Volatility Bounds: A Multivariate Inequality Approach

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I certify that this is my own original work and due acknowledgement is given to all other material used.

Signature of Author..

Supervisory Panel Professor Tom Smith, Chair for my parents...

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Abstract

This thesis re-examines the excess volatility debate by proposing an alternative variance bounds test based on multivariate inequality methodology, using conditioning information based on economic theory. We propose new variance bounds that are robust to non-stationarity issues of earlier studies. The tests are applied to annual U.S data over the 1871 to 2006 sample period using measures of perfect foresight stock prices, based on a geometric random walk model. We show that when stock price series are appropriately adjusted to ensure stationarity, volatility bounds are not violated. Those results hold both unconditionally and conditionally based on variables suggested by economic theory, such as dividends, real interest rates and consumption growth.

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Chapter 1

Introduction

This thesis examines the excess volatility debate using a novel multivariate hypothesis testing methodology and applying it to annual U.S. stockmarket data. The main thrust of the literature attempts to answer this basic question: are fluctuations in stock prices justified by changes in their fundamental determinants? In an efficient market with rational investors, stock prices are forward-looking variables that reflect anticipated changes in dividends as well as incorporate all relevant information. Hence, their volatility should reflect investors' expectations of changes in the determinants of stock prices. Given a model of stock prices, market efficiency places restrictions on the relative volatility of stock prices, which can be tested to yield insight on the validity of the underlying model. Shiller (1981) and Porter and LeRoy (1981) independently investigate this issue and find overwhelming evidence of excess volatility in stock prices.

If investors are rational, the stock price should equal the present value of the stock's expected dividend stream. By assuming that dividends follow a stationary stochastic process, it is possible to derive an upper bound for the variance of that stock price based on the subsequent stream of dividends.

The present value model with constant discount rates, as introduced by Miller and Modigliani (1961), defines stock prices as the present value of rationally expected future dividends:

$$p_t = \sum_{\tau=1}^{\infty} \frac{E_t\left(d_{t+\tau}|I_t\right)}{\left(1+r\right)^{\tau}}, \tag{1.1}$$

where r is the constant discount rate, d_t is dividend at time t, where agents base their expectations conditional on I_t, the unobservable vector of information set at time t.

The perfect foresight price is the ex-post rational price with perfect information about future dividend stream.

$$p_t^* = \sum_{\tau=1}^{\infty} \frac{d_{t+\tau}}{(1+r)^{\tau}}, \tag{1.2}$$

where d_t is the realised dividend at time t. The efficient markets hypothesis states that, under the assumption of rational expectations, the expected dividend stream should be equal to the realised dividend stream:

$$p_t = E_t \left[p_t^* | I_t \right], \tag{1.3}$$

where E_t refers to the mathematical expectation conditional on information available at time t. A direct implication of the efficient markets hypothesis is that p_t equals p_t^* plus a forecast error term, which is orthogonal to p_t^*. Otherwise, the forecast would not be optimal.

$$p_t^* = p_t + u_t \tag{1.4}$$

Hence, they differ by an unpredictable random error, u_t which measures the impact on p_t^* of information not available at the time that the expectations are formed.

Taking the variance on both sides of equation (1.4),

$$\begin{aligned} Var\left(p_t^*\right) &= Var\left(p_t + u_t\right) \\ Var\left(p_t^*\right) &= Var\left(p_t\right) + Var\left(u_t\right) + 2Cov(p_t, u_t) \end{aligned}$$

Since the error term is non-systematic, $E\left(u_t\right) = 0$ and given that u_t and p_t are uncorrelated with each other, $Cov(p_t, u_t) = 0$. It then follows that, under rational expectations, the variance of the sum of two uncorrelated random variables is simply the sum of their individual variances:

$$
\begin{aligned}
Var\,(p_t^*) &= Var\,(p_t) + Var\,(u_t) \qquad (1.5)\\
Var\,(p_t^*) &\geq Var\,(p_t)\,. \qquad (1.6)
\end{aligned}
$$

Equation (1.6) therefore places an upper bound on the variance of the observed price series, under the assumption that prices are formed according to equation (1.1). This is the best known of the three variance bounds that Shiller (1981) developed in his paper and it forms the basis of most studies in the excess volatility literature.

This simple variance bounds inequality condition can be stated in term of a null hypothesis that involves an inequality constraint of the following form.

$$Var\,(p_t^*) - Var\,(p_t) \geq 0 \qquad (1.7)$$

The extensive literature on variance bounds tests has highlighted that its econometric implementation is not as straightforward. In fact, any variance bounds testing approach must tackle the following issues:

- The stationarity properties of dividend and stock price series to ensure that the variance bounds tests are robust to the presence of unit roots.
- The computation of p_t^* or an alternative measure of the variance implied by the present value model.
- Tests of significance of the results.

This follows from Gilles and LeRoy (1991) and Cochrane (1992) who argue that all econometric tests are a joint test. The rejection of any null hypothesis could be due not only of the market efficiency hypothesis or present value model of stock prices, but also from a failure of the maintained assumptions underlying the econometric test itself.

Our research suggest that, within a proper statistical inference framework, i.e. the multivariate inequality constraints approach, one is able to reject the claim that the lack of volatility in the p_t^* is due to the constancy of the discount rate. In fact, under the restrictive assumption of constant discount rate, our variance bounds inequality conditions are not rejected by the data, unconditionally and conditionally.

This thesis aims to offer an alternative approach based on multivariate inequality restrictions hypothesis testing framework since the null hypothesis of interest is an inequality variance condition. In addition, the framework of Boudoukh, Richardson and Smith (1993) permits the use of conditioning information in order to restrict the information space under which conditional variance bounds can be tested. It is widely established in the excess volatility literature by Marsh and Merton (1986) and Kleidon (1986) that unconditional variances are not well defined for unit root processes. In this study, stationarity of the stock prices series is achieved by appropriate differencing.

When using conditional information, as dictated by economic theory, the econometric results, also support the variance bounds theorems. When conditioned on past dividends, the real interest rate and consumption growth, the multivariate inequality tests fail to find excess market volatility. It is also found that failing to account for stationarity will lead to a rejection of the variance bounds.

Following this introduction, the thesis is divided into four main chapters and a conclusion. Chapter 2 covers the excess volatility literature in terms of the different approaches to the econometric issues raised by Shiller (1981) and LeRoy and Porter (1981). Then, a brief overview of the multivariate inequality methodology, as applied to hypothesis testing is provided. The main aim is to bring together the two strands of literature in order to address the excess volatility debate and develop testable implications for conditional variance bounds. Chapter 3 develops a new variance bounds inequality condition and applies the multivariate inequality methodology to test the validity of conditional and unconditional variance bounds. Chapter 4 examines the properties of the annual data provided by Shiller, in order to access its suitability for our volatility bounds tests. We also investigate the suitability of alternative differencing strategies in terms of achieving stationary series. Chapter 5 applies the multivariate inequality methodology to annual U.S. data and analyzes the results. The results from the study yield unequivocal evidence in favour of conditional and unconditional volatility bounds. We find no evidence of excess volatility in stock prices, even after accounting for conditioning information. Concluding remarks are presented in Chapter 6.

Chapter 2

Literature Review

2.1 Overview

Since the the seminal work by Shiller (1981) and LeRoy and Porter (1981), variance bounds tests have been the focus of controversy in the literature. Both papers, using the present value model of stock prices, found that stock prices are too volatile to be consistent with the present value of rationally expected future dividends discounted by a constant rate. The violation of Shiller's variance inequality condition has been interpreted as rejection of the efficient market hypothesis. Cochrane(1991) argues that volatility tests are only tests of specific discount rate models and specific dividend models. Other extensive surveys of the literature can be found in West (1988b), Gilles and LeRoy (1991) and Gurkaynak (2008).

Studies which favour Shiller's conclusion include Mankiw, Romer and Shapiro(1985), Campbell and Shiller (1987, 1988, 1989), Poterba and Summers(1986) and West (1988a). Several studies have called into question the validity of Shiller's results for a variety of reasons, notably Flavin (1983), Kleidon (1986), Marsh and Merton (1985), Hoshi (1987), Cochrane(1991), Cuthbertson and Hyde (2002), Heaney (2004) among others. In summary, after two decades of research, the evidence on the violation of variance bounds rests largely on a myriade of model assumptions and weak statistical tests. Not surprisingly, several authors have expressed a perception of futility when reviewing the inconclusive state of research in the excess volatility literature See West(1988b), Gilles and LeRoy

(1991), LeRoy(2005) and Cochrane(1991) for extensive surveys of the literature.

The first half of this chapter examines the various themes present in studies undertaken in the excess volatility literature. The main themes are:

- The non-stationarity issue of stock market data and how they deal with it.
- Assumptions about the dividends process.
- Different classes of variance bounds theorems - conditional vs unconditional bounds.
- Computation of perfect foresight prices: theoretical and econometric issues.
- non-constant discount rates.

The chapter is organized as follows. Section 2 critically surveys the excess volatility literature in terms of the econometric issues raised by previous studies. Section 3 briefly introduces the multivariate inequality constraints literature and the subset that allows us to test linearity restrictions implied by variance bounds theorems. This thesis adopts two unique approaches to assess excess volatility by applying a new econometric methodology to test the variance bound inequality conditions typically found in the literature. The study will generate several benefits. Most important is the additional insight into the excess volatility debate by examining rolling variances bounds that are robust to unit roots.

2.2 The Excess Volatility Literature

2.2.1 The non-stationarity issue

Despite the intuitive appeal of the variance inequality just stated, empirical implementation is far from straightforward. An important econometric problem is that if the unconditional population variances change over time then sample variances, not being consistent estimates of the population variances that are subject to the inequality, are uninterpretable from the viewpoint of the variance-bounds theorems. Sample variances are of interest only if some data transformation can be found which ensures that the relevant variables are stationary and which at the same time preserves the variance-bounds theorems.

Shiller (1981) and LeRoy and Porter (1981) carried out their studies based on the assumption that the prices and dividends series are trend stationary. Nelson and Kang (1984) show that doing so does not resolve the non-stationarity issue if the data series still contain a unit root.

Heaney (2002) describes Shiller (1981) approach to achieving stationarity in the stock-market data as follows. The stock prices and dividends series are detrended by a long-run growth factor, $\lambda^{t-T} = (1+g)^{t-T}$, where g is rthe growth rate and T is the base year of the used stock price index, so that at $t = T$ nominal price equal real growth-adjusted price. The growth factor is estimated by regressing the natural log of stock prices on an intercept and a time trend, i.e. $p_t = c + \beta t + \varepsilon_t$ and setting $\lambda = e^{\beta}$. This involves dividing equation (1.1) by λ^{t-T} and multiplying it by $\frac{\lambda^{k+1}}{\lambda^{k+1}}$.After defining $\lambda\gamma = \left(\frac{1+g}{1+r}\right)$ and $\bar{\gamma} = \frac{1}{1+r}$ as the discount factor for the detrended data series, the following relation can be derived:

$$p_t = \sum_{\tau=1}^{\infty} (\lambda\bar{\gamma})^{k+1} E_t\left(d_{t+\tau}|I_t\right) = \sum_{\tau=1}^{\infty} (\bar{\gamma})^{k+1} E_t\left(d_{t+\tau}|I_t\right)$$

The growth-adjusted p and d series are given by

$$\begin{aligned} p_t &= \frac{P_t}{\lambda^{t-T}} = \frac{P_t}{(1+g)^{t-T}} \\ d_t &= \frac{D_t}{\lambda^{t-T}} = \frac{D_t}{(1+g)^{t-T}} \end{aligned}$$

The growth rate is defined as being less than the discount rate to ensure a finite price. The discount rate is calculated as the ratio of the mean growth-adjusted real dividend o the mean growth-adjusted real stock price index: $\bar{r} = E(d)/E(p)$.

The distributional assumptions of the first generation tests have been questioned by many. (Flavin, 1983; Kleidon, 1986, Marsh and Merton, 1986; Durlauf and Phillips, 1988). In particular, Shiller (1981) and Porter and LeRoy (1981) assume that both dividends and stock prices are trend stationary and use different detrending techniques before computing the point sample variances of the 2 stock prices. Shiller (1981) divides his data series by a long-term growth rate while LeRoy and Porter (1981) reversed the effect of inflation and retained earnings on dividends and stock prices using an algorithm to remove trends.

