Original paper

MATHEMATICAL FORMULA OF A CONE MODEL USED FOR CALCULATION OF SNAIL SHELL VOLUME

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Abstract

Problem statement: There are many species of *Helix* snails, each with its own unique exoskeleton shape, or shell shape. Principal analysis of measurement data from snail shells occupied by *Helix* points out the importance of the description and quantification of the snail shells. The present paper is aimed to compare some of the formulas in the literature used to determine the volume of snail shape.
Hypothesis: The methods and formulae which exist do not reflect the real shell volume (at least not precisely enough) but rather use the external measurement and on that basis they draw conclusions for the biomass and the development stage.
Organisms: 142 species of the Turkish snail (*Helix lucorum*).
Approach: The purpose of this study is to improve and to offer a much better and accurate formula for calculations of the shell volume.
Conclusion: Our results support the usage of the formulae, and confirm some of these formulas while disproving others. As a result we developed a formula which takes into consideration the varying shape and thickness of the shell and reflects the real shell volume in most of the cases.

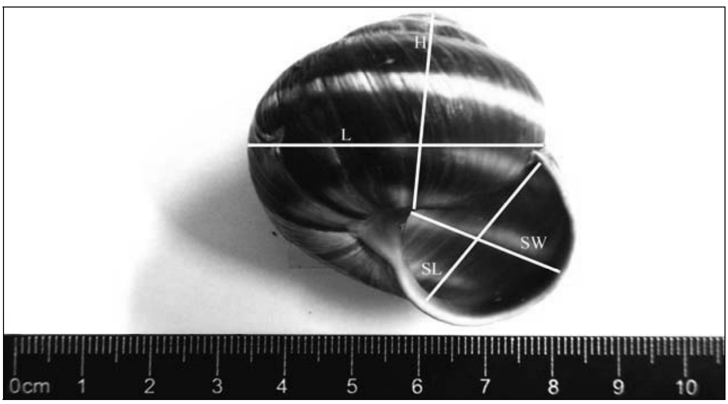
Keywords*:* *Helix lucorum, mathematical formula, snail shell*

Introduction

Scientific interest in land snails may have been caused by the fact that they are quite numerous species which inhabit different ecological niches and occupy different levels in food chains on land (Dedov, 2002). The growing interest in the technology of growing snails, the consumption of meat and their products cause massive interest in different types and their sizes (Georgiev and Atanasov, 2011). The research of specific types of features of the snail shells is essential for morphological characteristics and the relationship between volume and quantity of edible part of the snail meat. A number of models have been developed to describe the logarithmic form of coiled snail shells, some of which were also used to calculate the internal volume of the hull, surface area and material of shells (Graus and Raup, 1972; Stone, 1997; Williams and Price, 2008). Volume of the shells is rarely used in research, probably because it is not easy to use linear measurements. For the first time Moseley (1838) developed a series of equations to calculate volume, surface area and center of gravity of the spiral coiled shell. He intended these formulae to be applied in functional morphological studies, but the difficulty in measuring the necessary parameters made their use impossible. Trueman (1941) developed another set of equations based on simplifying assumptions. Raup and Chamberlain (1967) recognized and converted the more accurate inadequacies of Trueman`s equations. When used in research, capacity is often determined based on linear dimensions of the shell (Solem and Climo, 1985; McClain and Nekola, 2008), but rarely measured directly (Kemp and Bertness, 1984; Örstan, 2011). Internal volume of the snail shell is an important indicator because it defines the space available for vital activity of the snail (Örstan, 2006). Besides this the shell volume of the shell is related to quantity of gametes produced (Heath, 1985), as well as a meat quantity.

