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Research Article

Point and interval forecasts of age-specific life expectancies: A model averaging approach

Han Lin Shang

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Han Lin Shang[1]

Abstract

BACKGROUND
Any improvement in the forecast accuracy of life expectancy would be beneficial for policy decision regarding the allocation of current and future resources. In this paper, I revisit some methods for forecasting age-specific life expectancies.

OBJECTIVE
This paper proposes a model averaging approach to produce accurate point forecasts of age-specific life expectancies.

METHODS
Illustrated by data from fourteen developed countries, we compare point and interval forecasts among ten principal component methods, two random walk methods, and two univariate time-series methods.

RESULTS
Based on averaged one-step-ahead and ten-step-ahead forecast errors, random walk with drift and Lee-Miller methods are the two most accurate methods for producing point forecasts. By combining their forecasts, point forecast accuracy is improved. As measured by averaged coverage probability deviance, the Hyndman-Ullah methods generally provide more accurate interval forecasts than the Lee-Carter methods. However, the Hyndman-Ullah methods produce wider half-widths of prediction interval than the Lee-Carter methods.

CONCLUSIONS
Model averaging approach should be considered to produce more accurate point forecasts.

COMMENTS
This study is a sequel to another Demographic Research paper by Shang, Booth and Hyndman (2011), in which the authors compared the principal component methods for forecasting age-specific mortality rates and life expectancy at birth.

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1. Introduction

In many developed countries, concerns of population aging are concentrated on the sustainability of pensions, health and age care systems. These concerns have resulted in a surge of interest among government policy makers and planners in accurately modeling and forecasting age-specific mortality rates and age-specific life expectancies. Any improvement in the forecast accuracy of life expectancy would be beneficial for policy decision regarding the allocation of current and future resources. In particular, future life expectancy is of great interest to the health care, age care, life insurance, superannuation and pensions industries. In these industries, the viability of financial arrangements depends on knowing the likelihood that clients will live to older ages.

In the demographic literature, a number of parametric and nonparametric methods have been put forward for forecasting age-specific mortality rates and life expectancy at birth (see for example, Preston, Heuveline, and Guillot 2001; Rowland 2003; Alho and Spencer 2005; Hyndman and Ullah 2007; Torri and Vaupel 2012). In a recent paper by Shang, Booth, and Hyndman (2011), they compared the point and interval forecast accuracy for forecasting age-specific mortality rates and life expectancy at birth, among ten principal component approaches. Differing from Shang, Booth, and Hyndman (2011), this paper serves two purposes. First, we compare the point and interval forecast accuracy of age-specific life expectancies, instead of the life expectancy at birth. We argue that the prediction of life expectancy at birth is of limited use in some situations. For instance, given a person aged 60, the pension industry would like to know his/her remaining life expectancy. Such information can be adequately supplied by forecasting age-specific life expectancies. Second, we put forward the idea of model averaging to improve point forecast accuracy. We hope that this paper will motivate the readers to consider the model averaging approach in the context of demographic forecasting.

The structure of this article is given as follows. In Section 2., we briefly revisit the ten principal component approaches for forecasting age-specific mortality rates (see Shang, Booth, and Hyndman 2011, for more details). Of these methods, Lee and Carter (1992); Lee and Miller (2001) and Booth, Maindonald, and Smith (2002) used principal component analysis to extract a single time-varying index of the level of mortality rates, from which the forecasts are obtained by random walk with drift. Despite the simplicity and wide applicability, the Lee-Carter method and its variants consider only the first principal component, which may not be adequate to capture the underlying patterns of data. To address this problem, Bozik and Bell (1987), Bell and Monsell (1991) and Bell (1997) put forward a principal component method that utilized second or higher order of principal components. As emphasized by Bell (1997), it is desirable to use modeling and forecasting methods that capture a smooth shape over age for producing consistent forecasts and improving the accuracy of short-term forecasts. To solve this issue, Hyndman and Ullah (2007) combined the ideas of nonparametric penalized regression spline and functional principal component analysis using second or higher order functional principal components.

In Section 3., we introduce a model averaging approach by combining forecasts from different models. We explore four ways of combining forecasts:

(1) assigning weights equally to all 14 models investigated in this paper;
(2) assigning weights to all 14 models based on their forecast accuracy in the validation data set;
(3) assigning weights to the best two models based on their forecast accuracy in the validation data set; and
(4) assigning weights using the notion of Bayesian model averaging (BMA), with Akaike's (1974) weights.

Illustrated by the fourteen developed countries' data sets described in Section 4., we calculate the point and interval forecasts of ten principal component methods, two naïve random walk methods, and two univariate time-series models for forecasting age-specific life expectancies. In Section 5., the criteria used to measure point and interval forecast accuracy are discussed. We evaluate and compare the point and interval forecast accuracy among fourteen methods for forecasting age-specific life expectancies in Sections 6. and 7., respectively. In Section 8., we demonstrate the improvement of point forecast accuracy by combining forecasts with different weights. Conclusions are presented in Section 9., along with some thoughts on how the methods developed here might be further extended.

2. Revisiting the ten principal component approaches

The next subsections specify the principal component models and notations, and describe the calculations of their point and interval forecasts of mortality rates. To start, let $m_{x,t}$ be the log transformation of the mortality rate at age x in year t. For forecasting mortality rates, we are interested in obtaining estimates of age-specific mortality rates for one or more years of $t > n$ (where n is the last observed year) as well as their associated measures of uncertainty.

2.1 Lee-Carter (LC) method

The LC model is given by

$$m_{x,t} = a_x + b_x k_t + \epsilon_{x,t}, \quad t = 1, 2, \dots, n, \quad x = 0, 1, \dots, p, \tag{1}$$

where
1. a_x is the age pattern of the log mortality rates averaged across years;
2. b_x is the first principal component capturing relative change in the log mortality rate at each age x, and $\sum_{x=0}^{p} b_x = 1$;
3. k_t is the first set of principal component scores measuring general level of the log mortality rate at year t;
4. $\epsilon_{x,t}$ is the model residual at age x and year t, and $\sum_{t=1}^{n} k_t = 0$.

The LC model adjusts k_t by refitting to the total number of deaths. The adjusted k_t are then extrapolated by a random walk with drift method, from which point forecasts are obtained by (1) with the fixed a_x and b_x.

Two sources of uncertainty should be considered: errors in the parameter estimation of the LC model and forecast errors in the forecast principal component scores. Because of the orthogonality[2] between the first principal component and the error term in (1), the overall forecast variance can be approximated by the sum of the two variances. Conditioning on the past data points $\mathcal{I}$ and the first principal component b_x, we obtained the overall forecast variance of $m_{x,n+h}$,

$$\text{var}\left[m_{x,n+h}|\mathcal{I}, b_x\right] \approx b_x^2 u_{n+h|n} + v_x, \tag{2}$$

where b_x^2 is the variance of the first principal component, calculated as the square of the b_x in (1); $u_{n+h|n} = \text{var}(k_{n+h}|k_1, \dots, k_n)$ can be obtained from the time-series model; and the model residual variance v_x is estimated by averaging the residual squares $\{\epsilon_{x,1}^2, \dots, \epsilon_{x,n}^2\}$ for each x in (1).

There are two other methods that are closely related to the LC method. The first one is the LC method without adjustment of k_t, labeled as LCnone. The second one is the Tuljapurkar, Li, and Boe's (2000) method, labeled as TLB. The TLB method is the LC method without adjustment, and it also restricts the fitting period to 1950 onward.

2.2 Lee-Miller (LM) method

The LM method adjusts k_t so as to obtain the best forecasts of life expectancy at birth. It differs from the LC method in three ways:
1. The jump-off rates are the actual rates in the jump-off year instead of the fitted rates for all other methods studied in this paper;
2. The fitting period begins in 1950;
3. The adjustment of k_t involves fitting life expectancy at birth in year t.

[2] Because the error term in (1) contains the second and higher order principal components, they are orthogonal to (or vary independently of) the first principal component by construction.

2.3 Booth-Maindonald-Smith (BMS) method

As a variant of the LC method, the BMS method differs from the LC method in three ways:

1. The fitting period is determined by a statistical “goodness of fit" criterion, under the assumption that k_t is linear;
2. The adjustment of k_t involves fitting to the age distribution of deaths rather than to the total number of deaths;
3. The jump-off rates are the fitting rates under the fitting regime.

2.4 Hyndman-Ullah (HU) method

A possible weakness of the LC method and its variants is that they attempt to capture the patterns of age-specific mortality rates using only the first principal component and its associated scores. In addition, they do not implement a smoothing technique to smooth the mortality rates associated with old ages. To address this problem, Hyndman and Ullah (2007) proposed a functional data model that utilizes second and higher order ones to capture additional variation in mortality rates.

The method proposed by Hyndman and Ullah (2007) combines the nonparametric penalized regression spline of Ramsay (1988) with functional principal component analysis of Ramsay and Dalzell (1991) for forecasting mortality rates. The HU method differs from the LC method in three ways:

1. The log mortality rates are first smoothed using a penalized regression spline with a partial monotonic constraint (see Ramsay 1988, for detail). We assume that there is an underlying continuous and smooth function $f_t(x)$ that is observed with errors at discrete ages. Expressed mathematically,

$$m_t(x_i) = f_t(x_i) + \sigma_t(x_i)\epsilon_{t,i}, \quad i = 1, \dots, p, \quad t = 1, \dots, n, \tag{3}$$

   where $m_t(x_i)$ represents the log transformation of the observed mortality rate for age x_i in year t, $\sigma_t(x_i)$ allows the noise component to vary with x_i in year t, and $\epsilon_{t,i}$ is independent and identically distributed standard random variable.
2. More than one principal component is used. By functional principal component analysis, a set of continuous functions is decomposed into functional principal components and their associated scores. That is,

$$f_t(x) = a(x) + \sum_{j=1}^{J} b_j(x)k_{t,j} + e_t(x), \quad t = 1, \dots, n, \tag{4}$$

where

(1) $a(x)$ is the mean function estimated by empirical mean $\widehat{a}(x) = \frac{1}{n}\sum_{t=1}^{n} f_t(x)$;
(2) $\{b_1(x), \dots, b_J(x)\}$ represents a set of the first J functional principal components;
(3) $\{k_{t,1}, \dots, k_{t,J}\}$ represents a set of uncorrelated principal component scores;
(4) $e_t(x)$ is the residual function with mean zero and variance $v(x)$ estimated by averaging $\{e_1^2(x), \dots, e_n^2(x)\}$;
(5) $J < n$ is the optimal number of functional principal components used. Following Hyndman and Booth (2008) and Shang, Booth, and Hyndman (2011), we chose $J = 6$ which should be larger than any of the components required.

3. A broader range of univariate time-series models may be used to forecast the principal component scores. By conditioning on the past smooth curves $\mathcal{I} = \{m_1(x), \dots, m_n(x)\}$ and the set of functional principal components $\mathcal{B} = \{b_1(x), \dots, b_J(x)\}$, the h-step-ahead point forecast of $m_{n+h}(x)$ can be obtained by

$$\widehat{m}_{n+h|n}(x) = \mathrm{E}[m_{n+h}(x)|\mathcal{I}, \mathcal{B}] = \widehat{a}(x) + \sum_{j=1}^{J} b_j(x)\widehat{k}_{n+h|n,j},$$

where $\widehat{k}_{n+h|n,j}$ denotes the h-step-ahead forecast of $k_{n+h,j}$ using a univariate time-series model, such as the ARIMA model (Box, Jenkins, and Reinsel 2008) used in this paper. The optimal order of an ARIMA model is automatically selected using an algorithm of Hyndman and Khandakar (2008), which minimizes the Akaike information criterion (Akaike 1974) by default.

The forecast variance follows from (3) and (4). Due to the orthogonality[3] between the functional principal components and the error term, the overall forecast variance can be approximated by the sum of four individual variances. Conditioning on the past smooth curves $\mathcal{I}$ and the set of fixed principal components $\mathcal{B} = \{b_1(x), \dots, b_J(x)\}$, we obtained the overall forecast variance of $m_{n+h}(x)$,

$$\mathrm{var}[m_{n+h}(x)|\mathcal{I}, \mathcal{B}] \approx \widehat{\sigma}_a^2(x) + \sum_{j=1}^{J} b_j^2(x)u_{n+h|n,j} + v(x) + \sigma_{n+h}^2(x), \tag{5}$$

where

[3] Since the HU method first smooths the data for each year separately, the smoothing error in (3) does not correlate with the modeling error. Because the error term in (4) contains $J+1$ and higher order functional principal components, they are orthogonal to (or vary independently of) the first J functional principal components by construction.

(1) $\widehat{\sigma}_a^2(x)$ is the variance of the smooth estimate $\widehat{a}(x)$ that can be obtained from the smoothing method.
(2) $b_j^2(x)$ is the variance of the jth principal component;
$u_{n+h|n,j} = \text{var}(k_{n+h,j}|k_{1,j}, \dots, k_{n,j})$ can be obtained from the time-series model.
(3) the model residual variance $v(x)$ is estimated by averaging $\{e_1^2(x), \dots, e_n^2(x)\}$ for each x;
(4) the observational error variance $\sigma_{n+h}^2(x)$ is estimated by averaging $\{\widehat{\sigma}_1^2(x), \dots, \widehat{\sigma}_n^2(x)\}$ for each x (Hyndman and Ullah 2007).

By assuming that each of the four sources of uncertainty have a normal distribution and that they are uncorrelated, the $100(1-\alpha)\%$ prediction interval of $m_{n+h}(x)$ is constructed as $\widehat{m}_{n+h|n}(x) \pm z_\alpha \sqrt{\text{var}[m_{n+h}(x)|\mathcal{I}, \mathcal{B}]}$, where z_α is the $(1 - \alpha/2)$ standard normal quantile.

2.5 Robust Hyndman-Ullah (HUrob) method

Because the presence of outliers can seriously affect the performance of modeling and forecasting, it is important to eliminate the effect of outliers where possible. The HUrob method calculates the integrated squared error for each year,

$$\int_{x_1}^{x_p} \Big(f_t(x) - a(x) - \sum_{j=1}^{J} b_j(x) k_{t,j}\Big)^2 dx,$$

as this provides a measure of estimation accuracy for the functional principal component approximation of the functional data. Note that the continuous functional curves are bounded between x_1 and x_p. Outliers are those years that have a larger integrated squared error than the critical value calculated from a χ^2 distribution (see Hyndman and Ullah 2007, for detail). By assigning zero weight to outliers, we can apply the HU method to model and forecast mortality rates, from which forecasts of age-specific life expectancies are calculated without influence of possible outliers.

2.6 Weighted Hyndman-Ullah (HUw) method

The HUw method uses geometrically decaying weights in the estimation of $a(x)$ and $b_j(x)$, thus allowing the estimation of these quantities to be based more on recent data than on data from the distant past (Hyndman and Shang 2009; Shang, Booth, and Hyndman 2011).

The HUw method differs from the HU method in three ways:

1. The weighted functional mean $a^*(x)$ is estimated by

$$\widehat{a}^*(x) = \sum_{t=1}^{n} w_t f_t(x), \qquad \sum_{t=1}^{n} w_t = 1,$$

where $\{w_t = \kappa(1-\kappa)^{n-t}, t = 1, \dots, n\}$ denotes a set of weights, and $0 < \kappa < 1$ denotes a geometrically decaying weight parameter. Hyndman and Shang (2009) describe how to estimate the optimal value of κ empirically from data. In short, the optimal value of $\kappa \in (0, 1)$ is chosen by minimizing an overall forecast error measure within the validation data set among a set of possible candidates. In this paper, we utilize a one-dimensional optimization algorithm of Nelder and Mead (1965) to minimize the objective function and to find its corresponding optimal (exact) value of κ.

