the cost-corrected return share with capacity rewards model in the low-synergy case is a two-step piecewise linear increasing function where $a_1 = d^{\pi(1)} \cdot \tau_{r(2)}$, $a_2 = d^{\pi(1)} \cdot \tau_{r(1)}$, and $c = \bar{\kappa}^{\pi(2)}$. Since the facility $r(1)$ is more efficient, $\tau_{r(1)} < \tau_{r(2)}$ thus the linear function is concave. Based on our model, the best response function of producer 1 is essentially the global minimum of the total cost function $(x^1)_{t\mu}^{\alpha} + Q^1$. Hence, Proposition 5 follows immediately from formula (79), where $\kappa_0^{\pi(2)}$ is calculated based on the formula that calculates c_0 in the proof of Lemma 7. $\qquad\square$

Proof of Theorem 5. By definition, the equilibrium design profile is the intersection of the best response functions of the two producers. Hence, according to Figure 15, the equilibrium design profile is

$$
\begin{cases} \kappa_{ne}^{\pi(1)} = l_2 \\ \kappa_{ne}^{\pi(2)} = q_1 \end{cases} \text{if } q_1 > \kappa_0^{\pi(2)}; \qquad \begin{cases} \kappa_{ne}^{\pi(1)} = l_1 \\ \kappa_{ne}^{\pi(2)} = q_1 \end{cases} \text{if } q_1 \leq \kappa_0^{\pi(2)} \qquad (83)
$$

Notice that when $q_1 = \kappa_0^{\pi(2)}$, both of the two design profiles above are Nash equilibria. By definition, $l_1 = \kappa_{ind}^{*\pi(1)}$ and $q_1 = \kappa_{ind}^{*\pi(2)}$. $l_2 < \kappa_{ind}^{*\pi(1)}$ since $\tau_{r(2)} > \tau_{r(1)}$ and Q^1 is a convex function. Thus we can conclude that in the low-synergy case, $\kappa_{ne}^{\pi(i)} \leq \kappa_{ind}^{*\pi(i)}$ for both $i = 1, 2$. $\qquad\square$

Proof of Proposition 6. In the high synergy case, according to (67), the cost allocated to each producer i under cost-corrected return share with capacity rewards is a linear function of his design choice, and is independent of the other producer's design choice. Hence, in order to solve for the equilibrium design, we only need to compute the global minimum of the function $Q^i + (x^i)_{t\mu}^{\alpha}$ for both $i = 1, 2$. Since for both $i = 1, 2$, the total cost function $Q^i + (x^i)_{t\mu}^{\alpha}$ is a convex function, the global minimum is achieved where the

first order derivative of the function is zero. Hence, according to (67) and $\tau_{r(1)} < \tau_{r(2)}$, it is easy to calculate that $\kappa_{ne}^{\pi(i)} = (Q^{i\prime})^{-1}(-d^{\pi(i)} \cdot \tau_{r(1)}) \geq (Q^{i\prime})^{-1}(-d^{\pi(i)} \cdot \tau_{r(i)}) = \kappa_{ind}^{*\pi(i)}$ $\forall i = 1, 2$. $\square$

Proof of Proposition 7. The proof is based on Lemma 7. According to (70), the cost allocated to producer 2 based on the cost-corrected return share with capacity rewards model in the medium-synergy scenario is a piecewise linear increasing function where $a_1 = d^{\pi(2)} \cdot \tau_{r(2)}$, $a_2 = d^{\pi(2)} \cdot \tau_{r(1)}$, and $c = \bar{\kappa}^{\pi(1)}$. Since $\tau_{r(1)} < \tau_{r(2)}$, the linear function is concave. Hence, Proposition 7 follows immediately from formula (79), where $\kappa_0^{\pi(1)}$ is calculated based on the formula that calculates c_0 in the proof of Lemma 7. $\square$

Proof of Proposition 8. The proof is also based on Lemma 7. (70) is a piecewise linear function where $a_1 = d^{\pi(1)} \cdot \tau_{r(2)} - k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)})$, $a_2 = d^{\pi(1)} \cdot \tau_{r(1)}$, and $c = \bar{\kappa}^{\pi(2)}$. Since producer 1 is assumed to have enough independent capacity to process his own products, i.e., $k_{r(1)} \geq d^{\pi(1)}$, we can calculate that $d^{\pi(1)} \cdot \tau_{r(2)} - k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) = d^{\pi(1)} \cdot \tau_{r(1)} - (k_{r(1)} - d^{\pi(1)}) \cdot (\tau_{r(2)} - \tau_{r(1)}) \leq d^{\pi(1)} \cdot \tau_{r(1)}$. In particular, when $k_{r(1)} > d^{\pi(1)}$, $a_1 < a_2$ thus the cost allocated to producer 1 by cost-corrected return share with capacity awards is a piecewise linear convex function. Hence formula (72) follows from formula (80) in Lemma 7. In the special case when $k_{r(1)} = d^{\pi(1)}$, $a_1 = a_2$ thus the cost allocated to producer 2 becomes a linear function. Hence the best response function becomes a constant function, which is equal to $(Q^{1\prime})^{-1}(-a_1) = (Q^{1\prime})^{-1}(-a_1)$, i.e., l_1 or l_3. Notice that since $l_1 = l_3$ in this special case, formula (72) is also reduced to a constant function and thus is consistent with our conclusion. $\square$

Proof of Theorem 6. Figure 18 depicts the intersection of the best response functions of both producers and thus shows the equilibrium design profile in each of the three cases. Notice that the relationships $\kappa_0^{\pi(1)} \in (q_1, q_2)$ and $l_1 \leq l_3$ must hold in these graphs.

We first analyze the case where $l_1 \geq \kappa_0^{\pi(1)}$. We show that (l_1, q_1) is the only

equilibrium design profile (as demonstrated by Figure 18(a)). If producer 1 chooses l_1, then since $l_1 \geq \kappa_0^{\pi(1)}$, q_1 is the best response of producer 2 according to Proposition 7. Similarly, if producer 2 chooses q_1, we know from Proposition 8 that l_1 is the best response of producer 1 due to $q_1 < \kappa_0^{\pi(1)} \leq l_1$. By their definitions, it is easy to see that $l_1 = \kappa_{ind}^{*\pi(1)}$ and $q_1 = \kappa_{ind}^{*\pi(2)}$. Meanwhile, we can also show that (l_3, q_2) cannot be an equilibrium, since when producer 1 adopts l_3, producer 2's best response is $q_1 \neq q_2$ due to $l_3 > l_1 \geq \kappa_0^{\pi(1)}$. Hence, we conclude that when $l_1 \geq \kappa_0^{\pi(1)}$, the equilibrium design satisfies $\kappa_{ne}^{\pi(i)} = \kappa_{ind}^{*\pi(i)} \ \forall i = 1, 2$.

When $l_3 \leq \kappa_0^{\pi(1)}$ (Figure 18(b)), we can apply a similar argument to show that (l_3, q_2) is the only equilibrium; moreover, $l_3 \geq l_1 = \kappa_{ind}^{*\pi(1)}$ and $q_2 \geq q_1 = \kappa_{ind}^{*\pi(2)}$.

Combining the results from the above two cases, we conclude that if $\kappa_0^{\pi(1)} \in (0, l_1] \cup [l_3, \infty)$, $\kappa_{ne}^{\pi(i)} \geq \kappa_{ind}^{*\pi(i)} \ \forall i = 1, 2$.

We next analyze the case when $\kappa_0^{\pi(1)} \in (l_1, l_3)$ (Figure 18(c)). We first show that no equilibrium exists. Note that $\kappa_0^{\pi(1)} \in (l_1, l_3)$ implies that $l_1 < l_3$ thus $d^{\pi(1)} < k_{r(1)}$. Let producer 1 chooses an arbitrary design, to which the best response of producer 2 is either q_1 or q_2. Assume producer 2 responded by choosing q_1. Since $q_1 < \kappa_0^{\pi(1)} < l_3$, according to Proposition 8, producer 1's best response to q_1 is either l_1 or q_1, which is strictly less than $\kappa_0^{\pi(1)}$. By Proposition 7, producer 2 will then respond by choosing q_2. In this case, we can use the similar argument to show that producer 1's best response becomes either l_3 or q_2 since $q_2 > \kappa_0^{\pi(1)} > l_1$, which in turn leads to producer 2's choosing q_1. Hence, no (pure) equilibrium design profile exists.

We then show that the following mixed strategy design profile is a mixed Nash equilibrium: Producer 1 chooses $\kappa_0^{\pi(1)}$, and producer 2 chooses the mixed strategy such that the probabilities of choosing q_1 and q_2 equal

$$f^2(q_1) = \frac{-Q^{1'}(\kappa_0^{\pi(1)}) - a_1}{a_2 - a_1}; \quad f^2(q_2) = 1 - f^2(q_1) \tag{84}$$

where $a_1 = d^{\pi(1)} \cdot \tau_{r(2)} - k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)})$, $a_2 = d^{\pi(1)} \cdot \tau_{r(1)}$. We first show that the mixed strategy defined above for producer 2 is a valid one, i.e., $f^2(q_1) \in (0, 1)$.

Since $l_1 < \kappa_0^{\pi(1)} < l_3$, according to the convexity of the function Q^1, we know that $Q^{1'}(l_1) < Q^{1'}(\kappa_0^{\pi(1)}) < Q^{1'}(l_3)$, i.e., $-a_2 < Q^{1'}(\kappa_0^{\pi(1)}) < -a_1$. Hence, it is easy to see that $f^2(q_1) \in (0, 1)$.

Next we show that the mixed strategy design profile $(\kappa_0^{\pi(1)}, f^2)$ is a Nash equilibrium. According to our previous analysis, when producer 1 chooses $\kappa_0^{\pi(1)}$, both q_1 or q_2 are global minima of producer 2's total cost function, thus so is any of their convex combinations. The situation is more complex for producer 1 if producer 2 adopts the mixed strategy specified above. In this case, producer 1's expected cost allocation is calculated as $E_{f^2}[(x^1)_{t\mu}^\alpha(\kappa^{\pi(1)})] = f^2(q_1) \cdot (x^1)_{t\mu}^\alpha(\kappa^{\pi(1)}, q_1) + f^2(q_2) \cdot (x^1)_{t\mu}^\alpha(\kappa^{\pi(1)}, q_2)$, which is a piecewise linear function with three linear pieces.

$$E_{f^2}[(x^1)_{t\mu}^\alpha](\kappa^{\pi(1)}) = \begin{cases} a_1 \cdot \kappa^{\pi(1)} + b_1 & \text{if } \kappa^{\pi(1)} \le q_1 \\[2mm] (f^2(q_1) \cdot a_2 + f^2(q_2) \cdot a_1) \cdot \kappa^{\pi(1)} + (f^2(q_1) \cdot b_2^1 + f^2(q_2) \cdot b_1) \\[2mm] \qquad\qquad\qquad\qquad\qquad\qquad \text{if } \kappa^{\pi(1)} \in (q_1, q_2) \\[2mm] a_2 \cdot \kappa^{\pi(1)} + (f^2(q_1) \cdot b_2^1 + f^2(q_2) \cdot b_2^2) & \text{if } \kappa^{\pi(1)} \ge q_2 \end{cases}$$

$$(85)$$

where $a_1 = d^{\pi(1)} \cdot \tau_{r(2)} - k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)})$, $a_2 = d^{\pi(1)} \cdot \tau_{r(1)}$, $b_1 = d^{\pi(1)} \cdot \bar{c}^{\pi(1)}$, $b_2^1 = d^{\pi(1)} \cdot [\bar{c}^{\pi(1)} + q_1 \cdot (\tau_{r(2)} - \tau_{r(1)})] - k_{r(1)} \cdot q_1 \cdot (\tau_{r(2)} - \tau_{r(1)})$, $b_2^2 = d^{\pi(1)} \cdot [\bar{c}^{\pi(1)} + q_2 \cdot (\tau_{r(2)} - \tau_{r(1)})] - k_{r(1)} \cdot q_2 \cdot (\tau_{r(2)} - \tau_{r(1)})$. We analyze the best response of producer 1, i.e., the global minimum of the function $E_{f^2}[(x^1)_{t\mu}^\alpha] + Q^2$, as follows. First, we can show that the expected total cost function of producer 1, i.e., $E_{f^2}[(x^1)_{t\mu}^\alpha] + Q^2$, decreases in $(0, q_1]$ and increases in $[q_2, \infty)$. To see this, notice that $\forall \kappa^{\pi(1)} \in (0, q_1]$, $x \le q_1 < \kappa_0^{\pi(1)} < l_3$, thus the first order derivative of the expected total cost function is negative. Similarly, $\forall \kappa^{\pi(1)} \in [q_2, \infty)$, $x \ge q_2 > \kappa_0^{\pi(1)} > l_1$, thus the first order derivative of the expected total cost function is positive. When $\kappa^{\pi(1)} \in (q_1, q_2)$, we can calculate that there exists a local minimum at $x = (Q^{1'})^{-1}(-f^2(q_1) \cdot a_2 - f^2(q_2) \cdot a_1)$. Based on the definition of $f^2(q_1)$ and $f^2(q_2)$, it is easy to show that the local minimum

occurs at $x = \kappa_0^{\pi(1)}$. Since this is the only local minimum of the expected total cost function for producer 1, we can conclude that $\kappa_0^{\pi(1)}$ is the global minimum thus the best response for producer 1 to the mixed strategy f^2 as defined in (84). Hence, by definition, the mixed strategy design profile $(\kappa_0^{\pi(1)}, f^2)$ is a Nash equilibrium. Since $\kappa_0^{\pi(1)} > l_1 = \kappa_{ind}^{*\pi(1)}$, $q_1 = \kappa_{ind}^{*\pi(2)}$ and $q_2 > \kappa_{ind}^{*\pi(2)}$, the conclusion in result 2 of Theorem 6 follows. $\qquad\square$

Proof of Theorem 7. We prove this theorem by proving Lemma 4 - 6 as follows.

Proof of Lemma 4. We prove this lemma by duality theory. For the convenience of discussion, assume the routing computed by Algorithm 2 is nondegenerate. We first introduce the following notation. Let R_π be the set of processors to which the volume of a product π is assigned. Denote the largest (smallest) indexed member of the set R_π as $\bar{r}_\pi$ ($\underline{r}_\pi$) ,i.e., $\bar{r}_\pi \doteq r_{\max\{j:r_j \in R_\pi\}}$ ($\underline{r}_\pi \doteq r_{\min\{j:r_j \in R_\pi\}}$) is the most inefficient (efficient) processor in R_π. We compute a dual solution that shares the same basis as the routing solution obtained from the greedy algorithm, as follows. Define the marginal value of all partially-utilized processors as zero, i.e., $\alpha_r = 0 \; \forall r \notin \bigcup_{i=1}^m R_{\pi(i)}$, and $\alpha_{\bar{r}_{\pi(m)}} = 0$

$$\alpha_r = (c_r^{\pi(i)} - c_{\bar{r}_{\pi(i)}}^{\pi(i)}) + \sum_{j:\kappa^{\pi(j)} < \kappa^{\pi(i)}} (c_{\underline{r}_{\pi(j)}}^{\pi(j)} - c_{\bar{r}_{\pi(j)}}^{\pi(j)}) \quad \forall r \in R_{\pi(i)} \setminus \{\bar{r}_{\pi(i)}\} \quad \forall i \in M \quad (86)$$

$$\beta^{\pi(i)} = c_{\bar{r}_{\pi(i)}}^{\pi(i)} - \sum_{j:\kappa^{\pi(j)} < \kappa^{\pi(i)}} (c_{\underline{r}_{\pi(j)}}^{\pi(j)} - c_{\bar{r}_{\pi(j)}}^{\pi(j)}) \quad \forall i \in M \quad (87)$$

By the optimality condition of a transportation problem, the greedy algorithm produces the optimal routing as long as under the corresponding dual solution, $f_r^\pi = 0 \Rightarrow \alpha_r + \beta_\pi \leq c_r^\pi$. To check the condition, for every product $\pi(i)$, we first consider computing the value $\alpha_r + \beta_\pi - c_r^\pi$ for a processor r that is used to process a product $\pi(t)$ that is *less* expensive to recycle than $\pi(i)$, i.e., $\kappa^{\pi(t)} < \kappa^{\pi(i)}$. In order to write the following formula, for every producer i, we introduce another notation i' to denote the index of the product that is the most expensive to recycle among the set $\{j :$

$\kappa^{\pi(j)} < \kappa^{\pi(i)}\}$.

$$
\begin{aligned}
\alpha_r + \beta_{\pi(i)} - c_r^{\pi(i)} &= \left(c_r^{\pi(t)} - c_{\bar{r}_{\pi(t)}}^{\pi(i)}\right) + \sum_{j:\kappa^{\pi(j)}<\kappa^{\pi(t)}} \left(c_{\underline{r}_{\pi(j)}}^{\pi(j)} - c_{\bar{r}_{\pi(j)}}^{\pi(j)}\right) \\
&+ c_{\bar{r}_{\pi(i)}}^{\pi(i)} - \sum_{j:\kappa^{\pi(j)}<\kappa^{\pi(i)}} \left(c_{\underline{r}_{\pi(j)}}^{\pi(j)} - c_{\bar{r}_{\pi(j)}}^{\pi(j)}\right) - c_r^{\pi(i)} \\
&= c_r^{\pi(t)} - c_r^{\pi(i)} + \sum_{j:\kappa^{\pi(t)}<\kappa^{\pi(j)}\leq\kappa^{\pi(i)}} \left(c_{\bar{r}_{\pi(j)}}^{\pi(j)} - c_{\bar{r}_{\pi(j)}}^{\pi(j')}\right) \\
&= \sum_{j:\kappa^{\pi(t)}<\kappa^{\pi(j)}\leq\kappa^{\pi(i)}} \left[\left(c_{\bar{r}_{\pi(j)}}^{\pi(j)} - c_{\bar{r}_{\pi(j)}}^{\pi(j')}\right) - \left(c_r^{\pi(j)} - c_r^{\pi(j')}\right)\right] \\
&= \sum_{j:\kappa^{\pi(t)}<\kappa^{\pi(j)}\leq\kappa^{\pi(i)}} \left(\kappa^{\pi(j)} - \kappa^{\pi(j')}\right) \cdot \left(\tau_{\bar{r}_{\pi(j)}} - \tau_r\right) \leq 0 \qquad (88)
\end{aligned}
$$

Now we consider a processor that is used to process a product $\pi(t)$ that is *more* expensive to recycle than $\pi(i)$, i.e., $\kappa^{\pi(t)} > \kappa^{\pi(i)}$, and compute the value of $\alpha_r + \beta_\pi - c_r^\pi$ in a similar way.

$$
\begin{aligned}
\alpha_r + \beta_{\pi(i)} - c_r^{\pi(i)} &= \sum_{j:\kappa^{\pi(i)}<\kappa^{\pi(j)}\leq\kappa^{\pi(t)}} \left[\left(c_r^{\pi(j)} - c_r^{\pi(j')}\right) - \left(c_{\bar{r}_{\pi(j)}}^{\pi(j)} - c_{\bar{r}_{\pi(j)}}^{\pi(j')}\right)\right] \\
&= \sum_{j:\kappa^{\pi(i)}<\kappa^{\pi(j)}\leq\kappa^{\pi(t)}} \left(\kappa^{\pi(j)} - \kappa^{\pi(j')}\right) \cdot \left(\tau_r - \tau_{\bar{r}_{\pi(j)}}\right) \leq 0 \qquad (89)
\end{aligned}
$$

Combining (88) and (89), we show that $\alpha_r + \beta_\pi \leq c_r^\pi$ holds for all pairs of (r, π) such that $f_r^\pi = 0$ under the greedy algorithm. Hence, the routing computed by the greedy algorithm is optimal. $\qquad\square$

Proof of Lemma 5. The following two finding can be concluded in the low-synergy scenario: (i) the last unit of each product $\pi(i)$ cannot be processed at a facility more efficient than $r(i)$ due to the definition of the low-synergy scenario, i.e., $d^{\pi(i)} > \sum_{j<i} k_{r(j)} \ \forall i \in M$; (ii) the total capacity at the set of processors $\{r(1), ..., r(i)\}$ is sufficient to process the demand of products owned by the corresponding producers. Hence, according to the way that the greedy algorithm works, all of $\pi(i)$'s demand must be processed by $\{r(1), ..., r(i)\}$ if all products that are more expensive to recycle than $\pi(i)$ are owned by producers with a smaller index than i. We derive from these

finding the following remark on the structural property of the social optimal routing in the low-synergy scenario.

Remark 6. *Given any design profile $\mathcal{K}$, in the low-synergy scenario, Algorithm 2 satisfies the following for any producer $i \in M$.*

Observation 1: The last unit of product $\pi(i)$ can only be assigned to processors that are no more efficient than $r(i)$. In particular, it will be assigned to $r(i)$ if there does not exist producer $j > i$ such that $\kappa^{\pi(j)} > \kappa^{\pi(i)}$; otherwise, it will be assigned to $r(j_{\max})$ where $j_{\max} \doteq \max\{j : j > i, \kappa^{\pi(j)} > \kappa^{\pi(i)}\}$.

Observation 2: The last unit of capacity at processor $r(i)$ must be utilized by a product $\pi(j)$ such that $j > i$.

Observation 3: Any product $\pi(j)$ such that $j < i$ and $\kappa^{\pi(j)} > \kappa^{\pi}$ is only assigned to one processor. Similarly, Any product $\pi(j)$ such that $j < i$ and that is processed at less efficient facilities than $r(i)$ is only assigned to one processor.

Notice that under the sequential routing of the greedy algorithm, any change to the return volume of $\pi(i)$ only influences the optimal routing of products with $\kappa^{\pi} \geq \kappa^{\pi(i)}$. Similarly, increasing the capacity at $r(i)$ only has an impact on the optimal routing of products that are processed at less efficient facilities, i.e., $r(j)$ with $j \geq i$. Hence, based on Observation 3 in Remark 6, we conclude that both the marginal cost of $\pi(i)$ and the marginal value of $r(i)$ are not influenced by the design choices of any producer $j < i$. Moreover, according to Observation 1, we can write the marginal cost of $\pi(i)$ as

$$
\beta^{*\pi(i)} = \begin{cases} \kappa^{\pi(i)} \cdot \tau_{r(i)} + G(\{\kappa^{\pi(j)}, j > i\}) & \text{if } \{j : j > i, \kappa^{\pi(j)} > \kappa^{\pi(i)}\} = \emptyset \\ \kappa^{\pi(i)} \cdot \tau_{r(j_{\max})} + G(\{\kappa^{\pi(j)}, j > i\}) & \text{otherwise} \end{cases} \tag{90}
$$

where G is some function that only depends on the product design choices of producers with indices larger than i. Similarly, based on Observation 2, we can also write the marginal value of $r(i)$ as $\alpha^{*}_{r(i)}(\{\kappa^{\pi(j)}, j > i\})$. Hence, the second-stage cost allocation

155

to a producer i under the cost-corrected return share with capacity rewards model, i.e., $(x^i)^\alpha_{t\mu}$, is determined by only the design choices of a subset of producers $\{j : j \geq i\}$. □

Proof of Lemma 6. The first statement of the lemma that a design profile is a Nash equilibrium if and only if it can be solved as a solution to Algorithm 3 directly follows from Lemma 5 and the way that the algorithm is constructed.