However, when such series have unit roots, sample means and variances do not exist even after detrending, and therefore it is not valid to use the computed sample moments as a estimates of population moments. Flavin (1983) also argues that the sample variances will be biased due towards rejection of the null hypothesis due to the presence of serial autocorrelations in the actual stock prices and computed perfect foresight stock prices.

Most studies have undertaken formal tests for unit roots and the second generation variance bounds tests allow for non-stationartity of dividends and stock prices. Despite this, there has still been conflicting evidence on excess stock price volatility. LeRoy and Parke (1992) use price-dividend ratios and assume that the transformed series is stationary. However, Balke and Wohar (2005) apply augmented Dickey-Fuller tests to similar data but fail to reject the null hypothesis of a unit root in the log price-dividend ratio.

Following Shiller (1981) and LeRoy and Porter (1981), Grossman and Shiller (1981) relax the assumption of a constant discount factor and argue that fluctuations in discount factors are related to fluctuations in aggregate consumption. They conclude that even under perfect foresight, the large fluctuations in stock prices between 1949 and 1979, can not be explained by the fluctuations in aggregate consumption and dividends. However, if the stock prices series are non-stationary, their attempt to induce volatility in the price series is meaningless, as Kleidon (1986) has demonstrated. In LeRoy and Porter (1981), their test is essentially a vector autoregression (VAR) based test of the market fundamental prices. It is very similar to the approach in Campbell and Shiller (1987, 1988, 1989). Campbell and Shiller (1987, 1988a,b), using a VAR methodology, investigate a number of models of equilibrium returns, including the models considered in this paper, finding that the present valuation model of stock prices is rejected when each of the excess returns, volatility and consumption models is adopted. In addition, Azar (2004) re-examines the cointegrating relationship between real dividends and real stock prices and fails to reject the null hypothesis of no-cointegration.

Marsh and Merton (1986) argue that dividend smoothing most likely causes the dividend data to be non-stationary. Ackert and Smith (1993) suggest that focusing entirely on dividends omits many other important components of returns such as share repurchases. They advocate using a broader definition of cash flows, which seems to mitigate the excess volatility problem.

Heaney (2004) applies Shiller's methodology to the Australian data and finds excess volatility. However, using an alternative form of computing the perfect foresight price series, the severity of the violation is lessened.

Campbell and Shiller (1987), assuming that dividends are characterized by an arithmetic unit root, remove the linear stochastic trend in prices by using $p_t - \beta(1-\beta)^{-1}d_t$ in computing their variance bounds. As a critique to Campbell and Shiller (1987). Yuhn (1996) finds evidence of a non-linear cointegration relationship between the prices and dividend series. Using an error correction model, he finds that forecast errors in the current period are not transmitted to next period stock prices, implying that current stock prices reflect all available information on market fundamentals.

2.2.2 Modeling of the dividend process

West (1988b) categorizes variance bounds studies based on whether the tests are asymptotically valid with a arithmetic unit root or with a geometric unit root. In the former case, following Kleidon (1986), dividends can be assumed to follow the following process[1]:

$$d_t = \rho_a d_{t-1} + \eta_{at}, \tag{2.1}$$

where η_{at} is an error term that is independently and identically distributed with a zero mean and finite variance $\sigma^2_{\eta_a}$. Under the assumption that p_t are set according to equation (1.1), this implies that:

$$p_t = \rho_a p_{t-1} + a\eta_{at}, \tag{2.2}$$

where $a = \frac{\rho_a}{(1+r-\rho_a)}$. Therefore, the implicit assumption made about the stationarity nature of the dividend process has direct implications for the stationarity properties of its corresponding stock prices.

An alternative assumption is that the dividend process follows a geometric random

[1]the intercept term has been dropped for mathematical convenience as it does not affect the overall idea

walk process so that the logarithm of real dividends can be expressed as:

$$\ln d_t = \mu_d + \ln d_{t-1} + \eta_{gt}, \tag{2.3}$$

where η_{gt} is an error term that is independently and identically distributed with a zero mean and finite variance $\sigma^2_{\eta_g}$. Kleidon (1986) states that the implied stock prices is given by:

$$p_t = \left(\frac{1+g}{r-g}\right) d_t, \tag{2.4}$$

where r is the constant discount rate and g is the geometric growth rate of dividends, given by $(1+g) = \exp\left[\mu_d + \sigma^2_{\eta_g}\right]$ (Kleidon (1986)). In order for dividends to converge to a finite sum, r must be greater than g. According to West (1988b), studies that assume an arithmetic unit process for dividends include Mankiw *et al* (1985), Campbell and Shiller (1987) and West (1988a). Conversely, studies that conduct variance bounds tests based on a geometric random walk in dividends include Kleidon (1986), Campbell and Shiller (1989) and LeRoy and Parke (1992) among others. In this study, we follow Kleidon (1986) and assume that dividends follow a geometric random walk in the derivation of our perfect foresight price series.

2.2.3 Classes of Variance Bounds Theorems

This subsection documents some of the alternative variance bounds theorems put forward in the literature.

Mankiw, Romer and Shapiro (1985) proposed an alternative unbiased tests of variance bounds that do not rely on calculating the ex post rational prices, which are unobservable. They come up with the following variance inequalities:

$$E_0\left(p_t^* - p_t^0\right)^2 \geq E_0\left(p_t^* - p_t\right)^2 \tag{2.5}$$

$$E_0\left(p_t^* - p_t^0\right)^2 \geq E_0\left(p_t - p_t^0\right)^2, \tag{2.6}$$

where p_t^0 is defined as the naive forecast of stock prices based on dividends information available to agents at time t Assuming extrapolative and myopic expectations for the forecast of dividends, we get

$$d_{t+i} = d_i \, \forall i \tag{2.7}$$

Then, p_t^0 is defined as

$$p_t^0 = \frac{d_t}{1-\beta} \tag{2.8}$$

West (1988a) derives an inequality that the variances of innovations in actual stock prices must be less than or equal to the variance of the innovations in the forecasted present value of dividends based on a smaller subset of information available to the market. Therefore, even if dividends and stock prices are integrated of degree one, the innovations would have finite variances.

LeRoy and Parke (1992) attempt to resolve the non-stationarity issue by dividing dividends into stock prices. Given that current dividend at time t is in the information set I_t, the following variance bounds condition can be derived:

$$Var\left(p_t^*/d_t\right) \geq Var\left(p_t/d_t\right) \tag{2.9}$$

A recent paper by Engel (2005) argues that when expressing stock prices in first differences, the Shiller inequality condition is reversed. In particular, he shows that:

$$Var\left(p_t - p_{t-1}\right) \geq Var\left(p_t^* - p_{t-1}^*\right) \tag{2.10}$$

Upon further scrutiny, equation (2.10) contradicts the rational expectations model. To illustrate this, consider equation (1.4) which forms the basis of the efficient markets hypothesis that underlying all valid variance bound theorems:

$$p_t^* = p_t + u_t \tag{2.11}$$

If one substract p_{t-1}^* from the left hand side and p_{t-1} from the right hand side of equation (2.11) and then takes variances on both sides, then the equality condition in equation (2.11) may no longer holds.[2] As a result, there is no guarantee that $Var\left(p_t^* - p_{t-1}^*\right) \geq Var\left(p_t - p_{t-1}\right)$. Indeed, Engel (2005) analytically confirms that this is the case with equation (2.10). Therefore, Engel's new variance bounds are not at odds with the empirical results provided by Shiller (1981). Kleidon (1986) has warned that comparing $Var(p_t^*|p_{t-j}^*)$ with $Var(p_{t-1}|p_{t-j})$ as it will lead to misleading interpretations.

Conditional variance bounds

The actual stock price p_t at time t depends on the realization from the distribution of the error term at time t. Therefore, the variance bound has to hold cross-sectionally, as the information available at time $t-1$ determines the possible values of the present value of dividends. Therefore, the variance bounds relationships must hold in terms of conditional variances, i.e.

$$Var\left(p_t^*|I_{t-k}\right) \geq Var\left(p_t|I_{t-k}\right),$$

where I_t is the conditioning information available to agents at time $t-k$. In West (1988a), H_t is defined to be the information set consisting only of current and past dividends. $H_t = \{d_t,\ d_{t-1,}\ ...\}$ and $H_t \subseteq I_t$. Define $\hat{p}_t$ as the price that prevails conditional on H_t :

$$\hat{p}_t = E_t\left(p_t^*|H_t\right)$$

Assuming that I_t is at least as informative as H_t, i.e. $H_t \subseteq I_t$, the rule of iterated expectations implies that:

$$\hat{p}_t = E_t\left(p_t|H_t\right),$$

but the conditional expectation of any random variable is less volatile than the variable itself. This implies that:

[2] It is easy to show that $p_t^* - p_{t-1}^* \neq p_t - p_{t-1} + u_t$ whereas $p_t^* - p_{t-1} = p_t - p_{t-1} + u_t$, unless $p_{t-1}^* = p_{t-1}$.

$$Var\,(p_t) \geq Var\,(\hat{p}_t)$$

Kleidon (1986) points out that the basic unconditional variance bounds relationship holds cross-sectionally since information available at time $t-1$ restricts the possible values of present value of future dividends in different states of the economies at any given time t. In general, this implies the following conditional variance bound relationship for $k < \infty$:

$$Var\,(p_t^*|I_{t-k}) = Var\,(p_t|I_{t-k}) + Var\,(u_t|I_{t-k}) \quad (2.12)$$

$$Var\,(p_t^*|I_{t-k}) \geq Var\,(p_t|I_{t-k})\,, \quad k = 0, ..., \infty, \quad (2.13)$$

where $I_{t-k} \in I_t$,and rational expectations requires that $Var\,(u_t|I_{t-k}) = 0$. Akdeniz *et al* confirm the argument made by Kleidon (1986) that one can not made valid inferences from unconditional variance bounds. They generate simulated data based on an economic model that is consistent with efficient market hypothesis and apply it to Shiller (1981)'s variance test and find it is rejected by the test.

The information set

The fundamental concept of market efficiency relies heavily on the what goes into the information set. As pointed out by Sentana (1993), the concept of market efficiency is information dependent, since different information set will lead to different concept of informational efficiency (i.e. asset prices incorporate all relevant information).

In this sense, variance bound test are attempts at testing the weak form of efficiency, as described by Fama (1970). Weak efficiency is where the information set contains lagged values of the price and dividend serie,s as well as macroeconomic information such as past interest rates.

West (1988a) posits that the conditional variance would be smaller when the information set I_t contains additional variables useful in forecasting d_t than when the information set only contains past values of dividends.

2.2.4 Computation of unobservable ex post stock price p_t^*

Another controversial issue in the excess volatility literature is the computation of the perfect foresight price series, which is determined by the present values model of stock prices. Following Shiller (1981) and Flavin (1983), studies that compute an observable version of the perfect foresight use the following recursion:

$$p_t^* = \frac{p_{t+1}^* + d_{t+1}}{1+r}, \tag{2.14}$$

subject to a terminal condition that the terminal p_T^* is the last data point p_T. The constant

discount rate used is either computed from real data from the real interest rate over the whole sample, as in Shiller (1981, 2003) or incremental values of discount rates of 1 percent to 5 per cent are used (as in Kleidon (1986). In Shiller (1988) criticizes Kleidon's use of a discount rate that is lower than that observed in the data.

There has been theoretical issues with the direct computation of the perfect foresight price, p_t^* by many. Kleidon (1986) points out that, ex-ante, dividends are uncertain and there are several possible paths they can take. Hence, stock prices change as the probabilities of different dividend paths change as new information are known. Ex-post, there is no uncertainty and only one path is observed in historical data. By construction, $p_(t)^*$ must be smoother than the actual stock prices.

For instance, the computation of p_t^* involves an infinite sum of future dividends. Since any sample is finite, a terminal condition p_T^* must be used.

Although p_t^* is not observable, it is known that the ex post rational price is the solution to the recursive expression:

$$p_t^* = \beta\left(p_{t+1}^* + d_{t+1}\right) \tag{2.15}$$

that satisfies the condition

$$\lim_{t \to \infty} \beta^t p_t^* = 0. \tag{2.16}$$

Simply replace p_t^* by the solution to (2.15) that satisfies the terminal condition

This approach has been subject to various criticisms, mainly from Flavin (1983) and Kleidon (1986). The latter shows that Shiller's use of ex-post dividends to construct p_t^* is incorrect since he is assuming that agents know the future dividend stream at the time of the stock price valuation. Such dividends depends on different possible states of the economy. Therefore, the ex-post dividend series is only one of many possible realizations.

