

# Effect of non-structural masonry brick infill walls on the robustness of a RC framed building severely damaged due to a landslide

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## Abstract

Western countries are increasingly demanding for robust structures, i.e., structures capable of withstanding local damage caused by unforeseen extreme events without triggering a progressive collapse, thus reducing the magnitude and proportion of the resulting consequences.

In this paper, the robustness of framed RC buildings is analysed by comparing the reliability of the damaged structure with that of the original structure and considering (or not) the contribution of the masonry infill walls. To validate the adopted methodology, this is tested on a residential RC building severely damaged due to a landslide, herein considered as case-study. A numerical model of the original, as well as, of the damaged structure is defined using force-based finite elements with distributed plasticity. Masonry infill walls are modelled as equivalent internal struts. Monte Carlo simulation and FORM coupled with artificial neural networks and response surface polynomials are used in parallel to perform the reliability analyses of both original and damaged structures. Obtained results show that the masonry infill walls are fundamental to contain damage progression after the failure of a couple of columns. In fact, without these non-structural elements, the structure would lack in robustness, and the probability of failure would be above 99%. On the contrary, by considering the structural contribution of the masonry infill walls, the robustness of the structure would be circa of 30% corresponding to a failure probability of 6%.

**Keywords:** Robustness; Reliability; Probability of Failure; Reinforced Concrete; Masonry Infill Walls; Damage; Progressive Collapse.

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## 29 **1. Introduction**

### 30 **1.1 Background**

31 The interest on structural robustness has increased in the past 40 years, due to the occurrence of  
32 unforeseeable extreme events with resulting unacceptable consequences on structures and with high  
33 impact on society. The case of the partial collapse of the Ronan Point Building (UK,1968) [1] or the case  
34 of the total collapse of the World Trade Center (NY, 2001) [2], among other examples, have increased the  
35 discussion on the importance of structures to withstand inflicted local damages without triggering  
36 progressive collapse, thus resulting in disproportionate and catastrophic consequences.

### 37 **1.2 Research significance**

38 Taking into account the above-mentioned reasons, today it is mandatory to perform a structural  
39 robustness analysis for both new and existing structures. Special concern should be devoted to certain  
40 structural types, recognized as lacking in robustness (e.g., the Larsen–Nielsen building system [1] used in  
41 Ronan Point), or structures erected on a specific period related to poor quality of construction, as it is the  
42 case of a significant percentage of reinforced concrete (RC) framed buildings erected in Portugal between  
43 the 70's and the 90's [3].

44 In spite of the relevance of structural robustness, current codes and standards do not have a  
45 comprehensive approach for robustness, lacking in methods to check and/or to design for robustness.  
46 However, experience has shown that certain types of structures, although not having been specifically  
47 designed for robustness, exhibit an intrinsic capacity to withstand severe local damage without collapsing.  
48 This is the case of framed RC buildings with masonry infill walls which, although designed as non-  
49 structural elements, are capable of materializing struts that, combined with the RC frame, may provide  
50 extra strength to both horizontal and vertical actions.

### 51 **1.3 Approach**

52 A generic robustness analysis capable of considering a generic multi-story building subjected also to a  
53 generic and wide range of damage scenarios is unrealistic and simply not possible [4]. This paper presents  
54 a robustness analysis approach for multi-story RC framed buildings subjected to severe damage, based on  
55 the comparison of the reliability of the damaged and intact structure. The structure reliability is evaluated  
56 using advanced non-linear numerical models coupled with both simulation and gradient based reliability  
57 methods. Non-linear numerical models are rarely used on reliability studies. However, they are of

58 paramount importance in order to capture the potential alternate load paths developed during a structure  
59 failure analysis.

60 To depict the proposed approach, a 16 stories residential RC building, that experienced a local failure of a  
61 set of three columns at the ground level, after a landslide, is analysed. The structure inspection and  
62 retrofitting was already discussed in [3].

63

## 64 **2. Literature Review**

65 According to Murty and Jain [5] masonry infill walls in RC buildings may cause different undesirable  
66 effects under seismic loading, such as short-column effect, soft-story effect, torsion, and out-of-plane  
67 collapse. However, beneficial effects are reported in [5]–[7], including increase lateral stiffness and  
68 strength. Additionally, if appropriate reinforcement arrangements are provided in the masonry, and  
69 properly anchored into the RC frame, an out-of-plane response improvement may also be achieved. The  
70 configuration and presence of masonry infills significantly change the collapse mechanism.

71 The reported undesirable effects are mostly related to partially infilled frames and non-uniform  
72 distribution, in height and/or in plan, of such infills in buildings. Masonry fully infilled frames have been  
73 shown to have better seismic performance and lower collapse risk when compared to bare frames [7]. In  
74 the experimental tests conducted by Pujol et. al [8], consisting of full-scale 3-story flat-plate structure  
75 strengthened with masonry infill walls and tested under displacement reversals, infill walls were effective  
76 in increasing the strength (by 100%) and stiffness (by 500%) of the original and bare RC structure. The  
77 single story frames under in-plane lateral forces tested by Abdel-Hafez et al. [9] show that the presence of  
78 masonry infill walls changed the behaviour of the bare RC frame to a shear wall behaviour increasing the  
79 capacity by approximately 100%. Similar conclusions related to infilled frames strength and stiffness can  
80 be drawn from the experimental work conducted by Al-Chaar [10].

81 Regarding strategies to model the behaviour of infilled frames, extensive research has been dedicated to  
82 the topic [11]–[20]. Micro and macro models have been investigated. For the later, centered or eccentric  
83 struts or multi-struts have been proposed, validated by experimental work and numerical simulations.

84 In addition to the resistance to horizontal actions, namely to the seismic action, masonry infill walls have  
85 also demonstrated a positive influence on the behaviour of RC buildings severely damaged locally, such  
86 as in the case of the failure of a column or a set of columns [21]–[25]. However, research addressing this

87 issue is scarce and mostly numerical. The Unified Facilities Criteria (UFC) guidelines [26] of the  
88 Department of Defense of USA suggest that some comparisons can be established between the behaviour  
89 of RC frames submitted to e.g. the loss of a column and the behaviour of the same frames subjected to  
90 seismic actions. However, it must be noted that gravity loads are much different from seismic actions  
91 (base displacements) and that behaviour coefficients, widely used on seismic analysis and codes, are not  
92 valid in this context.

93 In the study conducted by Sasani (2008) [21], the response of a 6-story RC infilled frame structure was  
94 numerically evaluated following the simultaneous removal of two adjacent exterior columns. Three-  
95 dimensional Vierendeel frame action of the transverse and longitudinal frames with the participation of  
96 the infill walls was identified as the major mechanism for redistribution of loads in the structure. The  
97 response of the structure due to additional gravity loads, in the absence of infill walls, was also evaluated,  
98 and results have shown that, while the maximum vertical displacement of the structure was increased by  
99 almost 2.4 times, the system could still avoid progressive collapse.

100 Xavier et al. [24] performed a pushdown analysis of a 7-story composite steel-concrete benchmark  
101 building under sudden column loss scenarios and concluded that the use of masonry infill panels for  
102 building's envelope can considerably increase robustness.

103 A deterministic progressive collapse assessment following the Unified Facilities Criteria (UFC)  
104 guidelines [26] of the Department of Defense of USA was carried out for a typical 10-story RC framed  
105 structure by Helmy et al. [25]. Fully nonlinear dynamic analysis of the structure was carried out using the  
106 Applied Element Method following different damage scenarios such as the removal of a corner column,  
107 an edge column, an edge shear wall, internal columns and internal shear walls. It was found that  
108 neglecting the effect of the masonry infill walls conducts to incorrect structural behaviour assessment as  
109 they provide a valuable contribution in mitigating progressive collapse. Nevertheless, results indicate that  
110 the area of the opening has a significant effect on the wall's ability to resist the structure's collapse.

111 Tiago and Júlio (2010) [3] describe the inspection and assessment of a severely damaged RC residential  
112 building which withstood the loss of the first two levels of three external columns after a landslide caused  
113 by persistent heavy rain. These authors concluded that the masonry infill walls, acting as struts, combined  
114 with the slabs of the RC framed structure, acting as ties, have contained the building's progressive  
115 collapse.

