

FOCUS ON EXCEPTIONAL CHILDREN

MATHEMATICAL CONCEPTS AND SKILLS Diagnosis, Prescription and Correction of Deficiencies

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REMEDIAL MATHEMATICS FOR EXCEPTIONAL CHILDREN

Current sentiment emphasizes the uniqueness of the individual in terms of his rights, particularly to equal educational opportunity. The fact that each child is special presents a challenge to all educators, and especially to educators of exceptional children. For each child, whether retarded, emotionally disturbed, physically handicapped, gifted or normal, the goal for all educators is to provide the best possible educational environment from which the child can gain the maximum benefit.

It is a mistake to think of any group of children, exceptional, ethnic or whatever, collectively and then instruct all members of the group in an identical fashion. Children classified as learning disabled or emotionally disturbed have unique variances in their deficiencies in mathematical skills and concepts. To impose a single fixed curriculum for students classified in either group does not allow adequately for their individual differences. A curriculum must have flexibility and latitude to allow for adjustments in the learner's style, time required for mastery, sequencing and instructional mode. The basic finding of Stodolsky and Lesser (1967) indicates that groups do differ in patterns of mathematical abilities as well as in verbal abilities.

However, the skills and concepts taught to individuals and groups of individuals are much more similar than different. The hierarchy of skills may differ with a given teacher's objectives, but the sequence of content is based upon a sequence of subordinate learnings prerequisite to attaining the terminal objective. To build this hierarchy requires repeated questioning concerning what subordinate skills are needed to attain a given objective. In order to accommodate the mathematically handicapped child, one must carefully examine the existing hierarchy of skills and determine if further subdivisions of the skills in the sequence need to be made. Unless the curriculum was designed to be used with the mathematically handicapped, the steps or subordinate learnings probably need to be further subdivided. By translating the hierarchy into the smallest possible steps, there is a greater potential for a successful learning experience and a more positive attitude.

An example of a mathematical program that has attempted to decrease the difficulty level by expanding the number of steps in mastering mathematical concepts and skills

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is Project MATH. Project MATH, developed by Cawley and his associates (Cawley, Goodstein, Fitzmaurice, Lepore, Sedlak, & Althaus, 1975), has sequenced instructional objectives for mathematics instruction to the handicapped primarily by mathematical topics and has arranged all specifications within organizing matrices. While the sequence of content is not substantially different from that found in a mathematics program used in regular classroom settings, the nature of the hierarchy does differ.

Goodstein (1975) contends that the logical structure of mathematics is universal in nature. Thus, the term "mathematics for the handicapped" is misleading. It is not a case of a special mathematics for the handicapped but, rather, a case of how it is taught. The content does not differ substantially, but the time frame needed and instructional method do vary.

The use of concrete materials for introductory learning experiences is essential. Bruner's (1966) three levels of conceptual complexity (enactive-concrete; ikonic-pictorial; symbolic-abstract) particularly should be considered when dealing with mathematically handicapped children. Too often mathematics instruction centers on only the ikonic and symbolic response modes. Instructional materials must be modified to include all three response modes. Just as the rate of acquisition varies, there exists a variance in the modality used in the learning process for exceptional children. Armstrong (1970) proposed an intervention model which would emphasize an instructional environment that would be a function of the stage of cognitive development for the child (Piaget's component) and the level of mathematical concept acqui-

sition (Bruner's component). In two related studies on the mathematical learning of Trainable Mentally Retarded (TMR) and Educable Mentally Retarded (EMR), the TMR did significantly better under the manipulative, enactive representational mode. The EMR also learned better under the manipulative mode, but only on certain types of mathematical ideas.

The instructional triad of concrete-semiconcrete-abstract is critical to students with learning deficiencies in mathematical skills and/or concepts. *Many learning deficiencies might not occur in the first place if this instructional triad were the universal basis for all instruction in mathematical concepts.*

It is true that children can learn $8 \times 7 = 56$ by other procedures such as excessive repetition, but the question becomes, "Is this learning or training?" If one believes in the training premise, can it then be assumed that all hope for teaching exceptional children to reason or think is to be abandoned? For some students training may be adequate, but for most students learning is desired. All citizens, regardless of exceptionality, are entitled to reason to the best of their ability. Manipulative materials and experiences are an integral part of teaching such reasoning ability to students. Ultimately, mathematical skills and concepts must be applied in daily living. This requires judgment, reasoning, decision making and logic. Unfortunately, too many teachers of, and materials for, the mathematically handicapped rely on instruction that is abstract in nature and depends almost exclusively on repetition.