To prove the second part of the lemma, let us go back to the allocation by cost-corrected return share with capacity rewards to a producer i in the second-stage. Based on our analysis of the marginal cost of $\pi(i)$ and marginal value of $r(i)$, we conclude that, given others' design choices $\hat{\mathcal{K}}^{-i} = \{\hat{\kappa}^{\pi(j)}, j \neq i\}$, the cost allocation $(x^i)^\alpha_{t\mu}$ is a *concave* piecewise linear function of $\kappa^{\pi(i)}$. Specifically, the break points of the function correspond to the design choices of a set of producers $\{j_1, ..., j_T\} \subset \{j : j > i\}$ such that $j_1 \doteq \arg\max_{j>i} \hat{\kappa}^{\pi(j)}$ and $j_{t+1} \doteq \arg\max_{j_{t+1}>j_t} \hat{\kappa}^{\pi(j)}$ $\forall t \in \{t > 1, j_t < m\}$; it is easy to see that $j_T = m$, i.e., the index of the last producer. We further calculate the slope of $(x^i)^\alpha_{t\mu}$ in every linear piece to be

$$\frac{\partial (x^i)^\alpha_{t\mu}}{\partial \kappa^{\pi(i)}} = \begin{cases} d^{\pi(i)} \cdot \tau_{r(i)} & \forall \kappa^{\pi(i)} > \hat{\kappa}^{\pi(j_1)} \\ d^{\pi(i)} \cdot \tau_{r(j_t)} & \forall \kappa^{\pi(i)} \in [\hat{\kappa}^{\pi(j_{t+1})}, \hat{\kappa}^{\pi(j_t)}] \quad \forall t = 1, 2, ..., T-1 \\ d^{\pi(i)} \cdot \tau_{r(m)} & \forall \kappa^{\pi(i)} < \kappa^{\pi(m)} \end{cases} \quad \cdot \quad (91)$$

It can be observed that the function $(x^i)^\alpha_{t\mu}$ is a concave piecewise linear function. Hence, the total cost function $Q^i + (x^i)^\alpha_{t\mu}$ is a piecewise convex function. We show by contradiction that the global minimum point of $Q^i + (x^i)^\alpha_{t\mu}$ must be attained at the local minimum of one of the convex pieces of the function, i.e., where the first order derivative of the convex piece equals zero. If $(x^i)^\alpha_{t\mu}$ contains no more than two linear pieces, then the result is obvious according to Lemma 7. Otherwise, assume none of these local minima is the global minimum of the function. Then, since the $Q^i + (x^i)^\alpha_{t\mu}$ is a piecewise convex function, the global minimum can only be achieved at a break point between these convex pieces. Without loss of generality, assume $\kappa^{\pi(i)} = \hat{\kappa}^{\pi(j_t)}$ is

the global minimum for some $j_t \in \{j_1, ..., j_T\}$. Then this value is obviously the global minimum of the function $Q^i + (x^i)^\alpha_{t\mu}$ restricted on the interval $[\hat{\kappa}^{\pi(j_{t+1})}, \hat{\kappa}^{\pi(j_{t-1})}]$. This conclusion leads to a contradiction to Lemma 7, since $(x^i)^\alpha_{t\mu}$ contains only two linear pieces and is also concave, which completes the proof of this Lemma. $\square$

Continuing the proof to Theorem 7, since according to our assumption, $r(j)$ is less efficient than $r(i)$ $\forall j > i$, i.e., $\tau_{r(j)} > \tau_{r(i)}$ $\forall j > i$. Hence, due to the convexity of Q^i, it is easy to see that $(Q^{i\prime})^{-1}(d^{\pi(i)} \cdot \tau_{r(j)}) \leq (Q^{i\prime})^{-1}(d^{\pi(i)} \cdot \tau_{r(i)}) = \kappa^{*\pi(i)}_{ind}$ $\forall j \geq i$. Combining this result with Lemma 6, we can derive Theorem 7. $\square$

Proof of Theorem 8. It is easy to see that in the high-synergy case, the social optimal routing is to route all return volume to $r(1)$, the most efficient processor. Hence, the cost allocated to each producer i under cost-corrected return share with capacity rewards equals

$$(x^i)^\alpha_{t\mu}(\kappa^{\pi(i)}) = d^{\pi(i)} \cdot [\kappa^{\pi(i)} \cdot \tau_{r(1)} + \bar{c}^{\pi(i)}] \quad \forall i \in M \tag{92}$$

Hence, the total cost function for each producer i, $Q^i + (x^i)^\alpha_{t\mu}$, is a convex function. By taking the first order derivative of the function, we conclude that the only equilibrium design profile in the high-synergy scenario satisfies $\kappa^{\pi(i)}_{ne} = (Q^{i\prime})^{-1}(-d^{\pi(i)} \cdot \tau_{r(1)})$ $\forall i \in M$. Due to the convexity of Q^i and the fact that $\tau_{r(1)} \leq \tau_{r(i)}$ $\forall i \in M$, it is easy to conclude that $\kappa^{\pi(i)}_{ne} \geq \kappa^{*\pi(i)}_{ind}$ $\forall i \in M$. $\square$

Proof of Theorem 9. The existence of an equilibrium design profile can be concluded based on Figure 20. Figure 20 also demonstrates that the equilibrium design profile is achieved only at the local minimum of the convex pieces of each producer's total cost function. Hence, result 1 of the theorem regarding producer 2's design incentives can be proven based on the fact that the slopes of both linear pieces of his cost allocation by return share (Figure 19(b)) are smaller than $d^{\pi(2)} \cdot \tau_{r(2)}$, and the convexity of the investment function Q^2. Similarly, for producer 1, we can show that his cost allocation

by return share (Figure 19(a)) satisfies

$$
\begin{aligned}
slope_1^1 \quad &< \quad d^{\pi(1)} \cdot \tau_{r(1)} \\
slope_2^1 \quad &> \quad d^{\pi(1)} \cdot \tau_{r(1)} \quad \text{if } \frac{\tau_{r(1)}}{\tau_{r(2)}} < \frac{d^{\pi(1)}}{d^{\pi(2)} + d^{\pi(1)}}
\end{aligned}
\tag{93}
$$

Hence, result 2 follows due to the convexity of the investment function Q^1. □

Proof of Theorem 10. This theorem can be proven in the similar way as Theorem 9. In particular, we can calculate that when $\frac{\tau_{r(1)}}{\tau_{r(2)}} < \frac{d^{\pi(1)} + d^{\pi(2)} - k_{r(1)}}{d^{\pi(1)} + 2d^{\pi(2)} - k_{r(1)}}$, producer 1's cost allocation by return share (Figure 19(a)) satisfies $slope_2^1 > d^{\pi(1)} \cdot \tau_{r(1)}$. □

2.4 Future Research Directions

From its few-sentence principles to a state-wide program consisting of thousands of entities and influencing millions, EPR implementation is a complex process. During this process, multiple dimensions of environment, economics, politics and operations come into play, and the differences among them create challenges in achieving an efficient balancing of environmental and economic tradeoffs. Moreover, the exponential growth of the number of stakeholders involved poses additional challenges to coordinate and reconcile different individual agendas. These challenges raise a rich set of research questions in managing and optimizing decentralized systems, some of which are briefly outlined at the end of §2.1.3. In this section, we provide a more detailed discussion of the three issues that are closely related to the analytical study performed in this chapter on cost allocation mechanisms and product design implications under collective EPR.

First, the goal of our study in §2.2 is to understand the impact of operational factors (i.e., network synergy, product heterogeneity, stakeholder perspectives) in a given CRN on the implication of regulatory design choices such as cost allocations, and to propose policy recommendations accordingly. Considering practice where the cost is allocated ex-post after all return volume is observed, routed, processed, and

the total cost is incurred, it is reasonable to adopt a CRN model in this study that assumes full information of the network specifics, in particular the return volumes of the products. However, as it is mentioned at the end of §2.1.3, uncertainty in consumers' responses creates stochasticity in both the volume and the product mix of the returned electronics. This is a prominent feature of EPR implementation and plays a significant role in the operations of the CRN. Hence, it is an important future research to evaluate the *robustness* of group incentive compatibility under the cost allocation mechanisms proposed to such uncertainty of the CRN. In game theory literature, a set of cooperative game models with stochastic elements are proposed and various solution concepts for cost/benefit allocation mechanisms are studied (e.g., [15, 157, 29, 28, 24, 144, 70]). Cooperation under uncertainty is also studied in the operations literature (e.g., [30, 93]). In our problem, how the network synergy from capacity sharing in a collective system plays a role can be an interesting question to explore.

The EPR concept is regarded as an economic tool to incentivize more environmentally-friendly design choices of the producers, as by doing so, the producers can potentially reduce the post-use recycling cost of their products. In §2.3, we take an operational angle towards this issue, which suggests that the processing efficiency levels of the recycling facilities can exert a significant impact. This implies that the EPR legislation and its implementation may also potentially incentivize producers to invest in *process improvement*, e.g., investing in more advanced recycling technology and thus more efficient processing capacity. This is a practical and important issue. In practice, some major electronics producers have already established their own remanufacturing capacities as an essential sustainability strategy [80]. Capacity management issues are also extensively studied in sustainable operations (e.g., [153, 154, 85]). For example, in [132], the authors study how input and output prices affect material manufacturers'

incentives to invest in process improvement including improving input efficiency, capacity efficiency, and developing flexibility between the two; based on the analysis, the authors analyze the implication of a carbon tax or cap-and-trade policy on material manufacturer's incentives to improve energy efficiency. In our problem, the transportation network model used in §2.3 captures the impact of capacity efficiency on the total operational cost and thus offers a natural starting point for future research on this capacity investment issue in the EPR context.

In this chapter, our study is focused on the design and the implication of collective implementation of EPR. Although collective implementation is adopted in most states where the collection and recycling operations are directly managed by a central authority, developing a market-based approach may be more suitable in the US setting where industry is historically not open to direct state intervention and prefers market-based mechanisms. Given the CRN context where network effects are at work in determining individual preferences (e.g. no collector wants to be in charge of low-density areas), it may be especially effective for states to implement a more decentralized system where the state-run authority designs market mechanisms (resource exchange prices) and where each entity makes its own decisions about whom to transact with and at what volume. In the next chapter, we conduct a theoretical study of such market-based exchange mechanisms on general multicommodity networks with privately-owned resources that are motivated by the combined service networks in transportation alliances; we show that resource exchange mechanisms can be effective in coordinating individual participants' operations and generating sufficient payoffs to each of them. In addition, pricing mechanisms have also been extensively studied in the literature of routing games, where various ways to design traffic tolls that are effective in reducing network congestion are proposed (e.g., [130, 34, 104]). Hence, it will be an interesting research to evaluate the effectiveness of such market mechanisms in the EPR setting where a different set of practical complications exist,

e.g., the CRN has both producer-owned and exogenous resources. Further research questions include the robustness of the mechanism to return volume uncertainty and the impact of economies of scale.

CHAPTER III

MANAGING DECENTRALIZED RESOURCE SHARING IN MULTICOMMODITY NETWORKS

Network systems that consist of capacity privately owned by different entities are common in practice. For example, transportation networks often contain a pool of fleets owned by different companies; Internet is a collection of thousands of autonomous systems with their own bandwidth. In these combined networks, capacity is usually not restricted to the use of its owner and can be employed in the routing operations of others according to certain access rules. Hence, these systems have the potential for improving routing efficiency by exploiting synergies via capacity sharing. Indeed, consider the previous two examples: companies in transportation industry often form alliances and combine their service networks to reduce operational cost and improve service levels [3]; in Internet, end-to-end flows are delivered in an efficient manner with the joint effort among autonomous systems sharing bandwidth [128].

However, fully realizing the synergistic potential of a combined network can be challenging, as many practical systems are utilized by multiple users to deliver goods and information in a decentralized way. In particular, in these applications, the participants are entitled to make their own commodity routing and capacity management decisions, driven by their own interests and benefits. The resulting aggregate routing is not guaranteed to be efficient. In fact, it is widely observed that decentralized routing decisions can jointly undermine the overall efficiency of the network, reducing the network throughput or increasing the total congestion or routing cost (e.g., as noted in [139]). Such a problem is more complex and prominent in the setting where a limited amount of privately-owned capacity is shared among multiple users. First,

due to the existence of the capacity limits, the feasibility of the routes chosen by a user for his commodities depends on others' routings. However, the users normally decide individually and simultaneously and thus are unaware of each other's routing until all commodities are shipped. In this case, the aggregate routing may violate capacity limits and lead to overflow in the network. Second, sharing privately-owned capacities requires a well-defined and reasonable rule to regulate how others may access these capacities. In addition, in many applications, the users decide on their own volition whether to join a resource sharing agreement, for example, under collective implementation of EPR legislation that we analyzed in the previous chapter. Hence there exists a potential risk of players breaking away from a combined network, resulting in a fragmented system, thus undermining the synergies from resource sharing. Hence, it is desirable that the system benefit from resource sharing can be distributed in a reasonable way among players to incentivize participation.

The notion of mechanism design suggests an approach to tackle the above problems by incentivizing participants to adopt routing solutions that are aggregately efficient through imposing specially tailored operational guidelines. Taking into account the features of a combined network, one natural way to do this is to design the access rules according to which capacity can be used by individuals other than its owner. In this study, since market trade is one of the most common form of sharing private resources, we consider coordinating a combined network by designing a *capacity exchange* mechanism under which capacity is traded according to a set of centrally-designed unit prices. Specifically, every user pays the capacity exchange price for every unit of others' capacity utilized for the shipment of his own commodities on each edge in the combined network. Their individual routing decisions are then made in order to maximizes their own profits, i.e., the routing revenues generated from their own commodities plus their net gains from the capacity exchange prices received from and paid to others. Via such a mechanism, a central authority

of the system can influence individual participants' profits and thus their selfish operations by choosing the appropriate exchange prices. Note that under such a market exchange mechanism, producers essentially make side payments to each other for resources, thus the exchange prices essentially determine the allocation of the benefits from resource sharing among the players. Hence, it is also desirable that the prices are designed under which all players benefit and thus have the incentives to join a combined network.

Designing a mechanism that can well-coordinate a complex network system is a challenging task, especially taken into the operational complexities in practical applications. First, in many applications, there usually exist multiple owners of the same type of resources. For example, different airlines can operate fights between the same cities. Results from the cooperative game theory literature indicate that such multiple ownership of capacity on the same edge can give rise to diseconomies that undermine the resource-sharing incentives in combined networks (e.g., [90, 36]); yet there is little discussion on the impact of such multiple ownership conditions on the effectiveness of market exchange mechanisms. Second, zero exchange prices for privately-owned resources are generally not acceptable in practice. One of the main concerns is that they give rise to free-riders, i.e., players obtaining positive shipping revenues using others' capacities at zero cost. Hence, it will be desirable to design effective and strictly positive exchange prices. Third, it is ideal that the central authority has full knowledge of the network parameters such as capacity and demand levels, in order to tailor the mechanism accordingly. However, this is usually not the case in practice and one prominent example is demand uncertainty. Indeed, in many applications, a resource sharing mechanism is determined as a tactical decision before the actual demand is revealed; yet there is significant demand volatility, thus precise demand forecasting is difficult. Under such circumstances, *robustness* becomes an important and desirable property of a capacity exchange mechanism. Moreover, in

practice there is often a time lag between mechanism design and individual decision-making when more demand information is likely to be obtained by the players. Such information asymmetry between the central authority (the mechanism designer) and the players adds additional complexity to the problem.

In this chapter, we provide a systematic analysis of the use of market-based exchange mechanisms to motivate and regulate capacity sharing, focusing on the design of capacity pricing strategies in the presence of the aforementioned practical operational complexities. There are two main goals of this research stream: (i) to analyze how the coordination benefit of market-based mechanisms depends on the underlying network structure and characterize the network properties that matter; (ii) to propose efficient and effective pricing strategies to address different operational complexities. To this end, we adopt a non-cooperative game model based on a set of network optimization problems that represent individual operators' routing and capacity allocation decisions, and analyze the model using inverse optimization techniques.

The structure of this chapter is the following. We first provide a literature review in §3.1; note that some papers that are specific to individual topics covered in this research stream are discussed in the corresponding sections. In §3.2, we introduce the notation and mathematically define the combined network model and the capacity exchange mechanism studied in this chapter. We begin by analyzing the case with deterministic demands. In §3.3, we propose a dual-based pricing strategy that guarantees both the overall routing optimality and the voluntary participation of all players in the combined network via generating a group incentive compatible allocation of the total shipping revenue. We further discuss the potential diseconomies derived from multiple ownership of the same capacity. We further consider designing strictly positive prices in §3.4, where we show in certain networks the perfect alignment of the individual incentives with the goal to maximize the overall routing efficiency cannot be achieved using such prices, yet a certain level of partial alignment

can be guaranteed by a pricing algorithm we propose. In §3.5, we move on to study a stochastic case with unknown demand. Our main results in this section includes characterizing how network structure affects the *robustness* of the mechanism, and propose robust pricing strategies and algorithms

in any given network.

3.1 Review of Related Literature

The efficiency problem in decentralized network systems is intensively studied in the routing game literature where the increase in network congestion due to the selfish routings of individual travelers are analyzed mainly based on the measures of the *price of anarchy* (PoA) (e.g., [138, 35]) and the *price of stability* (PoS) (e.g., [8]). These two notions are defined, respectively, as the ratios of the congestion under the worst and the best routing equilibrium compared to the smallest congestion that can be achieved over the network. Collaboration mechanisms are also proposed in this literature to improve the PoA or PoS in decentralized networks, including the Stackelberg strategies [95, 136], taxation [130, 34] and rebates [104]. These papers study networks that are open to public such as the road systems, while we focus on networks where capacity are privately owned and users can decide whether to share their own capacity or not. Moreover, we study not only the equilibrium state of a combined network but also how an equilibrium can be achieved based on a decision-making model of the users that capture both their selfishness and unawareness of others operations.

In literature, resource-sharing in networks is often studied as cooperative games, called the *flow games* (e.g., in [90, 61, 40, 151, 128, 105]), where the value of a sub-coalition is defined as the optimal objective value of a certain flow problem on the sub-network formed by its members. A central issue in this literature is to characterize the set of fair allocations of benefits or costs among participants inside a combined

network. For example the *core* of a flow game is defined as the set of allocations under which no individual benefits less than his operating individually or inside a sub-coalition. It is a well-known result in this literature that a core payoff of a flow game can be calculated based on the optimal dual solutions of the corresponding flow problem (e.g., [123]). In addition, there also exists a set of papers that analyzes the diseconomies arising from the existence of multiple owners of the capacity on a single edge, under which a flow game may have an empty core (see a review by Sounderpandian [151]). However, while a cooperative game framework assumes the existence of a central authority who dictates the operations of a combined network and allocates the resulting benefits or costs, we investigate the design of a capacity exchange mechanisms aiming at achieving a fair distribution of the total revenue as the result of decentralized individual routings.

One collaboration mechanism that is similar in spirit to ours is the *competitive prices* studied in production economy literature, under which individual decisions made under selfish preferences synthesize a social optimum, called a competitive equilibrium [147], and a core payoff is guaranteed under mild conditions [37]. Yet the notion of competitive prices does not explicitly characterize how individuals interact so that a competitive equilibrium can be attained. In this study, we base our study on an individual decision-making model so that the routes chosen by the users are analytically tractable; and we show constructively how a collective optimal routing can be achieved under well-designed capacity exchange prices.

The most relevant study to this research stream is by Agarwal and Ergun [2], which introduces the mechanism based on capacity exchange prices. The authors show the general existence of prices that induce a collective optimal routing within a combined network, and that every set of such prices lead to a core payoff if the capacity on any edge is owned by a unique user. Our study in this research stream provide a more comprehensive analysis to the coordination power of the mechanism in general

situations where different practical operational complexities are incorporated. First, we study a general network setting that allows multiple capacity owners on the same edge. Second, we consider the potential existence of free riders, that can be a serious barrier to resource sharing in real-life situations. Third, we analyze stochastic network settings with demand uncertainty. In studying these issues, our results demonstrate not only the effectiveness but also the limitations of capacity exchange mechanisms under practical restrictions, and we propose price design solutions that represent a better balance between the practicality of the mechanism and the efficiency of the resulting system.

Finally, note that a large part of the complexity of collaboration mechanism design for combined networks is due to the mutual obliviousness among individual users to each other's behavior, as it is a significant barrier for effective coordination of the interactions among the users under a resource-sharing setting. In literature, various forms of individual obliviousness in games are modeled and studied, for example, by Halpern and Rêgo [72], Foster and Vohra [53] and Weintraub et al. [179], mainly focusing on characterizing the resulting equilibrium states. Our study contributes to the literature by considering using a mechanism design approach to minimize the negative impact of individual obliviousness within a decentralized system.

3.2 Preliminaries

In this section, we formally introduce a model of a multicommodity network with combined capacity (combined network for short) and the capacity exchange mechanism that this research stream is based on. The model is first introduced by Agarwal and Ergun [2], which captures many features of practical network systems. In §3.5, we also introduce a stochastic combined network by incorporating demand uncertainty.

A Multicommodity Network with Combined Capacity   Consider a directed network $G = (V, E)$ with multiple source-sink pairs and a set $N = \{1, 2, ..., n\}$ of

players, each representing a participant of the network system that owns capacity on G and/or commodity demand to be routed through G. Let c_e be the total amount of capacity on each edge $e \in E$. Each player i is assumed to own a γ_e^i percentage of the total capacity on e such that $0 \leq \gamma_e^i \leq 1$ and $\sum_{i \in N} \gamma_e^i = 1 \ \forall e \in E$. A commodity is defined by the triplet (o, d, i) that specifies its origin o, destination d and owner i. We assume that there exists at least one path in G for each commodity to be delivered. The demand and the unit routing revenue of commodity (o, d, i) are denoted by $d_{(o,d,i)}$ and $r_{(o,d,i)}$ respectively, and we assume $r_{(o,d,i)} > 0 \ \forall (o, d, i) \in D$. We also denote by D^i the set of commodities owned by a player i, and denote by $D \doteq \bigcup_{i \in N} D^i$ the set of all commodities.

G is a combined network when all its edge capacity are shared among all players. In other words, there is no restriction in capacity utilization in the routing of any commodity, thus the total shipping revenue generated over the entire network can be maximized. Such a maximum total revenue can be computed by the following weighted maximum flow problem on G, which we call the *centralized problem* (C). First, for modeling convenience, we create a fictitious edge (d, o, i) for each commodity (o, d, i) going from its destination node d to the origin node o with infinite capacity that is owned and only used by the corresponding commodity owner. Let $\delta^-(v) = \{(u, v) \in E : \forall u\}$ and $\delta^+(v) = \{(v, w) \in E : \forall w\}$ be the sets of incoming and outgoing edges for node v respectively. The variables $f_e^{(o,d,i)}$ denote the amount of commodity

(o, d, i) shipped via edge e.

$$(C) \quad \max \quad R(f) \;=\; \sum_{(o,d,i)\in D} r_{(o,d,i)} \cdot f_{(d,o,i)}^{(o,d,i)} \tag{94}$$

$$s.t. \sum_{e\in\delta^-(v)} f_e^{(o,d,i)} - \sum_{e\in\delta^+(v)} f_e^{(o,d,i)} \;\leq\; 0 \quad \forall v \in V \quad \forall(o,d,i)\in D \tag{95}$$

$$\sum_{(o,d,i)\in D} f_e^{(o,d,i)} \;\leq\; c_e \quad \forall e \in E \tag{96}$$

$$f_{(d,o,i)}^{(o,d,i)} \;\leq\; d_{(o,d,i)} \quad \forall(o,d,i)\in D \tag{97}$$

$$f \;\geq\; 0. \tag{98}$$

In words, the centralized problem (C) maximizes the total revenue $R(f)$ under a feasible routing f for all commodities in the network subject to the flow conservation constraints (95), the capacity constraints (96), the demand constraints (97), and the nonnegativity constraints. We denote an optimal solution to (C) by f^*, which we call the *social optimal routing* and which represents the highest level of system efficiency in G.

The above centralized problem implicitly assumes a *centralized* setting of the combined network, where the operation is directly managed by a central planner. In this case, it is easy to implement f^* by dictation. However, assuming such a centralized setting is generally not realistic, as in practice combined networks normally operate in a decentralized way where the players are entitled to independent management of their commodities and capacity. To coordinate such *decentralized combined networks* requires more subtle and indirect approaches. One way for the central planner to do this is to designing the operational rules of the network, which is often called a *mechanisms*. Considering our problem setting where the capacity is privately owned but generally accessible, a natural and practical mechanism is the market trade of capacity according to centrally-designed exchange prices, which we discuss next.

A Capacity Exchange Mechanism   In the following, we present a model of the capacity exchange mechanism, first studied by Agarwal and Ergun [2]. Let $cost_e \geq 0$

be the unit capacity exchange price on each edge $e \in E$ in G that is pre-determined by the central planner. Every network player pays for others' capacity that is used for the shipment of his own commodities according to these prices. We assume that the total amount of capacity exchange prices obtained on an edge e is allocated among the capcity owners on e proportionally to their ownership levels $\{\gamma_e^i\}$. Such an assumption makes analyzing the mechanism analytically tractable. Another interpretation of this assumption is that for each player, his demand for capacity on an edge e is allocated among all the capacity owners on e proportional to their ownership levels $\{\gamma_e^i\}$. Although such an assumption simplifies capacity buyers' individual preferences over the sellers, it captures the general relationship between one's capacity ownership level and his market share in resource exchanges. Note that under the above model of the capacity exchange mechanism, we essentially study the coordination of individual routing decisions under a fully-regulated market environment where no competition exists among capacity owners on the same edge for user demand[1].

Under the above model of the capacity exchange mechanism, we can calculate the payoff to the players given a routing f of all the commodities in network G and the set of capacity exchange prices $\{cost_e\}$. Specifically, we denote the payoff to a player i by $x_{cost}^i(f)$, which includes the shipping revenue from satisfying his commodity demand and the net profit from capacity exchanges over G.

$$x_{cost}^i(f) = \sum_{(o,d,i) \in D^i} r_{(o,d,i)} \cdot f_{(d,o,i)}^{(o,d,i)} + \sum_{e \in E} cost_e \cdot \left[\gamma_e^i \sum_{(o,d,j) \notin D^i} f_e^{(o,d,j)} - (1 - \gamma_e^i) \sum_{(o,d,i) \in D^i} f_e^{(o,d,i)} \right] .$$
(99)

In a decentralized setting, each player i plans for his own operations in order to maximize the individual profit $\{x_{cost}^i(f)\}$ under the mechanism based on the given capacity exchange prices. However, notice that according to (99), one player's profit

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[1]Competition over capacities and prices for user demand in networks among resource owners is also studied in literature, focusing on the inefficiency under an equilibrium state, for example, by [1].

is also dependent on the routings of others' commodities, which are assumed to be decided simultaneously and independently from that of player i. Estimation of others' behaviors is practically difficult. One major reasons is that it is often hard to access the revenue information of other companies. Hence, in order to model the incentives of individual players as well as their obliviousness to others' decisions, we adopt the following behavioral model where each player solves for a routing for all the commodities in G, including those of others, over the entire network in order to maximize his own profit. Mathematically, each player i solves an individual program (P^i_{cost})

$$(P^i_{cost}): \qquad \max \quad x^i_{cost}(f) \qquad s.t. \quad (95) - (98) \text{ in } (C), \qquad (100)$$

We call the optimal solution f^{i*}_{cost} player i's *individual optimal routing*, based on which player i routes the demand in his own commodity set D^i. Solving such an individual problem is practically plausible as it only requires the demand and capacity information of others, which is normally feasible to be accessed or estimated.

Given the above behavioral model of individual players, we conclude that in order to coordinate a decentralized combined network, the capacity exchange prices $\{cost_e\}$ should be designed to induce the individual optimal routings f^{i*}_{cost} towards f^*. This leads to the following definition of the *perfect coordinating prices*.

Definition 4. *A set of capacity exchange prices $\{cost_e\}$ is called perfect coordinating if $f^{i*}_{cost} = f^* \ \forall i \in N$ for some collective optimal routing f^*.*

Intuitively, Definition 4 indicates that perfect coordinating prices represent the strongest guarantee for the overall efficiency of a given combined network. Specifically, they induce all players to route their commodities according to the social optimal routing f^* and thus guarantees that the optimal total revenue $R(f^*)$ is achieved. In particular, under the influence of such prices, f^* also leads to the maximum individual

profit for every player that can ever be obtained within the combined network. Because of this, the individual incentives are *perfectly* aligned with the central planner's goal to maximize the total revenue. Moreover, it is also easy to see that f^* is guaranteed to be a *Nash equilibrium* under perfect coordinating prices, i.e., a stable routing from which no individual player would strictly benefit from unilateral deviation.