116 These works show that the masonry infills can provide alternative load paths to RC framed structures  
117 subjected to vertical support(s) local failure. For this reason, to understand the performance of buildings  
118 under severe damage scenarios, it is fundamental to evaluate their robustness analysis taking into account  
119 secondary elements, in particular masonry infill walls. Although several robustness indicators have been  
120 proposed in the literature [26], the probabilistic approach for robustness quantification proposed in [27]  
121 suits this purpose as it is given by the ratio between the reliability index of the damaged structure,  $\beta_d$ ,  
122 and the reliability index of the undamaged structure,  $\beta_i$ :

$$R = \frac{\beta_d}{\beta_i} \quad (1)$$

123

124 where:

$$\beta = -\Phi^{-1}(P_f) \quad (2)$$

125 and  $\Phi()$  is the normal distribution and  $P_f$  is the probability of failure. The maximum robustness,  $R$ ,  
126 computed according to Equation (1) can be 1, if the probability of failure of the damaged and undamaged  
127 structure is the same. This means that the damage inflicted has null impact on structural safety, which  
128 rarely occurs. The robustness index results null ( $R=0$ ) when the probability of failure of the damaged  
129 structure is exactly 50%. If this probability is above 50%, negative values can be expected when  
130 computing Equation (1), meaning that robustness is even lower. However, this distinction is not  
131 meaningful, since in the domain of structural engineering, probabilities within this range are considered  
132 extremely high and unacceptable, considering the possible consequences of a structural failure.

### 133 **3. Case Study**

#### 134 **3.1 Description of the accident**

135 In 2000, in Coimbra, Portugal, a landslide caused severe damage to the RC structure of a 16 stories  
136 residential building (Figure 1). The first two levels of three exterior columns were destroyed and the rear  
137 body of the building supported by these, with a dimension in plant of  $9.5 \times 6.7 \text{ m}^2$ , became a 7.0 m span  
138 cantilever with 12 stories above (Figure 1 (b)). Unlike what would be expected, no significant anomalies  
139 were identified, other than cracks on the masonry infill walls with a maximum width of 2 mm, located

140 predominantly at the openings' corners (Figure 2 (a)). After the inspection, the damaged part of the  
141 structure was consolidated using external prestress, the debris was removed, and the retrofitting works  
142 started (Figure 2 (b)).



Figure 1 –Severely damaged RC residential building after a land slide: (a) rear façade; (b) collapse of the outer columns of the rear body of building.

143



Figure 2 –Details of the accident: (a) cracks along the façade masonry infill walls; (b) removal of debris from the accident site.

144 According to [3], the progressive collapse of the structure was prevented due to the contribution of the  
145 non-structural infill walls. More specifically, the gravity loads, initially supported by the destroyed outer  
146 columns, were redistributed leading to the development of compressive stresses in the masonry infill  
147 walls (struts) and tension stresses (ties) in the slabs. The equilibrium to the adjacent part of the structure  
148 was ensured by a resultant tension force at the top slab, a resultant compressive force at the bottom slab,  
149 and a resultant compressive force at the columns, as depicted in Figure 3.

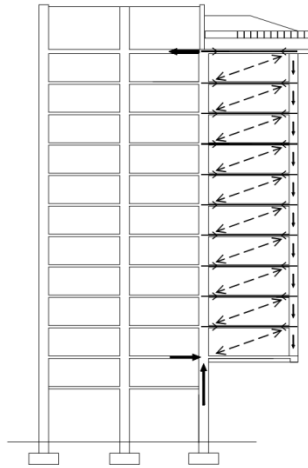


Figure 3 – Schematic drawing of the strut-tie system developed on the façade masonry infill walls and flooring slabs.

150

151 Cachado et al. [22] performed a 3D FEM analysis of the damaged structure either neglecting the  
 152 structural contribution of the masonry infill walls or considering it as shell elements. Following the frame  
 153 analysis, the authors have concluded that the failure mechanism of the rear body was controlled by the  
 154 cantilever beams since their flexural capacity was largely exceeded for the probable acting loads. Results  
 155 of the frame plus infill analysis showed the development of alternative load paths through a strut-tie  
 156 system, only possible due to the contribution of masonry infill walls. In this case, both the cantilever  
 157 beams, who acted as ties, and the masonry infill walls, who played the struts' role, did not exceed their  
 158 bearing capacity and the damaged structure was able to sustain the estimated acting loads.

159

### 160 3.2 Structural characterization

161

162 As already mentioned, the building has a RC framed structure settled on direct foundations. The floorings  
 163 are composed of ceramic blocks, supported by precast pre-stressed concrete joists, and topped by a cast  
 164 “in-situ” concrete layer (Figure 4). This type of flooring is typically limited to low-rise buildings, due to  
 165 deficient diaphragm effect, which tends to result in poor seismic behavior. A more detailed description of  
 166 the structure is presented next, although limited to the rear body of the building, that subjected to  
 167 extended damages after the landslide.

168

169 The arrangement of the structural elements per story is shown in Figure 5. The concrete joists are  
 170 supported by the V1 and V5 beams and spaced 0.33m apart. Ceramic blocks with 0.20m of height are  
 171 placed in the middle of the joist and topped by a cast in-situ concrete layer with 0.04m of thickness. The  
 concrete layer is reinforced with  $8\phi 6/m$  on both directions and over reinforced with  $8\phi 8/m$  in the joist

172 direction over the V5 beam to increase resisting hogging bending moments. V1 and V5 are continuous  
 173 beams with two spans. The former is supported by columns P1, P2 and P3, those destroyed by the  
 174 landslide in the first two stories. Beams V2 and V4 are parallel to the concrete joists and support the  
 175 exterior masonry infill walls. These are continuous beams which became cantilevers after the accident.  
 176 Beam V3 is a single span beam supported by columns P2 and P5.

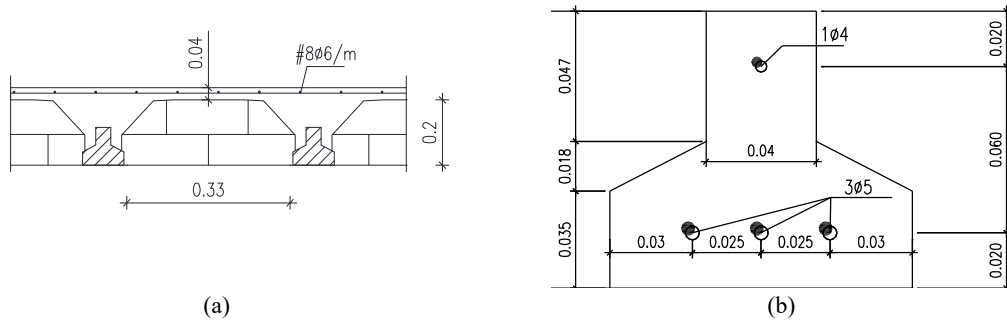


Figure 4 –Building flooring system (dimensions in m): (a) schematic drawing; (b) precast and pre-stressed concrete joist.

177

178 Beams V1 and V5 present a cross-section of  $450 \times 600 \text{ mm}^2$  but the number and diameter of reinforcing  
 179 bars is unknown. Beams V2 and V4 have a cross-section of  $300 \times 350 \text{ mm}^2$ . The top reinforcement is  
 180 composed by 4 bars with 12 mm of diameter, while bottom correspond to 4 bars with 10mm of diameter.

181 The corner columns, P1, P3, P4 and P6 have a cross-section of  $300 \times 600 \text{ mm}^2$  and 8 rebars with 16 mm  
 182 of diameter. The central columns, P2 and P5, have a cross-section of  $300 \times 700 \text{ mm}^2$  with 10 rebars with  
 183 16 mm of diameter. According to the project, C20/25 concrete class and S400 steel grade have been  
 184 adopted, as usual at that time and place.

