A State-Contingent Claim Approach to Asset Valuation

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Declaration

I hereby certify that this thesis is entirely the work of the author and has not been submitted to any other institution. All sources used in the preparation of this thesis have been acknowledged in the usual manner.

Kathryn Barraclough

4 May 2007

Acknowledgements

As we express our gratitude, we must never forget that the highest appreciation is not to utter words, but to live by them.

John Fitzgerald Kennedy

I'll keep this brief, but it must be said that I'm deeply, deeply grateful to Tom Smith. Without his invaluable support and encouragement, this thesis would never have been started, much less completed. I have learnt more from him than I ever bargained for, much of it completely unrelated to the study of finance. His dedication to students is truly inspiring, and my only hope is that I can pass on to others something of what he has taught me. Most valuable of all, I gained a wonderful friend in the process. I also want to thank my dear friend Emma Welch, who helped me out so many times, with both timely advice and practical assistance. Finally, I want to thank my friends and family, who (very wisely) had little to do with any of this, but are always supportive and, more importantly, always lots of fun!

Abstract

This thesis applies a state-contingent claim approach to asset valuation. First, prices are determined for options on the S&P 500 index. Market microstructure aspects such as minimum tick size are directly incorporated into the model, and empirical maximum and minimum returns are used to limit the range of the distribution. The state-contingent claim approach is shown to provide an overall improvement on the Black-Scholes (1973) formula and Stutzer's (1996) canonical valuation. The state-contingent claim approach is then applied to stock valuation. US stocks are valued annually, and the results are compared with Ohlson's (1995) residual income model, Stutzer's canonical valuation and the Sharpe (1964) and Lintner (1965) Capital Asset Pricing Model (CAPM). The state preference approach is found to provide a significant improvement on the residual income model, and a similar level of accuracy to the CAPM and the canonical valuation approach. Of the models under investigation, the CAPM provides the greatest overall accuracy in pricing stocks.

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Chapter 1

Introduction

First proposed by Arrow (1964) and Debreu (1959), the state preference approach to decision making under uncertainty represents one of the most important theoretical advances in modern financial economics. Arrow and Debreu formalised the time-state preference framework, where individuals evaluate alternative economic decisions both over time, and possible states of nature. In a competitive equilibrium price is not only determined by individual preferences for the physical characteristics of traded goods, but also individual preferences for the timing and risk characteristics of goods. This is easily understood in the context of financial markets, where individuals exhibit preferences for investments on the basis of the magnitude, timing, and riskiness of returns. Arrow and Debreu also introduced the concept of a state-contingent claim, an elementary security with a unit payoff in a given time and state, and zero elsewhere. The price of a state-contingent claim, or state price, reflects an individual's subjective determination of the likelihood of a particular state occurring in the next period. In this respect, state prices capture inherent uncertainty surrounding alternative states, and, as established by Ross (1976), in a complete market will be implicit in the price of all financial assets.

Arrow-Debreu state-contingent claims are now widely regarded as a foundation of modern asset pricing theory, including the Merton (1973), Breeden (1979), and Cox, Ingersoll and Ross (1985) models. In contrast to its theoretical success however, state preference theory has received relatively little empirical attention. Breeden and Litzenberger (1978) and Banz and Miller (1978) were among the first to estimate state prices. Following Ross (1976), who showed that even simple options will span the state space, Breeden and Litzenberger established that state-contingent claims may be priced as the second-derivative of a call option with respect to the strike price. Other than requiring complete markets, Breeden and Litzenberger's approach places few restrictions on the call option pricing function. Given a number of additional assumptions, a unique closed form solution to Breeden and Litzenberger's state pricing function is easily obtained from the Black-Scholes (1973) option pricing formula. While Breeden and Litzenberger were concerned primarily with determining state prices on aggregate consumption, Banz and Miller applied the same approach to the capital budgeting problem, where the underlying asset is the market portfolio. More recently the state preference approach has been used to uncover aggregate risk preferences (Jackwerth (2000)) and determine value-at-risk measures (Aït-Sahalia and Lo (2000)). Non-parametric methods of estimating state price densities have also been advanced (see, for example, Aït-Sahalia and Lo (1998)).

Despite state prices being incorporated into most theoretical models, state preference theory has not yet been empirically applied to the asset pricing question. The aim of this thesis is to investigate the application of state preference theory to pricing exchange-traded financial assets. Breeden and Litzenberger's approach is adopted to estimate the price of state-contingent claims on the S&P 500 index[1]. Once state prices have been determined, these may be used to price any asset where the payoff depends on the value of the index. Obtaining state prices from the second derivative of the Black-Scholes formula is computationally simple, and provides a number of appealing characteristics. Market microstructure features such as minimum tick size, or price increments, may be directly incorporated into the model, and the historical distribution may be used to proxy investor expectations regarding the range of possible returns on the underlying asset.