In literature, perfect coordinating prices are analyzed in [2], where they are shown to exist in any combined network G as modeled above. There, such prices are termed *inverse feasible prices* as they can be computed in polynomial time via inverse optimization techniques. Specifically, exchange prices $\{cost_e\}$ is perfect coordinating if and only if $\exists$ a dual solution to each individual problem (P^i_{cost}) that also satisfies the complementary slackness conditions with respect to f^*. The problem can be formulated as a feasibility problem with respect to a linear constraint system. Mathematically, let $\bar{D}^* = \{(o,d,i) \in D : f^{*(o,d,i)}_{(d,o,i)} = d_{(o,d,i)}\}$ and $\bar{E}^* = \{e \in E : \sum_{(o,d,i) \in D} f^{*(o,d,i)}_e = c_e\}$ to be the sets of commodities whose demand is fully shipped and edges where capacity is fully utilized under a social optimum f^*, and $\bar{F}^* = \{f^{*(o,d,i)}_e : f^{*(o,d,i)}_e > 0\}$ to be the set of strictly positive flow variables under f^*. The linear constraint system associated with the individual problem of player i,

denoted by I^i, can be written as follows.

$$(\pi^i)_v^{(o,d,i)} - (\pi^i)_u^{(o,d,i)} + (\alpha^i)_e \geq -(1 - \gamma_e^i)cost_e \quad (o,d,i) \in D^i, e \in E : f_e^{*(o,d,i)} \notin \bar{F}^* \quad (101)$$

$$(\pi^i)_v^{(o,d,i)} - (\pi^i)_u^{(o,d,i)} + (\alpha^i)_e = -(1 - \gamma_e^i)cost_e \quad (o,d,i) \in D^i, e \in E : f_e^{*(o,d,i)} \in \bar{F}^* \quad (102)$$

$$(\pi^i)_v^{(o,d,i)} - (\pi^i)_u^{(o,d,i)} + (\alpha^i)_e \geq \gamma_e^i cost_e \quad (o,d,i) \notin D^i, e \in E : f_e^{*(o,d,i)} \notin \bar{F}^* \quad (103)$$

$$(\pi^i)_v^{(o,d,i)} - (\pi^i)_u^{(o,d,i)} + (\alpha^i)_e = \gamma_e^i cost_e \quad (o,d,i) \notin D^i, e \in E : f_e^{*(o,d,i)} \in \bar{F}^* \quad (104)$$

$$(\pi^i)_o^{(o,d,i)} - (\pi^i)_d^{(o,d,i)} + (\beta^i)_{(o,d,i)} \geq r_{(o,d,i)} \quad (o,d,i) \in D^i : f_{(d,o,i)}^{*(o,d,i)} \notin \bar{F}^* \quad (105)$$

$$(\pi^i)_o^{(o,d,i)} - (\pi^i)_d^{(o,d,i)} + (\beta^i)_{(o,d,i)} = r_{(o,d,i)} \quad (o,d,i) \in D^i : f_{(d,o,i)}^{*(o,d,i)} \in \bar{F}^* \quad (106)$$

$$(\pi^i)_o^{(o,d,i)} - (\pi^i)_d^{(o,d,i)} + (\beta^i)_{(o,d,i)} \geq 0 \quad (o,d,i) \notin D^i : f_{(d,o,i)}^{*(o,d,i)} \notin \bar{F}^* \quad (107)$$

$$(\pi^i)_o^{(o,d,i)} - (\pi^i)_d^{(o,d,i)} + (\beta^i)_{(o,d,i)} = 0 \quad (o,d,i) \notin D^i : f_{(d,o,i)}^{*(o,d,i)} \in \bar{F}^* \quad (108)$$

$$(\alpha^i)_e = 0 \quad \forall e \notin \bar{E}^*; \quad (\beta^i)_{(o,d,i)} = 0 \quad \forall (o,d,i) \notin \bar{D}^* \quad (109)$$

$$\text{nonnegativity constraints}. \quad (110)$$

[2] indicates that the constraint set $\bigcup_{i \in N} I^i$ fully characterizes the set of perfect coordinating prices, which can thus be solved in polynomial time.

However, note that although players are guaranteed to achieve the maximum individual payoff in the combined network under perfect coordinating prices, such a payoff may be smaller than what they can achieve operating independently without sharing resources with others. In this case, a player can be incentivized to break away from the combined network. Such fragmentation, as we mentioned in Chapter 2, will undermine the synergies from resource sharing and thus the overall efficiency of the combined network. Hence, it is desirable that a set of perfect coordinating prices can also lead to payoffs that motivate the participation of all players in the combined network. Whether such perfect coordinating prices exist and if yes, how to design them, are open problems under the general setting of combined networks, especially with multiple players owning the capacity on a same edge. We discuss these problems in §3.3.

174

In §3.4, we further analyze the property of the perfect coordinating prices to ensure f^* to be an equilibrium. We show that this property of perfect coordinating prices can be incompatible with the requirement to avoid free riders. To resolve the conflict, we introduce in §3.4.2 a less restrictive pricing concept, i.e., *partial coordinating prices*. The basic idea is that the social optimal routing f^* is enforced only for commodities owned by player i. We show that strictly positive partial coordinating prices always exist; however, the tradeoff is that such a relaxation cannot guarantee that f^* is achieved as an equilibrium,

Also note that in the above model, the demand $d_{(o,d,i)}$ is assumed to be known information for every commodity. In §3.5, we generalize this deterministic model to a stochastic setting with demand uncertainty, and study the existence and the computation of *robust perfect coordinating prices*.

3.3 Dual Payoffs, Core and a Capacity Exchange Mechanism under the Multiple Capacity Ownership Condition

In this section, we focus on studying players' incentives to participate in a combined network and to share resources under perfect coordinating prices. To this end, we analyze their payoffs induced by perfect coordinating prices using concepts from the cooperative game framework.

We first define a cooperative game (N, v) based on the combined network model introduced in the last section. N is the set of players considered, which we call the *grand coalition*. The value of the grand coalition is set as the maximum total shipping revenue achievable on the combined network G, i.e., $v(N) \doteq R(f^*)$. For each subgroup of players $S \subset N$, which is called a *sub-coalition*, we define its value $v(S)$ as the maximum revenue that can be obtained by routing the commodity demand of its members on the sub-network of G with capacity of its members. Such a value can be computed by solving a weighted max-flow problem similar to the centralized problem

175

(C) (94)-(98) but restricted to the capacity and demand owned by members in S, i.e., $\gamma_e^S \doteq \sum_{i \in S} \gamma_e^i$ fraction of the total capacity on each edge e, and the demand with respect to the commodity set $D^S \doteq \bigcup_{i \in S} D^i$, respectively. We can interpret such a flow problem, denoted as (C^S), as the centralized problem within S. We call this cooperative game a *multicommodity flow game* (MCF game for short).

In a MCF game, an allocation of the total revenue $v(N) \doteq R(f^*)$ is defined as $\{x^i\}$ such that $\sum_{i \in N} x^i = v(N)$. Note that under a set of perfect coordinating prices $\{cost_e\}$, since all players are induced to the social optimal routing, the individual payoffs $\{x_{cost}^i(f^*)\}$ essentially give rise to an allocation of the maximum total revenue. The cooperative game theory literature provides a rich set of notions to study the performance of different allocation schemes. Many of them are defined based on the stand-alone values of sub-coalitions, i.e., the set of $\{v(S)\}$, as benchmarks. For example, as introduced in Chapter 2, the notion of the core of a cooperative game is defined as the set of allocations under which no sub-coalition S is allocated a smaller payoff inside the combined network compared to its stand-alone value $v(S)$ operating independently. A payoff allocation in the core provides participation incentives of players into the grand coalition and represents a very strong type of stability that is not threatened by the defection of sub-coalitions. Hence, in our problem, it will be ideal if perfect coordinating prices can also guarantee a core payoff among the players. This is the central issue we study in the rest of this subsection. In §3.3.1, we first present a dual-based pricing strategy to calculate a set of perfect coordinating prices under which a core payoff of the MCF game is also achieved given any combined network. In §3.3.2, we further analyze the relationship between the players' payoffs under any perfect coordinating prices, and the core of the MCF game. We show that in cases where there exist multiple owners of the capacity on a same edge, perfect coordinating prices do not generally guarantee a core allocation. This indicates a potential misalignment between the optimality and stability of a decentralized network

under the capacity exchange mechanism in the presence of multiple owners of the same resource. We show that such diseconomies of multiple ownership is mainly due to the asymmetry in the capacity levels among the multiple capacity owners on an edge relative to their commodity demand.

3.3.1 Dual Optimal Solutions as Capacity Exchange Prices.

In the following, we present a pricing scheme based on the dual optimal solution associated with the capacity constraint (96) on each edge e in the centralized problem (C), which we denote as α_e^*. The dual optimal solution α_e^* has a nice economic interpretation: It is the marginal value of the capacity on edge e in improving the maximum total shipping revenue over the combined network, and thus can be regarded as a benchmark for the pricing of the resource. In literature, they are termed as *market prices* in [46] and *equilibrium prices* in [123] to emphasize on their economic interpretation. In the cooperative game theory framework, it is a well-known result (e.g. in Owen 1975 [123]) that a core allocation of the maximum total revenue within a combined network can be computed using α_e^* and the optimal dual solution $\beta_{(o,d,i)}^*$ with respect to the demand constraint (97) in the centralized problem. Such an allocation is called a *dual allocation*, which we denote by $\{A_d^i\}$ and is computed as follows.

$$A_d^i = \sum_{e \in E} \alpha_e^* \cdot c_e \gamma_e^i + \sum_{(o,d,i) \in D^i} \beta_{(o,d,i)}^* \cdot d_{(o,d,i)} \quad \forall i \in N. \tag{111}$$

In this section, we show that $\{\alpha_e^*\}$ can be effective as capacity exchange prices to coordinate decentralized combined networks. We also call such prices *dual prices* in the subsequent discussions.

Theorem 11. *Given any combined network G, the capacity exchange prices $\{\alpha_e^*\}$ are perfect coordinating and give rise to a dual allocation as computed in (111).*

An important implication of Theorem 11 is that $\{\alpha_e^*\}$ are by nature *competitive prices*, if we regard the commodity routing process as a production economy with edge

capacity and commodity demand as resources, the routings as production plans, and the shipping revenues as products. According to Debreu and Scarf [37], competitive prices are defined as resource prices under which there exists a production plan and an allocation of the resulting products among participants that maximize the social welfare as well as satisfy individual preferences. This allocation is also proven to be in the core of the economy under mild conditions in the same paper. Such a relationship between the dual optimal solutions and competitive prices is also implied from the study by Owen [123]; yet the paper focuses on characterizing the core payoffs derived from the dual-based prices but does not consider how dual-based prices can induce a social optimal production plan that also satisfies individual preferences.

We also point out that there are several notable differences between Theorem 11 and the properties of competitive prices. In particular, first, in the capacity exchange mechanism, only edge capacity can be exchanged but not the commodity demand. Hence, Theorem 11 actually indicates that even by a *restricted* market mechanism where certain resources are not to be shared, we can still achieve efficiency of the economy and individual satisfaction using the dual optimal solutions $\{\alpha_e^*\}$. Second, we consider a decentralized setting where the overall routing, or the production plan, is composed by individual choices of the routes for their own commodities. Such choices are assumed to be made simultaneously and independently. However, the notion of competitive prices is not defined based on such a decentralized environment, and how a production plan that maximizes the social welfare can be constructed is not explicitly indicated. Third, in our setting, the allocation of the shipping revenue is achieved as a natural result of the market exchanges of capacities and individual commodity routing within the combined network. In summary, Theorem 11 not only indicates the existence of a collectively efficient routing (production plan) and a core payoff allocation under the prices $\{\alpha_e^*\}$, but also shows how they can be constructed in a practical decentralized environment under natural resource trading rules without

direct central intervention.

One main contribution of Theorem 11 is to provide insights regarding the general relationship between perfect coordinating prices, core payoff allocations, and dual optimal solutions $\{\alpha_e^*\}$ in the context of coordinating decentralized resource sharing in combined network. Specifically, given a network G as modeled in §3.2, we represent the set of individual payoffs that are derived under perfect coordinating prices as $\mathcal{P}$, i.e., $\mathcal{P} = \{\{x_{cost}^i(f^*)\} : \{cost_e\}$ is perfect coordinating$\}$. Let $\mathcal{C}$ and $\mathcal{D}$ denote the set of core payoffs and dual allocations. We conclude that $\mathcal{D} \subset \mathcal{P}$ (Theorem 11) and $\mathcal{D} \subset \mathcal{C}$ [123]. A natural question is whether $\mathcal{P} \subset \mathcal{C}$, i.e., whether a core payoff can be guaranteed under any perfect coordinating prices. We analyze this problem in the following subsection.

3.3.2 Individual Payoffs under Perfect Coordinating Prices

In the next proposition, we present a characterization of the general relationship between the stand-alone value of a sub-coalition S $v(S)$ and the payoff to S in the combined network operated under a set of perfect coordinating prices. Let α_e^{i*} and $\beta_{(o,d,i)}^{i*}$ denote the dual optimal solutions associated with the capacity and demand constraints respectively in the individual problem (P_{cost}^i) in (100) for each player i.

Proposition 9. *Given any combined network G operated under a set of perfect coordinating prices $\{cost_e\}$, $\forall$ sub-coalition $S \subsetneq N$, the payoff to S, $\sum_{i \in S} x_{cost}^i(f^*)$, satisfies*

$$\sum_{i \in S} x_{cost}^i(f^*) - v(S) \geq \sum_{e \in E} c_e(1 - \gamma_e^S) \cdot \sum_{i \in S} \alpha_e^{i*} + \sum_{(o,d,i) \notin D^S} d_{(o,d,i)} \cdot \sum_{i \in S} \beta_{(o,d,i)}^{i*} - \sum_{e \in E} cost_e(1 - \gamma_e^S) \cdot c_e \gamma_e^S.$$
(112)

The above formula (112) provides a lower bound on the difference between the payoffs to S operating inside a combined network and independently. Note that such a bound may be negative, hence $\sum_{i \in S} x_{cost}^i(f^*) \geq v(S)$ is not guaranteed. In

179
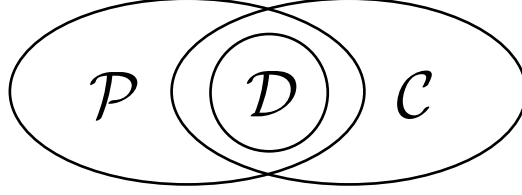
Figure 23: The general relation between the set of payoffs induced by perfect coordinating prices $\mathcal{P}$, the set of dual allocations $\mathcal{D}$, and the set of core payoffs $\mathcal{C}$.

fact, we can construct small network examples (e.g., Example 3) where the bound is negative and tight, indicating that some players' payoffs are strictly undermined participating in a combined network. Hence, it can be concluded that given a general combined network, perfect coordinating prices do not guarantee a core payoff of the corresponding MCF game. In other words, in general, $\mathcal{P} \not\subset \mathcal{C}$ (Proposition 9) and we can also find simple examples where $\mathcal{C} \not\subset \mathcal{P}$. Hence, we can summarize the general relationship between these three sets given any combined network in Figure 23.

Example 3. A network has two nodes o and d and an edge $e = (o, d)$. Player I owns $c_e \gamma_e^1$ units of the capacity, but his demand exceeds the total capacity, i.e., $d_{(o,d,1)} > c_e$. Player II owns no shipping demand but only $c_e(1 - \gamma_e^1)$ units of the edge resources. The unit shipping revenue is $r_{(o,d,1)} = 1$. The social optimal routing of this network instance is obviously to ship c_e units of the commodity owned by Player I. Consider the objective functions in the problems (P_{cost}^1) and (P_{cost}^2), i.e., $[1 - cost_e(1 - \gamma_e^1)]f_{(d,o,1)}^{(o,d,1)}$ and $cost_e(1 - \gamma_e^1)f_{(d,o,1)}^{(o,d,1)}$. We conclude that the exchange price on edge e, $cost_e$, is perfect coordinating if and only if $0 \leq cost_e \leq \frac{1}{1-\gamma_e^1}$.

Now calculate the payoff to Player I under an arbitrary inverse feasible exchange price $cost_e$.

$$x^1 = c_e[1 - \bar{cost}(1 - \gamma_e^1)] \tag{113}$$

$$= v(1) + c_e(1 - \gamma_e^1)(1 - cost_e). \tag{114}$$

Since in this example $\alpha_e^{*1} = 1 - cost_e(1 - \gamma_e^1)$, (113)-(114) indicates that the inequality

(112) holds tight for player I, and thus $x^1 < v(1)$ if $cost_e > 1$. Since $0 < \gamma_e^1 < 1$, $1 < \frac{1}{1-\gamma_e^1}$. Hence, a perfect coordinating price in the nonempty interval $(1, \frac{1}{1-\gamma_e^1}]$ leads to less profits for player I than his playing alone.

Figure 24 illustrates how player I accumulates his payoff when a perfect coordinating exchange price $cost_e$ strictly greater than 1 is adopted. The line segment ob represents the situation in which all resources in use are exchanged at a price of $cost_e$ and Player I earns a unit profit of $1 - cost_e(1 - \gamma_e^1)$ from routing his own commodity demand. Because the exchange price is perfect coordinating, $1 - cost_e(1 - \gamma_e^1) \geq 0$, indicating a nonnegative slope of ob. The situation becomes different if we consider the other way that player I accumulates his profit, as indicated by the right-hand-side of inequality (112), or equivalently by (114) in this example. Specifically, player I first operates alone to use up his own resources with a unit profit 1, which is the slope of the line segment oa. Then he joins the coalition and ships more using the capacity owned by player II. However, since player I needs to pay player II even for using his own capacity and in this example receives no income from capacity exchanges from player II, the unit profit he actually earns according to (114) after joining the grand coalition, which is the slope of ab, is $1 - cost_e$ which is negative when $cost_e > 1$. Hence the payoff to player I is undermined in the coalition with player II even if the total shipping revenue can be strictly increased.

Example 3 provides us with some insights into how the capacity exchange mechanism affects one's payoff especially under perfect coordinating prices, especially when there are multiple owners of the capacity on the same edge. Based on these insights, we analyze the bound provided in formula (112) in general cases as follows. The main observation is that the bound characterizes the potential additional profit and cost that a sub-coalition S would expect from joining the resource-sharing system in a combined network. First, consider the term $\sum_{i \in S} \alpha_e^{*i} + \sum_{i \in S} \beta_{(o,d,i)}^{*i}$. According to the sensitivity of a linear program to the changes in its right-hand-side parameters, the

181
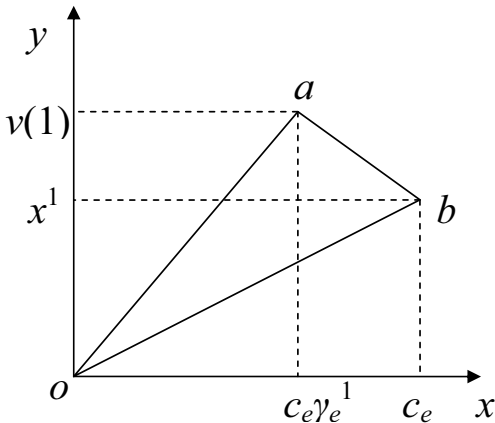
Figure 24: Payoff accumulation of Player I in Example 3, where x denotes the shipping amount as well as the resources used on e, and y denotes the profit earned.

individual payoff $x^i_{cost}(f^*)$ is a piecewise linear concave non-decreasing function of the amount of capacity on each edge or of the demand for each commodity. This indicates a non-increasing marginal profit to player i as the capacity or demand increases unit by unit. Hence, the value $\sum_{i \in S} \alpha^{i*}_e$ is a lower bound to the sum of the marginal profits to the members in S benefiting from each unit of the capacity on e owned by players outside S. The value $\sum_{i \in S} \beta^{i*}_{(o,d,i)}$ plays a similar role with regard to every commodity $(o, d, i) \notin D^S$. Hence, $\sum_{i \in S} \alpha^{*i}_e + \sum_{i \in S} \beta^{*i}_{(o,d,i)}$ essentially computes the minimum benefit that S can obtain by exploiting the synergies from resource sharing participating in a combined network.

The last term of the bound, i.e., $-\sum_{e \in E} cost_e(1 - \gamma^S_e) \cdot c_e \gamma^S_e$, indicates that the coalition S may need to pay extra cost on edges where $0 < \gamma^S_e < 1$ when participating in a combined network under the capacity exchange mechanism. To see why this happens, note that the stand-alone value of S, $v(S)$, is achieved under the implicit assumption that the capacity of its members is shared for free within themselves when S operates independently. However, it is no longer the case inside the combined network where a different capacity access rule is adopted. Specifically, the capacity exchange mechanism requires every participant to pay the other capacity owners for every unit of his own commodity routed through each edge e. For S, it implies an

additional capacity cost of $cost_e(1 - \gamma_e^S)$ per unit demand of its members routed on e. In cases where the total amount of flow routed by the members of S is no larger than their own capacity, i.e., $c_e\gamma_e^S$, on each edge e, S can choose to operates independently and completely avoid the capacity exchange prices. This provides an intuitive explanation for the extra cost that S will incur joining a combined network with capacity exchanges, and why this cost is upper bounded by $\sum_{e \in E} cost_e(1 - \gamma_e^S) \cdot c_e\gamma_e^S$ in formula (112).

In conclusion, Proposition 9 indicates that while one can benefit from the availability of a larger pool of resources and routing demand when participating in a combined network, he may also incur additional costs entering a capacity exchange market and collaborating with others who own the same capacity as he does. To understand the intuition behind such a cost, note that the purpose of capacity exchange prices is to benefit capacity owners from the shipment of others' commodities and thus motivate them to share resources. Hence, everyone who participates under such a mechanism may potentially yield part of the shipping revenue that she could achieve alone to others in the form of capacity exchange prices. Note that such a situation can be prevented for a sub-coalition S if on each edge, γ_e^S equals either 1 or 0, i.e., S either owns all the capacity on an edge or none, as in this case, no players outside S can claim a capacity payment if all commodities in S are routed using only edges where its members own capacity. This argument is substantiated by inequality (112) as the third term on the right-hand-side of the formula becomes zero when $\gamma_e^S = 0$ or 1. Moreover, we can observe that in this case, (112) implies $\sum_{i \in S} x_{cost}^i(f^*) \geq v(S)$ as α_e^{i*} and $\beta_{(o,d,i)}^{i*}$ are both nonnegative. This indicates that S is guaranteed to receive no less profit inside the combined network than operating by itself and thus has incentives to share resources with others, as long as a set of perfect coordinating prices is adopted. The above argument holds for all sub-coalitions when

there is a unique capacity owner on every edge in the network, thus perfect coordinating prices guarantees are guaranteed to yields a *core payoff* $\{x^i_{cost}(f^*)\}$. The same result (i.e., obtaining a core payoff under the unique ownership condition using a capacity exchange mechanism with perfect coordinating prices) is also reported in [2] as a stand-alone theorem using a different proof approach, while here we derive it as a corollary to a more general result of Proposition 9.

While the above situation is ideal, i.e., any perfect coordinating prices give rise to a core payoff, the unique ownership condition is usually not satisfied in practice. However, when there exist multiple owners of capacity on the same edge, the fact that the perfect coordinating prices do not generally guarantee individual payoffs in the core of the corresponding MCF game indicates a potential misalignment between the optimality and stability of a decentralized network under the capacity exchange mechanism. Our next result indicates that the impact of such potential diseconomies of multiple ownerships on a sub-coalition's payoff depends on the asymmetry in the capacity levels among the multiple capacity owners relative to their commodity demand. To present the result, we denote by $M_S = \{e \in E : 0 < \gamma^S_e < 1\}$ the set of edges where S has partial capacity. Let f_S be a feasible routing for sub-coalition S, i.e., a feasible solution to the problem (C^S), and let f^*_S be the optimal one.

Proposition 10. *Given any combined network G operated under a set of perfect coordinating prices $\{cost_e\}$, the payoff to a sub-coalition $S \subsetneq N$ is guaranteed to satisfy $\sum_{i \in S} x^i_{cost}(f^*) \geq v(S)$, if there exists a feasible routing $f_{N \setminus S}$ to the problem $(C^{N \setminus S})$ such that*

$$\frac{\sum_{(o,d,i) \in D^S} (f^*_S)^{(o,d,i)}_e}{c_e \gamma^S_e} \leq \frac{\sum_{(o,d,i) \notin D^S} (f_{N \setminus S})^{(o,d,i)}_e}{c_e (1 - \gamma^S_e)} \quad \forall e \in M^S. \tag{115}$$

The proof of Proposition 10 is intuitive: Consider a feasible routing $\bar{f}$ over G under which the commodities owned within S and $N \setminus S$ are routed according to f^*_S and $f_{N \setminus S}$ respectively. Then by (115) (after cross-multiplication), the cost that S pays

for capacity under $\bar{f}$ is guaranteed to be no larger than the amount that it receives from others on each edge in M^S. This indicates that given routing $\bar{f}$ under which the potential synergies from the scope economies is not yet exploited, the payoff to S is already no smaller than $v(S)$. The fact that a set of perfect coordinating prices is applied in the combined network ensures an even higher payoff to S under the social optimal routing f^* than $\bar{f}$ and this completes the proof.

Although formula (115) is derived based on a specific routing for the commodities owned within S and $N \setminus S$ respectively, we can gain some general insights from Proposition 10 into the factors that impact individual payoffs under the capacity exchange mechanism. Specifically, we can interpret the ratio of the left-hand-side in formula (115) as the optimal internal capacity utilization rate on e within S when it operates alone. Similarly, the term on the right-hand-side represents the internal capacity utilization rate within its counterpart $N \setminus S$ under some feasible routing for $N \setminus S$. Hence, according to Proposition 10, perfect coordinating prices can guarantee better individual profit within the combined network for sub-coalitions whose optimal internal capacity utilization rates are sufficiently low. To see why this is the case, note that a low internal capacity utilization rate indicates a large capacity ownership within the sub-coalition compared to the routing demand of its members; thus the participation of these players in the combined network increases the overall capacity availability for commodity routing. The nature of capacity exchange prices is to reward players for their capacity contribution to the combined network, thus the result of Proposition 10 follows. This analysis also indicates that under such a mechanism, the players who have higher internal capacity utilization rate and essentially contribute routing demand are not rewarded enough and thus may not receive a fair share of the total shipping revenue achieved by the joint effort of all players involved in the combined network. However our next result, which we derive as a corollary to Proposition 10, indicates that such an unfavorable situation will not happen in combined networks

under a homogeneous setting where all players have high internal capacity utilization rates.

