

Spin transport in an Aharonov-Bohm ring with exchange interaction

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(Received 10 August 2013; published 25 November 2013)

We investigate spin-dependent conductance through a quantum Aharonov-Bohm ring containing localized electrons which interact with the propagating flow of electrons via exchange interaction of the ferromagnetic or antiferromagnetic type. We analyze the conductance oscillations as a function of both the chemical potential (particle concentration) and external magnetic field. It is demonstrated that the amplitude of the conductance oscillations in the ballistic regime is determined by the value of the noncompensated spin localized in the ring. The results are in agreement with the concept of fractional quantization of the ballistic conductance, proposed by us earlier [*Phys. Rev. B* **71**, 113311 (2005)].

DOI: [10.1103/PhysRevB.88.195430](https://doi.org/10.1103/PhysRevB.88.195430)

PACS number(s): 73.23.Ad, 73.21.Hb, 73.63.Nm, 72.25.Dc

I. INTRODUCTION

Recent progress in nanotechnology allowed to prepare quasi-one-dimensional (1D) semiconductor systems containing high-mobility charge carriers, which exhibit ballistic behavior when the decoherence time is longer than the characteristic time needed for an electron or a hole to pass through the structure. In this case, the transport of charge carriers in such systems is of coherent nature. Since the ballistic transport is not accompanied by the Joule losses, the conductance of the quasi-1D semiconductor systems in a single-mode regime at small drain-source biases depends only on the transmission coefficient describing the elastic scattering in the ballistic region.^{1,2} The latter is determined by the geometry of the system and can depend on such parameters as the Fermi energy of the carriers and external electric or magnetic fields.

In some cases, the scattering becomes spin dependent. This happens if the ballistic region contains confined electrons thus possessing a noncompensated spin which interacts with the spins of the propagating electrons. A standard textbook example of this phenomenon is Kondo effect manifesting itself in transport through an individual quantum dot connected to one-dimensional leads.^{3,4} Another example is the formation of the “0.7 anomaly” in the ballistic conductance of an individual quantum point contact (QPC) split off from the first step in the quantum conductance staircase.^{5–7} Although the exact mechanism of the formation of the “0.7 anomaly” is still a matter of debate,⁸ several experimental observations have indicated the importance of the spin component for the behavior of this feature. First, an electron g factor is found to raise from 0.4 to 1.3 as the number of occupied 1D subbands decreases.⁵ Second, the height of the feature attains to a value of 0.5 in a strong external magnetic field.^{9,10}

These results have defined the spontaneous spin polarization of a 1D gas in a zero magnetic field as one of the possible mechanisms of the appearance of the 0.7 anomaly.^{11–15} The model of a quantum point contact containing only one localized electron is rather perspective for explanation of the feature.^{16,17} The localized spins affect the propagating

carriers via exchange interaction of either ferromagnetic or antiferromagnetic type. Since it is defined by the mutual orientation of spins, the transmission coefficient through the QPC becomes spin dependent. If the QPC contains a single localized electron, the eigenstates of the system consisting of localized and transmitted electrons are of singlet and triplet states. If the energy of the triplet state is lower than the energy of the singlet state, the potential barrier for the carriers in the singlet configuration becomes higher than one for the triplet state. Therefore, at small concentrations of carriers, the ingoing electron in the triplet configuration passes the barrier, while the carriers in the singlet configuration undergo reflection, thereby defining the principal contribution of the triplet pairs to the total conductivity. In a zero magnetic field, the probability of realization of the triplet configuration equals to $\frac{3}{4}$ against $\frac{1}{4}$ in the case of the singlet one; thus, the full conductance in the regime considered equals $G = 0.75(2e^2/h)$.¹⁶ In contrast, if the singlet configuration is energetically preferable, the conductance equals to $G = 0.25(2e^2/h)$.^{17,18} If the uncompensated spin of electrons localized in the QPC, $J > \frac{1}{2}$, the quantization pattern becomes more complicated, and the value of the fractional plateau becomes $G = (J + 1)/(2J + 1)(2e^2/h)$ for the ferromagnetic interaction and $G = (J)/(2J + 1)(2e^2/h)$ for the antiferromagnetic interaction.¹⁹

Spin-related effects become even more pronounced in the ballistic transport through non-single-connected objects, such as Aharonov-Bohm (AB) rings. Spin-orbit interaction of the Rashba type in the AB ring induces the Aharonov-Casher (AC) and Berry phase shifts between the spin waves propagating in the clockwise and anticlockwise directions, which results in the large conductance modulations due to the interference of the spin wave functions. Experimental observation of the AC oscillations²⁰ in the gate-controlled AB rings has been reported for both electrons^{21,22} and holes.²³ Moreover, formation of localized states with uncompensated spin can be also expected to affect the transport properties of the rings. It was shown that insertion of a quantum point contact (QPC) in one of the ring’s arms changes the conductance pattern of the AB ring significantly due to the exchange interaction between the electron localized inside the QPC and

a propagating electron. It manifests itself in the formation of “0.7 feature” on the quantum conductance staircase of the whole ring structure.²⁴

Moreover, the formation of the localized spin state can in principle strongly affect the pattern of the ballistic conductance of the ring as a function of the external magnetic field applied perpendicular to its interface. The presence of the vector potential in this case leads to the oscillatory dependence of the conductance on magnetic flux through the ring with a period of the elementary flux quantum $\Phi_0 = h/e$ known as Aharonov-Bohm oscillations.^{25–28} In addition, in the experimental measurements of the dependency of conductance on the external magnetic field, the component associated with the Altshuler-Aronov-Spivak (AAS) oscillations²⁹ has been routinely observed. These oscillations have a period $\Phi_0/2 = h/2e$ and in semiconductor rings they can be attributed to the processes of the round-trip interference.³⁰ The interplay between the spin-related phenomena and the pattern of the AB oscillations was a matter of extensive studies. In particular, it was shown that the spin-orbit interaction can drive the transition from the weak antilocalization (WAL) to the weak localization, which was observed as the crossover from the positive to the negative magnetoresistance.^{20,21,31,32} It affects periodicity of the AB oscillations.^{33–35} On the other hand, the possible effect of the exchange interaction between the conductance electrons on ballistic conductance in the presence of a magnetic field has never been considered in the literature to the best of our knowledge.