185 Both façade and partition walls were built using ceramic bricks connected and plastered with a  
 186 cementitious mortar. Façade walls and partition walls present a height of 2300 mm, being the formers  
 187 composed by double masonry panels with a thickness of 300 mm, and the latter composed by single  
 188 masonry panels with a thickness of 150 mm. There are two opening in the façade walls supported by the  
 189 V2 and V4 beams with the dimensions of  $2100 \times 1000 \text{ mm}^2$  and  $1100 \times 1000 \text{ mm}^2$  (see Figure 1) and  
 190  $2 \times 2100 \times 1000 \text{ mm}^2$  (see Figure 2), respectively.



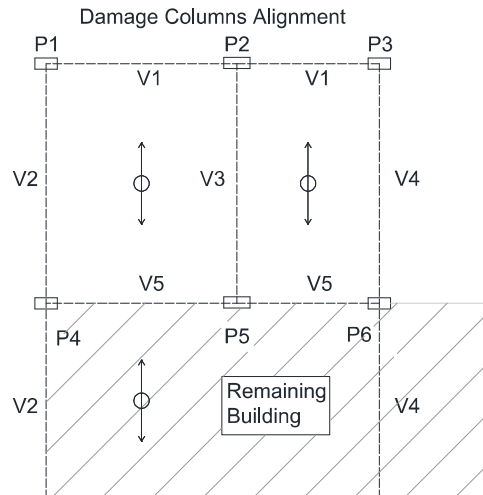


Figure 5 – Schematic drawing of the elements' arrangement on a typical floor plan of the damaged body.

191

### 192 3.3 Numerical Model

193 A non-linear numerical model of both the original structure and the damaged structure was developed  
 194 using OpenSees [28] aiming at performing a reliability analysis to characterize the structure prior and  
 195 after the accident. The event exact moment and the respective dynamic effects were, in this manner, not  
 196 considered in this paper. In the first case, the contribution of the masonry infill walls for structural  
 197 purposes was neglected, as this is the common practice in the design of framed RC structures. In the  
 198 second case, the masonry infill walls were first neglected and then taken into account, in order to  
 199 distinguish their impact on the structure's safety. The numerical model was limited to the rear body of the  
 200 building since, as according to Cachado et al. [22], collapse is controlled by the failure of the cantilever  
 201 beams (V2 and V4). Since the cantilevers are structurally isostatic, limiting the numerical model to the  
 202 rear body, and neglecting the deformation of the rest of the building, does not have relevant influence on  
 203 the structural strength, if rigid body global stability problems are disregarded.

204 On the numerical models of both the original (undamaged) structure and the damaged structure, force-  
 205 based frame elements were used to simulate the behaviour of beams and columns. The respective cross  
 206 sections were discretized into multiple fibres for which a constitutive relation was assigned according to  
 207 the type of material.

208 The structural effect of floorings was indirectly considered: the corresponding self-weight, including the  
 209 remaining dead and live loads, were applied directly to the V1 and V5 beams; on the damaged model of  
 210 the structure, the bending strength of the flooring system was neglected and only its tying effect,

211 described by [3], was considered and simulated using an equivalent tie at V2 and V4 levels, as foreseen in  
212 [26]. The flooring contribution to the tie effect was yet limited to a flooring band of 1m width next to V2  
213 and V4 beams, as already considered by [3] and recommended in [26].

214 This simplifying, however conservative hypothesis, was adopted to reduce the number of structural  
215 members to be modelled and, ultimately, to allow the reliability analysis to be performed. In the case of  
216 V1 and V5 beams, the area of the reinforcing bars had to be estimated based on REBAP [29], the  
217 prevailing legislation at that time, due to missing information.

218 Karsan-Jirsa [26] constitutive model was used to simulate the concrete behaviour, whereas for the  
219 reinforcing bars a uniaxial bilinear model with zero strain-hardening ratio was considered.

220 The contribution of the masonry infill walls was accounted for using the eccentric strut model proposed  
221 by Al-Chaar [1]. Although many other models have been proposed in the literature, this approach was  
222 selected since: it is computationally little time consuming; it is strongly supported by extensive  
223 experimental campaign; it has been defined based on the experiments of RC frames, while on other  
224 proposal steel frames have been used. However, and since this model was developed for horizontal loads,  
225 for the case study herein addressed it is necessary to adapt first the model for vertical loads, as suggested  
226 in the UFC manual [25]. In the original model, as shown in Figure 6 (a), the forces transmitted by the  
227 masonry infill walls are assumed to be resisted by the columns and the equivalent strut is connected to the  
228 column at a distance  $L_c$  from the face of the beams. The strut width,  $a$ , is dependent on the relative  
229 bending stiffness of the columns and the masonry panel,  $\lambda H$ . The distance,  $L_c$ , represents the length  
230 needed for the development of plastic hinges and is determined geometrically considering the strut width  
231 and the angle of the diagonal strut. The columns segments with length,  $L_c$ , are modelled as rigid to take  
232 into account the effect of the masonry panel.

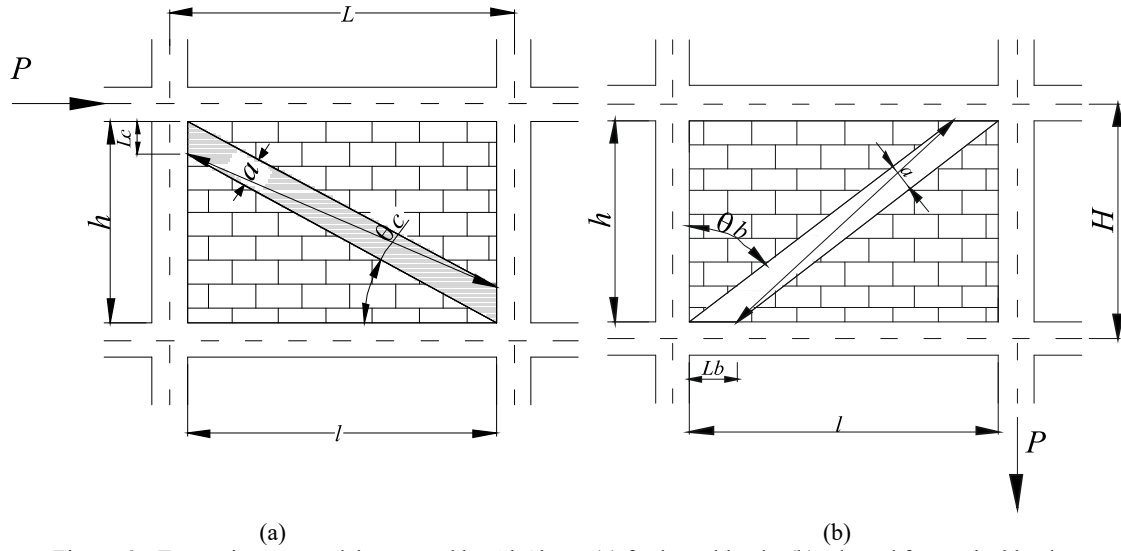


Figure 6 – Eccentric strut model proposed by Al-Chaar: (a) for lateral loads; (b) adapted for vertical loads.

233

234 For vertical actions, it is assumed that the load is distributed between the beams, instead of between the  
 235 columns as seen before for horizontal actions. Therefore, the equivalent strut must be anchored against  
 236 the beams positioned at a distance,  $L_b$ , from the edge of the columns, as depicted in Figure 6 (b). In this

237 case, the strut width,  $a$ , is dependent on the relative stiffness of the beams and masonry wall,  $\lambda L$ :

$$a = 0.175D(\lambda L)^{-0.4} \quad (3)$$

$$\lambda L = L^4 \sqrt[4]{\frac{E_m t \sin(2\theta_b)}{4E_c I_{beam} l}} \quad (4)$$