Corollary 2. *Given any combined network G operated under a set of perfect coordinating prices $\{cost_e\}$, the individual payoff $\{x^i_{cost}(f^*)\}$ is guaranteed to be a core payoff, if for each user i, there exists a feasible routing f_i to the problem (C^i) such that all capacity owned by i on edges in M_i is saturated, i.e., $\sum_{(o,d,i) \in D^i} (f_i)_e^{(o,d,i)} = c_e \gamma_e^i$ $\forall e \in M_i$.*

In words, the condition in the above corollary indicates that all players can achieve the highest internal capacity utilization rate of 1 on each edge where multiple capacity owners exist. This implies that all players are homogeneously under-capacitated. Under such a situation, Corollary 2 shows that any perfect coordinating prices can ensure a core payoff. This result further implies that the capacity exchange mechanism can be biased towards allocating better payoffs to players whose internal capacity utilization is low compared to others and whose participation enhances the capacity availability in the combined network.

3.3.3 Appendix

Proof of Theorem 11. Let $[\pi^*, \alpha^*, \beta^*]$ be the optimal dual solution of the centralized problem (C) associated with the constraints $(95)-(97)$ respectively. It must satisfy the dual constraints with respect to (C) $(116)-(118)$, and the complementary

186

slackness conditions together with the corresponding f^* (119)$-$(122).

$$\pi_v^{*(o,d,i)} - \pi_u^{*(o,d,i)} + \alpha_e^* \geq 0 \quad \forall (o,d,i) \in D \quad \forall e \in E \tag{116}$$

$$\pi_o^{*(o,d,i)} - \pi_d^{*(o,d,i)} + \beta_{(o,d,i)}^* \geq r_{(o,d,i)} \quad \forall (o,d,i) \in D \tag{117}$$

$$nonnegativity\ \ constraints \tag{118}$$

$$\pi_v^{*(o,d,i)} - \pi_u^{*(o,d,i)} + \alpha_e^* = 0 \quad \forall (o,d,i) \in D \quad \forall e \in E \text{ if } f_e^{*(o,d,i)} > 0 \tag{119}$$

$$\pi_o^{*(o,d,i)} - \pi_d^{*(o,d,i)} + \beta_{(o,d,i)}^* = r_{(o,d,i)} \quad \forall (o,d,i) \in D \text{ if } f_{(d,o,i)}^{*(o,d,i)} > 0 \tag{120}$$

$$\alpha_e^* = 0 \quad \forall e \in E \text{ if } \sum_{(o,d,i) \in D} f_e^{*(o,d,i)} < c_e \tag{121}$$

$$\beta_{(o,d,i)}^* = 0 \quad \forall (o,d,i) \in D \text{ if } f_{(d,o,i)}^{*(o,d,i)} < d_{(o,d,i)} \,. \tag{122}$$

Hence, it is easy to verify that when the capacity exchange prices $cost_e = \alpha_e^* \ \forall e \in E$, for each player $i \in N$, the vector $[\pi^i, \alpha^i, \beta^i]$ defined by (123)$-$(126) is a feasible dual solution with respect to player i's individual problem $(P_{\alpha*}^i)$, i.e., satisfying the dual constraints (137)$-$(141), due to inequalities (116)$-$(118).

$$\pi_v^{i(o,d,i)} \ \doteq \ \pi_v^{*(o,d,i)} \quad \forall (o,d,i) \in D^i \quad \forall v \in V \tag{123}$$

$$\alpha_e^i \ \doteq \ \alpha_e^* \gamma_e^i \quad \forall e \in E \tag{124}$$

$$\beta_{(o,d,i)}^i \ \doteq \ \beta_{(o,d,i)}^* \quad \forall (o,d,i) \in D^i \tag{125}$$

$$\beta_{(o,d,j)}^i \ \doteq \ \pi_v^{i(o,d,j)} = 0 \quad \forall (o,d,j) \notin D^i \quad \forall v \in V \,. \tag{126}$$

Moreover, due to equations (119)$-$(122), the above solution also satisfies the complementary slackness conditions together with f^* for $(P_{\alpha*}^i)$, i.e.,

$$\pi_v^{i(o,d,i)} - \pi_u^{i(o,d,i)} + \alpha_e^i = -(1 - \gamma_e^i)\alpha_e^* \quad \forall (o,d,i) \in D^i \quad \forall e \in E \text{ if } f_e^{*(o,d,i)} > 0 \tag{127}$$

$$\pi_v^{i(o,d,j)} - \pi_u^{i(o,d,j)} + \alpha_e^i = \gamma_e^i \alpha_e^* \quad \forall (o,d,j) \notin D^i \quad \forall e \in E \text{ if } f_e^{*(o,d,j)} > 0 \tag{128}$$

$$\pi_o^{i(o,d,i)} - \pi_d^{i(o,d,i)} + \beta_{(o,d,i)}^i = r_{(o,d,i)} \quad \forall (o,d,i) \in D^i \text{ if } f_{(d,o,i)}^{*(o,d,i)} > 0 \tag{129}$$

$$\pi_o^{i(o,d,j)} - \pi_d^{i(o,d,j)} + \beta_{(o,d,j)}^i = 0 \quad \forall (o,d,j) \notin D^i \text{ if } f_{(d,o,j)}^{*(o,d,j)} > 0 \tag{130}$$

$$\alpha_e^i = 0 \quad \forall e \in E \text{ if } \sum_{(o,d,i) \in D} f_e^{*(o,d,i)} < c_e \tag{131}$$

$$\beta_{(o,d,i)}^i = 0 \quad \forall (o,d,i) \in D \text{ if } f_{(d,o,i)}^{*(o,d,i)} < d_{(o,d,i)} \,. \tag{132}$$

Hence, by duality theory, we conclude that f^* and $[\pi^i, \alpha^i, \beta^i]$ defined by (123)$-$(126) are the primal and dual optimal solutions, respectively, to the individual problem $(P^i_{\alpha^*})$. Hence, by definition, the capacity exchange prices $\{\alpha^*_e\}$ are perfect coordinating. Moreover, by strong duality, the payoff to each player i under prices $\{\alpha^*_e\}$, $x^i_{\alpha^*}(f^*)$, equals $\sum_{e \in E} c_e \alpha^i_e + \sum_{(o,d,i) \in D} d_{(o,d,i)} \beta^i_{(o,d,i)}$, which can be shown to be identical to the dual allocation as computed in (111). $\qquad\square$

Proof of Proposition 9. Formulate as follows the dual program of the problem (C^S), i.e., the minimum cost problem from which the value of a sub-coalition $v(S)$ is computed.

$$(D^S): \quad \min \quad Z_S(\alpha, \beta) = \sum_{e \in E} \gamma^S_e c_e \alpha_e + \sum_{(o,d,i) \in D^S} \beta_{(o,d,i)} d_{(o,d,i)} \tag{133}$$

$$s.t. \quad \pi^{(o,d,i)}_v - \pi^{(o,d,i)}_u + \alpha_e \geq 0 \quad \forall (o,d,i) \in D^S \quad \forall e \in E \tag{134}$$

$$\pi^{(o,d,i)}_o - \pi^{(o,d,i)}_d + \beta_{(o,d,i)} \geq r_{(o,d,i)} \quad \forall (o,d,i) \in D^S \tag{135}$$

$$\alpha_e \geq 0; \quad \beta_{(o,d,i)} \geq 0; \quad \pi^{(o,d,i)}_v \geq 0 \quad \forall e \in E \quad \forall (o,d,i) \in D \quad \forall v \in V. \tag{136}$$

Consider the optimal dual solution to the individual problem (P^i_{cost}), i.e., $[\pi^{i*}, \alpha^{i*}, \beta^{i*}]$, which must satisfy the dual constraints with respect to (P^i_{cost}), i.e.,

$$\pi^{i(o,d,i)}_v - \pi^{i(o,d,i)}_u + \alpha^i_e \geq -(1 - \gamma^i_e) cost_e \quad \forall (o,d,i) \in D^i \quad \forall e \in E \tag{137}$$

$$\pi^{i(o,d,j)}_v - \pi^{i(o,d,j)}_u + \alpha^i_e \geq \gamma^i_e cost_e \quad \forall (o,d,j) \notin D^i \quad \forall e \in E \tag{138}$$

$$\pi^{i(o,d,i)}_o - \pi^{i(o,d,i)}_d + \beta^i_{(o,d,i)} \geq r_{(o,d,i)} \quad \forall (o,d,i) \in D^i \tag{139}$$

$$\pi^{i(o,d,j)}_o - \pi^{i(o,d,j)}_d + \beta^i_{(o,d,j)} \geq 0 \quad \forall (o,d,j) \notin D^i \tag{140}$$

$$nonnegativity \ \ constraints. \tag{141}$$

Hence, by adding the constraints (137) and (138) over all $i \in S$ with respect to each commodity and each edge, and adding (139) and (140) with respect to each commodity, we see that the vector $[\sum_{i \in S} \pi^{i*}, \sum_{i \in E} \alpha^{i*} + (1 - \gamma^S_e) cost_e, \sum_{i \in S} \beta^{i*}]$ is feasible to the problem of (D^S); hence it gives rise to an objective value no smaller

than the optimal value of (D^S), which equals $v(S)$ by strong duality. Mathematically,

$$Z_S(\sum_{i \in S} \alpha^{i*} + (1 - \gamma_e^S)cost_e, \sum_{i \in S} \beta^{i*}) \geq v(S). \tag{142}$$

Since $[\pi^{i*}, \alpha^{i*}, \beta^{i*}]$ is the optimal dual solution to the individual problem (P_{cost}^i), by strong duality we know that

$$\sum_{i \in S} x_{cost}^i(f^*) = \sum_{e \in E} \sum_{i \in S} \alpha_e^{i*} c_e + \sum_{(o,d,i) \in D} \sum_{i \in S} \beta_{(o,d,i)}^{i*} d_{(o,d,i)}$$
$$= Z_S(\sum_{i \in S} \alpha^{i*} + (1 - \gamma_e^S)cost_e, \sum_{i \in S} \beta^{i*})$$
$$+ \sum_{e \in E} c_e(1 - \gamma_e^S)(\sum_{i \in S} \alpha_e^{i*} - cost_e \gamma_e^S) + \sum_{(o,d,i) \notin D^S} \sum_{i \in S} \beta_{(o,d,i)}^{i*} d_{(o,d,i)} \tag{143}$$

where the second equality is obtained after some algebraic manipulations. Combining formula (142) and (143), we can derive the following relationship between $\sum_{i \in S} x_{cost}^i(f^*)$ and $v(S)$,

$$\sum_{i \in S} x_{cost}^i(f^*) \geq v(S) + \sum_{e \in E} c_e(1 - \gamma_e^S) \sum_{i \in S} \alpha_e^{i*} + \sum_{(o,d,i) \notin D^S} \sum_{i \in S} \beta_{(o,d,i)}^{i*} d_{(o,d,i)} - \sum_{e \in E} cost_e(1 - \gamma_e^S) \cdot c_e \gamma_e^S.$$
$$\tag{144}$$

By rearranging terms in the above inequality we obtained inequality (112) in Proposition 9. $\square$

Proof of Proposition 10. Consider a feasible routing $f_{N \setminus S}$ for $N \setminus S$ to operate alone that satisfies condition (115). Define a routing $f_S^* \oplus f_{N \setminus S}$ over the entire collective network G, such that all commodities owned by members in the sub-coalition S are routed according to f_S^* while the commodities owned within $N \setminus S$ are routed based on $f_{N \setminus S}$ (note that this routing is referred to as $\bar{f}$ in the paper). Given a set of capacity exchange prices $\{cost_e\}$, the payoff to S under such a routing can be computed as in (145).

$$\sum_{i \in S} x_{cost}^i(f_S^* \oplus f_{N \setminus S})$$
$$= v(S) - \sum_{e \in M^S} cost_e(1 - \gamma_e^S) \sum_{(o,d,i) \in D^S} (f_S^*)_e^{(o,d,i)} + \sum_{e \in E} cost_e \gamma_e^S \sum_{(o,d,i) \notin D^S} (f_{N \setminus S})_e^{(o,d,i)}. \tag{145}$$

According to (115), on each edge e in M^S we have

$$cost_e(1 - \gamma_e^S) \sum_{(o,d,i) \in D^S} (f_S^*)_e^{(o,d,i)} \leq cost_e \gamma_e^S \sum_{(o,d,i) \notin D^S} (f_{N \setminus S})_e^{(o,d,i)} \qquad (146)$$

and this leads to $\sum_{i \in S} x_{cost}^i(f_S^* \oplus f_{N \setminus S}) \geq v(S)$. The fact that $\{cost_e\}$ is perfect coordinating indicates that $\sum_{i \in S} x_{cost}^i(f^*) \geq \sum_{i \in S} x_{cost}^i(f_S^* \oplus f_{N \setminus S}) \geq v(S)$ and this completes the proof. $\qquad \square$

Proof of Corollary 2. Consider an arbitrary sub-coalition S. For its counterpart $N \setminus S$, there exists a feasible routing $f_{N \setminus S}$ defined as $\sum_{i \in N \setminus S}^{\oplus} f_i$ under which the commodities owned by player i is routed according to the routing f_i. Under the given condition that $\sum_{(o,d,i) \in D^i} (f_i)_e^{(o,d,i)} = c_e \gamma_e^i \ \forall e \in M^i$, we know that $\sum_{(o,d,i) \notin D^S} (f_{N \setminus S})_e^{(o,d,i)} = c_e(1 - \gamma_e^S)$ is guaranteed on all edges in M^S. This indicates that the RHS of inequality (115) is 1, which is the highest capacity utilization rate. Hence by Proposition 10 we know that $\sum_{i \in S} x_{cost}^i(f^*) \geq v(S)$. According to the definition of a core payoff, we conclude that $\{x_{cost}^i(f^*)\}$ is a core payoff. $\qquad \square$

3.4 Avoidance of Free Riders under a Capacity Exchange Mechanism

In the last section, Theorem 11 indicates that when the optimal dual solutions $\{\alpha_e^*\}$ are applied as the capacity exchange prices on the corresponding edges, a decentralized combined network can achieve its maximum efficiency in terms of the total shipping revenue, and all its participants can benefit from the synergies of capacity sharing. While such a situation is highly desirable, these dual solutions can have serious practical drawbacks as exchange prices. One of them is that, due to the complementary slackness of linear programs, $\alpha_e^* = 0$ on each edge e where the capacity is not fully used under the social optimal routing. Yet zero prices are usually unacceptable for privately-owned resources in practice, and if used, can discourage the capacity owners from sharing capacity. Moreover, the existence of zero exchange prices can

lead to *free riders* who gain positive shipping revenues by using others' capacity for free. The existence of free riders can cause perception of unfairness of the mechanism and becomes a significant barrier to its implementation.

The free rider problem is a classic problem in economics (e.g., in [74]) and has recently received attention in the algorithmic game theory literature. For example, Immorlica et al. [82] study the issue in the context of cost sharing mechanism design where the criterion of no free riders is found to be incompatible with those of budget-balancedness and group strategyproofness. While our study to the free rider problem also indicates incompatibility as capacity exchange prices that both avoid free riders and guarantee a social optimal routing do not generally exist, our result applies to capacity exchange mechanism design that manages selfish routings without direct central intervention in either network operations or payoff allocation. [87] show the existence of strictly positive competitive prices in digital goods pricing under the *non-satiation* assumption of individual utilities. However, such an assumption implies unlimited commodity demand, which is typically relevant to the digital market, yet is unrealistic under the setting of our problem.

In this section, we study the design of capacity exchange prices to coordinate decentralized combined networks under the requirement that no free riders should exist in the system. As a first study, we focus our discussion in cases where participation of all players in the combined network is enforced, for example by a binding agreement, since we find the problem challenging even under such a simplified situation. In particular, the requirement to avoid free riders may be incompatible with the goal to induce the decentralized individual routings towards the social optimal one within a combined network, as we show in certain cases (e.g., Example 4), free riders always exist under any perfect coordinating prices.

Example 4. Consider the network shown in Figure 25. with two network players a and b. Player a owns two thirds of the capacity on the edge AB and all capacity
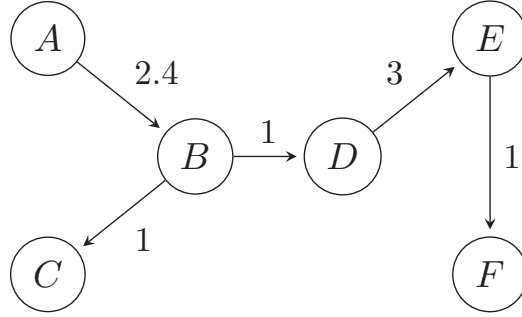
Figure 25: The combined network in Example 4. The numbers on the edges represent capacity levels.

on DE; player b owns the rest of the network capacity. There are four commodities, (A, F, a), (D, F, a), (B, E, a) and (A, C, b), each with 1 unit of demand and a shipping revenue of 1. The unique social optimal routing is to route 1 unit of (D, F, a), (B, E, a) and (A, C, b). It can be observed that if the exchange price on edge AB is set to be $cost_{AB} > 0$, the individual optimal routing for player b is to route 1 unit of commodity (A, F, a) and (A, C, b), which is different from the social optimal one. Hence all perfect coordinating prices must satisfy $cost_{AB} = 0$, which directly results in player a's providing free capacity for the shipment of player b's commodity (A, C, b), and thus b being a free rider.

In the light of the observation made in Example 4, our discussion in this section is centered around the following two studies. First, we analyze the factors that cause the non-existence of perfect coordinating prices that can also avoid free riders. Second, as a solution to this problem, we design and analyze capacity exchange prices that partially coordinate the behavior of individual players in the combined network, and under which no free riders exist. We will use strict positiveness of prices as the criterion for avoiding free riders throughout the rest of this section.

3.4.1 Existence of Strictly Positive Perfect Coordinating Prices

We first point out that according to Theorem 11 and strict complementary slackness of linear programs, at least one set of strictly positive perfect coordinating prices can be found in networks that are under-capacitated on every edge under the social optimal routing. Basically, we set the prices equal to the corresponding set of strictly positive dual prices. Hence, our discussion in this section on the existence of such prices focuses on the general setting where there can be excess capacity under the social optimal routing.

We start by analyzing Example 4 to gain some insights towards the cause of the incompatibility between perfect coordinating and free rider avoidance under the capacity exchange mechanism. In Example 4, when a unit of commodity (A, F, a) is shipped, player b can earn $\frac{1}{3}cost_{AB} + cost_{BD} + cost_{EF}$ from player a via capacity exchanges; whereas she only earns $cost_{BD} + cost_{EF}$ if commodities (D, F, a) and (B, E, a) are shipped instead. Hence, player b will prefer to route commodity (A, F, a) to (D, F, a) and (B, E, a) whenever $cost_{AB} > 0$.

The above reasoning is simple but it points to essential factors that undermine the existence of strictly positive perfect coordinating prices. First, compare the commodities (A, F, a) and (D, F, a). They create the same unit shipping revenue while (A, F, a) requires one more unit of capacity on edge AB and BD to ship. From the system efficiency perspective, commodity (D, F, a) is preferred as it uses less capacity to deliver and leaves space for the routing of commodity (B, E, a), which generates an additional unit of revenue. However, for player b as a capacity seller, the shipment of commodity (A, F, a) can benefit him more from capacity exchanges when $cost_{AB} > 0$, and thus is preferred under her own individual optimal routing. This indicates that strictly positive prices can deviate individual routings from the collective optimal one if certain inefficient routes exist, where more capacity is used to deliver commodities

with lower unit revenues compared to other routes in the network. The main underlying factor is that while the collective efficiency is measured based on the total shipping revenue, an individual's payoff is partially determined by the amount of his own capacity that can be sold under the capacity exchange mechanism. Second, notice that in this example, routing 1 unit of both commodities (D, F, a) and (B, E, a) requires more capacity on edge DE compared to when only (A, F, a) is shipped. However, player b does not own any capacity on edge DE, thus cannot benefit from capacity exchanges on that edge in either situation. This indicates that one's capacity ownership levels can bias his perception of the profitability of the edges and thus impact the individual optimal routing. In general, such individual bias can be hard to coordinate with positive prices especially when it is widely varied among the players.

We formalize the above discussion by showing that when the impact of both factors, i.e., existence of inefficient routes and heterogeneous individual ownership conditions, is eliminated, at least one set of perfect coordinating prices is guaranteed to be strictly positive. Before presenting the result, we first introduce the following definition that characterize the inefficient routes mentioned in the previous paragraph. Let $P_{(o,d,i)}$ be the set of all paths that go from the origin node o to the destination node d for commodity (o, d, i), and let $P = \bigcup_{(o,d,i) \in D} P_{(o,d,i)}$.

Definition 5. *Given a multicommodity network $G = (V, E)$ and the unit shipping revenues $\{r_{(o,d,i)}\}$ of the commodities, a set of paths $P_1 = \{p_t\} \subset P$ in G is dominated by another set of paths $P_2 = \{p_s\} \subset P$ if there exist constants $\{\lambda_t > 0\}$ and $\{\mu_s > 0\}$ such that*

$$\sum_{(o,d,i) \in D} \sum_{t: p_t \in P_1 \cap P_{(o,d,i)}} \lambda_t r_{(o,d,i)} \leq \sum_{(o,d,i) \in D} \sum_{s: p_s \in P_2 \cap P_{(o,d,i)}} \mu_s r_{(o,d,i)} \tag{147}$$

$$\sum_{t: p_t \in P_1} \lambda_t \chi_{\{e \in p_t\}} \geq \sum_{s: p_s \in P_2} \mu_s \chi_{\{e \in p_s\}} \quad \forall e \in E \tag{148}$$

$$\sum_{t: p_t \in P_1} \lambda_t \chi_{\{e \in p_t\}} > \sum_{s: p_s \in P_2} \mu_s \chi_{\{e \in p_s\}} \quad \text{for some } e \in E \tag{149}$$

194

where χ is an indicator function.

To interpret the above definition, regard the constants $\{\lambda_t\}$ and $\{\mu_s\}$ as the amount of flow routed through path p_t in P_1 and p_s in P_2 respectively. Hence, by choosing the routing scheme $\{\lambda_t\}$ over $\{\mu_s\}$, inequality (147) indicates that no more shipping revenue can be generated; yet no less capacity is used on any edge in the network (inequality (148)) and at least one edge exists where strictly more capacity is used (inequality (149)).

Theorem 12. *Given any combined network G, assume that for each player i, the capacity ownership levels are identical on all edges, i.e., $\gamma^i_{e^1} = \gamma^i_{e^2} \; \forall e^1, e^2 \in E$. Then strictly positive perfect coordinating prices are guaranteed to exist if there are no dominated paths.*

Theorem 15 is proven by inverse optimization techniques, first introduced and studied in [6] and the property of strict complementary slackness of linear programs [19]. The network setting of uniform capacity ownership for every user is relevant in practical situations where the ownership levels of an individual are not calculated edge by edge but are determined according to some aggregate measure over the entire network, such as her share in the total investment in establishing the infrastructure of the network. An example of such a situation occurs in sea cargo collaborations, where shipping companies often cooperate to operate a service route and their capacity ownership levels throughout the route are calculated as their percentage of the total TEUs (twenty-foot equivalent units) deployed along the route [121].

An implication of Theorem 15 is the limitation of the capacity exchange mechanism to coordinate heterogeneous combined networks under certain practical restrictions such as the strict positiveness of prices. To see this, note that both the conditions of uniform capacity ownership levels and the non-existence of dominated paths indicate a certain degree of homogeneity: The former one implies proportionally

identical resource ownership among the players over all the network edges, and the latter implies similar profitability of the routing paths. Such limitation is originated from the fact that this capacity exchange mechanism is based on a single price that applies to all users and all commodities on the same edge.

Although Theorem 15 provides insights into the underlying factors that results in the non-existence of strictly positive perfect coordinating prices in general networks, the sufficient conditions given are too restrictive in many practical applications. Hence, in order to provide a practical solution to this problem, in the next section, we study the design of strictly positive prices to *partially* coordinate the individual optimal routings so that a social optimal routing is guaranteed to be attained and can be maintained as an equilibrium under certain conditions or with the aid of an auxiliary mechanism to manage the capacity allocation among the players.

3.4.2 Design of Strictly Positive Partial Coordinating Prices

Recall that by Definition 4, a set of perfect coordinating prices $\{cost_e\}$ should be able to induce each individual optimal routing f_{cost}^{i*} towards the collective optimal one f^* for all commodities involved in the network. In this section, we relax this criterion such that for each player, only the routing of her own commodities should equal f^* under the influence of the capacity exchange prices.

Definition 6. *A set of capacity exchange prices $\{cost_e\}$ is called partial coordinating if $\forall i \in N$ $(f_{cost}^{i*})_e^{(o,d,i)} = f_e^{*(o,d,i)}$ $\forall e \in E$ $\forall (o,d,i) \in D^i$ for some social optimal routing f^*.*

The definition of partial coordinating prices is motivated by the fact that each player can only route her own commodities. Hence, in order to achieve a social optimal routing, it is sufficient to partially align every individual optimal routing with the social optimal one with respect to the commodity set owned by the corresponding player. In the following discussion, we show that unlike perfect coordination of

196

individual optimal routings, such partial coordination can always be achieved using strictly positive prices. However, the potential tradeoff is that the social optimal routing is not guaranteed to be a Nash equilibrium and thus may be unstable, since there is no guarantee under partial coordination that adopting f^* for his own commodities is the best response for a player given that others do so.

Theorem 13. *Given any combined network G, a set of strictly positive partial coordinating prices always exists and can be computed in polynomial time.*

The proof to Theorem 13 is a constructive one consisting of two steps (refer to Algorithm 4 for details). First, we compute a routing $\tilde{f}^i$ for each player i such that $\tilde{f}_e^{i(o,d,i)} = f_e^{*(o,d,i)} \ \forall e \in E \ \forall (o,d,i) \in D^i$, i.e., his own commodities being routed according to a social optimal routing f^*, plus some additional properties specially tailored for the proof. Second, we apply the techniques from inverse optimization to formulate a set of linear inequalities that identifies capacity exchange prices $\{cost_e\}$ such that the individual optimal routing f_{cost}^{i*} is identical to $\tilde{f}^i \ \forall i \in N$. The procedure is summarized in the following algorithm. The existence of a set of strictly positive prices that satisfies these inequalities is proven based on the theory of strict complementary slackness of linear programs (refer to the Appendix for proof details).