In this work, we analyze theoretically spin-dependent transport in a double-slit AB ring with confined delocalized spin-polarized electrons which interact with conductance electrons passing through the ring in the ballistic regime. The localized spins affect the propagating carriers via exchange interaction of either ferromagnetic or antiferromagnetic type. Since it is defined by the mutual orientation of spins, the transmission coefficient through the ring becomes spin dependent, which is reflected in the dramatic changes of the patterns of the conductance as a function of the Fermi energy and external magnetic field.

II. MODEL

A sketch of the device we consider is presented in Fig. 1. It represents a quantum AB ring containing spin-polarized interacting electron gas supplemented with two metal leads which serve as a source and a drain of electrons from the two opposite sides. Both the ring and the leads are considered 1D for simplicity, in order to make semianalytical treatment possible. This approximation is legitimate as long as the Fermi energy and the leads’ cross-section area are small enough and the condition that only the lowest subband of the dimensional quantization is occupied holds: $mL^2 E_F / \pi^2 \hbar^2 < 1$.²⁴

The AB ring and the leads are connected by means of two identical QPCs, scattering on which is considered to be of elastic nature, and spin independent, thus we assume that the spin of a carrier is conserved while passing through the QPCs. Thus, the propagating electrons are supposed to interact with the localized spin only in the region of the ring.

In our calculation we consider the case of zero temperature, hence the charge carriers (electrons) have a steplike

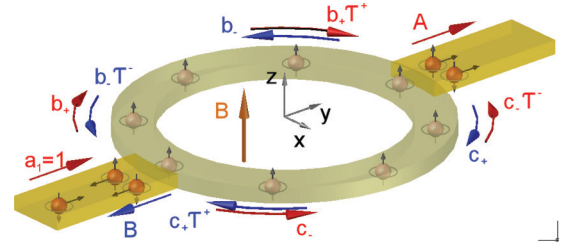


FIG. 1. (Color online) Sketch of the system: a double-slit AB ring with localized electrons interacting with the propagating charged carriers via the Coulomb and exchange interactions. The amplitudes of the waves which enter into equations for the determination of transmission and reflection probabilities are shown. A and B correspond to the transmission and reflection amplitudes. Due to the exchange interaction between propagating electrons and spin-polarized electrons in the ring, these amplitudes may become spin dependent as discussed in the main text.

distribution in the leads. The drain-source voltage V_d is also considered to be small enough, $eV_d \ll E_F$. Thus, only those electrons whose energy lies in the vicinity of the Fermi surface take part in the spin transport. The radius of the AB ring R is taken much smaller than the inelastic scattering length to ensure the validity of ballistic transport approximation and use of the well-known Landauer-Buttiker approach to calculate the conductance.^{36,37}

III. FORMALISM

The main parameter of a QPC is the amplitude of elastic backscattering of a carrier propagating inside the lead: σ , $|\sigma| < 1$, determined by the system geometry (a QPC becomes fully transparent if $\sigma = 0$). An external magnetic field B is applied perpendicularly to the plane of the AB ring. This field influences both the spatial and spin coordinates of the electrons propagating inside the AB ring and the leads and thereby defines the Aharonov-Bohm phase shift and the Zeeman splitting. The latter is neglected for simplicity of derivation and transparency of results.

Let us first consider the case when the AB ring contains a single localized electron. The full Hamiltonian of the system in the basis of uncoupled states corresponding to the spins of conductance (e) and localized (S) electrons ($|\uparrow_e \uparrow_s\rangle, |\downarrow_e \downarrow_s\rangle, |\uparrow_e \downarrow_s\rangle, |\downarrow_e \uparrow_s\rangle$) can be written in the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{int}} = \begin{pmatrix} \frac{\hbar^2 k^2}{2m} + V^- & 0 & 0 & 0 \\ 0 & \frac{\hbar^2 k^2}{2m} + V^- & 0 & 0 \\ 0 & 0 & \frac{\hbar^2 k^2}{2m} + V^+ & -2V_{\text{ex}} \\ 0 & 0 & -2V_{\text{ex}} & \frac{\hbar^2 k^2}{2m} + V^+ \end{pmatrix}, \quad (1)$$

where interaction of the localized and propagating electrons inside the QPC is modeled within the framework of the Heisenberg exchange Hamiltonian

$$\mathcal{H}_{\text{int}} = V_{\text{dir}} - 2V_{\text{ex}} \sigma_e \cdot \mathbf{J}. \quad (2)$$

Here, V_{dir} characterizes the Coulomb interaction between the moving and localized electrons plus the effect of the applied

gate voltage; V_{ex} corresponds to the exchange interaction; $V^{\pm} = V_{\text{dir}} \pm V_{\text{ex}}$; the spin operators σ_e and \mathbf{J} correspond to the propagating and localized electrons, respectively.

Hamiltonian (1) can be easily diagonalized by the canonical transformation,²⁴ and we obtain

$$\mathcal{H} = \begin{pmatrix} \frac{\hbar^2 k^2}{2m} + V^- & 0 & 0 & 0 \\ 0 & \frac{\hbar^2 k^2}{2m} + V^- & 0 & 0 \\ 0 & 0 & \frac{\hbar^2 k^2}{2m} + V^- & 0 \\ 0 & 0 & 0 & \frac{\hbar^2 k^2}{2m} + V_3 \end{pmatrix}, \quad (3)$$

where we used the notation $V_3 = V_{\text{dir}} - 3V_{\text{ex}}$.