In discussing diagnosis, prescription, and instruction for the mathematically handicapped, the three major considerations are: (1) the content to be learned, (2) the learner, and (3) the materials of instruction.

THE CONTENT

In terms of content, it seems imperative that educators of exceptional children agree upon what mathematical skills and concepts the exceptional child can be expected to learn. There does not seem to be universal agreement among educators about the inclusion of reasoning skills for the exceptional child. Rather, there appears to be an abundance of emphasis placed on the mechanical, computational component of mathematics. In addition, much of this element comes through instruction at the abstract level rather than at the concrete, manipulative level. There seems to be disharmony between the practice and the philosophy of educating exceptional children in mathematics.

Current practice seems to indicate agreement in teaching EMR students the basic computational skills

FOCUS ON EXCEPTIONAL CHILDREN is published monthly except June, July, and August as a service to those concerned with mentally retarded and emotionally disturbed children. This journal is abstracted and indexed in *Exceptional Child Education Abstracts*, and is also available in microfilm from Xerox University Microfilms, Ann Arbor, Michigan. Subscription rates, \$10.00 per year. Copyright 1977, Love Publishing Company. All rights reserved. Reproduction in whole or part without written permission is prohibited. Printed in the United States of America. Second class postage is paid at Denver, Colorado.

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involved in using the four operations with whole numbers. However, the fact that agreement exists does not justify the correctness of a practice. If 40 minutes per day are spent on instruction in mathematics, the student will devote approximately one-sixth of his school-life for eight years to achieve about fourth grade proficiency. At this level there can be only limited confidence in the accuracy and, consequently, limited application to any decision that might be crucial.

Would it not be wiser to teach EMR students how to use a hand calculator to do the computations? Then, in turn, use the instructional time to teach the student how to use the calculator in solving everyday problems and how to estimate the reasonableness of the answer. Consider an EMR student as an adult faced with finding the total cost of his utilities. The person could add the several numbers using pencil and paper, but the time involved would be considerable and the accuracy questionable. Better, he would have available a hand calculator and would use it to make the computation. The time involved would be much less, with greater accuracy.

The hand calculator provides the opportunity to decrease the emphasis on the mechanics of computation and place increasing emphasis on a more functional mathematics curriculum involving the solution to everyday problems. Educators of exceptional children need to consider the implications of the hand calculator and reach agreement on revising the mathematics curricula accordingly. Traditional expectations of mathematical experiences for exceptional children need to be re-examined.

Another example involving mathematical content deals with decimals. Traditionally, a mildly mentally handicapped child would have had little experience with decimal notation or decimal fractions. Two trends make it imperative that this cannot continue if the student is to be a functional citizen. First, calculators do not accommodate common fractions such as one-third, one-half, etc., unless they are expressed in decimal equivalents. Second, conversion to metric measurement makes it mandatory that all citizens understand decimal notation as it is used in expressing equivalent measurements. For example, to change 38.97 meters to centimeters, the decimal point is moved two places to the right. Similarly, to change 3003 centimeters to meters, the decimal point is moved two places to the left. To learn the metric system requires an understanding of decimal notation, a topic not previously taught to students with an intellectual handicap.

In answer to the question, "What mathematical content should be taught to the learning disabled, mentally retarded, or slow learner?", the authors would suggest a simply-stated practice: A greater amount of the instructional time should be directed to diagnosing, prescribing,

and instructing within those topics that are of the greatest relevance to the learner. This implies that not all content in the traditional curriculum is equally useful to all children, regardless of exceptionality. For some students, learning reasoning skills may be most helpful, while for others, learning reasoning skills may not be a practical or possible instructional goal. While reasoning skills would be useful and practical for the mildly retarded, such would not be the case for the TMR.