However, as we have mentioned before, although a social optimal routing is guaranteed to be attained under partial coordinating prices, the routing may not be an equilibrium. To see why this occurs, note that under partial coordinating prices $\{cost_e\}$, the social optimal routing f^* is guaranteed to be the best strategy for each player i to operate if others route their commodities according to his individual optimal routing f_{cost}^{i*}. Since f_{cost}^{i*} is not entirely coordinated with f^* for the demand of others, it is possible that player i can earn a better profit by deviating from f^*, given that others route according to f^* instead. This is a serious drawback of partial coordinating prices compared to perfect coordinating ones, as the lack of stability of a social optimal routing can pose serious threats to the long-term efficiency of the

197

ALGORITHM 4: Designing strictly positive partial coordinating prices in a general combined network

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Input: A combined network G

Output: A set of strictly positive partial coordinating prices $\{cost_e\}$

Find an optimal solution $\{\tilde{f}_\Delta^{i*}\}$ to the following program

$$\max \quad \sum_{e \in E} \sum_{j \in N} \left[\gamma_e^j \sum_{(o,d,i) \in D} (f_\Delta^j)_e^{(o,d,i)} - \sum_{(o,d,j) \in D^j} (f_\Delta^j)_e^{(o,d,j)} \right] \tag{150}$$

$$s.t. \quad f_\Delta^i \text{ is a feasible augmenting flow with respect to } f^* \quad \forall i \in N \tag{151}$$

$$\sum_{i \in N} \left[\sum_{(o,d,i) \in D^i} (1 - \gamma_e^i)(f_\Delta^i)_e^{(o,d,i)} - \sum_{(o,d,j) \notin D^i} \gamma_e^i (f_\Delta^i)_e^{(o,d,j)} \right] \leq 0 \quad \forall e \in E \tag{152}$$

$$\sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} (f_\Delta^i)_{(d,o,i)}^{(o,d,i)} = 0 \tag{153}$$

if *the optimal solution to the above program equals zero* **then**

Strictly positive perfect coordinating prices exist. Compute such prices $\{cost_e\}$ by solving a solution to $\bigcup_{i \in N} I^i$ where I^i is defined in (101) - (110). Such a solution can be solved efficiently (e.g., using the algorithm proposed in [19]). Output $\{cost_e\}$.

else

1. Consider the augmented flow $f^* + \tilde{f}_\Delta^{i*} \ \forall i \in N$ (refer to this flow $f^* + \tilde{f}_\Delta^{i*}$ as $\tilde{f}^i$).

2. Formulate a constraint set similar to $\bigcup_{i \in N} I^i$ where each I^i is defined in (101) - (110), but with the complementary slackness conditions changed to those with respect to the new flow $\tilde{f}^i$.

3. Find a solution to the program that contains a set of strictly positive prices $\{cost_e\}$ (similarly, this solution can be solved efficiently using the algorithm proposed in [19]). Output $\{cost_e\}$.

end

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network. Hence, it is important to understand the conditions under which f^* can be guaranteed as a Nash equilibrium using strictly positive partial coordinating prices in the capacity exchange mechanism. We discuss this issue in the following analysis. Moreover, we also propose auxiliary mechanisms to reinforce the stability of the social optimal routing under partial coordinating prices.

We first point out that since an individual player i can manage both his own capacity and demand, there are two ways for i to deviate from the social optimal routing f^* in operation: (i) by routing his commodities differently; and (ii) by refusing to

share his capacity according to f^*. While both situations can occur, in practice, the individuals' manipulation of how to share their own capacity within a combined network can be restricted, especially when a binding resource sharing agreement exists, which is usually the case in practical alliances. Hence, in the rest of this section, we focus on the routing decisions of players and study when and how all players can be prevented from adopting a different routing other than f^* for their own commodities under strictly positive partial coordinating prices, given others following f^*. We call f^* a *routing equilibrium* if such a condition is satisfied.

Definition 7. *Given a combined network and a set of capacity exchange prices* $\{cost_e\}$, *a routing f is called a routing equilibrium if for every player i, his individual payoff satisfies*

$$x^i_{cost}(f) \geq x^i_{cost}(f^{i\prime}, f^{-i}) \quad \forall \text{ routing for i's commodity } f^{i\prime} \text{ that is feasible given } f^{-i}$$

(154)

where $f^{-i} = \{f_e^{(o,d,i)}, \forall (o,d,i) \notin D^i\}$ denotes the part of the routing f that corresponds to commodities owned by players other than i.

Recall that we mentioned previously that perfect coordinating prices guarantee the social optimal routing f^* to be a routing equilibrium. The reason is that under such prices, each individual optimal routing is completely aligned with f^* for all commodities. Theorem 14 below indicates that such a strong condition is not necessary to ensure f^* to be a routing equilibrium. Specifically, given a set of strictly positive partial coordinating prices as defined in Theorem 13, a certain alignment between the set of saturated edges under the individual optimal routings and that under f^* is sufficient to prevent players from deviating from the social optimal routing. To present the result, recall that in the proof of Theorem 13, the strictly positive partial coordinating prices are computed so that the individual optimal routing for each player i equals a certain routing $\tilde{f}^i$ (refer to Algorithm 4). Let $\bar{E}^i = \{e \in E : \sum_{(o,d,i)\in D} \tilde{f}_e^{i(o,d,i)} = c_e\}$

and $\bar{E}^* = \{e \in E : \sum_{(o,d,i) \in D} f_e^{*(o,d,i)} = c_e\}$ be the set of fully-used edges under each $\tilde{f}^i$ and under the social optimal routing f^*.

Theorem 14. *Given any combined network G, assume the social optimal routing f^* is unique. There exists a set of strictly positive partial coordinating prices under which f^* is a routing equilibrium if either of the following two conditions holds.*

1. $\bar{E}^i \subset \bar{E}^* \ \forall i \in N$; or

2. On each edge $e \in \bar{E}^*$, $\gamma_e^i > 0 \ \forall i \in N$, i.e., all users own positive fraction of the capacity.

To see the connection between the two conditions in the above theorem, note that a perfect alignment between $\bar{E}^i$ and $\bar{E}^*$, i.e., $\bar{E}^i = \bar{E}^*$, requires that both $\bar{E}^i \setminus \bar{E}^* = \emptyset$ and $\bar{E}^* \setminus \bar{E}^i = \emptyset$. In Theorem 14, condition 1 is equivalent to $\bar{E}^i \setminus \bar{E}^* = \emptyset \ \forall i \in N$, which indicates that no edge with redundant capacity under the social optimal routing f^* is used to its full capacity under any individual routings. Condition 2, on the contrary, ensures the opposite situation that for each player i, $\bar{E}^* \subset \bar{E}^i$ and thus $\bar{E}^* \setminus \bar{E}^i = \emptyset$ (see Lemma 9 in the Appendix for a detailed proof). In this case, all edges that are fully used under f^* are also saturated under all individual optimal routings.

We can interpret Theorem 14 as follows. For each player i, the set $\bar{E}^i$ represents the set of edges where an additional unit of available capacity will lead to positive extra individual profit for him. The set $E \setminus \bar{E}^i$, on the other hand, represents the set of edges where the capacity is left redundant under the individual optimal routing. Intuitively, in order to maximize his own profit, player i tends to prefer a routing of his own commodities under which more capacity on edges in $\bar{E}^i$ and less capacity on edges in $E \setminus \bar{E}^i$ is used. Hence, if there is a large difference between $\bar{E}^i$ and $\bar{E}^*$, such incentives can result in player i deviating from f^* for his own commodities when others' commodities are routed according to f^*. Theorem 14 implies that, when such incentives are derived due to the existence of only *one* of the sets of $\bar{E}^i \setminus \bar{E}^*$ and

$\bar{E}^* \setminus \bar{E}^i$, strictly positive partial coordinating prices can be designed to prevent players from deviating from f^*, and thus guarantee a routing equilibrium at f^*. However, when both sets are nonempty, it may be impossible to devise a unique set of strictly positive partial coordinating prices to eliminate the negative impact of these two different types of edges at the same time.

We also point out that condition 2 in Theorem 14 is in fact stronger than the condition of $\bar{E}^* \setminus \bar{E}^i = \emptyset \ \forall i \in N$. In other words, the existence of a player i who owns no capacity on some edge $e \in \bar{E}^*$ may pose additional restrictions to our search for strictly positive partial coordinating prices under which f^* is a routing equilibrium, even if $\bar{E}^* \setminus \bar{E}^i = \emptyset$ is satisfied. To see this, note that such a player i will not profit from the capacity exchanges on this edge e due to his zero capacity ownership. Hence, in planning his individual optimal routing, the player is likely to deviate others' commodities from e to edges where he owns positive capacity in order to gain capacity exchange prices. This can result in more capacity on e, which can potentially induce player i to route more of his own commodities on e compared to that under f^*. Such a possibility needs to be eliminated according to the definition of partial coordinating prices, and thus creates an extra constraint for our price design problem.

The above discussion points to a direction for designing auxiliary mechanisms to guarantee the existence of strictly positive partial coordinating prices under which the social optimal routing f^* is a routing equilibrium. Specifically, we can impose an operational rule to limit the amount of commodities that the players can route on edges that are saturated under f^* if they own no capacity on these edges. Such flow caps help to explicitly eliminate the possibility that such a player may route more than he could under f^*. We prove that this mechanism is indeed effective under general network conditions, as shown in the following corollary to Theorem 14.

Corollary 3. *Given any combined network G, assume the social optimal routing f^* is unique. There exists a set of strictly positive partial coordinating prices under*

which f^* is a routing equilibrium if on each edge $e \in \bar{E}^*$, the flow that any player $i \in \{i \in N : \gamma_e^i = 0\}$ can route on e is upper bounded by $\sum_{(o,d,i) \in D^i} f_e^{*(o,d,i)}$.

As a final remark, the mechanism proposed above essentially determines capacity allotments on each of the saturated edges under the social optimal routing f^* to players who own no capacity on it. This is a realistic approach considering the current industry practice such as in carrier alliances (Houghtalen et al. [78]). We also mention that a similar mechanism is studied by Houghtalen et al. [78] where a capacity allocation scheme is pre-designed on every edge for all players. However, the study is different from ours in several notable ways. First, while their mechanism determines a capacity allocation on all edges for every player, ours only involves edges that are fully used under f^* and does not restrict the flow of any capacity owners on these edges, which is more practical. Second, their paper studies a setting where the capacity on each edge is uniquely owned and all individual optimal routings are computed to maximize the profit that can be obtained from the shipment of their own commodities (i.e., shipping revenue minus the capacity exchange prices paid) given the capacity allocated to them, ignoring the capacity exchange prices that they can earn from others; their result shows that perfect coordinating prices exist in such a setting given certain capacity allocation rules. Yet our study is based on a more general network setting with multiple capacity owners on an edge, and we focus on designing prices to coordinate the combined network under the practical restriction to avoid free riders.

3.4.3 Appendix

Proof of Theorem 15. To prove the theorem, we use the following lemma that provides a sufficient and necessary condition for the existence of a set of strictly positive perfect coordinating prices. First we define a feasible augmenting flow f_Δ with respect to f^* such that the augmented flow $f^* + f_\Delta$ is nonnegative and satisfies the flow

conservation constraint and demand constraint for each commodity, and the capacity constraint on every edge over all commodities.

Lemma 8. *Given any collective network G, strictly positive perfect coordinating prices exist if and only if the constraint (157) is binding $\forall e \in E$ in the following program for all the optimal solutions.*

$$\max \quad \sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)}(f_\Delta^i)_{(d,o,i)}^{(o,d,i)} \tag{155}$$

$$s.t. \quad f_\Delta^i \text{ is a feasible augmenting flow with respect to } f^* \quad \forall i \in N \tag{156}$$

$$\sum_{i \in N} \left[\sum_{(o,d,i) \in D^i} (1 - \gamma_e^i)(f_\Delta^i)_e^{(o,d,i)} - \sum_{(o,d,j) \notin D^i} \gamma_e^i(f_\Delta^i)_e^{(o,d,j)} \right] \leq 0 \quad \forall e \in E. \tag{157}$$

Proof of Lemma 8. $\Leftarrow$: According to duality theorem and complementary slackness for each linear program (P_{cost}^i), a set of capacity exchange prices $\{cost_e\}$ is perfect coordinating if and only if the union of the inequality sets $(137)-(141)$ (i.e., dual feasibility constraints) and $(127)-(132)$ (i.e., complementary slackness constraints) over all players i is feasible. Hence, we can solve for perfect coordinating prices using the following program, i.e.,

$$\min \sum_{i \in N} 0 \cdot [\pi^i, \alpha^i, \beta^i] + 0 \cdot cost$$

$$s.t. \quad (137) - (141) \text{ and } (127) - (132) \quad \forall i \in N; \quad cost_e \geq 0 \quad \forall e \in E \tag{158}$$

Every optimal solution to this program points to a set of perfect coordinating prices. After some manipulations [2], we can verify that the dual of (158) is exactly the one shown in $(155)-(157)$, and the primal variables $\{cost\}$ correspond to the constraint (157). Hence, according to strict complementary slackness of linear program, when constraint (157) is binding under all optimal solutions, there must exist an optimal solution to the primal problem which contains a set of strictly positive perfect coordinating prices.

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[2]For details, please refer to an online supplement at the author's webpage at www2.isye.gatech.edu/~lgui3.

$\Rightarrow$: Assume that there exists an optimal solution to the above program $(155)-(157)$ $\{f_\Delta^{i*}\}$ such that the constraint (157) is a strict inequality for at least one edge. We know that the optimal objective value of this program is zero due to the existence of perfect coordinating prices in any network (Agarwal and Ergun [2]). Then under any strictly positive prices $\{cost_e\}$, we conclude that $\sum_{i\in N} x^i_{cost}(f^* + f_\Delta^{i*}) > \sum_{i\in N} x^i_{cost}(f^*)$. This indicates that for at least one player i, $x^i_{cost}(f^*+f_\Delta^{i*}) > x^i_{cost}(f^*)$, which contradicts the condition that $\{cost\}$ is perfect coordinating. $\qquad\square$

Now going back to the proof of Theorem 15, let us assume that there does not exist a set of strictly positive perfect coordinating prices. Then according to Lemma 8, there exists at least one optimal solution to the program $(155)-(157)$, under which the constraint (157) holds as a strict inequality on at least one edge e. Denote such an optimal solution as $\{\hat{f}_\Delta^{i*}\}$. Under the uniform capacity ownership condition, let γ^i denote the capacity ownership of player i on every edge in G. Consider the following augmenting flow

$$\hat{f}_e^{(o,d,i)} \doteq \sum_{j\in N} \gamma^j (\hat{f}_\Delta^{j*})_e^{(o,d,i)} - (\hat{f}_\Delta^{i*})_e^{(o,d,i)} \quad \forall e \in E \cup \{(d,o,i) : \forall (o,d,i) \in D\} \quad \forall (o,d,i) \in D$$

$$(159)$$

Since each $\hat{f}_\Delta^{j*}$ is a feasible augmenting flow with respect to f^*, $\sum_{(o,d,i)\in D} r_{(o,d,i)}(\hat{f}_\Delta^{j*})_{(d,o,i)}^{(o,d,i)} \leq 0$ for each $j \in N$; otherwise it is a contradiction to the optimality of f^*. In addition, $\sum_{i\in N}\sum_{(o,d,i)\in D^i} r_{(o,d,i)}(\hat{f}_\Delta^{i*})_{(d,o,i)}^{(o,d,i)} = 0$ as the optimal objective value of the program $(155)-(157)$ is zero. Hence, we conclude that $\sum_{(o,d,i)\in D} r_{(o,d,i)}\hat{f}_{(d,o,i)}^{(o,d,i)} = \sum_{j\in N}\gamma^j \sum_{(o,d,i)\in D} r_{(o,d,i)}(\hat{f}_\Delta^{j*})_{(d,o,i)}^{(o,d,i)} - \sum_{i\in N}\sum_{(o,d,i)\in D^i} r_{(o,d,i)}(\hat{f}_\Delta^{i*})_{(d,o,i)}^{(o,d,i)} \leq 0$.

Second, it is easy to see that the total augmenting flow under $\hat{f}$ on any edge e is equal to the absolute value of the LFS in constraint (157), i.e., $\sum_{(o,d,i)\in D} \hat{f}_e^{(o,d,i)} = \sum_{j\in N}\left[\gamma_e^j \sum_{(o,d,i)\in D}(\hat{f}_\Delta^{j*})_e^{(o,d,i)} - \sum_{(o,d,j)\in D^j}(\hat{f}_\Delta^{j*})_e^{(o,d,j)}\right]$, and thus is guaranteed to be nonnegative on all edges and is strict positive at least one edge.

Finally, since each $\hat{f}_\Delta^{i*}$ is a feasible augmenting flow, we can verify that the augmenting flow $\hat{f}$ defined in (159) satisfies the flow conservation constraints for each commodity. Thus we can decompose $\hat{f}$ into flow on the $o - d$ paths for each commodity, and combine the paths with strictly positive and negative flow into two different sets. Such two sets cannot be empty, since we have shown that (i) $\hat{f}$ strictly increases the flow on at least one edge, and (ii) the revenue change is nonpositive under $\hat{f}$. Hence, according to the definition of dominated paths (Definition 5), it is easy to see that the set of path with negative flow is dominated. This is a contradiction to the condition of the theorem that no dominated paths exist. □

Proof of Theorem 13. To prove the theorem, we show in two steps that Algorithm 4 computes a set of prices $\{cost_e\}$ that is partial coordinating. First, we prove that if the algorithm is valid, then its outcome must be partial coordinating. Second, we show the validity of the algorithm. For each step we prove a related claim.

Claim 1. *Define routing $\tilde{f}$ such that $\forall i \in N \ \forall (o, d, i) \in D^i$*

$$\tilde{f}_e^{(o,d,i)} \doteq (f^* + \tilde{f}_\Delta^{i*})_e^{(o,d,i)} \quad \forall e \in E \cup \{(d, o, i) : \forall (o, d, i) \in D\}. \tag{160}$$

$\tilde{f}$ is a feasible and collective optimal routing to the centralized problem (C).

Proof of Claim 1. The feasibility of $\tilde{f}$ is proven as follows: Due to constraint (157) and the fact that $\tilde{f}_\Delta^{i*}$ is a feasible augmenting flow with respect to f^* for all i, the total flow on each edge e under $\tilde{f}$ satisfies

$$\sum_{(o,d,i) \in D} \tilde{f}_e^{(o,d,i)} = \sum_{i \in N} \sum_{(o,d,i) \in D^i} [f_e^{*(o,d,i)} + (\tilde{f}_\Delta^{i*})_e^{(o,d,i)}]$$
$$\leq \sum_{j \in N} \gamma_e^j \sum_{(o,d,i) \in D} [f_e^{*(o,d,i)} + (\tilde{f}_\Delta^{j*})_e^{(o,d,i)}] \leq \sum_{j \in N} \gamma_e^j c_e = c_e. \tag{161}$$

In addition, $\tilde{f}$ obviously satisfies the flow conservation and demand constraints as each $\tilde{f}_\Delta^{i*}$ is solved as a feasible augmenting flow with respect to f^*. Moreover, according to the last constraint in the program $(150)-(153)$, $\sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} \tilde{f}_{(d,o,i)}^{(o,d,i)} =$

$\sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} f^{*(o,d,i)}_{(d,o,i)}$. Since the flow $\tilde{f}$ generates the maximum total shipping revenue and is feasible, by definition $\tilde{f}$ is a collective optimal routing. $\square$

In the next step, we show the existence of strictly positive prices $\{cost_e\}$ under which the individual optimal routings f^{i*}_{cost} for each program (P^i_{cost}) equals $f^* + \tilde{f}^{i*}_\Delta$. To do that, we first take the dual of the program (158), where the complementary slackness conditions $(127)-(132)$ are with respect to the flows $\{f^* + \tilde{f}^{i*}_\Delta \ \forall i \in N\}$, and obtain a similar program to the one in $(155)-(157)$, i.e.,

$$\max \quad \sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} (f^i_\Delta)^{(o,d,i)}_{(d,o,i)} \tag{162}$$

$$s.t. \quad f^i_\Delta \text{ is a feasible augmenting flow with respect to } f^* + \tilde{f}^{i*}_\Delta \quad \forall i \in N \tag{163}$$

$$\sum_{i \in N} \left[\sum_{(o,d,i) \in D^i} (1 - \gamma^i_e)(f^i_\Delta)^{(o,d,i)}_e - \sum_{(o,d,j) \notin D^i} \gamma^i_e (f^i_\Delta)^{(o,d,j)}_e \right] \leq 0 \quad \forall e \in E \tag{164}$$

Claim 2. *The optimal objective value of the program $(162)-(164)$ is zero. Moreover, the constraint (164) is binding under all the optimal solutions to the program.*

Proof of Claim 2. Assume there exists a feasible solution $\{\tilde{\tilde{f}}^{i*}_\Delta\}$ to the program (162)-(164) that gives rise to strictly positive objective value. Then it is easy to see that the solution $\{\tilde{f}^{i*}_\Delta + \tilde{\tilde{f}}^{i*}_\Delta\}$ is feasible to the program $(155)-(157)$ and also generates a positive objective value. This is a contradiction to the fact that the program $(155)-(157)$ has a zero objective value, due to the general existence of perfect coordinating prices.

Now assume $\{\tilde{\tilde{f}}^{i*}_\Delta\}$ is an optimal solution to the program $(162)-(164)$ under which the constraint (164) is strictly positive on at least one edge. Then it is easy to see that the objective function (150) has a strictly higher value under the solution $\{\tilde{f}^{i*}_\Delta + \tilde{\tilde{f}}^{i*}_\Delta\}$ than $\{\tilde{f}^{i*}_\Delta\}$, a contradiction to the optimality of $\{\tilde{f}^{i*}_\Delta\}$ to the program $(150)-(153)$. $\square$

Given Claim 2, we conclude that, by strict complementary slackness of linear program, there must exist a feasible solution to the program (158) with respect to the flows $\{f^* + \tilde{f}^{i*}_\Delta \ \forall i \in N\}$ that contains strictly positive prices. These prices can

also be computed in polynomial time (e.g., using the algorithm given by Bertsimas and Tsitsiklis [19]). Moreover, we know by complementary slackness, such prices must induce each player i to the routing $f^* + \tilde{f}_\Delta^{i*}$, and give rise to a collective optimal routing $\tilde{f}$ according to Claim 1. Thus by definition, this set of prices is partial coordinating. $\qquad\square$

Proof of Theorem 14. In the previous proofs, we characterize the set of perfect (partial) coordinating prices by program (158) with respect to f^* ($f^* + \tilde{f}_\Delta^{i*} \ \forall i \in N$). Similarly, we provide here a characterization of the set of prices under which f^* is a routing equilibrium in the collective network. First, by definition, f^* is a routing equilibrium if and only if for each player i, the part of f^* with respect to her own commodity set D^i is optimal to the following program.

$$\max \sum_{(o,d,i)\in D^i} r_{(o,d,i)} f_{(d,o,i)}^{(o,d,i)} - \sum_{e\in E} cost_e (1-\gamma_e^i) \sum_{(o,d,i)\in D^i} f_e^{(o,d,i)} \tag{165}$$

$$s.t. \sum_{e\in\delta^-(v)} f_e^{(o,d,i)} - \sum_{e\in\delta^+(v)} f_e^{(o,d,i)} \le 0 \quad \forall v \in V \quad \forall (o,d,i) \in D^i \tag{166}$$

$$\sum_{(o,d,i)\in D^i} f_e^{(o,d,i)} \le c_e - \sum_{(o,d,j)\notin D^i} f_e^{*(o,d,j)} \quad \forall e \in E \tag{167}$$

$$f_{(d,o,i)}^{(o,d,i)} \le d_{(o,d,i)} \quad \forall (o,d,i) \in D^i \tag{168}$$

$$f \ge 0 . \tag{169}$$

It can be easily verified that the dual constraints with respect to the above program is identical to the constraints (137), (139) and (141); its complementary slackness conditions with respect to the f^* is exactly those in (127), (129), and (132) restricted to the commodity set D^i. Hence we conclude that a set of prices $\{cost_e\}$ is partial

coordinating and induces a routing equilibrium at f^* if

$$
\begin{cases}
\{cost_e\} \text{ is partial coordinating, i.e., } \exists \text{ values } [\pi_1^i, \alpha_1^i, \beta_1^i] \text{ such that inequalities} \\
\qquad (137) - (141) \text{ and } (127) - (132) \text{ with respect to flow } \{f^* + \tilde{f}_\Delta^{i*}\} \\
\qquad \text{are satisfied for all } i \in N; \\
\{cost_e\} \text{ induces a routing equilibrium at } f^*, \text{ i.e., } \exists \text{ values } [\pi_2^i, \alpha_2^i, \beta_2^i] \text{ such that} \\
\qquad \text{inequalities } (137), (139), (141) \text{ and } (127), (129), (132) \text{ restricted to} \\
\qquad D^i \text{are satisfied for all } i \in N.
\end{cases}
\tag{170}
$$

According to Theorem 13, it is known that strictly positive partial coordinating prices always exist. To prove Theorem 14, we first show that under condition 1 in Theorem 14, f^* is a routing equilibrium under any partial coordinating prices. To see this, note that under any partial coordinating prices, there exists feasible solutions $[\pi_1^i, \alpha_1^i, \beta_1^i] \ \forall i \in N$ to the first set of inequalities in (170). Since the collective optimal routing is unique, for each player i, $f^* + \tilde{f}_\Delta^{i*} = f^* \ \forall (o, d, i) \in D^i$. Hence, it is easy to check that setting $[\pi_2^i, \alpha_2^i, \beta_2^i] = [\pi_1^i, \alpha_1^i, \beta_1^i]$ generates a feasible solution to the second set of inequalities when $\bar{E}^i \subset \bar{E}^* \ \forall i \in N$.