238

239 where  $L$  is the span, assumed as the distance between the columns midlines,  $l$  is the width of the  
 240 masonry panel,  $t$  is the thickness of the panel,  $E_m$  is the Young's modulus of the masonry,  $E_c$  is the  
 241 Young's modulus of concrete,  $I_{beam}$  is the second moment of inertia of the beams, and  $D$  is the diagonal  
 242 length of the panel. The plastic hinges lengths ( $L_b$ ,  $L_c$ ) are [1]:

$$L_c = \frac{a}{\cos(\theta_c)} \quad (5)$$

$$L_b = \frac{a}{\sin(\theta_b)} \quad (6)$$

243 where:

$$\tan(\theta_c) = \frac{h-L_c}{l} \quad (7)$$

$$\tan(\theta_b) = \frac{h}{l-L_b} \quad (8)$$

244 To take into account the effect of the openings, the strut width should be reduced to  $a_{red}$ , through a  
 245 reduction factor [1],  $R_1$ , depending on the ratio between the area of the opening,  $A_{open}$ , and the total area  
 246 of the panel,  $A_{panel}$ :

$$a_{red} = a \times R_1 \quad (9)$$

$$R_1 = 0.6 \left( \frac{A_{open}}{A_{panel}} \right)^2 - 1.6 \frac{A_{open}}{A_{panel}} \quad (10)$$

247 The failure of the strut is controlled by the compressive or the shear strengths of the masonry. The strut  
 248 maximum strength is [1]:

$$R_{strut} = \min \left\{ \begin{array}{l} R_{cr} = a \times t \times f'_m \\ R_{shear} / \cos \theta_{strut} = A_n \times f'_v / \cos \theta_{strut} \end{array} \right\} = \min \left\{ \begin{array}{l} R_{cr} = a \times t \times f'_m \\ A_n \times f'_v / \cos \theta_{strut} \end{array} \right\} \quad (11)$$

$$\tan \theta_{strut} = \frac{h-2L_c}{l} \quad (12)$$

249 where  $\theta_{strut}$  is the angle between the eccentric strut and the horizontal,  $R_{cr}$  and  $R_{shear} / \cos \theta_{strut}$  are the  
 250 compressive and shear strength of the equivalent strut, respectively, and  $f'_m$  and  $f'_v$  are the compressive  
 251 and the shear strength of the masonry, respectively.

252 Figure 7 shows the numerical model of the rear body of the damaged structure considering the effect of  
 253 the masonry infill walls. For the case where this effect has been neglected, neither the masonry equivalent  
 254 struts nor the rigid elements, shown in Figure 7, were considered. In the original undamaged model of the  
 255 structure, the outer columns were extended up to the foundations and the masonry equivalent struts were

256 not considered since this is the current design practice. For the three cases, a static pushdown analysis was  
257 conducted.

258

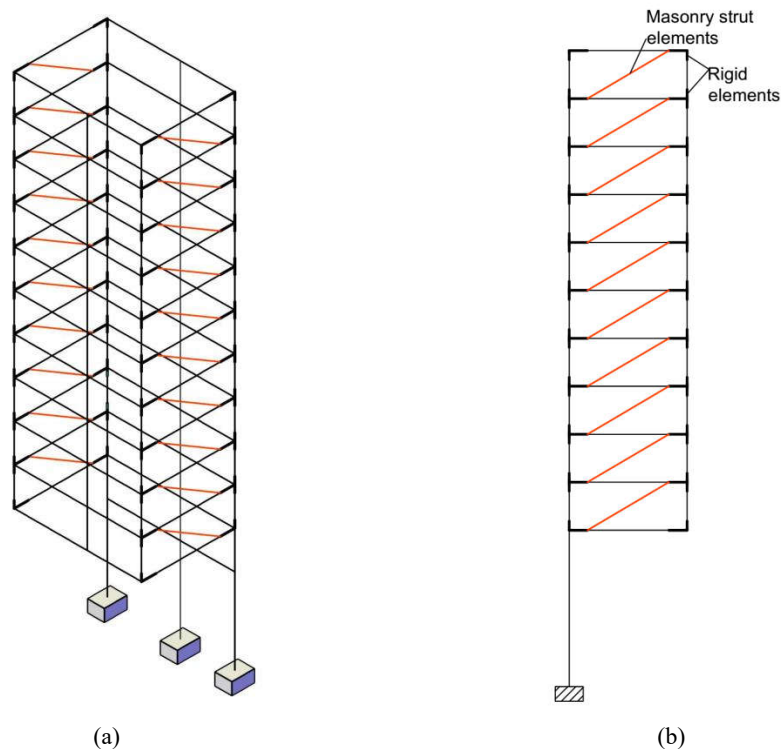


Figure 7 –Numerical model of the rear body of the damaged building: (a) 3D perspective; (b) lateral view.

259

260

## 261 4. RELIABILITY ANALYSIS

### 262 4.1 Random variables characterization

263 The uncertainties related to the strength of concrete, steel, and masonry were considered in their  
264 characterization from a probabilistic point of view, following the recommendations of the JCSS  
265 Probabilistic Model Code [30]. The concrete compressive strength,  $f_c$ , was modelled as a lognormal  
266 distribution with a mean value of 28 MPa (for a C20/25 concrete grade), and a coefficient of variation  
267 (CoV) of 15%, which results into a standard deviation of 4.2 MPa. The yielding stress of steel,  $f_y$ , was  
268 modelled as normally distributed, with a mean value equal to  $S_{nom} + 2\sigma$  MPa, where  $S_{nom}$  is the  
269 nominal yielding stress, and  $\sigma$  is the standard deviation (30 MPa). For an S400 steel grade,  $S_{nom}$  can be

270 considered equal to 400 MPa and the mean yielding stress results equal to 460 MPa. The compressive  
271 strength of the masonry wall,  $f_m$ , according the Probabilistic Model Code is defined by:

$$f_m = f'_m Y_1 \quad (11)$$

272 where  $Y_1$  is a a lognormal variable with a mean equal to 1.0 and a CoV of 17%. For this case study, the  
273 mean value of the masonry compressive strength,  $f'_m$ , was considered equal to 13 MPa, as suggested in  
274 [31].

275 In what concerns the actions, four additional random variables were considered associated to the self-  
276 weight of RC elements and masonry walls and to the live loads. A normal distribution was assumed for  
277 the concrete self-weight,  $g$ , with a mean value of 25kN/m<sup>3</sup> and a standard deviation of 0.75 kN/m<sup>3</sup> [30].

278 The self-weight of clay masonry walls,  $g_1$ , was modelled by a normal distribution with a mean value of  
279 2.9 kN/m<sup>2</sup> and CoV of 5%. Two types of live loads were considered according to the Probabilistic Model  
280 Code: the sustained live load  $q_s$ , Gamma distributed with a mean value of 0.30 kN/m<sup>2</sup>, a standard  
281 deviation of 0.31 kN/m<sup>2</sup> and renewal rate of one time each 7 years; and the intermittent live load  $q_i$ , of  
282 shorter duration, also described by a Gamma distribution with mean value of 0.30 kN/m<sup>2</sup>, a standard  
283 deviation of 0.36 kN/m<sup>2</sup>, and an average renewal time of 1 year and duration of 1 day.

284 Finally, a uncertainty related to the strength and actions models was considered, through two additional  
285 random variables: the uncertainty of the strength model,  $\theta_R$ , and the uncertainty of the actions models,  
286  $\theta_E$ . Both uncertainties were admitted lognormally distributed with mean values of 1.2 and 1.0, and  
287 standard deviations of 0.15 and 0.10, respectively.

288 Two combinations of actions were considered. The first corresponds to both live loads acting at the same  
289 time with an average occurrence rate of one day per year; and the second corresponds to the isolated  
290 action of the sustained live load on the remaining 364 days of the year.

291 The remaining variables were assumed as deterministic due to their relatively low impact on structural  
292 safety. The random variables, distributions, and parameters considered are summarized in Table 1.