The teacher is the key to determining what content the child will experience and how it is presented. Unfortunately, most teachers have had little, if any, preparation in diagnostic and prescriptive teaching in the area of mathematics. Specific attention needs to be devoted to this aspect of teacher preparation. Also, with appropriate use of modern technology, considerable change can be effected in the mathematics curriculum for exceptional children. Finally, teacher expectations must be altered. It is questionable whether instruction limited to basic computational skills constitutes sufficient preparation for functional citizenship. Educators may accept low achievement as normal, thereby curtailing the child's growth in mathematical skills and concepts. Limiting the child's exposure to mathematical experiences because of low expectations leads to "underteaching" by the teacher and underachievement for the child.

THE LEARNER

Ideally, the teacher should implant in the learner a predisposition to learn but, unfortunately, when children already have some mathematical handicaps, this task becomes difficult. Understanding the child, how he learns, and the structure of mathematics are important pieces of information which allow the teacher to manipulate the learning environment. Instruction should parallel the child's mathematical conceptual development, which implies that a teacher should have ways to assess a child's mathematical foundation.

The work of Piaget and his developmental theory of logical processes suggest means by which a teacher may learn more about the child's mathematical foundation. First, the teacher must be aware of the main mathematics delivery system to which the child has been exposed. Did it consist of appropriate manipulatives upon which the child could act and experience, or was it primarily more verbal in character with a large amount of printed materials? As Callahan and Glennon (1975) warn, from the Piagetian perspective a child may "know" more than he can verbalize, whereas from the functional-verbally oriented perspective, the child may verbalize more than he meaningfully "knows."

In "The Wisconsin Studies" (Van Engen, 1971), first-grade children were tested on arithmetic tasks and Piagetian tasks. All 100 children were near mastery on basic addition facts $2 + 3$ and $4 + 5$ in a verbal format, but only 50 were capable of conserving the equivalence relationship through a physical transformation. Other researchers have tested the relationship between Piagetian class inclusion tasks and missing-addends tasks and between conservation and achievement in mathematics. These studies also indicate a disparity between the child's ability to verbalize a concept and the understanding of the concept.

Rather than being overly concerned about a child's lack of achievement in mathematics and at what grade level he is performing, the teacher should investigate ways to determine at what logico-mathematical level the child is operating and then provide concrete experiences to develop these structures.

The problem may be in maturation rather than a learning deficiency. Whenever a child develops the cognitive intellectual structures to deal with such concepts, the "deficiency" may disappear. A mathematically handicapped child may have been moved too rapidly through the levels of representation; i.e., from the concrete to the abstract. Appropriate manipulatives for the student to experience and act upon in the development of psychomathematical, deductive processes is an extremely important step for remediation. Teachers should plan for learning activities on each of the response mode levels to ensure that students will gain knowledge of the mathematics skills or concepts.

How does a teacher diagnose a child's ability to respond in each of the levels? Many tests are available for diagnosing the child's cognitive ability at an abstract level. These range from standardized mathematics achievement tests or diagnostic tests to teacher-made informal tests. All paper-pencil tests measure the symbolic-abstract level and, to some extent, the pictorial level of learning. Even the most popular diagnostic tests on the market are highly content-oriented instruments requiring performance in the learner's least proficient response mode—the graphic symbolic. Diagnosing at the concrete level of learning becomes more difficult, more time consuming, and requires more interpretative skill by the teacher. For these reasons, along with the scarcity of this type of test, little diagnosis of the concrete level of learning has been done. Piagetian interviews are an excellent source of such information. Copeland (1974) has described how these interviews can be conducted and the teaching implications for various responses.

Other researchers also have developed structured oral interview techniques to learn about the child's ability to

deal with mathematical concepts and skills. Denmark (1976) has developed the Project for Mathematical Development of Children (PMDC) grade one and grade two. Cawley (1975) and his associates, as part of Project MATH, currently are field-testing the MATH Concept Inventory, a criterion reference instrument, and they also have developed a Clinical Mathematics Inventory.

Many researchers (Ashlock, 1972; Cawley, 1975; Glennon & Wilson, 1972) have praised the value of an oral interview with the child in which, through the child's verbalizations, one can discover the strategies the child uses to arrive at an answer. Once the flaws in the understanding are revealed, the teacher should use a variety of concrete materials and settings to see if the flaw persists.

