

Two probabilistic life-cycle maintenance models for the deteriorating pavement

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Abstract: Pavement maintenance management poses a significant challenge for highway agencies in terms of pavement deterioration over time and limited financial resources to keep the road condition at an acceptable level. In this paper two probabilistic maintenance models are proposed and compared for pavement deterioration and maintenance processes to evaluate different maintenance strategies. Firstly, the states of pavement condition are defined using the features of different pavement maintenance works, instead of using the traditional method of cumulative service index rating. Secondly, a Markovian model is presented to describe the pavement deterioration and maintenance process with some constraints on the number of interventions, the effect of interventions and etc. But for the complex scenarios, such as non-Markovian deterioration, dependencies between the different types of interventions and the usage of emergency maintenance for roads when the required budget for maintenance is unavailable, a simulation-based Petri-net model is built up to investigate the whole life-cycle evolution. Two examples are used to illustrate and compare the proposed models to demonstrate the merits and disadvantages of each model and its applicable conditions.

Keywords: Performance deterioration; pavement maintenance management; Markov process; Petri-net method

1. Introduction

Highway infrastructure is not only the most valuable asset for most countries, but also the largest asset used daily by the majority of their residents, which can strongly affect their quality of life. With increasing demands to cut the cost of road maintenance and maintain and even improve road conditions under reduced financial investments, highways agencies are placing more focus on establishing an efficient pavement management system (PMS). For example, FHWA (Federal Highway Administration) in the United States established a policy in 1989 which required all the states to have a PMS for managing interstate and principal highways ^[23]. Over 60% local councils have established PMS in United Kingdom before 2009^[5]. One of the main functions of the PMS is to support asset management decisions when a cost-effective road maintenance strategy needs to be found. However, this is a difficult task to achieve since such a strategy will depend on a number of factors ^[10,14], to name a few, the type and distress of pavement (such as rutting, cracking, ravelling and roughness, etc.), its location, climate and usage, effectiveness of pavement treatment technologies, etc. Therefore, over the last few decades a lot of research has been undertaken in the area of pavement deterioration and maintenance modelling. In general ^[2], pavement condition models can be divided into two categories: deterministic and probabilistic. In a deterministic model it is assumed that the pavement condition can be predicted as a single value on the basis of appropriate mathematical equations which relate the condition to a number of explanatory variables. For example, data on structural indicators^[7], such as rutting and roughness, or service indexes^[18], such as PSI (Present Serviceability Index), IRI (International Roughness Index) and CCI (Critical Condition Index), have been fitted to linear or nonlinear equations in order to predict the pavement condition. Such models cannot provide the PMS with a measure of uncertainty of future pavement condition, whereas probabilistic models can predict the condition as the probability of occurrence of a range of possible values, instead of a single value, and capture the uncertainty of pavement deterioration process and maintenance effects.

Markov model is a commonly used probabilistic state-based method for pavement condition, which can integrate several deterioration factors expressed in a transition probability matrix among discrete states ^[22]. It has been widely applied both in research and in practice for different types of pavement (rigid and flexible) to inform maintenance decisions at different levels of complexity (segment, project and network

[9,15,24]). However, the underlying assumption of 'no memory' for Markov model is not suitable for the application in more realistic scenarios of pavement maintenance, such as the fact that maintenance decision for the current state can be determined not only by the current condition but also by history of recent interventions before the current condition is reached. Also, an exponentially increasing number of states with the growth of the number of pavement sections can result in difficulties in finding a solution [17]. Thus some simulation models have been proposed to describe more realistic scenarios of transportation deterioration and maintenance process. A probabilistic model [11] based on Monte Carlo simulation has been developed to evaluate the cost-reliability trade-off in a flexible maintenance strategy with the uncertainty of parameters and the effects of maintenance actions on pavement condition. Similarly, Hong and Prozzi^[6] predicted pavement condition using Bayesian networks, which took account of three main factors of deterioration: structural indicators, environmental effects and traffic load, and Markov-chain Monte Carlo simulation was applied to estimate the parameter distribution. A simulation-based genetic algorithm (GA) approach was developed by Chootinan et al. [4] in order to plan maintenance activities over a planning period. A stochastic simulation was used to describe the uncertainty of future pavement condition, while the GA was used to handle the large number of combinations of maintenance actions for a network level problem.

Due to its features the Petri net (PN) formulation with a Monte Carlo solution routine has been widely used to model the combined deterioration and maintenance processes [21]. For example^[25], the PN method was used as a powerful analysis technique for the uncertainties in deterioration and maintenance process of multi-unit system, which has been combined with GA to get the solution of the optimal maintenance scheduling. Kowalski et al [12] extended the basic constructs of the high-level PN using the definition of immediate and timed transitions in order to model transportation system, where various factors such as redundancy, repair shop capacity and spare part inventory levels were discussed under different strategies in order to study effects of changing parameter values within the simulation. Prescott and Andrews^[19] built a model of track maintenance processes using the PN method, which considered the order of interventions and opportunistic maintenance under the limitations on the number of maintenance machines available. Application examples [1,8,16] of PN include workflow of a business, asset management of a supply chain, maintenance processes of infrastructure systems and the production of an industrial plant. So far, no PMS methods published in literature considered the PN method as a tool for simulating complex processes of pavement deterioration and maintenance.

There are two major contributions of this paper in the area of pavement maintenance management. First of all, the states of pavement condition are defined according to the features of different pavement maintenance interventions, when the grouping into model states is based on the type and extent of pavement distress, instead of basing it on the cumulative service index. Secondly, the newly defined states can be used in a Markov model with some constraints and more realistic scenarios of pavement maintenance are proposed to be modelled using the PN method. Such scenarios include situations when the pavement deterioration cannot satisfy the "no-memory" assumption and may depend not only on the usage of the asset but also on the effects of maintenance works carried out previously, such as the history of the number and type of routine and preventive repairs. In addition, emergency repairs of pavement might need to be carried out while the required budget for planned maintenance is unavailable, and this common strategy (when there are increasing pressures to cut the cost of road maintenance) cannot be modelled using Markov model. Using the proposed methods different maintenance strategies can be compared in terms of long-term cost and the extended pavement design life in order to inform pavement management decisions.

The structure of this paper is as follows. In Section 2 the states of the pavement are defined according to the type and extent of pavement distress followed by the features of the relevant interventions. Two pavement maintenance models are built in Section 3, using the newly defined states in the Markov model and the PN model. Section 4 illustrates the results of the two models using two examples. First of all, the Markov model and the PN model have been solved for a number of simple scenarios to demonstrate the validity of the two models developed. Then a number of more complex scenarios have been used to

demonstrate the flexibility of the PN model and the simulation method for maintenance planning. The sensitivity analysis has also been carried out in order to illustrate the influence of the main factors on the analysis. Some concluding remarks are given in Section 5.