293

Table 1 - Random variables distributions and parameters

Random Variable		Distribution	Mean value	Standard deviation
Concrete strength	$f_c$ (MPa)	lognormal	28.0	4.2
Steel strength	$f_y$ (MPa)	normal	460.0	30.0
Masonry strength	$f_m$ (MPa)	lognormal	13.0	2.21
Concrete self-weight	$g$ (kN/m <sup>3</sup> )	normal	25.0	0.25
Masonry self-weight	$g_1$ (kN/m <sup>2</sup> )	normal	2.9	0.15
Sustained live-load	$q_s$ (kN/m <sup>2</sup> )	gamma	0.3	0.31
Intermittent live-load	$q_i$ (kN/m <sup>2</sup> )	gamma	0.3	0.36
Resistance model uncertainty	$\theta_R$	lognormal	1.2	0.15
Load model uncertainty	$\theta_E$	lognormal	1.0	0.1

295

296 **4.2 Reliability assessment**

297 The reliability assessment of structures can be performed using simulation-based methods or gradient-  
298 based methods [32]. Simulation-based approaches such as the Monte Carlo method [33] may result  
299 unviable, if the deterministic structural analysis is time-consuming and low probabilities of failure are  
300 expected. In gradient-based methods such as FORM [33], the limit state function is approximated by a  
301 linear function in a normalized space at the design point vicinity, but insufficient (and thus unacceptably  
302 low) accuracy can result from strong nonlinear limit state functions. To overcome these problems  
303 different techniques to approximate complex and implicit limit state functions have been proposed and  
304 used with both simulation and gradient based methods. Among these techniques, response surface  
305 methods (RSM), consisting of first and second order polynomials, have been widely used by different  
306 researchers [34], [35] to solve structural reliability problems. Artificial Neural Network (ANN)  
307 algorithms have also proved to be versatile and efficient in this scope [34]–[37], in particular for the  
308 approximation of large domains of non-linear performance functions [38].

309 For the reliability assessment of the case study different and combined techniques aiming at approaching  
310 the results of the numerical analysis to the real structural behavior were tested, to ensure the consistency  
311 and accuracy of the results obtained.

312 For the reliability analysis, Monte Carlo simulation, as well as the First Order Reliability Method  
313 (FORM), were used. These techniques were combined with the Response Surface Method (RSM) and the  
314 Artificial Neural Network (ANN) in order to obtain a fast approach of the structural behaviour, given by

315 the structural analysis, and also to facilitate the reliability assessment by reducing the number of  
316 numerical analysis to be performed.

317 The limit state function,  $G$ , used for the reliability analysis was defined as follows:

$$G = \alpha \cdot \theta_R - \theta_E \quad (12)$$

318 where  $\alpha$  is the structural performance function, which depends on the defined random variables,

319  $\alpha(f_c, f_y, f_m, g, g_1, q_s, q_i)$ . It corresponds to the ratio between the resisting and the acting loads on the

320 structure of the building, obtained following a pushdown numerical analysis. As mentioned,  $\alpha$  needed

321 to be approximated by an ANN algorithm in order to reduce the number of numerical analyses vs. the

322 number of MC simulations. The probability of failure,  $P_f$ , was then computed as the ratio between failed

323 simulations, defined by  $G < 0$ , and the total number of simulations, and the reliability index resulted as

324  $\beta = -\Phi^{-1}(P_f)$ .

325 The MATLAB software [39] was used to construct two ANN algorithms of the multi-layer feed forward

326 type to simulate the damaged structure and the original (undamaged) structure. For the former, a 4456

327 size sample of the random variables  $(f_c, f_y, f_m, g, g_1, q_s, q_i)$  was generated and for each sample's element

328 the nonlinear analysis was performed. A large range of the  $f_m$  random variable was considered, in order

329 to observe the effect and importance of the masonry's mechanical properties on the structural safety. The

330 ANN algorithm was trained, validated, and then tested considering respectively 75%, 15%, and 10% of

331 the total data set chosen randomly. In this case, Levenberg-Marquardt back propagation algorithm [40]

332 was used to train the network was used. The architecture of the ANN for the damaged structure is

333 depicted in

334 Figure 8. The number of neurons used to define the ANN was selected in order to obtain the best fit when

335 comparing the output values with the target values. The network defined with 20 neurons resulted in the

336 minimum error. The input layer had six neurons, corresponding to the input variables, and the output

337 layer had just one neuron corresponding to the structural performance function. Since both the sustained,

338  $q_s$ , and intermittent,  $q_i$ , loads produce the same loading effect a single input variable,  $q$ , was used to

339 train the ANN. A mean square error equal to  $9.8 \times 10^{-4}$  and  $7.5 \times 10^{-4}$  was obtained for the validation and



340 testing phases, respectively. It is worth mentioning that selection of a network optimal architecture is not  
341 a simple task. The number of neurons and layers, including the size of the data used to train, validate and  
342 test the ANN, are in general determined based on a trial-and-error-process [32].

343 The same type of ANN was used to approach the load carrying capacity of the original (undamaged)  
344 structure (see  
345 Figure 8). As mentioned, the contribution of the masonry infill walls was not included in this model. A  
346 mean square error bellow  $10^{-5}$  was obtained for both the validation and testing phases, using only a 200  
347 size sample.

348 The FORM method was used in parallel with the MC simulation to obtain the probability of failure and  
349 the reliability index, including the design point and the respective direct cosines, which provide the  
350 relative weight of each random variable to structural safety. Two different formulations of the FORM,  
351 named method 1 and method 2, were adopted [33], depending on the process used to estimate the  
352 derivatives of the limit state function. Method 1 considers an explicit definition of the latter, which allow  
353 its analytic derivation. In this case, the limit state function is approached using the RSM and a linear  
354 polynomial, defined in the design point neighborhood, and redefined at each iteration of the method for  
355 improved accuracy. The support points, required for the polynomial definition, are obtained through  
356 direct structural analysis or, alternatively, obtained indirectly using the ANN. Method 2, uses centred  
357 finite differences to determine the partial derivatives of the limit state function. In this case, the trained  
358 ANN was used to approach structural response to allow the assessment of the finite differences. Table 2  
359 summarizes the different reliability methods used in this study.

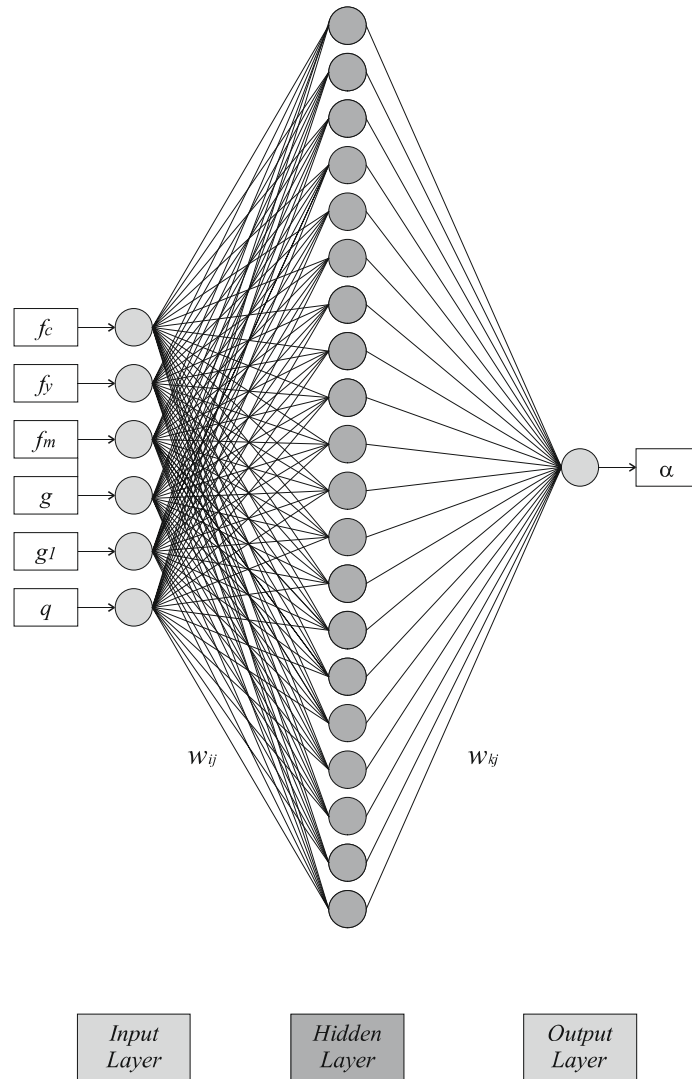


Figure 8 – Architecture of the ANN for the damaged structure.