Following this introductory chapter, Chapter 2 investigates the application of the state preference approach to pricing S&P 500 index options. For this initial application of the state preference approach, S&P 500 index options are selected because, as Rubinstein (1994) points outs, these options are more likely to conform to theoretical market conditions. S&P 500 index options are European rather than American, so payoffs are simpler to compute; the underlying asset is an index, which tends to exhibit smoother price movements than other underlying instruments such as commodities, currencies or stocks, and is more likely to follow a lognormal distribution; and the market is relatively liquid. Values determined with the state preference approach are compared to values obtained with the Black-Scholes formula and Stutzer's (1996)

[1] Breeden and Litzenberger determined state prices on consumption, however there are well-known econometric problems associated with aggregate consumption data. These are covered in detail by Breeden, Gibbons and Litzenberger (1989), and include the reporting of expenditures rather than consumption; infrequent reporting of consumption data relative to stock returns; sampling error resulting from taking a subset of the population of consumption transactions; and summation bias following from the reported integral of consumption rates rather than spot consumption. In addition, Campbell (1993) notes that measured aggregate consumption may be a poor proxy for consumption of equity market participants.

canonical valuation approach. These alternative option pricing methods provide an interesting contrast to the state preference approach. If state prices are estimated from the second derivative of the Black-Scholes formula, then, as increments in the price of the underlying asset approach zero, option values obtained with the state preference approach should approach Black-Scholes values. In practice however, price movements are discrete rather than continuous, and as previously noted, the state preference approach provides for this market microstructure feature to be directly incorporated into the model. Furthermore, the Black-Scholes formula will price the entire distribution of expected returns on the underlying asset, albeit with low probability attached to extreme values. Under the state preference approach the range of expected returns may be limited, excluding extreme values from the model.

Stutzer's canonical valuation approach is a non-parametric method that, in contrast to the Black-Scholes formula, places no restrictions on the stochastic process governing the underlying asset price. The empirical distribution of returns on the underlying asset is used to obtain risk-neutral probabilities, which are applied to expected payoffs to determine the value of the option. Risk-neutral probabilities are closely related to state prices. A risk-neutral probability distribution is obtained when relative marginal utilities in alternate states are normalised to sum to one, therefore assuming investors are indifferent to risk. Note that, similar to the state preference approach, canonical valuation does not price the entire distribution of expected returns on the underlying asset; rather, prices are determined from an empirical distribution. The results of Chapter 2 indicate that the state preference approach performs well in pricing S&P 500 index options, providing an overall improvement on both the Black-Scholes formula and canonical valuation.

The successful application of the state-contingent claim approach to pricing S&P 500 index options suggests that this approach should be applicable to asset pricing more generally. As such, the state preference approach is applied to pricing equities in Chapter 3. State prices on the S&P 500 index are determined in the same manner as in Chapter 2, and are used to value stocks traded on US exchanges over the period December 1996 through July 2004. Once again, market microstructure features such as minimum price movements on the underlying asset are directly incorporated into the model, and the historical distribution is used to determine the range of expected returns on the underlying asset. A test approach similar to that of the first chapter is also adopted. Values obtained under the state preference approach are compared to those determined using other equity valuation techniques; specifically, the residual income model popularised by the accounting literature (see, for example, Ohlson (1995), Feltham and Ohlson (1995) and Myers (1999)). In its simplest form, the residual income model respecifies the dividend discount model in terms of accounting variables, where stock price is expressed as a function of the book value of equity and expected earnings per share. Equities are also valued with Stutzer's canonical valuation approach, on the basis that if the state preference approach is applicable to pricing any exchange-traded asset, then it should also be possible to price securities using risk-neutral probabilities obtained via the canonical valuation approach in Chapter 2. Finally, equities are valued with the Sharpe (1964) and Lintner (1965) Capital Asset Pricing Model (CAPM). The state preference approach is shown to provide a significant improvement on the residual income model, which performs poorly in comparison to the alternative models. Consistent with the results for S&P 500 index options, the state-contingent claim approach provides an improvement on canonical valuation, however of the models under investigation, the CAPM is shown to provide the most accuracy in pricing equities. The final chapter concludes.

Chapter 2

A State-Contingent Claim Approach to Pricing S&P 500 Index Options

2.1 Introduction

State preference theory, first proposed by Arrow (1964) and Debreu (1959), provides a framework for evaluating individuals' economic decision making under uncertainty. Alternative investment decisions are evaluated over both time and uncertain states of nature, and investor preferences for the timing and riskiness of returns on financial assets are captured in the prices for state-contingent claims. State-contingent claims are elementary securities with a unit payoff in a given time and state, and zero elsewhere. The prices of state-contingent claims, or state prices, are widely viewed as the building blocks of modern asset pricing theory, including the Merton (1973), Breeden (1979) and Cox, Ingersoll and Ross (1985) models. However, despite state preferences being incorporated into most theoretical models, the empirical application of state preference theory has received relatively little attention in the literature. Breeden and Litzenberger (1978) and Banz and Miller (1978) were the first to quantify state prices as the second derivative of a call option price. While Breeden and Litzenberger were concerned with state prices on aggregate consumption, Banz and Miller determined state prices on the market portfolio and applied these to the capital budgeting problem. A number of other studies have investigated the state pricing question, including Jackwerth (2000), who applied the state preference approach to uncover aggregate risk preferences, and Aït-Sahalia and Lo (2000), who used state prices to determine an alternate value-at-risk measure.