Nomenclature

s_i : pavement states, $i=0,1,1',2,2',3,4$	P_{ij} : the transition probability between two states s_i and s_j at time t .
r_1 : the state threshold of routine repair	c_i : the agency cost of intervention i , $i=1,\dots,4$
r_2 : the state threshold of preventive repair	t_{in} : the time spent on inspection
T : the inspection interval	c_{ub} : the user cost when pavement remains in state b
N_1 : the maximum number of routine repair	c_d : the delay cost caused by the closure of pavement repair
N_2 : the maximum number of preventive repair	c_{in} : the cost of pavement inspection
Y_i : the deteriorating process of pavement	t_k : the decision epoch at k^{th} inspection
L : the planning horizon	$CA_i^{m_1,m_2}$: the mean agency cost calculated from time $E_i^{m_1,m_2}$ till the end of the planning horizon
λ_i : the transition (deterioration) rate from state s_{i-1} to s_i , $i=1,\dots,4$	$CU_i^{m_1,m_2}$: the user cost calculated from time $E_i^{m_1,m_2}$ till the end of the planning horizon
μ_i : the transition (repair) rate from state s_i to s_0 , $i=1,\dots,4$	$Q_{ibj}(T)$: the mean operation time of the pavement in state b , where $i \leq b \leq j$, during the interval $[0,T]$
$E_i^{m_1,m_2}$: the time when the section of the pavement has been identified to be in state s_i after m_1 routine repairs and m_2 preventive repairs	$L_i^{m_1,m_2}(T)$: the mean time from time $E_i^{m_1,m_2}$ to the completion of the next renewal
C_p : the long-term cost in the planning horizon	L_p : the expected life-cycle time in the planning horizon
N_{rr} : the number of routine repairs in the planning horizon calculated from the PN model	N_{em} : the number of emergency maintenance works
N_{pr} : the number of preventive repairs	N_{rec} : the number of reconstructions
N_{cm} : the number of corrective maintenance works	D_i : the duration of staying in state i , $i=3, 3'$ and 4
N_{in} : the number of inspections	

2. The process of pavement deterioration and maintenance

The states in deterioration model are defined using a rating score of some service indexes. For example^[10], the score of CCI can be grouped into five intervals from 0 to 100, where a range above 90 corresponds to excellent, 70-89 - good, 60-69 - fair, 50-59 - poor, 49 and below - very poor condition. However, the pavement distresses can be divided into three categories: structural deterioration (alligator cracking and rutting), environmental cracking (longitudinal/transverse cracking and edge cracking) and surface wear (ravelling/weathering and distortion). A service index like CCI can only represents the combination of several pavement distresses, which maybe not make the most of the information from the other distresses or maintenance process. According to highway maintenance manuals^[10], a specific maintenance action can correspond to the different type of distress with different extent and severity. Further details of the grouping can be found in Appendix A. In addition, most PMS has relatively complete records on maintenance type and time rather than on pavement distresses. Some recent developments in railway track^[3] and bridge asset management modelling^[13,19] illustrate how the states can be defined according to specific deterioration experienced and maintenance actions carried out to improve the usage of all collected information.

Therefore, In this paper it is proposed to group the types and severity of distress into four states according to decrease in pavement functionality and maintenance actions required, classified as routine repair (RR) (s_1 - cracking treatment and patching), preventive repair (PR) (s_2 - chip seal, slurry seal and micro-surfacing; thin overlay), corrective maintenance (CM) (s_3 - mill and fill; mill and overlay) and reconstruction (s_4). The state is revealed by periodic inspections; therefore, even if the transition occurs between two consecutive inspections, maintenance cannot be carried out until the inspection takes place. It is assumed that corrective maintenance and reconstruction restore the pavement to the new state, however, the routine and preventive repair can only improve the pavement to states s_1' and s_2' respectively, which are

not the new states but better than former ones. Therefore, the deterioration level of the pavement can be defined by one of the states in $S = \{s_0, s_1', s_2', s_1, s_2, s_3, s_4\}$, where s_0 represents pavement performance as new and no interventions are needed. The pavement starts in the new state and moves through the deteriorated states over time. Each maintenance strategy is defined by a vector (r_1, r_2, T, N_1, N_2) , where (r_1, r_2) denote the states for routine (RR) and preventive repairs (PR) respectively, T – the duration between two inspections, and (N_1, N_2) are the maximum number of routine and preventive repairs respectively. In general, three strategies are considered:

- (1) If $N_1 = N_2 = 0$, then no RR and PR on a section are possible; in the states s_3 and s_4 the CM or reconstruction is required respectively. In this case no routine or preventive works are carried out during the earlier levels of deterioration until corrective maintenance and reconstruction is necessary.
- (2) If $N_1 = 0, r_2 = 2$, then the PR only (not the RR) on a section is required when the pavement is in state s_2 ; in the following states CM and reconstruction is required. In this case preventive works (but not routine) are carried out during the earlier levels of deterioration, followed by corrective maintenance and reconstruction during the later levels of deterioration.
- (3) If $r_1 = 1, r_2 = 2, N_1 \neq 0, N_2 \neq 0$, then the RR and PR on a section are required (with the limitation on the maximum number) when the pavement is in state s_1 and s_2 respectively; in the following states CM and reconstruction is required. In this case all possible works are carried out during the pavement lifetime.

Each strategy can be evaluated in terms of its cost (agency and user cost^[3]) and in terms of its effects on pavement condition. In this paper it is proposed to use the long-term cost and the lifetime of the pavement as two evaluation criteria. It is assumed that agency cost includes maintenance and inspection costs while user cost considers vehicle operation costs, such as fuel and lubrication consumption, tyre wear and vehicle repair costs, but not travel delay costs due to maintenance. The agency cost is influenced by planning the investment to keep roads at an acceptable level, and the user cost increases if the pavement deterioration increases. Benefits of the chosen strategy are then measured in terms of the additional years of usage of pavement.

3. Pavement maintenance models

Using the proposed scheme for defining pavement states in Section 2, two models have been developed in this paper. First of all, a Markov model is proposed and used to illustrate the process of pavement deterioration and maintenance with some constraints, such as the maximum number of repairs allowed and the effect of different repairs. Secondly, a PN model is proposed when more complex situations of road maintenance planning are considered, such as the non-Markovian deterioration process, the inclusion of the history on the previous interventions and the emergency repairs if the budget for expensive interventions is unavailable.

3.1 Markov model

If one can assume that the transition between any two states is at a constant rate, the deterioration process of the pavement can be characterized by a continuous-time Markov process $\{Y_t\}_{t \geq 0}$ with state space S . If the planning horizon is denoted as $L = nT, n \in N$, then the sequence of inspections is denoted as $\{T, 2T, \dots, nT\}$. At each inspection the state of the pavement will be identified and the interventions, which will restore the pavement to the new state, will be modelled according to the chosen strategy. The rate of deterioration and the rate of maintenance are defined as λ_i and μ_i respectively, where i can be from 1 to 4. And the deterioration rates from s_1' and s_2' to s_1 are assumed to be λ_1 and λ_2 respectively. A dynamic programming model can be built in order to evaluate the expected long-term cost and expected lifetime

during the planning horizon. If the current decision epoch is $t_k = kT, k \in \{1, 2, \dots, n\}$, the actions at t_k must be one of following: (1) do nothing, if $i = 0, 1', 2'$ or $i = 1, m_1 = N_1$ or $i = 2, m_2 = N_2$, i.e. the maximum number of routine repairs or preventive repairs has been reached; (2) repair the pavement with the specified intervention required by current state – routine repairs if $i = 1, m_1 < N_1$, preventive repairs if $i = 2, m_2 < N_2$ corrective maintenance if $i = 3$ and reconstruction if $i = 4$. Then the long-term cost can be evaluated as $C_i^{m_1, m_2}(t_k) = CA_i^{m_1, m_2}(t_k) + CU_i^{m_1, m_2}(t_k)$. According to Bellman theorem ^[20], the cost can be calculated using the iterative approach described in Equations 1 and 2:

$$CA_i^{m_1, m_2}(t_{k+1}) = \begin{cases} c_{in} + \sum_{j \in S} P_{ij}(T) C_j^{m_1, m_2}(t_k), & i = 0, 1', 2' \text{ or } i = 1, m_1 = N_1 \text{ or } i = 2, m_2 = N_2 \\ c_{in} + c_1 + \sum_{j \in S} P_{1'j}(T) C_j^{m_1+1, m_2}(t_k), & i = 1, m_1 < N_1 \\ c_{in} + c_2 + \sum_{j \in S} P_{2'j}(T) C_j^{m_1, m_2+1}(t_k), & i = 2, m_2 < N_2 \\ c_{in} + c_3 + \sum_{j \in S} P_{0j}(T) C_j^{m_1, m_2}(t_k), & i = 3 \\ c_{in} + c_4 + \sum_{j \in S} P_{0j}(T) C_j^{0,0}(t_k), & i = 4 \end{cases} \quad (1)$$