Materials and methods

To fulfill the purpose of the study 142 shell snails of the genus *Helix lucorum* were examined. The used specimens were collected in the region of Stara Zagora, Bulgaria, during May and June 2011. Collected snails were treated initially with 20% solution of NaCl by the method described by Özogul et al. (2005). Afterwards, the internal organs were removed, and the shells were dried (Sarma, 2006).
To measure the volume [V (cm^3)] of the shell, the method with calibrated sand (3 mm) of Sarma (2006) and Yamaguchi et al. (2009) was modified, according to the snail type. When filled with water, the shell was tightly closed with foil, ensuring the formation of a vacuum and was weighed on an analytical balance. The difference between the value of this empty shell and shell filled with distilled water presents values for the volume of the cavity.
For the greatest length [GL (mm)] the distance from the dorsal part - sutures to the caudal part - outer lip of the shell was measured.
For the greatest width [GW (mm)] longest distance between the two sides of the shell was measured.
Height [H (mm)] distance was measured from the top - sutures to the deep pit in the center of the shell – umbilicus (the entire length of solumella). The width [W-mm] distance was measured between the two vertical sides of the shell, just above the operculum.
Short length [SL (mm)] and width [SW (mm)] are perpendicular lines which represent the distance between the outer lip and inner lip of the shell. All the measurements were performed using a digital caliper with the accuracy of 0.1 mm (Mihaylov and Dimitrov, 2010).



In theory, a different 3D objects can be generated by simply rotating plane curves. But the forms and shapes that occur in nature are much more complex and intrinsic which make them extremely unsuitable for simulation. In such a way one can think of generating the snail shell by rotating the logarithmic curve (spiral) around one of the coordinate axis. Some researches on different snail shapes point out sections which are almost spirally arranged. Logarithmic spiral is a plane curve and in polar coordinates (r,θ) it can be written as

$$r = ae^{b\theta} \quad \text{or} \quad \theta = \frac{1}{b}\ln\left(\frac{r}{a}\right)$$

e = base of natural logarithms; $a,\ b$ = arbitrary positive real constants.
If $y = f(x)$ is a continuous non-negative function in the interval $[a,b]$ and consider the 3D object V which one can get by rotating $y = f(x)$ around Ox in the points $x = a$ and $x = b$ perpendicular to the Ox then the volume of V can be found by well-known formula in analysis:

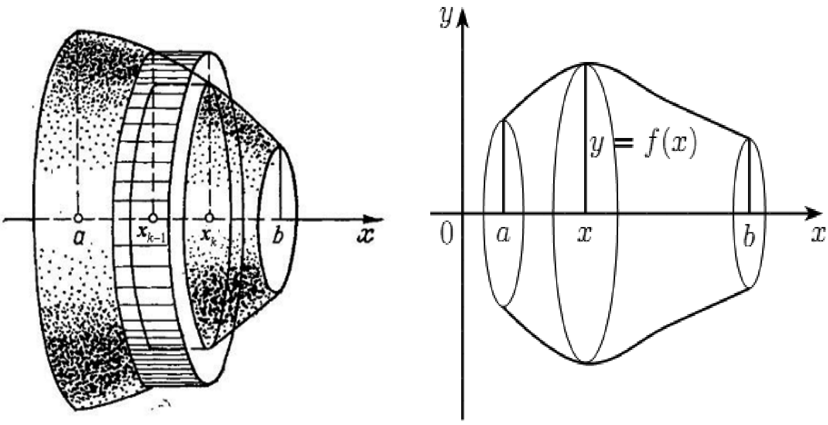


Figure 2. *Generation of a 3D object by rotating around the axis*

$$\mu(V) = \pi\int_a^b f^2(x)dx.$$

This classical approach, anyway, is quite inefficient for our purposes due to the irregularity of the snail shapes and the cavities inside the shells. Our approach is based on computation of the shell volumes using very basic measurements.
We measure four parameters of the snail shell, namely we measure *GL*, *GW, SL,* and *SW* (in *mm*) (see Figure 1), and we set the volume V_z of the snail shell as:

$$\mu(V_Z) = \frac{1}{3}\pi\left(\left(\frac{GW}{2}\right)^2 xGL + \left(\frac{SW}{2}\right)^2 xSL\right)$$

Our mathematical model is based on calculating the volumes of two cones.
Statistical methods. Data is reported as means ± Std.Dv. (N = number of snails), unless otherwise stated, and all statistical analyses were performed using STATISTICA version

6.0 (StatSoft Inc., 2002). Significant effects ($p<0.05$) were determined after applying t-test for dependent samples.

