

# **Technology, Leisure and Growth**

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**April, 2018**

**A thesis submitted for the degree of Doctor of Philosophy in  
Economics of The Australian National University**

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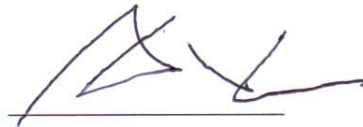
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# Declaration

This work in this thesis are my own except where otherwise stated. The vast majority of the research for this thesis was undertaken between January 2015 and April 2018 at the Crawford School of Public Policy, The Australian National University, Canberra, Australia.

This thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

A handwritten signature in black ink, consisting of several fluid, connected strokes, positioned above a horizontal line.

Anil Savio Kavuri

April, 2018



# Acknowledgements

I am particularly grateful to my supervisor, Professor Warwick McKibbin, for his unrivalled support and exceptional advice throughout my PhD journey. I wish to record that his help in polishing up Technology and leisure: Macro economic implications and the consequence of robots for economic wellbeing for publication purposes was critical. His guidance on the papers were also important. Professor Warwick McKibbin is more than a supervisor extending beyond the academic realm. I am especially grateful for Professor David Stern for his comments on all my papers. His help with refining the papers for publications was particularly vital. I also appreciate Professor Renee Fry-McKibbin for her helpfulness. I am thankful for the valuable advice from Professor Markus Hegland on the numerical analysis. I also thank the anonymous examiners for their useful comments, particularly in fuelling my mind for further research. I thank Dr Megan Poore for her aid on all student matters. I wish to express appreciation to Dr Ian Walker for his close friendship and care. I am thankful to Professor Peter Kanowski for his encouragement. I wish to dedicate this thesis to my beloved Father, Dr Seshu Babu Kavuri. I am truly indebted to him for his love and support throughout my life. I also wish to dedicate this thesis to my darling Mother, Dr Martha D'Mello who passed away whilst I was undertaking this PhD. I cannot express in words her strength, love and generosity.



# Abstract

This thesis develops models and methods to investigate leisure, technology and growth. Models in chapters two, three and four study the macroeconomic impacts of technology on the consumer side. The models allow for consumer habit formation for a technology good purchased for leisure. However, for the consumption good, habits are irrelevant. A method is introduced to determine the steady state of the technology good sector and consumption good sector independently. These chapters show that the models can contribute to the theoretical and empirical understanding of changes in consumption growth, interest rates, labour income share and wages. Models are constructed in chapters five and six to analyse technology on the production side in the form of job replacement by robots. Chapter five shows that the impact on welfare is ambiguous because leisure in the utility function can mitigate against wage decreases. In chapter six, policy to mitigate job losses from technology/robots is discussed.





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# Chapter 1

## Introduction

This thesis aims to make theoretical contributions by constructing models that are dynamically optimised in order to uncover characteristics of economies. The models involve leisure technology, consumption habits, endogenous technical change and interest rate determination. This thesis contains five self-contained chapters examining technology, leisure and growth.

A theoretical contribution of this thesis is to construct a macroeconomic model that allows for habit formation for a technology good purchased for leisure. While the impact of technology on production is widely researched, these theoretical models explore the economic implications of technology through the channel of enhancing leisure experience on the consumer side. Examples of the technology good include video games and smartphones. In contrast, for the normal consumption good, habits are assumed to be irrelevant. The overall aim of these chapters is to explore the economic implications of technology through the channel of enhancing leisure experience on the consumer side.

This thesis further theoretically contributes by developing an approach to investigate the steady state of an economy. In these chapters, equations are transformed into variables that are constant in the steady state. The steady state of the technology sector is defined when leisure technology, consumer habits and capital devoted to that sector grow at the same rate. In contrast,

the habits of the consumer are not applicable to the consumption sector. The steady state of this sector is defined as being when consumption and capital devoted to this sector grow at the same rate. A further theoretical contribution is to develop new macroeconomic models to understand this dynamics of job replacement by robots.

## **1.1 Structure and preview**

### **1.1.1 Technology and leisure: Macro economic implications**

This chapter primarily investigates the implications of leisure technology on interest rates and consumption growth. This chapter shows that a persistent fall in the relative price of the technology good and increased addiction to technology is shown to drive the real interest rate below the rate of time preference and to depress consumption of non-technology goods. Modelling the framework with US data illustrates that model predictions of falling interest rates and consumption growth are consistent with recent observations of declines in the relative price of the technology good and increases in technology good purchases.

### **1.1.2 Habits and labour income share: is there a link?**

The focus of this chapter is to investigate the implication of technology purchased to enhance the consumer's leisure on labour income shares. The theoretical contribution of this chapter is to develop a competitive equilibrium model, where consumer utility is separable in normal consumption and a technology good whose contribution to utility depends on habit formation. The chapter shows theoretically that leisure technology habits drive capital per hours worked and the labour income share downwards. Modelling the framework using US data shows that predictions match trends in wages, interest rates, and the labour income share.

### **1.1.3 Leisure and growth**

This chapter analyses the implications on output and consumption growth of the consumer learning to use the leisure technology good. When the importance of technology enhanced leisure is less than that of a consumer's habits, growth in technology-enhanced-leisure implies depressed interest rates and declines in consumption. However, if the importance of technology-enhanced-leisure is greater than that of a consumer's habit the reverse applies. In addition, technology-enhanced-leisure implies reductions in output of the technology sector if the growth is greater than the rate of time preference.

### **1.1.4 The consequence of robots for economic wellbeing**

The contribution of this chapter is to develop a general equilibrium framework to investigate the impact of robot development on economic well-being. The framework integrates leisure, endogenous technical change, substitution between robots purchased by the consumer, robots as a form of human replacement and heterogeneous skills of labour. This examination finds that incorporating leisure can imply increasing welfare in the presence of humans being replaced by robots.

### **1.1.5 The implications of singularity for workers**

This chapter contributes to this area by investigating the economic consequences of singularity. Singularity implies robots are better than humans in every way possible. Consequently, robots will have replaced workers for employment.





## Chapter 2

# Technology and Leisure: Macroeconomic Implications

This study explores the economic implications of technology on enhancing leisure. A theoretical model is developed which allows for habit formation for a technology good purchased to enhance leisure. A persistent fall in the relative price of the technology good and increased addiction to technology are shown to drive the real interest rate below the rate of time preference and depress consumption growth of non-technology goods. Using US data the model's prediction of falling interest rates and consumption growth are consistent with the recent observations of declining technology's relative prices and increases in technology good purchases.<sup>1</sup>

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1. This is based on a joint paper developed with Warwick McKibbin, see Kavuri and McKibbin (2017). In addition, I give special thanks to Adrian Pagan for his exceptional comments on various drafts of this paper. I also appreciate noteworthy advice from Bruce Preston and Ipppei Fujiwara and also thank Barry P. Bosworth, Megan Poore, David Stern and Peter J. Wilcoxon.

## 2.1 Introduction

In order to model the consequences on interest rates and consumption growth of the rising fascination with the use of digital technology in leisure, a habit formation model for technology-enhancing leisure purchases is introduced. This paper constructs a utility function for the consumer that separates normal consumption from the digital technology good used for leisure activities. The utility of the consumer depends on the level of the technology good purchased for leisure enhancement and on how these purchases compare to a habit stock. The impact of persistently falling technology prices and technology addiction<sup>2</sup> used to enhance leisure experience are examined. This analysis finds that the framework offers an explanation of observed interest rates and consumption growth over the past decades.

The remainder of the paper is organised as follows. In section 2.2, some stylised facts are introduced. In section 2.3, a simplified theoretical model is formulated. It involves a consumer who purchases a technology good to enhance leisure. The utility for the technology good involves habit formation.<sup>3</sup> Section 2.4 explores the steady state. Section 2.5 theoretically investigates the implications for interest rates and consumption growth of relative price change

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2. Ever since Stigler and Becker (1977) and Becker and Murphy (1988) there has been much economic literature on rational addiction. From a different perspective, we define addiction in terms of habits. In this regard, this investigation differs from habit persistence literature and economic addiction literature. For instance, Carroll, Overland and Weil (1997) and Overland, Carroll and Weil (2000) investigated the impact of habit persistence on a normal consumption good. The papers restrict  $\Theta$ , which indexes the importance of habits from between 0 and 1. Nonetheless, the conjecture in this paper is that addiction implies that a considerable amount of current technology is required to obtain a given utility. Consequently, the parameter should not be restricted to 1 and multiple times bigger.

3. The notion that an individual's utility depends on current consumption relative to a reference level is not new. See Constantinides (1990), Abel (1990), Carroll, Overland and Weil (1997), Overland, Carroll and Weil (2000), Deaton and Paxson (1992), Ferson and Constantinides (1991), Fuhrer (2000), Rayo and Becker (2007), and Atkin (2013) for interesting findings.

and technology addiction. Both of these perturbations drive the interest rate down below the rate of time preference. Section 2.6 studies the macroeconomic implications. The steady state equations of the model are applied to actual data. The predictions from the framework contribute quantitatively to the observed experience. Section 2.7 provides concluding remarks.

## 2.2 Stylised facts

### 2.2.1 Technology use in leisure activities

The time spent using a technology good<sup>4</sup> is significant. Lepp et al. (2015) found that 25 % of 454 US university students used their smartphone over 10 hours per day. Similarly, investigations by Junco and Cotten (2012) into the cell use of 1,649 college students found time spent per day is 118 minutes on the internet, 97 minutes on texting, 51 minutes talking, 49 minutes emailing and 41 minutes on Facebook.

Consequently, Psychologist Rosen (2012) in *'iDisorder: Understanding Our Obsession with Technology and Overcoming Its Hold on Us (2012)'* compares society's fascination with technology to the *habit of a drug addict*. Roberts, Pullig and Manolis (2015) suggest that there are similarities to substance and behavioural addictions with cell phone use including loss of control. The authors examine the relationship between personality traits and 'cell phone addiction' finding that impulsiveness is strongly associated with cell phone addiction. One important observation is that technology used to enhance leisure is almost exclusively where habits or addiction is forming. Deursen et al. (2015) found that those who use smartphones for leisure purposes develop smartphone habits faster. Lepp et al. (2013) found 88% of students used their phone primarily for leisure experience rather than for school. Despite the number of psychologists highlighting society's growing fascination with digital techno-

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4. A technology good is digital technology such as smartphones, video games and DVD players. These are mainly used to magnify leisure experience.

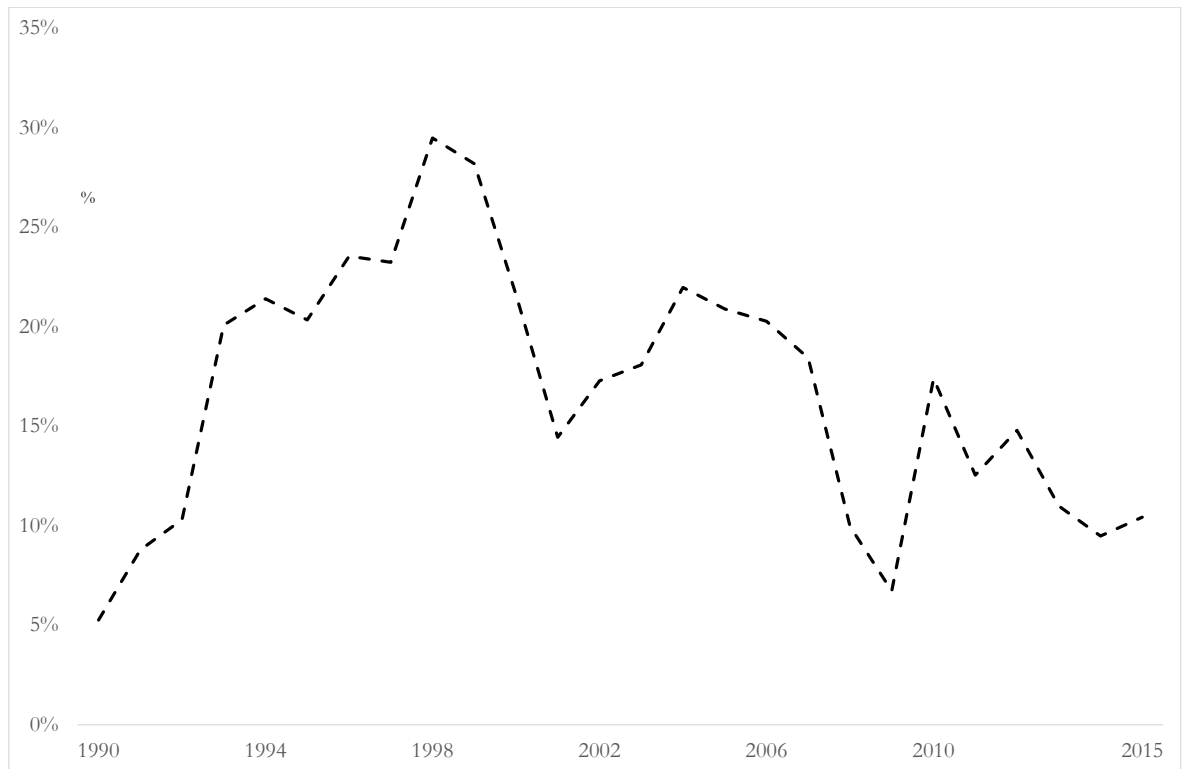
logy, there are limited economic studies that investigate the implications. An exception is Hurst (2016) who is one of the few economists who is investigating the implications of technology on leisure, primarily on labour supply.<sup>5</sup>

The annual percentage change in leisure technology consumed by the household is represented in Figure 4.1. This is the percentage change in the video, audio, photographic, and information processing equipment and media (VAPIM) chain-type quantity index devised by Federal Reserve Bank of St. Louis (2016c). Upward movements in the line indicate an increasing annual growth rate of consumed leisure technology. The data in Figure 1 consists of waves of technological innovation. This does not include watching programming distributed via the Web i.e., Netflix. Demand for personal computers was booming in the 1990s. After a slow start for the initial release of the Apple II (1977), by 1993, 4 million Apple II's were sold. There was another technology shift specifically in the electronic entertainment and video market in the late 1990s. DVDs were launched in 1996 (Seifert, Leleux and Tucci 2008), which was followed by the first DVD players in 1999. Huge uptake in DVD players followed. Further, the mobile phone market was strong. Nonetheless, the increase in leisure technology growth in 2001 could be due to a shift in the audio market. Apple was the engine for the transformation of mp3 technology. The iPod was officially released in October 2001 and growth soared, to 42 million sold in 2004 and, by April 2007, 100 million. Leisure technology growth fell drastically in the lead up to the Great Recession of 2008 and 2009.

---

5. His others investigating leisure time include Aguiar et al. (2017); Beraja, Hurst and Ospina (2016); Attanasio, Hurst and Pistaferri (2012); Aguiar, Hurst and Karabarbounis (2013) which may be of interest.

Figure 2.1: Percentage change in the quantity of technology in leisure activities



*Note:* The figure provides the percentage change in the chain-type quantity index for VAPIM.

*Source:* Federal Reserve Bank of St. Louis (2016c).

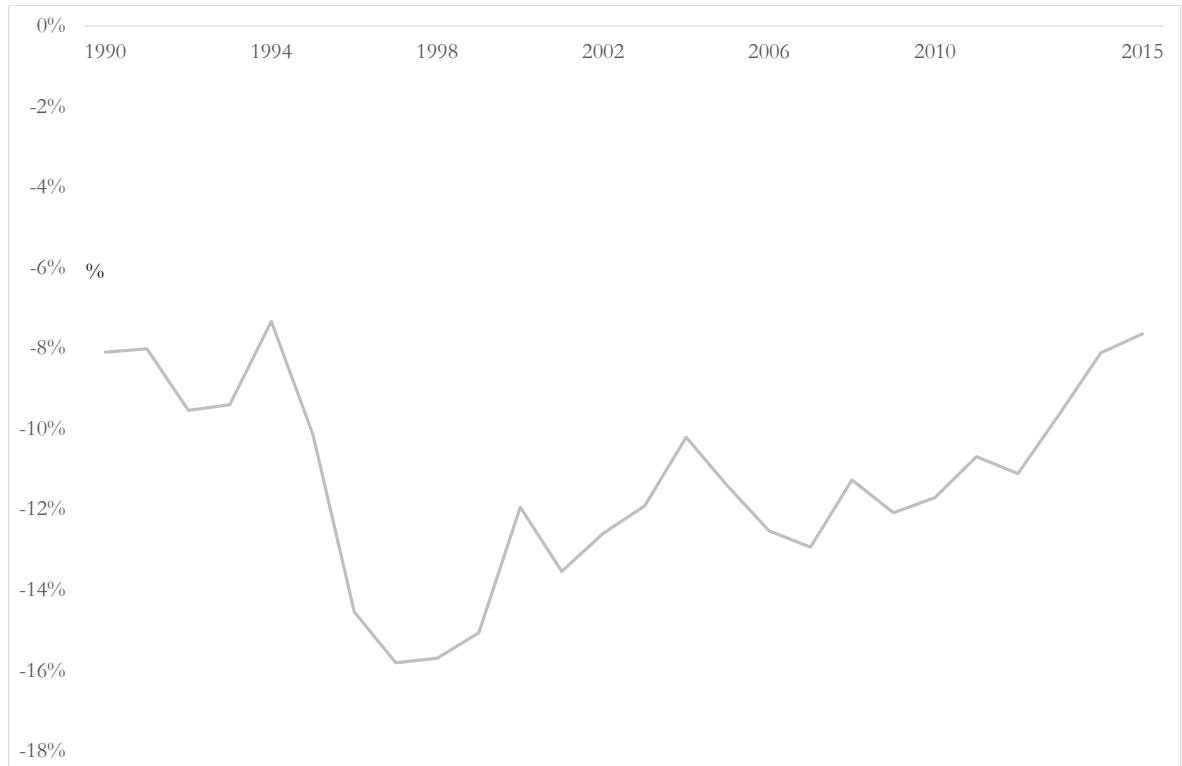
### 2.2.2 Percentage change in the relative price of technology

With the goal of creating a price series for technology used in leisure, I use data on price indices from the Federal Reserve Bank of St. Louis (United States). For leisure technology, VAPIM purchased for recreational uses by the consumer is used. To represent the change in the relative price of leisure technology an index based on the indices between technology in leisure activities and normal consumption is constructed. Specifically, this index is the ratio between the chain-type price index for VAPIM (Federal Reserve Bank of St. Louis 2016a) to the chain-type price index for total consumption (Federal Reserve Bank of St. Louis 2016b). The relative price of leisure technology is represented by  $p_R(t)$ . In the model below the annual percentage change in the relative price index is:

$$\left[ \frac{p_R(t+1) - p_R(t)}{p_R(t)} \right] \quad (2.1)$$

This is plotted in Figure 2.2 The relative price of technology has been constantly decreasing, mostly ranging between -10 % and -15 % per year. Jorgenson (2001) and Jorgenson and Vu (2007)) were the first to link the general price decline of technology to the economic growth of the United States and the G7. In a recent study, Jorgenson, Ho and Samuels (2016) investigate sub-periods of growth in the United States including 1973-1995, 1995-2000 (technology boom), 2000-2005 (post dot-com crash) and 2005-2010. Jorgenson, Ho and Samuels (2016) show that technology prices decline throughout all the time periods. For instance, relative to the GDP deflator, computers and equipment price growth was -15.9 percent (1973-1995), -26.3 percent (1995-2000), -17.6 percent (2000-2005) and -15.7 percent (2005-2010). Nonetheless in 1995, Jorgenson (2005) points out that the microprocessor price decline jumped to over ninety percent per year, which sparked IT prices to plummet. This had a domino effect on the prices of aircraft, automobiles and a multitude of other sectors that all use this technology. The study showed that even in the Great Recession, innovation was still substantial. Byrne, Oliner and Sichel (2015) show post-

Figure 2.2: Percentage change in the relative price of technology in leisure activities



*Note:*  $p_R(t)$  represents the ratio between the chain-type price indices for VAPIM and consumption.

*Source:* Federal Reserve Bank of St. Louis (2016a); Federal Reserve Bank of St. Louis (2016b).

Great Recession technology prices are still declining. They develop a hedonic index and show that the price of microprocessors declined by an average of forty-three percent per year from 2008 to 2013.

## 2.3 Model

In order to better understand the link between technology price declines and the impact on utility, a closed economy with an infinitely lived representative household is investigated.  $R(t)$  is the instantaneous flow of technology goods (e.g., ipads, Apple watches) for the representative household at time  $t$ . The economy is in discrete time with time in this period and the next period denoted by  $(t)$  and  $(t + 1)$ .  $h(t)$  is the stock of habits of the consumer for its purchases of the technology good. The household uses the technology good to enhance leisure experience ( $l$ ).  $\sigma$  is the coefficient of relative risk aversion and  $\rho$  the rate of time preference. For simplicity, the household does not supply labour and does not derive any income from work. This simplicity does not change the results themselves.<sup>6</sup> The household provides capital to firms to produce the consumption and the technology goods. The household maximises a discounted infinite discrete stream of utility:

$$\text{Max} \sum_{t=0}^{\infty} \left\{ \rho^t \left[ \frac{C(t)^{1-\sigma}}{1-\sigma} + \frac{(l(t)R(t)/h(t)^{\Theta})^{1-\sigma}}{1-\sigma} \right] \right\} \quad (2.2)$$

$$\sigma > 1$$

$$0 \leq l(t) \leq 1$$

$$\Theta \geq 0$$

Assume that the evolution of the habit stock of technology is taken as exogenous to the household (i.e., the household cannot influence the evolution of habits based on decisions). The habit stock of technology is a weighted average of past technology with  $\psi$  being the relative weight of technology at different times.  $\psi > 0$ , with larger values implying greater importance of the

---

6. The explicit trade-off between consumption and leisure was initially included, but this adds complications without changing the analysis in this paper. See next chapter for an example with labour employment.



recent past.  $\Theta$  indexes the importance of habits:<sup>7</sup>

$$h(t+1) = h(t) + \psi(R(t) - h(t)) \quad (2.3)$$

There are two goods produced in the economy: the consumption good (C) and the technology good (R). Assume the following production functions for technology and the consumption good is:

$$Y_R(t) = A_R(t)K_R(t) \quad (2.4)$$

$$Y_C(t) = A_C(t)K_C(t) \quad (2.5)$$

The household provides capital to the firms. Consequently, the following is the evolution of savings for the household:

$$\Delta K(t+1) = r(t)K(t) - \delta K(t) - p_R(t)R(t) - C(t) \quad (2.6)$$

$K(t) = K_R(t) + K_C(t)$ . The price of the technology good relative to the consumption good is  $p_R(t)$ . If the price of the consumption good is normalised to 1, then decreases in  $p_R(t)$  would mean the price of technology is decreasing compared to the price of the consumption good. Capital depreciates at  $\delta$ .

The consumer maximises a discounted infinite stream of utility as specified in equation (2.2). The choice variables are technology ( $R(t)$ ) and consumption ( $C(t)$ ). Neither leisure ( $l(t)$ ) nor habits ( $h(t)$ ) are choice variables. To optimise this dynamic discrete time problem, the constrained form of the Lagrangian is adopted. The problem is solved for each period given the respective constraints (i.e. equation (2.6)). (2.7) and (2.8) are obtained by reformulating the first order conditions:

$$\rho \left[ \frac{C(t)}{C(t+1)} \right]^{-\sigma} = [r(t+1) + (1 - \delta)] \quad (2.7)$$

---

7.  $\Theta$  represents the importance to the utility of current technology good purchases relative to habits. When  $\Theta = 0$ , the household cares only about the absolute level of the technology good. Its habits become irrelevant. As  $\Theta$  increases to compensate for its habits, more of the leisure technology good is required to obtain a given amount of utility.

$$\rho \left[ \frac{(l(t)R(t)/h(t)^\Theta)}{(l(t+1)R(t+1)/h(t+1)^\Theta)} \right]^{-\sigma} \left[ \frac{l(t)}{l(t+1)} \right] \left[ \frac{h(t+1)}{h(t)} \right]^\Theta \left[ \frac{p_R(t+1)}{p_R(t)} \right] = [r(t+1) + (1 - \delta)] \quad (2.8)$$

$$\frac{K(t+1)}{K(t)} = 1 + r(t) - \frac{p_R(t)R(t)}{K(t)} - \frac{C(t)}{K(t)} \quad (2.9)$$

$$\frac{h(t+1)}{h(t)} = 1 + \psi \left( \frac{R(t)}{h(t)} - 1 \right) \quad (2.10)$$

## 2.4 Steady state

The next task is to explore the steady state of the model. Following Mulligan and Sala-i-Martin (1992), the equations are transformed into variables that are constant in the steady state.<sup>8</sup> Define the steady state for the technology sector when  $R$  and  $h$  grow at the same rate.<sup>9</sup> For their consumption-based habit model Overland, Carroll and Weil (2000) also defined steady state in a similar fashion. The ratio of the technology good and leisure per habits ( $\frac{Rl}{h}$ ) are constant in the steady state. Mulligan and Sala-i-Martin (1992) point out that in most sectors the consumption good to consumption-sector capital is a constant ratio. As the consumption sector does not have habits, this is applicable here. The steady state for the consumption sector is defined when  $\frac{C}{K_C}$  is a constant ratio. This implies that the following holds:

$$g_R = g_h \quad (2.11)$$

$$g_C = g_{K_C} \quad (2.12)$$

where:

$$\left[ \frac{R(t+1)-R(t)}{R(t)} \right] = g_R, \quad \left[ \frac{C(t+1)-C(t)}{C(t)} \right] = g_C, \quad \left[ \frac{K_C(t+1)-K_C(t)}{K_C(t)} \right] = g_{K_C}.$$

Equations (2.7) to (2.10) are used to derive the following system applicable

8. As we are investigating the steady state,  $t$  for time is dropped.

9. More accurately, the technology good and leisure combined should grow at the rate of habits. Nonetheless, growth of leisure is zero in the steady state.

to the steady state. See Appendix for the full derivation.

$$g_R + \frac{\pi_R}{\sigma - \Theta\sigma + \Theta} = \frac{r_{ss} - \rho}{\sigma - \Theta\sigma + \Theta} \quad (2.13)$$

$$g_C = \frac{r_{ss} - \rho}{\sigma} \quad (2.14)$$

$$\pi_R = \lambda_R \quad (2.15)$$

Where  $\lambda_R$  is a constant growth/decline of the price of technology relative to consumption and  $r_{ss}$  is the interest rate in the steady state.<sup>10</sup>

The equations imply that the following holds with  $r_{ss}$  substituted out:

$$(\sigma - \Theta\sigma + \Theta)g_R + \lambda_R = \sigma g_C \quad (2.16)$$

It is important to recognise that  $r_{ss}$  may be described as a yield on capital.

### 2.4.1 Equilibrium

The growth rates for this benchmark steady state are as follows:

$$\pi_R = 0 \quad (2.17)$$

$$g_R = 0 \quad (2.18)$$

$$g_C = 0 \quad (2.19)$$

$$r_{ss} - \rho = 0, \quad (2.20)$$

## 2.5 Implications of perturbations

This section investigates two different exogenous perturbations. One is the impact of a sustained decline in the relative price of technology. The other is a sustained period of addiction. A relative price decline may occur from a technology shift. Addiction may result from an impulse to purchase a cutting-edge technology good. Equation (2.16) highlights the dynamics of movements of the steady states under the temporary perturbations. However, as a theoretical exploration, this paper investigates each perturbation in isolation. The

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10.  $r_{ss} = r(t+1) = r(t)$ .

appendix highlights the general case.<sup>11</sup>

### 2.5.1 Perturbation one: Relative price decline

**Proposition 1** (*Relative price proposition*): *If growth rate of leisure technology is zero, a persistent decline in the price of technology to consumption will lead to the interest rate falling below the rate of time preference.*

#### Proof

A sustained, decreasing relative price of technology to consumption  $\pi_R = \Lambda_R < 0$ . The economy is sent into a dynamic adjustment path. With this perturbation equations of the model shows that the following will apply:

$$g_R = 0 \quad (2.21)$$

$$\pi_R = \Lambda_R < 0 \quad (2.22)$$

$$g_C = \frac{\Lambda_R}{\sigma} < 0 \quad (2.23)$$

$$r_{ss} - \rho = \Lambda_R, \quad (2.24)$$

As  $\Lambda_R$  is negative, the last condition implies that  $r_{ss} < \rho$ .