360

361

Table 2 – Reliability Methods

Method	Reliability method	Structural performance approximation
MC/ANN	MC Simulation	Artificial Neural Network
FORM1/RSM	FORM method 1	Response surface method
FORM1/RSM+ANN	FORM method 1	Response surface method + Artificial Neural Network
FORM2/FD+ANN	FORM method 2	Centred finite differences + Artificial Neural Network

## 362 5. Results and Discussion

363 The reliability analysis was carried out taking into account the variables presented in Table 1 and using  
364 the different safety methods presented in Table 2. Results obtained for the damaged structure considering  
365 the effect of the masonry infill walls are presented in Table 3 for load cases 1 and 2. Results are referred  
366 to a period of one year. The annual probability of failure and the reliability indexes were also derived  
367 reflecting the average occurrence of both load cases in the same period.  $10^6$  simulations were used to  
368 ensure a reduced coefficient of variance for the probability of failure. It is worth mentioning that, for the  
369 damaged structure, when the contribution of the masonry infill walls is neglected, the probability of  
370 failure resulted approximately equal to 100%, corroborating the results presented in [7].

371 Results of MC/ANN approach are conservative in comparison to those of FORM1/RSM,  
372 FORM1/RSM+ANN and FORM2/FD+ANN approaches, reflecting the different nature (simulation vs  
373 gradient) of the methods used. By comparing the results obtained with the gradient-based approaches, it  
374 can be stated that both the RSM and the ANN provide similar approximations in terms of accuracy for the  
375 structural performance function. In addition, there are no significant differences between results obtained  
376 with FORM method 1 or method 2. Although load case 1 is associated with a higher probability of  
377 failure, due to the simultaneous action of both live loads, load case 2, corresponding to the sustained live  
378 load acting alone, is critical in the assessment of the annual probability of failure, due to the higher  
379 occurrence rate.

380 Results in Table 3 show that the probability of failure of the damaged structure considering the effect of  
381 the masonry infill walls ranges from 3.60% to 6.07%. These values correspond to reliability indexes  
382 varying between 1.80 and 1.55, which, although not being acceptable for structures in service, clearly  
383 explain why the building did not collapse after the accident. They also provide a valuable quantitative  
384 measure of the safety conditions during the repairing and retrofitting period that followed.

385

386

Table 3 –  $P_f$  and  $\beta$  for the damaged structure with masonry infill walls

387

Reliability Method	Case	$P_f$ (%)	$\beta$
MC/ANN	1	7.45	1.44
	2	6.07	1.55
	1+2	<b>6.07</b>	<b>1.55</b>
FORM1/RSM	1	5.23	1.62
	2	4.12	1.74
	1+2	<b>4.12</b>	<b>1.74</b>
FORM1/RSM+ANN	1	4.88	1.66
	2	3.61	1.80
	1+2	<b>3.60</b>	<b>1.80</b>
FORM2/FD+ANN	1	4.88	1.66
	2	3.59	1.80
	1+2	<b>3.58</b>	<b>1.80</b>

388

389 Table 4 provides the probability of failure of the original (undamaged) structure, computed using the  
 390 MC/ANN method. Due to the expected lower probability of failure of the latter, the number of  
 391 simulations was increased up to  $10^9$ .

392

393

Table 4 –  $P_f$  and  $\beta$  for the undamaged structure

Structure	Case	$P_f$ (%)	$\beta$
undamaged	1	$6.2 \times 10^{-6}$	5.29
	2	$3.6 \times 10^{-6}$	5.37
	1+2	<b><math>3.6 \times 10^{-6}</math></b>	<b>5.37</b>

394

395 The probability of failure and the reliability index of the original (undamaged) structure resulted equal to  
 396  $3.6 \times 10^{-8}$  and 5.37 respectively. It is worth mentioning that the computed reliability index is higher than  
 397 the value (4.7) prescribed by the Eurocode [41] for structures within the reliability class RC2,  
 398 consequences CC2, and a reference period of 1 year.

399 The robustness of the damaged structure, computed according to Equation (1) and considering the effect  
 400 of the masonry infill walls, resulted equal to  $R = \frac{1.55}{5.37} = 0.29$ , thus corresponding to 29 % of the  
 401 reliability of the original (undamaged) structure. Neglecting the contributions of masonry infill walls,

402 robustness results null being the reliability index negative. The probability of failure is around 100% and  
403 the collapse following the landslide would be certain.

404 To highlight the role played by the masonry infill walls in the safety of the damaged structure, Figure 9  
405 and Figure 10 show the influence of the mean value of the masonry compressive strength, on the  
406 probability of failure and reliability index, respectively. Calculations were performed using both the  
407 simulation (MC/ANN) and the gradient-based approaches (FORM1/RSM/ANN and FORM2/FD+ANN).

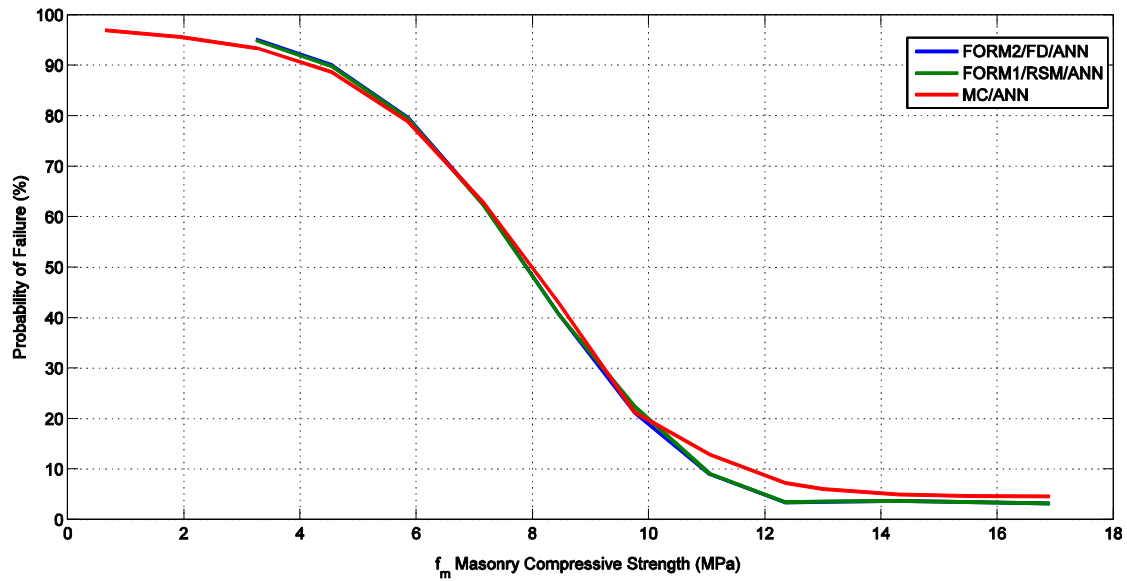


Figure 9 – Effects of the mean value of the masonry compressive strength to the damaged building probability of failure.

408

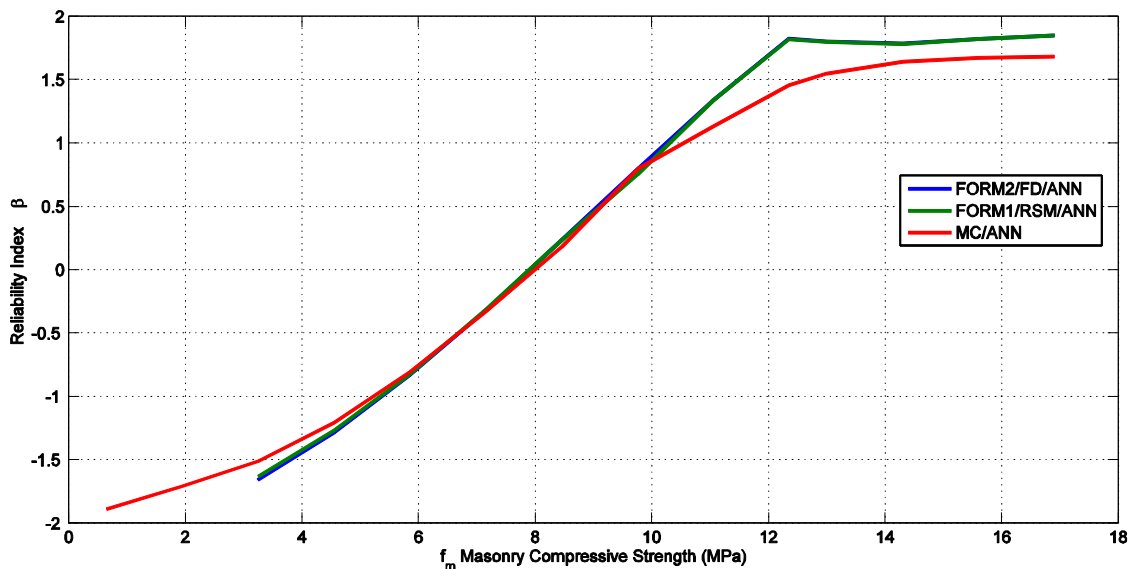


Figure 10 – Effects of the mean value of the masonry compressive strength to the damaged building reliability.