$$CU_i^{m_1, m_2}(t_{k+1}) = \begin{cases} \sum_{j \in S} P_{ij}(T) \left[\sum_{b=i}^4 Q_{ibj}(T) * C_{ub} + CU_j^{m_1, m_2}(t_k) \right], & i = 0, 1', 2' \text{ or } i = 1, m_1 = N_1 \text{ or } i = 2, m_2 = N_2 \\ c_d / \mu_1 + \sum_{j \in S} P_{1'j}(T) \left[\sum_{b=1'}^4 Q_{ibj}(T) * C_{ub} + CU_j^{m_1+1, m_2}(t_k) \right], & i = 1, m_1 < N_1 \\ c_d / \mu_2 + \sum_{j \in S} P_{2'j}(T) \left[\sum_{b=2'}^4 Q_{ibj}(T) * C_{ub} + CU_j^{m_1, m_2+1}(t_k) \right], & i = 2, m_2 < N_2 \\ c_d / \mu_3 + \sum_{j \in S} P_{0j}(T) \left[\sum_{b=0}^4 Q_{ibj}(T) * C_{ub} + CU_j^{m_1, m_2}(t_k) \right], & i = 3 \\ c_d / \mu_4 + \sum_{j \in S} P_{0j}(T) \left[\sum_{b=0}^4 Q_{ibj}(T) * C_{ub} + CU_j^{0,0}(t_k) \right], & i = 4 \end{cases} \quad (2)$$

When the transition rate from state s_i to s_j , λ_i , is known, the transition probability, $P_{ij}(t)$, can be calculated by solving the Kolmogorov equation^[20]. Let $Q_{ibj}(T)$ represent the mean operation time of the pavement in state b , where $i \leq b \leq j$, during the interval $[0, T]$, which can be calculated using Equation 3:

$$Q_{ibj}(T) = \int_0^T \frac{P_{ib}(t) P_{bj}(T-t)}{P_{ij}(T)} dt \quad (3)$$

Finally, the long-term cost can be computed as $C_0^{0,0}(nT)$.

In addition, assume that $L_i^{m_1, m_2}(T)$ is the mean time from time $E_i^{m_1, m_2}$ to the completion of the next renewal. Due to the limitation on the maximum number of routine and preventive repairs, only the CM or reconstruction can result in the renewal of the pavement. Therefore, $L_i^{m_1, m_2}(T)$ can be calculated using Equation 4. Finally, the lifetime in the planning horizon can be calculated as $L_0^{0,0}(T)$.

Using this method, the long-run cost and the lifetime under different maintenance strategies can be evaluated.

$$L_i^{m_1, m_2}(T) = \begin{cases} \sum_{j \in S} P_{ij}(T) [T + t_{in} + L_j^{m_1, m_2}(T)], & i = 0, 1', 2' \text{ or } i = 1, m_1 = N_1 \text{ or } i = 2, m_2 = N_2 \\ 1/\mu_1 + \sum_{j \in S} P_{0j}(T) L_j^{m_1+1, m_2}(T), & i = 1, m_1 < N_1 \\ 1/\mu_2 + \sum_{j \in S} P_{0j}(T) L_j^{m_1, m_2+1}(T), & i = 2, m_2 < N_2 \\ 1/\mu_3, & i = 3 \\ 1/\mu_4, & i = 4 \end{cases} \quad (4)$$

This process of evaluation of different maintenance strategies is only possible if a number of simplifying assumptions are made. However, the assumption of the constant deterioration rate might not be always true in real-world applications, i.e. the rate of deterioration of the pavement will increase if the level of distress increases. Also the number and timing of routine repairs (crack sealing and patching) will influence the effectiveness of preventive repairs (micro-surfacing and thin overlay). In addition, there might be a situation when emergency maintenance works need to be carried out, such as pothole patching, if corrective maintenance is needed according to the state of the pavement but the resources for it are unavailable. In this case such actions would be a temporary solution until a more permanent treatment can be carried out. Overall, using Markov model it appears to be impossible to model a range of situations, commonly observed in the practice of pavement maintenance, and one possibility for solving the problem is to develop a simulation model.

3.2 Petri-net model

The Petri-net (PN) method is commonly used to model the behaviour of dynamic systems in engineering, science and business context. The original concept of the PN is defined as a bi-partite directed graph with places and transitions linked by arcs. A place in the PN represents a particular state or condition of the system. A token present inside the place indicates that the state of the system is true. A transition through the PN moves tokens from one place to another mimicking the dynamic behaviour of the system^[19]. In the context of asset management, a PN can be used to replicate the processes of asset deterioration, inspection and maintenance. In this paper a PN model is proposed for modelling pavement deterioration and maintenance strategies, with the possibility to relax the assumptions needed for Markov approach.

3.2.1 the description of PN model

The PN model as shown in Figure 1 is built up to illustrate how the complex process, described in Section 2, can be modelled in a probabilistic manner and the results used to support pavement maintenance decisions. Places from P0 to P4 represent the four states of the pavement and the transitions from T0 to T3 – the transitions between the neighboring states. These transitions will fire while sampling values from appropriate distributions, which can be obtained from historical data of pavement maintenance. Every state is revealed by inspection, which is modelled by the loop P5→T4→P6→T5→P5. If since the last inspection the pavement has degraded to one of the states, now it is revealed and the token is moved to a place which represents one of the four revealed states (places from P7 to P10). Then the complex scenarios which cannot be described by Markov model are illustrated in details.

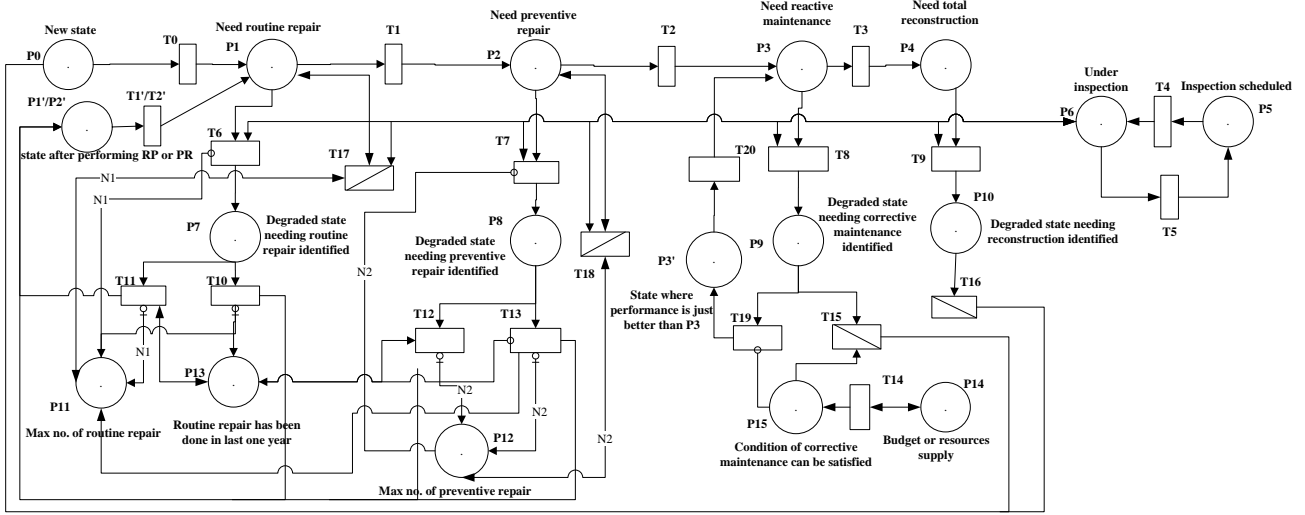
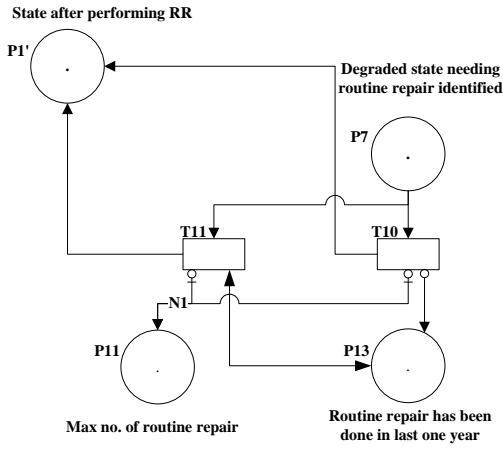