The aim of this thesis is to investigate the application of state preference theory to pricing exchange-traded financial assets. Breeden and Litzenberger's approach is adopted to estimate the prices of state-contingent claims on the S&P 500 index. Once state prices have been determined, these may be used to price any asset where the payoff depends on the value of the index. In this chapter, the state preference approach is applied to pricing S&P 500 index options. Values for S&P 500 index options from January 1990 through December 1993 are determined with the state preference approach, and compared to the Black-Scholes (1973) option pricing formula and Stutzer's (1996) canonical valuation method. These alternative option pricing methods provide an interesting contrast to the state preference approach. If state prices are estimated using the closed form solution of the second derivative of the Black-Scholes call option price then, as increments in the price of the underlying asset approach zero, option values should approach the Black-Scholes values. In practice however, price movements in the underlying asset are discrete rather than continuous, and one advantage of the state preference approach is that discrete price movements may be explicitly incorporated into the model. A further advantage of the state preference approach when compared to Black-Scholes is that the range of expected values of the underlying asset may be restricted. Whereas the Black-Scholes formula will price very small and very large expected values, albeit with low probability, these extreme values may be excluded altogether from the state price calculation. This has the additional practical advantage of reducing the computational burden.

Stutzer's canonical valuation method determines risk-neutral probabilities from the empirical distribution of returns on the underlying asset. Canonical valuation is a non-parametric method that, unlike the Black-Scholes formula, places no restrictions on the stochastic process governing the underlying asset price. The canonical valuation method has some similarities to the state preference approach. Both methods estimate subjective probabilities associated with possible values of the underlying asset, which are applied to expected payoffs to determine the value of the asset. As such, canonical valuation may be viewed as a non-parametric alternative to the state preference approach. An additional motivation for selecting canonical valuation is that this approach has been subject to little empirical testing.

This chapter is structured as follows. Section 2.1 provides an overview of the state preference approach and places it in the context of existing asset pricing models. Section 2.2 reviews alternative option pricing methods; in particular, the Black-Scholes option pricing formula and Stutzer's canonical valuation method. Section 2.3 describes the data and the methods used to generate state prices and canonical risk-neutral probabilities. Section 2.4 discusses the results and Section 2.5 summarises the main findings of the paper and suggests potential directions for further research.

2.2 The State Preference Approach

In modern finance theory, the price of any asset is understood to be a function of its payoff and a pricing kernel, or stochastic discount factor[2]. In a two-period context, the basic form of any asset pricing equation is expressed as follows:

$$P_{i0} = E\left(M_t X_{it}\right) \qquad \forall i,t \tag{1}$$

where P_{i0} is the price of any asset *i* in the current period, time *0*, E is an expectations operator conditioning on information available at time *0*, X_{it} is the payoff of asset *i* at time *t*, and M_t is the stochastic discount factor.

The stochastic discount factor is equivalent to a time discount factor, except in a world of uncertainty it will be a random variable. That is, if the outcome at time *t* were known with certainty, then the stochastic discount factor would collapse to a constant, and equation (1) would take the familiar present value form, where next period's (known) payoffs are discounted to today's dollars. This implies that, given a time-separable utility function and no uncertainty, the stochastic discount factor represents investors' discounted marginal rate of substitution between consumption in the current period, time *0*, and consumption in the next period, time *t*. In a world of uncertainty (and state-separable utility) the stochastic discount factor will also capture investors' consumption preferences across alternative states of nature, and the stochastic discount factor will represent investors' marginal rate of substitution between consumption in the current period and consumption in period *t*, state *s*. The appeal of the stochastic

[2] See Campbell (2000) for a comprehensive overview of the asset pricing literature and the role of the stochastic discount factor.

discount factor lies in its simplicity and broad applicability - any asset may be priced in terms of its payoff and the stochastic discount factor.

In a complete market the stochastic discount factor exists and may be characterised by the set of state-contingent claim prices. Ross (1976) observed that even simple options will span the entire state space, and therefore state-contingent claims will be implicit in the price of traded securities. If any one of S possible states occurs in the next period, then in a complete market, investors may form a portfolio with a positive payoff in state s, and zero elsewhere. Such a portfolio is entirely consistent with a state-contingent claim, and it is not necessary that an explicit market for state-contingent claims exists, so long as there exist enough traded securities so as to span the entire state space. Incorporating state prices into the general asset pricing equation (1) provides a basis for using the most basic of securities to price other, more complex assets.

Defining a state price as the individual's subjective probability assessment of the likelihood of a particular state s occurring, the price of a state-contingent claim will be given by investor k's true probability assessment of the occurrence of state s (π_{ts}^k) multiplied by their marginal rate of substitution between consumption in the current period and consumption in state s, time t. This is determined by solving the representative investor's two-period constrained optimisation problem, providing the following expression for the state price at time t, state s:

$$\Phi_{ts}^k = \frac{\pi_{ts}^k U_{ts}'^k \left(C_{ts}^k\right)}{U_0'^k \left(C_0^k\right)} \tag{2}$$

In this respect a state price can be interpreted as the price today of one unit of consumption in state s, time t. Equation (2) indicates state prices will exhibit probability-like characteristics – they are non-negative and, in the next period, will sum to one – and therefore constitute a legitimate probability distribution. However, since state prices are implicit in exchange-traded financial assets, and are therefore influenced by supply and demand, they will be subjective probabilities incorporating investor risk preferences. Relative to a true probability distribution, greater weight will be placed on outcomes with higher marginal utility of consumption, where wealth is more valuable to consumers. This provides a simple but powerful result: outcomes that occur with greater frequency or are improbable but undesirable attract a greater weighting, a result entirely consistent with our understanding of investor risk aversion.