409

410 Results in Figure 10 show that the probability of failure decreases as the mean value of the compressive  
 411 strength of the masonry increases up to 14MPa. This is particularly evident for masonry compressive  
 412 strengths within the range of 5 MPa to 10-11 MPa. For lower values, the material is too weak and  
 413 therefore the masonry equivalent strut does not provide a significant alternative load path to the RC  
 414 structure. For higher values, and in particular above 14 MPa, results do not show significant increase of  
 415 the damaged building's safety with the increase of the masonry compressive strength. The structural  
 416 analysis showed that too strong masonry infill walls do not increase the load-bearing capacity of the  
 417 damaged RC frame, as structural collapse is mainly controlled by the RC frame failure. Results presented  
 418 in Figure 11 show the square of the direction cosines, which translate the relative importance of each  
 419 random variable to the global safety. These results were obtained with FORM1/RSM+ANN method for  
 420 critical load case 2. As observed, for compressive strengths up to approximately 90% of the average value  
 421 (11-12 MPa), the uncertainty related to the masonry infill walls is critical for the safety of the damaged  
 422 structure. Above these values, the steel yield strength random square cosine shows a significant increase,  
 423 meaning that, for higher values of the compressive strength of the masonry, the structural collapse is  
 424 mainly controlled by the failure of the RC elements. Figure 11 also shows that uncertainties related to the  
 425 strength and load models are of paramount importance in the assessment of the damaged structure  
 426 reliability.

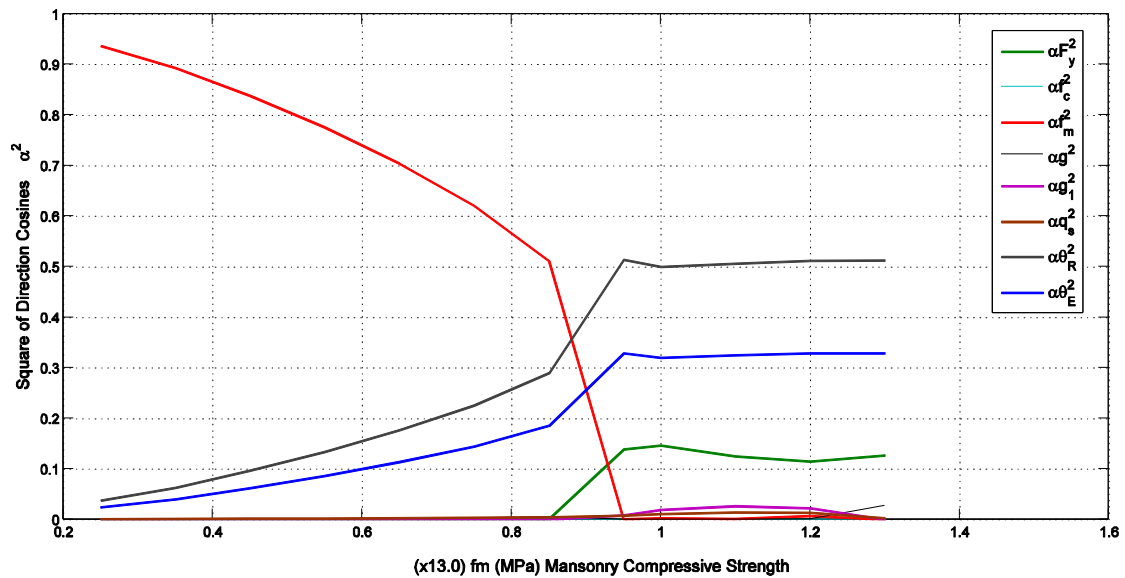


Figure 11 – Effects of the mean value of masonry compressive strength at the square of direction cosines.

427

428

## 429 **6. Conclusions**

430 The contribution of the masonry infill walls to the robustness of framed RC structures was investigated in  
431 this work by studying in detail a real building subjected to severe damage. Robustness was defined as the  
432 ratio between the reliability index of the damaged structure and the corresponding value of the original  
433 undamaged structure. The contribution of the masonry infill walls was either neglected or considered by  
434 means of an equivalent strut. The following conclusions can be drawn:

- 435 1. Even when neglecting the contribution of the masonry infill walls, the original undamaged  
436 structure exhibits an adequate safety level, with a reliability index equal to 5.4, thus significantly  
437 above the minimum recommended value in the Eurocode ( $\beta_{target} = 4.7$ );
- 438 2. When neglecting the contribution of the masonry infill walls, the structural robustness of the  
439 damaged structure is null, with a probability of failure very close to 100%;
- 440 3. Considering the contribution of the masonry infill walls, robustness is increased to 29% and the  
441 corresponding probability of having a global structural failure is reduced to 6%; this result  
442 clearly explains the survival of the damaged structure.
- 443 4. A sensitive analysis showed that the safety of the damaged structure is mainly dependent on the  
444 compressive strength of the masonry wall and by the yield stress of the steel bars of the RC  
445 cantilever beams. Below 5 MPa, the masonry infill walls are too weak and thus safety relies on  
446 the RC cantilevers. Likewise, above 11-12 MPa, the masonry infill walls are too strong and an  
447 increase in their compressive strength does not change the structure's safety, which depends  
448 again on the RC structure. Between the values referred to, the higher the compressive strength of  
449 the masonry infill walls, the safer the damaged structure is.

450

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## 454 **References**

- 455 [1] Cynthia Pearson and Norbert Delatte, “Ronan Point Apartment Tower Collapse and its Effect on  
456 Building Codes,” *J. Perform. Constr. Facil.*, vol. 19, no. 2, pp. 172–177, 2005.
- 457 [2] Z. P. Bazant and Y. Zhou, “Why did World Trade Center collapse?—Simple analysis,” *Arch. Appl.*  
458 *Mech.*, vol. 71, no. 12, pp. 802–806, 2001.