Figure 1. An overall PN model for pavement maintenance management

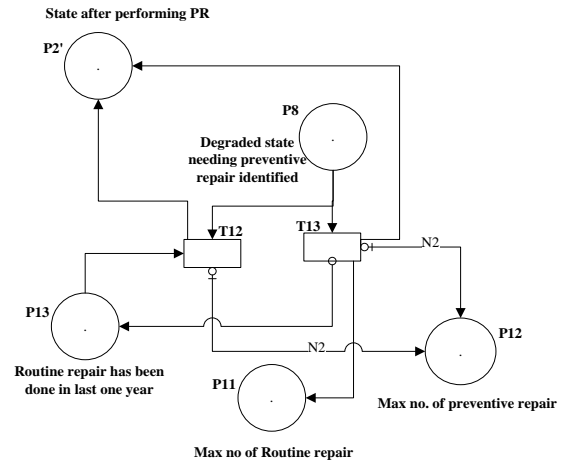
(1) The dependency between the routine and preventive repair

The routine repair, such as crack sealing or patching, is the first line of defense in pavement maintenance. It is generally recommended to be carried out within two years after the pavement renewal. It is assumed that following this routine or preventive repair, the pavement returns to a better but not new state, i.e. the token is transferred to Place P1' or P2'. In addition, the routine repair is required to be carried out as a pretreatment in the preparation phase before the preventive repair, if there are some cracks or potholes in the pavement. For example, cracks of 1/4 inch or wider should be treated (using crack sealing) prior to the chip sealing. However, if the routine repair has been carried out within one year after the renewal, the pretreatment before the preventive repair is unnecessary. The dependency between the routine and preventive repair has been modelled in Figure 2a and 2b. In Figure 2a the place P13 is used to represent an event when the routine repair has been carried out after the last inspection before state P2 (which requires the preventive repair) is reached. Note that commonly the pavement maintenance annuals recommend that the inspection should be carried out at least once per year. This process of the routine repair also influences the process of the preventive repair, as shown in Figure 2b, when the transition T13, which represents a situation when the preventive repair should be performed after the pretreatment of the routine repair, is inhibited. The transition T12 will be enabled when the preventive repair without a pretreatment is possible.

With time some interventions can become ineffective after being carried out for a number of times. For example, the overlay (preventive repair) can be carried out only for a limited number of times until a certain thickness of the pavement is reached. Thus places P11 and P12 are used to record the number of times the routine and preventive repairs were carried out, as shown in Figure 2. Once the maximum number of the intervention is reached, N1 and N2 for the routine and preventive repair respectively, further repairs are inhibited. In order to ensure that the deterioration process can continue in the model, two transitions T17 and T18 are used in Figure 3, also known as the reset transitions. Note that the reset transition is not a commonly used type of the transition in the PN method, as developed in [21]. In this model the reset transitions are used to represent the situation when, for example, the routine repair cannot be carried out (since the maximum number of repairs is reached) and all the tokens from the place P11 are removed in Figure 3a, so that the process of carrying out the routine repairs can restart after the next inspection. A similar situation is described in Figure 3b when the preventive repairs are considered.

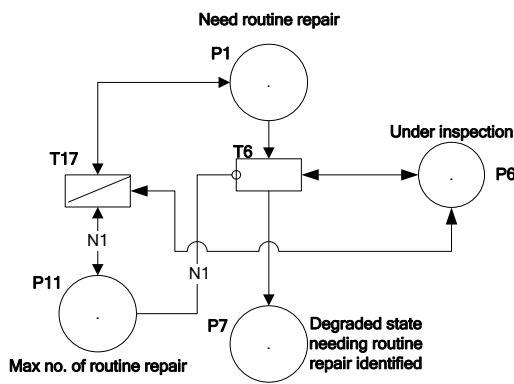


(a)

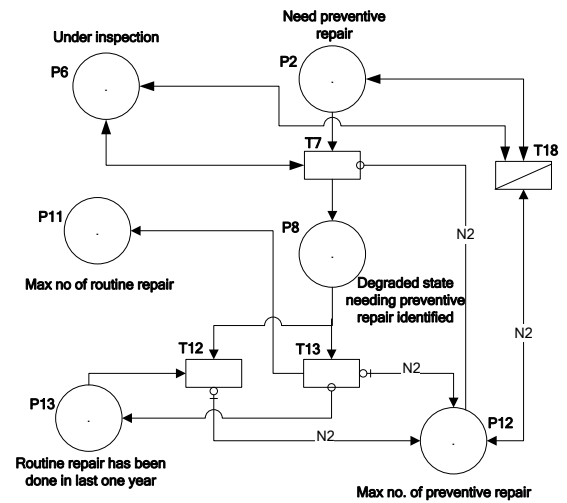


(b)

Figure 2. Two PN for the processes of routine (a) and preventive repairs (b)



(a)



(b)

Figure 3. Two PN for the introduction of the maximum number of routine (a) and preventive repairs (b)

(2) The emergency repairs if the resources are unavailable

If the corrective maintenance is needed, the significant levels of pavement deterioration need to be treated quickly to avoid further deterioration and resulting hazardous situations. If the resources for the corrective maintenance are unavailable (due to poor planning or unforeseen conditions, such as bad weather), a temporary treatment, known as the emergency maintenance (EM), should be performed in the meantime. For example, if large potholes appear in the section an intervention of hand patching can be carried out in order to hold the surface together until the mill & fill treatment (corrective maintenance) can be performed. This situation is modelled in Figure 4. If the budget is available to carry out the corrective maintenance, the places P9 and P15 are marked and after the repair the pavement returns to the new state. Otherwise, the emergency repair is carried out marking the place P3', which describes an intermediate state before it happens, and using the repair transition T20 to return to the state, denoted by P3, where corrective maintenance is necessary, but it can be carried out at a later date.

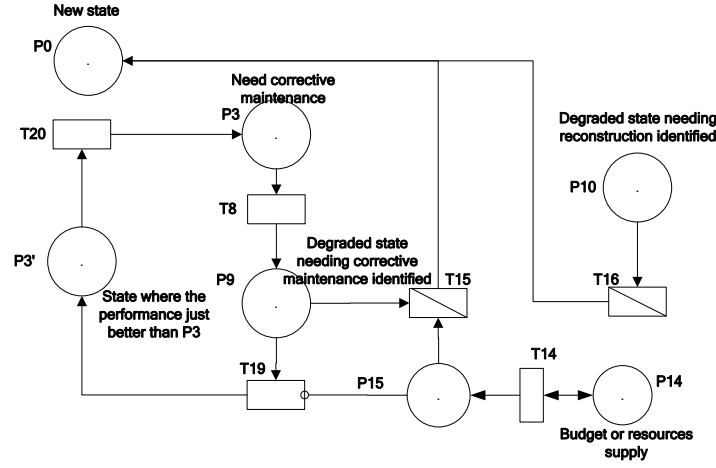


Figure 4. A PN for the processes of corrective maintenance and reconstruction

3.2.2 the analysis of PN model

As in Section 3.1, the PN model is used to evaluate the long-term cost and lifetime. The model is solved using a Monte Carlo simulation, when each simulation corresponds to a virtual experiment when one life history of the section evaluated throughout the planning horizon. For example, the long-term cost C_p in the planning horizon can be calculated using Equation 5:

$$C_p = c_1 N_{rr} + c_2 N_{pr} + c_3 N_{em} + c_3 N_{cm} + c_4 N_{rec} + \sum_{b \in \{3,3',4\}} c_{ub} D_b + c_{in} N_{in} \quad (5)$$

First of all, the number of different interventions and inspections are obtained from the PN model, by recording the number of times when relevant transitions have been enabled, such as T10 and T11 for the routine repair. These numbers are multiplied by the agency cost for each intervention and inspection. Secondly, the duration of being in the state that influences the user cost are obtained, by recording the time when relevant places have been marked for, such as P3 for corrective maintenance. These durations are multiplied by the user cost in each state. Note that the statistics are collected over the planning period.