Summing the set of state prices across the state space provides the price of an asset with a certain unit payoff in the next period: $\sum_{s=1}^{S} \Phi_{ts}^{k} = e^{-rt}$, where r is the risk-free rate of return. This is consistent with the conditional mean of the stochastic discount factor above[3]. The price of a risky asset j is expressed as the payoff in state s multiplied by the state price, summed over all possible S states:

$$P_0^j = \sum_{s=1}^{S} \Phi_{1s} d_{1s}^j \qquad \forall j = 1 \ldots N \tag{3}$$

[3] The conditional moments of the stochastic discount factor are easily determined. The conditional mean of the stochastic discount factor is given by price today of a riskless real asset with a certain unit payoff next period: $P = E\left(M_t\right) = e^{-r_f t}$.

2.2.1. Breeden and Litzenberger state prices

Breeden and Litzenberger (1978) and Banz and Miller (1978) first proposed that the price of an elementary security may be modelled as the second derivative of a call option price. This proposition is based on the construction of a simple butterfly spread, with a normalised unit payoff. If the current value of the asset is given by M, and ΔM is the step size between potential next period values, then the option portfolio replicating an elementary claim is created by purchasing two calls with strike prices $M-\Delta M$ and $M+\Delta M$ respectively, and selling two calls with a strike price of M. If the value of the underlying asset is M next period, then the payoff on the option portfolio will be ΔM, and zero otherwise. Dividing through by ΔM obtains a unit payoff:

$$1=\frac{\Delta M}{\Delta M}=\frac{C(M-\Delta M,T)-2C(M,T)+C(M+\Delta M,T)}{\Delta M} \tag{4}$$

With a step size of ΔM between possible next period values, and if M occurs in T periods then the cost of the call portfolio is $P(M,T;\Delta M)$. Dividing through by ΔM obtains the price of the call portfolio paying one unit:

$$\frac{P(M,T;\Delta M)}{\Delta M}=\frac{C(M-\Delta M,T)-2C(M,T)+C(M+\Delta M,T)}{\Delta M^2} \tag{5}$$

Taking the limit as the step size tends to zero provides the price of an elementary security evaluated at $X=M$:

$$\lim_{\Delta M\to 0}\frac{P(M,T;\Delta M)}{\Delta M}=\left.\frac{\partial^2 C(X,T)}{\partial X^2}\right|_{X=M} \tag{6}$$

Aside from the assumption of complete markets, Breeden and Litzenberger's approach places few restrictions on the option price. It is not necessary for the option pricing function to be twice differentiable as a discrete solution may be obtained from equation (5), and no restriction is placed on the stochastic process governing the time series behaviour of asset returns. With some additional assumptions[4], evaluating the second derivative of the Black-Scholes (1973) call option pricing formula provides the following explicit closed form solution for the price of an elementary claim:

$$\left.\frac{\partial^2 C}{\partial X^2}\right|_{X=M_T} = \frac{e^{-rT}}{M_T \sigma \sqrt{T}} n\left[d_2\left(X = M_T\right)\right] \tag{7}$$

where

$$d_2 = \frac{\ln\left[(M_0 - PVD)/M_T\right] + \left[r - 1/2\sigma^2\right]T}{\sigma\sqrt{T}}$$

M_T is the value of the underlying asset in T periods, M_0 its value today, PVD is the present value of dividends, and $n(d_2) = \frac{1}{\sqrt{2\pi}} e^{-d_2/2}$ is the standard normal probability density function.

Describing state prices as the second derivative of a call option with respect to the strike price makes the assumption that increments in the value of the underlying stock ΔM tend to zero. In reality however, price movements will be greater than zero and it will not be possible to obtain a limiting value for ΔM. Evaluating equation (7) at discrete intervals will not capture the range of state prices between each possible level of the strike price, X. Breeden and Litzenberger's "delta security" method provides a solution to this problem. The price of a state-contingent claim with a unit payoff if the value of the underlying asset at time t is greater than or equal to a pre-specified level Y,

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[4] The assumptions of the Black-Scholes formula are well known. Namely, asset returns are assumed to follow geometric Brownian motion with constant volatility.

and zero otherwise, will be simply the sum of the state prices for each asset value greater then Y. This is given by the cumulative pricing function:

$$G(Y) = \int_{Y}^{\infty} \frac{e^{-rT} n(d_2)}{X\sigma\sqrt{T}} dX = e^{-rT} N\left[d_2\left(X = Y\right)\right] \quad (8)$$

The cost of a security with a unit payoff if the value of the underlying asset is between two predetermined levels, say Y_i and Y_{i+1}, will be the difference between the cumulative pricing function at these levels:

$$\phi\left(Y_i, Y_{i+1}\right) = e^{-rT}\left\{N\left[d_2\left(X = Y_i\right)\right] - N\left[d_2\left(X = Y_{i+1}\right)\right]\right\} \quad (9)$$

The state price is therefore just the difference between consecutive delta security prices.