Hamiltonian \mathcal{H} corresponds to the new eigenstates of the coupled system: $|1\rangle = |\uparrow_e \uparrow_s\rangle$, $|2\rangle = |\downarrow_e \downarrow_s\rangle$, $|3\rangle = \frac{1}{\sqrt{2}}(|\uparrow_e \downarrow_s\rangle + |\downarrow_e \uparrow_s\rangle)$, $|4\rangle = \frac{1}{\sqrt{2}}(|\uparrow_e \downarrow_s\rangle - |\downarrow_e \uparrow_s\rangle)$. The first three states above correspond to the triplet configuration, whereas the fourth one to the singlet configuration. These modes are characterized by the following wave vectors:

$$k_t = \sqrt{\frac{2m}{\hbar^2} [\mu - V_{\text{dir}} + V_{\text{ex}}]}, \quad (4)$$

$$\begin{aligned} G \approx & \frac{e^2 \hbar}{2\pi m} \int_0^\infty \left\{ [P_{S\uparrow} |A_{e\uparrow S\uparrow \rightarrow e\uparrow S\uparrow}(k)|^2 + P_{S\downarrow} (|A_{e\downarrow S\downarrow \rightarrow e\downarrow S\downarrow}(k)|^2 + |A_{e\downarrow S\uparrow \rightarrow e\downarrow S\uparrow}(k)|^2)] \left(\frac{\partial n_{k\uparrow}}{\partial \mu} \right) \right. \\ & + [P_{S\downarrow} |A_{e\downarrow S\downarrow \rightarrow e\downarrow S\downarrow}(k)|^2 + P_{S\uparrow} (|A_{e\downarrow S\uparrow \rightarrow e\downarrow S\uparrow}(k)|^2 + |A_{e\downarrow S\uparrow \rightarrow e\uparrow S\downarrow}(k)|^2)] \left(\frac{\partial n_{k\downarrow}}{\partial \mu} \right) \\ & \left. + [P_{S\downarrow} |A_{e\uparrow S\downarrow \rightarrow e\downarrow S\uparrow}(k)|^2 - P_{S\uparrow} |A_{e\downarrow S\uparrow \rightarrow e\uparrow S\downarrow}(k)|^2] \left[n_{k\uparrow}(\mu) \left(\frac{\partial n_{k\downarrow}}{\partial \mu} \right) - n_{k\downarrow}(\mu) \left(\frac{\partial n_{k\uparrow}}{\partial \mu} \right) \right] k dk \right\}. \quad (6) \end{aligned}$$

In this formula, we made an assumption that spins of the propagating and localized electrons are uncorrelated. In principle, this is not always correct. The effect of strong correlations between the localized electrons in the ring and transport electrons in the leads becomes particularly important for the case of antiferromagnetic interaction below characteristic Kondo temperature. In this regime, Kondo cloud will be formed around the ring, magnetic moment will be screened, and transport properties drastically modified. For $V_{\text{ex}} < 0$, thus, our approach is valid above Kondo temperature only, which anyway can be assumed to be very small (it is about 1 K for quantum dot systems and can be expected to be even smaller for our structure where all characteristic energies are reduced due to larger size of the system).³⁸ For the ferromagnetic case, there is no analog of Kondo effect and one can expect that qualitatively our results remain valid in the whole range of the temperatures.

In Eq. (15) we have introduced a number of parameters. The mean number of electrons with a definite spin projection and wave vector in the leads $n_{k\uparrow,\downarrow}$ reads the same if we do not account for the Zeeman effect:

$$n_{k\uparrow,\downarrow} = \frac{1}{e^{\frac{E_k - \mu}{k_B T}} + 1}. \quad (7)$$

$$k_s = \sqrt{\frac{2m}{\hbar^2} [\mu - V_{\text{dir}} - 3V_{\text{ex}}]}. \quad (5)$$

In a zero magnetic field, the singlet and triplet state energies are split by $4V_{\text{ex}}$. Obviously, the energetic disposition of the states depends on the sign of the coupling constant V_{ex} : for “ferromagnetic” coupling ($V_{\text{ex}} > 0$), the triplet configuration becomes preferable energetically; the opposite case is valid for the “antiferromagnetic” coupling ($V_{\text{ex}} < 0$).

Due to the processes of exchange interaction between the localized and freely propagating electrons, the latter can either conserve their spin projection or undergo a spin flip. The probabilities of these processes depend on the mutual orientation of the two spins as well as on the exchange matrix element V_{ex} . If the spins of the localized and propagating electrons are parallel, only spin-conservative processes are allowed. On the other hand, if the spins are antiparallel, a spin-flip process becomes possible. Using the formalism of Ref. 24, we can write the expression for the ballistic conductance of the AB ring with exchange interaction (at finite temperature) as

Here, μ is the chemical potential in the quantum wire. $P_{S\uparrow}$ and $P_{S\downarrow}$ are probabilities of the localized electron findings in spin-up and spin-down states, given by the formulas

$$P_{S\uparrow} = \frac{e^{g\mu_B B/k_B T}}{e^{g\mu_B B/k_B T} + e^{-g\mu_B B/k_B T}}, \quad (8)$$

$$P_{S\downarrow} = \frac{e^{-g\mu_B B/k_B T}}{e^{g\mu_B B/k_B T} + e^{-g\mu_B B/k_B T}}, \quad (9)$$

and in the case of absent Zeeman splitting, $P_{S\uparrow} = P_{S\downarrow} = \frac{1}{2}$.

Parameters A (namely, $A_{e\uparrow S\uparrow \rightarrow e\uparrow S\uparrow}$, $A_{e\uparrow S\downarrow \rightarrow e\uparrow S\downarrow}$, $A_{e\downarrow S\downarrow \rightarrow e\downarrow S\downarrow}$, $A_{e\downarrow S\uparrow \rightarrow e\downarrow S\uparrow}$, $A_{e\downarrow S\uparrow \rightarrow e\uparrow S\downarrow}$) are the amplitudes of transmission through the AB ring, and the subscripts denote the spin states before and after the scattering event. To find the transmission amplitudes, we will address the scattering matrix approach discussed in Refs. 24 and 34. As the spin projection is assumed to be conserved during the scattering of a particle by the QPCs, the transport of particles with opposite spin projections through QPC can be treated separately.^{39,40}

The amplitudes of the transmitted and reflected waves can be found from the conditions of the conservation of the flux at the contacts connecting the AB ring and the leads. As we account for the exchange interaction inside the ring, we should

consider separately the cases when the spins of the propagating and localized electrons are aligned parallel or antiparallel.