2. By functional principal component analysis, a set of weighted functions $\{w_t[f_t(x) - \widehat{a}^*(x)]; t = 1, \dots, n\}$ is decomposed into weighted functional principal components and their uncorrelated principal component scores. That is,

$$f_t(x) = \widehat{a}^*(x) + \sum_{j=1}^{J} b_j^*(x) k_{t,j} + e_t(x),$$

where $\{b_1^*(x), \dots, b_J^*(x)\}$ is a set of weighted functional principal components.

3. Conditioning on the past smooth curves $\mathcal{I} = \{m_1(x), \dots, m_n(x)\}$ and the set of weighted functional principal components $\mathcal{B}^* = \{b_1^*(x), \dots, b_J^*(x)\}$, the h-step-ahead forecast of $m_{n+h}(x)$ is obtained by

$$\widehat{m}_{n+h|n}(x) = \mathrm{E}[m_{n+h}(x)|\mathcal{I}, \mathcal{B}] = \widehat{a}^*(x) + \sum_{j=1}^{J} b_j^*(x) k_{n+h|n,j}.$$

From the variance expression given by (5), the $100(1-\alpha)\%$ prediction interval of future mortality rates at year $n+h$ is constructed parametrically.

2.7 Random walk with drift (RWD) method

While Sections 2.1-2.6 introduce various principal component methods to forecast age-specific mortality rates and calculate survival probabilities, from which we can derive remaining life expectancy by age. In this subsection, we revisit two naïve random walk methods that are able to forecast the life expectancy for each age.

Given findings of linear life expectancy (White 2002; Oeppen and Vaupel 2002) and debate about its continuation (Bengtsson 2003; Lee 2003), it is pertinent to compare the point forecast accuracy of the principal component methods with linear extrapolation of life expectancy (see Alho and Spencer 2005, pp.274-276 for an introduction). The linear extrapolation of age-specific life expectancies was achieved by applying the random walk with drift (RWD) model for each age:

$$y_{x,t+1} = d + y_{x,t} + e_{x,t+1}, \quad t = 1, 2, \dots, n-1, \quad x = 0, 1, \dots, p. \tag{6}$$

where $y_{x,t}$ represents the life expectancy at age x in year t. The h-step-ahead point and interval forecasts are given by

$$\begin{aligned} \widehat{y}_{x,n+h|n} &= \text{E}[y_{x,n+h}|y_{x,1}, \dots, y_{x,n}] = dh + y_{x,n}, \\ \text{var}(\widehat{y}_{x,n+h|n}) &= \text{var}[y_{x,n+h}|y_{x,1}, \dots, y_{x_n}] = \text{var}(y_{x,n}) + \text{var}(e_{x,n+h}). \end{aligned} \tag{7}$$

Based on (6) to (7), the random walk without drift, labeled as RW, can be obtained simply by omitting the drift term. Computationally, the forecasts of RW and RWD methods are obtained by the `rwf` function in the *forecast* package (Hyndman 2012b) in R (R Development Core Team 2012).

2.8 Univariate time-series method

One popular approach to forecasting age-specific life expectancies involves the use of univariate time-series models. In this approach, we select a particular time-series model for the series to be forecasted, and use the fitted model to produce point and interval forecasts for each age. The ARIMA model discussed by Box, Jenkins, and Reinsel (2008) comprises one popular class of models (see also its equivalent exponential smoothing (ETS) state-space model of Hyndman, Koehler, Ord, and Snyder 2008). The ARIMA model has been applied to forecast life expectancy and its related problems by Torri and Vaupel (2012) for example.

The ARIMA model is designed to handle stationary and non-stationary stochastic processes. In general notation, we have an ARIMA(p, d, q) model, where p is the order of the autoregressive component, d indicates the order of integration, that is, how many times the series must be differenced in order to achieve the stationarity, and q is the order of the moving average component. For simplicity, we consider a stationary time series. The ARMA(p, q) model for a univariate time series $(y_{x,1}, y_{x,2}, \dots, y_{x,n})$ is given by

$$y_{x,t} - \mu = \underbrace{\sum_{i=1}^{p} \phi_i (y_{x,t-i} - \mu)}_{\text{AR}(p)} + \epsilon_{x,t} + \underbrace{\sum_{j=1}^{q} \theta_j \epsilon_{x,t-j},}_{\text{MA}(q)} \quad \max(p, q) + 1 \le t \le n$$

where the constant parameter μ is the drift term, representing the average change in the series over time; ϕ_i are the parameters of the autoregressive component, and likewise θ_j are the parameters of the moving average component, and $\epsilon_{x,t}$ is a sequence of independent and identically distributed random variables with mean zero and variance σ_w^2.

Following the early work by Box, Jenkins, and Reinsel (2008, p.218), the one-step-ahead point forecasts and overall variance are given by

$$\widehat{y}_{x,n+1|n} = \text{E}[y_{x,n+1}|y_{x,1}, y_{x,2}, \dots, y_{x,n}] = \widehat{\mu} + \sum_{i=1}^{p} \widehat{\phi}_i \left(y_{x,n+1-i} - \widehat{\mu} \right), \qquad (8)$$

$$\text{var}[y_{x,n+1}|y_{x,1}, y_{x,2}, \dots, y_{x,n}] = \widehat{\sigma}_w^2 \left(1 + \widehat{\theta}_1^2 + \widehat{\theta}_2^2 + \cdots + \widehat{\theta}_q^2 \right), \qquad (9)$$

from the overall variance, a $100(1-\alpha)\%$ prediction interval of $y_{x,n+1}$ can be constructed parametrically for each age. The parameters are commonly estimated by the maximum likelihood criterion. For the h-step-ahead forecasts, (8) and (9) can be applied iteratively.

A difficulty associated with the use of ARIMA model is the order selection. Hyndman and Khandakar (2008) developed an algorithm, named `auto.arima` in the *forecast* package in R, for automatically selecting the optimal orders based on a statistical criterion, such as Akaike's (1974) information criterion.

3. Model averaging approach

3.1 Literature review

Suppose a decision maker is forecasting the values of some variables, such as $m_{n+h|n}(x)$. He is given L different forecasts of the values $\widehat{m}_{n+h|n}(x)$, namely $\widehat{m}^{(1)}_{n+h|n}(x), \dots, \widehat{m}^{(L)}_{n+h|n}(x)$. Because these L different forecasts may reflect different assumptions, model structures and degree of model complexity, he does not want to simply choose the best one and discard the rest. Instead, he would like to combine these forecasts to get a better forecast accuracy.

This is the problem of model averaging, which has been studied intensively in statistics. The idea dates back to the work by Bates and Granger (1969) and Dickinson (1975), which stimulated a flurry of articles about combining predictions from different forecasting methods. See Clemen (1989) for a detailed review and annotated bibliography. In the seminal paper by Bates and Granger (1969), two separate sets of forecasts of airline passenger data have been combined to form a combined set of forecasts. They found that the combined forecasts can yield lower mean square error than either of the two original forecasts. Past errors of each of the two forecasts are used to determine the optimal weights to attach to these two original forecasts in forming the combined forecasts[4]. The review paper by Clemen (1989) collected a number of contributions on the idea of combining forecasts in the fields of forecasting, psychology, statistics, and management science. While Bates and Granger (1969) and Dickinson (1975) discussed a way of selecting optimal weights from a frequentist viewpoint, Bunn (1975) and Bordley (1982) provided an alternative Bayesian derivation of optimal weights, and showed the similarity with the frequentist viewpoint. An excellent review article on BMA is given by Hoeting et al. (1999).

By using the model averaging approach, one can obtain a better accuracy than any method alone, when the forecasts combined use different methods that capture different information, different specifications or different assumptions. Because the underlying data generating process is often unknown, combined forecasts are more robust toward model mis-specification and are more likely to produce accurate point forecasts. In the demographic literature, there has been little interest in combining forecasts from different models. Nonetheless, some notable exceptions include Smith and Shahidullah (1995); Ahlburg (1998, 2001) and Sanderson (1998), whose pioneering work, particularly in the context of census tract forecast, have done much to awaken others, including the present author. The contribution of this article is to apply the notion of model averaging to the problem of forecasting age-specific life expectancies.

In the work by Smith and Shahidullah (1995), it was found that a forecast of census tract population based on simple average of forecasts from four extrapolation techniques[5] was as accurate as the single most accurate method. By combining forecasts from two methods that were found to predict accurately for particular types of tracts, they asserted that using knowledge of historical forecast performance can further reduce forecast error when combined. Ahlburg (1998) also demonstrated that combined forecasts of births from an economic demographic model and from the U.S. Bureau of census' cohort-component model produced more accurate forecast than the official cohort-component forecasts alone, indeed with a 15% decrease in the mean absolute percentage error. Furthermore, Sanderson (1998) stated that combining casual economic-demographic models for developing countries produced better forecasts than demographic cohort-component forecasts. With a 21% reduction in the mean absolute percentage error, the combined forecasts produced better results than does reliance on only one forecast method.

[4] See also Section 3.2.1.
[5] linear, exponential, shift-share and share-of-growth

3.2 Weight selection

3.2.1 A frequentist viewpoint

There are two common questions raised from the model averaging approach, which may be of interest to demographers and statisticians. The first one is: how many models should be included in the combination? From the frequentist viewpoint, Schmittlein, Kim, and Morrison (1990) used the Akaike's (1974) information criterion to decide. From the Bayesian viewpoint, Madigan and Raftery (1994) suggested that if a model predicts the data far less well than the model which provides the best predictions, then it should be omitted from model combination. In what follows, we consider two situations, namely combining two best approaches and combining all approaches. The optimal weights are selected by their past forecast errors in the validation data set.

Two best models: Following the work of Bates and Granger (1969), our initial model averaging approach combines forecasts from two models given as follows:

$$\widehat{y}_{n+h|n,\text{combo}} = \lambda\widehat{y}_{n+h|n,M_1} + (1-\lambda)\widehat{y}_{n+h|n,M_2},$$

where $\widehat{y}_{n+h|n,M_1}$ and $\widehat{y}_{n+h|n,M_2}$ represent the point forecasts obtained from two models, labeled as M_1 and M_2 respectively; $\widehat{y}_{n+h|n,\text{combo}}$ represents the combined forecasts; and $0 < \lambda < 1$ is the weight parameter that needs to be estimated in light of data. As $\lambda \rightarrow 0$, $\widehat{y}_{n+h|n,\text{combo}} \rightarrow \widehat{y}_{n+h|n,M_2}$, as $\lambda \rightarrow 1$, $\widehat{y}_{n+h|n,\text{combo}} \rightarrow \widehat{y}_{n+h|n,M_1}$.

Libby and Blashfield (1978) reported that the majority of the improvement in accuracy was achieved with the combination of the top two forecasts. By combining forecasts from two methods, we have only one parameter λ to be determined from data, and this simplifies the optimization procedure in determining the value of λ.

Between the two best approaches, we calculate the point forecast accuracy as measured by the MAFEs in the validation set, and assign the weights to be the inverse of their MAFEs (that is $\frac{1}{\text{MAFE}}$). To avoid the possible model identification, the sum of the weights must be equal to 1, therefore, we normalize the weights $\left(\frac{\text{each weight}}{\text{sum of weights}}\right)$.

Conceptually, the method that performs better in the validation set receives a higher weight in the combined forecasts. Note that one can also iteratively update the optimal weights successively by taking into account the most recent data. Here, we implicitly assume that future age-specific life expectancies do not change significantly over years, which is a reasonable assumption (see also White 2002; Oeppen and Vaupel 2002).

All models: As suggested by a referee, combining only two models is quite restrictive. Therefore, we also combine the point forecasts from the 14 models investigated. The model averaging approach combines forecasts from 14 models given as follows:

$$
\begin{aligned}
&\widehat{y}_{n+h|n,\text{combo}} = \\
&\quad \eta_1\widehat{y}_{n+h|n,M_1} + \eta_2\widehat{y}_{n+h|n,M_2} + \eta_3\widehat{y}_{n+h|n,M_3} + \eta_4\widehat{y}_{n+h|n,M_4} + \eta_5\widehat{y}_{n+h|n,M_5} \\
&+ \eta_6\widehat{y}_{n+h|n,M_6} + \eta_7\widehat{y}_{n+h|n,M_7} + \eta_8\widehat{y}_{n+h|n,M_8} + \eta_9\widehat{y}_{n+h|n,M_9} + \eta_{10}\widehat{y}_{n+h|n,M_{10}} \\
&+ \eta_{11}\widehat{y}_{n+h|n,M_{11}} + \eta_{12}\widehat{y}_{n+h|n,M_{12}} + \eta_{13}\widehat{y}_{n+h|n,M_{13}} + (1 - \eta_1 - \eta_2 - \eta_3 - \eta_4 \\
&- \eta_5 - \eta_6 - \eta_7 - \eta_8 - \eta_9 - \eta_{10} - \eta_{11} - \eta_{12} - \eta_{13})\widehat{y}_{n+h|n,M_{14}},
\end{aligned}
$$

where $\widehat{y}_{n+h|n,M_1}, \widehat{y}_{n+h|n,M_2}, \dots, \widehat{y}_{n+h|n,M_{14}}$ represent the point forecasts obtained from the 14 models, labeled as $M_1, M_2, \dots, M_{14}$ respectively, $(\eta_1, \eta_2, \dots, \eta_{13})$ represents a set of weights.

For all approaches, we calculate the point forecast accuracy as measured by the MAFEs in the validation set, and assign the weights to be the inverse of their MAFEs (that is $\frac{1}{\text{MAFE}}$). To avoid the possible model identification, the sum of the weights must be equal to 1, therefore, we normalize the weights $\left(\frac{\text{each weight}}{\text{sum of weights}}\right)$.

3.2.2 A Bayesian viewpoint

The second question is: what is the best way of selecting optimal weights? While Section 3.2.1 presents a frequentist way of selecting optimal weights, in this subsection we discuss a Bayesian means of determining optimal weights. From the Bayesian viewpoint, BMA is another solution to the model uncertainty problem (see, for example, Wright 2008; Geweke and Amisano 2012). Let $M_1, M_2, \dots, M_K$ be a set of K possible models and one of the models is the true model. Let $\theta_1, \theta_2, \dots, \theta_K$ be the vector of parameters associated with each model. Denote Δ as the quantity of interest, such as a combined forecast of age-specific life expectancies, then its posterior distribution given data D is

$$
\begin{aligned}
Pr(\Delta|D) &= \sum_{k=1}^{K} Pr(\Delta|M_k, D) Pr(M_k|D), \\
&= \sum_{k=1}^{K} Pr(\Delta|M_k, D) \underbrace{\frac{Pr(D|M_k)}{\sum_{l=1}^{K} Pr(D|M_l) Pr(M_l)}}_{\text{weights}},
\end{aligned} \tag{10}
$$

where $\text{Pr}(D|M_k) = \int \text{Pr}(D|\theta_k, M_k)\text{Pr}(\theta_k|M_k)d\theta_k$. $\text{Pr}(\theta_k|M_k)$ is the prior density of θ_k under model M_k, $\text{Pr}(D|\theta_k, M_k)$ is the likelihood, $\text{Pr}(M_k)$ is the prior probability that M_k is the true model. Equation (10) is an average of the posterior distributions under each of the models considered, weighted by their posterior model probability (Hoeting et al. 1999).