2.2.2. Option pricing functions

The approach for determining state prices outlined above provides a number of empirical advantages. First, it provides the flexibility to select suitable increments in the value of the underlying asset for Y_i and Y_{i+1}, allowing market microstructure aspects such as minimum price movements and price limits to be incorporated into the model. To illustrate, over the sample period January 1990 to December 1993 the minimum tick size of the S&P 500 index futures contract was 5c, so increments of 5c for Y_i, Y_{i+1} are used. In addition, the range of expected values of the underlying asset may be bounded by selecting maximum and minimum values of Y. This not only reduces the computational burden, but also places limits on the range of index values priced by the model. In comparison, the Black-Scholes option pricing formula will price very large and very small values of the index, albeit with a very low probability.

With the mechanics for determining state prices outlined above, the option pricing formulae are easily specified. In equation (3), asset prices were determined as a function of the asset's payoff and the stochastic discount factor. This is easily understood in the context of options, where the payoff is determined by the price of the underlying asset at the option's expiry date. Following equation (3), the price of a call option will therefore be given by:

$$\begin{aligned} C &= \sum_h \max\left[\left(M_0 - PVD\right)\times R_h - X, 0\right]\phi_h \\ &= \sum_h \max\left[M_{Th} - X, 0\right]\phi_h \end{aligned} \tag{10}$$

and similarly for put options:

$$\begin{aligned} P &= \sum_h \max\left[X - \left(M_0 - PVD\right)\times R_h, 0\right]\phi_h \\ &= \sum_h \max\left[X - M_{Th}, 0\right]\phi_h \end{aligned} \tag{11}$$

The state prices on the S&P 500 index used to determine the price of S&P 500 index options are calculated in the following manner. For each option in the sample historical T-day returns on the S&P 500 index are determined from 1 January 1980, and the empirical maximum and minimum T-day return obtained. The empirical maximum and minimum return is then applied to the current level of the index (M_0) to determine the range of possible levels of the index in T days (M_T). As noted above, index levels between the maximum and minimum are calculated in increments of 5c, reflecting the minimum price movement in the S&P 500 index futures contract at the time of the sample. Once the state price associated with each potential level of the index in T-days is determined, the option price is calculated as the sum of the expected payoff at each level of the index multiplied by the respective state price.

Before turning to the results of pricing S&P 500 index options with the state preference approach, the following section outlines the alternative models under consideration, and provides a brief overview of the option pricing literature.

2.3 Alternative Option Pricing Models

In this chapter, the accuracy of the state preference approach in pricing options is assessed in comparison to alternative option pricing methods, including the Black-Scholes formula and Stutzer's (1996) canonical valuation. The similarities between the state preference approach and the Black-Scholes formula and Stutzer's canonical valuation provide the motivation for comparing these alternative valuation techniques. In addition, the canonical valuation approach has been subject to little empirical testing in the literature. This section provides an overview of the option pricing literature. In particular, the literature surrounding the Black-Scholes formula is reviewed, and Stutzer's canonical valuation approach is detailed.

2.2.1. The Black-Scholes formula

The Black-Scholes option pricing formula is arguably the most successful financial model in existence. It has been widely adopted by practitioners and its authors rewarded with justified acclaim. Moreover, it is a no-arbitrage model, and therefore is underpinned by a fundamental principle of finance theory. The Black-Scholes call option formula is given by:

$$C = (S - PVD)N(d_1) - Xe^{-rT}N(d_2) \tag{12}$$

where S is the value of the underlying asset, PVD is the present value of dividends expected to be paid over the life of the option, X is the option's strike price, T is the time to expiration, r is the instantaneous risk free rate of interest, σ is the expected volatility of the underlying asset over the option's life,

$$d_1 = \frac{\ln\left[\left(S - PVD\right)/X\right] + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

and $N(d)$ is the cumulative standard normal density function.

Since its inception in the early 1970s numerous studies have investigated the Black-Scholes option pricing formula's empirical performance, and its shortcomings have been well documented. Attention has been largely focused on the assumptions underpinning the Black-Scholes formula; in particular, the implications of assuming a particular stochastic process governing the underlying asset's price dynamic. Movements in asset prices are assumed to follow geometric Brownian motion with constant volatility, which implies asset prices are lognormally distributed (or alternatively, asset returns are normally distributed). If the true distribution deviates from lognormal and volatility is not constant, then the resulting Black-Scholes option price will be biased away from the true price. The constant volatility assumption implies that all options on a single underlying asset should have the same implied volatility. Empirical research shows that Black-Scholes implied volatilities differ across exercise prices and time to maturity, with the bias most noticeable for deep-in-the-money and deep-out-of-the-money options. Early studies (see, for example, Black (1975), MacBeth and Merville (1979) and Rubinstein (1985)) showed that although pricing biases in Black-Scholes do exist, the bias is non-constant through time. More recent studies (see Rubinstein (1994), Dumas, Fleming and Whaley (1998) and Bollen and Whaley (2004)) have focused on the performance of Black-Scholes post-1987 crash. Prior to the 1987 crash, Black-Scholes implied volatilities were understood to have a distinctive "smile" pattern, where deep-in-the-money and deep-out-of-the-money options have higher implied volatilities than at-the-money options. Post-crash, these studies find implied volatilities display a "sneer" or "smirk" pattern, where implied volatility monotonically decreases as the exercise price increases relative to the stock price. This pattern becomes more pronounced as the time to maturity decreases.