A. Single localized electron

First, let us consider the case in which the spins of both the propagating and the localized electrons are parallel, thus electrons are in the triplet configuration. Note that as exchange interaction conserves the total spin projection of the interacting electrons, in this case the spin-flip processes are impossible, and orientations $\uparrow\uparrow$ and $\downarrow\downarrow$ can be treated separately. The problem thus effectively reduces to one for the transport of the spinless particles. Introducing the spin-independent scattering matrix for the contacts \hat{S} , one can obtain the following system of linear algebraic equations connecting the transmission and reflection amplitudes $A = A_{e\uparrow S\uparrow \rightarrow e\uparrow S\uparrow} = A_{e\downarrow S\downarrow \rightarrow e\downarrow S\downarrow}$ and $B = B_{e\uparrow S\uparrow \rightarrow e\uparrow S\uparrow} = B_{e\downarrow S\downarrow \rightarrow e\downarrow S\downarrow}$ with the amplitudes of the waves propagating in the ring b_j, c_j ($j = +, -$), shown in Fig. 1:

$$\begin{pmatrix} b_- \\ A \\ c_+ \end{pmatrix} = \hat{S} \cdot \begin{pmatrix} b_+ \tau_t^+ \\ 0 \\ c_- \tau_t^- \end{pmatrix} = \begin{pmatrix} r & \varepsilon & t \\ \varepsilon & \sigma & \varepsilon \\ t & \varepsilon & r \end{pmatrix} \cdot \begin{pmatrix} b_+ \tau_t^+ \\ 0 \\ c_- \tau_t^- \end{pmatrix},$$

$$\begin{pmatrix} b_+ \\ B \\ c_- \end{pmatrix} = \hat{S} \cdot \begin{pmatrix} b_- \tau_t^- \\ 1 \\ c_+ \tau_t^+ \end{pmatrix}. \quad (10)$$

The QPCs connecting the ingoing and outgoing leads to the ring are considered to be identical. The scattering matrix is characterized by the parameters $r, t, \varepsilon, \sigma$ which have the following physical meaning. The parameters r and t are reflection and transmission amplitudes of the QPCs inside the AB ring; σ is the reflection amplitude from the lead to itself and ε is the transmission amplitude from a lead to the AB ring or from the AB ring to a lead. The scattering amplitudes r, t, σ , and ε are assumed real numbers. These parameters depend on the properties of the junction, in particular on the band mismatch between the leads and the AB ring that is electrically induced by the gate voltage. The condition of flux conservation resulting in the Hermiticity of the scattering matrix allows us to reduce the number of its independent elements. According to Buttiker,⁴¹ the following parametrization can be used:

$$r = \frac{\lambda_1 + \lambda_2 \sqrt{1 - 2\varepsilon^2}}{2},$$

$$t = \frac{-\lambda_1 + \lambda_2 \sqrt{1 - 2\varepsilon^2}}{2}, \quad (11)$$

$$\sigma = \lambda_2 \sqrt{1 - 2\varepsilon^2},$$

where $\lambda_{1,2} = \pm 1$. Therefore, the effect of the QPCs on the scattering of a particle in the AB ring turns out to be defined by only one parameter: $\varepsilon \in [-1/\sqrt{2}, 1/\sqrt{2}]$. The case $\varepsilon = 1/\sqrt{2}$ corresponds to the fully transmitting contact, and the case $\varepsilon = 0$ to the fully reflecting one.

The parameters τ_t^\pm are the phase shifts between the clockwise and anticlockwise traveling electron waves, correspondingly:

$$\tau_t^\pm = \exp \left[i \left(\pi k_t R \pm \frac{e\Phi}{2\hbar} \right) \right], \quad (12)$$

where $\Phi = \pi R^2 B$ is the magnetic flux through the ring, R is the radius of the AB ring, and e is the electron charge. The wave vector k_t corresponds to triplet configuration of the interacting spins and is defined by Eq. (16). In the absence of an external magnetic field, the phase shift is equal for the electrons moving in both the clockwise and anticlockwise directions.

Now, let us consider the situation when the spins of the localized and propagating electrons are antiparallel, and the spin-flip process can occur. For instance, consider the case when the propagating electron is initially in the spin-up state and the localized electron is in the spin-down state (the opposite case is fully equivalent). The amplitudes of scattering then become spin dependent and instead of the single transmission amplitude, one needs to introduce two amplitudes corresponding to the cases of the conservation of the spin of the propagating electron and its spin flip $A_{e\uparrow S\downarrow \rightarrow e\uparrow S\downarrow}$ and $A_{e\uparrow S\downarrow \rightarrow e\downarrow S\uparrow}$, respectively. Therefore, the system of the equations for the amplitudes becomes more complicated and reads as

$$\begin{pmatrix} b_{t-} \\ A_{e\uparrow S\downarrow \rightarrow e\uparrow S\downarrow} \\ c_{t+} \\ b_{s-} \\ A_{e\uparrow S\downarrow \rightarrow e\downarrow S\uparrow} \\ c_{s+} \end{pmatrix} = \begin{pmatrix} \hat{S} & 0 \\ 0 & \hat{S} \end{pmatrix} \cdot \begin{pmatrix} b_{t+} \tau_t^+ \\ 0 \\ c_{t-} \tau_t^- \\ b_{s+} \tau_s^+ \\ 1 \\ c_{s-} \tau_s^- \end{pmatrix},$$

$$\begin{pmatrix} b_{t+} \\ B_{e\uparrow S\downarrow \rightarrow e\uparrow S\downarrow} \\ c_{t-} \\ b_{s+} \\ B_{e\uparrow S\downarrow \rightarrow e\downarrow S\uparrow} \\ c_{s-} \end{pmatrix} = \begin{pmatrix} \hat{S} & 0 \\ 0 & \hat{S} \end{pmatrix} \cdot \begin{pmatrix} b_{t-} \tau_t^- \\ 1 \\ c_{t+} \tau_t^+ \\ b_{s-} \tau_s^- \\ 0 \\ c_{s+} \tau_s^+ \end{pmatrix}, \quad (13)$$

where

$$\tau_{t,s}^\pm = \exp \left[i \left(\pi k_{t,s} R \pm \frac{e\Phi}{2\hbar} \right) \right], \quad (14)$$

and $k_{t,s}$ are given by Eqs. (16) and (17). It should be noted that as initial configuration $|e\uparrow S\downarrow\rangle$ is not an eigenstate of the exchange Hamiltonian (2), both singlet and triplet configurations corresponding to different wave numbers of the propagating electrons are possible inside the ring.