Given diffuse priors and equal model prior probabilities, the BMA weights are approximately

$$w_k = \frac{\exp\left(-\frac{1}{2}\text{BIC}_k\right)}{\sum_{k=1}^{K} \exp\left(-\frac{1}{2}\text{BIC}_k\right)},$$

where

$$\text{BIC}_k = 2L_k + \log(n)\lambda_k,$$

where L_k is the negative log-likelihood, λ_k is the number of parameters in model k, BIC_k is the Bayesian information criterion for model k (see also Burnham and Anderson 2004).

The BMA estimator has a nice interpretation from a Bayesian viewpoint. The downside is that the “true" model must be in the set of K possible models. To remedy this situation, Burnham and Anderson (2002) suggested replacing BIC with AIC, and we follow this idea in this paper. The weights are given by

$$w_k = \frac{\exp(-\frac{1}{2}\text{AIC}_k)}{\sum_{k=1}^{K} \exp(-\frac{1}{2}\text{AIC}_k)}, \tag{11}$$

where

$$\begin{aligned} \text{AIC}_k &= 2L_k + 2\lambda_k, \\ &= n \log\left(\frac{\text{RSS}_k}{n}\right) + 2\lambda_k, \end{aligned}$$

where n is the sample size, and RSS is the residual sum of squares for each model (Burnham and Anderson 2004).

The individual AIC values are not interpretable, as they contain arbitrary constants and are much affected by sample size. Here, it is imperative to rescale AIC to

$$\text{AIC}_{\text{diff},k} = \text{AIC}_k - \text{AIC}_{\min}, \tag{12}$$

where $\text{AIC}_{\min}$ is the minimum of the K different AIC_k values. Plugging (12) into (11), we obtain the so-called Akaike’s weight (Burnham and Anderson 2004).

4. Data set

The data sets used in this study were taken from the Human Mortality Database (2012). Note that in the Human Mortality Database (2012), a full-fledged life table is calculated systematically and age-specific life expectancies are presented in the last column for each gender and country. Fourteen developed countries were selected, and thus 28 sex-specific populations were obtained for all analyses. The selected fourteen countries all have reliable data series commencing before 1950, which is the starting year of the fitting period for the LM method. The selected countries are presented in Table 1, along with their initial fitting period.

Table 1: Commencing years of the initial fitting period for different methods and countries

Country	LC	LCnone	TLB	LM	BMS[f]	BMS[m]	RWD	RW
Australia	1921	1921	1950	1950	1962	1958	1921	1921
Canada	1921	1921	1950	1950	1954	1954	1921	1921
Denmark	1835	1835	1950	1950	1948	1948	1835	1835
England	1841	1841	1950	1950	1952	1953	1841	1841
Finland	1878	1878	1950	1950	1962	1956	1878	1878
France	1816	1816	1950	1950	1946	1946	1816	1816
Iceland	1838	1838	1950	1950	1946	1947	1838	1838
Italy	1872	1872	1950	1950	1962	1958	1872	1872
Netherlands	1850	1850	1950	1950	1947	1947	1850	1850
Norway	1846	1846	1950	1950	1948	1948	1846	1846
Scotland	1855	1855	1950	1950	1960	1970	1855	1855
Spain	1908	1908	1950	1950	1952	1953	1908	1908
Sweden	1751	1751	1950	1950	1932	1953	1751	1751
Switzerland	1876	1876	1950	1950	1968	1948	1876	1876

Table 1: (Continued)

Country	HU	HU50	HUrob	HUrob50	HUw	ETS	ARIMA
Australia	1921	1950	1921	1950	1921	1921	1921
Canada	1921	1950	1921	1950	1921	1921	1921
Denmark	1835	1950	1835	1950	1835	1835	1835
England	1841	1950	1841	1950	1841	1841	1841
Finland	1878	1950	1878	1950	1878	1878	1878
France	1816	1950	1816	1950	1816	1816	1816
Iceland	1838	1950	1838	1950	1838	1838	1838
Italy	1872	1950	1872	1950	1872	1872	1872
Netherlands	1850	1950	1850	1950	1850	1850	1850
Norway	1846	1950	1846	1950	1846	1846	1846
Scotland	1855	1950	1855	1950	1855	1855	1855
Spain	1908	1950	1908	1950	1908	1908	1908
Sweden	1751	1950	1751	1950	1751	1751	1751
Switzerland	1876	1950	1876	1950	1876	1876	1876

Notes: Although the RW model depends only on the previous year, the variance of the RW model requires all data.

4.1 Training, validation, and forecasting data sets

We divide each data set into a fitting period $(1 : n-20)$ and a forecasting period $(n-19 : n)$, where n denotes the last year of observations in a data set. The commencing year of the fitting period differs by method, as seen in Table 1. We implement a rolling origin regime as follows: the forecasting period is set to be the last 20 years, all ending in 2007, while the remaining data are in the initial fitting period. Using the data in the fitting period, we compute the one-step-ahead and ten-step-ahead point and interval forecasts, and determine the forecast accuracy by comparing the forecasts with the holdout data in the forecasting period. Then, we increase the fitting period by one year, and compute the one-step-ahead and ten-step-ahead point and interval forecasts, and calculate the point and interval forecast accuracy. This process is repeated until the forecasts reach the last year of available data.

Within the fitting period, we further split the data into a training period $(1 : n-40)$ and a validation period $(n-39 : n-20)$. Using the data in the training period, we implement the rolling origin regime again to compute the one-step-ahead and ten-step-ahead point and interval forecasts, and calculate the forecast errors for all of the methods considered.

5. Measures of point and interval forecast accuracy

There exist a number of criteria for measuring point and interval forecast accuracy. In Sections 5.1 and 5.2, we present two criteria each for measuring the point and interval forecast accuracy.

5.1 Mean absolute forecast error and mean forecast error

By using the *demographic* package (Hyndman 2012a) in R, we calculate the point and interval forecasts for each method, and evaluate and compare their forecast accuracy. To measure point forecast accuracy and bias, we utilize the mean absolute forecast error and mean forecast error, labeled as MAFE and MFE. These two criteria have previously been implemented in Shang, Booth, and Hyndman (2011) and Booth et al. (2006), although one can also use mean absolute percentage forecast error and mean algebraic percentage forecast error to measure the accuracy and bias, respectively. The MAFE criterion measures how close the forecasts come to the actual values of the variable being forecast, regardless of the sign of error. In contrast, the MFE is the average of errors, (actual - forecast), and is a measure of bias. These measures can be expressed mathematically as

$$\text{MAFE} = \frac{1}{(p+1)\times q}\sum_{j=1}^{q}\sum_{x=0}^{p}\left|y_{x,j}-\widehat{y}_{x,j|j-h}\right|, \quad j=(n-20+h),\dots,n$$

$$\text{MFE} = \frac{1}{(p+1)\times q}\sum_{j=1}^{q}\sum_{x=0}^{p}\left(y_{x,j}-\widehat{y}_{x,j|j-h}\right),$$

where q represents the number of years in the forecasting period, $y_{x,j}$ represents the actual holdout sample for age x in year j, and $\widehat{y}_{x,j}$ represents the forecasts for the holdout sample. We consider the one-step-ahead and ten-step-ahead forecasts in this paper.

5.2 Coverage probability deviance and half-width

The evaluation of interval forecast accuracy is described as follows: variances for the LC method and its variants were calculated in (2); variances for the HU method and its variants were calculated in (5). Based on the normality assumption, we construct prediction interval of mortality rates, and thereby prediction interval of life expectancies (see Rowland 2003, Chapter 8). For each year in the forecasting period, the one-step-ahead prediction intervals were calculated at the customarily 0.8 (80%) nominal coverage probability, and were then tested against the empirical coverage probability (Swanson and Beck 1994; Tayman, Smith, and Lin 2007; Shang, Booth, and Hyndman 2011). The empirical coverage probability is defined as the actual proportion of out-of-sample data that fall into the calculated prediction intervals. To measure interval forecast accuracy, we calculated the coverage probability deviance, which is the absolute difference between the nominal coverage probability and the empirical coverage probability. With the nominal coverage probability of 0.8, the maximum coverage probability deviance is 0.8 when the empirical coverage probability is 0; while the minimum coverage probability deviance is 0 when the empirical coverage probability is 0.8.

Apart from the coverage probability deviance, we also calculate the size of the prediction interval. Following the early work by Lee and Tuljapurkar (1994) and Tayman, Smith, and Lin (2007), the size of the prediction interval is defined as half-width by dividing one-half of the difference between the upper and lower bounds of the interval forecasts. For the symmetric interval, the half-width reflects the distance between the point forecast (center) and the lower and upper bounds of the prediction interval.

6. Comparisons of the point forecasts

For simplicity, we will refer to the LC method and its variants[6] as the LC methods, and refer to the HU method and its variants[7] as the HU methods. Results are presented by country and for two averages: the simple average and a weighted average using weights based on population size in 2007. For each country, the weight is calculated as the country's population size in 2007 divided by the sum of each country's population size and scaled to sum to 14. For females, the weights are 0.87, 1.36, 0.23, 2.55, 0.21, 2.63, 0.01, 2.50, 0.68, 0.19, 0.22, 1.86, 0.38, 0.32 in the country order of Table 1. For males, the weights are 0.89, 1.39, 0.23, 2.55, 0.23, 2.56, 0.01, 2.46, 0.69, 0.20, 0.21, 1.88, 0.39, 0.31. Hereafter, the simple average is labeled by Mean, while the weighted average is labeled by Mean(w).

[6] all based on a single principal component
[7] all using a nonparametric smoothing technique and several principal components

Table 2: Point forecast accuracy of the female life expectancies by method and country, as measured by the MAFEs for the one-step-ahead forecasts

Country	LC	LCnone	TLB	LM	BMS	RWD	RW
Australia	0.2323	1.1877	0.2465	0.1621	0.1608	0.1586	0.1904
Canada	0.1395	0.6716	0.1978	0.0787	0.1169	0.0782	0.1155
Denmark	0.6228	0.7931	0.4014	0.1990	0.2846	0.1838	0.1886
England	0.2162	1.0086	0.2957	0.1374	0.1595	0.1370	0.1659
Finland	0.4093	1.7736	0.5839	0.1564	0.1952	0.1480	0.1741
France	0.5748	1.7898	0.2302	0.1366	0.1760	0.1357	0.1696
Iceland	0.4348	1.4165	1.2098	0.5037	1.3602	0.3804	0.3885
Italy	0.4032	1.4470	0.6114	0.1440	0.1540	0.1442	0.1855
Netherlands	0.3269	0.9000	0.2891	0.1251	0.1831	0.1184	0.1351
Norway	0.5514	1.1796	0.3364	0.1630	0.1852	0.1533	0.1666
Scotland	0.6219	1.3552	0.5605	0.1790	0.2309	0.1738	0.1986
Spain	0.4828	1.3948	0.7689	0.1612	0.2127	0.1534	0.1621
Sweden	0.5071	1.0276	0.2086	0.1421	0.2049	0.1309	0.1440
Switzerland	0.3530	1.3063	0.2485	0.1276	0.1492	0.1158	0.1496
Mean	0.4197	1.2322	0.4420	0.1726	0.2695	0.1580	0.1810
Mean(w)	0.3831	1.2821	0.3979	0.1394	0.1718	0.1363	0.1651
Country	**HU**	**HUrob**	**HU50**	**HUrob50**	**HUw**	**ETS**	**ARIMA**
Australia	0.2163	0.4880	0.1997	0.3074	0.2028	0.1449	0.1493
Canada	0.1068	0.1367	0.1137	0.1210	0.0995	0.0987	0.0896
Denmark	0.2278	0.5572	0.2227	0.2427	0.2315	0.2184	0.1967
England	0.2150	0.2848	0.1784	0.3173	0.1369	0.1433	0.1412
Finland	0.4767	1.4649	0.3559	0.5272	0.1823	0.1430	0.1610
France	0.5452	0.7534	0.1771	0.1918	0.1322	0.1478	0.1707
Iceland	0.4557	0.6922	0.5402	1.0132	0.4183	0.3415	0.3400
Italy	0.5265	0.7780	0.1535	0.2693	0.1486	0.1504	0.1899
Netherlands	0.1798	0.3873	0.2618	0.2085	0.1417	0.1466	0.1938
Norway	0.6440	0.6547	0.2870	0.4187	0.1955	0.1493	0.1614
Scotland	0.4560	1.0595	0.4033	0.3977	0.3137	0.1601	0.1906
Spain	0.2578	0.2695	0.1550	0.4092	0.1633	0.1439	0.1502
Sweden	0.4139	1.8243	0.1550	0.1816	0.1501	0.1688	0.1260
Switzerland	0.3233	0.6789	0.1480	0.2939	0.3370	0.1279	0.1319
Mean	0.3603	0.7164	0.2394	0.3500	0.2038	0.1632	0.1709
Mean(w)	0.3481	0.5527	0.1771	0.2735	0.1533	0.1436	0.1566

Table 3: Point forecast accuracy of the male life expectancies by method and country, as measured by the MAFEs for the one-step-ahead forecasts

Country	**LC**	**LCnone**	**TLB**	**LM**	**BMS**	**RWD**	**RW**
Australia	0.4412	1.6429	0.4716	0.1881	0.1746	0.1798	0.2254
Canada	0.3289	1.1484	0.4520	0.1243	0.2017	0.1219	0.1672
Denmark	0.3177	0.4883	0.6738	0.1983	0.3922	0.1613	0.1787
England	0.3300	1.6489	0.5275	0.1497	0.1542	0.1377	0.1995
Finland	0.4467	1.7786	0.6151	0.1793	0.2481	0.1520	0.1924
France	0.5316	1.8475	0.2457	0.1187	0.2248	0.1340	0.1724
Iceland	0.4693	0.6900	1.7789	0.1682	1.4659	0.4099	0.4173
Italy	0.3407	1.3609	0.8669	0.1468	0.2356	0.1331	0.1876
Netherlands	0.1995	0.4344	0.7279	0.1721	0.4500	0.1463	0.1814
Norway	0.3000	0.7189	0.8999	0.2205	0.3990	0.1887	0.2179
Scotland	0.7917	1.7288	0.6525	0.2252	0.2727	0.1890	0.2094
Spain	0.5477	0.9855	0.4276	0.1531	0.2153	0.1467	0.1502
Sweden	0.2409	0.4707	0.6778	0.1583	0.2511	0.1381	0.1681
Switzerland	0.3144	1.0141	0.3085	0.1518	0.1964	0.1361	0.1930
Mean	0.4000	1.1399	0.6661	0.1682	0.3487	0.1696	0.2043
Mean(w)	0.4040	1.3590	0.5323	0.1487	0.2255	0.1412	0.1825
Country	**HU**	**HUrob**	**HU50**	**HUrob50**	**HUw**	**ETS**	**ARIMA**
Australia	0.3478	0.6315	0.3054	0.5288	0.3050	0.1508	0.1636
Canada	0.3324	0.6582	0.3258	0.5517	0.1515	0.1080	0.1086
Denmark	0.4513	0.5524	0.5071	0.6019	0.3952	0.2331	0.2374
England	0.4508	0.5918	0.3079	0.5353	0.1684	0.1631	0.1966
Finland	0.9619	1.7533	0.5072	0.8587	0.2392	0.2875	0.3284
France	0.8984	1.2460	0.2685	0.3679	0.2888	0.1340	0.3174
Iceland	0.5048	0.7086	0.6526	1.0345	0.5376	0.5313	0.4443
Italy	0.8043	1.2068	0.3661	0.5797	0.2772	0.2130	0.3343
Netherlands	0.4320	0.3337	0.5156	0.5521	0.1786	0.2676	0.2197
Norway	0.4169	0.4796	0.3140	0.8530	0.4196	0.2510	0.2331
Scotland	0.7227	0.9819	0.6266	0.7011	0.5822	0.2187	0.2357
Spain	0.3512	0.3285	0.3165	0.3493	0.1636	0.1628	0.1437
Sweden	0.5420	0.7789	0.4691	0.6572	0.3945	0.2350	0.3957
Switzerland	0.6003	1.1496	0.3517	0.3479	0.4686	0.2210	0.2416
Mean	0.5583	0.8143	0.4167	0.6085	0.3264	0.2269	0.2571
Mean(w)	0.5800	0.8209	0.3411	0.5026	0.2443	0.1742	0.2367

Tables 2 and 3 provide summaries of the point forecast accuracy based on the MAFEs for the one-step-ahead forecasts of life expectancies. The summaries given in Tables 2 and 3 are averaged over different ages from 0 to 104, and years in the forecasting period for female and male data, respectively. As measured by the simple and weighted averages of MAFEs over the fourteen countries, the RWD method produces the most accurate point forecasts in both female and male data. The performance of RWD method is followed closely by the LM method.