To prove the theorem under condition 2, we show the existence of strictly positive prices under which all of the inequalities in (170) can be satisfied simultaneously, based on strict complementary slackness of linear programs. To do this, we first transform the inequality system (170) into a linear program with a zero objective function, which is similar to the program in (158).We denote the vector of dual variables associated with each of the two constraint sets in (170) by $f_{\Delta 1}^i$ and $f_{\Delta 2}^i$, and formulate the dual problem as follows. Note that while each $f_{\Delta 1}^i$ indicates an augmenting flow for all

commodities in D, $f_{\Delta 2}^i$ is only associated with i's own commodities in D^i.

$$\max \quad \sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} \cdot \left[(f_{\Delta 1}^i)_{(d,o,i)}^{(o,d,i)} + (f_{\Delta 2}^i)_{(d,o,i)}^{(o,d,i)} \right] \tag{171}$$

$s.t.$ $f_{\Delta 1}^i$ is a feasible augmenting flow with respect to $f^* + \tilde{f}_\Delta^{i*}$ $\forall i \in N$ (172)

$f_{\Delta 2}^i$ is a feasible augmenting flow with respect to f^* for D^i $\forall i \in N$ (173)

$$\sum_{i \in N} (1 - \gamma_e^i) \sum_{(o,d,i) \in D^i} \left[(f_{\Delta 2}^i)_e^{(o,d,i)} + (f_{\Delta 1}^i)_e^{(o,d,i)} \right] - \sum_{i \in N} \gamma_e^i \sum_{(o,d,j) \notin D^i} (f_{\Delta 1}^i)_e^{(o,d,j)} \le 0 \quad \forall e \in E \tag{174}$$

Note that the prices $\{cost_e\}$ are the primal variables associated with constraint (174) in the above dual program. Hence, in order to prove the existence of strictly positive $\{cost_e\}$, we show that given condition 2, under any optimal solution to the program (171)−(174), constraint (174) is binding. To achieve this goal, we first prove the following technical result.

Proposition 11. *If on each edge $e \in \bar{E}^*$, $\gamma_e^i > 0$ $\forall i \in N$, then any optimal solution $\{f_{\Delta 1}^{i*}, f_{\Delta 2}^{i*}\}$ to the program (171)−(174) satisfies $(f_{\Delta 1}^{i*})_e^{(o,d,i)} = (f_{\Delta 2}^{i*})_e^{(o,d,i)} = 0$ $\forall e \in E$ $\forall (o,d,i) \in D^i$ $\forall i \in N$.*

Proof of Proposition 11. We prove this result in three steps. First, we show that the optimal value to the program (171)−(174) is zero. To see this, we assume that the optimal solution to the program, denoted as $\{f_{\Delta 1}^{i*}, f_{\Delta 2}^{i*}\}$, generates a positive objective value. Then we construct the following flow $\hat{f}_\Delta^i$ for each player i such that

$$\hat{f}_\Delta^i \doteq \begin{cases} \frac{1}{2} \cdot [\tilde{f}_\Delta^{i*} + f_{\Delta 1}^{i*} + f_{\Delta 2}^{i*}] & \forall (o,d,i) \in D^i \\ \frac{1}{2} \cdot [\tilde{f}_\Delta^{i*} + f_{\Delta 1}^{i*}] & \forall (o,d,i) \notin D^i. \end{cases} \tag{175}$$

Since both $\tilde{f}_\Delta^{i*} + f_{\Delta 1}^{i*}$ and $f_{\Delta 2}^{i*}$ are feasible augmenting flow with respect to f^*, any flow that is a convex combination of them is also a feasible augmenting flow with respect to f^*. Since $\{f_{\Delta 1}^{i*}, f_{\Delta 2}^{i*}\}$ also satisfies (174), it is easy to verify that $\hat{f}_\Delta^i$ is a feasible solution to the program (155)−(157) that gives rise to a strictly positive objective value. However, it is known that the optimal value to the program (155)−(157)

209

should be zero, since perfect coordinating prices always exist. Thus by contradiction, we conclude that the optimal value to the program $(171)-(174)$ is zero.

Second, consider the routing resulting from every player i routing her own commodity according to $f^* + \hat{f}_\Delta^i$. According to the same reasoning as in Claim 1, such a routing is feasible and thus is a collective optimal one as we have shown previously that $\sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} \cdot (\hat{f}_\Delta^i)_{(d,o,i)}^{(o,d,i)} = 0$. Hence, due to the uniqueness of the collective optimal routing, we conclude for each player i, $(\tilde{f}_\Delta^{i*})_e^{(o,d,i)} + (f_{\Delta 1}^{i*})_e^{(o,d,i)} + (f_{\Delta 2}^{i*})_e^{(o,d,i)} = 0$ $\forall e \in E$ $\forall (o,d,i) \in D^i$. Since by Claim 1 and the uniqueness of the collective optimal routing, we know that $(\tilde{f}_\Delta^{i*})_e^{(o,d,i)} = 0$ $\forall e \in E$ $\forall (o,d,i) \in D^i$ $\forall i \in N$, we conclude that $(f_{\Delta 1}^{i*})_e^{(o,d,i)} = -(f_{\Delta 2}^{i*})_e^{(o,d,i)}$ $\forall e \in E$ $\forall (o,d,i) \in D^i$ $\forall i \in N$.

Now we prove the proposition by contradiction. Assume there exists an optimal solution where $f_{\Delta 2}^{i*} \neq 0$ for some i. Due to the uniqueness of the collective optimal routing, $\sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} (f_{\Delta 2}^{i*})_{(d,o,i)}^{(o,d,i)} < 0$ as each $f_{\Delta 2}^{i*}$ is a feasible augmenting flow with respect to f^*, and hence $\sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} (f_{\Delta 1}^{i*})_{(d,o,i)}^{(o,d,i)} > 0$. Then the routing $\hat{f}$ defined as

$$\hat{f}_e^{(o,d,i)} \doteq f_e^{*(o,d,i)} + m \cdot (f_{\Delta 1}^{i*})_e^{(o,d,i)} \quad \forall e \in E \quad \forall (o,d,i) \in D^i \quad \forall i \in N \qquad (176)$$

cannot be feasible for any constant $m > 0$, since otherwise it is a contradiction to the collective optimality of f^*. However, we know that each $f_{\Delta 1}^{i*}$ is a feasible augmenting flow with respect to $f^* + \tilde{f}_\Delta^{i*}$, and $(\tilde{f}_\Delta^{i*})_e^{(o,d,i)} = 0$ $\forall e \in E$ $\forall (o,d,i) \in D^i$. These two facts indicate that the routing $\hat{f}$ cannot violate any flow conservation or demand constraint. Hence, $\hat{f}$ being infeasible for any $m > 0$ implies that $\exists e \in \bar{E}^*$ such that

$$\sum_{i \in N} \sum_{(o,d,i) \in D^i} (f_{\Delta 1}^{i*})_e^{(o,d,i)} > 0. \qquad (177)$$

We show that inequality (177) is impossible under condition 2 of Theorem 14 as follows. First, we prove the following lemma that any edge in $\bar{E}^*$ also belongs in $\bar{E}^i$ if the player i owns capacity on it.

Lemma 9. *Given any collective network G, assume the collective optimal routing f^* is unique. Then $\bar{E}^* \cap \{e \in E : \gamma_e^j > 0\} \subset \bar{E}^j \ \forall j \in N$.*

Proof of Lemma 9. As we have argued before, due to the uniqueness of collective optimal routing and Claim 1, for each player i, $(\tilde{f}_\Delta^{i*})_e^{(o,d,i)} = 0 \ \forall e \in E \ \forall (o,d,i) \in D^i$. Hence, we obtain the following relation from constraint (157)

$$\sum_{j \in N} \gamma_e^j \sum_{(o,d,i) \in D} (\tilde{f}_\Delta^{j*})_e^{(o,d,i)} \geq \sum_{i \in N} \sum_{(o,d,i) \in D^i} (\tilde{f}_\Delta^{i*})_e^{(o,d,i)} = 0 \quad \forall e \in E. \tag{178}$$

Meanwhile, since each $\tilde{f}_\Delta^{j*}$ is a feasible augmenting flow with respect to f^*, we know that $\sum_{(o,d,i) \in D} (\tilde{f}_\Delta^{j*})_e^{(o,d,i)} \leq 0 \ \forall e \in \bar{E}^*$. Combining this inequality with (178), we conclude that on each edge $e \in \bar{E}^*$, for each player j with positive capacity ownership, i.e., $\gamma_e^j > 0$, $\sum_{(o,d,i) \in D} (\tilde{f}_\Delta^{j*})_e^{(o,d,i)} = 0$. Hence $\forall j \in N \ \bar{E}^* \cap \{e : \gamma_e^j > 0\} \subset \bar{E}^j$. $\qquad\square$

Now going back to the derivation of a contradiction to (177). On each edge $e \in \bar{E}^*$, we know that for each player i, $\sum_{(o,d,i) \in D^i} (f_{\Delta 2}^{i*})_e^{(o,d,i)} \leq 0$ and thus $\sum_{(o,d,i) \in D^i} (f_{\Delta 1}^{i*})_e^{(o,d,i)} \geq 0$. Under condition 2 in Theorem 14, i.e., all players own positive amount of capacity on each edge in $\bar{E}^*$, $\bar{E}^* \subset \bar{E}^i \ \forall i \in N$ by Lemma 9. Since each $f_{\Delta 1}^{i*}$ is a feasible augmenting flow with respect to $f^* + \tilde{f}_\Delta^{i*}$, this indicates that on every edge in $\bar{E}^*$, $\sum_{(o,d,j) \notin D^i} (f_{\Delta 1}^{i*})_e^{(o,d,j)} \leq 0$ and thus $\gamma_e^i \cdot \sum_{(o,d,j) \notin D^i} (f_{\Delta 1}^{i*})_e^{(o,d,j)} \leq 0$. Combine this inequality with constraint (174), which is reduced to $\sum_{i \in N} \gamma_e^i \sum_{(o,d,j) \notin D^i} (f_{\Delta 1}^{i*})_e^{(o,d,j)} \geq 0$ $\forall e \in E$ under any optimal solution as on any edge $f_{\Delta 1}^{i*} + f_{\Delta 2}^{i*} = 0$ for any commodity in $D^i \ \forall i \in N$. It can be concluded that for any edge in $\bar{E}^*$, $\sum_{(o,d,j) \notin D^i} (f_{\Delta 1}^{i*})_e^{(o,d,j)} = 0$ (since $\gamma_e^i > 0 \ \forall i \in N$). Thus we derive that

$$\sum_{(o,d,i) \in D^i} (f_{\Delta 1}^{i*})_e^{(o,d,i)} = 0 \quad \forall e \in \bar{E}^* \quad \forall i \in N, \tag{179}$$

since $\bar{E}^* \subset \bar{E}^i$ and $f_{\Delta 1}^{i*}$ is a feasible augmenting flow with respect to $f^* + \tilde{f}_\Delta^{i*}$. Hence, we obtain a contradiction to inequality (177), which completes our proof that any optimal solution to the program (171)−(174) satisfies that for each player i, $(f_{\Delta 1}^{i*})_e^{(o,d,i)} = (f_{\Delta 2}^{i*})_e^{(o,d,i)} = 0 \ \forall e \in E \ \forall (o,d,i) \in D^i$ under condition 2 in Theorem 14. $\qquad\square$

We finally show by contradiction that the above result implies that constraint (174) must be binding under all optimal solutions $\{f_{\Delta 1}^{i*}, f_{\Delta 2}^{i*}\}$. Assume constraint (174) is not binding under an optimal solution. Then since according to Proposition 11, $f_{\Delta 2}^{i*} = 0$, the flow $\{\tilde{f}_{\Delta}^{i*} + f_{\Delta 1}^{i*}\}$ is a feasible solution to the program $(150)-(153)$ that gives a higher objective value than $\{\tilde{f}_{\Delta}^{i*}\}$. This is a contradiction to the optimality of $\{\tilde{f}_{\Delta}^{i*}\}$ to the program $150-153$. Therefore, due to strict complementary slackness, we conclude that there must exist strictly positive prices $\{cost_e\}$ that satisfies the constraints in (170) and thus is partial coordinating and also induces a routing equilibrium at f^*. $\qquad\square$

Proof of Corollary 3. The proof of Corollary 3 follows the same argument as those in the proof of Theorem 14 under condition 2 except for the following difference: When deriving a contradiction to inequality (177), we first derive equation (179) on each edge $e \in \bar{E}^*$ for all players i such that $\gamma_e^i > 0$ in the same way as in the proof of Theorem 14. Yet for each player i who own no capacity on such an edge e, we conclude that $\sum_{(o,d,i)\in D^i} (f_{\Delta 1}^{i*})_e^{(o,d,i)} \leq 0$ since she is not allowed to route more than $\sum_{(o,d,i)\in D^i} f_e^{*(o,d,i)}$ on e and $f_{\Delta 1}^{i*}$ is a feasible augmenting flow with respect to $f^* + \tilde{f}_{\Delta}^{i*}$ where $(\tilde{f}_{\Delta}^{i*})_e^{(o,d,i)} = 0 \ \forall e \in E \ \forall (o,d,i) \in D^i$ due to Claim 1 and the uniqueness of the collective optimal routing. Hence we can conclude that

$$\sum_{i \in N} \sum_{(o,d,i)\in D^i} (f_{\Delta 1}^{i*})_e^{(o,d,i)} \leq 0 \quad \forall e \in \bar{E}^*, \tag{180}$$

which is also a contradiction to (177). $\qquad\square$

3.5 A Robustness Analysis of a Capacity Exchange Mechanism under Demand Uncertainty

In this section, we consider a practical challenge to coordinate a combined network using market exchange mechanisms that arises from uncertainties in demand. In many practical cases the mechanism is designed before the demand is revealed. Hence, it is

desirable that the capacity exchange mechanism is *robust*, i.e., it can effectively coordinate the network under all potential demand scenarios using a fixed set of exchange prices. Hence, we study the robustness of the capacity exchange mechanism assuming the existence of a number of potential demand scenarios of the combined network. Notice that the task of designing robust mechanisms becomes more challenging as in practice, there often exists a certain level of information asymmetry between the mechanism designer and the players about demand. In this study, we also capture the information asymmetry between the central authority and the participants by assuming that the exact demand is revealed for individual decision-making after the mechanism is imposed.

We conduct two analyses of the robustness problem from different angles. The first one revolves around how robust a capacity exchange mechanism can be under different network structures. It is motivated by the observation that in some networks, *no* fixed set of exchange prices can induce the maximum total routing revenue without violating capacity limits under every demand scenario, indicating that the robustness of the mechanism is undermined by intrinsic network features. We show that the undermining factors are the existence of inefficient path structures, which we call *dominated paths*, and the widely varied individual capacity ownership levels over the network. We also show that the negative impact of dominated paths can be countered and the robustness of the mechanism be reinforced by commodity-based heterogeneous pricing of the capacity on one edge (i.e., price discrimination).

Our second analysis is focused on the computational side of the problem, i.e., how to efficiently compute a set of capacity exchange prices that has certain robust properties in any given network. We investigate this question by proposing a polynomial pricing algorithm based on inverse optimization techniques, assuming a known probability distribution over the set of potential demand scenarios, and analyzing the routings induced by the mechanism. In particular, we bound the expected total

revenue and the maximum overflow relative to capacity limit on all edges under each demand scenario; one nice property observed about the prices computed is that they guarantee less overflow in demand scenarios that are more likely to occur. Based on these results, we mathematically illustrate the impact of demand uncertainty, characterized by the size of the set of potential demand scenarios and the probabilities associated with them, on the robustness of the capacity exchange mechanism.

Review of Related Literature Optimization problems with parameter uncertainty has been extensively addressed in literature, e.g., in stochastic optimization and robust optimization. The robust optimization framework, such as the one proposed by [17], are related to our study since the routing that maximizes the total revenue is computed by solving a network optimization problem. However, the decentralized setting of the collaborative network model makes our problem more complicated. There also exists a set of papers in the stochastic network flow literature that studies centralized management of resource sharing between different queuing systems with stochastic demand arrivals using fluid models, focusing on stylized network structures (e.g., [75]). In contrast, we consider a general multicommodity network setting with decentralized operators who manage the routing of their own demand, and study the design of a fixed robust mechanism that can coordinate the network under any demand realizations.

The challenge to manage individual incentives under information uncertainty is also recognized and addressed in the literature under different contexts. For example, [16] study the existence of robust incentive compatible mechanisms in the framework of Bayesian games. The issue of managing cooperative system under data uncertainty is also addressed in an inventory-based context using multi-stage stochastic programs, mainly focusing on collaborative recourse strategies among individuals and profit allocation schemes [63, 124]. In the routing game literature, there is a set of papers

that study stochastic network setting with uncertainties in travel time. For example, [120] show that a road toll mechanism can be used to induce risk-aversive network users towards a robust social optimal routing, i.e., a fixed routing that minimizes the worst-case congestion. This study is different from ours in the way that uncertainty is modeled: The authors study the setting where the central authority and the users face the same level of uncertainty in travel time, while in our model demand information is assumed to be asymmetric between the two parties. Due to such information asymmetry, it is generally implausible to induce a fixed routing solution under all demand scenarios in our problem, and we adopt a different approach in this study that allows for the computation of prices under which a different routing may be obtained in each demand scenario.

3.5.1 Model Description

In this study, we consider a stochastic combined network with uncertain demand. To this end, in this subsection, we first incorporate a demand uncertainty model into the multicommodity network model introduced in §3.2. We also generalize the perfect coordinating prices concept to such a stochastic setting by defining robust perfect coordinating prices. We then illustrate the robustness problem using an example, from which we derive intuition that sheds light on the general robustness analysis presented in the rest of this section.

We model the uncertainty in the routing demand of commodities by assuming the demand vector $\mathbf{d} \doteq \{d_{(o,d,i)}, \forall (o,d,i) \in D\}$ to be a discrete multivariant random variable with a sample space of size K, denoted by $\Omega = \{\mathbf{d}^1, \mathbf{d}^2, ..., \mathbf{d}^K\}$. Each element $\mathbf{d}^k$ in Ω is termed a *demand scenario*, where the demand level for each commodity (o,d,i) equals $d^k_{(o,d,i)}$. Given each $\mathbf{d}^k$, the maximum total routing revenue achievable without violating capacity limits can be computed as the optimal value of a weighted max-flow problem over the entire network G as if the routing of all commodities

can be centrally dictated. We call this problem the *centralized problem* in demand scenario k, denoted by (C^k) and formulated as follows.

$$(C^k) \qquad R(f) = \max \sum_{(o,d,i) \in D} r_{(o,d,i)} \cdot f_{(d,o,i)}^{(o,d,i)} \tag{181}$$

$$s.t. \sum_{\{e \in \delta^-(v)\}} f_e^{(o,d,i)} - \sum_{\{e \in \delta^+(v)\}} f_e^{(o,d,i)} = 0 \quad \forall v \in V \quad \forall (o,d,i) \in D \tag{182}$$

$$\sum_{(o,d,i) \in D} f_e^{(o,d,i)} \leq c_e \quad \forall e \in E \tag{183}$$

$$f_{(d,o,i)}^{(o,d,i)} \leq d_{(o,d,i)}^k \quad \forall (o,d,i) \in D \tag{184}$$

$$f \geq 0 \,. \tag{185}$$

The optimal solution to (C^k), denoted by f_k^*, is defined as the *social optimal routing* in demand scenario k, and represents the highest routing efficiency in that scenario. Hence, a natural goal of designing the capacity exchange mechanism in such a stochastic setting is to induce the individual routing decisions towards f_k^* in each demand scenario k.

The basic mechanics of the capacity exchange mechanism remain the same in the stochastic setting. It provides monetary side-payments to the players, and thus creates incentives for them to behave in a socially optimal manner. However, the implementation of the mechanism is affected by the demand uncertainty and information asymmetry between the mechanism designer and the individual decision makers. In order to highlight such an impact, we describe the sequence of events involved as follows.

The first step is the design of the mechanism, which is assumed to occur at the beginning of the planning horizon before the actual demand is observed. The central authority is assumed to have full information of the sample space Ω of the demand variable[3]. To design the mechanism, the authority chooses a unit exchange price

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[3]In section 3.5.3 we also assume that the authority knows the probability distribution over the sample space.

$cost_e \geq 0$ for the capacity on each edge e in the network.

In the second step, the demand scenario is revealed, according to which the players decide the routing for their commodities and exchange capacity, aiming at the best individual benefit. We adopt the same behavioral model considered previously, i.e., each player searches for a routing for all commodities over the entire network that maximizes his own profit, including the routing revenue obtained from satisfying his own commodity demand and the profit from capacity exchanges. Mathematically, given a demand scenario $\mathbf{d}^k$, each player i solves the following program, which we call the *individual problem under demand scenario* k (P_i^k)

$$(P_i^k) \quad \sum_{(o,d,i) \in D^i} r_{(o,d,i)} \cdot f_{(d,o,i)}^{(o,d,i)} + \sum_{e \in E} cost_e \left[\gamma_e^i \sum_{(o,d,i) \notin D^i} f_e^{(o,d,i)} - (1 - \gamma_e^i) \sum_{(o,d,i) \in D^i} f_e^{(o,d,i)} \right] \quad (186)$$

$$s.t. \quad (183) - (185) \text{ in problem } (C^k) \quad (187)$$

and route his own commodities (i.e., those in the set D^i) according to the *individual optimal routing* f_i^{k*}. It is obvious that the most desirable case is to design the mechanism such that $f_i^{k*} = f_k^* \ \forall i \in N$, i.e., all players are induced to the social optimal routing, in every demand scenario. This motivates the following definition, which generalizes the concept of the perfect coordinating prices in the deterministic case considered in the previous sections.

Definition 8. *A set of capacity exchange prices* $\{cost_e\}$ *is called* perfect coordinating *in demand scenario* k *if* $f_i^{k*} = f_k^* \ \forall i \in N$. *The exchange prices are called* robust perfect coordinating *if the above condition holds for all* $\mathbf{d}^k \in \Omega$.

As we have mentioned in §3.2, from the literature, we know that perfect coordinating prices exist in any combined network given a *fixed* demand scenario [2]. The primary method to show this result uses techniques from inverse optimization framework, which studies the design of parameters in optimization problem so that a target solution becomes optimal (e.g., [4]). Specifically, in the deterministic case, the perfect

coordinating prices are characterized by a set of dual feasibility and complementary slackness constraints of the individual problems with respect to the social optimal routing under the given demand scenario. According to optimization theory, these constraints only depend on the *basis*[4] of the social optimal routing, which, in our problem, can be fully characterized by the corresponding sets of (i) partially-used edges, (ii) commodities with partially-fulfilled demand, and (iii) strictly positive flow variables. Hence, we can easily derive the following result for the existence of robust perfect coordinating under demand uncertainty.

Proposition 12. *Given a combined network G and a set of demand scenarios Ω, robust perfect coordinating prices exist if $\forall \mathbf{d}^k \in \Omega$, the social optimal routing f_k^* has the same basis, i.e., the sets of $\bar{E}^k \doteq \{e \in E : \sum_{(o,d,i) \in D} (f_k^*)_e^{(o,d,i)} < c_e\}$ (partially-used edges), $\bar{D}^k \doteq \{(o,d,i) \in D : (f_k^*)_{(d,o,i)}^{(o,d,i)} < d_{(o,d,i)}^k\}$ (commodities with partially-fulfilled demand), and $\bar{F}^k \doteq \{(f_k^*)_e^{(o,d,i)} : (f_k^*)_e^{(o,d,i)} > 0\}$ (strictly positive flow variables) are identical for all k.*

Not surprisingly, the situation becomes complicated when the condition in the above proposition is violated, as robust perfect coordinating prices must simultaneously satisfy multiple sets of conditions with respect to different bases under the social optimal routings in different demand scenarios. These sets of conditions may contradict with each other and thus no robust perfect coordinating prices exist. In this case, the social optimal routing cannot be guaranteed for every possible demand scenario via a fixed capacity exchange mechanism: The individual decisions would lead to inefficient routing of commodities (e.g., inefficient route choices) and/or overflow on edges when directly aggregated. We illustrate such a situation in the following example, focusing on the overflow problem.

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[4]The basis of a solution to a linear program is defined as the set of nonzero variables under the standard form of the program, i.e., in the form $\min\{c^T x | Ax = b, x \geq 0\}$ (refer to [18] for details).
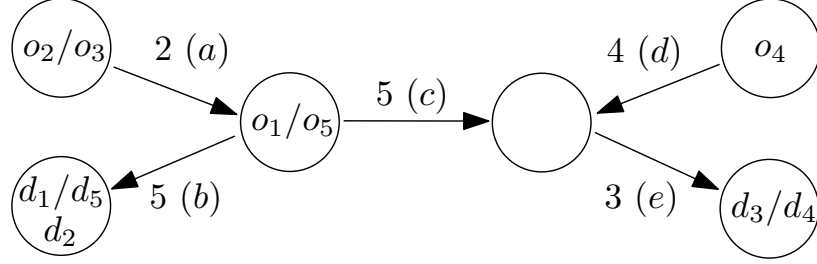
Figure 26: The network in Example 5

Example 5. Consider the network shown in Figure 26 with two players. The number on each edge is the total amount of capacity available (the letter after the number is used to denote the edge in the discussion.) Player I and II own all the capacity on edges a, c and edges d, e respectively, and share the capacity on edge b equally. There are five commodities; the origin and destination node of each commodity i is denoted by o_i and d_i respectively in the figure. Player I owns commodities 1, 2 and 3, and player II owns commodities 4 and 5. The unit revenue of the commodities are $r_1 = r_5 = 5$, $r_2 = 1$, $r_3 = 4$ and $r_4 = 2$. Two demand scenarios are considered, where the demand of commodity 1 and 5 increases from $d_1^1 = d_5^1 = 1$ (scenario 1) to $d_1^2 = d_5^2 = 3$ (scenario 2), while the demand of other commodities remains the same, i.e., $d_2^1 = d_2^2 = 4$, $d_3^1 = d_3^2 = 2$ and $d_4^1 = d_4^2 = 5$.

We show that robust perfect coordinating prices do not exist in this example. In fact, it can be observed that any set of perfect coordinating prices in one demand scenario leads to edge overflow in the other. To see this, first consider scenario 1 where it is social optimal to route 2 and 1 unit(s) of commodity 3 and 4 respectively. In order to induce player II to adopt this routing, the capacity exchange prices must satisfy $\frac{1}{2}cost_b + 2 \leq cost_e$, since otherwise routing 2 and 3 units of commodity 2 and 4 respectively instead of commodity 3 is profitable for player II. Meanwhile, $cost_e \leq r_3 = 4$ must also hold as otherwise player I pays a net cost to route commodity 3 and would refrain from doing it. These two constraints together lead to $cost_b \leq 4$. However, in scenario 2, such prices motivate both players to route 3 units of demand

of their own commodity through edge b (i.e., commodity 1 and 5 respectively), which creates 1 unit of overflow on b. On the other hand, we can calculate that in demand scenario 2, prices must satisfy $cost_b = r_1 = r_5 = 5$ and $cost_e \leq r_3 = 4$ to be perfect coordinating. Under these two conditions, it can be observed that in scenario 1, player II is induced to route 3 units of commodity 4 since $\frac{1}{2}cost_b + 2 > cost_e$, while player I continues to route 2 units of commodity 3. This leads to 2 unit of overflow on edge e.