Together, Eqs. (15), (10), and (13) allow for calculating the conductance in the ballistic regime.

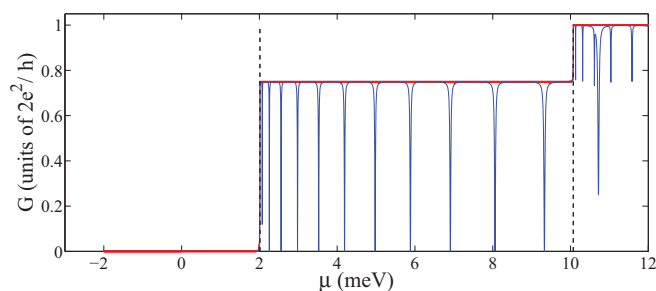
B. The case of several localized electrons

Let us suppose that the ring contains the total spin $J > \frac{1}{2}$ which appears as a result of spontaneous spin polarization provided by the exchange interactions. If the propagating electron enters the ring, the total spin S can be either $S_1 = J + \frac{1}{2}$ or $S_2 = J - \frac{1}{2}$. The number of the possible realizations of the state 1 appears to be $N_1 = 2S_1 + 1 = 2J + 2$, while the number of the realizations of the state 2 is $N_2 = 2S_2 + 1 = 2J$. Thus, there are $4J + 2$ possible mutual orientations of the spin of the propagating and localized electrons. For each of them after passing the region of the ring, the spin of the propagating electron can be either conserved or inverted due to the exchange interaction. Using the formalism described

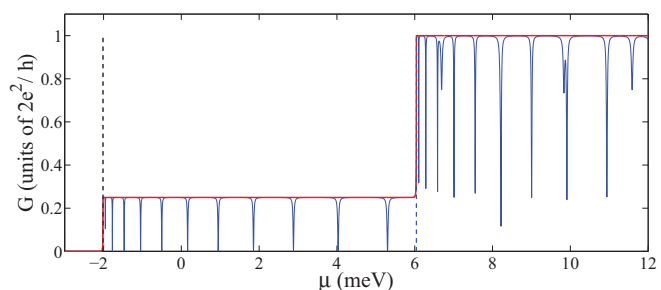
in Ref. 19 for arbitrary localized spin J , at zero temperature and neglecting the Zeeman splitting for simplicity, the ballistic conductance of the AB ring can be represented as

$$\begin{aligned}
 G_{T=0} = & \frac{e^2}{4h(2J+1)} \sum_{m=0}^{2J} \{ |A_{(-1/2; J-m+1) \rightarrow (-1/2; J-m+1)}|^2 \\
 & + |A_{(-1/2; J-m+1) \rightarrow (1/2; J-m)}|^2 \\
 & + |A_{(1/2; J-m) \rightarrow (1/2; J-m)}|^2 \\
 & + |A_{(1/2; J-m) \rightarrow (-1/2; J-m+1)}|^2 \}, \quad (15)
 \end{aligned}$$

where the amplitudes A can be found from the systems of equations similar to (10) and (13) in the following way. If spins of both the propagating and the localized electrons are parallel $A_{e\uparrow S\uparrow \rightarrow e\uparrow S\uparrow}$, they should be substituted by $A_{(1/2, J \rightarrow 1/2, J)}$ in Eq. (10) and $A_{e\downarrow S\downarrow \rightarrow e\downarrow S\downarrow}$ substituted by $A_{(-1/2, -J \rightarrow -1/2, -J)}$. In the case when the spin-flip process can occur, one should use Eq. (13) and the substitutions $A_{e\uparrow S\downarrow \rightarrow e\uparrow S\downarrow} \rightarrow A_{(1/2, J-m \rightarrow 1/2, J-m)}$, $A_{e\uparrow S\downarrow \rightarrow e\downarrow S\uparrow} \rightarrow A_{(1/2, J-m \rightarrow -1/2, J-m+1)}$. The amplitudes B should be changed in a similar way.



(a) Ferromagnetic interaction ($V_{\text{ex}} > 0$)



(b) Antiferromagnetic interaction ($V_{\text{ex}} < 0$)

FIG. 2. (Color online) The dependence of the conductance of the AB with single localized spin on the Fermi energy in the case of (a) ferromagnetic ($V_{\text{ex}} > 0$) and (b) antiferromagnetic ($V_{\text{ex}} < 0$) interaction between the spins of localized and ballistic electrons. The contacts connecting the AB ring to the leads are considered to be fully transparent, $\varepsilon = 1/\sqrt{2}$. Red curves correspond to the case of zero magnetic field ($B = 0$), blue curves to the case $B = 1.7$ T. For $B = 0$, the conductance reveals the same fractional quantization pattern as those of the individual QPC with localized spin. Magnetic field introduces additional AB phase shifts, and conductance reveals an oscillatory pattern.