Tables 4 and 5 show the corresponding MFEs for the one-step-ahead point forecasts. For the female data, the LCnone, HU, HUrob and RW methods consistently underestimate[8] the life expectancies for all countries, while the other methods exhibit a mixture of overestimation and underestimation for various countries. Of all the methods, the LM method performs the best as measured by both the simple and weighted averages. The performance of the LM method is followed by the BMS and RWD methods. For the male data, all methods consistently underestimate the life expectancies except the LC method. Cancelation of MFEs from different countries leads to the most superior performance of the LC method, which is followed by the RWD and LM methods. In contrast to the LC method, the MFEs of the RWD and LM methods are more centered around zero at all ages.

Table 4: Forecast bias of the female life expectancies by method and country, as measured by the MFEs for the one-step-ahead forecasts

Country	LC	LCnone	TLB	LM	BMS	RWD	RW
Australia	0.0533	1.1562	0.1714	0.0281	-0.0226	0.0460	0.1399
Canada	0.0026	0.6471	-0.1666	-0.0092	0.0090	-0.0129	0.0922
Denmark	-0.3730	0.7930	-0.3620	0.0022	0.0599	0.0126	0.0749
England	-0.0352	1.0084	0.2673	0.0369	0.0741	0.0169	0.1196
Finland	-0.1378	1.7457	0.4936	-0.0001	-0.0453	0.0472	0.1426
France	-0.2436	1.7898	0.1183	-0.0089	0.0154	0.0643	0.1384
Iceland	0.2496	1.3925	1.1813	0.2797	1.3262	0.0000	0.0876
Italy	-0.1577	1.4352	0.6083	0.0167	0.0135	0.0297	0.1439
Netherlands	-0.1329	0.8848	-0.2780	-0.0271	-0.0374	-0.0143	0.0657
Norway	-0.3223	1.1683	0.2193	0.0107	-0.0080	0.0402	0.1020
Scotland	-0.2684	1.3552	0.4348	0.0196	0.0852	0.0383	0.1063
Spain	-0.2360	1.3167	0.7206	-0.0208	0.1102	-0.0145	0.1204

[8] actual > forecast

Table 4: (Continued)

Country	LC	LCnone	TLB	LM	BMS	RWD	RW
Sweden	-0.3017	1.0144	-0.1267	-0.0064	-0.0254	0.0363	0.0870
Switzerland	-0.1650	1.2868	-0.0239	-0.0323	-0.0490	-0.0032	0.1129
Mean	-0.1477	1.2139	0.2327	0.0204	0.1076	0.0205	0.1095
Mean(w)	-0.1431	1.2631	0.2644	0.0046	0.0318	0.0224	0.1217
Australia	0.1467	0.4563	0.1393	0.2633	0.0128	0.0493	0.0660
Canada	0.0080	0.0760	-0.0091	-0.0233	-0.0013	-0.0159	-0.0154
Denmark	0.0667	0.5339	-0.0777	-0.0753	0.1722	0.0062	0.0196
England	0.1632	0.2577	0.1189	0.2742	0.0784	0.0788	0.0814
Finland	0.4319	1.4370	0.2610	0.4631	0.0332	0.0641	0.0837
France	0.5290	0.7473	0.0801	0.1528	0.0041	0.0643	0.1173
Iceland	0.0068	0.5514	0.4013	0.9595	0.2352	0.0619	0.0176
Italy	0.5189	0.7585	0.0619	0.2261	0.0867	0.0722	0.0944
Netherlands	0.0622	0.3352	-0.1936	-0.1417	0.0538	-0.0331	-0.1026
Norway	0.6257	0.5736	0.1882	0.3672	0.1287	0.0723	0.0791
Scotland	0.4352	1.0481	0.3214	0.2523	0.2674	0.0489	0.1110
Spain	0.1855	0.2229	0.0831	0.3268	0.0639	-0.0350	-0.0364
Sweden	0.3888	1.7578	-0.0842	0.0701	0.0834	0.1450	0.0456
Switzerland	0.2955	0.6572	0.0020	0.2429	0.3133	-0.0698	-0.0358
Mean	0.2760	0.6724	0.0923	0.2399	0.1094	0.0364	0.0375
Mean(w)	0.2995	0.5221	0.0655	0.1925	0.0612	0.0398	0.0513

Table 5: Forecast bias of the male life expectancies by method and country, as measured by the MFEs for the one-step-ahead forecasts

Country	LC	LCnone	TLB	LM	BMS	RWD	RW
Australia	0.0873	1.5445	0.4189	0.0956	0.0633	0.1212	0.1953
Canada	0.1906	1.0953	0.4129	0.0903	0.1656	0.0899	0.1501
Denmark	-0.0175	0.1804	0.6120	0.1042	0.3456	0.0679	0.1203
England	0.1309	1.6489	0.5142	0.1093	0.1054	0.0931	0.1733
Finland	0.1704	1.7777	0.5628	0.0474	0.0262	0.0945	0.1628
France	0.0847	1.8459	0.1907	0.0578	0.1805	0.1113	0.1605
Iceland	0.1855	0.4087	1.6180	0.0931	0.1801	0.0472	0.9256
Italy	0.0743	1.3601	0.8478	0.1015	0.1954	0.0894	0.1714

Table 5: (Continued)

Country	LC	LCnone	TLB	LM	BMS	RWD	RW
Netherlands	0.1024	0.2380	0.6596	0.1149	0.3958	0.0746	0.1320
Norway	0.0974	0.5883	0.8718	0.1370	0.3628	0.1143	0.1618
Scotland	-0.0669	1.7288	0.6016	0.0768	0.0072	0.0990	0.1513
Spain	-0.2013	0.7655	0.3979	0.0211	0.0909	0.0152	0.1121
Sweden	0.0205	0.3208	0.6333	0.1028	0.1913	0.1037	0.1458
Switzerland	0.0383	0.9659	0.2400	0.0650	0.1023	0.0674	0.1608
Mean	0.0640	1.0335	0.6130	0.0869	0.1723	0.0849	0.2088
Mean(w)	0.0593	1.2954	0.4960	0.0818	0.1578	0.0856	0.1574
Australia	0.2925	0.5529	0.2630	0.4874	0.2655	0.0658	0.0988
Canada	0.3015	0.6108	0.3006	0.5290	0.1248	0.0686	0.0651
Denmark	0.4062	0.4418	0.4728	0.5828	0.3808	0.1696	0.1750
England	0.4166	0.5624	0.2904	0.5245	0.1437	0.1413	0.1814
Finland	0.9540	1.6841	0.4719	0.8105	0.1963	0.2752	0.3122
France	0.8806	1.2440	0.2498	0.3446	0.2781	0.1113	0.3137
Iceland	1.3971	0.0830	0.3859	0.4743	0.3869	0.4389	0.2552
Italy	0.7981	1.2011	0.3426	0.5630	0.2620	0.2038	0.3013
Netherlands	0.3945	0.2734	0.4793	0.5208	0.1489	0.2366	0.0976
Norway	0.3755	0.2943	0.2609	0.8025	0.3837	0.2245	0.2037
Scotland	0.7215	0.9755	0.5885	0.6677	0.5747	0.1734	0.1863
Spain	0.3164	0.2993	0.2659	0.3077	0.0903	0.1230	0.0788
Sweden	0.5374	0.7665	0.4387	0.6183	0.3865	0.2292	0.3876
Switzerland	0.5779	1.1113	0.3067	0.3140	0.4566	0.1639	0.1956
Mean	0.5978	0.7215	0.3655	0.5391	0.2913	0.1875	0.2037
Mean(w)	0.5549	0.7902	0.3120	0.4769	0.2165	0.1445	0.2005

Figures 1a and 1b show the MAFEs of the one-step-ahead point forecasts for different methods. The MAFEs are averaged over countries and years in the forecasting period for the female and male life expectancies. Large errors occur at younger ages[9] for all methods, reflecting the difficulty in capturing the nadir of the mortality rates; errors from the LC, LCnone and HUrob methods are particularly large at these ages. At older ages[10], the MAFEs of all methods with exceptions of the RW and RWD methods display a kink. This reflects the difficulty in forecasting the mortality rates at old ages because of excessive variability.

Figures 1c and 1d show the MFEs of the one-step-ahead point forecasts for different methods. The MFEs are averaged over countries and years in the forecasting period for

[9] between age 0 and 5
[10] above age 95

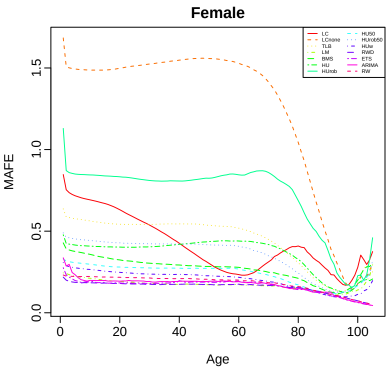
the female and male life expectancies. Apart from the LC method, all other methods underestimate life expectancies. The LCnone method performs the least accurately, while the LC method has minimal MFEs. In the LC method, the overestimation of life expectancies at younger ages is counterbalanced by the underestimation of life expectancies at older ages. Although the overall MFEs of the RWD and LM methods are not as small as the LC method, they remain constant across all ages.

In demography, mortality rate forecasts are primarily of value for a much longer horizon, since the modeling error becomes the dominant error source. Therefore, we also consider ten-step-ahead forecasts. Tables 6 and 7 provide summaries of the point forecast accuracy based on the MAFEs for the ten-step-ahead forecasts of life expectancies. The summaries given in Tables 6 and 7 are averaged over different ages from 0 to 104, and years in the forecasting period (that is 11 years) for female and male data, respectively. As measured by the weighted averages of MAFEs over the fourteen countries, the LM method produces the most accurate point forecasts for both female and male data. The performance of the LM method is followed closely by the RWD method.

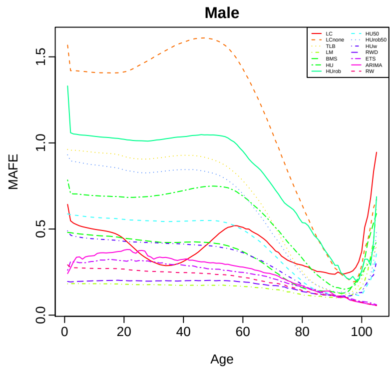
Tables 8 and 9 show the corresponding MFEs for the ten-step-ahead point forecasts. For the female data, the LCnone, RW and ETS methods consistently underestimate the life expectancies for all countries, while the other methods exhibit a mixture of overestimation and underestimation for various countries. Among all methods, the LM method performs the best as measured by both the simple and weighted averages. The performance of the LM method is followed by the BMS, RWD and ARIMA methods. For the male data, all methods consistently underestimate the life expectancies, including the LC method.

Figure 1: MAFEs and MFEs for the one-step-ahead point forecasts of the life expectancies by sex and method

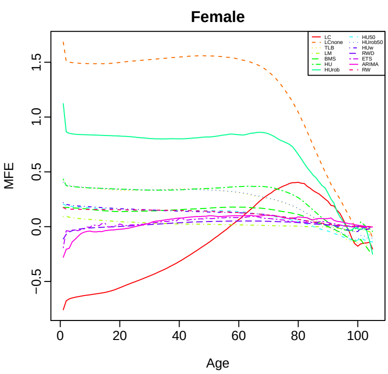
(a) MAFEs averaged over years in the forecasting period and countries



(b) MAFEs averaged over years in the forecasting period and countries



(c) MFEs averaged over years in the forecasting period and countries



(d) MFEs averaged over years in the forecasting period and countries

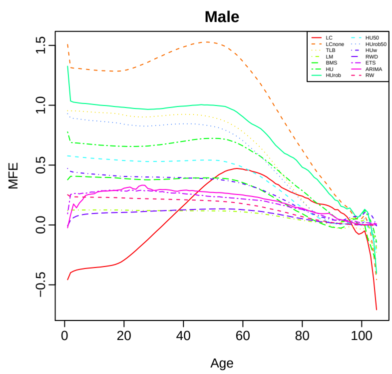


Table 6: Point forecast accuracy of the female life expectancies by method and country, as measured by the MAFEs for the ten-step-ahead forecasts

Country	**LC**	**LCnone**	**TLB**	**LM**	**BMS**	**RWD**	**RW**
Australia	0.8427	1.8795	0.6292	0.5351	0.2651	0.6278	1.5353
Canada	0.2740	0.7256	0.4788	0.3443	0.3404	0.3759	0.8764
Denmark	0.6133	1.2505	0.4661	0.4974	0.7114	0.4774	0.9272
England	0.3261	1.2874	0.4208	0.3283	0.3688	0.2883	1.1776
Finland	0.8040	2.4584	0.7116	0.3054	0.4603	0.6086	1.4833
France	0.5462	2.5393	0.2329	0.1994	0.2438	0.4963	1.2108
Iceland	0.7602	1.7770	2.2061	1.5307	2.0093	0.4306	1.0592
Italy	0.6072	2.1367	1.0866	0.3418	0.2224	0.4772	1.4581
Netherlands	0.4699	1.0033	0.6664	0.5635	0.6355	0.4822	0.5167
Norway	0.4717	1.6456	0.4603	0.3443	0.3636	0.4714	1.0508
Scotland	0.5283	1.8137	0.6524	0.3426	0.5321	0.5071	1.1398
Spain	0.4893	1.8884	0.9988	0.1738	0.3368	0.4404	1.1942
Sweden	0.3924	1.5559	0.3319	0.2298	0.2492	0.4319	0.9243
Switzerland	0.3724	1.8210	0.2657	0.2294	0.4028	0.2915	1.1443
Mean	0.5355	1.6987	0.6863	0.4261	0.5101	0.4576	1.1213
Mean(w)	0.4938	1.7734	0.6166	0.3108	0.3271	0.4384	1.1883