In addition, other results can be obtained from the PN model, such as the number of preventive repairs that were carried out with or without a pretreatment, the number of times when the budget has been exceeded, etc. Such statistics can provide the decision maker with detailed information, impossible to obtain using the analytical approach, about each strategy of pavement maintenance and support their investment decisions.

4. Numerical application

The two models developed in Section 3 are solved using numerical values for the parameters. For example, the length of the pavement section is assumed to be 1 km and the planning horizon is 50 years. Example 1 demonstrates that the results of the two models (Markov and PN) agree, if some complex scenarios cannot be considered in the PN illustrated in Figure 1; Example 2 demonstrates how the PN model can be useful in analyzing the maintenance strategies that are more complex than those described in the Markov model.

4.1 Example 1

4.1.1 Model implementation

The values of the deterioration and cost parameters are listed in Table 1. According to pavement maintenance manuals^[10] the recommended frequencies of the crack sealing (RR), thin overlay (PR), mill & fill (CM) and reconstruction are once in 2 years, once in 7 years, once in 15 years and once in 30 years respectively. The effect of cracking sealing maybe remain 1 year and thin overlay can extend the life of pavement about 5.5 years. For illustration purposes in this paper these frequencies have been used to derive the deterioration rates from the new state to a deteriorated state that requires each maintenance action

considered. The intervention cost and the mean time to its completion increases the worse the pavement becomes. It is assumed that the user cost is considered only if the states of the pavement need the corrective maintenance (s_3) or reconstruction (s_4), otherwise, they are ignored. The cost and duration of the yearly inspection is assumed to be constant at each state of deterioration. These assumptions have been taken to simplify the calculations; however, they can be relaxed to represent different scenarios of maintenance if required. Note that the time to the next level of deterioration and the time to the completion of the intervention follow the exponential distribution, necessary for Markov model.

Table 1. Input parameters for Example 1

State	from s_0 to s_1'	from s_0 to s_2'	from s_0 to s_1	from s_0 to s_2	from s_0 to s_3	from s_0 to s_4
Deterioration rate (per day)	0.00278	0.00185	0.00139	0.00035	0.00019	0.00009
Intervention cost (\$)	0	0	200	1500	3000	7000
State	s_1'	s_2'	s_1	s_2	s_3	s_4
Mean time of an intervention (day)	0	0	0	7	15	60
User cost (\$/day)	0	0	0	0	10	50
Inspection cost (\$)					10	
Inspection duration (day)					0.1	

A number of different maintenance strategies are consider in Table 2 and analyzed below.

Table 2. Pavement maintenance strategies

Strategy	r_1	r_2	N_1	N_2	T (year)	Description
1	3	3	0	0	1	No RR and PR, CM (in state s_3) and reconstruction (in state s_4)
2	2	3	1	0	1	Once RR (s_1), No PR, CM (s_3) and reconstruction (s_4)
3	2	3	2	0	1	Twice RR (s_1), No PR, CM (s_3) and reconstruction (s_4)
4	2	2	0	1	1	No RR, once PR (s_2), CM (s_3) and reconstruction (s_4)
5	2	2	0	2	1	No RR, twice PR (s_2), CM (s_3) and reconstruction (s_4)
6	1	2	1	1	1	Once RR (s_1), once PR (s_2), CM (s_3) and reconstruction (s_4)
7	1	2	2	1	1	Twice RR (s_1), once PR (s_2), CM (s_3) and reconstruction (s_4)
8	1	2	1	2	1	Once RR (s_1), twice PR (s_2), CM (s_3) and reconstruction (s_4)
9	1	2	2	2	1	Twice RR (s_1), twice PR (s_2), CM (s_3) and reconstruction (s_4)

The Markov model has been solved using an analytical approach and the PN model has been simulated for 3000 simulations for each strategy. Note that 3000 simulations gave the convergence of the simulation results, as demonstrated in Section 4.2. Two outputs have been recorded, the long-term cost and the lifetime, as shown in Figure 5. The curves from the two models match well and can be used to validate the correctness of the implementation of the models, with some marginal differences between the long-term cost curves for some strategies. These differences can be explained by using an approximation of the numerical integral when computing $Q_{ibj}(T)$ in Markov model.

It can be seen that the long-term cost decreases when the routine and preventive repairs are introduced instead of using the corrective maintenance and reconstruction only, i.e. considering the strategies from 1 (no RR and PR) to 9 (both RR and PR implemented twice). At the same time, the lifetime increases, i.e. the pavement lasts longer if the routine and preventive repairs are carried out while the pavement is in an acceptable condition (state s_1 and s_2), instead of relying on the major interventions only applied in the more critical states (state s_3 and s_4). In addition, it is possible to analyze whether the routine repairs (cheap and quick) or the preventive repairs (expensive and long) are more effective. According to the results in Example 1 it can be seen that the strategy with a higher number of the preventive repairs than the routine repairs (for example, strategy 8, $N_1=1$ and $N_2 = 2$) results in the same cost and a longer lifetime than the strategy with a lower number of the preventive repairs (for example, strategy 7, $N_1 = 2$, $N_1 = 1$).

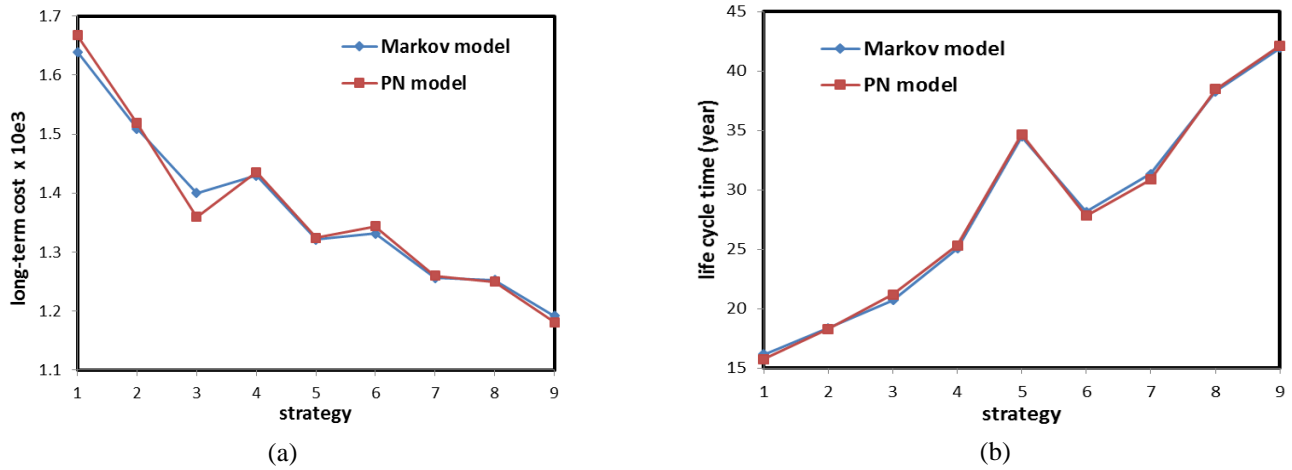


Figure 5. The results of the long-term cost (a) and the lifetime (b) in Example 1

4.1.2 Sensitivity analysis

In order to determine the effects on the long-term cost and lifetime resulted in by using the different parameters in the model, their sensitivity analysis has been carried out. For the illustration purposes only the results from one of the models (the PN model) has been used in this section.