Besides, in the case of the localized spin J the expressions for the wave vectors $k_{s,t}$ are changed:

$$k_t = \sqrt{\frac{2m}{\hbar^2} [\mu - V_{\text{dir}} + V_{\text{ex}} J]}, \quad (16)$$

$$k_s = \sqrt{\frac{2m}{\hbar^2} [\mu - V_{\text{dir}} - V_{\text{ex}}(J+1)]}. \quad (17)$$

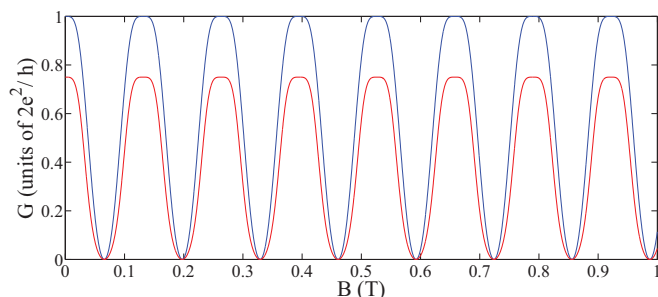
IV. RESULTS AND DISCUSSION

In the calculations presented below we used the following set of parameters: $m = 0.063 m_e$, $V_{\text{dir}} = 0.07e^2/t(\pi\epsilon_0 R)$, $V_{\text{ex}} = \pm 0.5V_{\text{dir}}$. The radius of the ring was taken equal to $R = 100$ nm. We also considered the cases of different J ($J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$). The temperature was set to zero. Results of modeling are presented in Figs. 2–5.

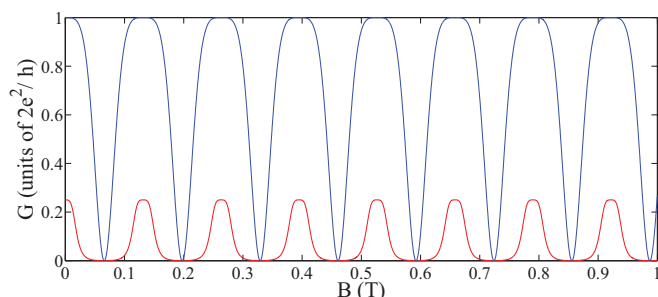
Figure 2 illustrates the influence of the many-body interaction on the AB ring conductance for the cases of zero and nonzero magnetic fields.

Both direct and exchange interactions of the moving and localized electrons form an effective potential barrier for the particles, thus influencing the conductance. In general, there are two such sub-barriers, corresponding to the energies $V_{\text{dir}} - 2V_{\text{ex}}J$ and $V_{\text{dir}} + 2V_{\text{ex}}(J+1)$.

Figure 2(a) corresponds to the case of ferromagnetic interaction ($V_{\text{ex}} > 0$ and $J = \frac{1}{2}$). The behavior of G has a



(a) Ferromagnetic interaction ($V_{\text{ex}} > 0$)



(b) Antiferromagnetic interaction ($V_{\text{ex}} < 0$)

FIG. 3. (Color online) Conductance of the AB ring with exchange interaction vs magnetic field for different Fermi energies: 5.0 (red curves) and 15.0 meV (blue curves) in the case of (a) ferromagnetic interaction ($V_{\text{ex}} > 0$) and (b) antiferromagnetic interaction ($V_{\text{ex}} < 0$). The value $\mu = 5$ meV corresponds to the second steps on the $G(\mu)$ staircases [see Figs. 2(a) and 2(b)]. Therefore, the maxima of the amplitudes of oscillations correspond to (a) $0.75(2e^2/\hbar)$ and (b) $0.25(2e^2/\hbar)$.

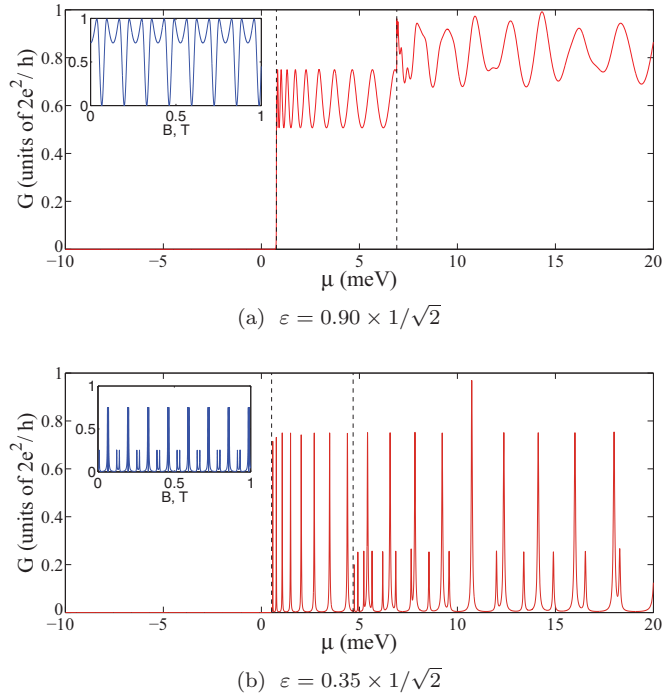


FIG. 4. (Color online) The influence of the transparency of the contacts on the conductance pattern of the ring with ferromagnetic exchange interaction. Two cases are considered: (a) $\varepsilon = 0.90 \times 1/\sqrt{2}$ and (b) $\varepsilon = 0.35 \times 1/\sqrt{2}$. Main plots depict the dependence $G(\mu)$ for $B = 0$; insets show the dependencies of the conductance on magnetic field for $\mu = 15$ meV.

clear explanation. Let us first consider the case when magnetic field is absent shown by the red line. The conductance G is zero below the $\mu = V_{\text{dir}} - V_{\text{ex}}$ (the lowest step of the barrier) since independently on the mutual orientation of the spins of propagating and localized electrons, the energy of a particle is not enough to overcome even the lowest barrier. Further, if the chemical potential lies in the range $[V_{\text{dir}} - V_{\text{ex}}; V_{\text{dir}} + 3V_{\text{ex}}]$ shown by two vertical lines on the plot, the carriers in triplet configuration can enter the ring while the carriers in the singlet configuration can not. As there are three states corresponding to the triplet configuration and one state corresponding to the singlet configuration and contacts are considered to be fully transparent, the conductance in these regimes equals to $G = 0.75(2e^2/h)$. If $\mu > V_{\text{dir}} + 3V_{\text{ex}}$, the electrons in both singlet and triplet configurations can enter the ring and conductance reaches the value of the elementary conductance quantum $G = 2e^2/h$. The conductance pattern is thus equivalent to those revealed by an individual QPC with localized spin studied in Refs. 17 and 19.