The existence of these anomalies has raised questions regarding the assumptions underlying the Black-Scholes formula, and alternatives have been proposed which should, it's claimed, overcome the limitations imposed by assuming a lognormal distribution with constant volatility. These alternative models have received a mixed response. Cox and Ross (1976) proposed a constant elasticity of variance model to allow volatility to vary with the asset price; however Emanuel and MacBeth (1982) find that, out of sample, this model performs no better than Black-Scholes. Rubinstein (1985) tested five alternative option pricing models that relaxed the constant volatility assumption over the period August 1976 through August 1978 and found that none of the models characterised stock option prices better than another. He also argues that while the bias from Black-Scholes is statistically significant, it is not "economically significant". In a later paper however, Rubinstein (1994) states "the Black-Scholes formula become increasingly unreliable over time", and proposes an alternative – an implied binomial tree. Other researchers have also proposed an implied tree approach (see Derman and Kani (1994) and Dupire (1994)) where volatility is assumed to be a deterministic function of asset price and time. Dumas, Fleming and Whaley investigate the performance of these models for S&P 500 index options over the period June 1988 through December 1993 and find that this approach performs no better than an "ad-hoc procedure that merely smooths Black-Scholes implied volatilities across exercise prices and time to expiration." In addition to the implied tree approach, a number of other non-parametric option pricing methods have been advanced. These include stochastic volatility models, with and without jumps (see Bakshi, Cao and Chen (1997) and Bates (2000)), kernel regressions (see Aït-Sahalia and Lo (1998)) and neural networks (see Hutchinson, Lo and Poggio (1994)). These alternatives can be data intensive and, out of sample, seem unlikely to provide a significant advantage over Black-Scholes. These approaches have also been criticised for not constituting a predictive theory of option pricing (Stutzer (1996)). Finally, an alterative solution to the implied volatility problem has recently been proposed by Bollen and Whaley (2004), who consider the relationship between the shape of the implied volatility function and the supply and demand for options contracts. Bollen and Whaley suggest that if implied volatilities reflect a series of market clearing prices, then imbalances between supply and demand could result in implied volatility functions that are not flat. Rather than relaxing the Black-Scholes assumptions, this approach provides an intuitive solution to the implied volatility problem, and suggests that prices may be formed in a manner not inconsistent with Black-Scholes once additional costs to the market maker are taken into account.

The Black-Scholes option-pricing formula is derived from the formation of an arbitrage portfolio, where a call option is replicated with a stock and bond portfolio. If a risk-free hedge may be created by buying the underlying asset and selling the option (or vice versa) then the value of the option will be independent of investor risk preferences, and a risk neutral investor will value the option in the same manner as a risk averse investor. The expression for the price of a state-contingent claim, equation (9) of the previous section, relies on this assumption; where, in a risk neutral world, the expected return on all assets is equal to the risk free rate (see Cox and Ross (1976) and Black and Scholes (1973)). That is, setting the instantaneous expected rate of change in stock price (the drift term in the stochastic process governing stock returns) equal to the risk free rate of interest will price options where investors are risk neutral. Pricing elementary claims with this form of the Black-Scholes option pricing formula will yield state prices comparable to a risk-neutral probability associated with each expected value of the underlying asset, Y.

More generally, a risk-neutral probability distribution is obtained when relative marginal utilities in alternative states are normalised to sum to one, assuming a risk-neutral world where investors are indifferent to risk. Risk-neutral probabilities will therefore reflect investors' subjective probability assessment of a particular outcome if they are risk neutral. Risk-neutral probabilities form the foundation of a number of option pricing methods, including the binomial tree approach of Cox, Ross and Rubinstein (1979), Rubinstein's (1994) implied trees, and Stutzer's (1996) canonical valuation. In general, these methods obtain a risk-neutral probability distribution from an empirical distribution of returns (canonical valuation) or simultaneously observed option prices (Rubinstein's implied trees). As noted above, this chapter provides a test of Stutzer's canonical valuation method, a non-parametric alternative to the Black-Scholes formula and the state preference approach. The following sub-section outlines the steps of the canonical valuation method.

2.2.2. Canonical valuation

Stutzer's canonical valuation approach does not assume any particular stochastic process governs asset returns; rather, an historical time series of asset returns is used to translate the empirical probability distribution into risk-neutral probabilities. These risk-neutral probabilities are then used to determine the expected discounted payoff of the asset, and consequently, the option price. Stutzer's approach is computationally simple, and relies on estimating only a single parameter. The data requirements are not extensive. The only input is an historical time series of underlying asset values, from which risk-neutral probabilities are calculated and applied to the range of possible future index values to determine the expected option payoff. The steps for determining option prices are as follows.