If a child's weakest mode is visual, the ubiquitous ditto worksheets will not be the answer. The tactile and auditory modes are greatly under-used and should be considered in modifying the curriculum and materials to suit the learner. Research is being conducted by Uprichard at the University of South Florida to study a child's modality strengths and how this information can be applied to learning content in mathematics.

Cognitive style is another area which merits consideration. If the teacher complains that the child uses his fingers to compute, perhaps the child's cognitive tempo is reflective behavior, and anxiety over making a mistake produces such action. On the other hand, if the child makes many computational errors and does not seem to display a consistent error pattern, the child may be impulsive and needs encouragement to slow down and reflect about the quality and accuracy of answers.

Another characteristic of the learner needing attention is referred to by cognitive theorists as "discrimination performance." From the performance on a discrimination task, something can be determined about a child's level of readiness for language, mathematics skills, and reasoning ability. Young, immature children attend to one feature of a stimulus at a time (as Piaget calls "centration") and perhaps are not aware of other less salient features. For example, young children tend to be more color dominate than shape dominate. If given a color-form task, the child would center on the more salient color dimension rather than on the shape.

The strategies employed by Zeaman and Hause (1963) to study discrimination learning among mentally handicapped children rely heavily upon the inclusion of patterns. Project MATH (Cawley, et. al., 1975) has included patterns based on the premise that patterns will parallel the extended readiness period demanded of the mentally handicapped child and the extended non-reading period of the learning disabled child.

Sternberg (1975) has developed an instrument, Pattern

Recognition Skills Inventory (PRSI), for use as a measuring and diagnostic tool to test a subject's ability to discriminate patterns. The clue to diagnosis in PRSI relates to a hierarchy in pattern recognition skills. Pattern discrimination appears to be a function of age and/or maturity in information processing.

It is imperative to know the learner before any corrective instructional process is begun. Too frequently, the teacher diagnoses the skill deficiencies and implements remedial instruction without assessing the learner's logico-mathematical development, response mode, cognitive style, or discriminatory learning abilities. The end result is failure to achieve any permanent change in the child's mathematical learning.

METHODS & MATERIALS

The synthesis of content and learner is the ultimate goal of mathematics instruction. Selection of appropriate methods and materials is critical to the successful outcome of this synthesis. Once the objectives have been determined in relation to our knowledge about the learner, selecting appropriate materials and instructional methods becomes critical to achieving the objectives. Many teachers minimize the importance of this aspect of the corrective process, evidenced by their over-reliance on drill sheets as a means of correcting the deficiency. For the mathematically learning disabled, paper-pencil tasks require the child to respond in one of his weakest response modes. This incompatibility between material and learner response mode results in limited retention on either a short- or long-term basis.

In selecting materials it is important to know something about the sense mode of the learner and the materials. In the routine of daily living, approximately 80 percent of our sensory intake is visual, 11 percent is auditory and 2 percent is tactile. Materials for and methods of instruction can be classified as being oriented toward these three sense modes. Textbook materials rely heavily on visual skills to interpret. Manipulative materials may be visual if used for demonstration, but when used by children, they involve tactile as well as visual skills in learning. Since many children with a learning disability in mathematics may also suffer from some perceptual disability, it seems reasonable to conclude that the materials and methods employed in the corrective process should minimize reliance on the visual mode.

Corrective procedures must be based on a more auditory and/or tactile approach. Whenever possible, beginning instruction for any concept at any age level should be based on concrete experiences before proceeding to the symbolic, abstract level. When using concrete, manipulative devices in initial or corrective instruction, it is of

utmost importance that the steps be verbalized and written in symbolic form *by the child* to help in the transition to more abstract levels.

The following example illustrates the manner in which a concept may be taught using each of the three senses independently. Assume you want a child to learn about "fourness." To teach the concept visually, a card with a pattern of four dots can be made as shown:



The card can be flashed and the student asked to identify the number. To teach the same concept auditorily, the teacher could, out of sight of the students, tap a specific note on the piano four times. Hopefully, the auditory approach also contributes to better listening skills on the part of the students. Finally, to teach the concept using a tactile approach, a card with four sandpaper discs could be placed in a closed container with an opening. The student is asked to reach through the opening and by "feel" determine the number of discs.