- 459 [3] P. Tiago and E. Júlio, “Case study: Damage of an RC building after a landslide—inspection,  
460 analysis and retrofitting,” *Eng. Struct.*, vol. 32, no. 7, pp. 1814–1820, Jul. 2010.
- 461 [4] E. S. Cavaco, J. R. Casas, L. A. Neves, and A. E. Huespe, “Robustness of corroded reinforced  
462 concrete structures—a structural performance approach,” *Struct. Infrastruct. Eng.*, vol. 9, no. 1, pp.  
463 42–58, 2013.
- 464 [5] C. Murty and S. K. Jain, “Beneficial influence of masonry infill walls on seismic performance of  
465 RC frame buildings,” in *12th World Conference on Earthquake Engineering (Auckland, New  
466 Zealand, January 30)*, 2000.
- 467 [6] P. Asteris, “Lateral stiffness of brick masonry infilled plane frames,” *J. Struct. Eng.*, vol. 129, no.  
468 8, pp. 1071–1079, 2003.
- 469 [7] S. Sattar and A. B. Liel, “Seismic performance of reinforced concrete frame structures with and  
470 without masonry infill walls,” in *9th US National and 10th Canadian conference on earthquake  
471 engineering*, 2010.
- 472 [8] S. Pujol, A. Benavent-Climent, M. E. Rodriguez, and J. P. Smith-Pardo, “Masonry infill walls: an  
473 effective alternative for seismic strengthening of low-rise reinforced concrete building structures,”  
474 in *14th World Conference on Earthquake Engineering (Beijing, China, Unknown Month October  
475 12)*, 2008.
- 476 [9] L. M. Abdel-Hafez, A. E. Y. Abouelezz, and F. F. Elzefer, “Behavior of masonry strengthened  
477 infilled reinforced concrete frames under in-plane load,” *HBRC J.*, vol. 11, no. 2, pp. 213–223,  
478 Aug. 2015.
- 479 [10] G. Al-Chaar, “Evaluating strength and stiffness of unreinforced masonry infill structures,” US  
480 Army Corps of Engineers, Engineer Research and Development Center, Construction Engineering  
481 Research Laboratory, 2002.
- 482 [11] G. Al-Chaar, “Non-ductile behavior of reinforced concrete frames with masonry infill panels  
483 subjected to in-plane loading,” DTIC Document, 1998.
- 484 [12] F. J. Crisafulli, A. J. Carr, and R. Park, “Analytical modelling of infilled frame structures - A  
485 general review,” Society for Earthquake Engineering, New Zealand, 2000.
- 486 [13] G. Al-Chaar and D. Abrams, “Parametric Studies on Seismic Behavior of Frame-Infill Systems,” in  
487 *Proceedings of the Ninth Canadian Masonry Symposium*, 2001.
- 488 [14] G. Al-Chaar, G. Lamb, and D. Abrams, “Seismic Behavior of a Multistory and Multibay Frame-  
489 Infill System,” in *Proceedings of the Ninth Canadian Masonry Symposium*, 2001.
- 490 [15] W. W. El-Dakhkhni, M. Elgaaly, and A. A. Hamid, “Three-Strut Model for Concrete Masonry-  
491 Infilled Steel Frames,” *J. Struct. Eng.*, vol. 129, no. 2, pp. 177–185, Feb. 2003.
- 492 [16] P. G. Asteris, “Numerical Investigation of the Effect of Infill Walls on the Structural Response of  
493 RC Frames,” *Open Constr. Build. Technol. J.*, vol. 6, no. 1, pp. 164–181, Oct. 2012.
- 494 [17] P. G. Asteris, I. P. Giannopoulos, and C. Z. Chrysostomou, “Modeling of Infilled Frames With  
495 Openings,” *Open Constr. Build. Technol. J.*, vol. 6, no. 1, pp. 81–91, Oct. 2012.
- 496 [18] P. G. Asteris, C. Z. Chrysostomou, and I. P. Giannopoulos, “MASONRY INFILLED  
497 REINFORCED CONCRETE FRAMES WITH OPENINGS,” in *Proc 3rd international conference  
498 on computational methods in structural dynamics and earthquake engineering*, Greece, 2011, p. 15.
- 499 [19] P. G. Asteris, D. M. Cotsovos, C. Z. Chrysostomou, A. Mohebkhah, and G. K. Al-Chaar,  
500 “Mathematical micromodeling of infilled frames: State of the art,” *Eng. Struct.*, vol. 56, pp. 1905–  
501 1921, Nov. 2013.
- 502 [20] P. G. Asteris, L. Cavaleri, F. Di Trapani, and V. Sarhosis, “A macro-modelling approach for the  
503 analysis of infilled frame structures considering the effects of openings and vertical loads,” *Struct.  
504 Infrastruct. Eng.*, vol. 12, no. 5, pp. 551–566, May 2016.
- 505 [21] M. Sasani, “Response of a reinforced concrete infilled-frame structure to removal of two adjacent  
506 columns,” *Eng. Struct.*, vol. 30, no. 9, pp. 2478–2491, Sep. 2008.
- 507 [22] Cachado, A. Grilo, I. Júlio, E. and Neves, L., “Use of Non-Structural Masonry Walls as Robustness  
508 Reserve,” in *IABSE-IASS Symposium-Taller, Longer, Lighter*, London, UK, 2011.
- 509 [23] S. Farazman, B. A. Izzuddin, and D. Cormie, “Influence of Unreinforced Masonry Infill Panels on  
510 the Robustness of Multistory Buildings,” *J. Perform. Constr. Facil.*, Sep. 2012.
- 511 [24] F. Xavier, L. Macorini, and B. Izzuddin, “Robustness of multistory buildings with masonry infill,”  
512 *J. Perform. Constr. Facil.*, vol. 29, no. 5, p. B4014004, 2014.
- 513 [25] H. Helmy, H. Hadhoud, and S. Mourad, “Infilled masonry walls contribution in mitigating  
514 progressive collapse of multistory reinforced concrete structures according to UFC guidelines,” *Int.  
515 J. Adv. Struct. Eng. IJASE*, vol. 7, no. 3, pp. 233–247, Sep. 2015.
- 516 [26] Department of Defense, USA, *Design of Structures to Resist Progressive Collapse*. 2013.
- 517 [27] E. Cavaco, L. A. C. Neves, and J. R. Casas, “Reliability-based approach to the robustness of  
518 corroded reinforced concrete structures,” *Struct. Concr.*, vol. 18, no. 2, pp. 316–325, Apr. 2017.



- 519 [28] S. Mazzoni, F. Mckenna, M. H. Scott, and G. L. Fenves, *Opensees command language manual*.  
520 Berkeley: Pacific Earthquake Engineering Research Center, 2015.
- 521 [29] Imprensa Nacional and Casa da Moeda, *Regulamento de Estruturas de Betão Armado e Pré-*  
522 *esforçado*. 1983.
- 523 [30] T. Vrouwenvelder, “The JCSS probabilistic model code,” *Structural Safety*, vol. 19. pp. 245–251,  
524 1997.
- 525 [31] R. Vicente, “Patologia Paredes Fachada. Estudo do comportamento mecânico das paredes de  
526 fachada com correcção exterior das pontes térmicas.,” MSc thesis, FCT-Universidade de Coimbra,  
527 Coimbra, 2002.
- 528 [32] A. A. Chojaczyk, A. P. Teixeira, L. C. Neves, J. B. Cardoso, and C. Guedes Soraes, “Review and  
529 application of Artificial Neural Networks models in reliability analysis of steel structures,” *Struct.*  
530 *Saf.*, 2014.
- 531 [33] A. Haldar and S. Mahadevan, *Probability, reliability, and statistical methods in engineering design*.  
532 John Wiley & Sons, Incorporated, 2000.
- 533 [34] H. M. Gomes and A. M. Awruch, “Comparison of response surface and neural network with other  
534 methods for structural reliability analysis,” *Struct. Saf.*, vol. 26, pp. 49–67, 2004.
- 535 [35] A. Hosni Elhewy, E. Mesbahi, and Y. Pu, “Reliability analysis of structures using neural network  
536 method,” *Probabilistic Eng. Mech.*, vol. 21, no. 1, pp. 44–53, Jan. 2006.
- 537 [36] A. T. Goh and F. H. Kulhawy, “Neural network approach to model the limit state surface for  
538 reliability analysis,” *Can. Geotech. J.*, vol. 40, no. 6, pp. 1235–1244, Dec. 2003.
- 539 [37] M. Papadrakakis, V. Papadopoulos, and N. D. Lagaros, “Structural reliability analysis of elastic-  
540 plastic structures using neural networks and Monte Carlo simulation,” *Comput. Methods Appl.*  
541 *Mech. Eng.*, vol. 136, no. 1, pp. 145–163, Sep. 1996.
- 542 [38] J. B. Cardoso, J. R. de Almeida, J. M. Dias, and P. G. Coelho, “Structural reliability analysis using  
543 Monte Carlo simulation and neural networks,” *Adv. Eng. Softw.*, vol. 39, no. 6, pp. 505–513, Jun.  
544 2008.
- 545 [39] D. Hanselman, B. Littlefield, and C. S. A. Martins, *Matlab 6: curso completo*. Prentice Hall, 2003.
- 546 [40] J. J. Moré, “The Levenberg-Marquardt algorithm: implementation and theory,” in *Numerical*  
547 *analysis*, Springer, 1978, pp. 105–116.
- 548 [41] CEN, “EN 1990: Eurocode: Basis of Structural Design,” *Bruss. Belg. Eur. Norm.*, 2002.
- 549