Shiller (1981) estimates the dividend process recursively by taking thr average growth adjusted real stock price from the full sample as the prevent value of dividends at the end of the sample. The price p_T^* is taken as being the most recent observation and p_t^* is then solved recursively back to the first observation using the equation:

$$p_t^* = \bar{\gamma}\left(p_{t+1}^* + d_t\right)$$

Heaney (2004) argues that stock prices can be expressed as simple perpetuity rather than requiring perfect knowledge of the future dividends stream. Under the assumption that dividends follow a random walk, this implies that the agents form their expectations on the basis of the last dividend payment. This follows that p_t^* can be re-written as:

$$p_{RW_t}^* = \frac{d_{t-1}}{\bar{r}}$$

This version of p_t^* is referred to by Mankiw et al (1985) as a myopic forecast of stock prices.

Following Amano and Wirjanto (1998), the perfect foresight price is calculated using Hansen's (1982) generalized method of moments estimate of a non-linear asset-pricing equation that allows for time-varying real asset returns. This relaxes the strong assumption of constant real asset returns and risk neutrality thathave characterise first generation models.

Other papers have attempted to bypass constructing an observable version of the p_t^* series altogether. For instance, West (1988a) focus on the variances of innovation implied by the present value model whereas Campbell and Shiller (1987, 1989) adopt the VAR methodology.

2.2.5 Non constant Discount Rates

From the outset, there has been several attempts to rationalize the evidence of excess volatility in earlier studies. One explanation put forward by LeRoy and La Civita (1981) and Grossman and Shiller (1981) is that most of stock prices variability is attributed to changes in the discount rate. Therefore, if one uses a present value model with constant discount rate, one would fail to adequately capture the variability in the unobserved stock prices.

A large proportion of first generation and second generation variance bounds tests rely on the crucial assumption of constant discount rates. The unobservable p_t^* series are either generated using an estimated value of β from the data (for instance, Shiller (1981)) or reasonable values as implied by economic theory (Kleidon (1986)). In both cases, the variability in p_t^* is predominantly explained by the variability in the dividends stream, ceteris paribus.

It has been pointed out that variance bounds tests depend on an implicit assumption of risk neutrality (LeRoy (2005)). LaCivita and LeRoy (1981), using Lucas (1978) model, show that allowing for risk aversion increase the predicted volatility of stock prices. Constructing the p_t^* series therefore involves a two-step estimation procedure. Following Amano and Wirjanto (1998), one can use GMM estimates of coefficient of risk aversion and the rate of time preference. Then, one uses the consumption and dividend series as well as a terminal stock price, p_T and recursively derive p_t^*. The relaxation of the constant discount rate is likely to induce less smoothness in the p_t^* series as well as more variability.

Grossman and Shiller (1981) compute the perfect foresight price based on asset pricing model with non-constant discount rate. They use aggregate consumption data to construct p_t^* for different values of risk aversion. They show that a relatively high value/degree of risk aversion to ensure that the p_t^* series match the p series. They still find evidence of excess volatility, primarily due to unsolved issue of non-stationarity in their price series.

2.2.6 Testing methodology

As documented in West (1988b), there is a wide range of econometric techniques used to test the variance bounds theorem in section 2.2.3. Shiller (1981) make use of simple point estimates of variances, whereas LeRoy and Porter use a VAR approach that allow them to obtain standard errors of their estimates. West (1988a) also provides standard errors of the variances of the innovations of returns. However, none of those tests directly test the null hypothesis of the inequality constraints implied by the variance bounds. Monte Carlo simulations of statistical and economic models have been used to test the variance bounds since Kleidon (1986) demonstrates that the variance bounds should hold cross-sectionally across different states of the world. Kleidon (1986) simulates several economies where the dividend process follows a geometric random walk and shows that across different economies, the variance bounds relationship derived by Shiller (1981) holds cross-sectionally. Akdeniz *et al* (2007) replicates Kleidon's approach by simulating cross-sectional data from a theoretical asset pricing model that satisfies the rational expectations assumption. They find that unconditional variance bounds are violating using Shiller's approach but conditional variance bounds are not rejected by the simulated data. LeRoy and Parke (1992) use Monte Carlo experiments to generate perfect foresight prices which are derived from a geometric random walk dividends process. Estimates of variances from price-dividend ratios are then computed from an analytical closed form expression. LeRoy and Parke (1992) are unable to reject the variance bounds condition implied by equation (2.9).

2.3 The Multivariate Inequality Constraints Literature

One recurrent criticism of Shiller's original work is the absence of test of significance of his point variance estimates. Similar criticisms are applicable to Mankiw *etal*(1985), LeRoy and Parke (1992) and any empirical studies that have replicated Shiller's variance bound tests (e.g. Heaney (2004)).

2.3.1 Background

The multivariate inequality restriction methodology seems suited to examine the excess volatility debate. The variance bounds theorems described in the previous sections, provide *apriori* information about the sign of the variance bounds conditions implied by rational expectations and the present value model of stock prices. Therefore, the main appeal of the multivariate inequality constraints test is that it provides a statistical test of the validity of *a priori* signs of the parameters where such *a priori* beliefs point to an inequality restriction, rather than an equality restriction (Wolak (1989)). Within the multivariate inequality constraints framework, moments conditions are jointly tested so that potential correlations among the moment conditions are taken into account.

This thesis draws upon a branch of the literature on statistical inference in which multiple inequality constraints are been tested as the null hypothesis. [3] In such cases, incorporating this *a priori* information in the hypothesis testing of the parameters of interest would yield more relevant inference. To illustrate this, consider the following null and alternate hypotheses:

$$H_0 : R\beta \geq r \quad \text{versus} \quad H_A : \beta \in R^K,$$

where $R\beta \geq r$ is a vector of inequality conditions being tested. The test involves computing an unrestricted estimate of $\hat{\beta}$ and a restricted estimate of $\hat{\beta}$ subject to $R\beta \geq r$.

Using the results provided in Wolak (1989), a Wald test statistic, W, can be derived. Such test statistic no longer has an asymptotic χ^2 distribution but is a weighted mixture of χ^2 distributions with different degrees of freedom under the null hypothesis (Gourieroux, Holly and Monfort, 1982). To compute the weights of the test statistic, Kudo (1963) provides an analytical expression for its special case. Gourieroux, Holly and Monfort (1982) use complex numerical simulation. Wolak (1987) derives closed form expression for the weights for dimensions of the inequality constraints tests less than 5. Kodde and Palm (1986), however, building on Perlman (1969) propose lower and upper bound critical values that correspond to a given level of significance without calculating the weights. For

[3]Extensive literature surveys are provided in in Gourieroux and Monfort (1995) and Sen and Silvapulle (2002, 2005).

a given level of significance, the null hypothesis is rejected if W exceeds the upper bound. Conversely, the null can not be rejected if the test statistic is less than the lower bound. For values of W between these bounds, Wolak (1989) develops an approximate numerical method of calculating the weights based on a Monte Carlo simulation.[4]

2.3.2 Literature developments and empirical applications

The multivariate inequality testing problem has roots in papers by Bartholomew (1961), Kudo (1963) and Perlman (1969). They focus on the multivariate one-sided hypothesis testing. Bartholomew (1959) proposes a hypothesis test for ordered alternatives. It was was expanded by Kudo (1963). The latter develops a multivariate equivalent of a one-sided significance test. The null hypothesis is that all parameters are jointly equal to zero against the alternative that at least one parameter is strictly positive under the alternative. Kudo (1963) applies the methodology to study the impact of development variables on birth deformity in Hiroshima and Nagasaki.

Yancey, Judge and Bock (1981) develop hypothesis tests that involve a combination of equality and inequality restrictions in a single test and constrast the critical regions with the conventional cases of two equality hypotheses. Other notable contribution to the multivariate inequality literature include Gourieroux, Holly and Monfort (1982) who examine multivariate one-sided hypothesis testing and investigate the equivalence and differences between the LR test, the Wald test and the Kuhn-Tucker tests. Wolak (1987, 1989, 1991) generalize the inequality constraints methodology to a wide range of econometric problems in economics and finance.

The monotonicity of term premiums was reconsidered by Richardson, Richardson, and Smith (1992) relying on recent econometric techniques for testing inequality constraints on linear models developed by Kodde and Palm (1986) and Wolak (1989). They combine the literature on conditional asset pricing models as well as unconditional multivariate inequality testing to create a testable framework to examine conditional inequality conditions. They replicate Fama (1984) results and can not refute the liquidity preference hypothesis. In Boudoukh, Richardson, and Smith (1993), the methodology has been expanded in

[4] A detailed outline of the methodology is provided in Chapter 3.

order to allow moments to be conditioned on a set of instrumental variables, which are observable and can be used to further restrict the information set. An appealing feature of their testing approach is that it does not require a full specification of the infromation. In order to preserve the inequality conditions being tested, the information vector has to be constrained to be a positive subset of variables. One of their objectives is to identify states in which the conditional *exante* equity premium is negative. Osdiek (1998) applies Boudoukh *et al* (1993) methodology to test whether the world *exante* risk premium is positive. Walsh (2006) tests the CAPM implications on the equity risk premium over different investment horizons.

2.4 Summary

The empirical implementation of variance bound tests has been more complicated than it appears on the surface. The literature is rife with attempts at ironing out several areas of controversy, both on theoretical grounds and on econometrical grounds. There are a myriad of differences in data modelling assumptions and econometric approaches that suggest why some variance bounds tests find excess volatility while others do not. Despite more than two decades of research, the jury is still out on the volatility bounds controversy. The principal issue of stationarity can be resolved by appropriate differencing and multivariate tests incorporating conditional information can be made by using advances in multivariate inequality testing literature to conduct valid volatility bounds tests. This is the main purpose of this thesis.

Chapter 3

Methodology

3.1 Overview

The main purpose of this chapter is to offer an alternative approach to existing tests of stock price volatility. Most of the focus in the volatility literature has been on alternative modeling of dividend and stock price processes. A novel feature of this thesis is to explicitly test the null hypothesis of the variance bound inequalities in an inequality restrictions framework. If the market is efficient, then the null hypothesis of the testable moment conditions should not be rejected. One potential problem in the existing literature is that sample estimates of variances are usually compared and no confidence intervals of the estimates are available. This thesis brings together the variance bounds literature into the modern world of multivariate statistical inference.

This chapter is organized as follows. Section 3.2 derive testable implications of unconditional and conditional variance bounds conditions that are valid even if stock prices and dividends have unit root processes. Section 3.3 describes the multivariate inequality testing methodology that was developed by Boudoukh, Richardson and Smith (1992). Section 3.4 concludes.

3.2 Variance Bounds Conditions

The present value model defines stock price as the present value of rationally expected future dividends:

$$p_t = \sum_{\tau=1}^{\infty} \frac{E_t\left(d_{t+\tau}|I_t\right)}{(1+r)^{\tau}}, \tag{3.1}$$

where r is the discount rate, d_t is dividend at time t, where agents base their expectations conditional on I_t, the information set at time t.

The perfect foresight price is the ex-post price with perfect information about future dividend stream.

$$p_t^* = \sum_{\tau=1}^{\infty} \frac{d_{t+\tau}}{(1+r)^{\tau}}, \tag{3.2}$$

where d_t is the realised dividend. Under the assumption of rational expectations, expected dividend stream should be equal to the realised dividend stream:

$$p_t = E\left[p_t^*|I_t\right]. \tag{3.3}$$

By definition, p_t equals p_t^* plus an error term, which is orthogonal to p_t^*.

$$p_t^* = p_t + u_t \tag{3.4}$$

Under rational expectations, it must be that:

$$Var\left(p_t^*\right) \geq Var\left(p_t\right). \tag{3.5}$$

Equation (3.5) therefore places an upper bound on the variance of the observed price series, under the assumption that prices are formed according to equation (3.1). There is widespread evidence in the literature that equation (3.5) can not be directly tested. Therefore, one must resort to making suitable data transformations to the p_t and p_t^* series to induce stationarity.