Consumption growth is negative and interest rates will fall persistently.

QED

### 2.5.2 Perturbation two: Addiction

To focus solely on technology addiction, consider the economy back at the benchmark growth rates with  $r_{ss} = \rho$ . Consider now the second proposition of this paper.

**Proposition 2** (*Addiction proposition*): *Other things being equal, technological addiction in leisure will cause the interest rate to fall below the rate of time preference.*

---

11. Notice that unless there are some adjustments during the perturbations in  $A_R, A_C$  or markups, steady state conditions on the production side will not apply.

**Proof**

This paper defines technology addiction as:

$$\frac{\sigma}{\sigma - 1} < \Theta \quad (2.25)$$

Notice, with addiction, the parameter  $(\sigma - \Theta\sigma + \Theta)$  is negative. To understand the impact, recall the relationship between  $g_R$  and  $r_{ss}$ :

$$(\sigma - \Theta\sigma + \Theta)g_R + \pi_R = r_{ss} - \rho \quad (2.26)$$

As  $\pi_R = 0$ , addiction will send consumption of non technology goods into decline.<sup>12</sup> Further, as can be seen in the equation above, it will drive interest rates below  $\rho$ .<sup>13</sup>

$$\pi_R = 0 \quad (2.27)$$

$$g_R = \bar{g}_R > 0 \quad (2.28)$$

$$g_C = \frac{(\sigma - \Theta\sigma + \Theta)\bar{g}_R}{\sigma} \quad (2.29)$$

$$r_{ss} - \rho = (\sigma - \Theta\sigma + \Theta)\bar{g}_R \quad (2.30)$$

QED

This model provides some interesting insights. Economies have been characterised by consumption growth falling, technology booming with plummeting interest rates. With addiction, more and more of an activity, product, drug, etc., is required for recreational benefit. Consequently, growth of non technology consumption is negative. This leads to depressed interest rates. The economy finds itself at this new state with consumption declining every year, interest rates depressed but with growth in the technology good growth.

---

12. We can express  $g_C$  in terms of  $g_R$ . With positive  $g_R$  addiction implies  $g_C < 0$ . Although  $g_R$  can be negative, here we investigate the theoretical impact of positive growth for our addiction case study. In the empirical section, as we are using real data we allow for negative growth in technology.

13. Note that  $r(t) > \bar{g}_R$

### 2.5.3 Perturbation: General case

Here we let both  $g_R$  and  $\pi_R$  to adjust and study the conditions for when  $r_{ss} < \rho$ . Four propositions for the general case are illustrated.

**Proposition 3** *General case: interest rate will fall below the rate of time of preference if the following holds:*

$$(\sigma - \Theta\sigma + \sigma)g_R + \pi_R < 0 \quad (2.31)$$

**Proposition 4** *(General case: Relative price decline) A persistent decline in the price of technology ( $\pi_R < 0$ ) to consumption will lead to the interest rate falling below the rate of time preference if the following holds:*

$$(\sigma - \Theta\sigma + \sigma)g_R < |\pi_R| \quad (2.32)$$

**Proposition 5** *(General case: Addiction) Regardless of movements in the relative price of technology, technological addiction in leisure will cause the interest rate to fall below the rate of time preference when the following holds:*

$$\pi_R < |(\sigma - \Theta\sigma + \sigma)g_R| \quad (2.33)$$

**Proposition 6** *The price elasticity of demand of leisure technology ( $g_R$ ) is as follows:*

$$\eta = \frac{-1}{\sigma - \Theta\sigma + \sigma} \frac{\pi_R}{g_R} \quad (2.34)$$

## 2.6 Macroeconomic implications

The previous section demonstrated the impact of technology in leisure on the macroeconomy in theory. One key result is that the two perturbations would drive the interest rate below the rate of time preference. This section

explores the macroeconomic implications generated through our model. First, to motivate this analysis, consider some stylised facts of the present economic environment.

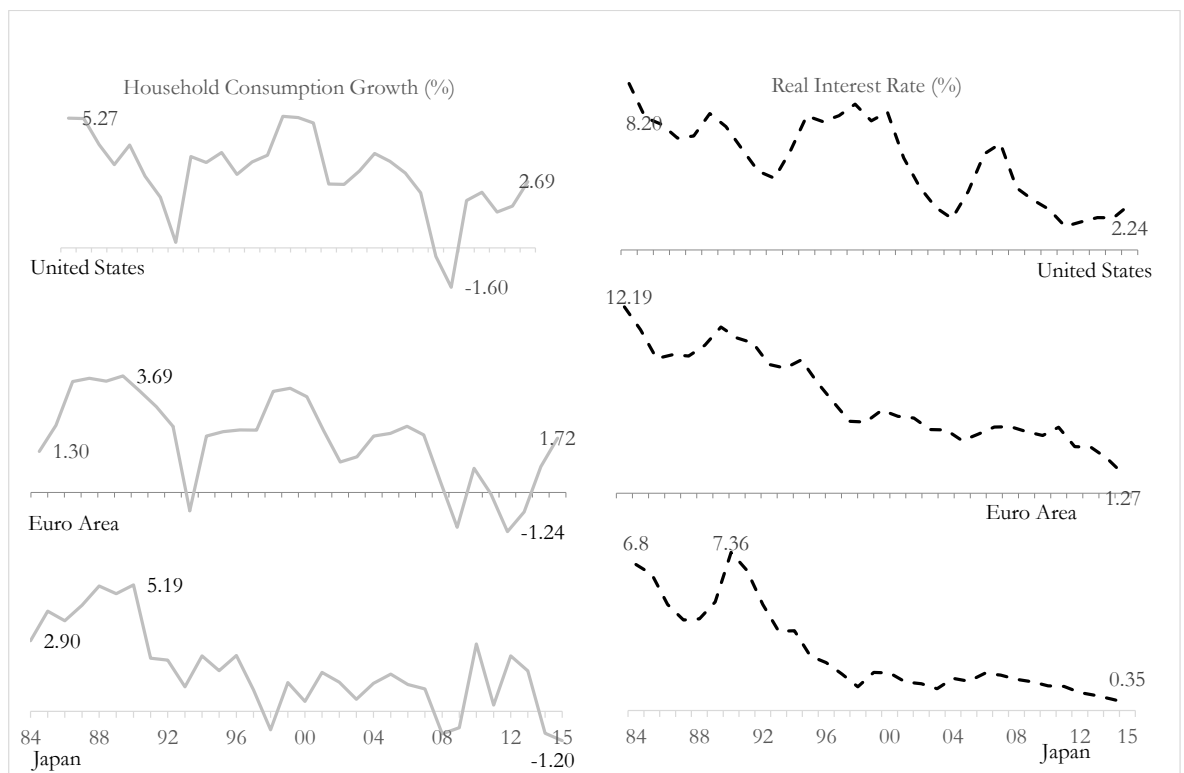
### **Stylised facts**

1. Low interest rate environment: There has been a period of persistently low real and nominal interest rates for OECD countries especially since 2000 (see Figure 2.3).
2. Low consumption growth: The decline started around 15-20 years ago for the vast majority of OECD economies. However, post-Great Recession, the trend has accelerated. Petev, Pistaferri and Saporta (2012) argues that out of all the recent US recessions consumption remains below the pre-recession levels for a longer period. Figure 2.3 presents household consumption growth and real interest rates.<sup>14</sup> over the last 30 years. As can be seen, both have been on a downward trend. Nonetheless, since the Great Recession, a new equilibrium appears to be emerging with depressed rates and low consumption growth.

---

14. We use World Bank's data for household consumption growth for the United States, Euro Area and Japan. We use World Bank's data for Japan's real interest rate and the real interest rate for the United States. The OECD interest rate estimations are used for the Euro Area. The World Bank's real interest estimations tend to be lower and have more fluctuations than the OECD calculations.

Figure 2.3: Decline of consumption growth and real interest rates



Source: World Bank (2016a); OECD (2016); World Bank (2016b).



A rising number of papers highlight various reasons for the low interest rates and lack of consumption growth, including demand-side secular stagnation (Summers 2015), supply side secular stagnation (Gordon 2015), overhanging debt (Rogoff and Reinhart 2010) and a liquidity trap (Bernanke 2016) In conjunction with these studies, we offer an additional explanation related to the increasing use of technology in leisure and the large fall in the relative price of this technology.<sup>15</sup>

Now plausible parameters in the theoretical model are used to generate paths of interest rates and real consumption growth given observed perturbations to the model.

### 2.6.1 Interest rates

Interest rates are computed using US data from 1990 to 2015 to investigate the dynamics associated with a moving steady state. This ignores the transitional adjustment between steady states.

$$(\sigma - \Theta\sigma + \Theta) \left[ \frac{R(t+1) - R(t)}{R(t)} \right] + \left[ \frac{p_R(t+1) - p_R(t)}{p_R(t)} \right] = r(t+1) - \rho \quad (2.35)$$

For Figure 2.4, 2.5, 2.6 and 2.7 and 2.8 we use 1.5 % (Evans and Sezer 2004) for  $\rho$  and 1.3 for  $\sigma$  (coefficient of relative risk aversion) (Zhuang et al. 2007). A 5-year moving average is provided to smooth the variability. In the figures, the black dash line is the computed interest rate. The grey solid line is the actual annual interest rate (World Bank 2016b). The consumers do not have addictions. However,  $\Theta$  is relatively high at 1.5.<sup>16</sup> Figure 2.4 and 2.5 shows the dynamics over the last 25 years. Figure 2.4 is calculated using the raw data. In figure 2.5 the data is smoothed using a 5-year moving average.

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15. Bosworth (2014) makes a valid point that it makes little sense to forecast interest rates within a closed-economy framework as markets are integrated globally. Nonetheless, we hope that the framework provides some useful insights.

16. Addiction implies  $\Theta > 4.33$ .

Figure 2.4: Computed and actual annual interest rates

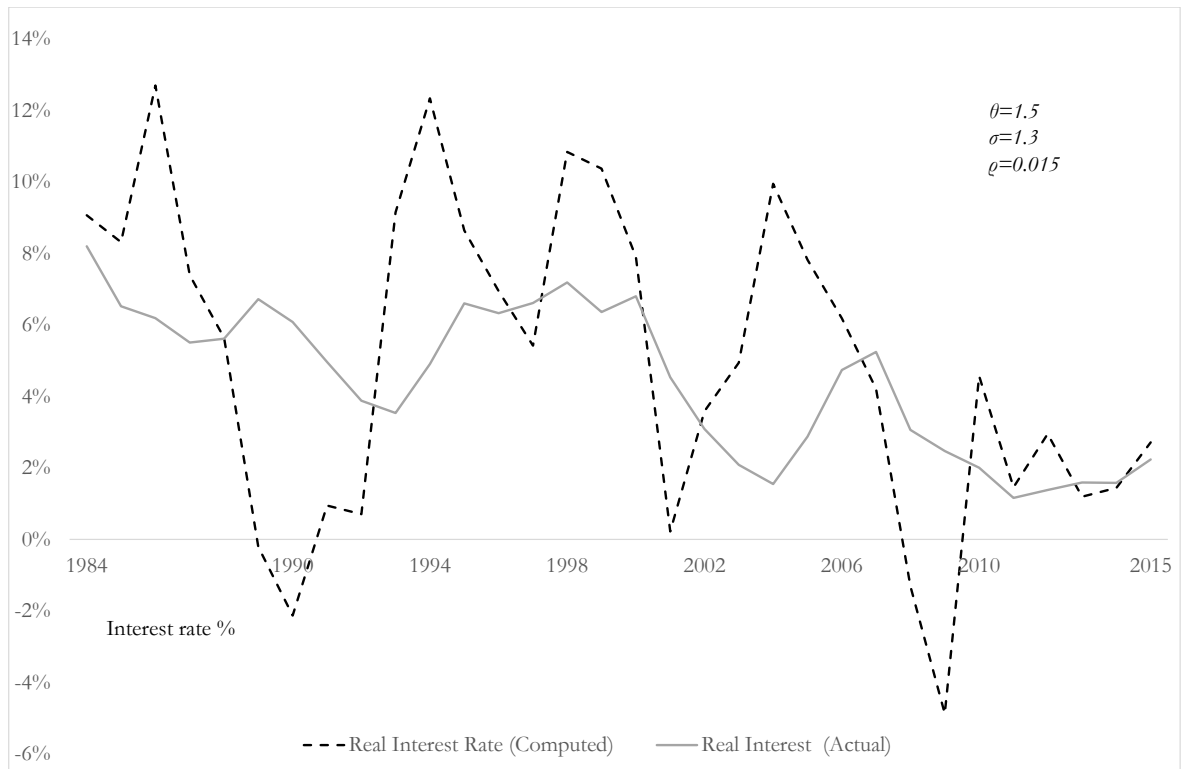
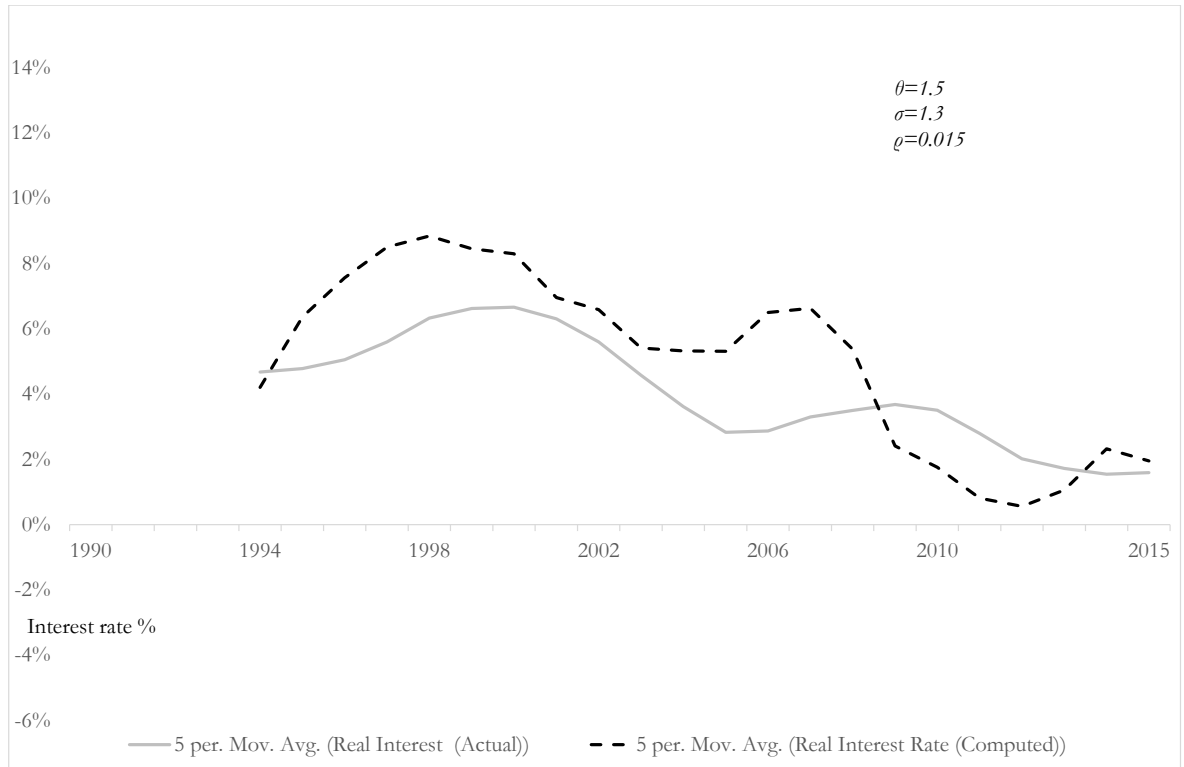


Figure 2.5: Computed and actual annual interest rates: 5-year moving average



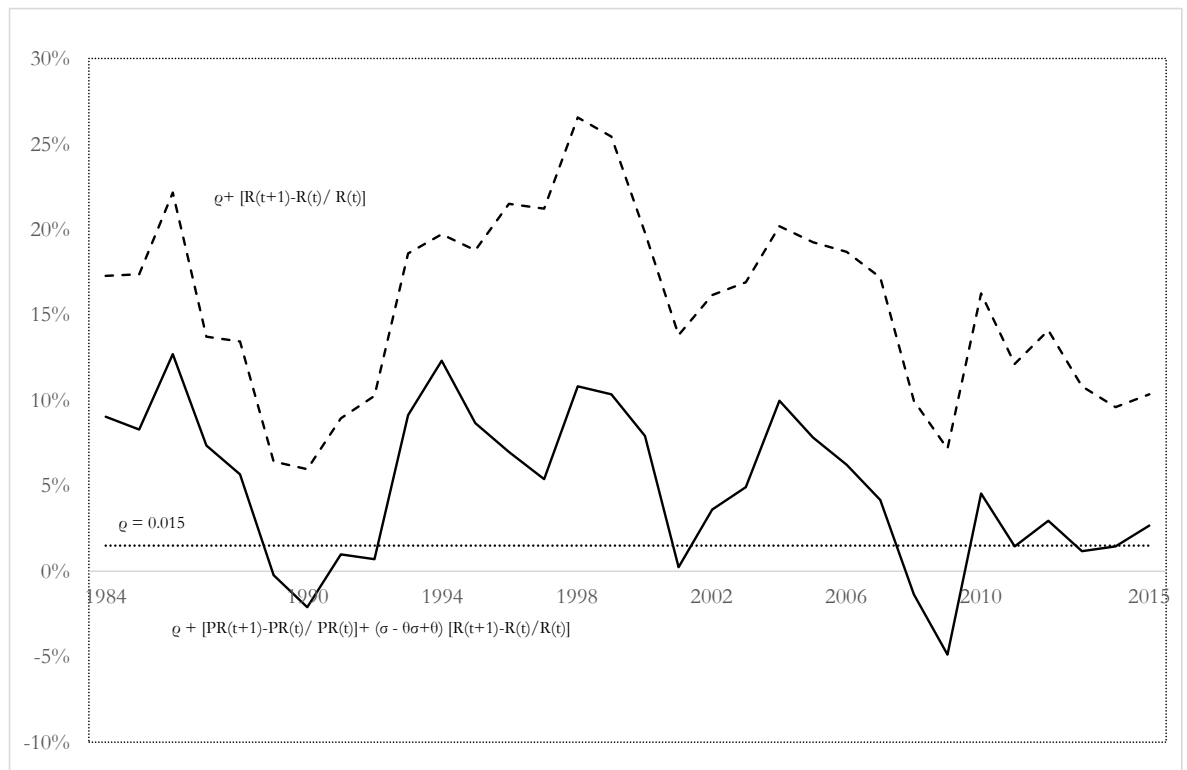
*Note:* The predicted interest rate are calculated using data from Federal Reserve Bank of St. Louis. The observed annual interest rate is obtained from the World Bank.

*Source:* Federal Reserve Bank of St. Louis (2016a); Federal Reserve Bank of St. Louis (2016b); World Bank (2016b).

In Figure 2.6 this paper decomposes the computed interest rate into  $\rho$ , the growth of leisure technology purchased and the change in the relative price of leisure technology. Intuitively, higher growth in leisure technology purchased leads to higher interest rates. Growth in leisure technology implies a production shift with businesses borrowing for future profit. The growth in produc-

tion ensures higher real interest rates. As the relative price of technology falls consumers shift from consumption of goods to consumption of leisure, which drives down the real interest rate.

Figure 2.6: Decomposition



Note: Actual/raw data.

### 2.6.2 Consumption growth

A two-step approach to compute consumption growth is taken. First, the computed interest rates from our model are obtained, After which, consumption growth is computed with the formula below. It has been suggested that computed rates from the Euler equation can be very different to the actual

interest rate.<sup>17</sup>

$$\frac{r(t+1) - \rho}{\sigma} = \left[ \frac{C(t+1) - C(t)}{C(t)} \right] \quad (2.36)$$

Using equation (2.32), we plot the calculated rate of consumption growth in Figure 2.7 and 2.8. Figure 2.7 is calculated using the actual data. Figure 2.8 is Figure 2.7 transformed using a 5-year moving average.<sup>18</sup>

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17. Hansen and Singleton (1982) and Mulligan (2004) show that aggregate consumption Euler equations are very poor fits to the empirical data. Furthermore, Canzoneri, Cumby and Diba (2007) compute interest rates implied by the consumption Euler equations for various models with different consumer preferences and compare them with money market rates. They find that the correlations between these Euler equation rates and the Federal Funds rate are generally negative. The models include CRRA preferences (-0.37), Abel (1999) (-0.36), Campbell and Cochrane (1995) (-0.37), Fuhrer (2000) (-0.07), Boldrin, Christiano and Fisher (2001), Edge (2002) and Christiano, Eichenbaum and Evans (2005) (-0.09).

18. The parameters chosen for equation 2.32 are  $\Theta = 1.5, \sigma = 1.3, \rho = 0.015$ .

Figure 2.7: Computed and actual consumption growth



Figure 2.8: Computed and actual consumption growth: 5-year moving average



*Note:* The predicted and actual consumption growth are calculated using data from the Federal Reserve Bank of St. Louis.

*Source:* Federal Reserve Bank of St. Louis (2016a); Federal Reserve Bank of St. Louis (2016b); Federal Reserve Bank of St. Louis (2016c); Federal Reserve Bank of St. Louis (2016d).

I acknowledge that the model makes better predictions over the last 25 years. This would imply that either the model is not appropriate prior to 1984 or changes in data estimation procedures led to better quality of data.

However, the figures show that the shifting steady state caused by the decline in the price of technology used in leisure and technology habits is

consistent with trend changes in real interest rates and consumption observed in the past 20 years.

## **2.7 Concluding remarks**

This paper has provided a theoretical framework to study the macroeconomic implications of a technology good purchased by the consumer to enhance leisure activities. Furthermore, predictions are made based on United States data to determine how much the framework can contribute to our understanding of recent economic trends.

There is a number of explanations for the global decline in real interest rates and consumption growth. This paper proposes that technology enhanced leisure and addiction to technology may be a further explanation that warrants further investigation.



## 2.8 Appendix

### 2.8.1 Technology: Sector R

#### Growth of $R(t)$

The steady state occurs for sector  $R$  when leisure and technology combined grow at the rate of habits growth. The following holds for the steady states

$$\frac{R(t+1)l(t+1)}{h(t+1)} - \frac{R(t)l(t)}{h(t)} = 0 \quad (2.37)$$

Obviously:

$$\frac{l(t+1)}{l(t)} \frac{R(t+1)}{R(t)} = \frac{h(t+1)}{h(t)} \quad (2.38)$$

Equation (2.8) is used to obtain the steady state level change of  $R(t)$  as below:

$$\rho \left[ \frac{p_R(t+1)}{p_R(t)} \right] \left[ \frac{R(t+1)}{R(t)} \right]^{\sigma-\Theta\sigma+\Theta} \left[ \frac{l(t+1)}{l(t)} \right]^{\sigma-\Theta\sigma+\Theta-1} = [1+r(t+1)] \quad (2.39)$$

With  $R$  the subject it follows:

$$\left[ \frac{R(t+1)}{R(t)} \right]^{\sigma-\Theta\sigma+\Theta} = \left[ \frac{1+r(t+1)}{\rho} \right] \left[ \frac{l(t+1)}{l(t)} \right]^{-(\sigma-\Theta\sigma+\Theta-1)} \left[ \frac{p_R(t+1)}{p_R(t)} \right]^{-1} \quad (2.40)$$

The growth of habits is substituted into the above formula to obtain:

$$\left[ \frac{1+r(t+1)}{\rho} \right] \left[ \frac{l(t+1)}{l(t)} \right]^{-(\sigma-\Theta\sigma+\Theta-1)} \left[ \frac{p_R(t+1)}{p_R(t)} \right]^{-1} = \left( \left( \psi \frac{R(t)}{h(t)} - \psi + 1 \right) \frac{l(t)}{l(t+1)} \right)^{\sigma-\Theta+\Theta} \quad (2.41)$$

The equation is reformulated. Natural logs are taken to obtain approximations.

$$\ln \left[ \frac{p_R(t+1)}{p_R(t)} \right] \approx \frac{p_R(t+1) - p_R(t)}{p_R(t)} \quad (2.42)$$

$$\ln \left[ \frac{1+r(t+1)}{\rho} \right] \approx r(t+1) - \rho \quad (2.43)$$

The steady state level of technology and leisure to habits is obtained as follows:

$$\frac{R(t)l(t)}{h(t)} = l(t) \left[ \frac{1}{\psi} \left( \frac{r(t+1) - \rho}{\sigma - \Theta\sigma + \Theta} - \frac{1}{(\sigma - \Theta\sigma + \Theta)} \left[ \frac{p_R(t+1) - p_R(t)}{p_R(t)} \right] \right) + 1 \right] \quad (2.44)$$

Note that in the steady state the following applies:

$$\left[ \frac{l(t+1) - l(t)}{l(t)} \right] = 0 \quad (2.45)$$

### 2.8.2 Equilibrium

With natural log the equations below apply:

$$r(t+1) - \rho = (\sigma - \Theta\sigma + \Theta) \left[ \frac{R(t+1) - R(t)}{R(t)} \right] + \left[ \frac{p_R(t+1) - p_R(t)}{p_R(t)} \right] \quad (2.46)$$

$$r(t+1) - \rho = \sigma \left[ \frac{C(t+1) - C(t)}{C(t)} \right] \quad (2.47)$$

As this is the stationary state,  $r(t)^* = r(t+1)^*$ . In that case steady state levels of technology to capital and consumption to capital simplify further. However, to avoid confusion,  $r_{ss} = r(t+1)$  is used to represent the interest rate in the steady state.

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## Chapter 3

# Could habits make the labour income share fall?

Could habits in the consumption of technology in leisure activities make the labour income share fall? A competitive equilibrium model is constructed, where consumer utility is separable to normal consumption and a technology good-such as video games or smartphones-whose contribution to utility depends on habit formation. This paper shows that leisure technology habits can theoretically drive capital per hours worked and the labour income share downwards. Modelling the framework with US data, predictions for major macroeconomic trends are consistent with actual observations. The model predicts trends in wages, interest rates, and the labour income share that all are compatible with the data since the mid-1990s.<sup>1</sup>

### 3.1 Introduction

Could habits make the labour income share fall? The first contribution of this paper is to show that habit formation in the consumption of leisure techno-

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1. I am particularly grateful for Warwick J. McKibbin, Renée McKibbin and David Stern for valuable constructive criticisms. I also thank Megan Poore, Paul Hubbard, Maxmillian Wakefield, Arjuna W. Mohottala and Benjamin Ascione for informative comments.

logy can cause labour's income share to fall. The second and most important contribution is to show with U.S data that the framework predicts trends in interest rates, wages and the labour income share that are consistent with observations since the mid-1990s.

The literature agrees that the labour income share decreased in nearly all of the most advanced economies in the world over the last twenty years (see International Labour Organization Organisation for Economic Co-operation and Development 2015; International Labour Office 2012; Piketty 2015 and Piketty, Goldhammer and Ganser 2014. Note that Rognlie (2016) and Rognlie (2014) disagrees due to the impact of housing in the US.). The decline has also been apparent in the US, particularly since the 2000s (Autor et al. 2017). Economists have explained this phenomenon by international trade and import shocks (Elsby, Hobijn and Sahin 2013), by the falling cost of capital relative to the cost of labour (Karabarbounis and Neiman 2014), and by 'superstar' firms (Autor et al. 2017<sup>2</sup>). However, there is no consensus for the decline in labour income shares (Autor et al. 2017). This paper provides a new theory for the decline in labour share. Integral to this theory is habit formation for the consumer, in which utility depends on a digital technology good purchased for leisure activities. Consequently, digital technology impacts preferences for leisure, consumption and work among other variables. Given that models with consumer habit formation have been adopted to explain diverse macroeconomic variables, habits may prove insightful for this analysis. For instance, models with consumer habit formation have successfully resolved the equity premium puzzle (Mehra and Prescott 1985) and they can show that increases in growth cause increased saving (Overland, Carroll and Weil 2000). Furthermore, an empirical test has rejected the hypothesis of no habit formation for consumers

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2. The authors introduce a superstar firm model which incorporates a 'winner take the most' idea. Large firms, take a larger share of the market but has lower shares due to the fixed amount of overheads amongst other aspects.

in a monetary-policy model (Fuhrer 2000). However, a recent study found that consumption commitments can also explain the empirical findings that consumption is smooth that is typically attributed to habit formation (Chetty and Szeidl 2016).