To price an option expiring in *T*-periods, first construct an historical time series of *T*-period returns on the underlying asset:

$$R_h = M_h / M_{T-h}, \qquad h = 1, 2, \ldots, H - T \tag{13}$$

providing *H-T* possible values of the underlying asset's price in *T*-periods,

$$M_T = M_t R_h, \qquad h = 1, 2, \ldots, H - T \tag{14}$$

with an estimated actual probability of $\hat{\pi}_h = \frac{1}{H-T}$. The estimated risk-neutral probabilities derived from the empirical probabilities must be non-negative and satisfy the following constraint, where *r* is the one-period riskless rate:

$$\sum_{h}^{H-T} \pi_h^* \frac{R_h}{r^T} = 1 \tag{15}$$

This constraint is entirely consistent with the conditional mean of the stochastic discount factor discussed in the previous section. That is, the probabilities must sum to one, and, under risk-neutrality, the expected return on the underlying asset must be equal to the risk free rate.

While there are many choices for π^* that would satisfy these twin constraints, Stutzer chooses an estimate $\hat{\pi}^*$ that minimises the Kullback-Leibler Information Criterion distance between the empirical probabilities $\hat{\pi}$, and the risk neutral probabilities π^*.

$$\hat{\pi}^* = \underset{\pi_h^*>0,\sum_{\forall h}\pi_h^*=1}{\arg\min}\, I\left(\pi^*,\hat{\pi}\right) = \sum_{h=1}^{H-T} \pi_h^* \ln\left(\frac{\pi_h^*}{\hat{\pi}_h}\right) \text{ s.t. } \sum_{h}^{H-T} \pi_h^* \frac{R_h}{r^T} = 1 \qquad (16)$$

The solution to the constrained maximisation problem in the equation above is obtained using the Lagrange multiplier method, providing the Gibb's canonical distribution:

$$\hat{\pi}_h^* = \frac{\hat{\pi}_h \exp\left[\gamma^*\left(R_h / r^T\right)\right]}{\sum_{\forall h} \hat{\pi}_h \exp\left[\gamma^*\left(R_h / r^T\right)\right]}, \qquad h = 1,2,\ldots,H-T \qquad (17)$$

The Lagrange multiplier, γ^*, is found by solving the unconstrained minimisation problem:

$$\gamma^* = \underset{\gamma}{\arg\min} \sum_{h} \exp\left[\gamma\left(R_h / r^T - 1\right)\right] \qquad (18)$$

The price of a European call option is then determined as:

$$C = \sum_{h} \frac{\max\left[\left(M_t - PVD\right)R_h - X, 0\right]}{r^T} \hat{\pi}_h^* \qquad (19)$$

European put options are similarly priced:

$$P = \sum_{h} \frac{\max\left[X - \left(M_t - PVD\right)R_h, 0\right]}{r^T} \hat{\pi}_h^* \qquad (20)$$

In this chapter historical T-day returns are calculated from January 1980 through to the option valuation date. Stutzer does not prescribe any particular sample length, although he does find including the 1987 crash produces larger values for in-the-money calls than when the crash is excluded from the sample. The impact of the 1987 crash is the subject of some discussion in the literature. Rubinstein (1994) suggests investor fears of another crash explain the deteriorating performance of the Black-Scholes formula, and Bollen and Whaley (2004) show that the downward sloping shape of the implied volatility function post-crash is driven by net buying pressure for index puts used for portfolio insurance.

This section detailed both the state preference approach for determining option prices, and the alternative methods against which the accuracy of the state preference approach is assessed; namely, the Black-Scholes formula and Stutzer's canonical valuation. Before turning to the results however, the followings section provides details of the data requirements of each approach.

2.4 Sample Description

The sample contains weekly quotes for options on the S&P 500 index traded on the Chicago Board of Options Exchange (CBOE) over the period January 1990 through December 1993. S&P 500 index options were chosen because, as Rubinstein (1994) points out, these options are most likely to conform to the Black-Scholes conditions. These options are European rather than American, and therefore payoffs are simpler to compute; the underlying asset is an index, which tends to exhibit smoother price movements than other underlying instruments such as commodities, currencies and stocks and may more closely follow a lognormal distribution; and finally the market is relatively liquid. Options with greater than 100 days to maturity and an absolute moneyness greater than 10% are excluded from the sample as these options are less frequently traded, and consequently quoted bid and ask prices may not be supported by actual trades. Moneyness is given by the ratio of the exercise price to the stock price less 1, $|X/S-1|$, and reported moneyness categories are consistent with those in Dumas, Fleming and Whaley (1998).

Valuing options with Black-Scholes requires inputs for the risk free rate of interest, expected dividend payments, and expected volatility of the underlying asset over the life of the option. The canonical valuation approach requires a time series of historical values of the S&P 500 index. As discussed in Section 2, the empirical distribution is also used to obtain historical maximum and minimum returns for determining the range of expected future values of the S&P 500 index for which state prices are calculated.