Country	**HU**	**HUrob**	**HU50**	**HUrob50**	**HUw**	**ETS**	**ARIMA**
Australia	1.0660	1.1911	0.5653	0.6006	0.5479	0.4665	0.6333
Canada	0.4622	0.3644	0.4562	0.4017	0.2820	0.4520	0.3825
Denmark	0.5741	1.1164	0.4180	0.4361	0.9005	0.5788	0.4448
England	0.9411	0.5005	0.4028	0.4317	0.3193	0.3462	0.3291
Finland	1.5539	2.3415	0.7603	0.7301	0.8253	0.5277	0.6582
France	1.8639	1.4266	0.4099	0.2171	0.4559	0.2511	0.4609
Iceland	0.5534	1.3926	1.2019	1.7684	1.1044	0.6355	0.5732
Italy	1.5976	1.4109	0.4649	0.5231	0.5463	0.5155	0.6599
Netherlands	0.3907	0.5530	0.6753	0.5700	0.4534	0.5378	0.6759
Norway	1.4951	1.1619	0.5021	0.8119	0.7560	0.4478	0.4552
Scotland	1.3979	1.5942	0.6322	0.6563	0.4707	0.3487	0.5731
Spain	1.0624	0.4387	0.2806	0.7619	0.3511	0.4942	0.5231
Sweden	1.0899	2.2663	0.2335	0.1900	0.5762	0.9749	0.2712
Switzerland	0.8316	0.9954	0.1447	0.4171	0.2595	0.3988	0.3542
Mean	1.0628	1.1967	0.5105	0.6083	0.5606	0.4983	0.4996
Mean(w)	1.2021	0.9852	0.4284	0.4735	0.4387	0.4320	0.4914

Table 7: Point forecast accuracy of the male life expectancies by method and country, as measured by the MAFEs for the ten-step-ahead forecasts

Country	LC	LCnone	TLB	LM	BMS	RWD	RWF
Australia	1.8568	3.0607	1.5374	1.2816	0.7977	1.4248	2.0912
Canada	1.3030	2.0440	1.1402	0.9144	0.8892	1.0226	1.5626
Denmark	0.8934	0.8200	1.6648	1.3428	1.6628	0.8759	1.3720
England	1.2279	2.5789	1.3863	1.0453	0.9813	1.0087	1.7481
Finland	1.4577	2.7815	1.2983	0.8331	1.1146	1.1318	1.7642
France	1.1499	2.8576	0.7895	0.5431	0.7693	1.0279	1.4964
Iceland	1.0845	1.0328	3.0191	0.9953	0.9953	0.7207	1.2171
Italy	1.2242	2.3058	1.9448	1.1021	1.1920	0.9783	1.7411
Netherlands	0.8755	0.7205	1.5911	1.0862	1.5768	0.7231	1.2554
Norway	1.3272	1.3658	2.1399	1.5580	1.8345	1.1879	1.6232
Scotland	1.1739	2.5241	1.4807	0.9696	1.3663	1.0562	1.5450
Spain	0.7054	1.5419	0.6608	0.3387	0.4310	0.3573	1.1902
Sweden	1.0893	1.0777	1.5519	0.9964	1.1885	1.0618	1.4630
Switzerland	1.1130	1.9875	1.1393	0.9278	0.8564	0.8531	1.7434
Mean	1.1773	1.9071	1.5246	0.9953	1.1183	0.9593	1.5581
Mean(w)	1.1653	2.2288	1.2888	0.8762	0.9486	0.9341	1.5859
Country	**HU**	**HUrob**	**HU50**	**HUrob50**	**HUw**	**ETS**	**ARIMA**
Australia	1.9652	2.4552	1.4055	1.5633	1.4487	0.8326	1.1702
Canada	0.9607	1.3871	1.0250	1.3290	0.9288	0.7481	0.7653
Denmark	1.0828	1.0858	1.6215	1.6030	1.4323	0.9687	0.8955
England	2.0794	1.3845	0.7991	1.3378	1.0314	0.9339	1.1342
Finland	2.6584	2.7545	1.3637	1.7096	1.3455	1.8616	1.7526
France	2.4812	2.8125	0.8560	1.0015	1.4291	1.5060	1.5251
Iceland	0.9953	2.5549	1.4513	1.1907	1.5717	1.6221	1.1523
Italy	2.2943	2.1435	1.3565	1.5986	1.6813	1.6645	1.9527
Netherlands	0.9503	0.9904	1.4583	1.5820	1.1502	1.3367	0.7339
Norway	1.4540	1.2805	1.4270	1.9955	1.5826	1.6695	1.2252
Scotland	2.0713	2.1025	1.6112	1.6043	1.7636	1.2111	1.2059
Spain	1.1940	0.5767	0.7037	0.8567	0.4680	1.0242	0.7571
Sweden	1.6559	1.5496	1.4764	1.6733	1.4919	1.5196	1.4889
Switzerland	1.5367	1.9608	1.2052	1.2995	1.1070	0.7689	0.9480
Mean	1.6700	1.7885	1.2686	1.4532	1.3166	1.2620	1.1933
Mean(w)	1.8573	1.7645	1.0604	1.3153	1.2104	1.2176	1.2592

Table 8: Forecast bias of the female life expectancies by method and country, as measured by the MFEs for the ten-step-ahead forecasts

Country	LC	LCnone	TLB	LM	BMS	RWD	RW
Australia	0.7461	1.8297	0.5354	0.4789	0.0745	0.6231	1.5257
Canada	-0.0022	0.6818	-0.4709	-0.2244	-0.1916	-0.2125	0.8516
Denmark	0.0600	1.2505	-0.2649	0.0717	0.2539	0.3115	0.9262
England	0.2704	1.2870	0.3737	0.2627	0.2957	0.1555	1.1742
Finland	0.6991	2.4227	0.5530	0.1690	0.0914	0.5394	1.4740
France	0.4506	2.5393	0.1419	-0.1025	-0.0215	0.4800	1.2101
Iceland	0.7201	1.7502	2.1932	1.4099	1.9936	0.1450	1.0164
Italy	0.5788	2.1247	1.0859	0.3239	0.1747	0.3267	1.4574
Netherlands	-0.1837	0.9809	-0.6664	-0.4914	-0.5953	-0.3166	0.4947
Norway	0.3034	1.6309	0.3886	0.2220	0.2424	0.4194	1.0254
Scotland	0.3762	1.8137	0.3408	0.1059	0.3288	0.4652	1.1312
Spain	0.2796	1.7999	0.9533	0.0586	0.2310	-0.1648	1.1925
Sweden	0.2604	1.5438	-0.3160	-0.1833	-0.0537	0.4240	0.9233
Switzerland	0.1466	1.7982	-0.0046	-0.1222	-0.2915	-0.0235	1.1388
Mean	0.3361	1.6752	0.3459	0.1413	0.1809	0.2266	1.1101
Mean(w)	0.3441	1.7493	0.3779	0.0799	0.0761	0.1949	1.1823

Country	HU	HUrob	HU50	HUrob50	HUw	ETS	ARIMA
Australia	0.9872	1.1629	0.4613	0.5290	0.5229	0.4665	0.6256
Canada	-0.3297	-0.1349	-0.4199	-0.2882	-0.1078	0.4520	-0.2493
Denmark	0.5704	1.1032	-0.0748	0.2442	0.8964	0.5788	0.1803
England	0.9256	0.4594	0.1718	0.3208	0.2123	0.3462	0.2373
Finland	1.5070	2.2997	0.6055	0.6481	0.7753	0.5277	0.5569
France	1.8415	1.4219	-0.1972	-0.0513	0.4271	0.2511	0.3894
Iceland	0.3473	1.3218	1.1449	1.7500	1.0699	0.6355	0.2124
Italy	1.5930	1.4079	0.1318	0.3938	0.5167	0.5155	0.4323
Netherlands	0.3356	0.4204	-0.6649	-0.5309	-0.1154	0.5378	-0.5382
Norway	1.4469	1.0662	0.4369	0.7788	0.6955	0.4478	0.3881
Scotland	1.3923	1.5872	0.3941	0.3051	0.3567	0.3487	0.5378
Spain	0.9856	0.4067	0.1763	0.6476	0.2069	0.4942	-0.1940
Sweden	1.0752	2.1968	-0.2323	0.0510	0.5627	0.9749	0.2355
Switzerland	0.8144	0.9586	-0.0436	0.3457	0.1308	0.3988	-0.1595
Mean	0.9637	1.1198	0.1350	0.3674	0.4393	0.4983	0.1896
Mean(w)	1.0971	0.9103	0.0103	0.2239	0.3155	0.4320	0.1842

Table 9: Forecast bias of the male life expectancies by method and country, as measured by the MFEs for the ten-step-ahead forecasts

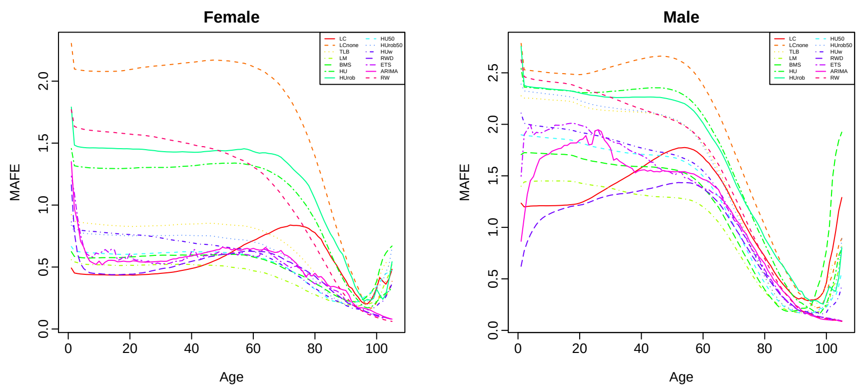
Country	LC	LCnone	TLB	LM	BMS	RWD	RWF
Australia	1.3904	2.9052	1.4429	1.2240	0.6932	1.4222	2.0825
Canada	1.1552	1.9571	1.0852	0.8829	0.8262	0.9950	1.5425
Denmark	0.8934	0.8121	1.5553	1.1867	1.6276	0.8551	1.3569
England	1.2279	2.5789	1.3648	1.0268	0.9388	0.9965	1.7432
Finland	1.4131	2.7807	1.1515	0.6600	0.4264	1.1086	1.7515
France	1.1428	2.8564	0.7538	0.5087	0.7241	1.0278	1.4947
Iceland	0.9191	0.8624	2.4072	2.5259	1.2828	0.5849	0.9191
Italy	1.2157	2.3049	1.9108	1.0595	1.1492	0.9497	1.7365
Netherlands	0.8609	0.6971	1.4517	1.0030	1.4563	0.6703	1.2255
Norway	1.3011	1.3428	2.1088	1.4685	1.8153	1.1418	1.5836
Scotland	1.1585	2.5241	1.3961	0.8820	-0.0818	1.0537	1.5445
Spain	0.1861	1.2384	0.6478	0.2853	0.4140	0.2250	1.1847
Sweden	1.0862	1.0747	1.4730	0.9248	1.0749	1.0536	1.4557
Switzerland	1.0088	1.9324	1.0902	0.8707	0.7514	0.8272	1.7318
Mean	1.0685	1.8477	1.4171	1.0363	0.9356	0.9222	1.5252
Mean(w)	1.0439	2.1661	1.2426	0.8297	0.8645	0.9012	1.5775
Country	**HU**	**HUrob**	**HU50**	**HUrob50**	**HUw**	**ETS**	**ARIMA**
Australia	1.7630	2.4491	1.3151	1.4997	1.4056	0.8326	1.1602
Canada	0.8888	1.3001	0.9715	1.2835	0.8884	0.7481	0.7297
Denmark	1.0816	1.0647	1.5559	1.5659	1.4253	0.9687	0.8585
England	2.0634	1.3287	0.7883	1.3095	0.9811	0.9339	1.1254
Finland	2.6579	2.6568	1.2331	1.5355	1.3272	1.8616	1.7372
France	2.4310	2.8095	0.8343	0.9658	1.4220	1.5060	1.5236
Iceland	1.0756	1.0646	2.0222	1.3571	1.4979	1.6221	1.0992
Italy	2.2758	2.1431	1.3270	1.5572	1.6715	1.6645	1.8509
Netherlands	0.9432	0.9121	1.3293	1.5016	1.1315	1.3367	0.5153
Norway	1.4192	1.1604	1.3888	1.9328	1.5347	1.6695	1.2007
Scotland	2.0713	2.0972	1.5541	1.5741	1.7622	1.2111	1.2023
Spain	1.0824	0.5506	0.6500	0.7439	0.3435	1.0242	0.6774
Sweden	1.6555	1.5401	1.3986	1.5825	1.4764	1.5196	1.4823
Switzerland	1.5106	1.9238	1.1533	1.2370	1.0544	0.7689	0.9257
Mean	1.6371	1.6429	1.2515	1.4033	1.2801	1.2620	1.1492
Mean(w)	1.8055	1.7325	1.0166	1.2598	1.1710	1.2176	1.2118

Figures 2a and 2b show the MAFEs of the ten-step-ahead point forecasts for different methods. The MAFEs are averaged over countries and years in the forecasting period for the female and male life expectancies. Large errors occur between ages 0 and 60 for most of the methods, and errors from the LCnone, HU, HUrob and RW methods are particularly large at these ages. At older ages[11], the MAFEs of all methods decrease gradually. For ages above 100, there is a sharp increase in MAFEs, which reflects the difficulty in forecasting the mortality rates at old ages because of excessive variability.

Figure 2: MAFEs and MFEs for the ten-step-ahead point forecasts of the life expectancies by sex and method

(a) MAFEs averaged over years in the forecasting period and countries

(b) MAFEs averaged over years in the forecasting period and countries



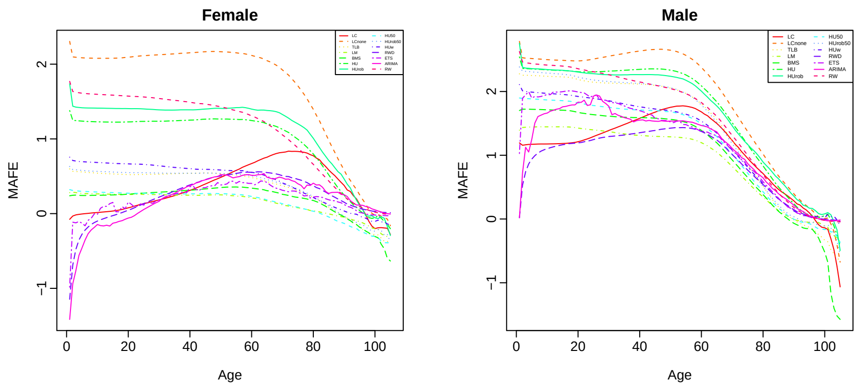
Figures 1c and 1d show the MFEs of the ten-step-ahead point forecasts for different methods. The MFEs are averaged over countries and years in the forecasting period for the female and male life expectancies. In the female data, apart from the LC, RW and two univariate time-series methods, all other methods underestimate life expectancies from ages 0 to 65, and the degree of underestimation becomes smaller and smaller as age increases. The LM and RWD methods perform similarly, but they are both more superior to the LC method.