If one applies an external magnetic field (inducing different AB phase shifts for the electrons moving clockwise and anticlockwise) and interference is not always constructive, the conductance as a function of the chemical potential reveals oscillations instead of the plateaus. The amplitude of these oscillations is increased from $0.75(2e^2/h)$ to $2e^2/h$ if the chemical potential is increased above the value $V_{\text{dir}} + 3V_{\text{ex}}$.

Figure 2(b) corresponds to the case of antiferromagnetic interaction ($V_{\text{ex}} < 0$). Now, the singlet configuration becomes energetically preferable. The conductance G is zero if the

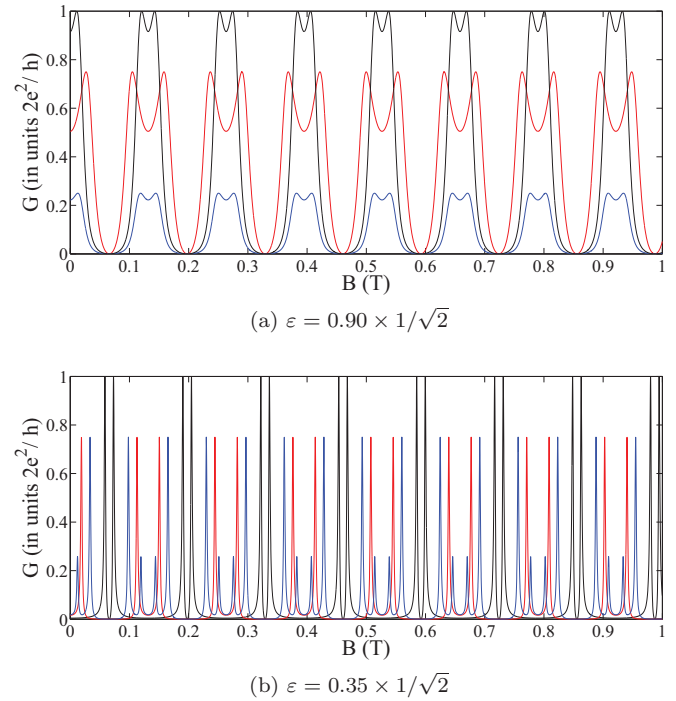


FIG. 5. (Color online) The influence of the exchange interaction on the oscillatory behavior of the dependence $G(B)$ for two distinct values of ε . Red curves correspond to ferromagnetic and blue to antiferromagnetic interaction; the black curve corresponds to the case of absent exchange interaction. It can be seen that at medium values of ε , the exchange interaction determines the periodicity of conductance oscillations, varying from h/e to $h/2e$ and $h/4e$.

chemical potential lies below the $\mu = V_{\text{dir}} + 3V_{\text{ex}}$. Further, if the chemical potential lies in the region $[V_{\text{dir}} + 3V_{\text{ex}}; V_{\text{dir}} - V_{\text{ex}}]$, the conductance becomes equal to $0.25(2e^2/h)$ since only electrons in the singlet configuration can enter the ring. Finally, at $\mu > V_{\text{dir}} - V_{\text{ex}}$ the ring becomes transparent for any spin orientation and the value $G = 2e^2/h$ is recovered.

Figure 3 illustrates the dependence of the conductance on the external magnetic field for different values of the chemical potential and $\varepsilon = 1/\sqrt{2}$. An oscillatory behavior revealing AB effect is observed. This happens since the conductance of the ring is governed by the phase factors for the clockwise (τ^+) and anticlockwise (τ^-) propagating waves. It should be noted that Figs. 3(a) and 3(b) are in good correspondence with Figs. 2(a) and 2(b). The amplitude of the AB oscillations is defined by the value of the chemical potential. It changes from $0.75(2e^2/h)$ to $2e^2/h$ when μ increases above $V_{\text{dir}} + 3V_{\text{ex}}$ for the ferromagnetic case and changes from $0.25(2e^2/h)$ to $2e^2/h$ when μ increases above $V_{\text{dir}} - V_{\text{ex}}$ for the antiferromagnetic case.

We have also analyzed the change of the conductance pattern if the transparency of the contacts described by the parameter ε is changed. The results are shown in Fig. 4 for the ferromagnetic case only (antiferromagnetic case is similar in this content). The deviation from the case of fully transparent contacts corresponding to $\varepsilon = 1/\sqrt{2}$ is revealed by the onset of the oscillatory pattern in the dependence of the conductance on the chemical potential. If $\mu \in [V_{\text{dir}} + 3V_{\text{ex}}; V_{\text{dir}} - V_{\text{ex}}]$, then only electrons in the triplet configuration can pass and

the amplitude of the oscillations is $0.75(2e^2/h)$. When μ exceeds the value $V_{\text{dir}} - V_{\text{ex}}$, the ring becomes transparent for both singlet and triplet configurations. The values of the wave number of the ballistic electron and thus the conditions of the constructive interference in these two configurations are different. If the deviation from the full transparency is small (e.g., $\varepsilon = 0.9/\sqrt{2}$), this results in the onset of broad oscillations with the amplitude a bit smaller than $2e^2/h$ in the region $\mu > V_{\text{dir}} - V_{\text{ex}}$, as shown in Fig. 4(a). However, if one decreases the transparency further, the resonances become sharper, and the conductance pattern at $\mu > V_{\text{dir}} - V_{\text{ex}}$ consists of the series of well-resolved peaks of the height $0.75(2e^2/h)$ and $0.25(2e^2/h)$ corresponding to resonant transmission of triplets and singlets, respectively.