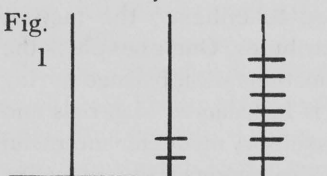
It is critical for the teacher to devise teaching strategies to utilize as many senses as possible in acquiring mathematical concepts. The two senses most neglected in teaching strategies and materials and, consequently, in most need of emphasis, are auditory and tactile. Teaching concepts via the auditory sense will require a bit of imagination on the part of the teacher, but it is possible to devise strategies for many of the mathematical skills and concepts. The University of Maryland's Arithmetic Center has been conducting research using exemplars which have been modified to enhance the tactile, kinesthetic, and auditory attributes. One example is the attachment of a bell to the mathematical balance to ring only when the instrument is in balance. Materials and methods may rely on various sensory modes for successful interpretation. One of the critical aspects of the corrective process centers on the selection of appropriate methods and materials in delivering the corrective instruction.

An example of the cruciality in selecting appropriate materials is apparent in working with regrouping in addition and subtraction. Suppose the child does not understand the regrouping procedure for addition and the teacher has many manipulative devices at her disposal. Which device is best to show how and when regrouping occurs in the example $53 + 29$: Cuisenaire rods, Unifix cubes, Dienes blocks, multi-base abacus, closed abacus (9 or 10 beads per column), place value chart, bundling sticks, chip trading, or bean sticks? Not all of these materials are equally appropriate for teaching this given concept. Consideration must be given to any additional information known about the student. Does he tend to count on his fingers? Does he have any visual or perceptual problems? Are his fine motor skills adequately

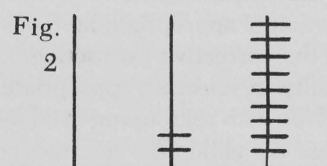
developed? Does the learner tend to persevere? Is he easily distracted?

Attention also must be directed toward the attributes of the exemplars being considered. Are there any irrelevant features which may be a distractor or serve as a hindrance to concept formation? Does the material clearly represent the concept to be extracted? Does the material provide for easy individual manipulation by the child? Does the device exploit as many senses as possible? Does the device allow for abstraction?

The implications of this materials selection task can be illustrated in the skill of regrouping. In the exchangeability idea of how 10 of the column on the right can be exchanged or traded for 1 of the column on its left (10 ones for 1 ten) which exemplars show the child when the trade is necessary? The closed abacus meets this criteria, but the child must possess additional skills when given this device to work the problem $27 + 8$. First the child shows the 27 in the proper columns (Figure 1). Then he begins to add 8 more in the ones column. Only 9 beads will fit in that column (to show the child when regrouping is necessary). He can fit only 2 more beads in the ones column (Figure 2). Then he must clear the ones column and add a bead to the tens column (Figure 3). Since the tenth bead does not fit in the ones column and is placed in the tens column, it may seem to the student that this is a trade of 9 for 1. The critical question is: Can the child remember how many of the 8 ones he added when the regrouping was necessary?

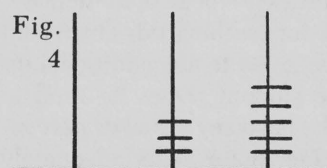
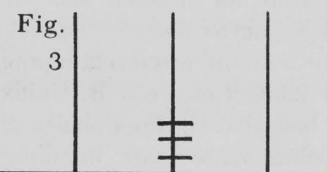


$$\begin{array}{r} 27 \\ + 8 \\ \hline \end{array}$$



1. Show 27 (Figure 1)
2. Add 8 ones
3. Only 2 fit (Figure 2). Take off all ones (Figure 3) and add 1 ten.
4. Add rest of the ones (Figure 4).

Problem: How many ones from 8 did you add when you needed to regroup?



Problem: Did you trade 9 ones for 1 ten or 10 ones for 1 ten?

Another question is whether the same aid can be used to show the regrouping process in subtraction. If one believes that subtraction should be represented as the inverse operation of addition, it seems valuable to use the same aid, with the reverse sequence of steps. Is the closed abacus appropriate? No. The regrouped ten as 10 ones cannot be placed on the ones column.