A habit formation model, in which utility depends on a technology good purchased for leisure activities (leisure technology) has gained little or no attention. However, there are reasons to believe that consumers form habits for leisure good technology purchases of smartphones, the Internet and video games. Rosen (2012) compares society's technology obsession to the 'habit of a drug addict'. Indeed, there are similarities to substance and behavioural addictions such as loss of control of cell phone use (Roberts, Pullig and Manolis 2015). What are the economic implications of consumers forming these sorts of habits? Aguiar et al. (2017) argues that leisure technology can have important effects on labour supply. The goal here is to investigate the impact of consumers' leisure technology habit formation on labour's income share.

A general equilibrium framework with two sectors and a representative consumer is constructed. The consumer's utility function distinguishes between normal consumption where habits are irrelevant and consumption of the digital technology good where the utility depends on the amount of the good consumed relative to previous habits. The digital technology good is used to enhance leisure.<sup>3</sup> The first contribution of this paper is to show, theoretically, that habits formed in leisure technology drive both capital per hours worked and the labour income share downwards. Other implications include reductions in normal consumption (World Bank 2016a) and depressed interest rates (World Bank 2016b). The most important contribution of this paper is to investigate the empirical relevance of the theory. Using U.S. data, predictions from the framework are consistent with trends in major macroeconomic vari-

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3. Notice that individuals using smartphones for social purposes develop smartphone habits faster (Deursen et al. 2015).

ables since the mid-1990s.

The rest of the paper is organized as follows. Section 3.2 builds the framework for the representative consumer and the sectors. Section 3.3 explores the steady state with section 3.4 formulating the income share-habit proposition. Section 3.5 investigates the impact of two perturbations (exogenous changes in relative prices and addiction to the leisure technology good<sup>4</sup>) on macroeconomic trends. Section 3.6 models the framework with US data. Ever since the mid-1990s, predictions from the framework are correlated with actual trends. Section 3.7 concludes.

## 3.2 The model

The representative consumer derives instantaneous utility at time  $t$  as specified in equation (3.1):<sup>5</sup>

$$u(t) = \frac{C(t)^{1-\sigma}}{1-\sigma} + \frac{(l(t)R(t)/h(t)^\Theta)^{1-\sigma}}{1-\sigma} \quad (3.1)$$

$$0 \leq l(t) \leq 1$$

$$\Theta \geq 0$$

Normal consumption and the technology good are represented by  $C(t)$  and  $R(t)$  respectively. The utility function is separable in normal consumption where habits are inconsequential and the technology good where habits are critical. For the normal consumption good, only the absolute level is relevant to the consumer. In contrast, the consumer cares about technology goods relative to a stock of habits denoted by  $h(t)$ .<sup>6</sup> Use of these goods enhance time spent on leisure,  $l(t)$ .  $\sigma > 1$  is the coefficient of relative risk aversion.<sup>7</sup>  $\Theta$  specifies the

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4. Addiction is a perturbation as habits are many times stronger than typical habit formation.

5. Kavuri and McKibbin (2017) originally developed the utility function to investigate implications on interest rates and consumption growth. The framework was in discrete time.

6. Abel (1990), Carroll, Overland and Weil (1997) and Overland, Carroll and Weil (2000) for their consumption based habit models in a similar way relate consumption to a habit stock.

7. Overland, Carroll and Weil (2000) highlight that the literature has tended to find it greater than one

importance of habits. For instance, when  $\Theta = 0$ , habits drop out of the utility function and only current purchases of the technology good is relevant. The higher  $\Theta$  is, the more purchases of the technology good are needed to achieve a given level of utility, given the stock of habits. Habits evolve as follows:

$$\dot{h}(t) = \psi(R(t) - h(t)) \quad (3.2)$$

Habits are a weighted average of past technology. The associated weight at various times is represented by  $\psi$ .  $\psi > 0$ . The budget constraint for the consumer is as follows:

$$\dot{K}(t) = r(t)K(t) - p_R(t)R(t) - C(t) + w(t)(1 - l(t)) \quad (3.3)$$

Capital is denoted by  $K(t)$ .<sup>8</sup>  $r(t)$  represents the interest rate. The consumer provides capital to the sector that produces the normal consumption good and to the technology good. The price of the consumption good is normalized to one.<sup>9</sup>  $p_R(t)$  represents the price of the technology good relative to the normal consumption good. The consumer maximises a discounted infinite stream of utility:

$$U = \int_0^{\infty} u(C(t), h(t), R(t), l(t)) e^{-\rho t} dt \quad (3.4)$$

The current value Hamiltonian is used to solve the consumer problem. The maximization problem is illustrated in equation (3.5). The choice variables of the consumer are  $C(t)$ ,  $R(t)$  and  $l(t)$ . The state variable for the maximization problem is  $K(t)$ :

$$H = \frac{C(t)^{1-\sigma}}{1-\sigma} + \frac{(l(t)R(t)/h(t)^\Theta)^{1-\sigma}}{1-\sigma} \quad (3.5)$$

$$+ v_K(t)(r(t)K(t) + w(t)(1 - l(t)) - C(t) - p_R(t)R(t))$$

Appendix provides the complete solution to the problem. There are eight dynamic equations of motion. Appendix A provides the dynamic equations

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8. Note that the theory here depends on the web-based consumer products how comparative capital intensity. Further research should provide evidence for this analysis.

9.  $p_C(t) = 1$ .

for the evolution of the technology good, habits, the relative price of technology, the consumption good, capital provided to the technology good sector, capital provided to the normal consumption good sector and wages. There are two production sectors: one producing the technology good and the other producing the consumption good. The production functions take the form as below:

$$Y_R(t) = [\gamma_{K_R}(A_{R_K}(t)K_R(t))^{\frac{\epsilon-1}{\epsilon}} + (1 - \gamma_{K_R})(A_{R_L}(t)N_R(t))^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}} \quad (3.6)$$

$$Y_C(t) = [\gamma_{K_C}(A_{C_K}(t)K_C(t))^{\frac{\beta-1}{\beta}} + (1 - \gamma_{K_C})(A_{C_L}(t)N_C(t))^{\frac{\beta-1}{\beta}}]^{\frac{\beta}{\beta-1}} \quad (3.7)$$

Where  $\gamma_{K_R}$  and  $\gamma_{K_C} \in (0, 1)$  are the cost share of capital.  $N_R(t) = (1 - l_i(t))$  and  $K_R(t)$  are the use of labour and capital by the technology goods producing sector. Similarly,  $N_C(t)$  and  $K_C(t)$  is labour and capital use by the consumption good producing sector.  $A_{C_K}(t)$ ,  $A_{C_L}(t)$ ,  $A_{R_K}(t)$  and  $A_{R_L}(t)$  are technology terms.  $\epsilon$  and  $\beta \in (0, \infty)$  represent the elasticity of substitution.  $p_R(t)$  is the relative price of the technology good with the price of the consumption good normalized to one. Firms maximize profit.

### 3.3 The steady state

Following Mulligan and Sala-i-Martin (1992), Overland, Carroll and Weil (2000), and Kavuri and McKibbin (2017), the dynamic system is transformed into variables that are constant in the steady state. Consequently, as habits are formed for the technology good,  $R$  needs to grow at the same rate as  $h$ .

$$g_h = g_R \quad (3.8)$$

Where  $\frac{\dot{h}(t)}{h(t)} = g_h$ ,  $\frac{\dot{R}(t)}{R(t)} = g_R$ ,  $\frac{\dot{C}(t)}{C(t)} = g_C$ ,  $r(t) = r_{ss}$  in the steady state. Appendix C outlines the steps undertaken in order to determine the steady state. The dynamical equations that are required are:

$$g_R + \frac{\pi_R}{(\sigma - \Theta\sigma + \Theta)} = \frac{r_{ss} - \rho}{(\sigma - \Theta\sigma + \Theta)} \quad (3.9)$$

$$g_R + \frac{\pi_w}{(\sigma - 1 - \Theta\sigma + \Theta)} = \frac{r_{ss} - \rho}{(\sigma - 1 - \Theta\sigma + \Theta)} \quad (3.10)$$

$$g_C = \frac{r_{ss} - \rho}{\sigma} \quad (3.11)$$

$$\pi_R = \Lambda_R \quad (3.12)$$

$$\pi_w = \Lambda_w \quad (3.13)$$

Where  $\pi_R = \frac{\dot{p}_R(t)}{p_R(t)}$ , which is a constant relative price change of leisure technology.  $\pi_w = \frac{\dot{w}(t)}{w(t)}$ , which is a constant change in wages.

From the above equations we can derive the following:

$$(\sigma - \Theta\sigma + \Theta)g_R + \Lambda_R = (\sigma - 1 - \Theta\sigma + \Theta)g_R + \Lambda_w = \sigma g_C \quad (3.14)$$

This equation shows that the steady state depends on the interaction among all the variables ( $g_R$ ,  $\Lambda_R$ ,  $\Lambda_w$  and  $g_C$ ) and parameters ( $\sigma$  and  $\Theta$ ).

### 3.4 Income share-habit proposition

**Proposition 7** (*Income share-habit proposition*) *Income share trends are influenced by the importance of habits ( $\Theta$ ) and growth rate of habits ( $h$ ).*

**Proof:**

To derive the income share proposition, we use the worker's aggregate supply of labour derived from the consumer's first order condition and equate this to the sectors demand. The real wage that equates aggregate supply and demand is substituted out of the equations to form capital per hours worked. Capital income per labour income is the following:

$$\frac{r_{ss} + \delta}{w} \frac{K_R}{(1 - l_R)} = \frac{r_{ss} + \delta}{w} \left[ \left( \frac{\gamma_{KR}}{1 - \gamma_{KR}} \right) \left( \frac{A_{R_K}}{A_{R_L}} \right)^{\frac{\epsilon-1}{\epsilon}} \left( \frac{C}{l} \right)^\sigma \left( \frac{R^{1-\sigma}}{h^{\Theta-\sigma\Theta}} \right) \left( \frac{1}{r_{ss} + \delta} \right) \right]^\epsilon \quad (3.15)$$

$$\frac{r_{ss} + \delta}{w} \frac{K_C}{(1 - l_C)} = \frac{r_{ss} + \delta}{w} \left[ \left( \frac{\gamma_{KC}}{1 - \gamma_{KC}} \right) \left( \frac{A_{C_K}}{A_{C_L}} \right)^{\frac{\beta-1}{\beta}} \left( \frac{C}{l} \right)^\sigma \left( \frac{R^{1-\sigma}}{h^{\Theta-\sigma\Theta}} \right) \left( \frac{1}{r_{ss} + \delta} \right) \right]^\beta \quad (3.16)$$

QED

In the absence of consumer habits, the final three terms in parentheses inside the brackets equations in (3.15) and (3.16) would be absent.

**Proposition 8** *The derivative of capital income per labour income with respect to importance of habits using equation (3.15) is as follows:*

$$\begin{aligned} \epsilon \frac{r_{ss} + \delta}{w} \left[ \left( \frac{\gamma_{KR}}{1 - \gamma_{KR}} \right) \left( \frac{A_{R_K}}{A_{R_L}} \right)^{\frac{\epsilon-1}{\epsilon}} \left( \frac{C}{l} \right)^\sigma \left( \frac{R^{1-\sigma}}{h^{\Theta-\sigma\Theta}} \right) \left( \frac{1}{r_{ss} + \delta} \right) \right]^{\epsilon-1} \\ \left[ \left( \frac{\gamma_{KR}}{1 - \gamma_{KR}} \right) \left( \frac{A_{R_K}}{A_{R_L}} \right)^{\frac{\epsilon-1}{\epsilon}} \left( \frac{C}{l} \right)^\sigma \left( \frac{R^{1-\sigma}}{h^{\Theta-\sigma\Theta}} \right) \left( \frac{1}{r_{ss} + \delta} \right) \right] \\ \ln(h)(\sigma - 1) \end{aligned}$$

## 3.5 Impact of perturbations

This section provides an analysis of the impact of two exogenous shocks (relative price decline and technology addiction) to a benchmark case (steady state). In the benchmark case, the interest rate is equal to the rate of time preference. In this case, the income share of labour can only change if the elasticity of substitution is non-unitary.

### 3.5.1 Perturbation one: Relative price declines

In the first perturbation the relative price of the technology good declines due to a technological breakthrough. The price decline of information technology equipment has been relentless (Jorgenson 2001). Relative to the GDP deflator, computers and equipment price growth was -15.9 % (1973-1995), -26.3 % (1995-2000), -17.6 % (2000-2005) and -15.7 % (2005-2010) respectively (Jorgenson 2005; Jorgenson, Ho and Samuels 2016). Although prices have been



decreasing for many years, Jorgenson (2005) emphasizes that in 1995 prices of microprocessors took a sharp plunge down-wards by 90 per cent. Since then prices have been declining significantly every year causing IT prices to also nose-dive. Since the Great Recession the prices of semiconductors declined by an average of forty-three percent per year from 2008 to 2013 (Byrne, Oliner and Sichel 2015).

First, assume the economy is in the benchmark steady state. See Appendix A.D for all the equations applicable to the benchmark case. The following applies in the benchmark case:

$$\sigma > 1 \quad (3.17)$$

$$0 < \Theta < 1 \quad (3.18)$$

To understand the impact of the shocks, we assume the following growth rates for the benchmark steady state case:

$$\pi_R = 0 \quad (3.19)$$

$$g_R = 0 \quad (3.20)$$

$$g_C = 0 \quad (3.21)$$

$$\pi_w = 0 \quad (3.22)$$

$$r_{ss} = \rho \quad (3.23)$$

Assume that  $\pi_R = \Lambda_R < 0$ . In contrast still assume that  $g_R = 0$ . (3.21), (3.22) and (3.23) are allowed to adjust. Equations (3.9)-(3.13) imply the following:

$$\pi_R = \Lambda_R < 0 \quad (3.24)$$

$$g_R = 0 \quad (3.25)$$

$$g_R + \frac{\pi_R}{\sigma - \Theta\sigma + \Theta} = \frac{r_{ss} - \rho}{\sigma - \Theta\sigma + \Theta} \quad (3.26)$$

$$g_R + \frac{\pi_w}{(\sigma - 1 - \Theta\sigma + \Theta)} = \frac{r_{ss} - \rho}{(\sigma - 1 - \Theta\sigma + \Theta)} \quad (3.27)$$

$$g_C = \frac{\Lambda_R}{\sigma} < 0 \quad (3.28)$$

$$\pi_w = \Lambda_R \quad (3.29)$$

$$r_{ss} - \rho = \Lambda_R \quad (3.30)$$

As can be seen in the above equations, consumption growth rate is negative. As  $g_R=0$ , many of the other variables such as  $\pi_w$  and  $\pi_R$  are pinned down. It is clear that as  $\Lambda_R$  is negative  $r_{ss}(t) < \rho$ . Furthermore, the relative price decline also drives wages to decline ( $\pi_w$ ).<sup>10</sup> With  $g_R=0$ , in discrete time the following holds:

$$\frac{\Delta w}{w} = \frac{\Delta p_R}{p_R} \quad (3.31)$$

In regards to capital per hours worked, the model has the flexibility to enable capital deepening or shallowing. The flexibility does not necessarily depend on the elasticity. To understand we note that the stationary state ratio of  $R(t)$  to  $h(t)$  is still one.<sup>11</sup> Consequently, it is possible that as consumption declines, there would be a switch from capital shallowing to capital deepening:

$$\frac{K_R}{(1 - l_R)} = \left[ \left( \frac{\gamma_{KR}}{1 - \gamma_{KR}} \right) \left( \frac{A_{R_K}}{A_{R_L}} \right)^{\frac{\epsilon-1}{\epsilon}} \left( \frac{C}{l} \right)^\sigma \left( \frac{1}{\rho + \delta + \Lambda_R} \right) \left( \frac{1}{h^{\sigma - \sigma\Theta + \sigma - 1}} \right) \right]^\epsilon \quad (3.32)$$

10. As  $g_R = 0$

11. As explained by Mulligan and Sala-i-Martin (1992) and Overland, Carroll and Weil (2000) for steady state to exist the system needs to be transformed into variables that are constant in the steady state. As habits are formed for the digital technology good, the steady state ratio of  $R(t)$  to  $h(t)$  must equate to one.

$$\frac{K_C}{(1-l_C)} = \left[ \left( \frac{\gamma_{KC}}{1-\gamma_{KC}} \right) \left( \frac{A_{Ck(t)}}{A_{CL}} \right)^{\frac{\beta-1}{\beta}} \left( \frac{C}{l} \right)^\sigma \left( \frac{1}{\rho+\delta+\Lambda_R} \right) \left( \frac{1}{h^{\sigma-\sigma\Theta+\sigma-1}} \right) \right]^\beta \quad (3.33)$$

Conventional economic wisdom has long argued that the effect of declines in the interest rate on capital per hours worked and income share depend entirely on elasticity of substitution. We find that the resulting ratio and shares are driven by the interaction between consumer preferences and sector production functions. This is a rather interesting result. A falling relative price of leisure technology, such as video games, increases utility from leisure. There will be declining wages, negative consumption growth, and depressed interest rates. Workers forgo consumption of normal goods for a leisure experience.

### 3.5.2 Shock two: Technology addiction

Addiction to leisure technology is very much a real phenomenon. Internet gaming addiction may be as high as 50 % within Korean teenagers (Kuss 2013). 60 % of students feel they are addicted to cell phones (Roberts and Pirog III 2012). The popularity of online gaming rose during the 2000s and since then internet gaming addiction has increasing (Griffiths, Kuss and King 2012). Individuals are addicted to the technology good to enhance leisure purposes (leisure technology) rather than to use it for work. For instance, cell-phone addiction is driven primarily by the desire to socially connect (Junco and Cotten 2012; Roberts, Yaya and Manolis 2014), which is for leisure purposes. The same applies to addiction to other technology goods such as on-line gaming. Shock two models this phenomenon. Define technology addiction as:

$$\frac{\sigma}{\sigma-1} < \Theta \quad (3.34)$$

At this point the behaviour of the model changes and hence we use this level as an indicator when habits turn into addiction. At this point significant amount of technology is required to satisfy the consumers needs. The intuition behind the inequality is for  $\Theta$  to be related to risk aversion. Recall that as  $\Theta$

increases more purchases of technology are required to gain a given level of utility. Research shows that more risk averse individuals have a low addiction threshold (i.e. low  $\Theta$ ). The reasoning is that, as found by Cain (2013) introverts are more risk adverse. In her book, she describes introverts as:

*'reflective, cerebral, bookish, unassuming, sensitive, thoughtful, serious....,shy, risk-averse, [and] thick-skinned.'*

Nonetheless, Kuss (2013) determined that extraversion acts as a protective mechanism against addiction to video games.<sup>12</sup> To isolate addiction, we assume that  $\pi_R = 0$  and  $g_R = \bar{g}_R > 0$ . (3.21), (3.22) and (3.23) are allowed to adjust. Equations (3.19)-(3.23) imply the following:

$$\pi_R = 0 \quad (3.35)$$

$$\pi_w = 0 \quad (3.36)$$

$$g_R = \bar{g}_R > 0 \quad (3.37)$$

$$g_C = \frac{(\sigma - \Theta\sigma + \Theta)\bar{g}_R}{\sigma} \quad (3.38)$$

$$r_{ss} - \rho = (\sigma - \Theta\sigma + \Theta)\bar{g}_R \quad (3.39)$$

Relative to the benchmark case with  $r_{ss}=\rho$ , the interest rate decreased by  $(\sigma - \Theta\sigma + \Theta)\bar{g}_R$ . In regards to capital per hours, whether there is capital deepening or capital shallowing will depend on the interaction between the various brackets. Subsequently, the model offers the ability to capture both capital deepening and capital shallowing. This can be seen below:

$$\frac{K_R}{(1-l_R)} = \left[ \left( \frac{\gamma_{KR}}{1-\gamma_{KR}} \right) \left( \frac{A_{RK}}{A_{RL}} \right)^{\frac{\epsilon-1}{\epsilon}} \left( \frac{C}{l} \right)^\sigma \left( \frac{R^{1-\sigma}}{h^{1-\sigma}} \right) \left( \frac{1}{h^{\Theta-\sigma\Theta+\sigma}} \right) \left( \frac{h}{1} \right) \left( \frac{1}{(\sigma-\Theta\sigma+\Theta)\bar{g}_R+\rho+\delta} \right) \right]^\epsilon \quad (3.40)$$

### 3.6 Predictions are consistent with the data

In the previous section, the theoretical effect of perturbations on the steady state was highlighted. In this section, the empirical relevance of the theory in

<sup>12</sup> There is some counter evidence but only with addiction to smartphones. Nonetheless, the evidence results from the more socially active being more likely to be addicted to social network sites.

this paper is investigated. The framework is modelled with United States data obtained from the Federal Reserve Bank of St. Louis (2016c) and the World Bank (2016b) to examine the implications of habit formation on wages, interest rates and particularly on labour income share. First, it is shown that stronger importance of habits ( $\Theta$ ) can drive the labour income share down. After which it is demonstrated with U.S data that the framework matches trends in wages, interest rates and the labour income share since the mid-1990s.

Before considering the analysis, it is informative to provide some background on the leisure technology market. Figure 3.1 shows the growth rates in leisure technology, normal consumption (net of leisure technology) and leisure hours. Figure 3.2 shows the spending on leisure technology and the growth rate of leisure technology throughout the period was very high. In the 1990s growth rates were above 20 percent due to a technological shift in the personal computer market. In 1996 there was another technological shift but this time in the entertainment market with the launch of the DVD (digital versatile disc) technology. The growth rates of leisure technology slowed subsequently in the 2000s. However, the growth rate is still dramatic, with chained dollars of leisure technology rising exponentially.

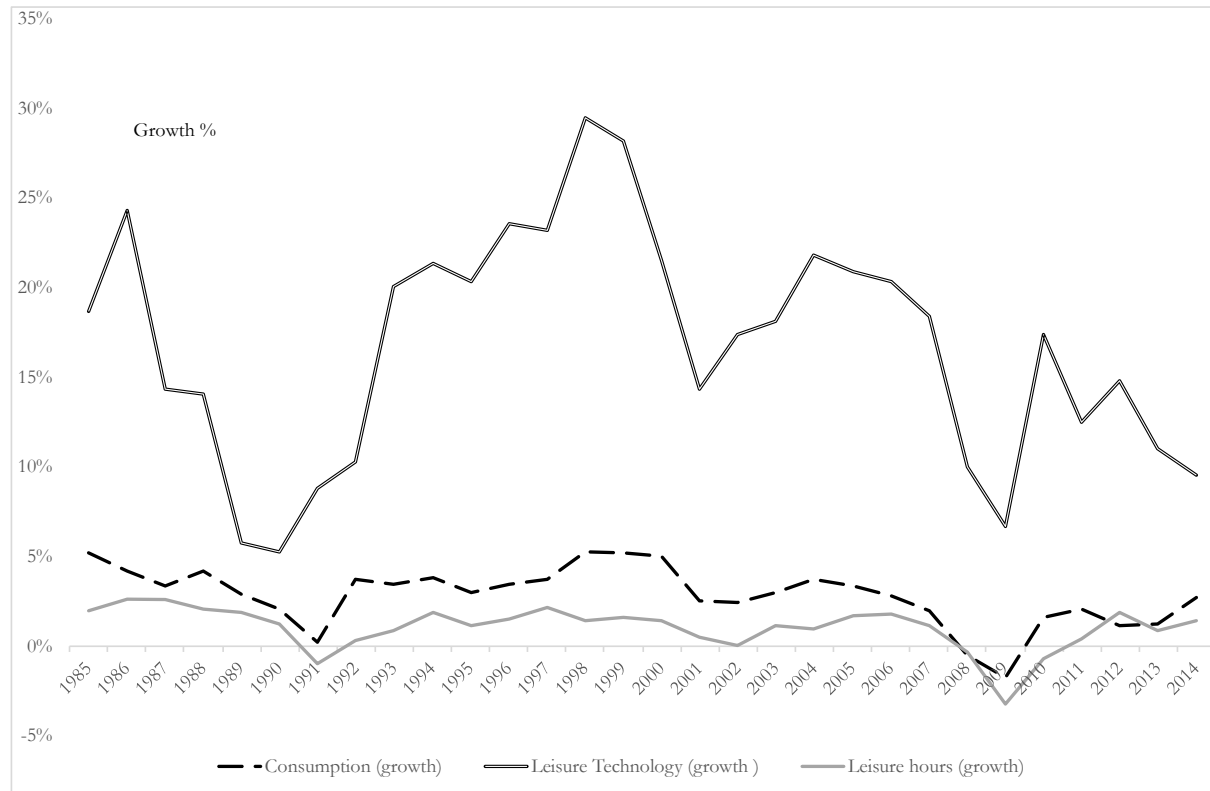


Figure 3.1: Growth rates of consumption, leisure technology and leisure hours from 1985 to 2014

*Note:* Leisure technology purchased ( $R(t)$ ) and consumption ( $C(t)$ ) obtained from Federal Reserve Bank of St. Louis (2016h)<sup>13</sup> and Federal Reserve Bank of St. Louis (2016g) respectively.<sup>15</sup> Annual hours worked (Federal Reserve Bank of St. Louis 2016a) and the number of persons engaged (Federal Reserve Bank of St. Louis 2016d) is used to calculate leisure hours.

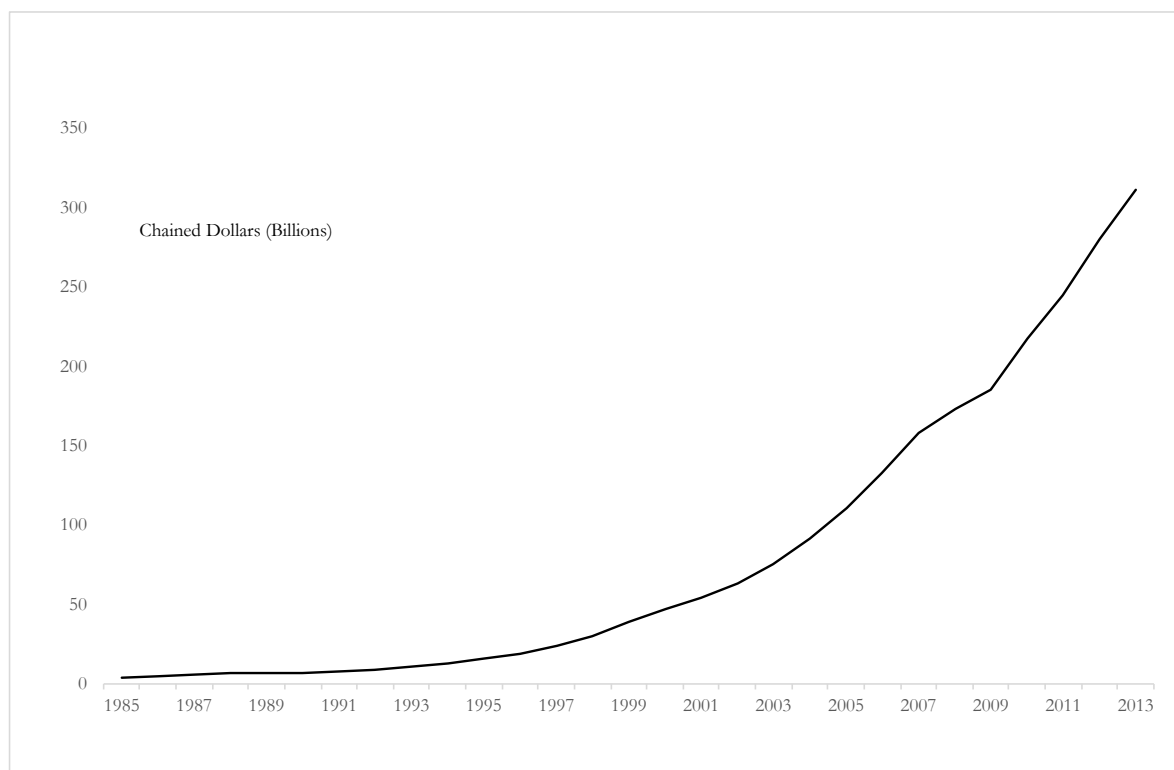


Figure 3.2: Spending on leisure technology (Billions) from 1985 to 2013

It should be noted that leisure technology has risen considerably as a percentage of aggregate spending. Nonetheless, it is still only around 3 percent. Given that leisure technology is such a small percentage of aggregate consumption a key question is whether it is possible consumption of leisure technology has played a role in shaping the labour income share?