[11] between 66 and 100

Figure 2: (Continued)

(c) MFEs averaged over years in the forecasting period and countries

(d) MFEs averaged over years in the forecasting period and countries



7. Comparisons of the interval forecasts

As was emphasized by Chatfield (1993, 2000), it is important to provide interval forecasts as well as point forecasts, so as to (1) assess future uncertainty level, (2) enable different strategies to be planned for the range of possible outcomes indicated by the interval forecasts, (3) compare forecasts from different methods more thoroughly, and (4) explore different scenarios based on different assumptions.

In recent years, many authors have provided interval forecasts to measure the uncertainty associated with the point forecasts; see for example, Lutz and Scherbov (1998) for Austria, Alho (1998) for Finland, Keilman and Pham (2000) for Norway, and Tayman, Smith, and Lin (2007) for the United States. These methods have been motivated by earlier work on stochastic forecasts, by for instance, Lee (1974), Stoto (1983), Alho and Spencer (1985), Alho (1990) and Lee and Tuljapurkar (1994). The current paper is a contribution to the literature in this area.

To obtain prediction intervals for the future life expectancies, we utilize a simulation approach of Shang, Booth, and Hyndman (2011). In short, the simulated forecasts of log mortality rates were obtained by adding disturbances to the forecast principal component scores, which were then multiplied by the fixed functional principal components. Life expectancies were then calculated for each set of the simulated log mortality rates (see

Rowland 2003, Chapter 8 for detail). Prediction interval was customarily constructed by 80% percentile of the simulated sets of the life expectancies.

Table 10: Coverage probability deviances of the one-step-ahead forecast female life expectancies by method and country. Minus sign indicates that the empirical coverage probability is less than the nominal coverage probability

Country	LC	LCnone	TLB	LM	BMS	RWD	RW
Australia	0.2500(−)	0.7929(−)	0.4048(−)	0.1243(−)	0.0324(−)	0.1790(+)	0.1162(+)
Canada	0.2624(−)	0.7881(−)	0.4800(−)	0.0714(−)	0.3081(−)	0.1929(+)	0.1648(+)
Denmark	0.6519(−)	0.7595(−)	0.2648(−)	0.2519(−)	0.5181(−)	0.2000(+)	0.1995(+)
England	0.1086(−)	0.7929(−)	0.5086(−)	0.1805(−)	0.1957(−)	0.1890(+)	0.1805(+)
Finland	0.5433(−)	0.7881(−)	0.6390(−)	0.0443(−)	0.1267(−)	0.1819(+)	0.1667(+)
France	0.5824(−)	0.8000(−)	0.3167(−)	0.0824(−)	0.1333(−)	0.1752(+)	0.1729(+)
Iceland	0.0629(−)	0.6476(−)	0.6776(−)	0.5843(−)	0.7248(−)	0.2000(+)	0.2000(+)
Italy	0.4319(−)	0.7943(−)	0.7648(−)	0.1300(−)	0.1243(−)	0.1757(+)	0.1729(+)
Netherlands	0.3414(−)	0.7514(−)	0.3938(−)	0.1457(−)	0.2595(−)	0.2000(+)	0.2000(+)
Norway	0.6881(−)	0.7890(−)	0.3481(−)	0.2505(−)	0.2924(−)	0.1938(+)	0.1900(+)
Scotland	0.6238(−)	0.7890(−)	0.4395(−)	0.1576(−)	0.1519(−)	0.1648(+)	0.1643(+)
Spain	0.5390(−)	0.7981(−)	0.7981(−)	0.2362(−)	0.3700(−)	0.1971(+)	0.1924(+)
Sweden	0.5667(−)	0.7824(−)	0.2471(−)	0.1810(−)	0.2419(−)	0.2000(+)	0.2000(+)
Switzerland	0.4100(−)	0.7795(−)	0.0919(−)	0.1095(−)	0.1648(−)	0.1948(+)	0.1810(+)
Mean	0.4330(−)	0.7752(−)	0.4553(−)	0.1821(−)	0.2603(−)	0.1889(+)	0.1787(+)
Mean(w)	0.3983(−)	0.7918(−)	0.5205(−)	0.1432(−)	0.2042(−)	0.1858(+)	0.1754(+)
Country	**HU**	**HUrob**	**HU50**	**HUrob50**	**HUw**	**ETS**	**ARIMA**
Australia	0.2000(+)	0.0119(−)	0.1981(+)	0.0152(+)	0.2000(+)	0.1867(+)	0.1767(+)
Canada	0.2000(+)	0.2000(+)	0.2000(+)	0.1990(+)	0.2000(+)	0.1667(+)	0.1433(+)
Denmark	0.2000(+)	0.1343(+)	0.1890(+)	0.1819(+)	0.2000(+)	0.1757(+)	0.1857(+)
England	0.1981(+)	0.1538(+)	0.1362(+)	0.0548(−)	0.2000(+)	0.1490(+)	0.1562(+)
Finland	0.2000(+)	0.2614(−)	0.1557(+)	0.0962(+)	0.2000(+)	0.1795(+)	0.1681(+)
France	0.0395(+)	0.1143(−)	0.1476(+)	0.1343(+)	0.2000(+)	0.1776(+)	0.1710(+)
Iceland	0.2000(+)	0.2000(+)	0.2000(+)	0.2000(+)	0.2000(+)	0.2000(+)	0.2000(+)
Italy	0.1438(+)	0.0590(+)	0.1500(+)	0.0700(+)	0.2000(+)	0.1795(+)	0.1743(+)
Netherlands	0.2000(+)	0.2000(+)	0.1110(+)	0.1948(+)	0.2000(+)	0.1995(+)	0.1971(+)
Norway	0.0671(−)	0.0919(+)	0.1667(+)	0.1038(+)	0.2000(+)	0.1548(+)	0.1471(+)
Scotland	0.1129(+)	0.3705(−)	0.1200(+)	0.1643(+)	0.2000(+)	0.1714(+)	0.1886(+)
Spain	0.1976(+)	0.2000(+)	0.1719(+)	0.0300(−)	0.2000(+)	0.1952(+)	0.1938(+)
Sweden	0.2000(+)	0.5748(−)	0.1976(+)	0.1862(+)	0.2000(+)	0.2000(+)	0.2000(+)
Switzerland	0.2000(+)	0.1576(+)	0.2000(+)	0.1767(+)	0.2000(+)	0.1548(+)	0.1790(+)
Mean	0.1685(+)	0.1950(+)	0.1674(+)	0.1291(+)	0.2000(+)	0.1779(+)	0.1772(+)
Mean(w)	0.1561(+)	0.1590(+)	0.1490(+)	0.0991(+)	0.2001(+)	0.1755(+)	0.1721(+)

Table 11: Coverage probability deviance of the one-step-ahead forecast male life expectancies by method and country. Minus sign indicates that the empirical coverage probability is less than the nominal coverage probability

Country	LC	LCnone	TLB	LM	BMS	RWD	RW
Australia	0.4329(–)	0.7929(–)	0.6867(–)	0.2305(–)	0.1886(–)	0.1457(+)	0.0824(+)
Canada	0.6452(–)	0.7805(–)	0.7629(–)	0.4471(–)	0.6957(–)	0.1267(+)	0.0533(+)
Denmark	0.4886(–)	0.6138(–)	0.6433(–)	0.4943(–)	0.6471(–)	0.2000(+)	0.1995(+)
England	0.4219(–)	0.8000(–)	0.7400(–)	0.2271(–)	0.2610(–)	0.1724(+)	0.1348(+)
Finland	0.3605(–)	0.7686(–)	0.5333(–)	0.0767(–)	0.2171(–)	0.1910(+)	0.1862(+)
France	0.5086(–)	0.7933(–)	0.3367(–)	0.0714(–)	0.3681(–)	0.1824(+)	0.1800(+)
Iceland	0.0981(–)	0.2081(–)	0.6410(–)	0.3011(–)	0.7257(–)	0.2000(+)	0.2000(+)
Italy	0.4419(–)	0.7895(–)	0.7552(–)	0.2876(–)	0.4752(–)	0.1857(+)	0.1833(+)
Netherlands	0.0690(–)	0.2676(–)	0.7729(–)	0.5052(–)	0.7524(–)	0.1981(+)	0.1981(+)
Norway	0.4738(–)	0.6490(–)	0.7771(–)	0.5357(–)	0.5419(–)	0.1467(+)	0.1395(+)
Scotland	0.7248(–)	0.7995(–)	0.5852(–)	0.3195(–)	0.3119(–)	0.1562(+)	0.1452(+)
Spain	0.4771(–)	0.7500(–)	0.6086(–)	0.1767(–)	0.4595(–)	0.1943(+)	0.1919(+)
Sweden	0.1571(–)	0.3500(–)	0.7329(–)	0.3062(–)	0.4600(–)	0.2000(+)	0.2000(+)
Switzerland	0.3057(–)	0.7248(–)	0.3329(–)	0.2367(–)	0.3105(–)	0.1714(+)	0.1671(+)
Mean	0.4004(–)	0.6491(–)	0.6363(–)	0.3011(–)	0.4582(–)	0.1765(+)	0.1615(+)
Mean(w)	0.4494(–)	0.7412(–)	0.6357(–)	0.2485(–)	0.4250(–)	0.1754(+)	0.1557(+)

Country	HU	HUrob	HU50	HUrob50	HUw	ETS	ARIMA
Australia	0.1286(+)	0.0881(–)	0.1133(+)	0.1581(–)	0.2000(+)	0.1462(+)	0.1110(+)
Canada	0.0429(–)	0.3033(–)	0.0295(–)	0.2205(–)	0.2000(+)	0.1200(+)	0.1276(+)
Denmark	0.0967(+)	0.0638(+)	0.0300(–)	0.0757(–)	0.2000(+)	0.1029(+)	0.1162(+)
England	0.0786(–)	0.2043(–)	0.0724(–)	0.3738(–)	0.2000(+)	0.1352(+)	0.0814(+)
Finland	0.0700(+)	0.2781(–)	0.1386(+)	0.0871(–)	0.2000(+)	0.1738(+)	0.1695(+)
France	0.1881(–)	0.3529(–)	0.0733(+)	0.0129(–)	0.2000(+)	0.1805(+)	0.1738(+)
Iceland	0.1995(+)	0.2000(+)	0.2000(+)	0.2000(+)	0.2000(+)	0.1967(+)	0.2000(+)
Italy	0.0205(+)	0.2729(–)	0.0305(+)	0.1176(–)	0.2000(+)	0.1848(+)	0.1629(+)
Netherlands	0.0662(+)	0.2000(+)	0.2305(–)	0.2657(–)	0.1971(+)	0.1800(+)	0.1810(+)
Norway	0.1171(+)	0.1205(+)	0.0238(+)	0.3157(–)	0.2000(+)	0.0929(+)	0.0729(+)
Scotland	0.1576(–)	0.3081(–)	0.1686(–)	0.1867(–)	0.2000(+)	0.1176(+)	0.0467(+)
Spain	0.1905(+)	0.2000(+)	0.0495(+)	0.1157(+)	0.2000(+)	0.1914(+)	0.1910(+)
Sweden	0.2000(+)	0.0281(–)	0.1495(–)	0.3300(–)	0.1990(+)	0.1986(+)	0.1900(+)
Switzerland	0.1119(+)	0.2738(–)	0.1776(+)	0.1695(+)	0.2000(+)	0.1419(+)	0.1257(+)
Mean	0.1191(+)	0.2067(-)	0.1062(+)	0.1878(-)	0.1997(+)	0.1545(+)	0.1393(+)
Mean(w)	0.1085(+)	0.2411(-)	0.0740(+)	0.1748(-)	0.1998(+)	0.1623(+)	0.1442(+)

The average coverage probability deviances for the one-step-ahead forecasts of the female and male life expectancies are shown in Tables 10 and 11, respectively. The HUrob50 method performs the best with the minimum average coverage probability deviance, followed by the HU method. These results show that the prediction intervals of the LC methods are too narrow, which lead to underestimate[12] the coverage probability. In contrast, the HUw, RW and RWD methods tend to overestimate the coverage probability, since their prediction intervals are too wide in both female and male data. With the exception of the LM method, the HU methods generally provide more accurate interval forecasts than the LC methods.

For the one-step-ahead forecasts, Tables 12 and 13 show that the HUw method has a larger half-width of the prediction interval than the others. The better coverage probability of the HUw method may due to a larger half-width of the prediction interval. By contrast, the RW, RWD, ETS and ARIMA methods have slightly large half-widths than the LC methods, but with much smaller coverage probability deviance. This indicates that the simple method tends to work quite well for short-term interval forecasts.

The average coverage probability deviances for the ten-step-ahead forecasts of the female and male life expectancies are shown in Tables 14 and 15, respectively. In the female data, the ARIMA model performs the best, with the minimum overall coverage probability deviance. Moreover, the HUw and RWD methods tend to overestimate the coverage probability deviance for each country, since their prediction intervals are too wide. In the male data, the RWD model performs the best, with the minimum overall coverage probability deviance. In addition, the LC methods tend to underestimate the coverage probability deviance for each country except Iceland, since their prediction intervals are too narrow (see also Lee and Carter 1992).

For the ten-step-ahead forecasts, Tables 16 and 17 show that the HUw method has a larger half-width than the others for both females and males. Among all the methods, the LM method produces the narrowest half-widths for both female and male data. Such a narrower half-width of prediction interval is informative, but could be at the cost of larger coverage probability deviance, especially for the male data. Again, the simple methods like the ARIMA and RWD methods tend to perform well for long-term interval forecasts.