To derive a variance bounds condition that is theoretically consistent with equation (2.11), one may substract p_{t-1} from both sides. This implies:

$$p_t^* - p_{t-1} = p_t - p_{t-1} + u_t \tag{3.6}$$

Taking unconditional variance leads to:

$$Var\left(p_t^* - p_{t-1}\right) = Var\left(p_t - p_{t-1}\right) + Var\left(u_t\right) \tag{3.7}$$

Under the assumptions that $Var\left(u_t|p_t\right) = 0$ and $Cov\left(u_t, u_{t-1}\right) = 0$, we arrive at the following unconditional variance bounds condition:

$$Var\left(p_t^* - p_{t-1}\right) \geq Var\left(p_t - p_{t-1}\right) \tag{3.8}$$

In order obtain valid sample variance estimates from testing equation (3.8), we require that both $(p_t^* - p_{t-1})$ and $(p_t - p_{t-1})$ to be stationary, even if p_t^* and p_t may not be. Given that there is overwhelming evidence of the random walk nature of the actual stock prices given by p_t, we need to formally test that $(p_t^* - p_{t-1})$ also satisfies the stationarity property. Assume that p_t has a unit root in levels[1] and is given by:

$$\begin{aligned} p_t &= p_{t-1} + e_t, && (3.9) \\ p_t - p_{t-1} &= e_t && (3.10) \end{aligned}$$

where e_t is i.i.d. $(0, \sigma_e^2)$. Substituting (3.10) into equation (3.6) yields:

$$p_t^* - p_{t-1} = e_t + u_t. \tag{3.11}$$

Therefore, on theoretical grounds, one expects the sum of two stationary processes to be stationary as well. In Chapter 4, the stationarity properties of the data series used in computing variances will be investigated in more depth.

An alternative specification of rational expectations assumption that defines the relationship between the actual stock price and the perfect foresight stock price is to express equation (3.3) as follows:

$$p_t^* = p_t e^{\varepsilon_t}. \tag{3.12}$$

[1]For sake of simplicity, a time trend and/or drift term has been omitted since it does not affect the result.

It is straightforward to show that taking expectations of equation (3.12) will yield equation (3.3).

Taking logs on both sides yields:

$$\ln p_t^* = \ln p_t + \varepsilon_t, \tag{3.13}$$

where $\varepsilon_t \tilde{} i.i.d \left(0, \sigma_\varepsilon^2\right)$. The log specification indicates a multiplicative error structure, compared to an additive error structure as in equation (3.4). To induce stationarity, we substract $\ln p_{t-1}$ from both sides of equation (3.13). This yields:

$$\ln p_t^* - \ln p_{t-1} = \ln p_t - \ln p_{t-1} + \varepsilon_t, \tag{3.14}$$

Taking variances on both sides gives rise to the following variance inequality condition:

$$Var\left(\ln p_t^* - \ln p_{t-1}\right) \geq Var\left(\ln p_t - \ln p_{t-1}\right) \tag{3.15}$$

This can be generalized to conditional variances of the form:

$$Var\left(\ln p_t^* - \ln p_{t-1} | I_{t-1}\right) \geq Var\left(\ln p_t - \ln p_{t-1} | I_{t-1}\right). \tag{3.16}$$

Kleidon (1986) argues that the variance bounds in equation (3.10) has to hold cross-sectionally since the information set I_{t-1} determines the possible values of the present value of dividends stream. Therefore, the variance bounds inequality should also hold for conditional variances:

$$Var\left(\ln p_t^* - \ln p_{t-1} | I_{t-k}\right) \geq Var\left(\ln p_t - \ln p_{t-k} | I_{t-k}\right). \tag{3.17}$$

where I_{t-k} denotes the conditional information set at time $t-k$.

Similarly, using equation (3.8), the theoretical variance bounds can be written in first differences as:

$$\underset{t-j}{Var}\left(p_t^* - p_{t-j}|I_{t-k}\right) \geq \underset{t-j}{Var}\left(p_t - p_{t-j}|I_{t-k}\right) \quad (3.18)$$

$$\underset{t-j}{Var}\left(p_t^* - p_{t-j}|I_{t-k}\right) - \underset{t-j}{Var}\left(p_t - p_{t-j}|I_{t-k}\right) \geq 0 \quad (3.19)$$

The fundamental hypothesis behind the volatility literature is whether this condition is ever violated. Secondly if violations take place, what are the instruments that are responsible? Third, has there been historical episodes where the variance bounds were violated? Are there time horizons implications?

Notice in equation (3.19) we subtract p_{t-j} from both sides of the inequality condition. This is necessary to preserve the inequality. If the first differences of p_t^* and p_t were used instead, it possible, as shown by Engel (2004), that the inequality is reversed. There has also been some debate regarding the plausibility of the empirical results using reasonable parameter values. Nevertheless, it remains an empirical question as to whether this condition actually occurs. In this section, the testable inequality restrictions implied by the variance bounds condition are deduced.

Due to the nature of the condition (i.e. in order to maintain the initial inequality condition and not reverse the sign in equation (2.8), only instrumental variables, which are non-negative for all t(denoted by z_t^+) are used. Such instruments can include the level of real interest rate, dividends, or past volatilities of stock prices and so forth. These instruments ought to be based on existing economic theory, which provides some information about the stock volatility.

The set of instruments, z_t^+, are non-negative so that multiplying both sides of equation (2.9) will not change the sign. Any random variable z_t can be separated into two positive variables, $\mathrm{z}_{1t}^+ = \max(0, z_t)$ and $\mathrm{z}_{2t}^+ = \max(0, -z_t)$,which captures all postive states of the world. Consistent with Boudoukh, Richardson and Smith (1993), each instrument is normalised by dividing through by the expected value of z_{jt} to yield z^+ for each instrument.

Therefore, using an instrumental variables approach, it is possible to rewrite equation (3.19) as:

$$\underset{t-j}{E}\left[Var\left(p_t^* - p_{t-j}\right) \otimes z_{t-j}^+ - Var\left(p_t - p_{t-j}\right) \otimes z_{t-j}^+\right] = \varepsilon \otimes z_{t-j}^+ \geq 0 \qquad (3.20)$$

Re-arranging (2.9) and applying the law of iterated expectations,

$$\underset{t-j}{E}\left[\left\{Var\left(p_t^* - p_{t-j}\right) - Var\left(p_t - p_{t-j}\right)\right\} \otimes z_{t-j}^+ - \theta_{\varepsilon z^+}\right] = 0, \qquad (3.21)$$

$$E\left[\left\{Var\left(p_t^* - p_{t-j}\right) - Var\left(p_t - p_{t-j}\right)\right\} \otimes z_{t-j}^+ - \theta_{\varepsilon z^+}\right] = 0 \qquad (3.22)$$

where

$$\theta_{\varepsilon z^+} = E\left[\varepsilon \otimes z_{t-j}^+\right] \geq 0 \qquad (3.23)$$

Equation (3.23)[2] provides a set of moment conditions for which the vector of parameters, $\theta_{\varepsilon z^+}$, is to be estimated. The various benefits of this approach are as follows. There is no need for an explicit model of conditional expectations. The stationary and ergodity assumptions are implictly satisfied since we are computing variances of lagged difference of the stock prices, not the level of stock prices. There is also an existing literature on the determinants of stock price volatility, which would be useful candidates as instrumental variables. Here, there is no assumed functional form, sos this is not a potential problem. Finally, the multivariate inequality restrictions framework, developed by Wolak and used by Boudoukh *et al* (1993) is perfectly suitable to analyse the hypotheses. In particular, the inequality restrictions implied by the variance bounds condition can be jointly tested and will take into account any correction across the estimators $\theta_{\varepsilon z^+}$. For example, in evaluating the significance of the estimators, the relevant factors are no only the magnitudes of the estimaees but also whether these magnitudes are consistent with the covariance matrix of $\hat{\theta}_{\varepsilon z^+}$.

[2]It is straightforward to show that taking unconditional expectations implies that $Var(X) = E\left[VarX|\Phi_{t-j}\right] + Var\left[E\left(Var(X|\Phi_{t-j}\right)\right]$
$= E\left[VarX|\Phi_{t-j}\right]$

3.3 Econometric Methodology

In this section, the test statistic for testing inequality restrictions is described. It follows closely Boudoukh *et al* (1993), which draws upon Wolak (1989) and Kodde and Palm (1986).

Suppose that there are T observations on the lagged variances $Var\,(p_t^* - p_{t-j}) - Var\,(p_t - p_{t-j})$ and a N-vector z_t^+. Assume these random variables are stationary and ergodic, with finite variances. Let the variance-covariance marix of the sample moment vector, $\frac{1}{T}\sum_{t=1}^{T}\left[\left\{Var\,(p_t^* - p_{t-j}) - Var\,(p_t - p_{t-j})\right\} \otimes z_{t-j}^+\right]$, be defined as Ω. As described by Wolak (1989), this matrix can take quite general forms and can account for cross-section, autocovariances or heteroskedasticity in the series.

The restrictions given in equations (3.21) and (3.23) can be written as a system of N-moment conditions:

$$
\begin{aligned}
\underset{t-j}{E}\left[\left\{Var\,(p_t^* - p_{t-j}) - Var\,(p_t - p_{t-j})\right\} z_{1t-j}^+ - \theta_{\varepsilon z_1^+}\right] &= 0,\\
&\vdots\\
\underset{t-j}{E}\left[\left\{Var\,(p_t^* - p_{t-j}) - Var\,(p_t - p_{t-j})\right\} z_{Nt-j}^+ - \theta_{\varepsilon z_N^+}\right] &= 0,\ \forall j,
\end{aligned}
$$

The null and alternate hypothesis can be expressed as follows:

$$
\begin{aligned}
H_0 &: \quad \theta_{\varepsilon z_i^+} \geq 0\ \forall i = 1, \cdots, N. \qquad (3.24)\\
&\qquad \text{versus}\\
H_A &: \quad \theta_{\varepsilon z_i^+} \in R^N.
\end{aligned}
$$

With respect to testing the hypothesis in (3.24), the first step is to estimate the sample lagged variances of the product of the observable variables. In particular,

$$\theta_{\varepsilon z_i^+} = \frac{1}{T}\sum_{t=1}^{T}\left[\left\{Var\left(p_t^* - p_{t-j}\right) - Var\left(p_t - p_{t-j}\right)\right\} z_{it-j}^+\right], \quad \forall j,\ i = 1, \cdots, N. \tag{3.25}$$

There is no restriction on the sign of the difference variances. In other words, they may be negative to sampling error or the possible rejection of the null hypothesis. It is important to note that the vector $\hat{\theta}_{\varepsilon z^+}$ is asympotically normal wiht mean $\bar{\theta}_{\varepsilon z^+}$ and variance-covariance matrix Ω. The Ω can be estimated using Newey and West (1987) heteroskedastic-consistent techniques.

Under the null hypothesis restriction in (3.24), the parameter estimates must be non-negative. Following Perlman (1969) and Wolak (1989), estimates are derived under the restriction by minimising the deviations from the unrestricted model:

$$\min_{\theta_{\varepsilon z}} \left(\hat{\theta}_{\varepsilon z^+} - \bar{\theta}_{\varepsilon z^+}\right)' \Omega^{-1} \left(\hat{\theta}_{\varepsilon z^+} - \bar{\theta}_{\varepsilon z^+}\right)$$

subject to $\theta_{\varepsilon z^+} \geq 0$.

Let $\tilde{\theta}_{\varepsilon z}$ be the solution to this quadratic program.

The aim is to test how close the restricted estimates $\tilde{\theta}_{\varepsilon z^+}$ are to the unrestricted estimates $\hat{\theta}_{\varepsilon z^+}$. Under the null, the difference should be small. In particular, the test statistic is given by:

$$W \equiv T\left(\tilde{\theta}_{\varepsilon z^+} - \hat{\theta}_{\varepsilon z^+}\right)' \hat{\Omega}^{-1} \left(\tilde{\theta}_{\varepsilon z^+} - \hat{\theta}_{\varepsilon z^+}\right) \tag{3.26}$$

Wolak(1989) shows that W does no longer have an asymptotic chi-squared distribution in the presence of inequality restrictions. Instead, the statistic is distributed as a weighted sum of chi-squared variables with different degrees of freedom. Specifically, the asymptotic distribution of W is given by:

$$\sum_{k=0}^{N} \Pr\left[\chi_k^2 \geq c\right] w\left(N, N-k, \frac{\hat{\Omega}}{T}\right), \tag{3.27}$$

where c$\in R^+$ is the critical value for a given size and the weight $w\left(N, N-k, \frac{\hat{\Omega}}{T}\right)$ is the probability that $\tilde{\theta}_{\varepsilon z}$ has exactly $N-k$ positive elements and χ_0^2 is a point mass at the origin.