(1) User cost

If the user cost is not included in the analysis, i.e. each strategy is evaluated in terms of agency cost only, it can be seen that it is hard to distinguish between the different strategies of pavement maintenance and hardly any benefits can be seen from the strategies that allow smaller and more frequent repairs in order to reduce the long-term cost, as shown in Figure 6. Therefore, the user cost should be included in the analysis. Note, that the user cost does not influence the lifetime in this study, therefore, no sensitivity analysis on this model output is carried out.

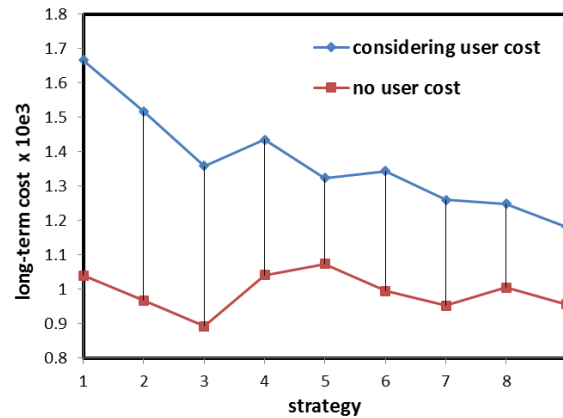


Figure 6. The results of the long-term cost with the user cost and without the user cost

(2) Cost of preventive repairs

The chosen value of the preventive repair cost can influence the comparison of the strategies that consider a different number of the routine repairs and preventive repairs, say strategy 7 (twice RR and once PR) and strategy 8 (once RR and twice PR). In the analysis presented in this section it is assumed that the cost of preventive repair is \$1500, as stated in Table 1, and the long-term cost of strategies 7 and 8 are the same, as shown in Figure 7.

If the cost of the preventive repair can be reduced, the long-term cost for strategy 8 (with a higher number of preventive repairs) is lower than for strategy 7 (with a lower number of preventive repairs). Alternatively, if the cost is higher, an advantage in terms of the long-term cost can be seen from strategy 7 instead of strategy 8, as shown in Figure 7. Note that as for the analysis of the user cost above, the change in the preventive repair cost has no influence on the life-time cycle of the pavement. Therefore, in terms of the lifetime strategy 8 is better than strategy 7 (as shown in Figure 5b) despite of the increase in the preventive

repair cost.

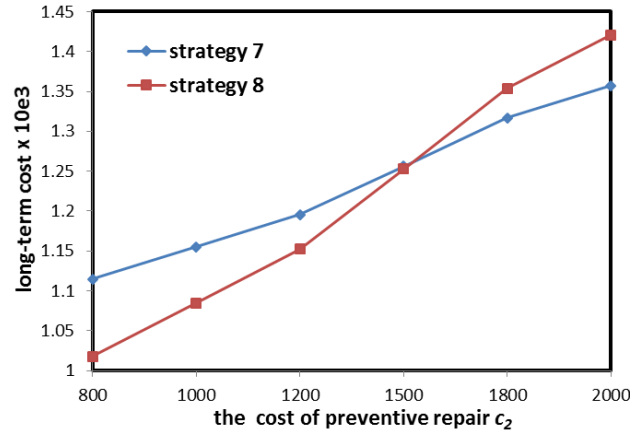


Figure 7. Influence of cost of preventive repairs to the long-term cost for the strategies 7 and 8

(3) Inspection interval

The inspection interval T is an important factor that can influence maintenance strategies. In Figure 8, the long-term cost and the lifetime are considered when the inspection interval is one year (Inspection 1) and 6 months (Inspection 2). When the inspections are carried out more frequently (Inspection 2), the states of the pavement are revealed and the relevant interventions carried out more frequently.

Therefore, in this study it can be seen that the additional cost of inspection due to more frequent inspections (Inspection 2) results in a lower long-term cost (Figure 8a) than in the situation of less frequent inspections (Inspection 1). In terms of the lifetime (Figure 8b), the situation with more frequent inspections gives an increase (although only a marginal one) for the strategies that consider a certain number of routine repairs and preventive repairs, such as strategies 7 and 8, instead of mainly relying on the corrective maintenance and reconstruction.

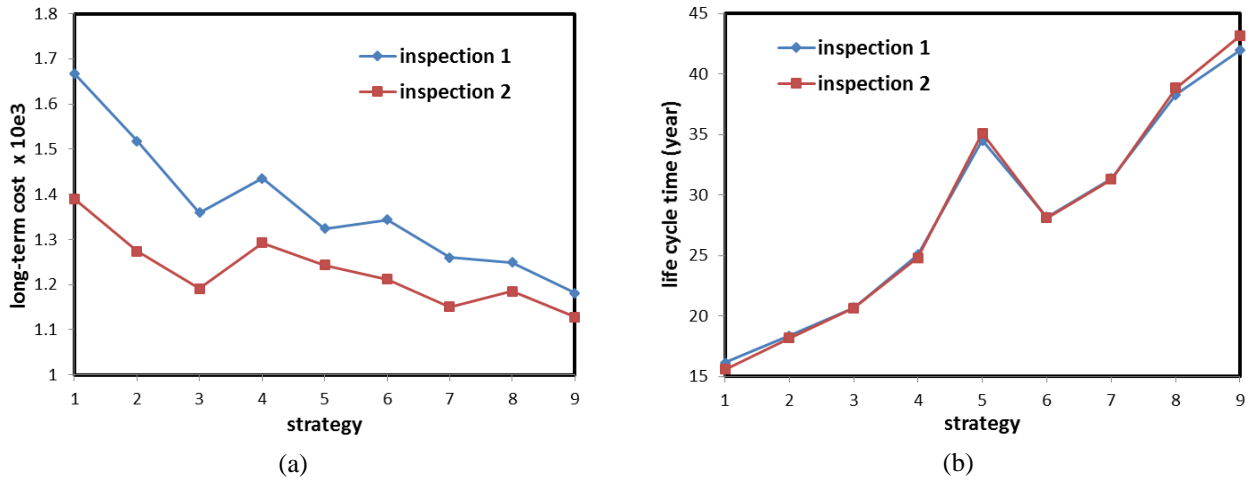


Figure 8. The results of the long-term cost (a) and the lifetime (b) for the two values of the inspection interval

4.2. Example 2

4.2.1 Model implementation

In this example, the PN model in Figure 1 is used to obtain the results of the analysis. A number of assumptions have been made. A two-parameter Weibull distribution has been assumed to describe the time to reach each deteriorated state, as shown in Table 3. Note that in addition to the six states considered in Example 1, state 3' is introduced which is used to model the state obtained after an intervention of emergency repairs. The parameters of the Weibull distribution have been assumed and the scale parameter η was chosen according to the recommended frequencies of maintenance actions, used in Example 1.