#### **Increasing importance of habits drives labour income shares down**

Recall that  $\Theta$  models the importance of habits.

#### **Data for Interest rates and wage**

The aggregate level of capital per labour income is approximately the

following:

$$\frac{r_{ss} + \delta}{w} \frac{K}{(1-l)} = \frac{r_{ss} + \delta}{w} \left[ \left( \frac{\gamma_K}{1-\gamma_K} \right) \left( \frac{A_K}{A_L} \right)^{\frac{\epsilon-1}{\epsilon}} \left( \frac{C}{l} \right)^\sigma \left( \frac{R^{1-\sigma}}{h^{\Theta-\sigma\Theta}} \right) \left( \frac{1}{r_{ss} + \delta} \right) \right]^\epsilon \quad (3.41)$$

Define the aggregate level of capital per labour income as  $\lambda(t)$ . Labour income share as a percentage of total income is the following:

$$\left( \frac{\lambda(t)+1}{\lambda(t)} \right) * 100. \quad (3.42)$$

$l(t)$  is estimated by calculating the total number of annual hours for all those engaged in employment in the United States.<sup>16</sup> Consumption net of technology is determined by subtracting the amount spent on leisure technology.<sup>17</sup> Federal Reserve Bank of St. Louis (United States) data is used to calculate the quantity and relative price changes. A relative price index ( $p_R(t)$ ) is created in order to capture the change in the relative price of leisure technology. Further, a relative price index is created for the ratio of the chain-type price index for VAPIM (Federal Reserve Bank of St. Louis 2016e) to the chain-type price index for total consumption (Federal Reserve Bank of St. Louis 2016f). From equation (3.8), the growth rate of leisure technology must equate to the growth rate of habits for the leisure technology sector to be in a steady state. Equation (3.43) specifies that the growth rate of  $R(t)$  equals  $h(t)$ . Given the growth rate of habits in (3.43) a steady state level of  $h(t)$  is calculated by equation (3.44).<sup>18</sup>

$$\frac{\dot{R}(t)}{R(t)} = \frac{\dot{h}(t)}{h(t)} \quad (3.43)$$

$$h(t) = \frac{R(t)}{1 + \frac{\dot{R}(t)}{R(t)} \frac{1}{\psi}} \quad (3.44)$$

16. This involves first calculating the total number of annual hours worked by all those engaged in the United States. After which the total annual hours worked is subtracted from total hours in a year to obtain  $l(t)$ . i.e.,  $l(t) = (\text{hours in a year} - \text{average annual hours worked by those engaged in employment}) / \text{total engaged in employment in US}$ .

17. Prior to 1999, the quantity indexes for  $R(t)$  (Federal Reserve Bank of St. Louis 2016i) and  $C(t)$  (Federal Reserve Bank of St. Louis 2016i) are used to estimate billions of chained dollars.

18. A constant level of  $\psi = 0.1$  for simplicity.



The constants of  $A_K = 1$ ,  $A_L = 1$ ,  $\gamma_K = 0.5$  are assumed so that they do not bias our results. A value of 1.3 for  $\sigma$  (CRRA) is chosen based on Zhuang et al. (2007) is followed.  $\delta = 0.05$  is assumed. There is no consensus on the appropriate value of the elasticity of substitution. Consequently, 0.9 is used as this number leads to predicted and actual labour income share being the most similar. Labour income share is predicted by equation (3.42).<sup>19</sup> Different values of  $\Theta$  are used to illustrate the impact on the predicted labour income share. Figure 3.3 presents the predictions with various levels of  $\Theta$ .<sup>20</sup> Figure 3.3 also presents the actual labour income share for the U.S. from OECD statistics. As can be seen, higher levels of  $\Theta$  imply a steeper decline of labour income share. The steepest decline is when  $\Theta$  equates to 4.3. Clearly,  $\Theta$  at this level overpredicts the depth of the decline. When  $\Theta$  equates to 0, the labour income share is predicted to increase which clearly is not consistent with the data. Labour income share is predicted to increase as habit and leisure technology play a minor role with changes in consumption driving the change in labour income share. Despite leisure technology being such a small percentage, changing  $\Theta$  dramatically changes the predictions on labour income share. Why is this?

The importance placed on habits ( $\Theta$ ) compounds/multiplies across the equation (3.41). Specifically, increases in  $\Theta$  increase the following component:

$$\left( \frac{R(t)^{1-\sigma}}{h(t)^{\Theta(1-\sigma)}} \right) \quad (3.45)$$

This has a significant impact on labour share as it compounds/multiplies across the other brackets implying increasing  $\lambda(t)$  and decreasing labour share  $\left( \frac{\lambda(t)}{\lambda(t)+1} \right)$ . When  $\Theta$  is relatively high, then small changes in leisure technology can dramatically impact the dynamics of the economy.

Now, the economic intuition for the reasons why  $\Theta$  impacts the labour

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19. The real interest rate for the United States from the World Bank (2016b) and average hourly earnings from Federal Reserve Bank of St. Louis 2016b is utilized.

20. As there is a stock of leisure technology in 1984 the figures do not start at the same point.

income share is discussed. In this framework, consumer's utility depends on how leisure technology compares to a habit stock. This habit stock is a standard of reference.  $\Theta$  is the importance placed on this standard of reference, not on the actual amount or the growth of the habit stock. It encapsulates the importance placed by consumers on keeping up with latest leisure technology. The consumer's optimization problem differs from traditional growth model because of the existence of habits. As  $\Theta$  increases, more leisure technology, leisure, and even consumption are required to provide a given amount of utility to the consumer. Consequently, this influences the dynamics of the economy considerably. It will change labour supply, savings, wages and interest rates. All these interacting together will impact labour income share.<sup>21</sup>

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21. To understand the impact of  $\Theta$ , the predicted labour income share for the different  $\Theta$  levels are assumed to start at the actual labour income share in 1984.

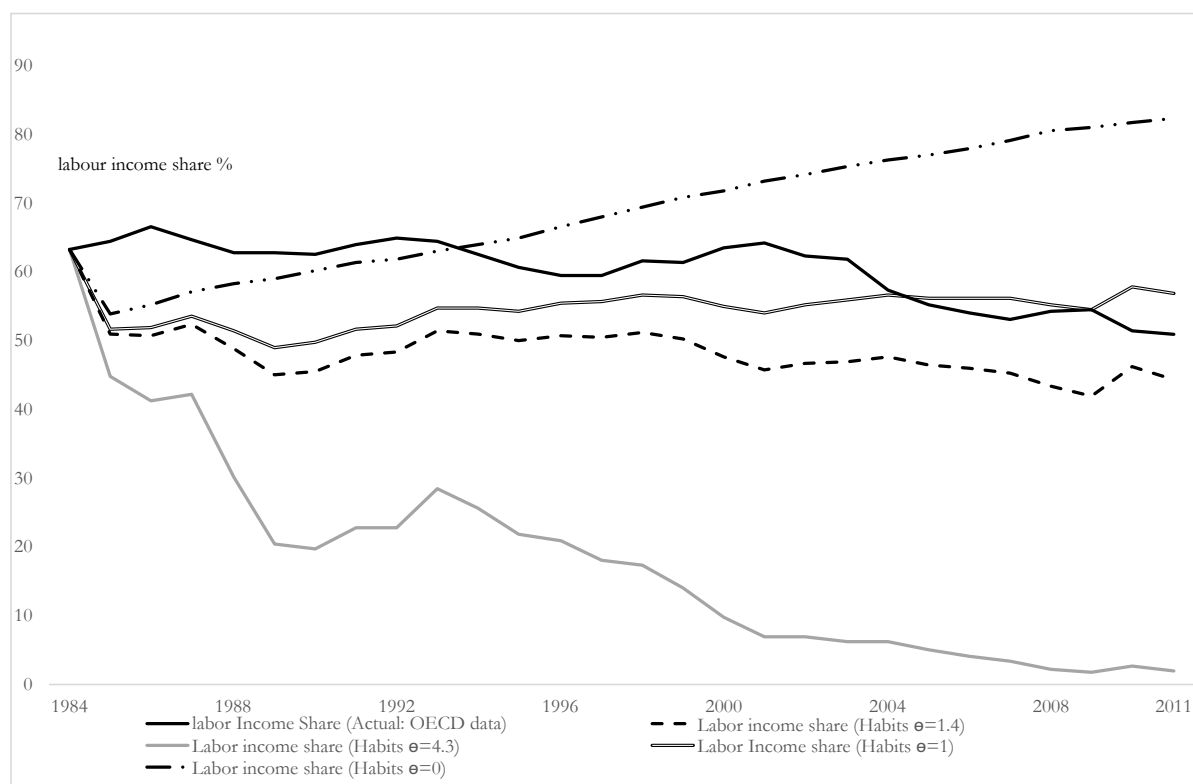


Figure 3.3: Impact of  $\Theta$  on labour income share

From this empirical analysis, the income share-habit proposition is derived below.

**Proposition 9** (*Income share-habit proposition*) *Given a data set with a non-unitary elasticity of substitution, a perturbation of an exogenous increase in  $\Theta$  implies a steeper decline and lower labour income shares. In contrast, higher growth of habits leads to slightly higher labour income share.*

The impact of  $\Theta$  on labour income share is much greater than from changes in the growth of habits. Whereas marginal changes in  $\Theta$  changes trends dramatically, doubling the growth of habits only slightly increases labour income

share.

### Predictions of interest rates, wages and the labour income share

Despite being only 3 percent of aggregate consumption, it is now illustrated that this framework with consumer habits predict interest rates, wages and labour income that are consistent with the data. The implied interest rate is determined by the equation below.

$$(\sigma - \Theta\sigma + \Theta)g_R + \rho + \pi_R(t) = r(t) \quad (3.46)$$

Subsequently this implied interest rate is used to calculate the wage rate ( $\pi_w(t)$ ).<sup>22</sup>.

$$r(t) - \rho - (\sigma - 1 - \Theta\sigma + \Theta)g_R = \pi_w(t) \quad (3.47)$$

Figure 3.4 shows the computed interest rate with  $\Theta$  of 1.43 and  $\epsilon = 0.9$ . Predicted interest rates have higher volatility than actual interest rate. The reason is that to compute interest rate the framework utilises growth rate of leisure technology and change in the relative price of technology index. A swing in any one variable (i.e. price of consumption, the price of leisure technology and growth rate of leisure technology) will lead to volatility in the computed interest rate. Figure 3.5 shows the computed wage with  $\Theta$  of 1.47 only for illustrative purposes. Figure 3.6 computes wage, interest rate and labour share combined all with  $\Theta$  of 1.43 and  $\epsilon = 0.9$ . Computing labour income with  $\Theta = 1.43$  and elasticity of  $= 0.9$  leads to the closest fit with actual labour income. Table 3.1 highlights the model parameters.

There are limitations to the framework's predictions. Firstly, prediction of labour share of income from the framework is a relatively poor prior to the mid-1990s. This would imply that either the model is not appropriate prior to 1984 or changes in data estimation procedures led to better quality of data. In addition, although the framework predicts the level of the labour share consistently, it appears that the change in labour share of income is anti-correlated

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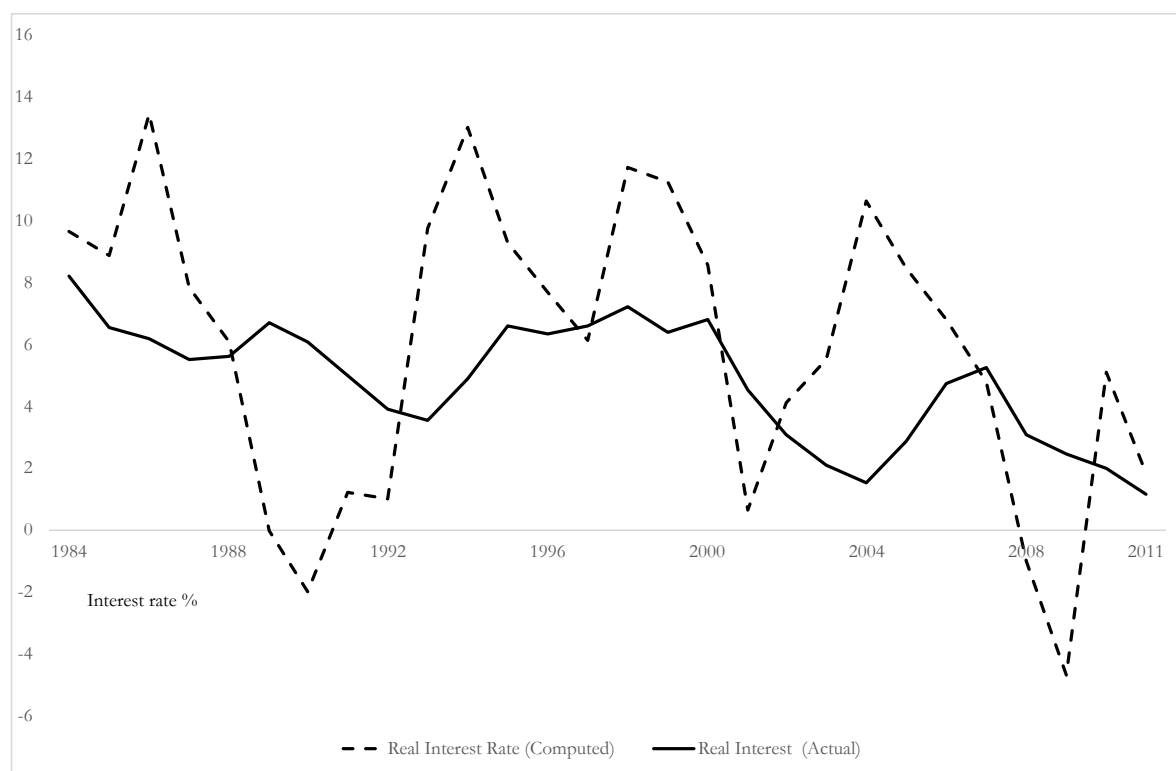
22.  $\pi_w$  is the percentage change. Consequently, 1984, is chosen as the arbitrary start date of when the wage equates to wage from the data

with the data. Furthermore, only if the model is correct relatively strong importance of habits is applicable. If the framework is correct, then an aggregate level of elasticity between capital and labour of approximately one is viable.

As can be seen in the figures, the computed trends are consistent with actual trends. However, if habits played no role in the utility function, the framework would calculate a much greater level of labour income share than in the data. It is important to note that data is not from the steady state. Nonetheless, the equations can be considered steady state. However, the approximations should be viable given there is not significant overshoots from adjustments.

Table 3.1: Model Parameters

| Parameter  | Meaning                    | Value | How chosen |
|------------|----------------------------|-------|------------|
| $\Theta$   | Importance of habits       | 1.43  | Gussed     |
| $A_K$      | Technology term            | 1     | Gussed     |
| $A_L$      | Technology term            | 1     | gussed     |
| $\sigma$   | CRRA                       | 1.3   | Literature |
| $\delta$   | Depreciation               | 0.05  | Gussed     |
| $\epsilon$ | Elasticity of substitution | 0.9   | Gussed     |

Figure 3.4: Computed and actual interest rate,  $\Theta = 1.43$ 

*Note:* Actual real interest rate from the World Bank (2016b).

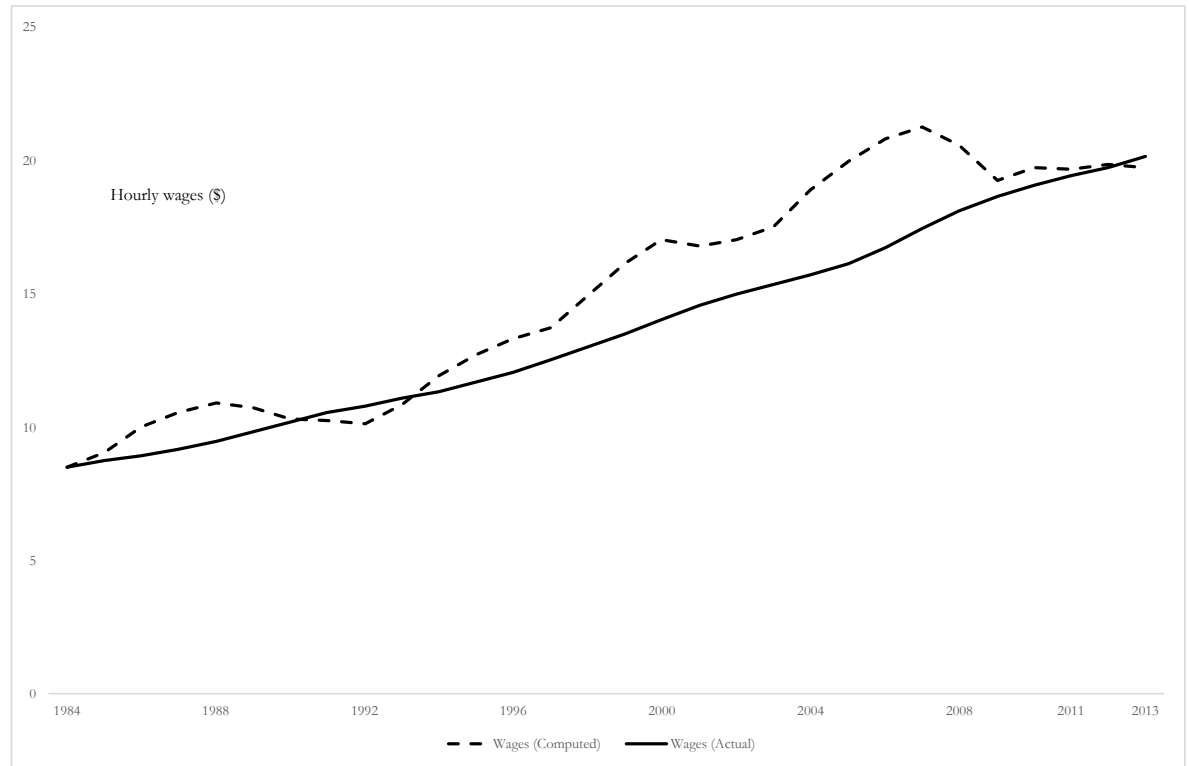


Figure 3.5: Computed and actual hourly wage \$ with  $\Theta = 1.47$

*Note:* Actual real wages are obtained from Federal Reserve Bank of St. Louis (2016b)

$\theta=1.47$  used for illustrative purposes. Full model uses  $\Theta=1.43$

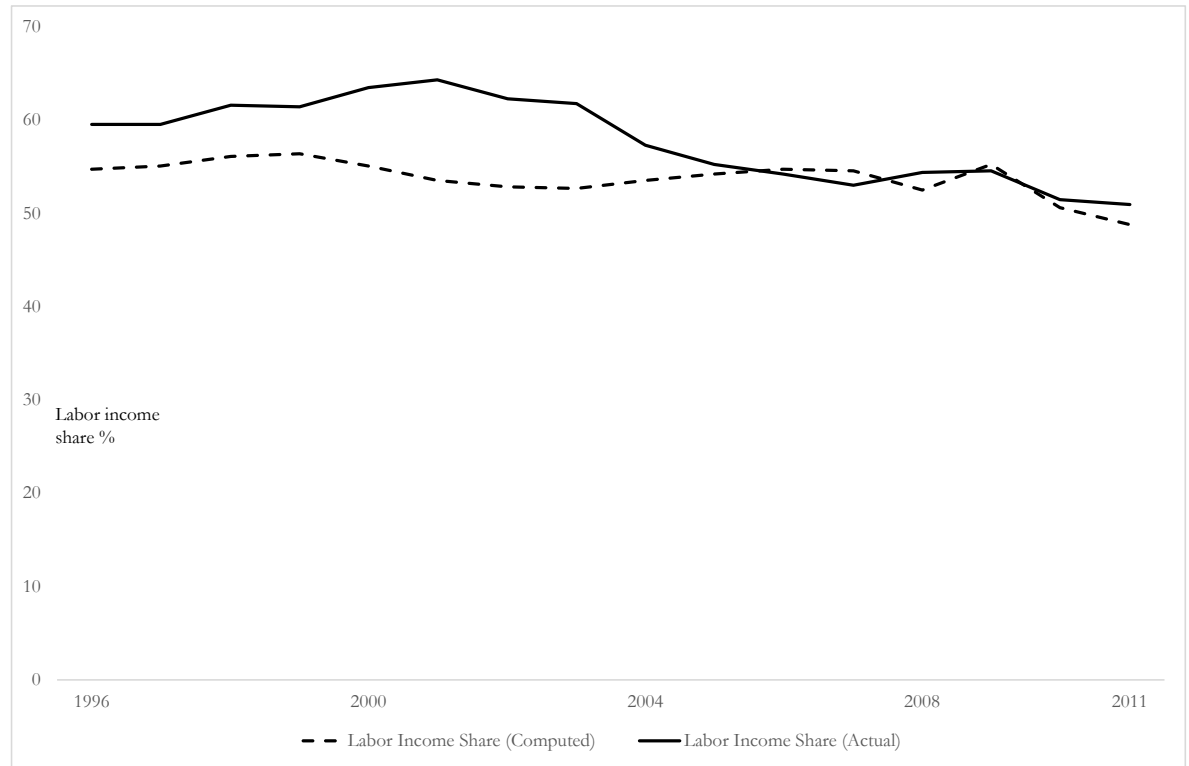


Figure 3.6: Labour income share,  $\Theta = 1.43$ ,  $\epsilon = 0.9$

*Note:* Actual level of US labour income share obtained from the Organisation for Economic Co-operation and Development (2016).

### 3.7 Conclusion

Is habit formation in leisure technology a contributor to the decline in labour income share? Theoretically, it seems plausible that it would have numerous impacts. For one it would influence the consumers' desire to experience leisure. This novel theory has appeal when investigating the predictions from the framework. The predictions are relatively consistent with at least three economic variables. If utility depends on how leisure technology compares to a habit stock, this model can predict trends in wages, interest rates and the



labour income share that are consistent with the data. Nonetheless, if habits played no role in the utility function, the framework would calculate a much greater level of labour income share than in the data. Consequently, if one is willing to believe that consumers do have habits for leisure technology then this framework may be useful to explain some changes in labour income share.

## 3.8 Appendix A

### 3.8.1 Optimisation

#### Consumer solution

The current value Hamiltonian is set up below. The maximisation problem is equated to marginal utility subject to the budget constraint. Choice variables are  $C(t)$ ,  $R(t)$  and  $l(t)$ . The state variables for the maximisation problem are  $K(t)$ :

$$H = \frac{C(t)^{1-\sigma}}{1-\sigma} + \frac{(l(t)R(t)/h(t)^\Theta)^{1-\sigma}}{1-\sigma} + v_K(t)(r(t)K(t) + w(t)(1-l(t)) - C(t) - p_R(t)R(t)) \quad (3.48)$$

The first order conditions apply:

$$\frac{\partial H}{\partial C(t)} = C(t)^{-\sigma} - v_K(t) = 0 \quad (3.49)$$

$$\frac{\partial H}{\partial l(t)} = \left[ \frac{l(t)R(t)}{h(t)^\Theta} \right]^{-\sigma} \frac{R(t)}{h(t)^\Theta} - v_K w(t) = 0 \quad (3.50)$$

$$\frac{\partial H}{\partial R_i(t)} = \left[ \frac{l(t)R(t)}{h(t)^\Theta} \right]^{-\sigma} \frac{l(t)}{h(t)^\Theta} - v_K(t) p_R(t) = 0 \quad (3.51)$$

$$\frac{\partial H}{\partial K(t)} = \rho v_K(t) - \dot{v}_K(t) = v_K r(t) \quad (3.52)$$

The transversality condition is:

$$\lim_{t \rightarrow \infty} e^{-\rho t} v_K(t) K(t) = 0 \quad (3.53)$$

By using the first order conditions for the three chosen variables of leisure, technology and consumption, three dynamic equations can be derived relating to  $r(t)$ . As such the whole dynamical system of equations for consumer can be considered as per below. I split up capital devoted to the technology sector and the consumption sector.  $a$  and  $b$  are fractions of wage income that augments the capital stock devoted to each sector. It does not change the results in the

paper by including fraction  $a$  and  $b$ .

$$\frac{\dot{R}(t)}{R(t)} = \frac{1}{\sigma} \left( r(t) - \rho + (\Theta\sigma - \Theta) \frac{\dot{h}(t)}{h(t)} - \frac{\dot{p}_R(t)}{p_R(t)} \right) \quad (3.54)$$

$$\frac{\dot{h}(t)}{h(t)} = \psi \left( \frac{R(t)}{h_i(t)} - 1 \right) \quad (3.55)$$

$$\frac{\dot{p}_R(t)}{p_R(t)} = \pi_R \quad (3.56)$$

$$\frac{\dot{C}(t)}{C_i(t)} = \frac{1}{\sigma} [r(t) - \rho] \quad (3.57)$$

$$\frac{\dot{K}_R(t)}{K_R(t)} = r(t) - \frac{p_R(t)R(t)}{K_R(t)} + \frac{aw}{K_R(t)} \quad (3.58)$$

$$\frac{\dot{K}_C(t)}{K_C(t)} = r(t) - \frac{C(t)}{K_C(t)} + \frac{bw(t)}{K_C(t)} \quad (3.59)$$

$$\frac{\dot{R}(t)}{R(t)} = \frac{1}{\sigma-1} \left( r(t) - \rho + (\Theta\sigma - \Theta) \frac{\dot{h}(t)}{h(t)} - \frac{\dot{w}(t)}{w(t)} \right) \quad (3.60)$$

$$\frac{\dot{w}(t)}{w(t)} = \pi_w \quad (3.61)$$

### Firms solution

As suggested in the paper, there are two sectors, the technology good and consumption, and their production functions take the following forms:

$$Y_R(t) = [\gamma_{K_R} (A_{R_K}(t) K_R(t))^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma_{K_R}) (A_{R_L}(t) N_R(t))^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{\epsilon-1}} \quad (3.62)$$

$$Y_C(t) = [\gamma_{K_C} (A_{C_K}(t) K_C(t))^{\frac{\beta-1}{\beta}} + (1-\gamma_{K_C}) (A_{C_L}(t) N_C(t))^{\frac{\beta-1}{\beta}}]^{\frac{\beta}{\beta-1}} \quad (3.63)$$

$p_R(t)$  represents the relative price of the technology good with the price of the consumption good normalized to one.  $\Phi_i(t)$  is used for profit.  $\mu_R(t)$  and  $\mu_C(t)$  are the markup in the industry.  $w(t)$ ,  $r(t)$  and  $\delta$  is assumed to be equal for both sectors.

$$\Phi_R(t) = \frac{p_R(t)}{\mu_R(t)} Y_R(t) - (r(t) + \delta) K_R(t) - w(t) N_R \quad (3.64)$$

$$\Phi_C(t) = \frac{1}{\mu_C(t)} Y_C(t) - (r(t) + \delta) K_C(t) - w(t) N_C \quad (3.65)$$

The first order condition implies:

$$MPK_R(t) = p_R(t)\gamma_{K_R}A_{R_K}(t)^{\frac{\epsilon-1}{\epsilon}}\left(\frac{Y_R(t)}{K_R(t)}\right)^{\frac{1}{\epsilon}} = \mu_R(t)(r(t)+\delta) \quad (3.66)$$

$$MPK_C(t) = \gamma_{K_C}A_{C_K}(t)^{\frac{\beta-1}{\beta}}\left(\frac{Y_C(t)}{K_C(t)}\right)^{\frac{1}{\beta}} = \mu_C(t)(r(t)+\delta) \quad (3.67)$$

$$MPL_R(t) = p_R(t)(1-\gamma_{K_R})A_{R_L}(t)^{\frac{\epsilon-1}{\epsilon}}\left(\frac{Y_R(t)}{N_R(t)}\right)^{\frac{1}{\epsilon}} = \mu_R(t)w(t) \quad (3.68)$$

$$MPL_C(t) = (1-\gamma_{K_C})A_{C_L}(t)^{\frac{\beta-1}{\beta}}\left(\frac{Y_C(t)}{N_C(t)}\right)^{\frac{1}{\beta}} = \mu_C(t)w(t) \quad (3.69)$$

With the consumer and the sectors are outlined as per above, the next step in the study is to investigate the steady state

### 3.8.2 Steady state

#### Steady state growth of technology

Following Mulligan and Sala-i-Martin (1992), Overland, Carroll and Weil (2000), and Kavuri and McKibbin (2017) the dynamical system are transformed into variables that are constant in the steady state. Consequently,  $R$  and  $h$  need to grow at the same rate.

$$g_h = g_R \quad (3.70)$$

$\frac{\dot{R}(t)}{R(t)} = g_R$ ,  $\frac{\dot{C}(t)}{C(t)} = g_C$ ,  $r_{ss} = r(t)$  in the steady state.