As discussed in Wolak(1989), calculating the weights for larger sets of restrictions and non-zero estimators covariances become analytically intractable. As an alternative, Kodde and Palm(1986) compute upper and lower bound critical values which do not require calculation of the weights. They are given by:

$$\begin{aligned} \alpha &= \frac{1}{2}\Pr\left(\chi_1^2 \geq c_l\right), \\ \alpha &= \frac{1}{2}\Pr\left(\chi_{N-1}^2 \geq c_u\right) + \frac{1}{2}\Pr\left(\chi_N^2 \geq c_u\right), \end{aligned}$$

where c_l and c_u are the lower and upper bounds respectively for the critical values of the test. It is necessary to compute the weights for values between these bounds. Wolak(1989) proposes an approximate method of Monte Carlo simulation to calculate the weights in these cases. A multivariate normal distribution with zero mean and covariance$\left(\frac{\hat{\Omega}}{T}\right)$ is simulated. We note the realised vector from each replication by $\theta^*_{\varepsilon z^+}$. The idea is to find the vector $\hat{\theta}_{\varepsilon z^+}$ which solves the following minimisation problem:

$\min \ \left(\theta^*_{\varepsilon z^+} - \tilde{\theta}_{\varepsilon z^+}\right)\left(\frac{\hat{\Omega}}{T}\right)^{-1}\left(\theta^*_{\varepsilon z^+} - \tilde{\theta}_{\varepsilon z^+}\right)$

subject to $\tilde{\theta}_{\varepsilon z^+} \geq 0$.

For each replication, the number elements in the N vector $\widehat{\tilde{\theta}}_{\varepsilon z^+}$ that are greater than zero is counted. According to Wolak(1989), the approximate weight $\hat{w}\left(N, N-k, \frac{\hat{\Omega}}{T}\right)$ is the fraction of replications in which $\widehat{\tilde{\theta}}_{\varepsilon z^+}$ has exactly $N-k$ elements greater than zero.

3.4 Summary

This chapter has derived new variance bounds theorems and described the multivariate inequality methodology to derive testable moment conditions for conditional variance bounds tests that address the econometric issues raised in the literature. Our variance bounds tests also allow us to investigate the statistical significance of the results and provide an adequate framework where the importance of conditional information can be assessed.

Chapter 4

Data Analysis

4.1 Overview

This chapter investigates the stationarity properties of dividends and stock prices data. First, the raw data employed in this study are described.Particular attention is paid to the importance of the stationarity properties of the data as they influence the derivation and use of conditional variance bound tests to be implemented in Chapter 5. Flavin (1983), Marsh and Merton (1983) and Kleidon (1986) among others. were the first to critique Shiller (1981) detrending of his data series in order to achieve stationarity. Kleidon (1986) also showed that in the presence of a random walk, unconditional variance bounds are undefined.

A critique of the excess volatility literature is that some studies directly assume that their transformed data is stationary without carrying out formal unit root tests on them. Doing so may lead to questions about the validity of their empirical results.

The next section briefly investigates the null hypothesis that the stockmarket data contains a unit root. While the sample size may too small to permit a conclusive test, we find some evidence consistent with this hypothesis. Then, we conduct a series of unit root tests on two alternative data transformations and this will decide which variance bounds specifications (described in the previous chapter) that will be used in our empirical study.

4.2 A look at the data

The stock prices data used in this study consists of annual Standard & Poor's composite stock price index from 1971 through to 2006. It is compiled by Shiller and updated on his website. The S&P data is extended back to 1871 by using the data in Cowles (1939). Nominal stock prices are converted into real terms by deflating the January price of the stock index with the annual average of the consumer price index (CPI) at 2000 prices. The nominal dividend series, from 1926, is dividends per share adjusted to index for the Standard and Poor's composite index. Prior to 1926, the dividend is also taken from Cowles (1939). Real dividends are similarly calculated by dividing the total dividend per share accruing to the stock price index with the CPI.[1]

Stockmarket data is generally available on an annual and monthly frequency for the same length of time. In order to avoid dealing with seasonality issues and to enable comparison with previous studies, annual data is generally preferred. Moreover, our dataset extends further to 2006 compared to most studies in this literature, which were undertaken in the 1980s. This longer time series will allows us to examine whether the worldwide asset price bubble in the 1990s will significantly affect our results.

Table 4.1 Summary Data Descriptive Statistics

	p_t	p_t^*	d_t	r_{1t}	r_{10t}	C_t
Mean	324.04	222.99	10.94	1.03	1.03	9540.52
Std. Dev	323.25	115.73	4.81	0.07	0.06	6729.89
Min	65.67	94.26	4.05	0.85	0.86	2384.12
Max	1709.49	552.84	24.88	1.25	1.24	26723.87
Skewness	2.341	1.053	0.555	0.427	0.153	0.942
Kurtosis	8.469	3.222	2.288	4.668	4.704	2.714
Jacque-Berra	293.75	25.39581	9.864507	19.88969	16.9826	17.854

Table 4.1: includes the mean, standard deviation (Std. Dev), minimum (Min), maximum (Max), Skewness, Kurtosis and Jacque-Berra statistics for the real stock prices, the perfect foresight stock prices, real dividends, 1-year real interest rate, 10-year real interest rate and real consumption. All real values are converted using the consumer price index.

Table 4.1 presents a summary of descriptive statistics of the annual data used in this study. Sample means and standard deviations, minimums and maximums as well as

[1] A detailed description of the data can be found in Chapter 26 of Shiller (1989).

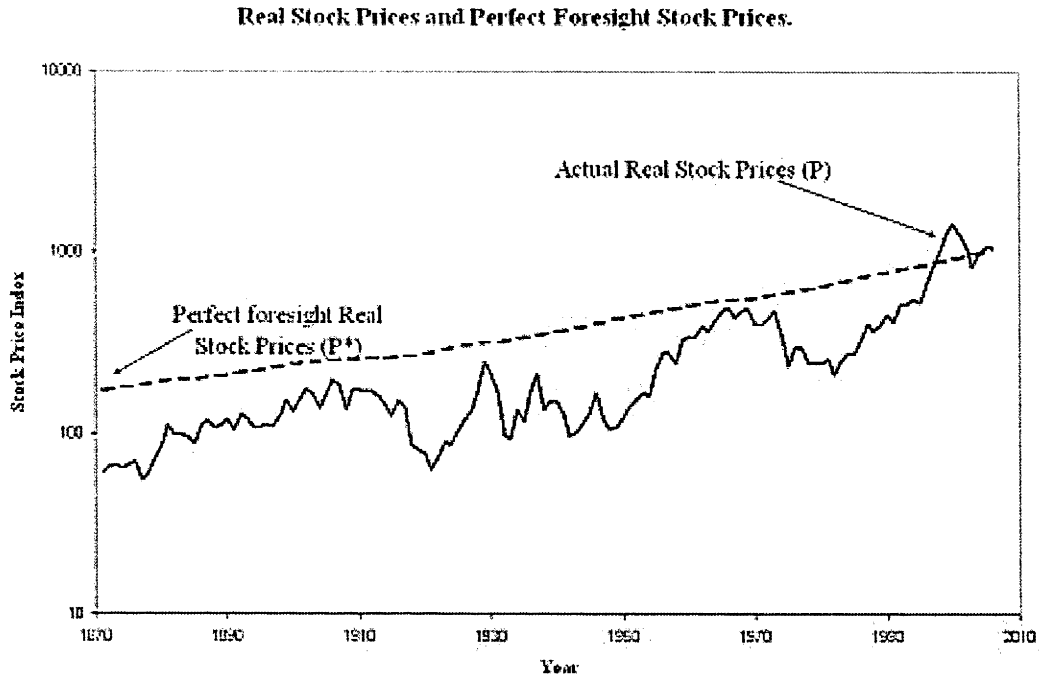

Figure 4.1: Real S&P Composite Stock Price Index (solid line P) and perfect foresight Real Stock Price (dotted line P*), 1871-2006, as used by Shiller (1981) and updated on his website. p_t^* is constructed using the present value of real dividends (which grow at the geometric-average historical rate of 1.2 percent) and discounted by the geometric-average rate of return of 6.7 percent for the entire sample.

Kurtosis and Jacque-Berra statistics are reported. The distributional properties of our dataset appear non-normal. All the series appear to be positively skewed. The kurtosis is relatively small, indicating the absence of extreme observations. Finally, the Jacque-Berra(1981) statistics suggest that the null hypothesis of normal distribution for all series can be rejected at conventional levels of significance.

4.3 Computing the perfect foresight price

As discussed in Section (2.23) The computation of the unobservable perfect foresight stock prices has been subject to various criticisms, mainly by Flavin (1983). Here, we compare two approaches proposed by Kleidon (1986) to compute the perfect foresight series. Kleidon (1986) shows that the present valuation model implies the following relationship between stock prices, dividend and a constant discount rate.

$$p_t^* = \frac{p_{t+1}^* + d_{t+1}}{1+r}, \tag{4.1}$$

where r is defined as the discount rate, which is assumed to be constant over the whole sample. Following Shiller (2003), it is given by the geometric-average real stock return, which is estimated to be 6.7 percent for the sample 1871-2006. The estimated value of β is therefore 0.936. The terminal p_T^* is equated to the terminal price p_T where terminal year is 2006. The perfect foresight series p_t^* is then estimated recursively back to the first observation (in 1871) using equation (4.1).

An alternative approach used by Kleidon (1986) is to assume that the dividend process follows a geometric random walk model and is given by:

$$\ln d_t = \alpha + \ln d_{t-1} + \epsilon_t, \tag{4.2}$$

where ϵ_t is i.i.d. $N\left(0, \sigma_\epsilon^2\right)$. Given that stock prices are generated by equation (1.1), the implied price can be expressed as:

$$p_t = \left(\frac{1+g}{r-g}\right) d_t, \tag{4.3}$$

where g is defined as the geometric average real dividend growth and can be given by $(1+g) = \exp\left[\alpha + \frac{\sigma_\epsilon^2}{2}\right]$ and r is the geometric average real stock return. In order for the discount sum of dividends to be finite, r must be greater than g. To derive p_t^* based on a geometric random walk model, we define the terminal perfect foresight price as:

$$p_T^* = \left(\frac{1+g}{r-g}\right) d_T \tag{4.4}$$

The price series is then recursively reconstructed using equation (4.1). Both versions of the perfect foresight prices are plotted in Figure 4.2.

A close visual inspection of the graph reveals that there does not seem to be much of a difference between the two computer series. Therefore, the second approach, which underlies a geometric random walk model will be used in our empirical analysis.

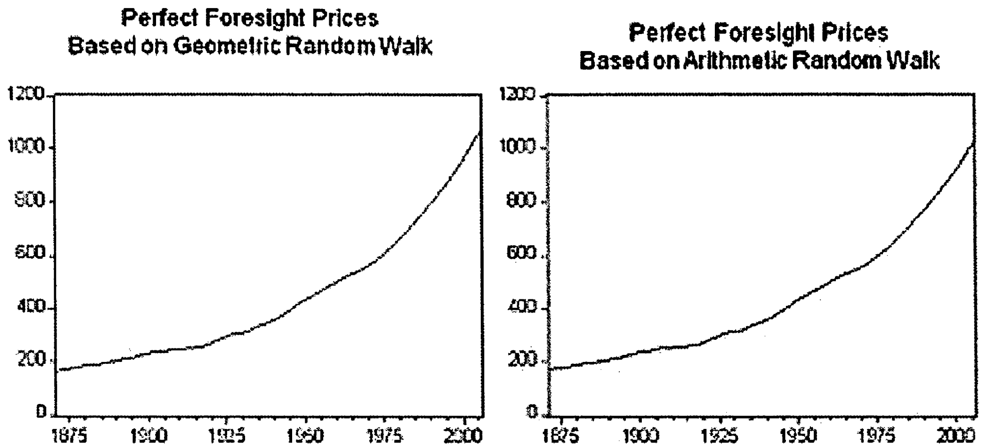

Figure 4.2: Graph plots of two perfect-foresight prices based on the geometric random walk (left graph) and based on recursion defined by (4.1) and terminal condition $p_T^* = p_T$ (right graph)

4.4 Instrumental variables

In addition to the financial data, a number of macro-finance variables have been selected as forming part of the information set in the setting of stock prices. In no particular order, these are the short-term(1-year) real interest rate, the real per capita consumption for non-durables and services as well as past values of dividends.