Results and discussion

V_A indicate the results of calculations by the formula:

$$V_A = \frac{1}{3}\pi\left(\frac{Shell\ width}{2}\right)^2 x(Shell\ height)$$

V_B are the results of calculations of the second formula for land snails (McClain and Nekola, 2008):

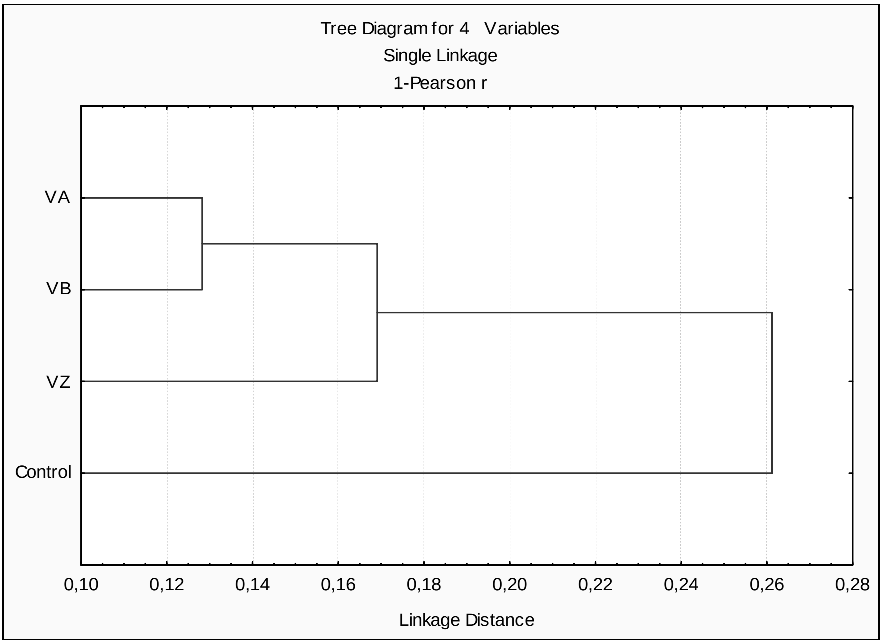
$$V_B = ((\pi r^2(1-t))h + \frac{\pi r^2 th}{3}.$$

The results of the calculations in both formulae are substantially different from actually determined volume of the shell with distiled water V (control group). Most probably these deviations are due to the various geometric shapes of the shell of this kind of land snail.
To determine whether the proposed formula provides similar results with practical application various parameters and their replacement in the formula have to be taken in consideration.

Table 1. *Statistical parameters for the groups*

	Mean	**Std.Dv.**	**N**	**Diff.**	**Std.Dv.**	**t**	**df**	**p**
V_A	-4.07308	1.803816						
V_Z	1.29597	1.528375	142	-5.36904	1.522663	-22.5780	40	< 0.05
V_B	-4.70156	1.582109						
V_Z	1.29597	1.528375	142	-5.99752	1.200752	-31.9824	40	< 0.05

At the end a Cluster analysis is presented (Dendogram 1). The results show that grouping a set of objects in such a way that objects in the V_Z group (called a cluster) are more similar V (0.17) comparing groups V_A and V_B (0.13). Correlation effects can be also seen in the dendogram.



Dendogram 1. *Correlation effect*

When comparing the results obtained from both formulae, very close values or minor deviations are present.

Conclusion

This investigation provides practical and useful information for calculation of snail shell volume. This study contributes to a description of the new mathematical formula of a cone model which could be used to extend existing information and improve calculation of the internal space of the shell available for vital activity. These results could be important for the snail producers and researchers for improving process of cultivation of land snails**.**

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