These conditions imply that following are constant in the steady state.

$$\frac{Rl}{h} \quad (3.71)$$

The time derivative of  $\left[\frac{R(t)}{h(t)}\right]$  needs to be taken and set to zero, after which the equation for the evolution of habits is substituted into the formula.

$$\frac{\dot{h}}{h} = \frac{\dot{R}}{R} \quad (3.72)$$

Consequently, the steady state equation relating technology to  $r(t)$  is the following:

$$g_R + \frac{\pi_R}{\sigma - \Theta\sigma + \Theta} = \frac{r_{ss} - \rho}{\sigma - \Theta\sigma + \Theta} \quad (3.73)$$

### 3.8.3 Income share-habit proposition

The next step is to combine the households and sectors to form equilibrium.

#### Capital market

Capital market clearing requires that on each date the following holds.

$$K_R(t)^d + K_C(t)^d = K(t)^s \quad (3.74)$$

$$K(t) \geq 0 \quad (3.75)$$

The supply of capital comes from the consumer.

#### Labour market equilibrium

For labour market equilibrium the real wage equates the demand from the firm and worker's supply of labour. Labour market clearing requires at each date the following:

$$N_R(t)^d + N_C(t)^d = N(t)^d = N(t)^s \quad (3.76)$$

As a result, the wage adjusts to clear the labour market with the equilibrium wage denoted by  $w(t)^*$ . The factor price differential is specified as follows:

$$\frac{w^*(t)}{r(t)+\delta} = \frac{(1-\gamma_{KR})}{\gamma_{KR}} \left( \frac{A_{R_L(t)}}{A_{R_K(t)}} \right)^{\frac{\epsilon-1}{\epsilon}} \left( \frac{(1-I_R(t))}{K_R(t)} \right)^{\frac{-1}{\epsilon}} \quad (3.77)$$

$$\frac{w^*(t)}{r(t)+\delta} = \frac{(1-\gamma_{KC})}{\gamma_{KC}} \left( \frac{A_{C_L(t)}}{A_{C_K(t)}} \right)^{\frac{\beta-1}{\beta}} \left( \frac{(1-I_C(t))}{K_C(t)} \right)^{\frac{-1}{\beta}} \quad (3.78)$$

The aggregate supply of consumer is derived from the first order condition for the consumer.  $w^*(t)$  clears the market, equating the sector's demand and the worker's supply. Consequently, by substituting out  $w(t)$ , the capital per hours worked for each of the sectors can be determined as follows:

#### Capital per hours worked

$$\frac{K_R(t)}{(1-l_R(t))} = \left[ \left( \frac{\gamma_{KR}}{1-\gamma_{KR}} \right) \left( \frac{A_{R_K(t)}}{A_{R_L(t)}} \right)^{\frac{\epsilon-1}{\epsilon}} \left( \frac{C(t)}{l(t)} \right)^\sigma \left( \frac{R(t)^{1-\sigma}}{h(t)^{\Theta-\sigma\Theta}} \right) \left( \frac{1}{r(t)+\delta} \right) \right]^\epsilon \quad (3.79)$$

$$\frac{K_C(t)}{(1-l_C(t))} = \left[ \left( \frac{\gamma_{KC}}{1-\gamma_{KC}} \right) \left( \frac{A_{C_K(t)}}{A_{C_L(t)}} \right)^{\frac{\beta-1}{\beta}} \left( \frac{C(t)}{l(t)} \right)^\sigma \left( \frac{R(t)^{1-\sigma}}{h(t)^{\Theta-\sigma\Theta}} \right) \left( \frac{1}{r(t)+\delta} \right) \right]^\beta \quad (3.80)$$

From inspection, first, notice that first two brackets result from the production function. Indeed, in the directed and biased technical change literature, these two brackets are fundamental to factor differentials and allocations. Nonetheless, with consumers habits we can observe the importance of the interaction between steady state level of technology ( $R(t)$ ) and habits ( $h(t)$ ) as well as between consumption ( $C(t)$ ) and leisure ( $l(t)$ ).

### Goods market

Equilibrium requires that prices (i.e.  $p_C(t) = 1$  and  $p_R(t)$ ) clear the market at every point in time. Total aggregate demand for technology and consumption goods equates to their respective supplies. Aggregate supply is determined from the production functions.

$$Y_R(t)^d = R(t) + i_R(t) = Y_R^s \quad (3.81)$$

$$Y_C(t)^d = C(t) + i_C(t) = Y_C^s \quad (3.82)$$

### 3.8.4 Impact of shocks

In the benchmark case the parameters are:

$$\sigma > 1 \quad (3.83)$$

$$0 < \Theta < 1 \quad (3.84)$$

The growth rates for the benchmark state are:

$$\pi_R = 0 \quad (3.85)$$

$$g_R = 0 \quad (3.86)$$

$$g_C = 0 \quad (3.87)$$

$$\pi_w = 0 \quad (3.88)$$

$r_{ss} - \rho$

The parameters above and the growth rates imply that  $r_{ss}$  equates to  $\rho$ .

### Capital

The capital devoted to each sector is presented below. Logically, the difference at any point in time is determined by the difference between habits and consumption.

$$K_R = \frac{h - aw}{\rho} \quad (3.89)$$

$$K_C = \frac{C - bw}{\rho} \quad (3.90)$$

### Capital per hours worked

The penultimate step is to investigate the capital-labour ratio. Where  $r_{ss} = \rho$  the capital per hours worked the equation can be simplified as per below. Specifically, in the benchmark case  $R(t)$  to  $h(t)$  reduces to one.

$$\frac{K_R}{(1-l_R)} = \left[ \left( \frac{\gamma_{KR}}{1-\gamma_{KR}} \right) \left( \frac{A_{R_K}}{A_{R_L}} \right)^{\frac{\epsilon-1}{\epsilon}} \left( \frac{C}{l} \right)^\sigma \left( \frac{1}{\rho+\delta} \right) \left( \frac{1}{h^{\sigma-\sigma\Theta+\sigma-1}} \right) \right]^\epsilon \quad (3.91)$$

$$\frac{K_C}{(1-l_C)} = \left[ \left( \frac{\gamma_{KC}}{1-\gamma_{KC}} \right) \left( \frac{A_{C_K}}{A_{C_L}} \right)^{\frac{\beta-1}{\beta}} \left( \frac{C}{l} \right)^\sigma \left( \frac{1}{\rho+\delta} \right) \left( \frac{1}{h^{\sigma-\sigma\Theta+\sigma-1}} \right) \right]^\beta \quad (3.92)$$

### Income Share

$B$  is subscript for the benchmark case.

$$\frac{w^B (1-l_R)^B}{(\rho+\delta)K_R^B + w^B (1-l_R)^B} \quad (3.93)$$

**$r_{ss}$  and growth**

Given that  $g_R$  and  $g_C$  are zero, total growth  $g$  will also be zero. This implies  $r_{ss} > g$ . The gap is constant and does not continue expanding as in the analysis of Piketty (2014).

**3.9 Appendix B**

Predicted and actual hourly wages with  $\Theta=1.43$  as in full model to calculate the labour income share.

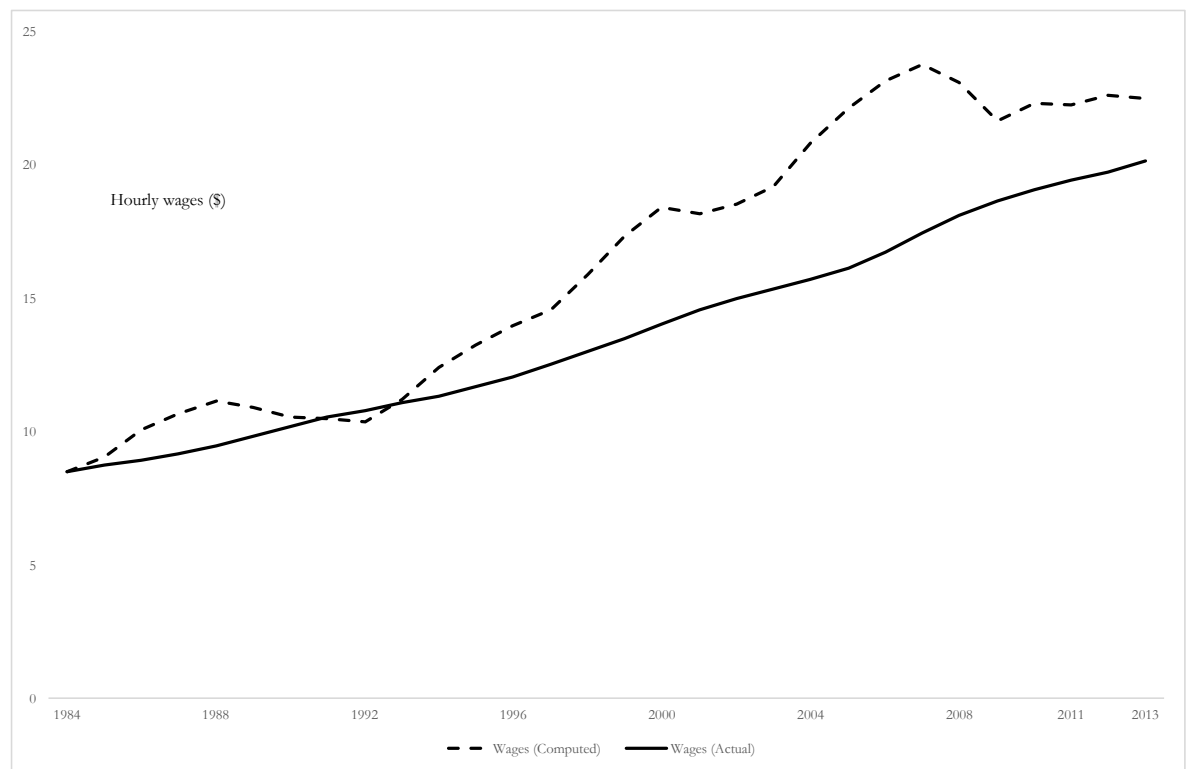


Figure 3.7: Predicted and actual hourly wages



### 3.9.1 Subsistence level of consumption

The following introduces a subsistence level requirement for consumption.  $C_i\bar{(t)}$  is a subsistence requirement for consumption.

$$U(t) = \frac{(C(t) - C\bar{(t)})^{1-\sigma}}{1-\sigma} + \frac{(l(t)R(t)/h(t)^\Theta)^{1-\sigma}}{1-\sigma} \quad (3.94)$$

$\sigma > 1$

$$\frac{\dot{C}(t)}{C(t) - C\bar{(t)}} = \frac{1}{\sigma} [A_C(t) - \delta_C - \rho] \quad (3.95)$$

$$\frac{\dot{C}(t)}{C(t) - C\bar{(t)}} = \frac{1}{\sigma} [r(t) - \rho] \quad (3.96)$$

Under Shock One: Relative Price Declines the following is obtained:

$$\frac{C(t)}{C_i(t) - C\bar{(t)}} = \frac{\Lambda_R}{\sigma} < 0 \quad (3.97)$$

Given that  $C\bar{(t)}$  is satisfied, the consumption growth over the subsistence level is negative.

#### Capital per hours worked

$$\frac{K_R(t)}{(1-l_R(t))} = \left[ \left( \frac{\gamma_{KR}}{1-\gamma_{KR}} \right) \left( \frac{A_{R_K}(t)}{A_{R_L}(t)} \right)^{\frac{\epsilon-1}{\epsilon}} \left( \frac{C(t) - C\bar{(t)}}{l(t)} \right)^\sigma \left( \frac{R(t)^{1-\sigma}}{h(t)^{\Theta-\sigma\Theta}} \right) \left( \frac{1}{r(t)+\delta} \right) \right]^\epsilon \quad (3.98)$$

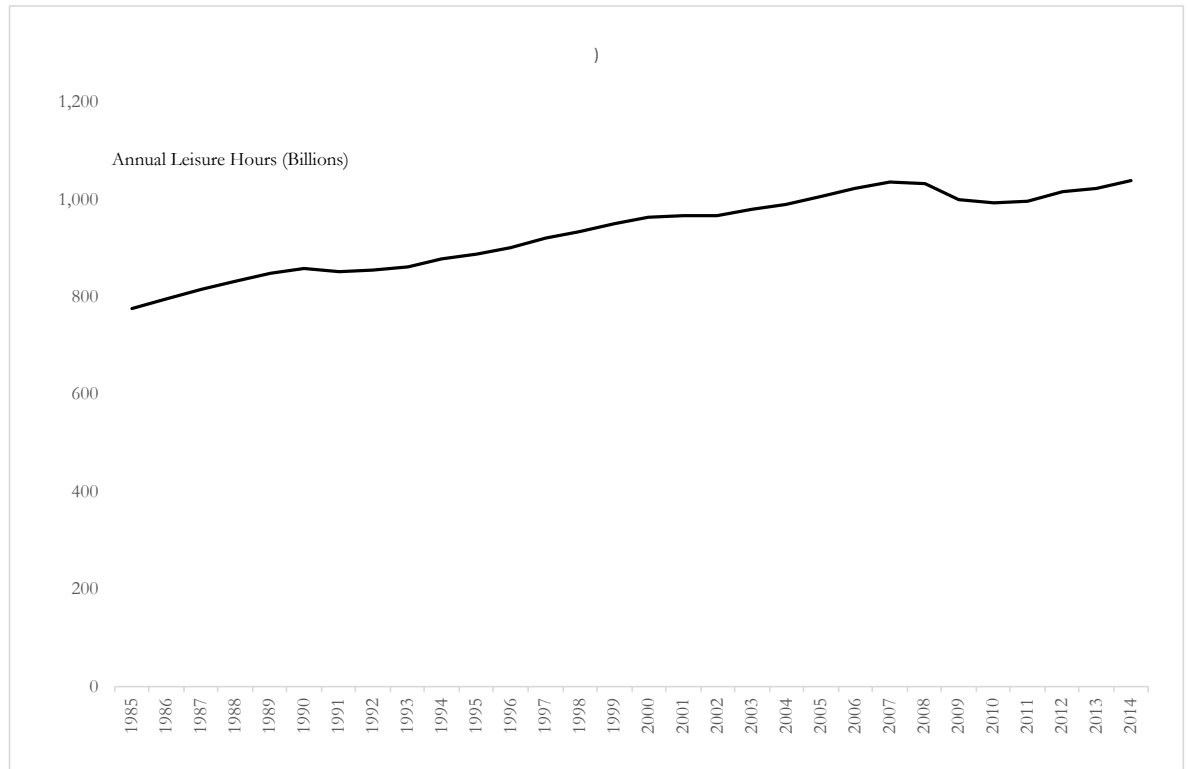


Figure 3.8: Hours of leisure for employed individuals (Billions)

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## Chapter 4

# Leisure and growth

Low-interest rates, depressed consumption and low output growth are concerns for the EU, the United States and Japan. This paper adopts a theoretical model with habit formation for the consumer in a leisure technology good to explore the parameters that are consistent with this environment. The focus is on the macroeconomic implications of a consumer who is learning to use the technology good for leisure-enhancing purposes (i.e. technology-enhanced-leisure). When the importance of technology-enhanced-leisure is less than that of a consumer's habits, growth in technology-enhanced-leisure implies depressed interest rates and consumption declines. Consequently, the impact of technology-enhanced-leisure growth on interest rate results from whether the importance of the new (technology-enhanced-leisure) to the old (habits). Technology-enhanced-leisure growth is compatible with contractions in the output of the technology sector if the growth is greater than the rate of time preference. Unfortunately, the implications of technology-enhanced-leisure growth on the output of the consumption sector are not so simple. The impact depends on all the parameters concerned (importance on habits, importance on technology-enhanced-leisure, CRRA and technology-enhanced-leisure growth).

## 4.1 Introduction

Novel, stimulating and challenging experiences tend to be more pleasurable for an individual (Scitovsky 1976, Berlyne 1970). According to Scitovsky (1976), leisure activities, unlike consuming goods, can perpetually stimulate and challenge. Nonetheless, some leisure activities such as watching television may quickly become mundane. On the other hand, others such as going to the opera can keep providing opportunities to stimulate. A consumer learns to appreciate opera which is challenging and stimulating. Consequently, even after the the event of going to the opera has passed, opera can provide opportunities to challenge. Scitovsky (1976) emphasises that one requirement is to be skilled or competent at consuming an experience. For instance, as suggested a consumer needs to learn to appreciate opera. Furthermore, Taylor and Gratton (2002) point out that as the consumption needs of the individual are satisfied, the individual will look for stimulating ‘skill acquisition’ leisure activities such as sport rather than low-level activities such as watching television.<sup>1</sup> Like culture and sport, the use of a technological good may also be continuously stimulating. It is well-known that the digital revolution has provided consumers with the opportunity to continuously find stimulating experiences to relieve boredom.<sup>2</sup> The internet, online gaming, mobile phones and mobile apps, use of technology involve the consumer learning to make the good more stimulating.

Nonetheless, incorporating this notion into traditional economic models is limited. The contribution of this article is to investigate the impact of this notion of technology-enhanced-leisure on interest rates and output growth. Con-

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1. See Kahneman and Deaton (2010) for an alternative perspective on a lasting shift in happiness.

2. Obviously, users of the digital technology have also increased significantly. Consumers of the internet per 100 people in the World has increased from 0.050 (1990) to 6.77 (2000) to 40.69 (2014). Similarly, mobile cellular subscriptions increased from 0.00 (1980), 0.21 (1990), 12.08 (2000), 76.57 (2010), to 96.89 (2014) (World Bank 2016c, World Bank 2016d).

sequently, we develop the idea of technology-enhanced-leisure, which is the ability of the consumer to take advantage of the technology good for leisure-enhancing activities. Given the recent declines in interest rates and consumption growth<sup>3</sup> our motivation is to investigate the parameters for which increases in the ability of consumers to enhance leisure is consistent with these declines. A theoretical model is developed in which habit formation for the representative consumer in the leisure technology good is employed to pursue this objective. A consumer cares only about how leisure technology compares to his habits. The model is viable for this analysis as habits capture the increasing need for more technology. However, for the normal consumption good, the consumer only cares about the absolute level with habits inconsequential.

If the importance placed on habits is greater than the importance placed on technology-enhanced-leisure, technology-enhanced-leisure growth implies lower consumption growth and reductions in the interest rate. What this means conceptually is that the distaste for breaking 'old' habits is more important to the consumer than the benefits of increases in 'new' technology-enhanced-leisure. If the technology-enhanced-leisure growth rate is greater than the rate of time preference, the output of the technology sector will decline.

The next section outlines the framework. It involves habit formation for the consumer in leisure technology.<sup>4</sup> Section 4.3 discusses the applicable steady state in the system. In the steady state, the variables are transformed into constant ratios. Section 4.4 provides a theoretical exploration of when increases in technology-enhanced-leisure will imply interest rates below the rate of time preference. Furthermore, it analyses the parameters

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3. See World Bank (2016b) and World Bank (2016a) for the decline in interest rates and consumption growth respectively.

4. The framework introduced by Kavuri and McKibbin (2017) is adopted as it originated the utility function. The representative consumer owns the two firms. This paper builds on the leisure term in utility function introducing the notion of technology-enhanced-leisure.

what will cause a contraction in output when there is a rise in technology-enhanced-leisure. The last section concludes and provides suggestions for further research. Possible extensions include introducing substitution between technology-enhanced-leisure and normal consumption purchases. Technology-enhanced-leisure is an unorthodox concept. Leisure in economics has traditionally been studied in the context of substitution. Specifically, leisure, work and allocation of time (see Dickinson 1999; Connolly 2008; Zivin and Neidell 2014). Nonetheless, the framework is simplified and it may provide a starting point for analysis.

## 4.2 Model

The economy is in continuous time with an infinitely lived representative consumer. The consumer derives utility from the function (4.1). This function distinguishes between normal consumption and leisure technology. The utility of the consumer depends on the level of technology ( $R(t)$ ) purchased for enhancement of technology-enhanced-leisure ( $\eta(t)$ ) and on how these purchases compare to habits ( $h(t)$ ). In addition, only the level rather than habits of normal consumption ( $C(t)$ ) is important. There is no habit formation for normal consumption.  $\sigma$  measures the degree of relative risk aversion.  $\alpha$  and  $\Theta$  measure the importance of technology-enhanced-leisure and habits respectively. To enable the full exploration of technology-enhanced-leisure, the consumer does not work and technology-enhanced-leisure is not a choice variable.<sup>5</sup> Notice that the framework differs from the view of Scitovsky (1976).<sup>6</sup> In direct contrast, we consider a consumer that has technology habits that constitute the leisure-enhancing component.

$$U(t) = \frac{C(t)^{1-\sigma}}{1-\sigma} + \frac{(\eta(t)^\alpha R(t)/h(t)^\Theta)^{1-\sigma}}{1-\sigma} \quad (4.1)$$

5. For contrasting views on the allocation of time see Becker 1965; Linder 1970; Scitovsky 1976; Phelps 1973.

6. Scitovsky (1976) highlights how habits are formed on consumption. If  $R(t)$  is culture, then in order to model Scitovsky (1976) appropriately would require  $C(t)$  to have  $h(t)$ .

Where  $0 < \Theta < 1$

$0 < \alpha < 1$

The evolution of the technology habits for the representative consumer evolves as follows:

$$\dot{h}(t) = \psi(R(t) - h(t)) \quad (4.2)$$

Technology-enhanced-leisure is not a random variable. It captures the ability of the representative consumer to learn to use technology more effectively for leisure purposes. Assume that there is a maximum level of productivity that a technology can achieve for an individual. A current example may be Apple watches. The consumer learns how to use the watch effectively by playing with it but also absorbing knowledge by reading on the internet and talking to friends.

$$\dot{\eta}(t) = \kappa(\hat{\eta}(t) - \eta(t)) + \Lambda\eta(t) \quad (4.3)$$

where  $\kappa \in (0, \infty)$

$\Lambda \in [0, g_{\hat{\eta}})$

The consumer absorbs knowledge from this maximum level of productivity at a exogenous rate  $\kappa$ . The consumer can also learn themselves.  $\Lambda$  captures that ability.

$$g_{\hat{\eta}} = \frac{\dot{\hat{\eta}}(t)}{\hat{\eta}(t)} \quad (4.4)$$

Now consider the full model. The production functions for the technology and the consumption good are:

$$Y_R(t) = A_R(t)K_R(t) \quad (4.5)$$

$$Y_C(t) = A_C(t)K_C(t) \quad (4.6)$$

The consumer provides capital to the firms. The evolution of capital for the consumer is as follows:

$$\dot{K}(t) = r(t)K(t) - R(t) - C(t) \quad (4.7)$$

The consumer maximises a discounted infinite stream of utility. The choice variables are the technology ( $R(t)$ ) and consumption ( $C(t)$ ) goods. The state variable is capital. Technology-enhanced-leisure ( $\eta(t)$ ) and habits ( $h(t)$ ) are not choice variables. The equation below is maximised subject to the budget constraint of the consumer.

$$U = \int_0^{\infty} u(C(t), h(t), R(t), \eta(t)) e^{-\rho t} dt \quad (4.8)$$

### 4.3 Steady state

The equations are transformed into variables that are constant in the steady state. Please see appendix A for the full optimising solution. The steady state is defined when variables are constant in the steady state (Mulligan and Sala-i-Martin 1992; Overland, Carroll and Weil 2000; Kavuri and McKibbin 2017). A model has been outlined for a consumer that uses the technology good produced by sector  $R$  to enhance technology-enhanced-leisure. Consequently, for sector  $R$  the growth rate of technology and technology-enhanced-leisure combined per habits need to equate to zero. The steady state is defined as when the following equation holds:

$$\left[ \frac{\eta(t)\dot{R}(t)}{h(t)} \right] = 0 \quad (4.9)$$

$$\left[ \frac{K_R(t)}{h(t)} \right] = 0 \quad (4.10)$$

$$\left[ \frac{C(t)}{K_C(t)} \right] = 0 \quad (4.11)$$

The growth rates in the steady state are the following:

$$(\sigma - \Theta\sigma + \Theta) \frac{\dot{R}(t)}{R(t)} + (\alpha\sigma - \alpha + \Theta - \Theta\sigma) g_{\hat{\eta}} = r(t) - \rho \quad (4.12)$$

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\sigma} \quad (4.13)$$

$$\frac{\dot{\hat{\eta}}(t)}{\hat{\eta}(t)} = \frac{\dot{\eta}(t)}{\eta(t)} \quad (4.14)$$

Equations (4.12) to (4.14) imply that:

$$(\sigma - \Theta\sigma + \Theta)\frac{\dot{R}(t)}{R(t)} + (\alpha\sigma - \alpha + \Theta - \Theta\sigma)g_{\hat{\eta}} = \sigma\frac{\dot{C}(t)}{C(t)} \quad (4.15)$$

As  $\frac{\dot{\eta}(t)}{\eta(t)}$  can be viewed as exogenous technical change this implies  $\frac{\dot{\eta}(t)}{\eta(t)}$  is exogenous growth.

#### 4.4 Implications of perturbation

The parameters involved that imply technology-enhanced-leisure growth are consistent with decreases in  $r(t)$  and contractions in economic growth are now investigated. First, a base case is developed that implies that  $r(t)$  equates to  $\rho$ . Then an exogenous perturbation of increases in technology-enhanced-leisure is induced. Note that depending on  $A_R$  and  $A_C$  under this state the interest rate may not equate to marginal product of capital. Consider a perturbation of increases in technology-enhanced-leisure in isolation.

**Proposition 10** (*Technology-enhanced-leisure proposition*): *Technology-enhanced-leisure increases are consistent with declines in the interest rate and consumption growth when the importance of habits is greater than the importance of productive leisure.*

**Proof**

$$\alpha\sigma - \alpha + \Theta - \Theta\sigma < 0 \quad (4.16)$$

This implies:

$$\alpha < \Theta \quad (4.17)$$

$\Theta$  and  $\alpha$  are the importance of habits and technology-enhanced-leisure respectively.

The growth rates for the base case are:

$$\hat{g}_{\eta} = 0 \quad (4.18)$$

$$\frac{\dot{R}(t)}{R(t)} = 0 \quad (4.19)$$

$$\frac{\dot{C}(t)}{C(t)} = 0 \quad (4.20)$$

With these,  $r(t)$  equates to  $\rho$ .