4.5 Sample autocorrelations

As a preliminary step in conducting variance bounds tests, sample autocorrelation functions for the levels and first differences of the stockmarket data are computed. Autocorrelation coefficients, up to the fifth order, are computed for stock prices and dividends in levels and logarithmic form. For any given random variable x_t, ρ_n is the covariance between x_t and x_{t-n} normalized by the variance of x_t and can be expressed as:

$$\rho_n = \frac{\sum_{t=n+1}^{T} \frac{1}{T-n} \left(x_t - \bar{x}\right) \left(x_{t-n} - \bar{x}_{t-n}\right)}{\sum_{t=1}^{T} \frac{1}{T} \left(x_t - \bar{x}\right)^2}, \tag{4.5}$$

where $\bar{x}_{t-n} = \sum_{t=1}^{T} \frac{x_{t-n}}{T-n}$. Table 4.2 reports the results for the sample autocorrelations. There seems to be little difference in the autocorrelation functions of the real stock prices and dividends and their logarithm equivalents. The slow decay of the autocorrelations for the data in levels and natural logs suggests that the data may have a unit root. First differencing of the four data series yield autocorrelation coefficients consistent with a stationary autoregressive process of order one.

4.2: Sample Autocorrelations of Stock Market Data

Stock Market Data.					
Series	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$
In Levels:					
p_t	0.940	0.864	0.796	0.748	0.685
d_t	0.938	0.877	0.832	0.798	0.773
$\ln p_t$	0.946	0.891	0.846	0.798	0.75
$\ln d_t$	0.936	0.869	0.819	0.782	0.753
In First Differences:					
Δp_t	0.327	-0.043	-0.159	-0.146	-0.09
Δd_t	0.224	-0.145	-0.088	-0.107	-0.11
$\ln \Delta p_t$	0.05	-0.178	0.084	-0.085	-0.1
$\ln \Delta d_t$	0.13	-0.144	-0.124	-0.112	-0.09

4.6 Unit Root Tests

Motivated by the autocorrelatiosn results, the data can be formally be tested for the presence of unit roots. There is some contention in the literature about the non-stationarity properties of stock prices. Although the random walk model of stock prices have firm theoretical support, some studies have rejected such model. See Poterba and Summers (1988). In the excess volatility literature, there is supporting evidence that stock prices follow a random walk. See Kleidon (1986), Campbell and Shiller (1988) among others. Shiller and Perron (1985) and Chaudhuri and Wu (2003) argue that conventional unit root tests lack power in smaller samples, especially in cases where the data may have a near-unit root process. In practice, the test procedures may also be affected by structural breaks (Perron (1989) and Byrne and Perman (2006)). As a result, two different unit root tests are used, namely the augmented Dickey-Fuller (1979) test and the Kwiatkowski, Phillips, Schmidt and Shin (1992) test. The tests are performed on both the levels and the first differences of the stock market data. The combined use of the ADF and KPSS tests provide a more comprehensive picture of the order of integration of the data.

4.6.1 Augmented Dickey- Fuller Tests

The augmented Dickey-Fuller tests are based on the following auxillary regressions:

$$\Delta x_t = \alpha_0 + \beta_0 x_{t-1} + \sum_{i=1}^{q} \beta_i \Delta x_{t-i} + \varepsilon_{1t} \quad (4.6)$$

$$\Delta x_t = \alpha_0 + \delta t + \beta_0 x_{t-1} + \sum_{i=1}^{q} \beta_i x_{t-i} + \varepsilon_{2t}, \quad (4.7)$$

where α_0 and δ are the intercept and time trend term, q is the number of lagged terms and Δx_t is the lagged first differences to accommodate serial correlation in the errors. Equation (4.6) tests for the null hypothesis of a unit root against a mean stationary alternative, whereas equation (4.7) tests the null hypothesis of a unit root against a trend stationary alternativc. In both cases, the null and alternative hypotheses for a unit root in x_t are:

$$H_0 : \beta_0 = 0 \qquad H_1 : \beta_0 < 0.$$

The test statistic does not have an asymptotic standard normal distribution but follows a non-standard limiting distribution. If the estimate of β_0 is not significantly different from zero, then the null hypothesis of a unit root can not be rejected. But if $\beta_0 < 0$, then the alternative hypothesis holds. MacKinnon's (1994) critical values are used in order to determine the significance of the test statistic associated with β_0.The critical values at the 10%, 5% and 1% levels are −2.57, −2.86 and −3.44 for equation (4.6).type tests. For equations (4.7) with time trend, the critical values at the at the 10%, 5% and 1% significance levels are −3.12, −3.41 and −3.66 respectively. For each series, the lag length i chosen by the minimum values of the Akaike Information Criterion.

The null hypothesis of a unit root can not be rejected for p_t at the conventional significance levels, with or without the time trend. Testing for unit root in the first diffferenced series, i.e Δp_t, leads to a rejection of the null at 1% level. It can therefore be concluded the real stock prices can be modelled as a random walk. Similar conclusions are drawn for the log of real stock prices.

Table 4.3: **ADF Test Statistics for Stock Market Data**

Variable	With Constant	With Constant and Trend
	Panel A: In Levels	
p_t	-0.683	-2.102
d_t	0.072	-3.086
lnp_t	-0.827	-2.363
lnd_t	-1.468	-4.475***
	Panel B: In First Differences	
Δp_t	-8.113***	-8.1787***
Δd_t	-8.340***	-8.527***
Δlnp_t	-10.829***	-10.935***
Δlnd_t	-8.856***	-8.943***

Table 4.3: presents the results of the ADF tests for the levels, log-levels and the first differences of the series for real stock prices and dividends series. The KPSS unit root test hypotheses are H_0: unit root, H_1: no unit root (stationary). Test statistics are reported for regression with a constant (Panel A) and a constant and time trend (Panel B). */**/*** indicates coefficient (or test statistic) is statistically significant at the 10/5/1 percent level of significance respectively.

When examining the ADF test statistic for real dividends with or without a time trend, the null hypothesis of a unit root can be comfortably rejected at conventional significance levels. However, when transformed into logarithms, the log of real dividends is found to be stationary at 1% significance level.

4.6.2 KPSS Tests

Kwiatkowski, Phillips, Schmidt and Shin (1992) introduce a unit root test, which adopts stationarity as the null hypothesis. This involves modelling a time series as a sum of deterministic trend, a random walk and a stationary error, and then testing for the random walk having zero variance. The KPSS test statistic is derived from the residuals of the X_t on an exogenous vector Y_t.

$$X_t = Y_t'\lambda + u_t \tag{4.8}$$

For mean trend stationary and time trend stationary series, this can be written as:

$$X_t = \alpha_0 + u_t \tag{4.9}$$

$$X_t = \alpha_0 + \delta t + u_t \tag{4.10}$$

The LM statistic is defined as:

$$LM = \sum_{t}^{T} \frac{1}{T^2} \frac{S(t)^2}{f_0}, \tag{4.11}$$

where f_0 is an estimator of the residual spectrum at frequency zero and $S(t)$ is a cumulative residual function defined as

$S(t) = \sum_{r=1}^{t} \hat{u}_r$ where $\hat{u}_t = X_t - Y_t'\hat{\lambda}$. f_0 is also interpreted as a consistent estimate of the long-run variance and is given by:

$$f_0 = \frac{1}{T}\sum_{t=1}^{T} \hat{u}_t^2 + \frac{2}{T}\sum_{i}^{1} w(i,l) \sum_{t=i+1}^{T} \hat{u}_t \hat{u}_{t-i},$$

where $w(i,l)$ is the Bartlett kernel, l is the order of serial correlation allowed. The lag window suggested by Newey and West (1987) is used to ensure positive semidefiniteness. The critical values for the KPSS statistics are provided in Kwiatkowski *et al.* (1992).

From Table 4.4, the results of KPSS tests are consistent with those of the ADF tests. In this case, the null hypothesis is that of stationarity. The major result is that, in levels, with or without the time trend, all the data appear to reject the null hypothesis of stationarity at the 5% significance levels.

4.7 Inducing stationarity in stock market data

The results of unit root tests in the previous section are consistent with findings in the literature (e.g. Nelson and Plosser (1982)). In order to derive variance bounds conditions that are robust to unit roots, we investigate two alternative price adjustments to ensure stationarity. The first price adjustment involves substract lagged values of p_{t-j} from both the levels of the actual stock prices p_t and perfect foresight stock prices, p_t^*. If the transformed stock prices series are stationary, then variance bounds inequality conditions

Table 4.4: KPSS Test Statistics of Stock Market Data

Variable	With Constant	With Constant and Trend
	Panel A: In Levels	
p_t	0.925***	0.205**
d_t	1.324***	0.204**
$\ln p_t$	1.181***	0.161
$\ln d_t$	1.344***	0.088
	Panel B: In First Differences	
Δp_t	0.246	0.068
Δd_t	0.186	0.064
Δlnp_t	0.080	0.050
Δlnd_t	0.061	0.059

Table 4.4: presents the results of the KPSS tests for the levels, log-levels and the first differences of the series for real stock prices and dividends series. The KPSS unit root test hypotheses are H_0: no unit root (stationary), H_1: unit root. Test statistics are reported for regression with a constant (Panel A) and a constant and time trend (Panel B). The asymptotic critical values for the KPSS LM test statistic at the 0.10, 0.05 and 0.01 levels are 0.1190, 0.1460 and 0.2160, respectively. */**/*** indicates coefficient (or test statistic) is statistically significant at the 10/5/1 percent level of significance respectively.

from equation (3.19).

Table 4.5 ADF Test Statistics for stock market data

Variable	With Constant	With Constant and Trend
	Panel A: Actual Stock Prices	
$p_t - p_{t-1}$	-8.11***	-8.18***
$p_t - p_{t-2}$	-3.93***	-4.07***
$p_t - p_{t-5}$	-1.70*	-1.97
$p_t - p_{t-10}$	-1.64*	-2.56
	Panel B: Perfect Foresight Stock Prices	
$p_t^* - p_{t-1}$	-2.196**	-3.38**
$p_t^* - p_{t-2}$	-2.129**	-3.38**
$p_t^* - p_{t-5}$	-1.483	-2.18
$p_t^* - p_{t-10}$	-0.274*	-1.509

Table 4.5: presents the results of the ADF tests for differenced series $(p_t^* - p_{t-j})$ and $(p_t - p_{t-j})$ for $j = 1, 2, 5, 10$. Test statistics are reported for regressions with a constant (2nd Column) and a constant and time trend (3rd Column). */**/*** indicates coefficient (or test statistic) is statistically significant at the 10/5/1 percent level of significance respectively.

Table 4.5 present the Augmented Dickey Fuller test results for $j = 1, 2, 5$ *and* 10 for transformed stock prices in levels. In Panel A, we find that the lagged differencing achieves stationarity in the transformed actual stock price series for j= 1 and 2 at conventional levels of significance. At higher levels of lags, the null hypothesis of unit root is barely rejected at 10 percent. In Panel B, subtracting p_{t-1} from the p_t^* series achieves stationarity only for lag j= 1 and 2.

The second price adjustment involves taking natural logs of both the actual stock prices and perfect foresight prices and substracting lagged values of the log of actual stock prices. [2] If one find enough statistical evidence to fail to accept the null of unit root in these transformed price series, then variance inequality conditions given by equation (3.17) will be used.

Table 4.6 ADF Test Statistics for stock market data

Variable	No Constant	With Constant
	Panel A: Actual Stock Prices	
$\ln p_t - \ln p_{t-1}$	-10.83***	-10.94***
$\ln p_t - \ln p_{t-2}$	-4.25***	-4.51***
$\ln p_t - \ln p_{t-5}$	-2.80***	-2.99**
$\ln p_t - \ln p_{t-10}$	-2.57**	-2.78*
	Panel B: Perfect Foresight Stock Prices	
$\ln p_t^* - \ln p_{t-1}$	-1.63*	-2.40
$\ln p_t^* - \ln p_{t-2}$	-1.61	-2.47
$\ln p_t^* - \ln p_{t-5}$	-1.51	-2.10
$\ln p_t^* - \ln p_{t-10}$	-1.37	-2.82*

Table 4.6: presents the results of the ADF tests for log differenced series $(lnp_t^* - lnp_{t-j})$ and $(lnp_t - lnp_{t-j})$ for $j =$ 1, 2, 5, 10. Test statistics are reported for regressions without a constant (2nd Column) and a constant (3rd Column). */**/*** indicates coefficient (or test statistic) is statistically significant at the 10/5/1 percent level of significance respectively.