[12] nominal coverage probability $>$ empirical coverage probability

Table 12: Averaged half-width of the one-step-ahead forecast female life expectancies by method and country

Country	LC	LCnone	TLB	LM	BMS	RWD	RW
Australia	0.25	0.16	0.21	0.23	0.26	0.40	0.40
Canada	0.14	0.14	0.15	0.11	0.09	0.30	0.30
Denmark	0.23	0.22	0.51	0.20	0.16	0.80	0.80
England	0.33	0.18	0.18	0.18	0.20	0.53	0.52
Finland	0.23	0.24	0.33	0.21	0.22	0.73	0.72
France	0.31	0.35	0.24	0.19	0.24	1.02	1.01
Iceland	0.85	0.98	0.07	0.05	0.20	3.49	3.48
Italy	0.31	0.39	0.19	0.17	0.20	1.16	1.16
Netherlands	0.31	0.35	0.27	0.17	0.18	0.92	0.91
Norway	0.18	0.18	0.29	0.16	0.18	0.68	0.68
Scotland	0.23	0.19	0.43	0.24	0.29	0.70	0.70
Spain	0.23	0.30	0.19	0.18	0.16	1.08	1.07
Sweden	0.32	0.40	0.25	0.16	0.20	1.35	1.35
Switzerland	0.28	0.34	0.37	0.18	0.18	0.66	0.66
Mean	0.30	0.32	0.26	0.17	0.20	0.99	0.98
Mean(w)	0.28	0.28	0.22	0.18	0.19	0.84	0.83
Australia	0.61	0.69	0.53	0.56	3.65	0.36	0.36
Canada	0.56	0.60	0.46	0.51	3.96	0.27	0.26
Denmark	1.11	1.38	0.96	1.01	4.44	0.67	0.67
England	0.58	0.62	0.39	0.43	2.40	0.41	0.41
Finland	1.29	1.52	0.97	1.00	7.62	0.67	0.65
France	0.79	0.87	0.45	0.50	4.17	0.92	0.90
Iceland	3.85	4.31	2.74	2.89	10.26	2.73	2.68
Italy	0.96	1.11	0.50	0.52	5.29	1.00	0.99
Netherlands	1.11	1.22	0.55	0.57	6.28	0.83	0.80
Norway	0.95	1.27	0.68	0.74	4.04	0.63	0.61
Scotland	0.87	1.04	0.79	0.87	6.16	0.57	0.58
Spain	0.89	1.00	0.53	0.60	0.96	0.95	0.95
Sweden	1.26	1.56	0.60	0.65	8.88	1.30	1.21
Switzerland	1.09	1.26	0.80	0.85	7.31	0.57	0.56
Mean	1.14	1.32	0.78	0.84	5.39	0.85	0.83
Mean(w)	0.82	0.92	0.51	0.55	3.96	0.73	0.72

Table 13: Averaged half-width of the one-step-ahead forecast male life expectancies by method and country

Country	LC	LCnone	TLB	LM	BMS	RWD	RW
Australia	0.24	0.13	0.20	0.21	0.21	0.40	0.40
Canada	0.11	0.10	0.11	0.08	0.06	0.27	0.27
Denmark	0.22	0.22	0.11	0.09	0.07	0.74	0.74
England	0.24	0.15	0.15	0.15	0.16	0.48	0.48
Finland	0.35	0.35	0.35	0.22	0.22	1.82	1.81
France	0.41	0.43	0.19	0.18	0.17	1.64	1.63
Iceland	0.82	0.97	1.16	1.16	0.32	3.33	3.32
Italy	0.27	0.33	0.12	0.13	0.13	1.27	1.27
Netherlands	0.38	0.38	0.07	0.09	0.07	1.15	1.14
Norway	0.17	0.17	0.08	0.07	0.09	0.70	0.70
Scotland	0.15	0.13	0.40	0.15	0.23	0.64	0.64
Spain	0.33	0.37	0.20	0.19	0.13	1.12	1.12
Sweden	0.35	0.42	0.17	0.12	0.14	1.42	1.42
Switzerland	0.31	0.40	0.29	0.16	0.17	0.77	0.76
Mean	0.31	0.32	0.26	0.21	0.16	1.12	1.12
Mean(w)	0.29	0.29	0.16	0.15	0.14	0.99	0.99
Australia	0.64	0.77	0.59	0.64	2.15	0.36	0.36
Canada	0.54	0.65	0.47	0.54	2.13	0.25	0.25
Denmark	1.04	1.22	0.83	0.85	2.29	0.63	0.62
England	0.63	0.68	0.42	0.49	2.39	0.42	0.42
Finland	1.46	2.05	1.12	1.30	5.33	1.67	1.64
France	1.05	1.13	0.55	0.60	3.69	1.57	1.52
Iceland	4.37	4.26	2.96	3.16	9.99	2.66	2.63
Italy	1.10	1.30	0.60	0.67	3.55	1.24	1.14
Netherlands	1.06	1.22	0.56	0.58	4.00	1.04	1.01
Norway	0.97	1.32	0.63	0.82	2.56	0.67	0.65
Scotland	0.90	1.06	0.87	0.89	3.80	0.58	0.58
Spain	1.12	1.23	0.64	0.78	4.73	1.06	1.06
Sweden	1.28	1.35	0.62	0.70	6.60	1.37	1.26
Switzerland	1.12	1.33	0.80	0.84	4.59	0.62	0.63
Mean	1.23	1.40	0.83	0.92	4.13	1.01	0.98
Mean(w)	0.93	1.06	0.57	0.65	3.43	0.93	0.90

Table 14: Coverage probability deviance of the ten-step-ahead forecast female life expectancies by method and country. Minus sign indicates that the empirical coverage probability is less than the nominal coverage probability

Country	LC	LCnone	TLB	LM	BMS	RWD	RW
Australia	0.3775(−)	0.7818(−)	0.1567(−)	0.0346(+)	0.0771(+)	0.1931(+)	0.5264(−)
Canada	0.0251(−)	0.6952(−)	0.2398(−)	0.3541(−)	0.2225(−)	0.1991(+)	0.1229(−)
Denmark	0.2026(−)	0.7126(−)	0.1853(+)	0.0606(−)	0.2571(−)	0.2000(+)	0.2000(+)
England	0.1723(+)	0.7706(−)	0.0398(−)	0.0667(+)	0.0857(+)	0.2000(+)	0.1498(+)
Finland	0.3654(−)	0.7645(−)	0.0251(−)	0.0130(+)	0.0519(−)	0.2000(+)	0.0918(+)
France	0.1879(−)	0.8000(−)	0.0900(+)	0.0961(+)	0.1056(+)	0.2000(+)	0.2000(+)
Iceland	0.1126(+)	0.0519(+)	0.7463(−)	0.7463(−)	0.7610(−)	0.2000(+)	0.2000(+)
Italy	0.1706(−)	0.7784(−)	0.7489(−)	0.0035(+)	0.1143(+)	0.2000(+)	0.1775(+)
Netherlands	0.0684(+)	0.2684(−)	0.1152(−)	0.3749(−)	0.2926(−)	0.2000(+)	0.2000(+)
Norway	0.2597(−)	0.7697(−)	0.0944(+)	0.0433(+)	0.0156(−)	0.2000(+)	0.2000(+)
Scotland	0.2390(−)	0.8000(−)	0.1368(+)	0.1238(+)	0.1160(+)	0.2000(+)	0.2000(+)
Spain	0.2052(−)	0.7905(−)	0.7662(−)	0.0095(−)	0.1818(−)	0.2000(+)	0.2000(+)
Sweden	0.1108(−)	0.6797(−)	0.0632(+)	0.0987(+)	0.0545(+)	0.2000(+)	0.2000(+)
Switzerland	0.0173(−)	0.7368(−)	0.1342(+)	0.0649(+)	0.0684(−)	0.2000(+)	0.1359(+)
Mean	0.1796(−)	0.6714(−)	0.2530(−)	0.1493(−)	0.1717(−)	0.1994(+)	0.2003(+)
Mean(w)	0.1732(−)	0.7453(−)	0.3105(−)	0.0953(−)	0.1313(−)	0.1996(+)	0.1967(+)
Country	**HU**	**HUrob**	**HU50**	**HUrob50**	**HUw**	**ETS**	**ARIMA**
Australia	0.2026(−)	0.3645(−)	0.1498(−)	0.1429(−)	0.2000(+)	0.1307(+)	0.1931(−)
Canada	0.1784(+)	0.2000(+)	0.1290(−)	0.0147(+)	0.2000(+)	0.0701(−)	0.0000(−)
Denmark	0.1896(+)	0.0779(−)	0.1680(+)	0.1931(+)	0.2000(+)	0.0139(−)	0.0771(+)
England	0.2398(−)	0.1887(+)	0.1714(+)	0.2000(+)	0.2000(+)	0.0260(−)	0.0052(+)
Finland	0.3567(−)	0.5074(−)	0.1524(+)	0.0771(+)	0.2000(+)	0.1004(+)	0.0459(−)
France	0.6589(−)	0.3039(−)	0.0961(+)	0.1939(+)	0.2000(+)	0.1887(+)	0.1056(+)
Iceland	0.2000(+)	0.2000(+)	0.2000(+)	0.2000(+)	0.2000(+)	0.2000(+)	0.2000(+)
Italy	0.5584(−)	0.1351(−)	0.0710(−)	0.0502(−)	0.2000(+)	0.1238(+)	0.0675(−)
Netherlands	0.2000(+)	0.1723(+)	0.3455(−)	0.2641(−)	0.2000(+)	0.1931(+)	0.1489(+)
Norway	0.5584(−)	0.0727(+)	0.0675(+)	0.3177(−)	0.2000(+)	0.0338(+)	0.0675(−)
Scotland	0.5905(−)	0.5100(−)	0.1680(+)	0.1117(+)	0.2000(+)	0.0909(+)	0.0745(−)
Spain	0.0623(−)	0.2000(+)	0.1861(+)	0.1671(+)	0.2000(+)	0.1645(+)	0.1100(+)
Sweden	0.1576(+)	0.6563(−)	0.1835(+)	0.1957(+)	0.2000(+)	0.1913(+)	0.1931(+)
Switzerland	0.1463(+)	0.0866(+)	0.2000(+)	0.1827(+)	0.2000(+)	0.0597(−)	0.1048(+)
Mean	0.3071(−)	0.2625(−)	0.1635(+)	0.1651(+)	0.2000(+)	0.1134(+)	0.0995(+)
Mean(w)	0.3482(−)	0.2304(−)	0.1436(+)	0.1472(+)	0.2001(+)	0.1188(+)	0.0785(+)

Table 15: Coverage probability deviance of the ten-step-ahead forecast male life expectancies by method and country. Minus sign indicates that the empirical coverage probability is less than the nominal coverage probability

Country	LC	LCnone	TLB	LM	BMS	RWD	RW
Australia	0.7481(–)	0.7818(–)	0.7662(–)	0.7515(–)	0.2952(–)	0.3879(–)	0.5619(–)
Canada	0.7385(–)	0.7749(–)	0.7532(–)	0.7584(–)	0.7775(–)	0.3801(–)	0.5619(–)
Denmark	0.5074(–)	0.4615(–)	0.5229(–)	0.5229(–)	0.6294(–)	0.1879(+)	0.1195(+)
England	0.5887(–)	0.8000(–)	0.6892(–)	0.7039(–)	0.7203(–)	0.0762(+)	0.4078(–)
Finland	0.4407(–)	0.7169(–)	0.3446(–)	0.3152(–)	0.4208(–)	0.1758(+)	0.0571(+)
France	0.3818(–)	0.7844(–)	0.4433(–)	0.0961(–)	0.5264(–)	0.2000(+)	0.1662(+)
Iceland	0.0623(+)	0.0926(+)	0.0528(–)	0.5510(–)	0.6182(–)	0.2000(+)	0.2000(+)
Italy	0.6104(–)	0.7714(–)	0.7169(–)	0.6848(–)	0.7108(–)	0.2000(+)	0.1048(+)
Netherlands	0.0771(–)	0.0485(–)	0.7550(–)	0.6381(–)	0.7654(–)	0.2000(+)	0.2000(+)
Norway	0.7489(–)	0.7472(–)	0.7403(–)	0.6926(–)	0.7550(–)	0.0139(+)	0.1377(–)
Scotland	0.5515(–)	0.7991(–)	0.3429(–)	0.6727(–)	0.3030(–)	0.1030(+)	0.0372(–)
Spain	0.1887(–)	0.5662(–)	0.3169(–)	0.0052(–)	0.3065(–)	0.2000(+)	0.2000(+)
Sweden	0.2874(–)	0.2026(–)	0.6589(–)	0.6675(–)	0.6918(–)	0.2000(+)	0.2000(+)
Switzerland	0.4848(–)	0.6494(–)	0.3420(–)	0.6537(–)	0.5100(–)	0.1273(+)	0.1801(–)
Mean	0.4583(–)	0.5855(–)	0.5318(–)	0.5510(–)	0.5736(–)	0.1894(+)	0.2239(–)
Mean(w)	0.4877(–)	0.6919(–)	0.5917(–)	0.4886(–)	0.5908(–)	0.2010(+)	0.2664(–)

Country	HU	HUrob	HU50	HUrob50	HUw	ETS	ARIMA
Australia	0.5870(–)	0.5342(–)	0.6061(–)	0.5870(–)	0.2762(–)	0.1022(–)	0.4303(–)
Canada	0.5446(–)	0.5463(–)	0.5974(–)	0.5913(–)	0.1593(+)	0.2918(–)	0.2996(–)
Denmark	0.2788(–)	0.2485(–)	0.5238(–)	0.5195(–)	0.0156(+)	0.3065(–)	0.2987(–)
England	0.6372(–)	0.4970(–)	0.5861(–)	0.6061(–)	0.0736(–)	0.3965(–)	0.4909(–)
Finland	0.5385(–)	0.4442(–)	0.4874(–)	0.4485(–)	0.1957(+)	0.2199(–)	0.2104(–)
France	0.6390(–)	0.6511(–)	0.4182(–)	0.4147(–)	0.2000(+)	0.1273(–)	0.2277(–)
Iceland	0.2000(+)	0.2000(+)	0.2000(+)	0.1654(+)	0.2000(+)	0.1991(+)	0.2000(+)
Italy	0.5680(–)	0.3879(–)	0.3905(–)	0.5957(–)	0.2000(+)	0.1801(–)	0.2996(–)
Netherlands	0.0814(–)	0.0294(–)	0.5896(–)	0.5558(–)	0.1775(+)	0.0641(–)	0.0190(–)
Norway	0.4502(–)	0.1541(–)	0.4234(–)	0.5688(–)	0.0424(–)	0.3602(–)	0.3489(–)
Scotland	0.6571(–)	0.6641(–)	0.6035(–)	0.5853(–)	0.2000(+)	0.3368(–)	0.3351(–)
Spain	0.0571(–)	0.2000(+)	0.2000(+)	0.1333(+)	0.2000(+)	0.1905(+)	0.1835(+)
Sweden	0.4182(–)	0.3411(–)	0.5905(–)	0.5792(–)	0.1983(+)	0.0450(–)	0.2320(–)
Switzerland	0.5437(–)	0.5619(–)	0.5342(–)	0.5316(–)	0.2000(+)	0.3472(–)	0.4225(–)
Mean	0.4429(–)	0.3900(–)	0.4822(–)	0.4916(–)	0.1670(+)	0.2262(–)	0.2856(–)
Mean(w)	0.4893(–)	0.4399(–)	0.4657(–)	0.4931(–)	0.1713(+)	0.2193(–)	0.3007(–)

Table 16: Averaged half-width of the ten-step-ahead forecast female life expectancies by method and country

Country	LC	LCnone	TLB	LM	BMS	RWD	RW
Australia	0.84	0.51	0.74	0.82	1.04	1.38	1.30
Canada	0.46	0.46	0.54	0.38	0.36	1.05	0.98
Denmark	0.72	0.69	2.20	0.73	0.59	2.64	2.56
England	1.07	0.58	0.61	0.64	0.74	1.81	1.69
Finland	0.73	0.73	1.11	0.73	0.82	2.43	2.33
France	0.98	1.12	0.81	0.65	0.87	3.34	3.25
Iceland	2.80	3.16	1.51	0.24	1.10	11.50	11.14
Italy	1.00	1.25	0.63	0.58	0.70	3.88	3.72
Netherlands	0.98	1.13	0.95	0.60	0.78	3.03	2.93
Norway	0.56	0.56	0.96	0.57	0.60	2.25	2.18
Scotland	0.73	0.56	1.56	0.89	1.08	2.32	2.24
Spain	0.70	0.94	0.62	0.57	0.52	3.68	3.48
Sweden	1.02	1.27	0.91	0.57	0.66	4.39	4.30
Switzerland	0.90	1.10	1.29	0.60	0.69	2.21	2.12
Mean	0.96	1.00	1.03	0.61	0.75	3.28	3.16
Mean(w)	0.89	0.89	0.75	0.61	0.71	2.80	2.68