Suppose the wooden Cuisenaire rods were selected instead of the closed abacus. The child clearly could see when regrouping is necessary and the exchangeability of groups of 10. In the example $27 + 8$, the steps would be as follows:

1. Show 27 (Figure 5).
2. Add 8 ones (Figure 6). The combined length of 7 and 8 shows that there is more than 10.
3. Regroup the 7 and 8 into a 10 and a 5 (Figure 7).
4. Answer 3 tens and 5 ones (Figure 8).

Fig. 5

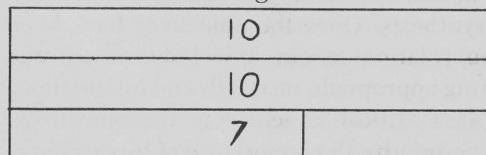


Fig. 6

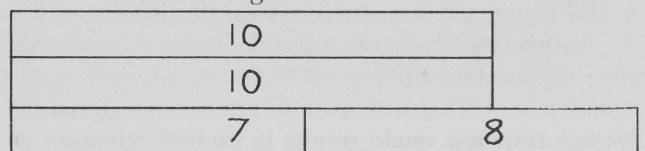


Fig. 7

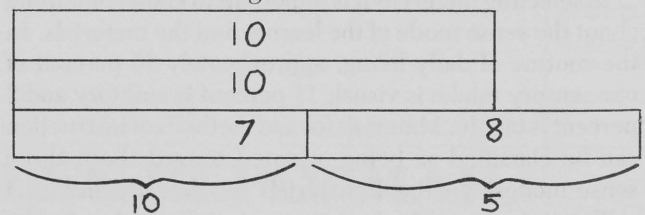
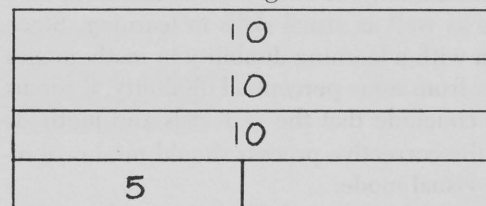


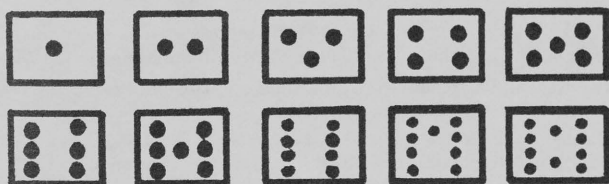
Fig. 8



Using the rods, the child readily can see at what point the regrouping is necessary and that the ones can be exchanged for one ten-rod, leaving 5 ones remaining. The rods can be manipulated easily and allow for abstraction of the concept. However, the teacher must be aware of two

irrelevant attributes which are associated with the rods and which, if abstracted by the learner, might cause misconceptions—color and size. In our notational system, neither color nor size is an attribute associated with the system. We do not write a 7 in black ink and a 5 in yellow. Neither do we write a 7 as a larger numeral than the 5. The astute teacher must be alert to the possibility that when selecting the Cuisenaire rods, additional attributes are being introduced as a part of our notational system which may cause problems and erroneous concept formation in the child. It is important to the learning process that the appropriate exemplar be chosen in relation to the skill being taught.

As another example, consider the disability of over-reliance on rote counting. At the initial stages of learning an addition fact, counting is quite natural. However, the student who becomes over-reliant on counting is headed for almost certain difficulty when higher level addition skills are required in problems such as $368 + 249$. For this reason, those involved in corrective teaching must be especially alert to students who exhibit an over-reliance on counting. One appropriate corrective procedure is to prepare cards with standard configurations of dots as illustrated:



These cards can be flashed briefly, and students can be asked to display a numeral card to tell how many dots they saw. Once a student can do this with speed and accuracy, reliance on counting will begin to diminish.

Another manipulative aid that may be employed to diminish over-reliance on counting is the wooden Cuisenaire rods, in which length and color assume a value. These rods have advantages over the cards for several reasons: The rods are not scored into individual units; thus, they would be useful with the rote counter because there is no opportunity to count to find the answer. The attributes of size and color that were irrelevant in teaching place value become beneficial in teaching numerosity. Other materials which frequently are used to teach addition and subtraction are chip trading, Unifix cubes, abaci, bundling sticks, and bean sticks. Each of these materials has discrete elements which encourage counting. Again the selection of the exemplar is critical to correcting the deficiency.