#### 4.4.1 Impact of technology-enhanced-leisure growth for the interest rate

Now consider a perturbation, with  $g_{\eta} = \bar{\lambda}_{\eta}$ . With the perturbation  $\frac{\dot{R}(t)}{R(t)}$  is assumed to be zero. However, resulting from technology-enhanced-leisure growth the interest rate and consumption growth adjusts. Under this condition the following applies in the perturbation state<sup>7</sup>:

$$\hat{g}_{\eta} = \bar{\lambda}_{\eta} > 0 \quad (4.21)$$

$$\frac{\dot{R}(t)}{R(t)} = 0 \quad (4.22)$$

$$\frac{\dot{C}(t)}{C(t)} = \frac{(\alpha\sigma - \alpha + \Theta - \Theta\sigma)\bar{\lambda}_{\eta}}{\sigma} \quad (4.23)$$

$$r(t) - \rho = (\alpha\sigma - \alpha + \Theta - \Theta\sigma)\bar{\lambda}_{\eta} \quad (4.24)$$

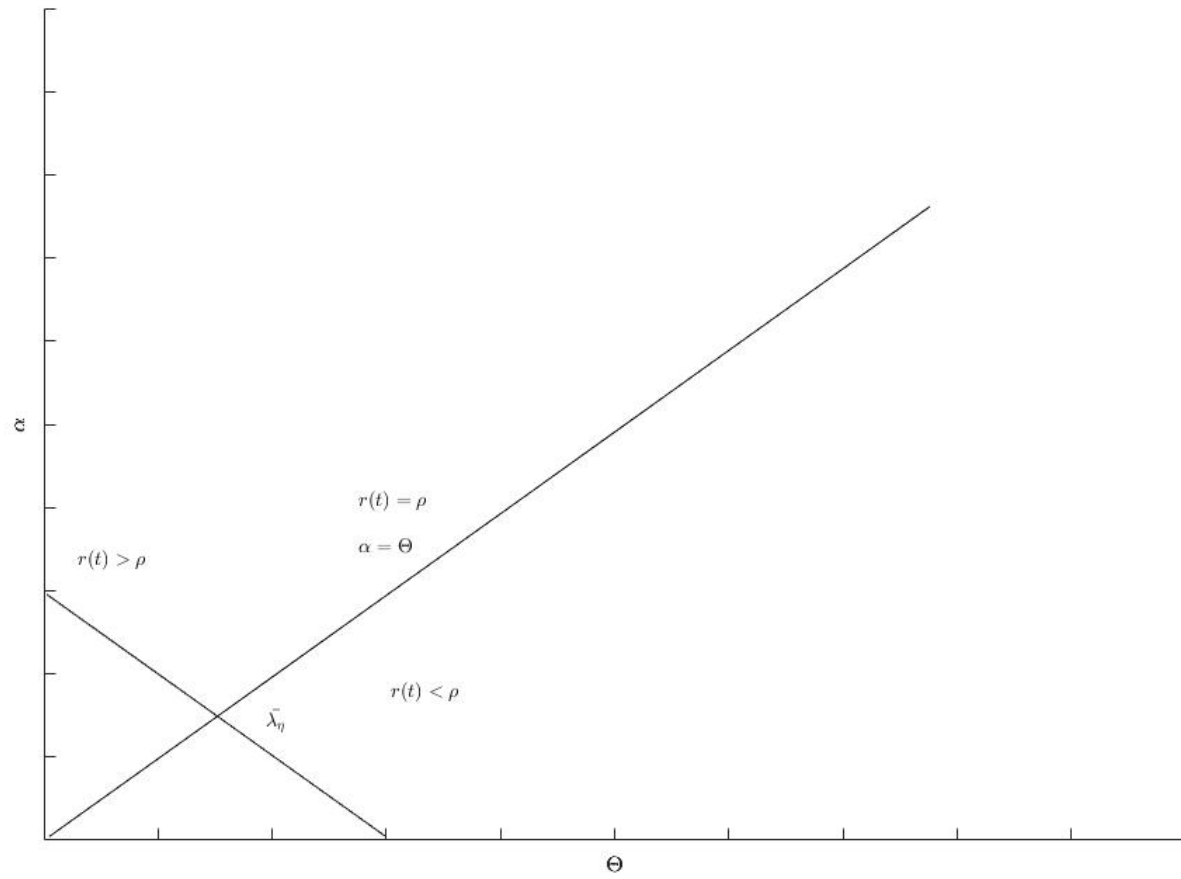
Figure 4.1 represents the relationship between  $\alpha, \Theta$  and  $r(t)$ . The 45° degree line represents  $r(t) = \rho$ .<sup>8</sup> With technology-enhanced-leisure  $\bar{\lambda}_{\eta}$  the economy shifts from the origin to any point on the line marked by  $\bar{\lambda}_{\eta}$ . When  $\Theta > \alpha$ , consumption will be declining and the interest rate will fall below the rate of time preference. The economy can revert back to the origin under three scenarios: firstly, if  $\bar{\lambda}_{\eta}$  falls back to zero and there is no increase in productive leisure in 'knowledge'; secondly, if demand for technology  $R(t)$  grows; thirdly, if the price of technology was to increase relative to the consumption good.

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7.  $r(t) > \bar{\lambda}_{\eta}$

8. Notice that there is no scale and no parameter values are chosen. The figure shows the relationship between the variables concerned.



Figure 4.1:  $\Theta$  and  $\alpha$

It is useful to consider what the proposition means conceptually.<sup>9</sup>  $\Theta$  indexes the importance of the habit stock.  $\Theta = 0$ , then only the absolute level of the technology good matters. On the other hand,  $\alpha$  measures the importance of technology-enhanced-leisure. Hence what does this result mean? Although the importance and weight of the stock of habit technology are higher for the consumer than the benefit of increases in technology-enhanced-leisure, technology-enhanced-leisure still increases. This is a representative result as it is also consistent with the depressed environment. If  $\alpha > \Theta$ , then increases in technology-enhanced-leisure that the consumer values more than habits would conversely imply a brighter environment with higher interest rates and consumption growth. Although there is no substitution and technology-enhanced-leisure is not a choice variable, this is an interesting result nonetheless. See Appendix C for special cases.

#### 4.4.2 Implications of technology-enhanced-leisure growth for output

This section shows that growth in technology-enhanced-leisure is consistent with decreases in sector  $R$  when productive leisure growth is greater than the rate of time preference. The implication on output of sector  $C$  is not as simple and it will be outlined in this section. We need to determine the difference explicitly. Here, output in the base case and the perturbation state is expressed, after which, the difference between output in the two states is illustrated. The production functions as specified in equation (4.5) and (4.6):

##### Value of economy wide output

$$Y_v = A_R(t)K_R(t) + A_C(t)K_C(t) \quad (4.25)$$

To determine change explicitly, we need to look at the stationary level of capital devoted to each sector.  $K_R(t)$  and  $K_C(t)$  in the base case as specified below. Subscript  $B$  indicates the base case.  $r_B(t) = \rho$ .

$$K_{C_B}(t) = \frac{C_B(t)}{\rho} \quad (4.26)$$

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9. Scitovsky (1976) 'distaste' of breaking old consumption habits may be of interest.

$$K_{R_B}(t) = \frac{h_B(t)}{\psi_B} \left[ \frac{(\psi_B)}{\rho} \right] \quad (4.27)$$

$$Y_{C_B}(t) = A_{C_B}(t) \left( \frac{C_B(t)}{\rho} \right) \quad (4.28)$$

$$Y_{R_B}(t) = A_{R_B}(t) \left( \frac{h_B(t)}{\rho} \right) \quad (4.29)$$

The level of capital devoted to each sector in the perturbation state is as follows. Whether the levels of capital fall depend on the parameters in the equations below:

#### Perturbation

$$K_C(t) = \frac{\sigma C_B(t)(1 + (\alpha\sigma - \alpha + \Theta - \Theta\sigma)(\bar{\lambda}_\eta/\sigma))}{\sigma(\rho + (\alpha\sigma - \alpha + \Theta - \Theta\sigma)\bar{\lambda}_\eta) - (\alpha\sigma - \alpha + \Theta - \Theta\sigma)\bar{\lambda}_\eta} \quad (4.30)$$

$$K_R(t) = \frac{h(t)}{\psi} \left[ \frac{(\alpha\sigma - \alpha + \Theta - \Theta\sigma)\bar{\lambda}_\eta + (\psi)(\sigma - \Theta\sigma + \Theta) - (\alpha\sigma - \alpha + \Theta - \Theta\sigma)\bar{\lambda}_\eta + (\sigma - \Theta\sigma + \Theta)\bar{\lambda}_\eta}{(\rho + (\alpha\sigma - \alpha + \Theta - \Theta\sigma)\bar{\lambda}_\eta)(\sigma - \Theta\sigma + \Theta) - (\alpha\sigma - \alpha + \Theta - \Theta\sigma)\bar{\lambda}_\eta - (\sigma - \Theta\sigma + \Theta)\bar{\lambda}_\eta} \right] \quad (4.31)$$

The impact on (value of) output is as follows:

$$A_C(t)K_C(t) - A_{C_B}(t)K_{C_B}(t) \quad (4.32)$$

$$p_R(t)A_R(t)K_R(t) - p_{R_B}(t)A_{R_B}(t)K_{R_B}(t) \quad (4.33)$$

For simplicity of notation, assume:

$$\Gamma = (\alpha\sigma - \alpha + \Theta - \Theta\sigma) \quad (4.34)$$

$$\Phi = (\sigma - \Theta\sigma + \Theta) \quad (4.35)$$

The implication on value of output is as follows.  $\Gamma$ ,  $\Phi$  and  $\Gamma\Phi$  are taken out of the brackets.

#### Sector C

$$\frac{\Gamma[\rho A_C(t)C_B(t)\bar{\lambda}_\eta - \sigma A_{C_B}(t)C_B(t)\bar{\lambda}_\eta + A_{C_B}(t)C_B(t)\bar{\lambda}_\eta] + \sigma\rho C_B(t)A_C(t) - \sigma\rho A_{C_B}(t)C_B(t)}{\sigma\rho^2 + \Gamma[\sigma\rho\bar{\lambda}_\eta - \rho\bar{\lambda}_\eta]} \quad (4.36)$$

To simplify the solution, assume that  $A_C(t) = A_{C_B}$

$$\frac{\Gamma[\rho A_C(t)C_B(t)\bar{\lambda}_\eta - \sigma A_C(t)C_B(t)\bar{\lambda}_\eta + A_C(t)C_B(t)\bar{\lambda}_\eta]}{\sigma\rho^2 + \Gamma[\sigma\rho\bar{\lambda}_\eta - \rho\bar{\lambda}_\eta]} \quad (4.37)$$

The change in output of sector C is negative with either: (i) a negative numerator and positive denominator, or (ii) a positive numerator and negative denominator. For instance, for (i) the following conditions would obtain the result:

$$1 + \rho > \sigma \quad (4.38)$$

$$\frac{\sigma\rho^2}{\sigma\rho\bar{\lambda}_\eta - \rho\bar{\lambda}_\eta} > -\Gamma \quad (4.39)$$

Note that  $\Gamma < 0$  is necessary for a decline in the interest rate.

### Sector R

$$\frac{\Phi[\psi A_R(t)h(t) + A_R(t)h(t)\bar{\lambda}_\eta - \psi\rho A_{RB}(t)h_B(t) + \psi A_{RB}(t)h_B(t)\bar{\lambda}_\eta] + \Gamma[\psi A_{RB}(t)h_B(t)\bar{\lambda}_\eta] - \Gamma\Phi[A_{RB}(t)h_B(t)\bar{\lambda}_\eta]}{\Phi[\rho^2\psi - \rho\psi\bar{\lambda}_\eta] - \Gamma[\rho\psi\bar{\lambda}_\eta] + \Phi\Gamma[\rho\psi\bar{\lambda}_\eta]} \quad (4.40)$$

Assume  $A_R(t) = A_{R_B}(t)$ . The equation with  $A_R(t) = A_{R_B}(t)$  is provided in Appendix B .

The parameters<sup>10</sup> ensure the numerator is positive.

$$\rho < 1 \quad (4.41)$$

$$\psi < 1 \quad (4.42)$$

$$\Phi > 1 \quad (4.43)$$

The change in value of output for sector R is negative with a negative denominator. This is induced by the specification below:

$$\rho \leq \bar{\lambda}_\eta \quad (4.44)$$

What this means is that the growth in technology-enhanced-leisure is sufficiently high to ensure the output of R is negative. Unfortunately, the para-

10. Also assume that  $h(t) = h_B(t)$ .

meters that are consistent with a decline in sector  $C$  output are not straightforward.

#### 4.5 Concluding remarks and extensions

Increases in technology-enhanced-leisure are consistent with declines in the interest rate and consumption when the distaste for breaking old habits is more important than the benefits of increases in new productive leisure. This is surprisingly representative of the depressed situation. Although technology-enhanced-leisure ( $\alpha$ ) is less valued by the consumer than habits ( $\Theta$ ), it still increases. Would this result change if an elasticity of substitution between leisure and consumption in the utility function is introduced? This may be of interest for further research. When the growth in technology-enhanced-leisure is greater than the rate of time preference, the change in the output of  $R$  is negative.

This investigation is unorthodox and simplified. Nonetheless, it aims to provide a starting point for further investigations. There are a number of ways which in the analysis could be extended. The next step would be to incorporate substitution into the framework. Furthermore, in order to understand the applicability, it would be desirable to test this framework with real numbers. A proxy for technology-enhanced-leisure could be used for the simulation.

## 4.6 Appendix A

The current value Hamiltonian is below.

$$H = \frac{C(t)^{1-\sigma}}{1-\sigma} + \frac{(\eta(t)^\alpha R(t)/h(t)^\Theta)^{1-\sigma}}{1-\sigma} + \nu(t)(r(t)K(t) - R(t) - C(t)) \quad (4.45)$$

The first order conditions:

$$\frac{\partial H}{\partial C(t)} = C(t)^{-\sigma} - \nu(t) = 0 \quad (4.46)$$

$$\frac{\partial H}{\partial R(t)} = \left[ \frac{\eta(t)^\alpha R(t)}{h(t)^\Theta} \right]^{-\sigma} \frac{\eta(t)^\alpha}{h(t)^\Theta} - \nu(t) = 0 \quad (4.47)$$

$$\frac{\partial H}{\partial K(t)} = \rho \nu(t) - \dot{\nu}(t) = \nu r(t)$$

The transversality condition is:

$$\lim_{t \rightarrow \infty} e^{-\rho t} \nu(t) K(t) = 0 \quad (4.48)$$

The co-state variables are solved out of the first order conditions. Consequently, the whole dynamical system of equations that characterises the economy is stated below. To make it simpler to determine the steady state level of

capital devoted to each sectors, the motion of capital is split up as below:

$$\frac{\dot{R}(t)}{R(t)} = \frac{1}{\sigma} \left( r(t) - \rho + (\Theta\sigma - \Theta) \frac{\dot{h}(t)}{h(t)} + (\alpha - \alpha\sigma) \frac{\dot{\eta}(t)}{\eta(t)} \right) \quad (4.49)$$

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\sigma} [r(t) - \rho] \quad (4.50)$$

$$\frac{\dot{K}_R(t)}{K_R(t)} = r(t) - \frac{R(t)}{K_R(t)} \quad (4.51)$$

$$\frac{\dot{K}_C(t)}{K_C(t)} = r(t) - \frac{C(t)}{K_C} \quad (4.52)$$

$$\frac{\dot{h}(t)}{h(t)} = \psi \left( \frac{R(t)}{h(t)} - 1 \right) \quad (4.53)$$

$$\frac{\dot{\eta}(t)}{\eta(t)} = g_\eta \quad (4.54)$$

### Steady state

As suggested the system of equations in the steady state is defined as:

$$\left[ \frac{\eta(t)\dot{R}(t)}{h(t)} \right] = 0 \quad (4.55)$$

$$\left[ \frac{K_R(t)}{h(t)} \right] = 0 \quad (4.56)$$

$$\left[ \frac{C(t)}{K_C(t)} \right] = 0 \quad (4.57)$$

Steady state occurs when the growth rate of technology and leisure combined per habits equate to zero. i.e  $\left[ \frac{\eta(t)\dot{R}(t)}{h(t)} \right] = 0$ . To determine the steady state values, average growth rates apply. The time derivative of  $\left[ \frac{\eta(t)R(t)}{h(t)} \right]$  is taken and the evolution of habits is substituted into this time derivative.

$$\left[ \frac{\eta(t)\dot{R}(t)}{h(t)} \right] = \frac{\eta(t)R(t)}{h(t)} \left[ \frac{\dot{\eta}(t)}{\eta(t)} + \frac{\dot{R}(t)}{R(t)} - \frac{\psi R(t)}{h(t)} + \psi \right] \quad (4.58)$$

This time derivative is set to zero. The equation is then substituted into the formula for the growth rate of technology. Define  $\frac{\dot{\eta}(t)}{\eta(t)} = g_\eta$ . The station-

any growth of leisure technology and the steady state level of technology and leisure to habits are the following:

$$\frac{\dot{R}(t)}{R(t)} = \frac{r(t) - \rho}{\sigma - (\Theta\sigma - \Theta)} - \frac{(\alpha\sigma - \alpha + \Theta - \Theta\sigma)g_\eta}{\sigma - (\Theta\sigma - \Theta)} \quad (4.59)$$

$$\frac{R(t)\eta(t)}{h(t)} = \eta(t) \left[ \frac{1}{\psi} \left( \frac{r(t) - \rho}{\sigma - (\Theta\sigma - \Theta)} + \frac{(\sigma - (\Theta\sigma - \Theta))g_\eta - (\alpha\sigma - \alpha + \Theta - \Theta\sigma)g_\eta}{\sigma - (\Theta\sigma - \Theta)} - \frac{\pi_R}{(\sigma - (\Theta\sigma - \Theta))} \right) + 1 \right] \quad (4.60)$$

To ensure the growth of  $K_R(t)$  is a constant ratio in the steady state requires the growth rate of capital to habits to go to zero. The time derivative of capital to habits is taken. Further the evolution of habits and the motion of capital is substituted into the equation which obtains:

$$\left[ \frac{K_R(t)}{h(t)} \right] = \frac{K_R(t)}{h(t)} \left( r(t) - \psi \left( \frac{R(t)}{h(t)} - 1 \right) \right) - \frac{R(t)}{h(t)} \quad (4.61)$$

The steady state level of capital to habits is the following.

$$\frac{K_R(t)}{h(t)} = \frac{1}{\psi} \left[ \frac{(r(t) - \rho) + (\psi)(\sigma - \Theta\sigma + \Theta) - (\alpha\sigma - \alpha + \Theta - \Theta\sigma)g_\eta + (\sigma - \Theta\sigma + \Theta)g_\eta}{r(t)(\sigma - \Theta\sigma + \Theta) - r(t) + \rho + (\alpha\sigma - \alpha + \Theta - \Theta\sigma)g_\eta - (\sigma - \Theta\sigma + \Theta)g_\eta} \right] \quad (4.62)$$

To obtain a constant ratio the time derivative of consumption/capital devoted to sector is set to zero. i.e.  $\left[ \frac{\dot{C}(t)}{K_C(t)} \right] = 0$ .

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\sigma} [r(t) - \rho] \quad (4.63)$$

$$\frac{\dot{K}_C(t)}{K_C(t)} = r(t) - \frac{C(t)}{K_C(t)} \quad (4.64)$$

Observe that the constant ratio of consumption  $C(t)$  per capital  $K_C(t)$  is:

$$\frac{C(t)}{K_C(t)} = \left[ r(t) - \frac{r(t)}{\sigma} + \frac{\rho}{\sigma} \right] \quad (4.65)$$

#### Steady state growth of $\eta(t)$

The next task is to determine growth in  $\eta(t)$  the steady state. The steady state for the evolution of  $\eta(t)$  occurs when the growth between the knowledge of the representative consumer and the maximum knowledge stops changing. The evolution of  $\eta(t)$  is reformulated to obtain:

$$\frac{\dot{\eta}(t)}{\eta(t)} = \kappa \left( \frac{\eta(\hat{t})}{\eta(t)} - 1 \right) + \Lambda \quad (4.66)$$



Define  $b$  as the fraction between consumer knowledge and maximum level of knowledge.

$$b = \frac{\eta(t)}{\hat{\eta}(t)} \quad (4.67)$$

The time derivative of the above is taken and is substituted into the differential for  $\eta(t)$ . The time derivative is set to zero indicating that the change in growth rate between individuals  $i$  knowledge and maximum knowledge stops. A steady state occurs with the following:

$$b = \frac{\kappa}{\kappa + g_{\hat{\eta}} - \Lambda} \quad (4.68)$$

Further at this steady state, the following applies:

$$\frac{\dot{\eta}(t)}{\eta(t)} = g_{\eta} = \frac{\dot{\hat{\eta}}(t)}{\hat{\eta}(t)} = g_{\hat{\eta}} \quad (4.69)$$

## 4.7 Appendix C

### 4.7.1 Scenario One

$\alpha = 1$ ,  $\frac{\dot{\eta}(t)}{\eta(t)} = 0$ , technology-enhanced-leisure is not a choice variable.

$$(\sigma - \Theta\sigma + \Theta) \frac{\dot{R}(t)}{R(t)} + = r(t) - \rho \quad (4.70)$$

### 4.7.2 Scenario Two

$\alpha = 0$ .  $\eta$  disappears.

$$(\sigma - \Theta\sigma + \Theta) \frac{\dot{R}(t)}{R(t)} + = r(t) - \rho \quad (4.71)$$

### 4.7.3 Scenario Three

$0 < \alpha < 1$ , leisure not a choice variable,  $\frac{\dot{\eta}(t)}{\eta(t)} > 0$ .

$$(\sigma - \Theta\sigma + \Theta) \frac{\dot{R}(t)}{R(t)} + + (\alpha\sigma - \alpha + \Theta - \Theta\sigma) g_{\hat{\eta}} = r(t) - \rho \quad (4.72)$$

#### 4.7.4 Scenario Four

$0 < \alpha < 1$ , leisure is not choice variable. The same equation applies if technology-enhanced-leisure is a choice variable)  $\frac{\dot{\eta}(t)}{\eta(t)} > 0$ .

$$(\sigma - \Theta\sigma + \Theta) \frac{\dot{R}(t)}{R(t)} + (\sigma + \Theta - \Theta\sigma) g_{\dot{\eta}} = r(t) - \rho \quad (4.73)$$

As there is no labour supplied, there is no extra first order condition resulting from leisure being a choice variable.

#### 4.7.5 Scenario Five

$0 < \alpha < 1$ , leisure choice variable,  $\frac{\dot{\eta}(t)}{\eta(t)} > 0$  and labour supplied. This is clearly non logical as  $\frac{\dot{\eta}(t)}{\eta(t)}$  cannot grow forever  $\eta$  must be constrained by  $0 < \eta < 1$  as it will have to be fraction of day supplied for work.

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## Chapter 5

# The consequence of robots for economic wellbeing

This study develops a general equilibrium framework to investigate the impact of robot development on economic wellbeing. The framework integrates leisure, endogenous technical change, substitution, robots purchased by the consumer, robots as a form of human replacement and heterogeneous skills of labour. This examination finds that incorporating leisure can imply increasing welfare in the presence of robots replacing jobs of humans.<sup>1</sup>

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## 5.1 Introduction

Whether robots will be a curse or boon for human welfare is extremely topical. The majority of research has focused on the potentially ominous consequences. Sachs and Kotlikoff (2012) demonstrate that a rise in machine productivity could lead to machines substituting for unskilled young generations depressing wages and limiting investment in skill acquisition. This, in turn, drives down wages further. Benzell et al. (2015) use an overlapping generation model with high-tech and low-tech workers and determine that wages are likely to decrease due to the use of robots. Sachs, Benzell and LaGarda (2015) also argue that robotic productivity could lower wages and consumption in the long run. Frey and Osborne (2013) estimate that over the next twenty years 47 % of jobs are at risk to automation (see also Susskind (2017)). The thorough study by Acemoglu and Restrepo (2015) is less pessimistic. A valuable feature of their framework is to distinguish between tasks easily automated and complex tasks in which labour has a comparative advantage. The authors show inequality increases during the transition process but may self-correct due to technology's endogenous response. According to Gordon (2014) the impact of new innovations has fallen over the years. He highlights that robots have been in existence since General Motors with its first industrial robots. Consequently, Gordon argues in a series of recent articles that overall, jobs are not under threat by robots.

The concern about the detrimental impacts of technology is not new. Benzell et al. (2015) acknowledge this and highlighted the case of Ned Ludd in 1779 who destroyed two knitting frames. Ludd was the impetus for the revolts of 1813 by luddites against technology. Academically, one of the first to raise concerns was Karl Marx who emphasised the impact of technology on inequality. In the 20th Century, Keynes (1933) and Schumpeter (1939) also stressed some of the negative consequences of technological progress. More recently, important contributions to the impacts on wage inequality include

those by Katz and Murphy (1992), Goldin and Katz (1998), Autor, Katz and Krueger (1998), Acemoglu (2002) , Autor, Levy and Murnane (2003), Autor and Dorn (2013), Autor, Katz and Kearney (2008), Autor and Dorn (2009) , Goldin and Katz (2009), Acemoglu and Autor (2011), Katz and Margo (2013) and Brynjolfsson and McAfee (2014). These significant contributions are both theoretical and empirical.

Katz and Murphy (1992) developed a demand and supply framework that matched the relative increase in wage and quantities of more educated workers in the US between 1963 and 1987. Goldin and Katz (1998) further showed that between 1910 and 1940 the increase in the supply of skills may have mitigated the rising inequality resulting from technological change. More recently Autor and Dorn (2013) argued that when the production elasticity of substitution between computer capital and routine labour is higher than the elasticity of substitution in consumption between goods and services, then falling prices of computers causes wages for low-skill labour undertaking routine jobs to worsen relative to wages for low-skill labour performing manual tasks. Although most research points to technology as facilitating the rising inequality, Mishel, Schmitt and Shierholz (2013) in *Don't blame the robots* highlight their scepticism of the analysis by Autor and Dorn 2013.

Nonetheless, these studies focus only on the impacts on consumption and wages with leisure receiving no attention. Yet, the inclusion of leisure is surely of consequence for evaluating the welfare effects. Indeed, the trade-off between leisure and work is the cornerstone of economic theory. Furthermore, policy makers have become increasingly aware of leisure and variables including health as a critical component of welfare. As basic needs are increasingly satisfied, arguably this trend will continue. According to Costanza et al. (2014) there is a consensus that society should strive for a high quality of life that is equitable and sustainable. Costanza et al. (2014) further highlight that major groups including the European commission and Frederick S. Pardee Center for

the Study of the Longer-Range Future concur. The quality of growth, not just the amount of growth is important (Barro 2002). Consequently, it is no surprise that numerous national indicators exist that include variables other than consumption. These include the Australian Unity Well-Being Index, Gallup-Healthways Well-Being Index, Gross National Happiness, and Human Development Index.

Secondly, an important dimension that current studies neglect is the potential for robots purchased by consumers to enhance leisure. Technologies such as smartphones and computers, which might be seen as a precursor to robots, are increasingly being used to enhance leisure activities. Cell phones (and potentially robots) are now an indispensable component of leisure. A survey of 305 university students by Lepp et al. (2013) found that 88.2% reported using their cell phone primarily for leisure. The activities involved by students include playing games, surfing the net and social networking. Foley, Holzman and Wearing (2007) determined that cell phones can increase access to leisure in public spaces for adolescent females. The authors found that cell phones gave confidence and converted daunting spaces into comfortable areas due to potential connections on the phone. White and White (2007) investigated the effect of cell phone use on the tourist experience and found that phones influence experiences. For instance, tourists can feel away and at home at the same time. The random survey by Lepp (2014) of US university students found that high users of cell phones are now actually dependent on cell phones to experience leisure. Consequently, it is no surprise that personal demand for robots by consumers is expected to grow the most out of all consumption categories. Boston Consulting Group (Sander and Wolfgang (2014)) forecast spending on robots to increase from US\$15 billion in 2010 to US\$67 billion in 2025 with the demand from consumers (personal demand) the fastest growing. The estimated compound annual growth rate (CAGR) of robots for the years 2010 to 2015 by Boston Consulting Group for the personal market, commercial, indus-



trial and military is 17.4%, 12.3 %, 7.6% and 8.1% respectively.

This paper contributes to the debate on the impacts of robots or technology on economic wellbeing.<sup>2</sup> Leisure in the utility function and robots purchased by the consumer is introduced. Furthermore, the ingredients of substitution, mobility, endogenous technical change and an analysis of various segments of society is included. This paper finds that including leisure in the utility function enables utility to increase in the response to rising robot productivity despite decreasing wages and consumption. The importance of leisure as found in this research has considerable policy implications.

This paper is structured as follows. Section 5.2 sets out the model. There are 4 sectors, 2 types of workers and the owner of the firms and supplier of R&D which is a 'Bill Gates' type of entrepreneur. I call the owner 'entrepreneur'. The equilibrium of the model is derived. In section 5.3, the impact of a rise in the productivity of robots under two different parameterizations of the model is explored. The results for the cases of with and without robots included in the utility functions through the impact of enhancing leisure is discussed.

## 5.2 Model framework

There are four production sectors ( $i \in [C,H,M,R]$ ). Table one highlights the sector characteristics. Sector  $C$  combines the intermediate goods to produce the final good. Sector  $H$  uses labour to produce the intermediate good. Sector  $M$  uses robots to produce an intermediate good. Sector  $R$  uses workers to produce robots. Agents are comprised of several types of workers and an entrepreneur. The set-up is an infinite-horizon continuous time economy. The entrepreneur owns all the production sectors and is the capitalist. The entrepreneur supplies labour for research and development.<sup>3</sup> Workers supply

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2. Robots and technology will be used interchangeably in this paper.