Since the deterioration rate of the pavement is to increase with time (due to the wear-out

characteristic), the shape parameter β is chosen to be greater than 1 and it is increasing with each state, as given in Table 3. The intervention costs and the mean time to complete an intervention are assumed to be as in Example 1, including the cost of the emergency repair and the time to complete it (given in the final column) to be equal to the parameters of the routine repair (given in the second column). Since the EM is used to recover from some serious damage using a temporary solution and cannot improve the performance of the pavement, the user cost for the EM has been assumed to be similar to the value during the CM. According to pavement maintenance manuals^[10] the budget planning period is assumed to be 10 years, since it is recommended to carry out the CM every 10 to 15 years. The time to the end of the budget planning period is assumed to follow the exponential distribution with $\lambda = 0.1$. Note that in this paper the time to the next level of deterioration follows the Weibull distribution, and the time to the completion of the intervention follows the exponential distribution. However, there are no limitations on the type of distribution to be used, which could be derived from the analysis of historical data of pavement deterioration and maintenance records.

Table 3. Input parameters for Example 2

State	from s_0 to	from s_1' to	from s_2' to	from s_0 to	from s_0 to	from s_0 to	from s_3' to
	s_1	s_1	s_1	s_2	s_3	s_4	s_3
Weibull parameter β	1.3	1.2	1.2	1.5	1.8	2.2	2
Weibull parameter η (days)	720	360	540	2880	5400	10800	360
State	s_1	s_1	s_1	s_2	s_3	s_4	s_3'
Intervention cost (\$)	200	200	200	1500	3000	7000	200
Mean time to complete an intervention (day)	1	1	1	7	15	60	1
User cost (\$/day)	0	0	0	0	10	50	8
Inspection cost (\$)					10		
Inspection duration (day)					0.1		
Mean of the budget planning period (year)					10		

Using the Monte Carlo simulation method to solve the PN model in Figure 6, a number of statistics have been collected in order to test convergence of the results. These include the duration of staying in particular states (Figure 9) and the number of interventions (Figure 10) over the planning horizon of 50 years. Note that strategy 9 was chosen for the illustration purposes in Figures 9 and 10. Based on these results the number of simulations for each strategy has been chosen to be 3000, as the outputs of the model have converged after this number of simulations.

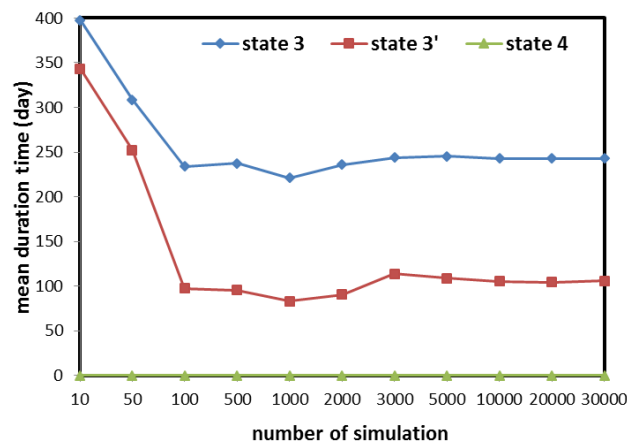


Figure 9. The relationship between the mean duration of staying in the different states and the number of simulations

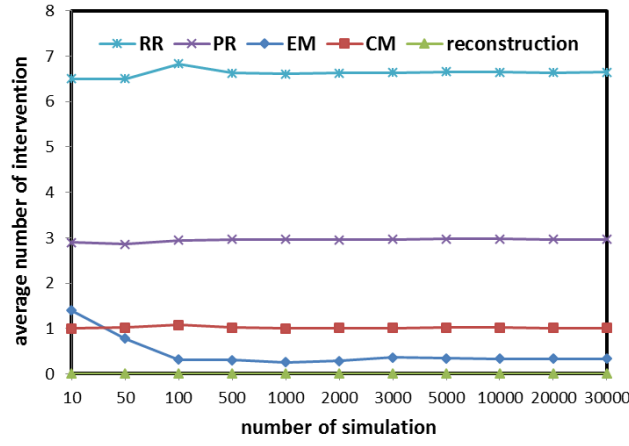


Figure 10. The relationship between the average number of interventions in planning horizon and the number of simulations

Further conclusions about the chosen strategy (strategy 9 in this case) could be drawn from Figure 9. For example, if the last three states (state s_3 , s_3' and state s_4) are considered as hazardous and potentially causing road safety risks, over the planning horizon the pavement stays in these states for around 350 days if strategy 9 is implemented. In other words, the probability to stay in a hazardous state (if strategy 9 is implemented) is under 2%. This example illustrates one of the additional criteria (in addition to the long-term cost and the lifetime) of comparing different strategies using the PN method, which was not possible using the Markov method. In addition to the long-term cost and the lifetime, such conclusions on other outcomes of the model can also be used to inform maintenance decisions.

The long-term cost and the lifetime under different strategies (as listed in Table 2) are given in Figure 11 with 3000 simulations. As in Example 1, it can be seen the routine repairs and preventive repairs can reduce the long-term cost and extend the lifetime of the pavement, since the strategies with a higher number of these (such as strategies 7, 8 and 9) result in lower cost and greater lifetime.

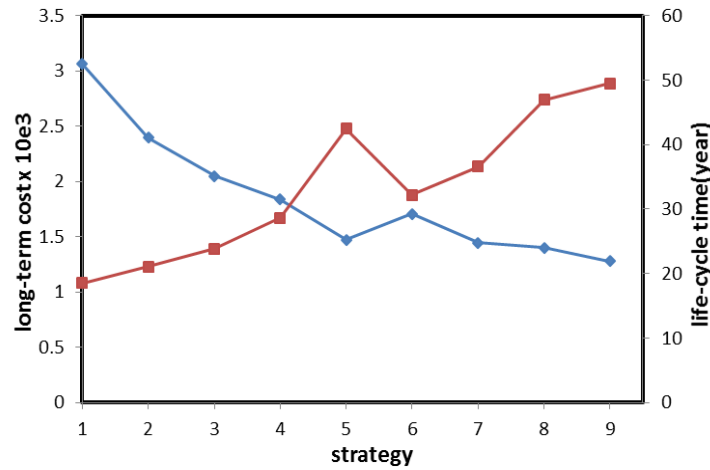


Figure 11. The long-term cost and lifetime in Example 2

4.2.2 Sensitivity analysis

The PN model in Figure 6 has been developed with the focus of illustrating the possibility to model the emergency maintenance and the dependencies between the routine and preventive repairs, which were impossible to take account of using the Markov model. Therefore, the budget planning period and the inspection interval, as the two parameters that influence these maintenance actions, have been chosen in the sensitivity analysis.

(1) The budget planning period

The chosen value of the budget planning period, which affects how often the corrective maintenance can be carried out or the emergency repairs are needed due to budget limitations, can influence the long-term cost (Figure 12a) and the lifetime (Figure 12b) for the different strategies. Three mean values of this period

were chosen: 5 years, 10 years and 15 years. If the period is shortened from 15 years to 5 years, savings in the long-term cost are observed for strategy 1, where only the corrective maintenance and reconstruction are possible.

For the other strategies, saving in the long-term cost will become less apparent if this period is reduced. This could be due to a larger number of emergency repairs required during the less frequent budget planning process, which offset the cost saving caused by the routine and the preventive repairs. The larger the budget planning period the higher the lifetime, since a larger number of emergency repairs delay the occurrence of corrective repairs and reconstruction and delay the renewal. If both criteria are of importance (minimize the cost and maximize the lifetime) the value for the budget planning period needs to be chosen carefully.