The non-existence of robust perfect coordinating prices in the above example indicates limitation of a fixed capacity exchange mechanism in coordinating the combined network shown in Figure 26 under demand uncertainty. This motivates us to ask the following questions: What factors influence the robustness of the capacity exchange mechanism? How is such influence originated considering the nature of the mechanism? We investigate these questions in the next section.

3.5.2 Limitation of the Mechanism's Robustness due to Underlying Network Structure

In order to build intuition for more general discussions, we start this section by analyzing the elements that contribute to the non-existence of robust perfect coordinating prices in Example 5. From our description of the example, it can be observed that in demand scenario 1, the exchange price should be low on edge b (i.e., no more than 4) to be perfect coordinating; yet in scenario 2, it requires a relatively high price on b (i.e., equal to 5) to induce both players towards the social optimal routing. To see how such a distinction occurs, note that in scenario 1, the exchange price on edge b is designed to divert player II from routing commodity 2, while in scenario 2 the role of $cost_b$ is to coordinate the individual routing decisions regarding commodity 1 and 5. The underlying factor that makes it impossible to accomplish both goals using the same price is that commodity 2 generates less unit revenue compared to commodity 1 and 5 yet uses more capacity to route. In other words, path $a - b$ is less efficient than path b, and the robustness of the capacity exchange mechanism is undermined in

220

the presence of this inefficient path (which is not even used under the social optimal routing in either demand scenarios). We generalize the above observation regarding the influence of path structure in the network in §3.5.2.1.

3.5.2.1 Network Structure and the Existence of Robust Perfect Coordinating Prices.

The key observation in this subsection is that the robustness of the capacity exchange mechanism can be reinforced in networks where inefficient paths, such as path $a - b$ in Example 5, do not exist. In particular, in this case, we can guarantee the existence of robust perfect coordinating prices under certain homogeneity conditions of the individual capacity ownership levels over the network. To present the result, we first introduce a definition that formally characterizes inefficient paths in a network. Let $\mathcal{P}_{(o,d,i)}$ be the set of all paths that go from the origin node o to the destination node d for commodity (o, d, i), and let $\mathcal{P} = \bigcup_{(o,d,i) \in D} \mathcal{P}_{(o,d,i)}$.

Definition 9. *Given a combined network G and the unit routing revenue $r_{(o,d,i)}$ of all commodities, a set of paths $\mathcal{P}_1 = \{p_t\} \subset \mathcal{P}$ in G is* dominated *by another set of paths $\mathcal{P}_2 = \{p_s\} \subset \mathcal{P}$ (called the* dominating *path set) if there exist positive constants $\{\lambda_t > 0\}$ and $\{\mu_s > 0\}$ such that*

$$\sum_{(o,d,i) \in D} \sum_{t:p_t \in \mathcal{P}_1 \cap \mathcal{P}_{(o,d,i)}} \lambda_t r_{(o,d,i)} \quad < \quad \sum_{(o,d,i) \in D} \sum_{s:p_s \in \mathcal{P}_2 \cap \mathcal{P}_{(o,d,i)}} \mu_s r_{(o,d,i)} \tag{188}$$

$$\sum_{t:p_t \in \mathcal{P}_1} \lambda_t \chi_{\{e \in p_t\}} \quad \geq \quad \sum_{s:p_s \in \mathcal{P}_2} \mu_s \chi_{\{e \in p_s\}} \quad \forall e \in E, \tag{189}$$

where χ is an indicator function.

To interpret the above definition, regard the constants $\{\lambda_t\}$ and $\{\mu_s\}$ as the amount of flow routed through paths in $\mathcal{P}_1$ and $\mathcal{P}_2$ respectively. Definition 9 essentially says that by choosing the routing scheme $\{\lambda_t\}$ along paths in $\mathcal{P}_1$ over the one $\{\mu_s\}$ with respect to $\mathcal{P}_2$, strictly less routing revenue is generated (inequality (188)) using no less capacity on any edge in the network (inequality (189)), indicating inefficiency of paths in $\mathcal{P}_1$. Note that the definition of path dominance in this

221

section is different from that in Definition 5 in §3.4.

Theorem 15. *Given a combined network G where for each player i, the capacity ownership levels are identical on all edges, i.e., $\gamma_{e^1}^i = \gamma_{e^2}^i \ \forall e^1, e^2 \in E$, robust perfect coordinating prices $\{cost_e\}$ exist for any set of demand scenarios Ω if there are no dominated path sets in G.*

We prove this theorem in a constructive way where we propose a method to compute a set of robust perfect coordinating prices, and show the existence of such prices are related to the non-existence of dominated paths by duality theory. Please refer to Appendix 3.5.5 for details. Note that Theorem 15 assumes the network condition of *uniform* capacity ownership for every user. Such a setting is relevant in practical situations where the ownership levels of an individual are not calculated edge by edge but are determined according to some aggregate measure over the entire network. For example, in some transportation networks, the capacity ownership of player i on each edge depends on the proportion of i's asset contribution compared to the total assets deployed in the entire network.

To see how the undermining effect of dominated path structures on the robustness of the capacity exchange mechanism arises, note that the incentives provided by the mechanism is based on capacity payments. Hence, while dominating paths are preferred under the social optimal routings due to higher routing revenues associated with them, the capacity owners tend to favor the dominated ones as they use more capacity, which, under a homogeneous exchange price on each edge, imply a larger capacity payment. Consistency between capacity payment and revenue is crucial for the effectiveness of a capacity exchange mechanism especially under uncertainty. Indeed, it can be observed that when dominated paths do not exist, the robust perfect coordinating prices constructed in the proof to Theorem 15 are such that the total unit price of the capacity along a path equals the unit revenue of the corresponding

222

commodity. We can also use a similar argument to explain the impact of the individual capacity ownership levels in the network on the robustness of the mechanism: Heterogeneous capacity ownership levels imply different potential for capacity payments to the player over the edges and hence bias his evaluation of the profitability of network capacity. Such individual biases are hard to coordinate using a fixed set of prices given different demand levels.

3.5.2.2 Enhancing the Robustness of the Capacity Exchange Mechanism.

In this section, we discuss an approach to enhance the robustness of the capacity exchange mechanism by commodity-based price discrimination. The approach is motivated by our analysis in the previous section: It works by designing prices to reflect the revenue of routing each commodity through the corresponding edges and thus encountering the negative influence of dominated paths.

Price discrimination has been a traditional topic in economic theory literature as an important pricing strategy (e.g., [129]). Recently, it has inspired intensive research in operations research on the design and implementation of such strategies from both a theoretical and practical point of view, e.g., in service industries [98]. The heterogeneous pricing principle is also studied in the setting of communication network control. For example, [159] study the network equilibrium in the presence of multiple congestion control protocols that respond to different pricing signals.

Given the setting of the capacity exchange mechanism on a combined network, we propose a commodity-based pricing model as follows. On each edge e, we let $cost_e^{(o,d,i)}$ to be the unit exchange price charged for the routing of commodity (o,d,i). Under these prices, the profit of every player i is calculated using a formula different from (187).

$$\sum_{(o,d,i)\in D^i} r_{(o,d,i)} \cdot f_{(d,o,i)}^{(o,d,i)} + \sum_{e\in E}\left[\gamma_e^i \sum_{(o,d,i)\notin D^i} cost_e^{(o,d,i)} f_e^{(o,d,i)} - (1-\gamma_e^i)\sum_{(o,d,i)\in D^i} cost_e^{(o,d,i)} f_e^{(o,d,i)}\right],$$

$$(190)$$

and the individual problems (P_k^i) is changed accordingly. The definition of being robust perfect coordinating for such heterogeneous prices is the same as for homogeneous ones in Definition 8. In the next theorem, we show that the existence of robust perfect coordinating prices is not affected by the existence of dominated paths in the network if such a commodity-based pricing strategy is adopted.

Theorem 16. *Given a combined network G where for each player i, the capacity ownership levels are identical on all edges, i.e., $\gamma_{e^1}^i = \gamma_{e^2}^i \ \forall e^1, e^2 \in E$, heterogeneous robust perfect coordinating prices $\{cost_e^{(o,d,i)}\}$ exist for any set of demand scenarios Ω.*

An important result that leads to the above theorem is as follows.

Lemma 10. *Dominated paths do not exist in any single-commodity network with one source-sink pair.*

The proof of lemma 10 is based on Definition 9 of dominated paths and the max-flow-min-cut theorem in single-commodity networks. Since commodity-based pricing is essentially equivalent to designing a set of exchange prices over the network considering the routing between each source-sink pair separately, it is easy to see that Theorem 16 follows from Theorem 15 and Lemma 10. We also mention that the observation made in Lemma 10 is enlightening by itself as it sheds light on the potential link between the existence of dominated path structures and the max-flow-min-cut relation in a multicommodity network, which is an interesting direction for future study.

It should be noted that the heterogeneous pricing approach trades price equity for the robustness of the capacity exchange mechanism. Next, we investigate the minimization of such price differentiation while guaranteeing the prices to be robust perfect coordinating. We measure the degree of price discrimination under a set of heterogeneous prices $\{cost_e^{(o,d,i)}\}$ by the largest price difference on the same edge.

$$\Delta_{cost} = \max_{e \in E} \max_{(o,d,i)_1, (o,d,i)_2 \in D} \left| cost_e^{(o,d,i)_1} - cost_e^{(o,d,i)_2} \right|. \tag{191}$$

224

Theorem 15 implies that under the condition of uniform capacity ownership levels, robust perfect coordinating prices can be designed such that $\Delta_{cost} = 0$ (i.e., homogeneous) if no dominated paths exist. We extend this observation by showing that in networks with a general path structure, the minimum degree of price discrimination under the heterogeneous robust perfect coordinating prices is upper bounded by a certain normalized measure of the maximum inefficiency of dominated paths in the network. Mathematically, given any path set $\mathcal{P}_1$ dominated by another $\mathcal{P}_2$, let $\varphi(\mathcal{P}_1, \mathcal{P}_2)$ be the absolute difference between the right- and left-hand-side in inequality (188) in Definition 9, which calculates the additional revenue achieved if capacity is used for routing along paths in $\mathcal{P}_2$ instead of $\mathcal{P}_1$ under the corresponding parameters $\{\lambda_t\}$ and $\{\mu_s\}$. We also compute the difference in the amount of capacity used on edge e for routing each commodity (o, d, i), denoted by $\kappa_e^{(o,d,i)}(\mathcal{P}_1, \mathcal{P}_2) \doteq \sum_{p_s \in \mathcal{P}_2 \cap \mathcal{P}_{(o,d,i)}} \mu_s \chi_{\{e \in p_s\}} - \sum_{p_t \in \mathcal{P}_1 \cap \mathcal{P}_{(o,d,i)}} \lambda_t \chi_{\{e \in p_t\}}$. We measure the *normalized inefficiency* of the dominated path set $\mathcal{P}_1$ with respect to $\mathcal{P}_2$ by the following function

$$\Phi(\mathcal{P}_1, \mathcal{P}_2) = \frac{\varphi(\mathcal{P}_1, \mathcal{P}_2)}{\sum_{e \in E} \sum_{(o,d,i) \in D} \max\{\kappa_e^{(o,d,i)}(\mathcal{P}_1, \mathcal{P}_2), 0\}} . \tag{192}$$

Note that the denominator in (192) is guaranteed to be strictly positive, since otherwise we can find at least one commodity in D and derive dominated paths on the corresponding single commodity network, which is a contradiction to Lemma 10.

Theorem 17. *Given a combined network G where for each player i, the capacity ownership levels are identical on all edges, i.e., $\gamma_{e^1}^i = \gamma_{e^2}^i \ \forall e^1, e^2 \in E$, the least price discriminating heterogeneous robust perfect coordinating prices $\{cost_e^{(o,d,i)}\}$ satisfy*

$$\Delta_{cost} \leq \max_{\mathcal{P}_1 \text{ dominated by } \mathcal{P}_2} \Phi(\mathcal{P}_1, \mathcal{P}_2) \tag{193}$$

for any set of demand scenarios Ω.

We close this section by mentioning a research direction on minimizing another

dimension of price discrimination under heterogeneous exchange prices in a multi-commodity network, i.e., the number of different sets of prices employed. Note that even if homogeneous robust perfect coordinating prices do not exist, it is possible to coordinate a *subset* of the commodities using the same prices. This motivates the following combinatorial problem: How to pack the commodities into the smallest number of subsets, for each of which a single set of prices is designed, such that the resulting prices are guaranteed to be robust perfect coordinating? Inspired by our discussion on the impact of path dominance in this section, an initial step is to analyze a network decomposition problem, i.e., partitioning the commodity set D such that the sub-network corresponding to each subset is free of dominated path sets.

3.5.3 Mechanism Design in General Networks under Demand Uncertainty

In the previous section, we investigate the influence of network structure on the robustness of a capacity exchange mechanism. While this analysis provides insights into the nature and effectiveness of the mechanism under demand uncertainty, two practical problems remain unsolved. First, since the conditions given in both Theorem 15 and 16 are quite restrictive considering most practical applications, a natural question is how robust the mechanism can be in a general network. The second question concerns the computation of the exchange prices. Although the proof to Theorem 15 provides a way to construct robust perfect coordinating prices under certain network structures (i.e., compute prices such that the total unit price along each path equals the unit revenue of the corresponding commodity), solving an equation system of exponential size is not an efficient choice. Extending the question to general networks, we are interested in developing an efficient method to compute prices that guarantee certain robustness properties for the resulting mechanism.

In literature, [2] provide a polynomial method to compute perfect coordinating

prices given a static network with fixed demand using the inverse optimization framework. Specifically, as we have briefly described in §2.3.2, a set of dual and complementary slackness conditions of the individual problems with respect to the social optimal routing, called an *inverse problem*, are used to characterize such prices. The constraint set has a polynomial size and is guaranteed to be feasible in the static setting, thus can be directly solved by linear programming techniques. We generalize this approach to the stochastic setting considered in this paper by first combining all the inverse problems that can arise under a given set of demand scenarios Ω. However, the central difficulty is that the resulting constraint set, which obviously fully defines robust perfect coordinating prices, is usually infeasible, as illustrated by Example 5 and our discussion in the previous section, meaning that the social optimal routing cannot be achieved in every scenario under one set of prices. The contribution of our algorithm is to resolve this situation by identifying a set of new routings, one for each individual problem (P_i^k), with respect to which the new inverse problem is feasible. Each of these new routings is obtained by augmenting the social optimal routing in the corresponding demand scenario. The augmenting directions are computed by a program specially designed using the dual program of the original infeasible inverse problem in order to maintain a certain level of efficiency under the new routings[5]. To do this, the algorithm requires a set of weight parameters associated with the demand scenarios that represent their relative importance in the central authority's price design decision. This is an intuitive approach since the algorithm essentially tries to manage multiple objectives using one set of variables. Under the context of uncertainty, a natural candidate for the weight parameters is the probabilities that the potential scenarios will occur.

Formally, we assume for the rest of this section a prior knowledge of the probability

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[5][4] recognize that the dual of an inverse problem can be formulated so that every feasible point of it represents a feasible way to alter the target solution based on which the inverse problem is defined. They call this dual problem the *0-centered dual inverse problem*.

distribution of the demand variable $\mathbf{d}$ over its sample space Ω. Mathematically, let p^k be the probability that $\mathbf{d} = \mathbf{d}^k$, i.e., demand scenario k occurs. We focus on designing *homogeneous* exchange prices that are unique on every edge, and Algorithm 5 outlines the steps to do this in a general network. First, we denote the dual variables associated with the constraints in each individual problem (P_i^k) as $[\pi_i^k, \alpha_i^k, \beta_i^k]$. We let Inv_i^k denote the inverse problem with respect to the social optimal routing f_k^* and an individual problem (P_i^k), i.e., the following set of linear constraints with $[\pi_i^k, \alpha_i^k, \beta_i^k, cost_e]$ as variables.

$$(\pi_i^k)_v^{(o,d,i)} - (\pi_i^k)_u^{(o,d,i)} + (\alpha_i^k)_e \geq -(1-\gamma_e^i)cost_e \quad (o,d,i) \in D^i, e \in E : (f_k^*)_e^{(o,d,i)} \notin \bar{F}^k \quad (194)$$

$$(\pi_i^k)_v^{(o,d,i)} - (\pi_i^k)_u^{(o,d,i)} + (\alpha_i^k)_e = -(1-\gamma_e^i)cost_e \quad (o,d,i) \in D^i, e \in E : (f_k^*)_e^{(o,d,i)} \in \bar{F}^k \quad (195)$$

$$(\pi_i^k)_v^{(o,d,i)} - (\pi_i^k)_u^{(o,d,i)} + (\alpha_i^k)_e \geq \gamma_e^i cost_e \quad (o,d,i) \notin D^i, e \in E : (f_k^*)_e^{(o,d,i)} \notin \bar{F}^k \quad (196)$$

$$(\pi_i^k)_v^{(o,d,i)} - (\pi_i^k)_u^{(o,d,i)} + (\alpha_i^k)_e = \gamma_e^i cost_e \quad (o,d,i) \notin D^i, e \in E : (f_k^*)_e^{(o,d,i)} \in \bar{F}^k \quad (197)$$

$$(\pi_i^k)_o^{(o,d,i)} - (\pi_i^k)_d^{(o,d,i)} + (\beta_i^k)_{(o,d,i)} \geq r_{(o,d,i)} \quad (o,d,i) \in D^i : (f_k^*)_{(d,o,i)}^{(o,d,i)} \notin \bar{F}^k \quad (198)$$

$$(\pi_i^k)_o^{(o,d,i)} - (\pi_i^k)_d^{(o,d,i)} + (\beta_i^k)_{(o,d,i)} = r_{(o,d,i)} \quad (o,d,i) \in D^i : (f_k^*)_{(d,o,i)}^{(o,d,i)} \in \bar{F}^k \quad (199)$$

$$(\pi_i^k)_o^{(o,d,i)} - (\pi_i^k)_d^{(o,d,i)} + (\beta_i^k)_{(o,d,i)} \geq 0 \quad (o,d,i) \notin D^i : (f_k^*)_{(d,o,i)}^{(o,d,i)} \notin \bar{F}^k \quad (200)$$

$$(\pi_i^k)_o^{(o,d,i)} - (\pi_i^k)_d^{(o,d,i)} + (\beta_i^k)_{(o,d,i)} = 0 \quad (o,d,i) \notin D^i : (f_k^*)_{(d,o,i)}^{(o,d,i)} \in \bar{F}^k \quad (201)$$

$$(\alpha_i^k)_e = 0 \quad \forall e \in \bar{E}^k; \quad (\beta_i^k)_{(o,d,i)} = 0 \quad \forall (o,d,i) \in \bar{D}^k \quad (202)$$

$$nonnegativity \quad constraints. \quad (203)$$

Note that the sets $\bar{E}^k$, $\bar{D}^k$ and $\bar{F}^k$ in the above formulas are as defined in Proposition 12. By duality theory, every feasible solution to Inv_i^k identifies a set of prices under which $f_i^{k*} = f_k^*$.

It is easy to see that Algorithm 5 is polynomial. We can also guarantee that the algorithm outputs a set of prices by the following proposition.

Proposition 13. *In Algorithm 5, the constraint set $\bar{Inv} = \bigcup_{k=1}^{K} \bigcup_{i \in N} \bar{Inv}_i^k$ where each $\bar{Inv}_i^k$ is defined with respect to the routing $\bar{f}_i^k$ is guaranteed to be feasible.*

ALGORITHM 5: Price design under demand uncertainty in a general combined network

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Input: A combined network G, a set of demand scenarios Ω and the probabilities $\{p^k\}$ on Ω

Output: A set of capacity exchange prices $\{cost_e\}$

Solve the constraint set $Inv = \bigcup_{k=1}^{K} \bigcup_{i \in N} Inv_i^k$

if *a feasible solution $\{cost_e\}$ is found* **then**

    $\{cost_e\}$ is robust perfect coordinating. Output $\{cost_e\}$ and the algorithm stops.

else

    1. Find an optimal solution $\{\delta f_i^{k*}\}$ to the following program

$$(DInv) \quad \max \quad \sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} \cdot \sum_{k=1}^{K} p^k \cdot (\delta f_i^k)_{(d,o,i)}^{(o,d,i)}$$

$$s.t. \quad \delta f_i^k \text{ is a feasible augmenting flow w.r.t. } f_k^* \quad \forall i \in N \quad \forall k = 1, 2, ..., K$$

$$\sum_{i \in N} \sum_{(o,d,i) \in D^i} \sum_{k=1}^{K} p^k \cdot (\delta f_i^k)_e^{(o,d,i)} \leq \sum_{i \in N} \gamma_e^i \cdot \sum_{(o,d,j) \in D} \sum_{k=1}^{K} p^k \cdot (\delta f_i^k)_e^{(o,d,j)} \quad \forall e \in E \,.$$

    2. Calculate the routing $\bar{f}_i^k = f_k^* + \delta f_i^{k*} \ \forall i \in N \ \forall k = 1, 2, ..., K$. Solve the constraint set $I\bar{n}v = \bigcup_{k=1}^{K} \bigcup_{i \in N} I\bar{n}v_i^k$ where each $I\bar{n}v_i^k$ is defined with respect to the routing $\bar{f}_i^k$, and output any feasible solution $\{cost_e\}$.

end

In summary, Algorithm 5 computes in polynomial time a set of prices that is either robust perfect coordinating as long as such prices exist, or coordinates every player i towards the routing $\bar{f}_i^k$ in each demand scenario k, i.e., the individual optimal routing $f_i^{k*} = \bar{f}_i^k \ \forall i \in N \ \forall \mathbf{d}^k \in \Omega$. Note that in the latter situation, it is not guaranteed that given any demand scenario $\mathbf{d}^k$, all players are induced to the same routing as in the definition of perfect coordinating prices. Hence, we introduce a new notation f'_k to denote the (direct) aggregate routing in this case, defined such that $\forall (o, d, i) \in D^i$, $(f'_k)_e^{(o,d,i)} = (\bar{f}_i^k)_e^{(o,d,i)} \ \forall e \in E$. Moreover, we call the prices computed by Algorithm 5 *partial coordinating* with respect to f'_k in demand scenario k, since they only motivate each player i to follow f'_k for his own commodities.

In the following analysis, we evaluate the potential total routing revenue implied by f'_k, i.e., $R(f'_k)$ as computed in the objective function (181) of the centralized problem (C^k). Furthermore, we also note that partial coordination is not as strong as perfect

coordination; one direct consequence is that the aggregate routing f'_k may violate capacity limits and cause overflow on the edges. Our next theorem provides analytical results on both the revenue and capacity violation implications of the routings $\{f'_k\}$. These results characterize the effectiveness of the capacity exchange mechanism under demand uncertainty in networks that do not admit robust perfect coordinating prices. In the theorem, we adopt a relative capacity usage measure, i.e., the ratio between the total flow routed on an edge e and its capacity, to evaluate the degree of capacity violation in a network[6].

Theorem 18. *Consider a combined network G where robust perfect coordinating prices do not exist, and a set of demand scenarios Ω with probabilities $\{p^k\}$. Algorithm 5 computes a set of capacity exchange prices under which the induced aggregate routing f'_k in a demand scenario k satisfies*

1. *the expected total revenue under $\{f'_k\}$ is no less than that under the social optimal routings $\{f^*_k\}$, i.e.,*

$$\sum_{k=1}^{K} p^k \cdot R(f'_k) > \sum_{k=1}^{K} p^k \cdot R(f^*_k) \qquad (204)$$

2. *the maximum relative capacity usage over all edges in G under f'_k satisfies*

$$\max_{e \in E} \frac{1}{c_e} \cdot \sum_{(o,d,i) \in D} (f'_k)_e^{(o,d,i)} \leq \frac{1}{p_k} \quad \forall k = 1, 2, ..., K . \qquad (205)$$

We briefly mention the proof to Theorem 18. Inequality (204) in result (1) is due to the way that the objective function of the program $(DInv)$ in Algorithm 5 is designed, i.e., the program aims at maximizing the change in the expected total revenue in search for qualified augmenting flows. Result (2) can be proven by showing that the expected routing $\sum_{k=1}^{K} p^k \cdot f'_k$ is a feasible one in terms of capacity usage, which is due to the constraints of $(DInv)$. Please refer to Appendix 3.5.5 for details.

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[6]The same formula is used to calculate the *congestion ratio* on an edge in the congestion game literature that studies a different setting where all demand is to be routed through the network, and exceeding the capacity results in lengthening routing delay.

Theorem 18 provides a lower bound of the network efficiency under the direct aggregation of the individual routings induced under the prices calculated by Algorithm 5 in the unfavorable cases where the network does not admit robust perfect coordinating prices. However, note that for any demand scenario k, the total revenue $R(f'_k)$ may not be readily achievable in the given network due to the existence of overflow under f'_k; yet we observe that in any network, the prices guarantee a lower degree of capacity violation in demand scenarios that are more likely to occur. Both of the results are due to the way that the program $(DInv)$ is constructed in Algorithm 5. Below we provide an illustration of Theorem 18 using the network setting in Example 5.

Example 6. Consider the combined network presented in Example 5 where robust perfect coordinating prices do not exist. We show that exchange prices can be designed in this network to induce routings that satisfy both (204) and (205). We discuss two cases. First, assume the probability associated with scenario 1 $p^1 \geq \frac{1}{4}$. We design the prices to be perfect coordinating in scenario 1, e.g., $cost_b = 1$ and $cost_e = 3$ (since capacity exchanges only occur on edge b and e), which will induce the social optimal routing in scenario 1. If scenario 2 occurs, it can be calculated that such prices lead to the aggregate routing f'_2 such that 3 units of both commodity 1 and 5, as well as 2 and 1 unit(s) of commodity 3 and 4 respectively, are routed. This results in a higher revenue in scenario 2 (thus a higher expected revenue since the social optimal routing is achieved in scenario 1), yet an overflow on edge b. However, the relative capacity usage equals $\frac{6}{5} < \frac{1}{p_2}$, and the expected flow on edge b can be calculated to be no more than the amount of capacity there.