Table 4.6 present ADF results on $(lnp_t^* - lnp_{t-j})$ and $(lnp_t - lnp_{t-j})$ for $j =$ 1, 2, 5, 10 for ADF regressions with and without a constant. In Panel A, all the lagged differences of log stock prices are found to be stationary at 5 percent level of significance. However, in Panel B, the null hypothesis of a unit root is barely rejected for the $\ln p_t^* - \ln p_{t-1}$ series at 10 percent significance level. For higher lag levels, one fail to reject the null.

[2]The log specification indicates a multiplicative error structure (i.e. $p_t^* = p_t e^{u_t}$) rather than an additive error structure (i.e. $p_t^* = p_t + u_t$)

4.8 Summary

Using the two unit root tests, we find that the stock price indices are characterized by random walk models. This is consistent with Kleidon (1986) and other studies in the excess volatility literature. We also find that it is possible to make appropriate adjustments to the data, such as price differencing, to obtain stationary stock price series that can be used. Our results suggest that subtracting lagged values of p_t from levels of actual and perfect foresight price series is most suitable. However, the log specification is better suited to analyzing cases where the use of potentially non-stationary data may invalidate multivariate inequality constraints tests.

Chapter 5

Empirical Results and Analysis

5.1 Overview

This chapter applies the conditional multivariate inequality methodology described in Chapter 3 to the dataset in Chapter 4 in order to investigate the testable implications of our variance inequality conditions. It is organized as follows. Section 5.2 describes the rolling variances approach to computing the unconditional variance series. Unconditional variance bounds tests are presented in Section 5.3. Section 5.3 test the conditional variance relationships for different subsets of information. Section 5.4 concludes.

5.2 Computing Unconditional Variance

From Section (3.2), our conditional variance bounds inequality conditions can be written in the form of:

$$Var\left(lnp_t^* - lnp_{t-j}|I_{t-k}\right) - Var\left(lnp_t - lnp_{t-j}|I_{t-k}\right) \geq 0 \tag{5.1}$$

Using transformed data series of $(lnp_t^* - lnp_{t-j})$ and $(lnp_t - lnp_{t-j})$ for $j = 1$, 2, 5 and 10 lags, rolling variance estimates over a 10 year window are computed. For each of the forementioned series, we compute the sample variance for observations 1 *to* 10, then observations 2 *to* 11, then 3 *to* 12 ... up to $(T - 9)$ to T, where T is the final observation for the series. Rolling variances are often used in financial studies of stock price volatility. (See Officer (1973) and more recently Schwert (2002)). There are several advantages to using rolling variances in testing the variance bounds inequality restrictions. First, rolling variance approach assumes constant variance over the window. This is a plausible assumption particularly in the context of low frequency annual time series. Second, it allows us to observe the time-varying changes in observed log stock prices variance and the perfect foresight price variance over decades. As an illustration, Figure 5.1 and Figure 5.2 depict the rolling variances of $(lnp_t^* - lnp_{t-j})$ and $(lnp_t - lnp_{t-j})$ respectively.

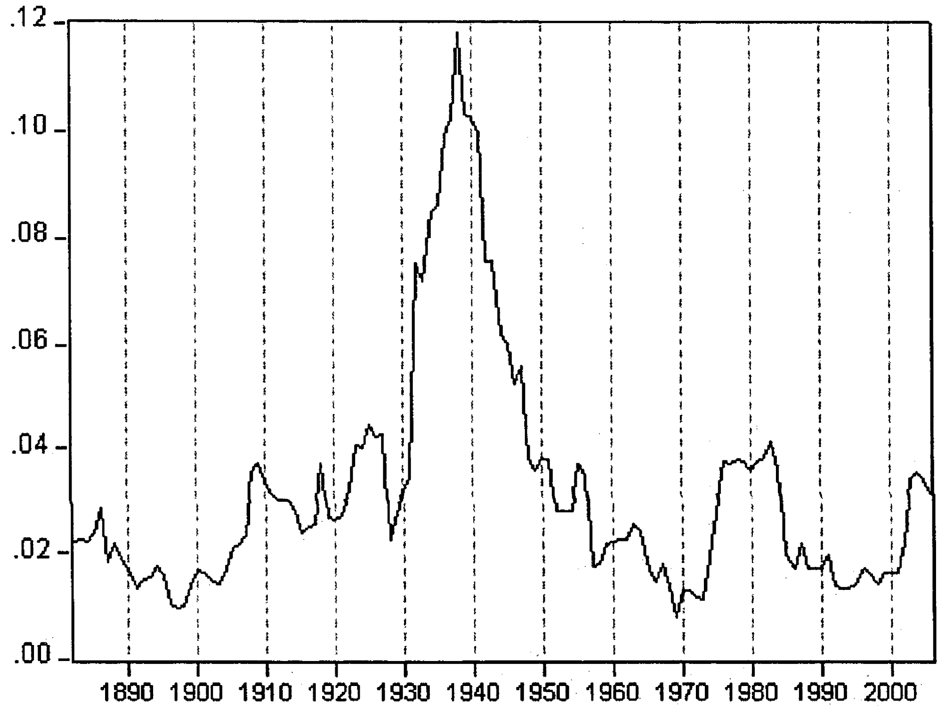

Figure 5.1: Historical Rolling variances of first differences of Actual Log Real Stock Prices, using a 10-year window from 1971 through to 2006. The Y-axis refers to the rolling variance estimates and the X-axis are years.

Visual inspection of Figure 5.1 and Figure 5.2 reveals that the rolling unconditional variances of actual stock prices and perfect foresight, adjusted for stationarity tend to fluctuate a lot. There are periods when both variances are unusually high and periods when they are unusually low. In particular, the plot in Figure 5.1 for the $(lnp_t - lnp_{t-1})$ data series is consistent with early findings by Officer (1973). The latter, using monthly index of stock returns on the New York Stock Exchange, finds abnormal level of stock price variability in the 1930s.and subsequent reduced volatility in the pre-war and post-war

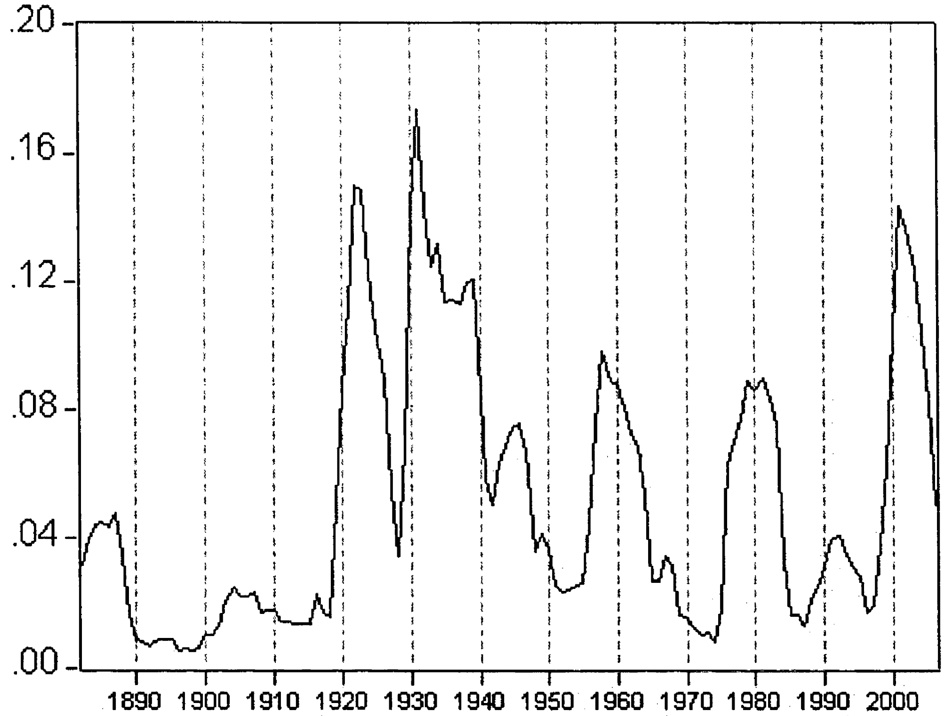

Figure 5.2: Historical Rolling variances of Stationary log Perfect Foresight Stock Prices, using a 10-year window from 1971 through to 2006. The Y-axis refers to the rolling variance estimates and the X-axis are years.

period. The plot for the rolling variances of $(lnp_t^* - lnp_{t-1})$ tells a very interesting story. In particular, the periods of peaks of the perfect foresight rolling variances correspond roughly to important episodes in the U.S. economic history such as the Great Depression of the 1930s, the oil crisis in the late 1970s and the dot-com bubble of the late 1990s. Finally, the magnitude of the rolling variances estimates for $(lnp_t^* - lnp_{t-1})$ is significant higher than those for $(lnp_t - lnp_{t-1})$. Overall, the visual evidence seems to indicate that unconditional variances are not violated. However, as Kleidon (1986) points out when comparing the correspondence between the p_t^* and p_t plots, this impression can often be misleading.

5.3 Unconditional Variance Bounds Tests

Table 5.1 presents the empirical results of inequality constraints tests of the following variance inequality conditions for selected values of j in the first column. Column 2 gives the respective estimated average variance differential with standard errors in parentheses. Column 3 provides the W-Statistics and the corresponding p-values. For $j = 1$, the estimate of unconditional variance differential, denoted by $\widehat{\theta}$, is positive and statistically significant at 1 percent level. Moreover, the W-statistic is almost zero, implying that the null hypothesis of the inequality constraint can not be rejected. For $j = 2$, the $\widehat{\theta}$ is negative but statistically insignificant. However, the W-statistic does not reject the null hypothesis that the variance inequality condition is non-negative. For higher lag levels, the estimates of the unconditional variance differential become negative and the inequality constraint statistic W becomes highly significant at 1 percent level of significance. Such finding is not surprising because as the lag increases, the transformed series get closer to a unit root series. In the subsequent section, we will conduct conditional variance bounds tests by

Table 5.1:Unconditional Variance Bound Tests

Inequality Variance Conditions	$\widehat{\theta}$ (Std. Error)	W Statistic [p-value]
$\text{Var}(lnp_t^* - lnp_{t-2}) - Var\,(lnp_t - lnp_{t-2})$	0.0204	0.000
	(0.0059)	[1.000]
$\text{Var}(lnp_t^* - lnp_{t-2}) - Var\,(lnp_t - lnp_{t-2})$	-0.0089	1.782
	(0.0067)	[0.124]
$\text{Var}(lnp_t^* - lnp_{t-5}) - Var\,(lnp_t - lnp_{t-5})$	-0.046	12.67***
	(0.0136)	[0.0004]
$\text{Var}(lnp_t^* - lnp_{t-10}) - Var\,(lnp_t - lnp_{t-10})$	-0.0549	15.16***
	(0.0154)	[0.00013]

Table 5.1: presents the estimated unconditional variance differentials (denoted by $\widehat{\theta}$) for $j = 1, 2, 5, 10$. In this case, the statistic W is a univariate test of the inequality restriction hypothesis which is given by the non-negative inequality conditions in the first column. The standard errors are adjusted for conditional heteroskedasticity and serial correlation using Newey and West (1987) method. */**/*** indicate coefficient (or test statistic) is statistically significant at the 10/5/1 percent level of significance respectively. 2) Figures in the parentheses are standard errors. p-values are in brackets.

5.4 Conditional Variance Bounds Tests

Kleidon (1986) has shown that conditional variance bounds are preferred over unconditional ones since the latter may not be well defined in the presence of a random walk in the data series. In this chapter, the main conditional variance bounds framework follows Kleidon (1986):

$$Var\,(\,lnp_t^* - \ lnp_{t-j}|I_{t-1}) \geq Var\,(\,lnp_t - \ lnp_{t-j}|I_{t-1})\,, \tag{5.2}$$

where j refers to the order of lagged in the level of actual stock prices in order to achieve stationarity in the data series. From Chapter 4, the main conclusion from our unit root tests supports the use of subtracting the first lagged level of stock prices in order to achieve stationarity series for the variances.

5.4.1 The importance of conditioning information.

Since Hansen and Singleton (1982) and Gibbons and Ferson (1985), there has been increased recognition of the role of conditioning information in empirical analysis of asset pricing models. In basic intertemporal asset pricing models, information accumulates over time and investors in asset markets make trades based on such information so that the latter becomes embedded in asset prices. As a result, asset prices are not only random payoffs but must also satisfy some form of informational constraints. West (1988a) mentioned that the larger the information set faced by the investor, the smaller the variance of the asset prices would be. Since there is no a priori rationale as to how big the vector of our instrumental variables can be, we will experiment with various lags of instruments and various subsets of the conditioning information.