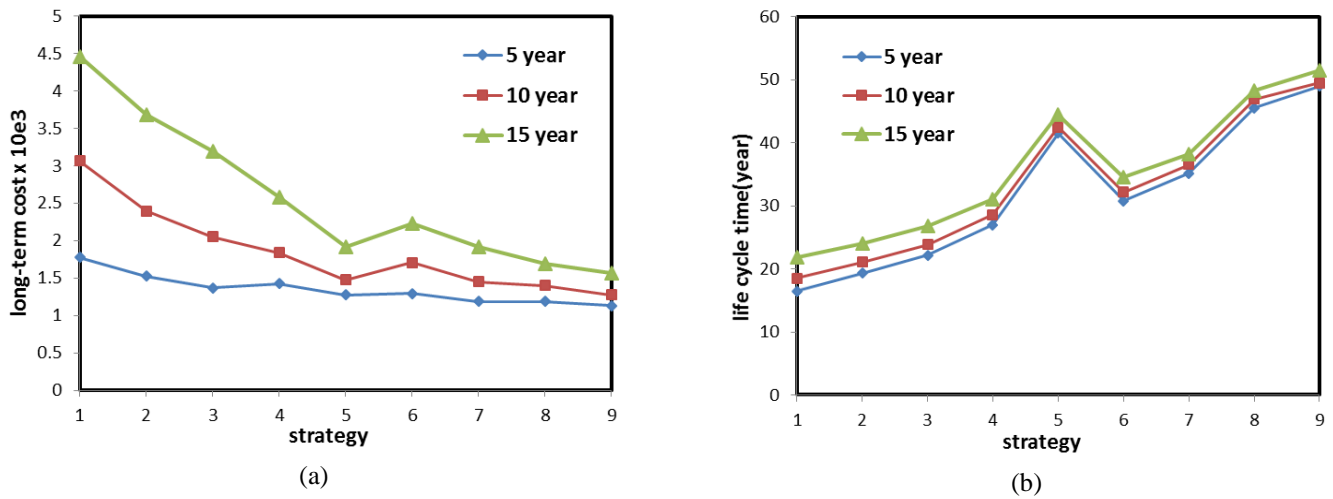


Figure 12. The results of the long-term cost (a) and the lifetime (b) for the three values of the budget planning period

(2) Inspection interval

In Figure 13, the long-term cost and the lifetime are considered when the inspection interval is 1.5 years (inspection 1), 1 year (inspection 2) and 6 months (Inspection 3). As in Example 1, when the inspections are carried out more frequently, the states of the pavement are revealed and the relevant interventions carried out more frequently.

Therefore, in this study it can be seen that more frequent inspections (Inspection 2) result in a lower, long-term cost (Figure 13a) and longer, although marginally, lifetime (Figure 13b) than in the situation of less frequent inspections (Inspection 1), especially for the strategies with fewer routine and preventive repairs. If the frequency of inspections continues to increase (inspection 3), the savings on the long-term cost become smaller and the lifetime even decreases than in the situation with less frequent inspections (inspection 2). The reduction in the savings on the cost is caused by an increase of cost due to additional inspections, whereas a shorter lifetime could be explained by an increased number of corrective maintenance actions when the poor state of the pavement is revealed (and rectified) more often. When choosing the frequency of the inspection interval, the balance between the long-term cost and the lifetime should be investigated carefully.

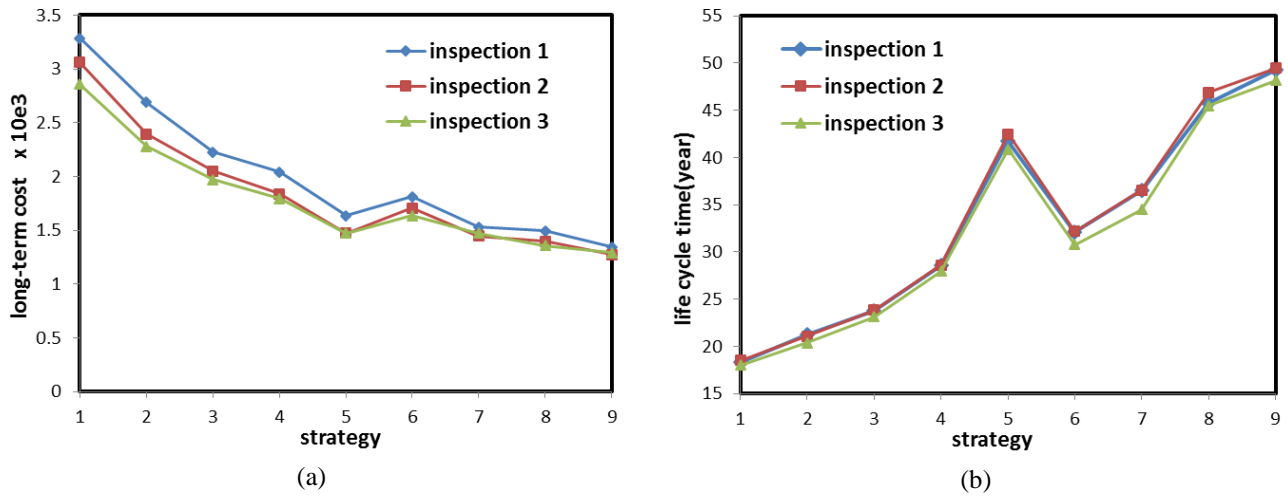


Figure 13. The results of the long-term cost (a) and lifetime (b) for the two values of the inspection interval

5. Conclusions

Efficient pavement management is of great importance for the governing transport bodies in terms of maintenance and reconstruction costs and pavement deterioration. This study is on modelling pavement deterioration and maintenance processes in order to evaluate different strategies for pavement management. An analytic model based on Markov process and a simulation model based on Petri-net have been developed to support pavement management decisions under different scenarios. The results from two models are compared in a numerical example to evaluate a number of characteristics suitable for comparing different pavement management strategies, such as the long-term cost and the lifetime of the pavement. For the values of the different parameters chosen it has been demonstrated that the strategies where the routine repairs and preventive repairs are carried out while the pavement is still in the good state can result in savings in the long-term cost and extend the pavement design life, in comparison to the strategies where the corrective maintenance and reconstruction only are carried out (when the more critical states of the deterioration have been reached). Additional outputs of the PN model can be obtained, such as the time spent in critical pavement states under a chosen strategy, and used to compare different maintenance approaches.

The sensitivity analysis has been carried out in order to investigate the effects of the main factors used in the analysis, such as the user cost, the cost of preventive repairs, the inspection interval and the budget planning period. It has been demonstrated how these main factors influence the results of the analysis and should be carefully considered when using the proposed method to inform maintenance decisions. One way of testing a range of parameter values is to formulate and solve an optimization routine where such values can be analyzed in an automatic way.

In future works we could extend the models by using in-field data to define the probability distributions for describing the process of pavement deterioration and the effect of different interventions, which are more accordant with practical circumstances. Further complexity in road maintenance practice and system-level road maintenance optimization can be analyzed by improving the simulation model with coloured PN method.

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Appendix 1

The selection of interventions can depend on a number of factors, such as the type, severity and extent of distress, traffic volume and climate conditions, and realized by decision matrix or decision tree as listed in Table A.1. Possible pavement interventions include pavement crack treatment, patching, chip seal, micro-surfacing, overlay, fill & mill, reconstruction and etc.

Table A.1 maintenance action matrix for the different distresses

Flexible pavement distress	low		moderate		High	
	Occasional	Frequent	Occasional	Frequent	Occasional	Frequent
Alligator cracking	3	3	4	4	6	6
Longitudinal cracking	2	2	4	4	6	6
Rutting	1	1	5+4	5+4	7	7
Transverse cracking	2	2	4	4	4	4
Random/block cracking	2	2	4	4	7	7
Raveling/weathering	3	3	4	4	7	7
Edge cracking	2	2	6	6	6	6
Distortion	1	1	5	5	5	5
Excess asphalt	1	1	4	4	5+4	5+4

1: do nothing, 2: crack seal/fill, 3: fog seal, 4: chip seal/armor coat, 5: mill, 6: overlay , 7: mill&fill

Extent of distress: occasional- area or length affected<30%, frequent- area or length affected>=30%