Wiison (1976) and his colleagues at the University of Maryland have devised a rating system to analyze the

many exemplars available for teaching a given concept. The accompanying chart is a capsule analysis of several of the available materials, and lists some of the major features.

In materials selection, the importance of proper matching of the exemplar with the learner's style, the content to be taught, and the deficiency to be corrected must be emphasized. Also the teacher who is diagnosing and prescribing for a learning deficiency must give careful thought to the sequencing of instruction. No commercially prepared materials will do the job alone. The teacher must be knowledgeable and astute in her observations.

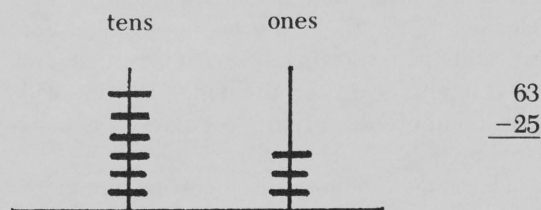
The role of preventive teaching merits attention. Summarizing the relationship of preventive teaching to remediation: The best remediation for learning problems in mathematics is quality teaching in the first place.

How many deficiencies are created by inadequate instruction when concepts are introduced for the first time? A child's initial experience with a concept is the time when it is most critical for the lesson to be well presented with concrete representation, as it is the most likely time for success. Presenting the initial lesson poorly spawns much "undoing" in terms of attitude and/or confusion.

Students often work this subtraction problem as follows:

$$\begin{array}{r} 63 \\ -25 \\ \hline 42 \end{array}$$

Note that the student failed to regroup and took the difference between the 3 and 5. This reveals a great deal about the manner in which the student was introduced to the regrouping necessary. Representation of the problem on the abacus is:



Pointing out the 63 shown and asking the student to remove 5 ones certainly could not result in taking the difference between 3 and 5. Students who make that error have been taught how to do the problem beginning at the symbolic level. Proper initial instruction utilizing an abacus or rods would have helped to avoid this error. Greater emphasis needs to be placed on the role of preventive teaching—in preservice teacher training programs, and (an even greater need) among teachers already in service. The availability of a learning disabilities teacher often allows regular classroom teachers an escape mechanism from dealing with a skill deficiency. By refer-

EVALUATION OF FOUR EXEMPLARS USED IN TEACHING PLACE VALUE

	Positive Attributes	Limitations
Cuisenaire Rods	Discourages counting Emphasizes 10 to 1 regrouping Adaptable to many grade levels Adaptable to many concepts Color cueing helps remember value of rod Easily manipulated	Color is irrelevant in terms of place value Size is irrelevant in terms of place value No fixed location to represent place value
Unifix Cubes	Easily manipulated Possible to structure various subsets using color Adaptable to many grade levels Adaptable to many concepts	May encourage counting Fixed location to represent place value is not apparent 10 to 1 regrouping may be obscured
Dienes Blocks	10 to 1 regrouping is obvious Adaptable to many grade levels Adaptable to many concepts Easily manipulated	No fixed location to represent place value May encourage counting
Closed Abacus (9 units)	Has fixed location to show place value Color cueing may help understand place value Easily manipulated	Cannot show more than 9 units Encourages counting Has limited application across concepts 10 to 1 regrouping may be obscured Limited application across grade levels Color is irrelevant in terms of place value

ring the student out of their realm of concern, classroom teachers fail to learn how to help the student and how to change their own instructional style to avoid or minimize the possibility of a similar learning problem occurring in the future.

The learning disabilities teacher has a moral obligation to aid the regular classroom teachers in changing their instructional style if it will result in better learning for the students. Correcting the 63-25 regrouping problem for the student, and then showing the regular classroom teacher how it might have been prevented initially will improve the competence of the regular classroom teacher.

The role of learning disabilities teachers involves more than appearing on the scene, administering a battery of tests, diagnosing the difficulty, and submitting the results to the classroom teacher in a lengthy and sometimes incomprehensible written report. The resource teacher must be able to prescribe activities and materials to be used to correct the deficiency, and then must help the regular classroom teacher implement the prescription. The implementation process with the regular classroom teacher is the key to changing the instructional style of that teacher.

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