3. The basic insights do not change by separating the highest skilled individual from the firm owner. In addition, Autor (2015) and Frey and Osborne (2013) highlight that some jobs, which

labour for employment. There are two types of labour workers. For example,  $L_H$  specifies a labour worker in sector  $H$ . Only the labour worker  $H$  is in direct competition with robots. Labour worker  $R$  produces robots and these workers are harder to replace than in sector  $H$ .<sup>4</sup>

The entrepreneur and the workers maximise lifetime utility  $U_{j_i}(0)$  subject to their budget constraints ( $j \in [N, L]$  with  $N$  denoting the entrepreneur,  $L$  denoting workers).  $u_N$  is the utility for the entrepreneur.  $u_{L_i}$  is the utility for a worker in sector  $i$ . There is no government and there is no mobility of workers between sectors.<sup>5</sup>

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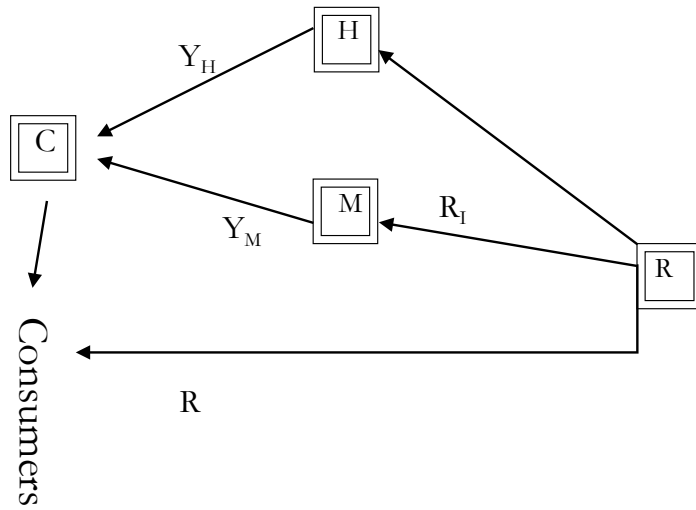
involve creativity or social intelligence, are harder to automate.

4. Interestingly, Autor, Levy and Murnane (2003) and Autor (2015) highlight that middle-skilled worker jobs that require some cognitive ability or manual work are most at risk. Some manual work such as food preparation and jobs in security are harder to replace with robots.

5. This is strong assumption. However, it is included to determine the welfare implications for worker and the entrepreneur.

Table 5.1: Sector Characteristics

| Sector | Characteristics   |
|--------|---|
| C      | Final good sector (uses $Y_H$ and $Y_M$ to produce unique consumption good (C))           |
| H      | Intermediate good sector (employs workers $H$ to produce $Y_H$ )                          |
| M      | Intermediate good sector (employs robots ( $R_I$ ) to produce $Y_M$ )                     |
| R      | Intermediate & final good sector (employs worker $R$ to produce robots ( $R_I$ and $R$ )) |



| Sectors | Output      |
|---------|-------------|
| R       | $R_I$ & $R$ |
| H       | $Y_H$       |
| M       | $Y_M$       |
| C       | $C$         |

### 5.2.1 The firm's problem

The unique consumption good is produced by sector  $C$ . Sector  $C$  combines output from the sector that use machines/robots (defined as sector  $M$ ) and the sector that employs workers (defined as sector  $H$ ). Labour workers in sector  $R$  produce robots for use in sector  $M$ . Each firm is assumed to maximise lifetime value ( $V_i(0)$ ) subject to technology and profit functions as specified below:

$$V_i(0) = \int_0^{\infty} e^{-\int_0^t r(s) ds} \pi_i(t) dt \quad (5.1)$$

$r$  is interest rate and  $\pi_i$  is profit for firm  $i$ . The production functions are listed in equations (5.2) to (5.5). Whereas sectors  $H$  and  $R$  use labour workers. Sector  $M$  uses robots ( $R_I$ ) as form of human replacement. Sector  $R$  produces the robots ( $R_I$ ).

$$Y_C(t) = (\gamma Y_H(t)^{\frac{\epsilon-1}{\epsilon}} + (1-\gamma) Y_M(t)^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} \quad (5.2)$$

$$Y_H(t) = A_H(t) L_H(t) \quad (5.3)$$

$$Y_R(t) = A_R(t) L_R(t) \quad (5.4)$$

$$Y_M(t) = A_M(t) R_I(t) \quad (5.5)$$

Where  $\gamma \in (0,1)$  is the distribution parameters and represents the intensity of  $Y_H$  in production.  $\epsilon \in (0,\infty)$  represent the elasticity of substitution.  $A_i$  represents the productivity of the respective sectors. Assume initially that there is no productivity growth in the sectors.

$$\dot{A}_M = \dot{A}_H(t) = \dot{A}_R(t) = 0 \quad (5.6)$$

The profit functions vary between the firms.

$$\pi_C(t) = P_C(t) Y_C(t) - P_H(t) Y_H(t) - P_M(t) Y_M(t) \quad (5.7)$$

$$\pi_M(t) = P_M A_M(t) R_I(t) - P_R(t) R_I(t) - w_N(t) N(t) \quad (5.8)$$

$$\pi_H(t) = P_H A_H(t) L_H(t) - w_{L_H}(t) L_H(t) \quad (5.9)$$

$$\pi_R(t) = P_R A_R(t) L_R(t) - w_{L_R}(t) L_R(t) \quad (5.10)$$

$P_R(t)$  is price of robots (i.e., output from sector  $R$ ), with  $P_C(t), P_H(t)$  and  $P_M(t)$  being the price of output from the respective sectors.  $w_{L_i}$  represents the wages for the labour workers. The entrepreneur only obtains wages for the research and development they supply. However, as owners of the firm, the profits from all the firms go to the entrepreneur. All four sectors maximise lifetime value subject to their own technology and production functions. The price of final consumption good,  $P_C(t)$  is below:

$$P_C(t) = (\gamma^\epsilon (P_H(t))^{1-\epsilon} + (1-\gamma)^\epsilon (P_M(t))^{1-\epsilon})^{\frac{1}{1-\epsilon}} \quad (5.11)$$

The demand from the final good sector is a CES function that is isoelastic demand. Hence, intermediate sectors  $M$  and  $H$  will charge a constant mark-up over costs. The relative price difference is a function of the state of the technology in these two sectors.

$$\frac{P_H(t)}{P_M(t)} = \frac{\gamma}{1-\gamma} \left( \frac{Y_H(t)}{Y_M(t)} \right)^{-\frac{1}{\epsilon}} \quad (5.12)$$

$$\frac{Y_M(t)}{Y_H(t)} = \left( \frac{\gamma}{1-\gamma} \frac{A_M(t)w_{L_H}(t)}{A_H(t)P_R(t)} \right)^\epsilon \quad (5.13)$$

$$P_H(t)A_H(t) = w_{L_H}(t) \quad (5.14)$$

$$P_M(t)A_M(t) = P_I(t) \quad (5.15)$$

$$P_R(t)A_R(t) = w_{L_R}(t) \quad (5.16)$$

When there is a rise in productivity of robots,  $\nu_M(t)A_M(t) = w_N(t)$ . This is obtained from profit maximisation in sector  $M$ .  $\nu_M(t)$  is the shadow price of entrepreneur in sector  $M$  providing one more unit for research and development.

### 5.2.2 The consumer's problem

The entrepreneur and the workers maximise lifetime utility subject to their budget constraints. There are different workers in each sector.  $\varphi$  and  $\delta$  are the weights (elasticities) both of which are greater than zero. The weights

are the same for the entrepreneur and the worker.

Maximise:

$$U_j(0) = \int_0^{\infty} e^{-\rho t} u_j(t) dt \quad (5.17)$$

**Entrepreneur**

$$u_N(t) = \varphi \log C_N(t) + \delta \log((1-N(t))) \quad (5.18)$$

Subject to:

$$\dot{a}_N(t) = r(t)a_N(t) + w_N(t)N(t) - P_C(t)C_N(t) + \Pi(t) \quad (5.19)$$

**Workers**

$$u_{L_i}(t) = \varphi \log C_{L_i}(t) + \delta \log((1-L_i(t))) \quad (5.20)$$

Subject to:

$$\dot{a}_{L_i}(t) = r(t)a_{L_i}(t) + w_{L_i}(t)L_i(t) - P_C(t)C_{L_i}(t) \quad (5.21)$$

$a_N(0)$  and  $a_{L_i}(0)$  are known

$\rho$  represents the discount rate,  $a_j(t)$  represents asset holdings and  $r(t)$  denotes the interest rate.  $(1-N(t))$  denotes the fraction of the day that is used for leisure.  $N(t)$  represents the fraction of the day supplied by the entrepreneur for research and development.  $\Pi(t)$  is the sum of profits  $\pi_i(t)$  from the four sectors and goes to the capitalist owner, the entrepreneur.

The problems are solved by using the current value Hamiltonian method. The choice variables are consumption and for labour workers the fraction of day supplied for work. Work is not a choice variable for the entrepreneur.  $L_i(t)$  represents the fraction of the day supplied for labour work. The state variable is asset holdings. The first order conditions for the workers and the entrepreneur are below.  $\pi_C(t)$  is the constant price change of consumption purchased. Where  $j$  is either  $N$  or  $L$ .  $i$  is the index for sector.

$$\frac{\varphi}{P_C C_{L_i}(t)} = \frac{\delta}{(1-L_i(t))w_{L_i}(t)} \quad (5.22)$$

$$\frac{\dot{C}_j}{C_j} + \pi_C(t) = r(t) - \rho \quad (5.23)$$

The transversality condition is as follows.  $v_{a_j}(t)$  is the costate variable.

$$\lim_{t \rightarrow \infty} e^{-\rho t} v_{a_j}(t) a_{j_i}(t) = 0 \quad (5.24)$$

### 5.3 The Impact of rise in robot productivity

In this section we investigate a rise in robot productivity in two cases. The first case is with robots used for human replacement by sector  $M$  and without robots purchased for leisure purposes. The second case is with robots used by sector  $M$  and with robots purchased for leisure by the consumer.

#### 5.3.1 Human replacement in sector $M$

There is no productivity growth in sector  $H$  and  $R$ . However, there is growth in the technology function for sector  $M$  due to research and development. The entrepreneur only undertakes R&D in sector  $M$ . Notice that the consumers' problem is the same as before. Equations (5.17) -(5.21) still apply. When there is rising robot productivity equations (5.25) and (5.26) apply. The equations apply during the transition process when  $\dot{A}_M(t) > 0$ . During this transition process  $N(t) > 0$ . When  $N(t)$  reverts back to zero,  $\dot{A}_M(t)$  also reverts back to zero. Nonetheless, this leads to a new steady state with a higher level of  $A_M(t)$ . Consequently, it is assumed that equations (5.6) and (5.8) are modified to equations (5.25) and (5.26) during the transition process.

$$\dot{A}_M(t) = A_M(t)N(t) \quad (5.25)$$

$$\pi_M(t) = P_M A_M(t) R_I(t) - P_R(t) R_I(t) - w_N(t) N(t) \quad (5.26)$$

Dynamic optimisation determines the path of research and development in sector  $M$ . The state variable is  $A_M(t)$  with the choice variable being  $N(t)$ .

$$r(t) = P_M(t) \left( \frac{R_I(t)}{T(t)} \right) \left( \frac{A_M(t)}{w_N(t)} \right) + \left( \frac{N(t)}{T(t)} \right) + \frac{\dot{w}_N(t)}{w_N(t)} - \frac{\dot{A}_M(t)}{A_M(t)} \quad (5.27)$$

Before proceeding, it is useful to note what this equation implies. The interest rate is positively related to the proportion of robots in sector ( $R_I$ ) per



capita ( $T$ ). The interest rate is positively correlated with the proportion of the time the entrepreneur spends on research and development in sector  $M$  to the total population. Understandably, only the proportion of the entrepreneur's time devoted for research and development is important. In equilibrium, the rate of return for undertaking research and development in sector  $M$  and the return to savings must equate. The superscript  $j$  denotes the labour worker ( $L$ ) and the entrepreneur ( $N$ ).  $i$  indicates the sectors.

$$\frac{\dot{C}_j}{C_j} + \pi_C(t) = P_M(t) \left( \frac{R_I(t)}{T(t)} \right) \left( \frac{A_M(t)}{w_N(t)} \right) + \left( \frac{N(t)}{T(t)} \right) + \bar{w}_N - \bar{A}_M - \rho \quad (5.28)$$

There are wage differentials between all the workers. To reduce the number of equations, here, we only highlight the differential between workers in sector  $H$  and the entrepreneur undertaking research and development.

$$\frac{w_{L_H}(t)}{w_N(t)} = \frac{\gamma}{1} \left( \frac{Y_C(t)}{A_H(t)L_H(t)} \right)^{\frac{1}{\epsilon}} \frac{A_H(t)}{v_{M(t)}A_M(t)} \quad (5.29)$$

Define  $\lambda_H(t)$  as the proportion of sector  $H$  labour out of total population  $T$  (i.e.  $L_H/T(t)$ ). Given the definition, the following provides an intuitive way of reading the wage differential.

$$\frac{w_{L_H}(t)}{w_N(t)} = \frac{\gamma}{1} \left[ \left( \frac{Y_C(t)}{T(t)} \right) \left( \frac{1}{A_L(t)} \right) \left( \frac{1}{\lambda_H(t)} \right) \right]^{\frac{1}{\epsilon}} \frac{1}{v_{M(t)}} \frac{A_H(t)}{A_M(t)} \quad (5.30)$$

Notice that the wage differential depends on the relative productivity of the worker,  $A_H(t)$ , to the shadow price for undertaking research and development multiplied to the productivity of robots  $v_{M(t)}A_M(t)$ . Further output per capita ( $Y_C(t)/T(t)$ ) and the proportion of sector workers in  $H$  impacts the differential. The following proposition shows an insight obtained from the consumers' utility function in this section. It highlights when a consumer will gain in net welfare.

**Proposition 11** *Regardless of the rise in robot productivity there will be a net welfare gain for any given consumer from  $t_0$  to  $t_1$  if the following condition holds:*

$$\frac{w(t_0)}{w(t_1)} > \left( \frac{C(t_0)}{C(t_1)} \right)^{(\rho+\delta)/\delta} \quad (5.31)$$

As long as the proposition holds, the consumer will always benefit even if the productivity of robots rises.

### 5.3.2 Robots purchased by consumers

An innovation of this paper is to explicitly investigate the impact of the ability of robots to enhance leisure activities. To distinguish between the robots sold to the consumer and to the sector we use  $R$  and  $R_I$ .  $R$  represents the demand for robots from all the consumers. There is no price discrimination, hence  $P_R(t) = P_I(t)$ .

The consumers maximise:

$$U_{j_i}(0) = \int_0^{\infty} e^{-\rho t} u_{j_i}(t) dt \quad (5.32)$$

#### Entrepreneur

$$u_N(t) = \varphi \log C_N(t) + \delta \log((1-N(t))R_N(t)) \quad (5.33)$$

Subject to:

$$a_N \dot{(t)} = r(t)a_N(t) + w_N(t)N(t) - P_C(t)C_N(t) - P_I(t)R_N(t) + \Pi(t) \quad (5.34)$$

#### Workers

$$u_{L_i}(t) = \varphi \log C_{L_i}(t) + \delta \log((1-L_i(t))R_{L_i}) \quad (5.35)$$

Subject to:

$$a_{L_i} \dot{(t)} = r(t)a_{L_i}(t) + w_{L_i}(t)L_i(t) - P_C(t)C_{L_i}(t) - P_I(t)R_{L_i}(t) \quad (5.36)$$

$a_N(0)$  and  $a_{L_i}(0)$  are known

The following now applies:

$$\frac{\varphi}{P_C C_{j_i}(t)} = \frac{\delta}{(1-j_i(t))w_{j_i}(t)} = \frac{\delta}{P_I R_{j_i}(t)} \quad (5.37)$$

$$\frac{\dot{R}_{j_i}}{R_{j_i}} + \pi_I(t) = r(t) - \rho \quad (5.38)$$

$$\frac{\dot{C}_{j_i}}{C_{j_i}} + \pi_C(t) = r(t) - \rho \quad (5.39)$$

Logically, one can view the allocation of robot as the contest between the consumer requirements to enhance leisure and the sector demand for human

replacement. Notice that allocation boils down to the consumer weights,  $\varphi$  and  $\delta$ , relative demand for normal consumption versus sector requirements especially sectoral elasticity of substitution.

$$\frac{R_I(t)}{R(t)} = \frac{\varphi P_I}{\delta P_C C(t)} \frac{A_H(t)}{A_M(t)} \left[ \frac{L_H(t)}{(w_{LH}(t) A_M(t) (1-\gamma) / P_R(t) A_H(t) \gamma)^{-\epsilon}} \right] \quad (5.40)$$

Importantly, robots purchased for leisure augments the welfare benefits shown previously. Even under the case of robot productivity greater than workers, there could be a net welfare gain for all groups in society. A rise in robot productivity implies initially a fall in the price of robots. This leads to consumers demanding more leisure and robots purchased for this purpose. Consequently, the price of robots for human replacement and for leisure purchases increases. As the price of robots for human replacement increases, this ensures demand for workers in sector  $H$ . This insight and reasoning is applicable to any utility function. Now a proposition that highlights this insight for the utility function in this paper is derived. The subscripts and use  $L$  for labour to reduce unnecessary notation is dropped.

**Proposition 12** *Regardless of the rise in robot productivity there will be a net welfare gain for any given consumer from  $t_0$  to  $t_1$  if either of the following conditions hold:*

$$\frac{w(t_0)}{w(t_1)} > \left( \frac{C(t_0)}{C(t_1)} \right)^{(\rho+\delta)/\delta} \left( \frac{P_C(t_0)R(t_0)}{P_C(t_1)R(t_1)} \right) \quad (5.41)$$

$$\frac{w(t_0)}{w(t_1)} > \left( \frac{C(t_0)}{C(t_1)} \right)^{(\rho+\delta)/\delta} \left( \frac{P_I(t_0)}{P_I(t_1)} \right) \left( \frac{R(t_0)}{R(t_1)} \right)^2 \quad (5.42)$$

**Proof:**

$$u(t_0) = \varphi \log C(t_0) + \delta \log((1-L(t_0))R(t_0)) \quad (5.43)$$

$$u(t_1) = \varphi \log C(t_1) + \delta \log((1-L(t_1))R(t_1)) \quad (5.44)$$

Labour ( $L$ ) is substituted out with  $C$  by using the first order condition for the consumer in (5.37). This obtains:

$$u(t_0) - u(t_1) = \varphi \log \left[ \left( \frac{C(t_0)}{C(t_1)} \right) \right] + \delta \log \left[ \left( \frac{P_C(t_0)}{P_C(t_1)} \right) \left( \frac{C(t_0)}{C(t_1)} \right) \left( \frac{R(t_0)}{R(t_1)} \right) \left( \frac{w(t_1)}{w(t_0)} \right) \right] \quad (5.45)$$

$u(t_1) > u(t_0)$  if the following holds:

$$(\varphi + \delta) \log\left(\frac{C(t_0)}{C(t_1)}\right) + \delta \log\left(\frac{P_C(t_0)R(t_0)}{P_C(t_1)R(t_1)}\right) - \delta \log\left(\frac{w(t_0)}{w(t_1)}\right) < 0 \quad (5.46)$$

Condition (5.41) can be derived from here. Condition (5.42) can be obtained with  $L$  substituted out for robots ( $R$ ) instead of consumption. Consequently, the following applies:

$$u(t_0) - u(t_1) = \varphi \left(\frac{C(t_0)}{C(t_1)}\right) + \delta \left(\frac{P_I(t_0)}{P_I(t_1)}\right) \left(\frac{R(t_0)}{R(t_1)}\right) \left(\frac{R(t_0)}{R(t_1)}\right) \left(\frac{w(t_1)}{w(t_0)}\right) \quad (5.47)$$

QED

There are numerous variables in conditions (5.41) and (5.42). Hence, determining when the conditions will hold is tough. However, note that increases in the price of robots, consumer purchases of robots or consumption will reduce the right hand side.

Besides employment and economic wellbeing, robots purchased to enhance leisure activities could have major macroeconomic impacts. For instance, Kavuri and McKibbin (2017) originated a framework that included habit formation for a digital technology good purchased for leisure enhancement activities. Leisure and the digital technology good are not separable in their paper. The framework was shown to match trends in interest rates and normal consumption good purchases in the U.S.

In summary, this section shows that robots purchased by the consumer can improve welfare, even in the presence of reductions in wages. As suggested Boston Consulting Group estimated that the compound annual growth rate of personal demand for robots to increase the fastest out of any market for robots. This result is promising in regards to the welfare impact of robots. Consequently, policy could augment this welfare benefit by supporting the supply side and demand side of the market. Demand side policies such as educating consumers on how to use digital technology and robots may have other welfare benefits.

## 5.4 Conclusion

This paper has investigated the effect of more productive robots on economic wellbeing. The most important finding is that regardless of the rise in productivity of robots,, the welfare of workers can increase because of the rise in leisure activity. Robots purchased by the consumer is likely to imply net welfare gains. Policy could be directed to educate consumers on using digital technology.

The findings of the gains in utility from leisure, robots to enhance leisure and the negative impact of workers supplying hours excessively are important. Arguably, as needs become increasingly satisfied, leisure and other variables ( e.g., health) will become a more important element to the utility of the consumer than normal consumption. Clark et al. (2016) find that mental health and physical health are significantly more important than economic variables in impacting the happiness of individuals. Indeed, it is quite clear that robot development will coincide with consumption needs being increasingly met. There is an ongoing debate amongst academics and policy makers about GDP being an adequate measure of a country's well-being. There are numerous well-being measures with variables such as sustainability included. Nonetheless, leisure is critical. For instance, Stiglitz, Sen and Fitoussi (2009) stress the importance of leisure in their report for President Sarkozy:

*'The question of leisure arises. Consuming the same bundle of goods and services but working for 1500 hours a year instead of 2000 hours a year implies an increase in ones standard of living. Although valuation of leisure is fraught with difficulties, comparisons of living standards over time or across countries needs to take into account the amount of leisure that people enjoy.'*

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## Chapter 6

# The implications of singularity for workers

This paper investigates two scenarios in which the worker remains in existence despite artificial intelligence overtaking human thinking-singularity. In this analysis, the Entrepreneur (capitalist) has replaced the worker with a robot. In order to isolate the reason for the worker to remain in existence, different models are developed. The only similarity is that they both involve the entrepreneur (who owns the robot) and the worker, who has been replaced by the robot. The scenarios considered focus on altruism and the role of economies of scale. In the altruistic case, a model is developed that endogenises a concept called acceptable living standards. The worker lobbies the entrepreneur to increase acceptable living standards and receives a transfer allowance from the entrepreneur. This case is introduced to contribute to the debate on basic universal income. The paper also explores whether a government with corresponding taxation can lead to a similar outcome as the altruistic case i.e., transfer allowance by the entrepreneur to the worker without government. The second scenario explores whether there can be profit motivation for the entrepreneur to keep the worker alive in order to sell products.

## 6.1 Introduction

Singularity implies robots are better than humans in every way possible (Bostrom 2014). Economic studies tend not to investigate the impact of singularity. There are a growing number of articles and books mostly from computer scientists including renowned theoretical physicist Kaku (2012) and mathematicians advocating those disruptive technologies will lead to singularity (Kurzweil 2005). Robotist Moravec (1988), in *Mind Children*, argues that human equivalence by machines is just the beginning with endless possibilities to come. Others make predictions on dates. For instance, Kurzweil (1999) in *Age of Spiritual Machines* outlines his law of accelerating returns. In *Singularity is Near*, he makes a concrete prediction that by the year 2045 technological singularity will occur. Rigorous economic scholarly testing of the singularity hypothesis is lacking in the literature (Eden et al. 2012). An exception is an empirical test undertaken by Nordhaus (2015) within the framework of an economic growth model. Nordhaus (2015) defines singularity as the time when the economic growth rate is greater than 20% per year. After undertaking tests to determine substitutability between conventional inputs and information, Nordhaus (2015) concludes that singularity is not near. He extrapolates and finds that it will be at least 100 years before the US economy could ever reach the singularity. There are others who go further and deny the possibility of singularity (Huebner 2005, Modis 2003).

This paper investigates two scenarios in which the worker will remain in existence despite singularity. The two scenarios involve the entrepreneur, who owns the robot, and the worker, who has been replaced by the robot. The first scenario is the altruistic case. There is a basic requirement for the worker to obtain an acceptable living standard which is defined as Minimum Acceptable Living Standards (MALS). MALS is endogenised with the choice variable being a transfer allowance from the entrepreneur to the worker. Optimisation determines that the worker equates the rate of time preference to the marginal

benefit of time devoted to influencing MALS divided by consumption. Ever since the analysis by Böhm-Bawerk (1890) the rate of time preference represented how individuals discount present relative to future consumption. Hence this result may appear surprising. Nonetheless, it is quite logical. The notion of time-preference is built on the concept of marginal utility (i.e., from Menger). More specifically, according to Olson and Bailey (1981), the concept provided by Böhm-Bawerk (1890) represents two main notions. The first notion is diminishing the marginal utility of consumption and the second notion is the discounting of the future compared to the present utility. These two notions are crucial to highlight that the rate of preference is a trade-off between utility at different points of time. In the model, this trade-off equates to the trade-off for the worker of giving up leisure time to influence the acceptable living standards to obtain a transfer allowance. This analysis may contribute to the philosophical discussions over singularity and the existential risk of artificial intelligence. Bostrom (2003) highlights in regards to the cognitive study of ethics an artificially intelligent entity may be better than humans.<sup>1</sup> In this scenario, an altruistic case where the entrepreneur cares about the worker is compared to a situation where a government intervenes to impose income transfers. The analysis shows that a government introducing a profit tax on a firm using robots with a corresponding transfer to the worker induces the same consumption outcome as does altruism by the entrepreneur without government. It could imply greater welfare for the worker as the worker benefits

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1. Bostrom (2003) argues that to ensure the superintelligent entity has a beneficial impact it should be installed with philanthropic values and human-friendly intentions. Note that Bostrom (2002) emphasises a badly programmed superintelligent entity could end up annihilating humankind. In the model, one could replace the acceptable standard of living with philanthropic values. These values may need to be installed into a robot by the capitalist as the control mechanism. Nonetheless, I am aware of some of the ethical considerations, including whether robots should be given moral consideration as discussed by Lin, Abney and Bekey (2011).

from leisure experience. In contrast, wage and consumption taxes are distortionary and yield less benefits. Furthermore, a tax on robot use should also be avoided, as it could increase the price of the consumption good and also be distortionary. In the second scenario, there may be a profit incentive for the entrepreneur to keep the worker alive in order to sell products. In this scenario, I also explore the conditions under which there may be a revolution with the worker taking control of the robot.

The rest of the paper is structured as follows. Section 6.2 introduces the altruistic case and compares it to a situation with a government. Economies of scale and revolution is investigated in section 6.3. Section 6.4 provides the conclusion.

## 6.2 Altruism: Transfer allowance

There is one unique representative entrepreneur and representative labour worker denoted by  $N$  and  $L$ . The worker can influence the acceptable living standard in society ( $F(t)$ ), but cannot influence the subsistence level ( $F(\bar{t})$ ) nor the transfer itself ( $\psi(F(t))$ ).  $F(t)$  can be viewed as the level of fairness in society. Consequently, although the entrepreneur suffers negatively from providing ( $\psi(F(t))$ ) out of his/her resources, in the altruistic case, the entrepreneur feels it desirable to provide the worker with income.  $\psi(F(t))$  is the transfer from the Entrepreneur from his/her assets, which is a function of accepted standard of living ( $F(t)$ ). Notice that both  $\psi(F(t))$  and ( $F(\bar{t})$ ) is in the utility function of the entrepreneur.