Consider the other case where $p^1 < \frac{1}{4}$ and the exchange prices $cost_b = 5$ and $cost_e = 4$. It can be calculated that these prices induce the following aggregate routing in each demand scenario (here we simplify the notation and use $(f'_k)^j$ to denote the amount of commodity j routed under f'_k). In scenario 1, $(f'_1)^1 = (f'_1)^5 = 1$, $(f'_1)^3 = 2$

231

and $(f_1')^4 = 3$; note that due to a high $cost_b$, player II is motivated to give routing priority to commodity 2 and 4 instead of to commodity 3. In scenario 2, the same prices can partially coordinate both players to route their own commodities according to f_2' such that $(f_2')^1 = (f_2')^5 = \frac{3}{2p_2}+1$, $(f_1')^3 = 2-\frac{2p_1}{p_2}$ and $(f_1')^4 = 1$. We calculate that the expected revenue under f_1' and f_2' equals $31p_1 + 35p_2$, which is strictly larger than that under the social optimal routings in these two scenarios, i.e., $20p_1 + 35p_2$, since demand is uncertain and $p_1 > 0$. Moreover, under these two routings, the expected flow on any edge in the network does not exceed its capacity and thus inequality (205) also holds.

The possibility of capacity violation when individual routings of their own commodities are directly aggregated, as indicated by Theorem 18, implies additional complexity in the operations and management of a combined network with capacity exchanges. In particular, from an individual participant's perspective, it can suggest the existence of potential competition for capacity in the network, which is one of the prominent factors that influence one's operational strategies in practice. From the perspective of the central authority who coordinates the network, the managerial implication of Theorem 18 is the potential necessity of a flow control and/or capacity management mechanism to solve the overflow problem if it does occur. In fact, such mechanisms are widely employed in practice. For example, airlines often adopt policies to redirect passengers to flights other than their initial choices with certain compensations in case of ticket oversales due to the widely-used overbooking policy in the industry [135].

In the rest of this section, we provide a preliminary discussion on the impact of the potential existence of overflow in a general decentralized network under the capacity exchange mechanism studied in this paper, focusing on situations with central flow control. We first present a simple flow synthesizing mechanism based on uniform scaling, and discuss its efficiency implication in eliminating the overflow that

can arise under the capacity exchange prices computed by Algorithm 5. By doing so, we are able to incorporate the negative effect of capacity violation on network efficiency and provide a more comprehensive evaluation of the robustness of the capacity exchange mechanism. We then discuss a number of research directions, including design and analysis of other flow control/capacity management mechanisms, as well as incorporating and studying competitive behavior of individual participants.

A Flow Synthesizing Mechanism   Consider a network where robust perfect coordinating prices do not exist, and the partial coordinating exchange prices computed by Algorithm 5 are used and induce an aggregate routing f'_k in each demand scenario k. A flow synthesizing mechanism regulates how individual routings can be composed into a feasible one to prevent overflow in the network. One intuitive and straightforward method is by *uniform scaling*: A scale factor between 0 and 1 is designed on every edge e, by which the flow of all commodities on e is multiplied so that a feasible *adjusted* aggregate routing is achieved. We analyze the case where the flow synthesizing mechanism is imposed after the individual routings are decided and thus a set of scaling factors $\eta_k = \{\eta_k^e\}$ can be designed accordingly for each demand scenario k. We denote by $\eta^* \doteq \{\eta_k^*\}$ the optimal scaling factors under which the adjusted aggregate routings, denoted by $\{f'_k(\eta_k^*)\}$, maintain the maximum percentage of the expected total routing revenue under $\{f'_k\}$. We can derive the following result based on Theorem 18.

Proposition 14. *Consider a combined network G where robust perfect coordinating prices do not exist, and a set of demand scenarios Ω with probabilities $\{p^k\}$. Algorithm 5 computes a set of capacity exchange prices under which the adjusted aggregate routings by the optimal uniform scaling factors η^* under the flow synthesizing mechanism satisfy*

$$\sum_{k=1}^{K} p^k \cdot R(f'_k(\eta_k^*)) > \min_k p^k \cdot \sum_{k=1}^{K} p^k \cdot R(f_k^*) \tag{206}$$

In words, Proposition 14 indicates that under demand uncertainty, we can guarantee at least $\min_k p^k$ percentage of the highest expected revenue without violating capacity limits in any demand scenario. We discuss two implications of this result. First, this lower bound is the largest when the demand vector $\mathbf{d}$ follows a uniform distribution over the sample space Ω, indicating that the capacity exchange mechanism, with prices designed by Algorithm 5, can be more effective dealing with demand uncertainty that is more "balanced". This is intuitive, since in cases with widely-varied probabilities associated with different demand scenarios, the pricing decision tends to favor the scenarios that are more likely to occur (as modeled by the high weights assigned to these scenarios in Algorithm 5), and can result in low routing efficiency when the "worst-cases", i.e., scenarios with small probabilities, happen. Second, we also note that in the case of uniform distribution, the capacity exchange mechanism, together with a flow synthesizing mechanism based on uniform scaling, can guarantee an expected revenue no less than $\frac{1}{K}$ of the highest level attainable when coordinating K different demand scenarios with one single set of prices. This well illustrates how the robustness level of the mechanism diminishes as the set of potential demand scenarios is enlarged.

3.5.4   Final Remark

Robustness of mechanisms based on market exchange of resources is an important and complex issue. The discussion in this subsection summarizes our first study of the problem in the context of managing decentralized networks with combined capacity under demand uncertainty, and illustrates the challenges in doing so under general network conditions. In particular, the issue of capacity violation opens up a rich set of research questions, of which we briefly discuss as follows. First, the flow synthesizing mechanism studied previously is assumed to be designed *ex post* after the individual routings are determined. A natural extension of the study (and a more practical one)

is to assume that the mechanism is imposed *ex ante* and analyze whether it can divert players from overly utilizing capacity for their own commodities spontaneously. Second, instead of using flow control methods, another common approach used in practice to solve the overflow problem is to purchase additional capacity from the spot market, usually at higher rates than the exchange prices within the centrally-regulated combined network. We can potentially use Theorem 18 to evaluate the profitability of such a capacity outsourcing mechanism; one particular question to investigate is at how high a price the capacity should be purchased to maintain a certain level of the system profit (i.e., total routing revenue − cost of additional capacity). Finally, as we have mentioned, overflow implies competition among participants for capacity, which often results in price premium in practice. In order to incorporate such competitive behavior into the model and capture the interaction between prices and supply/demand balance, we can study a generalization of the capacity exchange mechanism considered in this section, where players choose exchange prices for their own capacity in each demand scenario within a fixed centrally-designed price range on each edge.

3.5.5 Appendix

Proof of Theorem 15. First, we show that under the conditions of the theorem, a set of prices $\{cost_e\}$ exists such that $\sum_{e \in p} cost_e = r_{(o,d,i)} \ \forall p \in \mathcal{P}_{(o,d,i)}$. This is because otherwise the dual problem to the program $\min\{0 \cdot cost | \sum_{e \in p} cost_e = r_{(o,d,i)}, \forall p \in \mathcal{P}_{(o,d,i)}$ and $cost \geq 0\}$ must admit a solution with strictly positive objective value. Mathematically, there exists a set of values $\{\theta_p\}$, which can be either positive or negative, such that

$$\sum_{p \in \mathcal{P}} \chi_{\{e \in p\}} \theta_p \leq 0 \quad \forall e \in E \quad \text{and} \quad \sum_{(o,d,i) \in D} \sum_{p \in \mathcal{P}_{(o,d,i)}} r_{(o,d,i)} \theta_p > 0. \quad (207)$$

Hence, according to Definition 9, the set of paths $\mathcal{P}_1 = \{p : \theta_p < 0\}$ is dominated by the set $\mathcal{P}_2 = \{p : \theta_p > 0\}$, and both sets must be non-empty due to (207), which is a

contradiction to the condition of non-existence of dominated path sets in the network.

We then prove that the prices constructed above is robust perfect coordinating. Given $\{cost_e\}$ such that $\sum_{e\in p} cost_e = r_{(o,d,i)} \ \forall p \in \mathcal{P}_{(o,d,i)}$ and uniform capacity ownership levels $\{\gamma^i\}$, we can rewrite the objective function (187) in each individual program (P_i^k) as

$$(187) = \gamma^i \cdot \sum_{(o,d,i)\in D} r_{(o,d,i)} \cdot f^{(o,d,i)}_{(d,o,i)} = \gamma^i \cdot (181), \tag{208}$$

where (181) is the objective function of the centralized problem (C^k). In other words, the objectives of all individual players are perfectly aligned with that of the central authority. Since (P_i^k) and (C^k) share the same constraints, it is guaranteed that their optimal solutions coincide. Hence, by Definition 8, $\{cost_e\}$ is robust perfect coordinating for any set of demand scenarios. □

Proof of Lemma 10. Consider a single commodity network G with a source o and a sink d. Assume there exists a set of paths $\mathcal{P}_1$ that is dominated by another one $\mathcal{P}_2$ under weights $\{\lambda_t\}$ and $\{\mu_s\}$. Define the capacity on edges in G as

$$c_e = \sum_{p_t\in\mathcal{P}_1} \chi_{\{e\in p_t\}}\lambda_t \quad \forall e \in E. \tag{209}$$

It is easy to show that the value of the minimum cut in G must equal $\sum_{p_t\in\mathcal{P}_1} \lambda_t$. However, by definition of dominated paths, we can utilize the network G capacitated as in (209) to deliver $\sum_{p_s\in\mathcal{P}_2} \mu_s > \sum_{p_t\in\mathcal{P}_1} \lambda_t$ units of flow from o to d, which is a contradiction to the max-flow-min-cut theorem. □

Proof of Theorem 17. We first calculate the minimum degree of price discrimination under heterogeneous prices $\{cost_e^{(o,d,i)}\}$ such that $\sum_{e\in p} cost_e^{(o,d,i)} = r_{(o,d,i)} \ \forall p \in \mathcal{P}_{(o,d,i)}$ as the optimal value of the following program. For notation simplicity, we index the commodities as $l = 1, 2, ..., |D|$, and replace (o, d, i) by l in all notations to indicate

236

their association with the l-th commodity.

$$\min \quad z \tag{210}$$

$$s.t. \quad z - cost_e^{l_1} + cost_e^{l_2} \geq 0 \quad \forall e \in E, \forall l_1 \neq l_2, l_1, l_2 \in \{1, 2, ..., |D|\} \tag{211}$$

$$\sum_{e \in p} cost_e^l = r_l \quad \forall p \in \mathcal{P}_l, \forall l \in \{1, 2, ..., |D|\} \tag{212}$$

$$cost_e^l \geq 0 \quad \forall e \in E, \forall l \in \{1, 2, ..., |D|\}. \tag{213}$$

The dual problem of the above program can be written as

$$\max \quad \sum_{l=1}^{|D|} r_l \sum_{p \in \mathcal{P}_l} \theta_p \tag{214}$$

$$s.t. \quad \sum_{p \in \mathcal{P}_l} \chi_{\{e \in p\}} \theta_p \leq \sum_{l' \neq l} (\zeta_{ll'}^e - \zeta_{l'l}^e) \quad \forall e \in E, \forall l \in \{1, 2, ..., |D|\} \tag{215}$$

$$\sum_{e \in E} \sum_{l_1 \neq l_2} (\zeta_{l_1 l_2}^e + \zeta_{l_2 l_1}^e) = 1 \tag{216}$$

$$\zeta \geq 0. \tag{217}$$

First, adding constraints (215) over all commodities (i.e., all l) generates the condition $\sum_{l=1}^{|D|} \sum_{p \in \mathcal{P}_l} \chi_{\{e \in p\}} \theta_p \leq 0 \ \forall e \in E$. Hence, we conclude that every feasible solution to the dual program that gives rise to a strictly positive objective value corresponds to a path set $\mathcal{P}_1$ dominated by another $\mathcal{P}_2$. Moreover, the θ_p values should satisfy $\theta_p = c \cdot \lambda_t$ if $p = p_t \in \mathcal{P}_1$ and $\theta_p = c \cdot \mu_s$ if $p = p_s \in \mathcal{P}_2$ (others are zero), where c is a constant such that there exist $\{\zeta\}$ values that satisfy

$$\sum_{l' \neq l} (\zeta_{ll'}^e - \zeta_{l'l}^e) \geq c \cdot \kappa_e^l(\mathcal{P}_1, \mathcal{P}_2) \quad \forall e \in E, \forall l \in \{1, 2, ..., |D|\} \tag{218}$$

and (216)-(217). It is easy to see that given $\mathcal{P}_1$, $\mathcal{P}_2$ and the corresponding parameters $\{\lambda_t\}$ and $\{\mu_s\}$, the maximum value of the objective function (214) is obtained by maximizing the constant c. This is equivalent to solving the following minimization problem

$$\min \quad \sum_{e \in E} \sum_{l_1 \neq l_2} (\zeta_{l_1 l_2}^e + \zeta_{l_2 l_1}^e) \quad s.t. \quad \text{(218) with } c \text{ replaced by 1, and (217)}, \tag{219}$$

and set c equal to the inverse of its optimal solution. We again take the dual of the above minimization program and obtain the following.

$$\max \sum_{e \in E} \sum_{l=1}^{|D|} \kappa_e^l(\mathcal{P}_1, \mathcal{P}_2) \cdot \varphi_e^l \quad s.t. \quad \varphi_e^{l_1} - \varphi_e^{l_2} \in [-1, 1] \quad \forall e \in E, \forall l_1 \neq l_2 \text{ and } \varphi \geq 0 \tag{220}$$

The constraints in the above maximization problem indicate that the variables φ_e^l are within an interval of length at most 1 for each edge e. Since we know that $\sum_{l=1}^{|D|} \kappa_e^l(\mathcal{P}_1, \mathcal{P}_2) \leq 0 \ \forall e \in E$, the optimal solution is to set $\varphi_e^l = 0$ if $\kappa_e^l(\mathcal{P}_1, \mathcal{P}_2) \leq 0$ and $\varphi_e^l = 1$ otherwise. This leads to the maximum constant c given $\mathcal{P}_1$ and $\mathcal{P}_2$ (and their corresponding parameters λ and μ)

$$c^*(\mathcal{P}_1, \mathcal{P}_2) = \frac{1}{\sum_{e \in E} \sum_{l=1}^{|D|} \max\{\kappa_e^l(\mathcal{P}_1, \mathcal{P}_2), 0\}} \tag{221}$$

Therefore, we conclude that the optimal value to the program (214)-(217) equals $\max_{\mathcal{P}_1 \text{ dominated by } \mathcal{P}_2} \Phi(\mathcal{P}_1, \mathcal{P}_2)$, and this completes our proof. $\qquad \square$

Proof of Proposition 13. We first write out the dual of the program $\min\{0 \cdot cost | \bar{Inv}\}$.

$$\max \quad \sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} \cdot \sum_{k=1}^{K} (\delta f_i^k)_{(d,o,i)}^{(o,d,i)}$$

$$s.t. \quad \delta f_i^k \text{ is a feasible augmenting flow w.r.t. } \bar{f}_i^k \quad \forall i \in N \quad \forall k = 1, 2, ..., K$$

$$\sum_{i \in N} \sum_{(o,d,i) \in D^i} \sum_{k=1}^{K} (\delta f_i^k)_e^{(o,d,i)} \leq \sum_{i \in N} \gamma_e^i \cdot \sum_{(o,d,j) \in D} \sum_{k=1}^{K} (\delta f_i^k)_e^{(o,d,j)} \quad \forall e \in E \quad (222)$$

We show by contradiction that the above dual program has a zero objective value, indicating that $\bar{Inv}$ is feasible. Assume that it has a feasible solution $\bar{\delta} f_i^k$ that gives rise to a strictly positive objective value. Hence, we can construct a feasible solution to program $(Dinv)$ in Algorithm 5, i.e., $\delta f_i^k = \delta f_i^{k*} + \frac{\bar{\delta} f_i^k}{p^k \cdot M}$, where $M > 0$ is chosen big enough such that each $\frac{\bar{\delta} f_i^k}{p^k \cdot M}$ is a feasible augmenting path w.r.t. $\bar{f}_i^k$ for all k. To see its feasibility, first, it is easy to verify that this solution $\{\delta f_i^k\}$ is a feasible augmenting path w.r.t. f_k^* since $f_k^* + \delta f_k^{i*} + \frac{\bar{\delta} f_i^k}{p^k \cdot M} = \bar{f}_i^k + \frac{\bar{\delta} f_i^k}{p^k \cdot M}$ must be a feasible flow $\forall i, k$. Next,

due to the last inequality in both $(DInv)$ and constraint (222),

$$\sum_{i \in N} \sum_{(o,d,i) \in D^i} \sum_{k=1}^{K} p^k \cdot (\delta f_i^k)_e^{(o,d,i)} - \sum_{i \in N} \gamma_e^i \cdot \sum_{(o,d,j) \in D} \sum_{k=1}^{K} p^k \cdot (\delta f_i^k)_e^{(o,d,j)}$$

$$= \sum_{i \in N} \sum_{(o,d,i) \in D^i} \sum_{k=1}^{K} p^k \cdot (\delta f_i^{k*})_e^{(o,d,i)} - \sum_{i \in N} \gamma_e^i \cdot \sum_{(o,d,j) \in D} \sum_{k=1}^{K} p^k \cdot (\delta f_i^{k*})_e^{(o,d,j)}$$

$$+ \sum_{i \in N} \sum_{(o,d,i) \in D^i} \sum_{k=1}^{K} p^k \cdot (\frac{\bar{\delta f}_i^k}{p^k \cdot M})_e^{(o,d,i)} - \sum_{i \in N} \gamma_e^i \cdot \sum_{(o,d,j) \in D} \sum_{k=1}^{K} p^k \cdot (\frac{\bar{\delta f}_i^k}{p^k \cdot M})_e^{(o,d,j)} \leq 0 . \quad (223)$$

We then show that δf_i^k gives rise to an objective value of $(DInv)$ greater than that under the solution $\{\delta f_i^{k*}\}$ and thus contradicts the optimality of $\{\delta f_i^{k*}\}$.

$$\sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} \cdot \sum_{k=1}^{K} p^k \cdot (\delta f_i^{k*} + \frac{\bar{\delta f}_i^k}{p^k \cdot M})_{(d,o,i)}^{(o,d,i)}$$

$$= \sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} \cdot \sum_{k=1}^{K} p^k \cdot (\delta f_i^{k*})_{(d,o,i)}^{(o,d,i)} + \sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} \cdot \sum_{k=1}^{K} p^k \cdot (\frac{\bar{\delta f}_i^k}{p^k \cdot M})_{(d,o,i)}^{(o,d,i)}$$

$$= \sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} \cdot \sum_{k=1}^{K} p^k \cdot (\delta f_i^{k*})_{(d,o,i)}^{(o,d,i)} + \frac{1}{M} \cdot \sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} \cdot \sum_{k=1}^{K} (\bar{\delta f}_i^k)_{(d,o,i)}^{(o,d,i)}$$

$$> \sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} \cdot \sum_{k=1}^{K} p^k \cdot (\delta f_i^{k*})_{(d,o,i)}^{(o,d,i)} \quad (224)$$

Hence, we conclude that the dual program to the problem $\min\{0 \cdot cost | \bar{Inv}\}$ has a zero optimal value, which indicates that $\bar{Inv}$ is feasible by strong duality. $\qquad \square$

Proof of Theorem 18. To show result (1), note that the optimal value of $(DInv)$ is strictly positive when robust perfect coordinating prices do not exist. Hence,

$$\sum_{k=1}^{K} p^k \cdot \sum_{(o,d,i) \in D} r_{(o,d,i)} \cdot (f_k')_{(d,o,i)}^{(o,d,i)}$$

$$= \sum_{k=1}^{K} p^k \cdot \sum_{(o,d,i) \in D} r_{(o,d,i)} \cdot (f_k^*)_{(d,o,i)}^{(o,d,i)} + \sum_{i \in N} \sum_{(o,d,i) \in D^i} r_{(o,d,i)} \cdot \sum_{k=1}^{K} p^k \cdot (\delta f_i^{k*})_{(d,o,i)}^{(o,d,i)}$$

$$> \sum_{k=1}^{K} p^k \cdot \sum_{(o,d,i) \in D} r_{(o,d,i)} \cdot (f_k^*)_{(d,o,i)}^{(o,d,i)} \quad (225)$$

239

To show result (2), by the last constraint in $(DInv)$, we know that for each edge $e \in E$

$$\sum_{(o,d,i) \in D} \sum_{k=1}^{K} p^k \cdot (f'_k)_e^{(o,d,i)} = \sum_{i \in N} \sum_{(o,d,i) \in D^i} \sum_{k=1}^{K} p^k \cdot (f_k^i)_e^{(o,d,i)}$$

$$= \sum_{i \in N} \sum_{(o,d,i) \in D^i} \sum_{k=1}^{K} p^k \cdot (f_k^* + \delta f_i^{k*})_e^{(o,d,i)} \leq \sum_{i \in N} \gamma_e^i \cdot \sum_{(o,d,j) \in D} \sum_{k=1}^{K} p^k \cdot (f_k^* + \delta f_i^{k*})_e^{(o,d,j)} \leq c_e \quad (226)$$

The last inequality is obtained since each $f_k^* + \delta f_i^{k*}$ is a feasible routing. Hence, we conclude that the routing $\sum_{k=1}^{K} p^k \cdot f'_k$ is a feasible routing, and (205) follows. □

3.6   *Conclusion and Future Research Directions*

In this chapter, we study the use of market-based exchange mechanisms to motivate and regulate capacity sharing so as to achieve the optimal overall routing efficiency in a general decentralized multicommodity network such as a transportation or a telecommunication system. We focus on the design of capacity pricing strategies in the presence of several practical operational complexities, including multiple ownership of the same capacity, uncertainty in network specifications, and information asymmetry between the central coordinator and individual operators.

Our study is based on a general multicommodity network where the capacities are privately owned by individual players; we allow multiple capacity owners to exist on a single edge. We also incorporate demand uncertainty into the network model in order to analyze the robustness of exchange mechanism. We adopt a non-cooperative game theory approach where the game model is defined based on on a set of network optimization problems that represent individual operators' routing and capacity allocation decisions, and analyze the model using inverse optimization techniques. Combining game theory analysis and network optimization models, our study provides insights into the interaction between operational features of networks and individual incentives in resource sharing, and to design effective mechanisms accordingly.

To summarize, we show two sets of results.

1. We demonstrate the impact of operational features of decentralized networks on the effectiveness of using market-based exchange mechanisms to coordinate resource sharing and to allocate the resulting synergistic benefit. Specifically, in §3.3, we show that perfect coordinating prices may not give rise to a core allocation under the multiple ownership of the capacity on a single edge. This indicates that the effectiveness of the capacity exchange mechanism is potentially undermined in this situation. Analyzing the cause of this problem, we show that it is due to the heterogeneity in capacity ownership levels of the multiple owners relative to their return volume. In particular, under the capacity exchange mechanism which essentially rewards capacity contribution, players who have abundant capacity compared to their own volumes make significant capacity contribution when participating in a combined network and thus are likely to be allocated a better payoff. This study characterizes the diseconomies of multiple capacity ownerships, which has been discussed in literature in other contexts, under our problem setting of designing capacity exchange mechanisms. Furthermore, in both §3.4 and §3.5, we show that network structure, in particular the existence of inefficient paths, can undermine the coordination benefit of the capacity exchange mechanism. For example, in some networks, it requires zero prices on certain edges to achieve perfect coordination, which potentially lead to free riders. Such network conditions can also strongly influences the robustness of the capacity exchange mechanism under demand uncertainty.

2. As the second set of results, we propose efficient and effective pricing policies and other mechanism design strategies to address different operational complexities. We develop dual-based capacity exchange prices that are perfect coordinating and also guarantees a core allocation. In order to mitigate the incompatibility between prefect coordination of individual routing decisions and free rider

avoidance, we propose the notion of partial coordinating prices and an algorithm to efficiently compute such prices. We also propose a capacity control mechanism to tackle the potential instability of the social optimal routing under partial coordination. In the presence of demand uncertainty, we propose a general robust pricing algorithm. We also evaluate different pricing strategies such as commodity-based price discrimination, which is shown to have an advantage in coordinating decentralized resource sharing in combined networks under uncertainty.

Coordination of decentralized resource sharing in general networks is a very broad topic and has many practical applications. These applications raise a rich set of challenging research questions that can motivate interdisciplinary studies combining frameworks in operations research, game theory, design and analysis of algorithms, machine learning, etc. Our research in this chapter has motivated a set of open questions in designing market-based exchange mechanisms. For example, first, since players' payoffs greatly influence their participation incentives in a combined network, it is important to study the payoff implication of adopting the partial coordinating prices and the robust prices computed by the algorithms proposed in this chapter. Second, the dual prices are shown to be perfect coordinating and to guarantee a core payoff. Yet such prices are zero on edges where the capacity is not fully utilized, which can lead to free riders and may mute the resource-sharing incentives of capacity owners. In literature, this is known that dual-based approaches in payoff allocation may discourage players to share their resources [63]; a common approach to tackle this problem is to use methods based on the average marginal contribution of players, e.g., Shapley value. Our preliminary results indicate that capacity exchange prices based on the Shapley value of the players can be effective in preventing free riders; yet it remains an open question of how these prices can coordinate the decentralized routing decisions of players. Third, as we mention at the end of §3.5, capacity violation is a

prominent problem in a stochastic decentralized network, which indicates that players may be incentivized to compete for capacity by the capacity exchange mechanism under demand uncertainty. This motivates us to study different capacity management methods such as capacity outsourcing, and auction mechanisms. In addition, it also motivates us to consider incorporating stochastic network approaches, where the excess demand for edge capacity is stored in a buffer in queues to be processed later, and to analyze a multi-period routing model on the network.

Lastly, recall that in §3.5, we assume individual players have full knowledge of the demand when making their routing decisions. Such a model highlights the information asymmetry between the central planner (the mechanism designer) and the individual decision makers. One direction for further research is to generalize this model by incorporating demand uncertainty that individual players themselves can often experience in practice. One particular example is the uncertainty associated with online demand that is to be routed in the order of its reception without knowing the total demand to be received over the network in the entire time horizon, especially in many applications of service operations. Hence, we are motivated to study the robustness of the capacity exchange mechanism when players decide their individual routings using online routing algorithms, and whether adopting approaches such as online learning techniques when designing the exchange prices can reinforce the robustness of the mechanism. In our initial study of the problem, we start by analyzing centralized routing strategies of online demand. We adopt the notion of oblivious routing from computer science literature [13, 71], and study the design of online oblivious routing algorithms to maximize network throughput.

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