Country	HU	HUrob	HU50	HUrob50	HUw	ETS	ARIMA
Australia	1.13	1.14	0.74	0.78	1.73	0.82	0.74
Canada	0.93	1.05	0.63	0.70	2.89	0.63	0.60
Denmark	1.37	1.52	1.34	1.17	7.41	1.06	0.95
England	0.99	1.11	1.13	1.06	2.37	0.73	0.60
Finland	1.63	1.88	1.44	1.39	7.50	1.42	1.29
France	1.25	1.36	0.75	0.88	4.40	1.57	1.40
Iceland	4.46	4.90	3.58	3.20	6.27	3.36	3.58
Italy	1.31	1.57	0.81	0.80	7.36	1.55	1.37
Netherlands	1.62	1.75	0.68	0.71	7.41	1.58	1.29
Norway	1.26	1.81	0.80	0.84	4.17	1.14	0.98
Scotland	1.06	1.25	1.24	1.56	4.39	0.90	0.92
Spain	1.25	1.43	1.42	1.46	5.30	1.65	1.65
Sweden	1.63	1.86	0.69	0.77	8.80	2.64	1.63
Switzerland	1.33	1.55	0.92	1.09	7.70	0.88	0.85
Mean	1.52	1.73	1.16	1.17	5.55	1.42	1.27
Mean(w)	1.21	1.37	0.94	0.97	4.80	1.28	1.13

Table 17: Averaged half-width of the ten-step-ahead forecast male life expectancies by method and country

Country	LC	LCnone	TLB	LM	BMS	RWD	RW
Australia	0.81	0.40	0.71	0.73	0.81	1.36	1.27
Canada	0.33	0.29	0.35	0.27	0.22	0.91	0.86
Denmark	0.73	0.71	0.25	0.31	0.07	2.44	2.36
England	0.77	0.44	0.51	0.52	0.56	1.66	1.55
Finland	1.11	1.10	1.26	0.76	1.02	6.07	5.82
France	1.29	1.37	0.67	0.63	0.62	5.37	5.22
Iceland	2.77	3.18	3.18	3.18	3.18	10.99	10.65
Italy	0.88	1.05	0.34	0.43	0.41	4.25	4.08
Netherlands	1.27	1.26	0.18	0.27	0.14	3.80	3.67
Norway	0.58	0.54	0.09	0.10	0.14	2.30	2.23
Scotland	0.46	0.38	1.47	0.49	2.66	2.12	2.04
Spain	1.11	1.24	0.71	0.65	0.45	3.83	3.62
Sweden	1.16	1.38	0.53	0.42	0.49	4.63	4.54
Switzerland	1.00	1.33	1.00	0.53	0.60	2.56	2.46
Mean	1.02	1.05	0.80	0.66	0.81	3.73	3.60
Mean(w)	0.93	0.91	0.55	0.51	0.52	3.31	3.17
Country	**HU**	**HUrob**	**HU50**	**HUrob50**	**HUw**	**ETS**	**ARIMA**
Australia	1.26	1.36	0.85	0.93	1.96	1.07	0.92
Canada	0.69	0.82	0.64	0.67	1.83	0.72	0.73
Denmark	1.18	1.33	0.92	0.94	2.31	0.99	0.87
England	0.94	1.19	0.59	0.66	2.00	0.82	0.78
Finland	1.94	2.73	1.34	1.90	4.35	3.24	2.70
France	1.54	1.75	0.72	0.77	5.05	3.86	2.53
Iceland	6.36	6.37	6.36	6.37	9.69	3.53	3.73
Italy	1.78	2.12	0.92	0.79	4.46	3.58	2.56
Netherlands	1.28	1.55	0.73	0.77	2.57	1.94	1.41
Norway	1.32	1.96	0.77	0.85	2.37	1.53	1.17
Scotland	1.03	1.16	0.93	1.05	3.35	1.21	1.26
Spain	1.69	1.81	1.61	2.17	5.92	2.51	2.49
Sweden	1.66	1.53	0.70	0.83	2.48	2.90	1.68
Switzerland	1.24	1.38	0.87	1.01	4.82	0.80	0.80
Mean	1.71	1.93	1.28	1.41	3.80	2.05	1.69
Mean(w)	1.37	1.59	0.88	0.98	3.68	2.27	1.75

8. Result of the model averaging approach

In the demographic literature, methods have been developed to combine forecasts from different approaches. For example, the forecasts from time-series models have been combined with expert opinions (Lee and Tuljapurkar 1994) or combined with target levels and age distribution of fertility and mortality (Lutz, Sanderson, and Scherbov 2001). In Section 3., we introduce a model averaging approach to combine point forecasts, in which the optimal weights are determined by past forecast error in the validation set or BMA or equal weighting. Table 18 presents the MAFEs of the one-step-ahead combined point forecast accuracy. As expected, the equal weighting performs poorly, whereas the combination of two best models weighted by their past forecast accuracy performs the best, followed closely by the weighting based on the notion of BMA.

Table 18: Point forecast accuracy of the combined female and male life expectancies by country, as measured by the MAFEs for the one-step-ahead forecasts

	Equal		**Frequentist$_{\text{all}}$**		**Frequentist$_2$**		**BMA**	
Country	Female	Male	Female	Male	Female	Male	Female	Male
Australia	0.2314	0.3762	0.1912	0.2967	0.1588	0.1796	0.1489	0.1846
Canada	0.0934	0.3231	0.0817	0.2262	0.0773	0.1211	0.0777	0.1224
Denmark	0.1708	0.3132	0.1639	0.3147	0.1838	0.1613	0.1808	0.1760
England	0.2112	0.3674	0.1850	0.2914	0.1362	0.1377	0.2052	0.1636
Finland	0.3965	0.5837	0.2291	0.2824	0.1473	0.1511	0.1486	0.1661
France	0.2666	0.4446	0.1613	0.2880	0.1327	0.1187	0.1324	0.1428
Iceland	0.5382	0.8412	0.4320	0.5609	0.3802	0.4058	0.3829	0.4002
Italy	0.2891	0.4687	0.2312	0.4065	0.1420	0.1330	0.1411	0.1573
Netherlands	0.1482	0.3039	0.1309	0.2568	0.1184	0.1463	0.1181	0.1799
Norway	0.2713	0.3799	0.1943	0.3253	0.1533	0.1887	0.1551	0.2176
Scotland	0.3527	0.4772	0.2883	0.4176	0.1730	0.1890	0.1762	0.2082
Spain	0.2452	0.2462	0.1803	0.1989	0.1531	0.1467	0.1421	0.1400
Sweden	0.2486	0.3550	0.1393	0.3124	0.1309	0.1381	0.1314	0.1513
Switzerland	0.2131	0.3684	0.1298	0.2851	0.1157	0.1359	0.1143	0.1374
Mean	0.2626	0.4178	0.1956	0.3188	0.1573	0.1681	0.1611	0.1820

The advantage of the model averaging approach becomes more apparent as the forecast horizon increases to $h = 10$. In Table 19, we again found improvement of the point forecast accuracy produced by the model averaging approach, in comparison to the naïve random walk methods.

Table 19: Point forecast accuracy of the combined female and male life expectancies by country, as measured by the MAFEs for the ten-step-ahead forecasts

	Equal		$\mathbf{Frequentist_{all}}$		$\mathbf{Frequentist_{2}}$		BMA	
Country	Female	Male	Female	Male	Female	Male	Female	Male
Australia	0.7983	1.6240	0.7363	1.6282	0.5384	1.2863	0.6189	1.5798
Canada	0.2914	1.1317	0.3240	1.1126	0.3408	0.9144	0.3639	1.0296
Denmark	0.4637	1.2100	0.4571	1.3177	0.4442	0.8759	0.6337	1.2713
England	0.4773	1.3261	0.4195	1.2772	0.2722	1.0087	0.3712	1.1853
Finland	0.9738	1.6643	0.8602	1.6135	0.3054	0.8331	0.6924	1.1119
France	0.6238	1.4360	0.3216	1.2323	0.1842	0.5432	0.2010	1.1331
Iceland	1.1282	1.5610	1.1235	1.2613	0.4306	0.7208	0.7718	1.2259
Italy	0.7817	1.6360	0.6248	1.6320	0.3400	0.9783	0.3468	1.3412
Netherlands	0.3759	1.1204	0.4275	1.0502	0.4822	0.7231	0.4540	1.1087
Norway	0.7178	1.5352	0.6377	1.4943	0.3443	1.1879	0.6293	1.5819
Scotland	0.7310	1.5216	0.6194	1.5644	0.3426	0.9696	0.4790	1.2783
Spain	0.4919	0.7146	0.4010	0.6689	0.1738	0.3301	0.4728	0.8131
Sweden	0.5557	1.3620	0.2267	1.3740	0.1434	0.9964	0.4176	1.2511
Switzerland	0.3561	1.2345	0.2378	1.1530	0.2183	0.8531	0.2809	0.9071
Mean	0.6262	1.3627	0.5298	1.3128	0.3258	0.8729	0.4810	1.2013

9. Discussion

The above comparative analysis of fourteen methods for forecasting age-specific life expectancies is the most comprehensive to date. It constitutes an evaluation of point and interval forecasts for life expectancies based on ten principal component methods, two random walk methods, and two univariate time-series methods, using age- and sex-specific of the fourteen developed countries. The methods include the LC method and four LC variants, the HU method, itself an extension of the LC method, and four HU variants, RW and RWD, ETS and ARIMA methods. The evaluation of point and interval forecasts of life expectancies is a sequel of Shang, Booth, and Hyndman (2011), who considered a similar forecasting exercise, but with application to mortality rates and life expectancy at birth. The evaluation of interval forecast accuracy is novel in the context of forecasting age-specific life expectancies.

9.1 Point forecasts

Our overall findings regarding the direction of bias is that the LM has the smallest amount of bias among all methods for the female data[13]. The bias performance of the LM method is followed closely by the BMS and RWD methods. For the male data, all methods consistently underestimate the life expectancies with an exception of the LC method. Cancelation of MFEs from different countries leads to the most superior performance of the LC method[14], which is followed by the RWD and LM methods. Unlike the LC method, the RWD and LM methods are more centered around zero at all ages (see Figure 1).

Regarding the point forecast accuracy of life expectancies, the RWD method is the most accurate method for forecasting the one-year-ahead life expectancies. The performance of the RWD method is followed closely by the LM method[15]. In comparison to the RWD method, the slightly less accurate performance of the LM method may due to the inadequacy of truncating data from 1950 onward. As emphasized by Booth, Maindonald, and Smith (2002), the optimal fitting period should be selected in a data-driven manner, by using a statistical “goodness of fit" criterion. Truncating data from 1950 onward in the LM method serves as a very rough guide for obtaining the optimal fitting period. As the number of observations increases over years, truncating data from 1950 onward may no longer be adequate to cope with recent structural change that happens after 1950. A future research direction is to explore the possibility of combining the advantages of the LM and BMS methods. To be more specific, the optimal fitting period is selected by a statistical “goodness of fit" criterion, but the jump-off rates are the actual rates in the jump-off year.

9.2 Interval forecasts

Another finding is that all LC methods underestimate the variability in the life expectancies; in other words, on average the prediction intervals are too narrow. This finding is not new, and has been pointed out by Lee and Carter (1992). This underestimation occurs for all populations consistently. By contrast, the HUw, RW and RWD methods consistently produce much wider prediction intervals than all other methods, which also lead to inferior interval forecast accuracy. The remaining methods exhibit a mixture of overestimation and underestimation of the coverage probability for various countries.

[13] refer to Table 4
[14] refer to Table 5
[15] refer to Tables 2 and 3

As measured by the simple and weighted averages of the coverage probability deviance, the HU50 method produces the best interval forecast accuracy, which is followed by the HU and HUrob methods[16]. For both female and male data, the HU methods produce more accurate interval forecasts than the LC methods. The main reason for their narrower prediction intervals is that the LC methods consider only two sources of uncertainty (see also Cairns 2000; Alho and Spencer 2005, p.215). In contrast, the HU methods consider four sources of uncertainty. The additional two sources of uncertainty include: First, the HU methods allow more principal components to be included in the model. These additional components capture the underlying patterns of data, which are not explained by the first principal component (Booth, Maindonald, and Smith 2002; Renshaw and Haberman 2003). Second, the HU methods use a nonparametric penalized regression spline with the monotonic constraint of Ramsay (1988), to smooth the noisy log mortality rates. Oftentimes, this results in a reduction of the overall variability.

It should be pointed out that the prediction intervals of life expectancies are constructed nonparametrically via a simulation approach. Therefore, they are asymmetric, and this occasionally results in a wider upper bound only. In addition, the coverage probability deviance does not distinguish between the upper and lower bounds because it is an absolute measure. Concretely, the coverage probability deviance would be the same, when the empirical coverage probability is 0.9 or 0.7 at the nominal coverage probability of 0.8. Further research is needed to examine other measures of interval forecast accuracy.

9.3 Comparison with some previous findings

Our findings differ from those of Shang, Booth, and Hyndman (2011) on several factors. First, we use a rolling origin from 1988 to 2007, and examine the one-step-ahead and ten-step-ahead point and interval forecast accuracy. Second, we consider the problem of forecasting age-specific life expectancies rather than the life expectancy at birth studied in Shang, Booth, and Hyndman (2011). Third, we use data from age 0 to 104+, rather than 0 to 89+.

9.4 Limitations

Though including fourteen methods, this comparative analysis is limited to methods based mainly on principal component approaches and univariate methods. Comparison of these methods with other forecasting approaches is beyond the scope of this paper. However, the analyses presented here provide a basis for the selection of principal component methods to forecast age-specific life expectancies.

[16] refer to Tables 10 and 11

The comparison of the fourteen methods is solely based on the successive one-step-ahead and ten-step-ahead point and interval forecasts. The use of the one-step-ahead evaluations is a standard practice in forecasting, especially when a rolling origin is used. While the one-step-ahead forecasts are customary in some time-series contexts, it may not be the most relevant one in demography. Therefore, we also consider the ten-step-ahead forecasts in this paper. However, it would be useful to examine a variety of forecast horizons, such as $h = 20$ or $h = 30$.

The comparison of forecast accuracy is specific to the forecasting period from 1988 to 2007. It is possible that different results would be obtained for a different forecasting period. In addition, the difference in forecast accuracy among methods may not be statistically significant. To determine if the differences among methods are statistically significant, a hypothesis testing is often performed. However, statistical tests of the results are problematic, because the number of countries and the number of years in the forecasting period are rather small, and cannot be considered as random samples. Therefore, we did not test the statistical significance among the methods investigated.

The conclusion of this comparative analysis are based on age- and sex-specific simple and weighted averages across the fourteen countries. As can be seen from the tables, some methods perform well for certain countries, but may perform poorly overall. Similarly, a method may perform best overall, but performs poorly for a certain country.

9.5 Future research

There are numerous ways in which the results of the present paper can be extended, and we briefly mention a few of them at this point.

1. There are a number of problems, not just in the relatively narrow context of the present paper but in age-specific mortality rate forecasting, in forecasting age-specific fertility rates and in forecasting population size, to which the concept of model averaging is applicable.
2. One might consider the model averaging approach to improve the accuracy of interval forecast. Note that we cannot simply combine the lower and upper bounds of prediction interval, as the underlying distribution is unknown.
3. One might consider different ways of selecting optimal combination weights.

9.6 Implementation

Implementation of the methods used in this article is straightforward using the readily available R package *demography* (Hyndman 2012a). This package provides all fourteen principal component approaches to model and forecast age-specific mortality rates and age-specific life expectancies. The data requirements are historical mortality rates, population numbers and life expectancies in a complete matrix format by age and year. Such data are readily available for many developed countries from the Human Mortality Database (2012).

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