### Production

The unique consumption good is produced by the robot.  $R(t)$  represents the robot. As we are considering singularity, neither the worker nor the entrepreneur are required to produce the robot. It may seem odd that the robot appears from nowhere. Nonetheless, by the definition of singularity, humans

are superfluous to produce robots.  $w$  is a wage for the entrepreneur to ensure the smooth running of the robot.  $wN$  is a fixed cost. Total output,  $Y(t)$  is consumed by the entrepreneur and the worker,  $Y(t) = C_N(t) + C_L(t)$ .

$$Y(t) = A(t)R(t) \quad (6.1)$$

$$\Pi(t) = A(t)R(t) - wN \quad (6.2)$$

$\Pi(t)$  is cash flow provided to the entrepreneur.

The entrepreneur and the workers maximise lifetime utility  $U_j(0)$  subject to their budget constraints ( $j \in [N, L]$  with  $N$  denoting the entrepreneur,  $L$  denoting workers).

Maximise:

$$U_j(0) = \int_0^{\infty} e^{-\rho t} u_j(t) dt \quad (6.3)$$

### Entrepreneur

The choice variables for the representative entrepreneur are  $\psi(F(t))$  and  $C_N(t)$  with state variable  $a(t)$ . The entrepreneur cannot choose the subsistence level ( $F(\bar{t})$ ) nor the accepted standard of living ( $F(t)$ ), but he/her can choose the level transfers:

$$u_N(t) = b_1 \log(C_N(t)) - b_2 \log(\psi(F(t)) - F(\bar{t})) + b_3(1 - N(t)) \quad (6.4)$$

Subject to:

$$\dot{a}_N(t) = r(t)a_N(t) - \psi(F(t)) - C_N(t) + wN + \Pi(t) \quad (6.5)$$

$a(0)$  known

$\psi(F(t)) > F(\bar{t})$

Solving the optimisation problems leads to two equations that relate to

$r(t) - \rho$ :

$$\frac{(\psi(F(t)) - F(t))}{(\psi(F(t)) - F(t))} = \rho - r(t) \quad (6.6)$$

$$\frac{\dot{C}_N(t)}{C_N(t)} = r(t) - \rho \quad (6.7)$$

Equation (6.6) shows that the growth rate of the difference between transfers and the subsistence level required to survive equates to  $\rho - r(t)$ . This equation is a reverse of equation (6.7), which is the Euler consumption equation.

### Worker

Although the worker has no employment they need to consume. Consequently, the worker provide  $L_F(t)$  to lobby to increase the acceptable standards of living. This may be in the form of campaigns. The accepted living standards are not included in the utility function. The choice variables for the worker are  $L_F(t)$  and  $C_L(t)$ <sup>2</sup> with state variable  $F(t)$ .

$$u_L(t) = b_1 \log(C_L(t)) + b_2 \log(1 - L_F(t)) \quad (6.8)$$

Subject to:

$$\dot{F}(t) = F(t)L_F(t) \quad (6.9)$$

$$F(0) = F_0 \quad (6.10)$$

$$\psi(F(t)) = C_L(t) \quad (6.11)$$

**Proposition 13** *The worker optimises by equating the the rate of time preference to the growth rate of the (current value) shadow price of one more unit devoted to lobbying plus the ratio of the marginal benefit of transfers from the entrepreneur to the transfer itself.*

### Proof

The first order conditions for the worker imply the following:

$$\rho = \frac{\dot{v}(t)}{v(t)} + L_F(t) + \frac{b_1}{v(t)} \frac{\psi'(F(t))}{C_L(t)} \quad (6.12)$$

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2. However, note that  $\psi(F(t)) = C_L(t)$ .



where  $\psi'(F(t))$  is the marginal benefit to the worker of providing one more unit to  $L_F(t)$  to lobbying.  $v(t)$  is the costate variable, the current value shadow price. (See appendix for derivation and complete proof)

Exploring this equation in detail is enlightening. Consider the following, which defines the evolution of the acceptable living standards.

$$\dot{v}(t) = \frac{b_2(\rho - L_F(t))}{(1 - L_F(t))F(t)} - \frac{b_1\psi'(F(t))}{C_L(t)} \quad (6.13)$$

$$\dot{F}(t) = F(t)L_F(t) \quad (6.14)$$

**Proposition 14** *When  $L_F(t) = 0$  ( $\dot{F}(t) = 0$ ) and  $\dot{v}(t)=0$ ,<sup>3</sup> optimality implies that the rate of time preference equates to the ratio of the marginal benefit to the worker of devoting time to lobbying (multiplied by the acceptable standard of living) divided by the transfer allowance from the entrepreneur.*

#### Proof

Substituting  $L_F(t) = 0$  ( $\dot{F}(t) = 0$ ) and  $\dot{v}(t)=0$  in the above equations leads to the following:

$$\rho = \frac{b_1\psi'(F(t))F(t)}{b_2C_L(t)} \quad (6.15)$$

At this point the worker could increase fairness and transfers by lobbying more. Nonetheless, equation (6.15) shows it is not optimal to do so as they are giving up leisure time.

### 6.2.1 Government: Taxation

A government is now introduced to investigate whether the outcome in the case of taxation is the same as in altruism.

#### Sector

The unique consumption good is produced by the robot as before, with  $Y(t) =$

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3. This is applied to provide insight into the equation.

$C_N(t) + C_L(t)$ .

$$Y(t) = A(t)R(t) \quad (6.16)$$

$$\Pi(t) = A(t)R(t) - wN \quad (6.17)$$

The government taxes gross cash flow  $\Pi(t)$  to give after flow tax  $(\pi(t))$ . This after tax cash flow is distributed to the Entrepreneur:

$$\pi(t) = (1 - \tau_\Pi)\Pi(t) \quad (6.18)$$

### Government

The government cannot borrow and satisfies the budget constraint.  $T$  is a lump sum transfer to the worker.  $\tau_a(t)$ ,  $\tau_C(t)$ ,  $\tau_\Pi(t)$  and  $\tau_w(t)$  are taxations on assets, consumption, profit and wages respectively.

$$T = \tau_a(t)a(t) + \tau_C(t)C(t) + \tau_\Pi(t)\Pi(t) + \tau_w(t)wN \quad (6.19)$$

### Entrepreneur

The entrepreneur and the worker maximises lifetime utility  $U_j(0)$  subject to their budget constraints ( $j \in [N, L]$  with  $N$  denoting the entrepreneur,  $L$  denoting workers).

Maximise:

$$U_j(0) = \int_0^\infty e^{-\rho t} u_j(t) dt \quad (6.20)$$

The choice variables for the representative entrepreneur are now both  $C_N(t)$  and  $N(t)$  with state variable  $a(t)$ . Note that previously  $N$  was not a choice variable. The entrepreneur has no desire to transfer to the worker. Nonetheless, the utility function is the same as in the altruistic case:

$$u_N(t) = b_1 \log(C_N(t)) - b_2 \log(\psi(F(t)) - \bar{F}(t)) + b_3(1 - N(t)) \quad (6.21)$$

Subject to:

$$\dot{a}_N(t) = r(t)(1 - \tau_a)a_N(t) - \psi(F(t)) - (1 + \tau_C)C_N(t) + (1 - \tau_w(t))w(t) + (1 - \tau_\Pi(t))\Pi(t) \quad (6.22)$$

$a(0)$  is known.

### Worker

As in the altruistic case assume the same utility function and that the workers have no employment. However, the worker no longer needs to lobby to increase the acceptable standards of living. Consequently  $L_F(t) = 0$ . The government provides a lump sum  $T$  based on the acceptable standards of living:

$$u_L(t) = b_1 \log(C_L(t)) + b_2 \log(1 - L_F(t)) \quad (6.23)$$

Subject to:

$$F'(t) = F(t)L_F(t) \quad (6.24)$$

$$F(0) = F_0 \quad (6.25)$$

$$T + \psi(F(t)) = (1 + \tau_C)P_C(t)C_L(t) \quad (6.26)$$

### 6.2.2 Impact of taxation

Here each tax in turn is considered.

**Proposition 15** *If  $\tau_a, \tau_C, \tau_w = 0$ ,  $T = \psi(F(t))$ , and the government makes necessary transfers to the worker, the welfare of the entrepreneur is the same as in the altruistic case. In addition, the welfare of the worker is higher as he/she does not need to devote any time to lobbying to obtain a transfer, ( $L_F(t) = 0$ ).*

#### Proof

$\tau_a, \tau_C, \tau_w = 0$ . Consequently,  $T = \tau_\Pi(t)\Pi(t)$ . Suppose  $T = \psi(F(t))$  and the government applies  $\tau_\Pi(t)$  and transfer  $T$  based on MALS. The impact on the utility function and on the entrepreneur's assets is the same as in the altruistic case.<sup>4</sup> Although the worker's assets and consumption is the same as the altruistic case, welfare is higher as he/she does not need to devote any time to lobbying.

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4.  $\Pi(t) - \psi(F(t))$  equals  $\Pi(1 - \tau_\Pi)$ .

**Proposition 16** *If  $r, \tau_C(t), \tau_w(t), \tau_\Pi(t) = 0$  and  $\tau_a > 0$ , welfare is higher for the worker. However, consumption growth rate for the entrepreneur is less than in the altruistic case.*

**Proof**

If  $\tau_a > 0$  and  $\tau_C(t), \tau_w(t), \tau_\Pi(t) = 0$  this implies that  $T = \tau_a a(t)r(t)$ . Hence the consumption growth rate for the entrepreneur is less than in the altruistic case. Nonetheless, it does not distort the intratemporal choices of the entrepreneur. As the worker does not save, if  $T = \psi(F(t))$ , this implies as in the previous case that the worker's welfare is higher. Note that the worker does not save and hence the asset tax does not impact them:

$$\frac{\dot{C}_N(t)}{C_N(t)} = r(t) - \rho \quad (6.27)$$

**Proposition 17** *Consumption taxes are distortionary. It would impact the intratemporal choices and could lead to lower welfare for both the entrepreneur and the worker.*

**Proof**

Notice that the first order condition for the entrepreneur is now the following:

$$N(t) = 1 - \frac{b_3(1-\tau_C)C_N(t)}{b_1(1-\tau_w)w(t)} \quad (6.28)$$

These taxations could have unintended consequences. As the entrepreneur is critical to ensure smooth running of the robot, which produces the unique consumption good, a tax on wage should be avoided.

A consumption tax would impact the worker and the entrepreneur. Although all taxation revenue is provided to the worker, it leads to unnecessary payments and transfers. In reality there would be an administration cost and could lead to lower welfare for the worker. Furthermore, it is distortionary for the choices of the entrepreneur.

$$T + \psi(F(t)) = (1 + \tau_C)C_L(t) \quad (6.29)$$

Lastly, it is important to note that a tax on robot use should also be avoided. In the appendix, this case is considered in more detail. Nonetheless, a tax on robot use implies a higher price of the consumption good. Consequently, it would be distortionary to the entrepreneur's intratemporal choices. Hence, it should also be avoided. In conclusion, the most preferable taxation choice is on profit and ownership of a firm using robots.

### 6.3 Profit motivation: Economies of scale

In this section, a different model is considered in which there is a profit motivation for the entrepreneur to ensure that the worker survives. There is increasing returns to scale in this section. Consequently,  $\alpha > 1$  with  $R(t)^\alpha$ . Furthermore, there is a marginal cost to produce the robot which is defined as  $m_R(t)$ .

#### Production

$$Y(t) = A(t)R(t)^\alpha \quad (6.30)$$

$$\Pi(t) = A(t)R(t)^\alpha - wN - m_R(t)R(t) \quad (6.31)$$

$\Pi(t)$  is profit provided to the entrepreneur. The first order conditions imply:

$$R(t) = \left( \frac{m_R(t)}{\alpha A(t)} \right)^{\frac{1}{\alpha-1}} \quad (6.32)$$

The entrepreneur and the worker maximise lifetime utility  $U_j(0)$  subject to their budget constraints ( $j \in [N, L]$  with  $N$  denoting the entrepreneur,  $L$  denoting workers).

Maximise:

$$U_j(0) = \int_0^\infty e^{-\rho t} u_j(t) dt \quad (6.33)$$

#### Entrepreneur

The utility function of the entrepreneur is provided below. Utility and

the budget constraint is increasing with higher  $\Pi(t)$ . The choice variables are  $C_N(t)$  and  $\psi(F(t))$ .

$$u_N(t) = b_1 \log(C_N) + b_2 \log(\Pi(t) - \psi(F(t))) + b_3(1 - N(t)) \quad (6.34)$$

Subject to:

$$\dot{a}_N(t) = r(t)a_N(t) - \psi(F(t)) - C_N(t) + wN + \Pi(t) \quad (6.35)$$

$a(0)$  is known

$F(t)$  is known and it is minimum requirement to keep workers alive. The first order conditions imply:

$$\frac{\psi(F(t)) - \Pi(t)}{C_N(t)} = \frac{b_2}{b_1} \quad (6.36)$$

In addition, two equations relate to  $r(t) - \rho$ :

$$\frac{(\Pi(t) - \dot{\psi}(F(t)))}{(\Pi(t) - \psi(F(t)))} = \rho - r(t) \quad (6.37)$$

$$\frac{\dot{C}_N(t)}{C_N(t)} = r(t) - \rho \quad (6.38)$$

### Worker

$$u_L(t) = b_1 \log(C_L(t)) + b_2 \log(1 - L(t)) \quad (6.39)$$

Subject to:

$$\psi(F(t)) = C_L(t) \quad (6.40)$$

The worker spends all the transfer on consumption. Specifically,  $\psi(F(t)) = C_L(t)$ . The worker does not devote any time to employment or lobbying.

**Proposition 18** *It is more profitable for entrepreneur to ensure the worker survives o purchase the products when the right hand side of equation 6.41 is increasing as the market increases.*

$$A(t) \left( \left( \frac{m_R(t)}{\alpha A(t)} \right)^{\frac{1}{\alpha-1}} \right)^\alpha - m_R(t) \left( \left( \frac{m_R(t)}{\alpha A(t)} \right)^{\frac{1}{\alpha-1}} \right) > w(t)N(t) \quad (6.41)$$

**Proof**

As  $\Pi(t)$  increases the utility for the entrepreneur, consider now when profit is increasing. Introducing  $R(t)$  into the profit function yields:

$$\Pi(t) = A(t) \left( \left( \frac{m_R(t)}{\alpha A(t)} \right)^{\frac{1}{\alpha-1}} \right)^\alpha - w(t)N(t) - m_R(t) \left( \left( \frac{m_R(t)}{\alpha A(t)} \right)^{\frac{1}{\alpha-1}} \right) \quad (6.42)$$

Rearranging equation 6.42 leads to 6.41.

**6.3.1 Revolution**

The possibility of revolution is now considered. A fixed cost  $I(t)$  is introduced in order to keep control of workers. In addition,  $P_N$  and  $P_L$  are introduced in the utility functions as a preferences for power and control. The entrepreneur and the worker maximise lifetime utility  $U_j(0)$  subject to their budget constraints.

Maximise:

$$U_j(0) = \int_0^\infty e^{-\rho t} u_j(t) dt \quad (6.43)$$

**Entrepreneur**

The utility function of the entrepreneur is modified below.

$$u_N(t) = b_1 \log(C(t)) + b_2 \log(\Pi(t) - \psi(F(t))) + b_3(1 - N(t)) + P_N \quad (6.44)$$

Subject to:

$$a_N(t) = r(t)a_N(t) - \psi(F(t)) - C_N(t) + wN + \Pi(t) \quad (6.45)$$

$a(0)$  is known

**Worker**

The utility function of the worker is as follows:

$$u_L(t) = b_1 \log(C_L(t)) + b_2 \log(1 - L(t)) + P_L \quad (6.46)$$

Subject to:

$$\psi(F(t)) = C_L(t) \quad (6.47)$$

Where:

$$P_L = \begin{cases} 1, & \text{if the worker is in control.} \\ 0, & \text{otherwise.} \end{cases} \quad (6.48)$$

$$P_N = \begin{cases} 1, & \text{if the entrepreneur is in control.} \\ 0, & \text{otherwise.} \end{cases} \quad (6.49)$$

**Proposition 19** *If equation (6.50) holds, the worker and the entrepreneur co-exist. If equation (6.51) holds there is revolution.*

$$\Pi(t) - I(t) \geq F\bar{t} \quad (6.50)$$

$$\Pi(t) \leq F\bar{t} + I(t) \quad (6.51)$$

### Proof

The entrepreneur remains in control of workers when equation (6.50) holds. The entrepreneur's transfer,  $\psi(F(t))$ , equates to  $F\bar{t}$ . The worker obtains a minimum level requirement to survive. However, equation (6.51) holds, there is revolution. As the entrepreneur cannot pay  $I(t)$  to oversee the workers, workers take control of the robot. Although there is a revolution, the worker cannot wipe out the entrepreneur. The worker requires the entrepreneur to maintain the robots and produce a consumption good. As suggested  $wN$  is the fixed cost paid to the entrepreneur for maintaining the robot. As there is a preference for power the worker prefers revolution.

It is a simple model. There are many ways that imply a preference for power. For instance, the worker could take profit or reduce the wage for the entrepreneur. In addition, more complexity could be added including an optimising decision for revolution. Nonetheless, the aim here is to initiate debate around this area.



## 6.4 Conclusion

The results are particularly noteworthy and relevant.<sup>5</sup> Debates exist on whether the singularity is possible but limited research tackles it economically. In the event of singularity, ownership of the firm using robots is important. Nonetheless, even in the worst case scenario of no jobs, no ownership and no government, workers are not necessarily going to die out. First of all, there is a profit motivation for the entrepreneur to ensure that the worker can purchase the unique consumption good. Secondly, the transfer adheres to altruistic motivations. In this scenario, acceptable minimum living standards are endogenised. The model proposed is quite novel given that endogenising standards has not been undertaken this way previously. Nonetheless, standards have been shown to be important for profitability of wage cuts (Kahneman, Knetsch and Thaler (1986)) and tax evasion (Andreoni, Erard and Feinstein (1998)). A surprising result occurs when the endogenous variables (shadow price of the worker devoting one more unit to lobbying and this accepted standards) are no longer changing. The result is that at this point the rate of time preference equates to the ratio of the marginal benefit to the worker of devoting time to lobbying (multiplied by the acceptable standard of living) divided by the transfer allowance from the entrepreneur. On reflection, the result is intuitive. At this point, one more unit devoted to lobbying has been driven down to zero. Consequently, the rate of time preference that relates consumption today relative to the future, which is intratemporal, is now also equated to the

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5. This analysis may also be relevant to the philosophical discussions over singularity. Bostrom (2003), a celebrated philosopher and a leading thinker, argues that superintelligence may be better than human in the study of ethics. He also argues that to ensure that the superintelligence is beneficial for society it should be framed with 'philanthropic values'. Consequently, this investigation may have wider appeal than the economic spectrum. Indeed, one could replace the acceptable standard with philanthropic values that may need to be installed in robots as a means of control. I am also aware of the ethical considerations as raised by Lin, Abney and Bekey (2011), including whether robots should be given rights.

substitution between one more unit of lobbying and the allowance transfer.

This paper also shows that welfare of the worker could be higher with a government and taxation. The worker does not need to lobby to obtain the transfer and hence could gain from health benefits from more leisure (Sparks et al. (1997)). For instance, a summary of research undertaken by White and Beswick (2003) suggests that there is a positive association between working long hours and fatigue, cardiovascular problems and poor physical health. However, importantly, the preferred tax is a profit tax on a firm using robots. A tax on wages, consumption and on the use of robots should be avoided as it would be distortionary and impact intratemporal choices.

## 6.5 Appendix

### 6.5.1 Tax on robot use

Here, there is a cost in using robots denoted by  $m_R(t)$ . Total profit is as follows:

$$\Pi(t) = A(t)R(t) - m_R(t)R(t) - w(t)N(t) \quad (6.52)$$

Profit maximisation implies:

$$A(t) = m_R(t) \quad (6.53)$$

Suppose the government taxes the use of robots. The tax is denoted by  $\tau_M(t)$ .

$$\Pi(t) = A(t)R(t) - (1 + \tau_M)m_R(t)R(t) - w(t)N(t) \quad (6.54)$$

Profit maximisation implies:

$$A(t) = (1 + \tau_M)m_R(t) \quad (6.55)$$

With  $A(t)$  and  $m_R(t)$  as in the case without taxation, this necessarily implies that price of consumption is higher. The first order condition of the entrepreneur is as follows:

$$N(t) = 1 - \frac{b_3 C_N(t)}{b_1 w(t)} \quad (6.56)$$

A higher price of the unique consumption benefit, will impact labour supplied by the entrepreneur. Subsequently it may impact output and consequently welfare. It is distortionary and hence should be avoided.

### 6.5.2 Transfer allowance

$$u_L(t) = b_1 \log(C_L(t)) + b_2 \log(1 - L_F(t)) \quad (6.57)$$

Subject to:

$$\dot{F}(t) = F(t)L_F(t) \quad (6.58)$$

$$F(0) = F_0 \quad (6.59)$$

$$\psi(F(t)) = C_L(t) \quad (6.60)$$

The current value Hamiltonian is below:

$$H = u_L(t) = b_1 \log(C_L(t)) + b_2 \log(1 - L_F(t)) + v(t)[F(t)L_F(t) - C_L(t) + \psi(F(t))] \quad (6.61)$$

The first order conditions:

$$\frac{\partial H}{\partial C_L(t)} = \frac{b_1}{C_L(t)} - v(t) = 0 \quad (6.62)$$

$$\frac{\partial H}{\partial L_F(t)} = -\left(\frac{b_2(t)}{1-L_F}\right) + v(t)F(t) = 0 \quad (6.63)$$

$$\frac{\partial H}{\partial F(t)} = \rho v(t) - \dot{v}(t) = v(t)[L_F(t) + \psi'(F(t))] \quad (6.64)$$

Equation 6.64 provides the following:

$$\rho = \frac{\dot{v}(t)}{v(t)} L_F(t) + \psi'(F(t)) \frac{v(t)}{v(t)} \quad (6.65)$$

Substitute the first order condition for  $C_L(t)$  into equation (6.65) leads to:

$$\rho = \frac{\dot{v}(t)}{v(t)} + L_F(t) + \frac{\psi'(F(t))}{v(t)} \frac{b_1}{C_L(t)} \quad (6.66)$$

Substitute equation (6.63) into equation (6.66) gives

$$\dot{v}(t) = \frac{b_2(\rho - L_F(t))}{(1-L_F(t))F(t)} - \frac{b_1\psi'(F(t))}{C_L(t)} \quad (6.67)$$

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## Chapter 7

# Concluding Remarks

The thesis develops a compilation of five independent papers that provide original interpretations of some global macroeconomic trends over the last 25 years. There are also some predictions for the future. The novel theoretical and empirical frameworks are based on neoclassical economic theory utilising dynamic optimisation tools. All the original interpretations involve either a digital technology good that enhances leisure experience or robotic use. In Chapter 2, I construct a utility function in which there is habit formation on a digital technology good. The framework shows that lower relative prices of the digital technology good and the growth in its demand can drive down interest rates and imply lower consumption growth of another good sector. This is consistent with the experience of many developed countries around the world. Chapter 3 introduces multiple sectors, labour employment and a CES production function to the framework. This paper shows that it can imply the labour income share to fall. Chapter 4 investigates the impact on output growth under a similar framework as in Chapter 2. In this Chapter, the consumer can augment the ability of the digital technology good to enhance leisure experience. Chapter 5 investigates the implications of robotic use on welfare under a different framework. There are several types of workers, a capitalist owner and various sectors. Along with consumption, leisure experience is found to be instrumental to welfare. Chapter 6 introduces the case when labour employment is no longer required due to the superiority of robots in production.

My thesis has limitations and possible areas where future research can be dir-

ected. The focus of this thesis is to assume that this new technology revolution is fundamentally different from previous industrial revolutions. Many observers would agree. Indeed, the literature suggests that the digital technology is transforming the world (e.g., Kurzweil 2005; Goldin 2017). For instance, Van Zeebroeck (2017) emphasises five digital idiosyncrasies that make the digital technology revolution different than previous revolutions. The five idiosyncrasies are digital technology is reasoning, is exponential, is programmable, is modular and is ubiquitous. He points out that comparisons with previous revolutions are hazardous. Anthes (2017) highlights three ways that the digital revolution is reshaping the workforce. The three pressing problems are the potential to increase worker exploitation, increase the digital skills gap problem and displacement of skilled workers by machine learning.

However, I have not proven that the digital technology revolution is fundamentally different. It is important to note that academics such as Gordon (2015) would disagree. He argues that previous revolutions are far greater than this fourth digital revolution. Empirical work is required to fully evaluate whether digital leisure technology goods are different.

Leisure is central in my thesis. Nonetheless, Bridgman (2018) finds that although the value of leisure is large, it has become less important over time. Hence the conclusions of this thesis are subject to this caveat. Analysis of time use surveys including on the use of digital technology goods and other non-technology leisure goods would be a fruitful extension.

In Chapters 2, 3 and 4 of my thesis, I focus on digital technology as a leisure enhancing aid. Nonetheless, it can also impact business costs. Note that in Chapters 5 and 6, I investigate the impact of technological change in production on macroeconomic variables. However, possible further research is to empirically disentangle the impact of technology on providing more leisure time through efficiency and that which is leisure enhancing.

It is important to note that web-based services are currently badly measured. This may impact the results particularly in Chapter 3 'Habits and labour income share: is there a link?'. Many questions arise. For instance, how do we measure this digital technology? In addition, labour that supplies the web-based services is often provided free of charge i.e., Youtube, web-based blog. A possible extension is to develop a model that incorporates and accounts for labour that supplies the web-based



services free of charge. There are many other empirical extensions with new models that could augment this work. For instance, a valuable comment on this paper is to develop a model that divides the population between the 'competent' and those prone to addiction. Those prone to addiction have less ability to raise their living standards and consequently increase inequality. Empirical analysis of this nature would provide further support to the results in Chapter 3.

Further research into other models that captures the results in chapter 4 'Leisure and growth' paper would be useful. For instance, instead of the model in this chapter, one could develop a productivity growth model of online service providers that could increase utility.

Future research would test empirically the assumption in Chapter 5 that leisure is critical to the utility. Are people with more leisure happier? Robertson (2016) emphasises the growth in numerous '*Institutes of Happiness*' and '*National Surveys of Well-being*'. Leisure is a key component of many of them. Haworth (2011) highlights that the UK Cabinet Office's report on Life Satisfaction (2002) found a strong link between leisure and overall life satisfaction. McHugh et al. (2016) analysed two surveys of subjective perceptions of leisure and happiness. The first was in 1938 and the second was in 2014. They found leisure rose in its importance to happiness from eighth place in 1938 to third place in 2014. Current research appears to suggest that leisure is important. However, an extension of this thesis is to contribute to this topic.

Chapter 6 is very restrictive. Simple changes in the model could change the results significantly. For instance, heterogeneous preferences amongst workers could lead to some workers seeking revolution and for some to be controlled by a robot. In addition, it is well beyond the scope of this thesis to answer the main philosophical questions inherent in that paper. Nonetheless, this paper is an attempt to touch much of the key issues on this subject.

A critical part of this thesis is the relationship between leisure and digital technology. However, I do not prove robustly the interactions between leisure, digital technology and welfare. To fully convince of the conclusions in this thesis would require a major new empirical research project. Do people who have more leisure have more digital technology? Do these individuals spend more time using digital technology? Are they happier? Nonetheless, to answer these would spur new research questions including what is the appropriate measurement tool for web-based digital

technology?

It is important to consider the implications of the thesis and the context in the broader picture. Current debates on the implications of the digital technology revolution and automation tend to be rather extreme. Either it is going lead to massive unemployment and riots or it will imply a utopian vision. However, there is still much work to be done before we understand what has been the impact and what changes are likely to come. As shown in this thesis, there could be some surprising implications of the digital revolution which only more research will uncover. New theoretical frameworks are required. In addition, data for these new theoretical frameworks is just as critical. Governments and private institutions need to collect data for the research. This thesis investigates specific aspects of this subject. However, the broader picture of this thesis is to determine whether technological change is magnifying the big pressing problems of our time. These include growing inequality. What is optimal government policy to safeguard the poor and prevent them from being undermined by the technological revolution? What is the appropriate combination of safety nets, a program of reskilling workers and taxation? There is much research to be done to understand the impact of of technological change. However, it is hoped that this thesis provides an adequate introduction to the subject and sets out a number of areas where new research is required.

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