In the dependence of the conductance on magnetic field shown in the insets of Fig. 5, the decrease of the transparency of the contacts leads to appearance of the higher harmonics connected with the increased probabilities of the round trips inside the ring characteristic to the transition from the Aharonov-Bohm to the Aharonov-Altshuler-Spivak oscillations, typical for the weak localization regime.^{29,34}

The exchange interaction strongly affects the patterns of the dependence of the ballistic conductance on the magnetic field due to the possibility of the opening of the additional scattering channels. To analyze this dependence, we compared the spectrum of the AB oscillations for the case of $V_{\text{ex}} = 0$ when scattering is spin conservative and $V_{\text{ex}} \neq 0$. The results are shown in Fig. 6 for the cases of both ferromagnetic ($V_{\text{ex}} > 0$) and antiferromagnetic ($V_{\text{ex}} < 0$) interactions. One

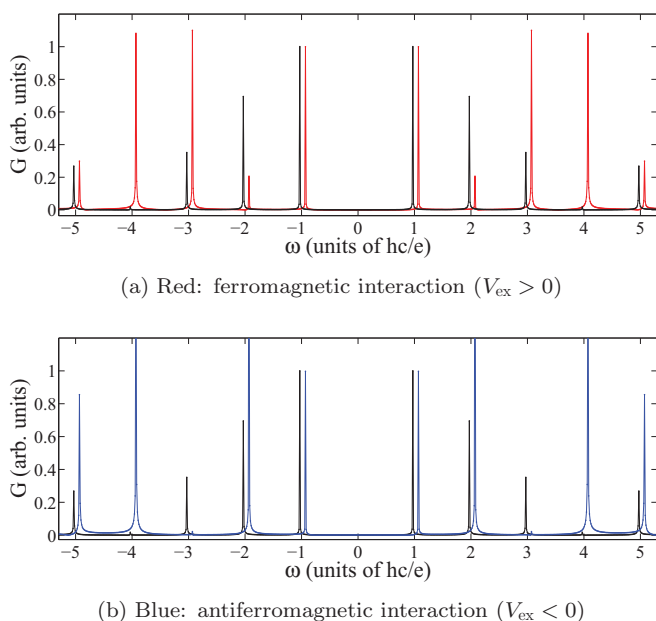
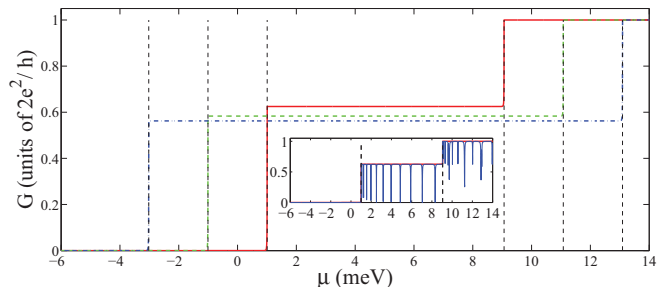
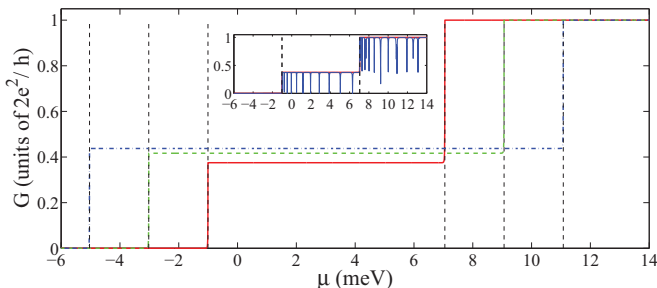


FIG. 6. (Color online) Illustration of influence of the exchange interaction on the dependence of the ballistic conductance on magnetic flux through the ring $G(B)$. $\varepsilon = 0.35 \times 1/\sqrt{2}$. Red curve corresponds to ferromagnetic and blue one to antiferromagnetic interaction; the black curves correspond to the case of absent exchange interaction ($V_{\text{ex}} = 0$). Exchange interaction increases the importance of the higher-order harmonics due to the opening of additional scattering channels.



(a) Ferromagnetic interaction ($V_{\text{ex}} > 0$)



(b) Antiferromagnetic interaction ($V_{\text{ex}} < 0$)

FIG. 7. (Color online) Conductance of the AB ring dependence on the Fermi energy for different values of the uncompensated spin localized in the ring J : $J = \frac{3}{2}$ (red), $J = \frac{5}{2}$ (green), and $J = \frac{7}{2}$ (blue) for the cases of (a) ferromagnetic and (b) antiferromagnetic interaction. Main plots correspond to zero magnetic field. Insets illustrate the magnetic field dependence: $B = 0$ (red) and $B = 1.7$ T (blue) for the case $J = \frac{3}{2}$. The transparency parameter $\varepsilon = 1/\sqrt{2}$.

can see that in both cases, the importance of higher-order harmonics grows if the exchange interaction is taken into account. If $V_{\text{ex}} = 0$, first harmonics corresponding to h/e oscillations is dominant. On the contrary, for the ferromagnetic exchange, the third and the fourth harmonics corresponding to the frequencies $h/3e$ and $h/4e$ become dominant. For the antiferromagnetic exchange, the second harmonics with the period $h/2e$ characteristic for AAS oscillations gives major impact to the conductance pattern.

Finally, we have analyzed how the increase of the localized spin affects the conductance patterns of the AB rings with exchange interaction. The results are shown in Fig. 7. In the case of transparent contacts, the increase of the spin leads to the decrease of the value of the substep from $0.75(2e^2/h)$ to $[(J+1)/(2J+1)](2e^2/h)$ if we consider the ferromagnetic interaction and increase from $0.25(2e^2/h)$ to $[J/(2J+1)](2e^2/h)$ if we deal with the antiferromagnetic case. This is in agreement with the picture of the fractional quantization of the ballistic conductance presented in Ref. 19. The application of the external magnetic field leads to the onset of the oscillatory behavior, similar to that observed for $J = \frac{1}{2}$.

V. CONCLUSIONS

In conclusion, we have considered ballistic conductance of an Aharonov-Bohm ring containing localized uncompensated spin accounting for the exchange interaction between the spins of propagating and localized electrons. We have shown

that exchange interaction drastically modifies the conductance as a function of magnetic field and chemical potential. The obtained results are in agreement with the concept of the fractional quantization of the ballistic conductance proposed earlier.

ACKNOWLEDGMENTS

The work was supported by FP7 IRSES project “SPIN-MET.” R.G.P. thanks the University of Iceland for hospitality. I.G.S. acknowledges support of the Eimskip Fund.

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