The tests of conditional variance inequality conditions are conducted as tests of multivariate inequality tests and the results are presented in Table 5.2, 5.3, 5.4 and 5.5.

5.4.2 Subsets of conditioning information

5.4.3 Dividends

The influence of past dividends on stock price volatility is captured in two ways. First, we follow West (1988a) and use lagged values of dividends as instruments. $H_t = \{d_{t-j}|j = 1, 2, 3$ The specific variance bound inequality conditions tested in Table 5.2 are given by the following moment conditions:

$$Var\left(lnp_t^* - lnp_{t-1}|d_{t-1}\right) - Var\left(lnp_t - lnp_{t-1}|d_{t-1}\right) \geq 0$$

$$Var\left(lnp_t^* - lnp_{t-1}|d_{t-2}\right) - Var\left(lnp_t - lnp_{t-1}|d_{t-2}\right) \geq 0$$

$$Var\left(lnp_t^* - lnp_{t-1}|d_{t-3}\right) - Var\left(lnp_t - lnp_{t-1}|d_{t-3}\right) \geq 0$$

Table 5.2: Conditional Variance Bounds Tests - Real Dividends

The table presents the estimated conditional variance differentials using lagged real dividends as instruments. The unconditional variance differentials are computed over a ten-year period window. Conditional variance differentials are conditioned on information at t-k, k=1-5. The statistic W is a joint test of multiple inequality restrictions conditional on d_{t-1}, d_{t-2} and d_{t-3}. All estimates are adjusted for conditional heteroskedasticity and serial correlation using Newey and West (1987) method.

	Conditional			
Lagged of Dividends	$k=1$	$k=2$	$k=3$	W Statistic (p-value)
$j=1$	0.206	0.2047	0.2046	0.0000
	(0.0633)	(0.644)	(0.0650)	1.000

Table 5.2: */**/*** indicate coefficient (or test statistic) is statistically significant at the 10/5/1 percent level of significance respectively. 2) Figures in the parentheses are standard errors.

Following West (1988a), we use the subset of I_t, i.e. H_t where H_t include past values of dividends. In Table 5.2, the set of conditioning variables is defined as z_{1t} $=(d_{t-1}, d_{t-2}, d_{t-3})$. In this simplest form, we compute and test conditional variance bounds conditions by varying the time horizon, denoted by $j = 1$. The results of inequality tests for the first specification are presenting in Table I. The overall result shows that individual inequality conditions are all positive, which is consistent with the conditional variance bounds In addition, the Wald statistic is zero. Therefore, there is no evidence of excess volatility for the conditional variance bound conditions, conditional on past values of dividends.

5.4.4 The Real Interest Rate

Most instrumental approach to conditional asset pricing have used the real interest rate in the information set for estimation purposes. In this section, we use past values of the

Table 5.3: Conditional Variance Bounds Tests - Real Interest Rate

The table presents the estimated conditional variance differentials using lagged real interest rateConditional variance differentials are conditioned on information at t-k, k=1-5. The statistic W is a joint test of multiple inequality restrictions conditional on $r-t-1$, $r-t-2$ and $r-t-3$. All estimates are adjusted for conditional heteroskedasticity and serial correlation using Newey and West (1987) method.

	Conditional			
Lagged of Real Interest rate	$k=1$	$k=2$	$k=3$	W Statistic (p-value)
$j=1$	0.02079	0.02062	0.02055	0.0000
	(0.00626)	(0.00619)	(0.00613)	1.000

Table 5.3: */**/*** indicate coefficient (or test statistic) is statistically significant at the 10/5/1 percent level of significance respectively. 2) Figures in the parentheses are standard errors.

short term (1-year) real interest rate as instruments, where $z_{2t}=(r_{t-1}, r_{t-2}, r_{t-3})$. In this simplest form, we compute and test conditional variance bounds conditions by varying the time horizon, denoted by $j=1$. The specific variance bound inequality conditions tested in Table 5.2 are given by the following moment conditions:

$$Var\left(lnp_t^* - lnp_{t-1}|r_{t-1}\right) - Var\left(lnp_t - lnp_{t-1}|r_{t-1}\right) \geq 0$$
$$Var\left(lnp_t^* - lnp_{t-1}|r_{t-2}\right) - Var\left(lnp_t - lnp_{t-1}|r_{t-2}\right) \geq 0$$
$$Var\left(lnp_t^* - lnp_{t-1}|r_{t-3}\right) - Var\left(lnp_t - lnp_{t-1}|r_{t-3}\right) \geq 0$$

The results of inequality tests for the first specification are presenting in Table 5.3. The overall result shows that individual inequality conditions are all positive, which is consistent with the conditional variance bounds In addition, the Wold statistic is zero. Therefore, there is no evidence of excess volatility for the conditional variance bound condition, conditional on past values of the real interest rates.

5.4.5 Consumption Growth

In Consumption-based asset pricing models (see Abel, 1990; Campbell and Cochrane, 1999 for instance), aggregate consumption plays a crucial role in the determination of stock prices. LeRoy and LaCivita (1981) highlights the importance of risk aversion in stock price volatility. They argue that risk-averse agents, when faced with stock prices appropriate to a world of risk neutrality, will attempt to smooth their consumption stream over time by trading in assets in good and bad states of the economy. Consequently, stock prices will be more volatile than would be the case if economic agents are risk neutral. The magnitude of the impact of consumption variability is directly correlated with the degree of risk aversion. In particular, we use (C_t/C_{t-1}) as a measure of the smoothness of the consumption series in order to capture variations that may be attributed to varying discount factor. The conditional information is given by $z_{3t} = \left(\frac{C_{t-1}}{Ct-2}, \frac{C_{t-2,}}{C_{t-3}}, \frac{C_{t-3}}{C_{t-4}}\right)$.

The specific variance bound inequality conditions tested in Table 5.2 are given by the following moment conditions:

$$
\begin{aligned}
Var\left(lnp_t^* - lnp_{t-1}|\frac{C_{t-1}}{C_{t-2}}\right) - Var\left(lnp_t - lnp_{t-1}|\frac{C_{t-1}}{C_{t-2}}\right) &\geq 0 \\
Var\left(lnp_t^* - lnp_{t-1}|\frac{C_{t-2,}}{C_{t-3}}\right) - Var\left(lnp_t - lnp_{t-1}|\frac{C_{t-2,}}{C_{t-3}}\right) &\geq 0 \\
Var\left(lnp_t^* - lnp_{t-1}|\frac{C_{t-3}}{C_{t-4}}\right) - Var\left(lnp_t - lnp_{t-1}|\frac{C_{t-3}}{C_{t-4}}\right) &\geq 0
\end{aligned}
$$

The results of inequality tests for the first specification are presenting in Table 5.4. The overall result shows that individual inequality conditions are all positive, which is consistent with the conditional variance bounds In addition, the Wold statistic is zero. Therefore, there is no evidence of excess volatility for the conditional variance bound condition, conditional on past consumption growth.

Table 5.4: Conditional Variance Bounds Tests -Consumption Growth

The table presents the estimated conditional variance differentials using lagged real consumption as instruments. Conditional variance differentials are conditioned on information at t-k, k=1-5. The statistic W is a joint test of multiple inequality restrictions conditional on r(t-1) and p(t-1). All estimates are adjusted for conditional heteroskedasticity and serial correlation using Newey and West (1987) method.

	Conditional			
Lagged of Consumption Growth	$k = 1$	$k = 2$	$k = 3$	W Statistic (p-value)
$j = 1$	0.0243	0.0244	0.0243	0.0000
	(0.00683)	(0.00682)	(0.00680)	1.000

Table 5.4: */**/*** indicate coefficient (or test statistic) is statistically significant at the 10/5/1 percent level of significance respectively. 2) Figures in the parentheses are standard errors.

5.4.6 Unconditional and Conditional Variance bound tests

In this section, we conduct a series of multivariate conditional and unconditional variance bounds tests, using the multivariate inequality methodology. As seen in Table 5.5, for $j = 1$, the multivariate tests all jointly fail to reject the null hypothesis of the inequality conditions. When considered jointly, the Wald statistic is quite high and we are able to reject the null hypothesis of non-negativity at the 1% significance level.

Based on our previous results, we compute a different sets of conditional variance bounds using 1 year lags of real interest rate and dividends as instruments. The main results support the null hypothesis that the conditional variance bound inequality for $j = 1, 2$.

Table 5.5: footnotesize */**/*** indicate coefficient (or test statistic) is statistically significant at the 10/5/1 percent level of significance respectively. 2) Figures in the parentheses are standard errors.

Table 5.5: Conditional and Unconditional Variance Bounds Tests

The table presents the estimated conditional variance differentials using lagged real interest rate and lagged real dividends as instruments. The unconditional variance differentials are computed over a ten-year period. Conditional variance differentials are conditioned on information at t-1. The statistic W is a joint test of multiple inequality restrictions conditional on r(t-1) and p(t-1). All estimates are adjusted for conditional heteroskedasticity and serial correlation using Newey and West (1987) method.

		Conditional		
Instruments	Unconditional	Lagged Real Interest Rate	Lagged Dividends	W Statistic (p-value)
$j = 1$	0.0204	0.0207	0.0448	0.0000
	(..00592)	(0.00606)	(0.0132)	(1.000)
$j = 2$	-0.00893	-0.00912	-0.0141	1.782*
	(0.00669)	(0.00697)	(0.0412)	(0.0998)
$j = 10$	-0.0549	-0.0563	-0.111	15.401***
	(0.0154)	(0.0160)	(0.0284)	(0.000065)

Bounds Tests

Table 5.5, the set of conditioning variables is defined as $z_{1t} = (1, d_{t-1}, r_{t-1})$. In this simplest form, we compute and test conditional and unconditional variance bounds conditions by varying the time horizon, denoted by $j = 1$. The evidence, similar to Table 5.1, suggest that conditional variance inequality conditions, when appropriately adjusted to ensure stationarity by subtracting the log price series with $\ln p_{t-1}$ in the first row, will fail to reject the null hypothesis of the variance bounds inequality relationships, both conditionally and unconditionally. However, for $j = 2$, the effects of non-stationarity will begin to invalidate our variance bounds tests. In row 2, the estimated log variance differentials, both unconditionally (see column 2) and conditionally on last period values of dividends and real interest rate, become negative. However, jointly, the W-statistic is still significant at the 10 percent level. However, when the 10^{th} lag of log of stock prices is used, the transformed price series come closer to a unit root, leading to an negative estimates of the log variance differentials in all cases and the W-statistic becoming highly significant at the 1 percent level of significance.

5.5 Summary

The results in this chapter show that when stock price series are appropriately adjusted to ensure stationarity, volatility bounds are not violated. Those results hold both unconditionally and conditionally based on variables suggested by economic theory, such as dividends, real interest rates and consumption growth. Furthermore, there is compelling evidence that using non-stationary transformed data with the multivariate inequality methodology will lead to spurious results and misleading interpretations.

Chapter 6

Conclusion

The aim of this thesis has been to propose an alternative approach to existing variance bounds tests. After more than two decades of research on excess volatility, there are still unsolved issues with non-model based volatility tests. The novel approach is to reformulate existing variance bounds theorems into multiple inequality conditions that can be jointly tested using the multivariate inequality testing methodology developed by Boudoukh, Richardson and Smith (1993) that also allows conditioning information to be used as instruments. Based on Shiller's long-term annual stockmarket data and a present value model with constant discount rates, we are not able to reject variance inequality conditions that explicitly account for non-stationarity issues raised by previous studies. This holds much promise for the efficient markets theory.

The methodology is easy to implement and does not require an explicit formulation of conditional expectations. Moreover, the multivariate inequality framework enables us to account for heterogeneity in the conditional information vector and provides a test statistic and critical values to properly test for variance bounds conditions. Various studies only provide point estimates of their respective variance bounds conditions.

Consistent with previous studies, we accept the random walk model as being a proper representation of the real stock prices series and subtract lags of the real stock prices to achieve stationarity.

Our results are comparable to Kleidon (1986) who disputes several earlier findings by Shiller and advocates the computation of conditional variance bounds over unconditional variance bounds. In particular, when accounting for non-stationarity, we fail to reject the variance bounds inequality conditions and thus do not find evidence of excess volatility in real stock prices. Further analysis of the data also reveals the crucial importance of correcting for non-stationarity in real stock prices since the inequality tests tend to reject the null hypothesis of no excess volatility the closer the transformed stock prices are to a unit root process.

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