Historical returns are determined from a time series of daily observations on the S&P 500 index from January 1980 through December 1993 obtained from CRSP. The riskless interest rate is proxied by US T-bill rates reported in the *Wall Street Journal*, and the present value of dividends paid during the life of the option are discounted daily cash dividends for the S&P index portfolio collected from the *S&P 500 Information Bulletin*. Expected volatility on the S&P 500 index is proxied by the previous trading day's CBOE Market Volatility Index (VIX) level. The VIX represents the market's consensus view on expected future stock market volatility, and is regarded as a benchmark of US stock market volatility. On this basis the VIX should provide a more appropriate proxy for expected volatility than an historical volatility estimate, as it is both forward-looking and market determined. In addition, the VIX has been found to demonstrate characteristics of observed stock market volatility. Fleming, Ostdiek and Whaley (1995) investigate the properties of the index, and find it to be a useful proxy for expected stock market volatility. The VIX is strongly related to future realised stock market volatility, and exhibits a negative, asymmetric relationship with contemporaneous market return. This is consistent with the observed pattern of increases stock prices being associated with a reduction in volatility, and vice versa; and with falling stock prices being associated with larger absolute changes in volatility, than stock price increases of the same magnitude. In September 2003 the CBOE changed the methodology for calculating the VIX. The underlying instrument was changed from the S&P 100 index to the S&P 500 index, as S&P 500 derivatives are more actively traded. In addition, options over a range of strike prices are used, including out-of-the-money puts and calls, rather than only at-the-money options, and the calculation is no longer based on the Black-Scholes option pricing formula[5]. The CBOE has reproduced the VIX values prior to September 2003 using the new methodology, and continues to report values based on the old methodology. Since the new volatility index is based on S&P 500 index options it is used in this paper. To ensure state price values are robust to alternative volatility measures an historical volatility estimate based on daily returns on the S&P 500 index over the previous 40 trading days is used as a second proxy for expected volatility over the life of the option. Comparison of option values using each volatility measure also provides an interesting insight into the appropriateness of each measure, which is discussed in the following section.

[5] For all these changes the new VIX provides values largely similar to the previous index. This is to be expected due to the high correlation between the underlying indices.

2.5 Results

Comparison is made between the state preference approach, the Black-Scholes formula and Stutzer's canonical valuation on the basis of three goodness of fit measures. Mean squared error (MSE), which measures the average squared deviation of model values from the midpoint of the bid-ask spread, provides a good overall measure of the goodness of fit of each model. Mean squared outside error (MSOE) assesses the extent to which model values lie within the quoted bid-ask spread. This is measured as the squared difference between the estimated model value and the option's ask (bid) price if the model value is greater (less) than the ask (bid) price, averaged over the sample. If the model value lies within the spread then the model is considered to have accurately priced the security, and the error will be zero. Mean squared error and mean squared outside error, while useful in assessing overall goodness of fit, provide little information on the degree to which each model over- or under-prices an option relative to the quoted spread. Dumas, Fleming and Whaley use mean outside error (MOE) to assess the extent of model over- or under-pricing. MOE measures the average over- or under-pricing outside of the option's quoted bid-ask spread. Similar to MSOE, if the model value lies between the bid and ask prices then the error measure will be zero. If the model value exceeds the ask price then the error is the positive difference between the model value and the ask, and if the model value is less than the bid price then the error will be the negative difference between the model value and the bid. MOE will therefore be greater than zero if the model overprices options on average, and less than zero if the model underprices options on average. As Dumas, Fleming and Whaley point out, MOE is a useful determinant of pricing biases within moneyness and maturity categories.

All three goodness of fit measures indicate that the state preference approach represents an overall improvement on both the Black-Scholes formula and canonical valuation. The tables contain results of the three goodness of fit measures for each model. Results are provided for the full sample, for total puts and total calls in the sample, and for puts and calls categorised by maturity and moneyness. As discussed in the previous section, two measures of expected volatility are used in calculating the Black-Scholes and state price estimates. Tables 2.1 to 2.3 contain MSEs, MSOEs and MOEs respectively where the proxy for expected volatility is the previous day's VIX index value, and in Tables 2.4 to 2.6 expected volatility is proxied with an historical volatility estimate based on daily returns on the S&P 500 index over the previous 40 trading days. Canonical valuation errors in Tables 2.4 to 2.6 will be the same as those in Tables 2.1 to 2.3, and are included for comparative purposes.

The results in Table 2.1 indicate that for the sample as a whole the state preference approach provides lower MSEs than either of the alternative models. Squared errors on the state price values are 1.599 on average, compared to 1.712 and 6.376 for Black-Scholes and canonical valuation respectively. The state preference approach represents an overall improvement of approximately 7% on the Black-Scholes formula and 75% on canonical valuation. Considering puts and calls separately indicates a similar improvement, and the state preference approach generally outperforms the alternative models when puts and calls are categorised by maturity and moneyness. The Black-Scholes formula provides better estimates for out-of-the-money puts across most maturities than either canonical valuation or state price estimates. Stutzer's canonical valuation method generally performs poorly when compared to both

Black-Scholes and the state preference approach, with the exception of out-of-the-money calls with fewer than 60 days to maturity.

Table 2.2 contains MSOEs for the three models, and the results are consistent with the MSEs in Table 2.1. Overall MSOEs on the state price, Black-Scholes and canonical valuation estimates are 1.165, 1.262 and 5.464 respectively, and the state preference approach provides an 8% improvement on Black-Scholes and a 79% improvement on canonical valuation. Comparing the MSOEs in Table 2.2 to MSEs in Table 2.1 indicates a greater proportion of state price option values lie within the spread than either Black-Scholes or canonical values, and, on average, there is less dispersion in state price values than for the alternative models.

MOEs are reported in Table 2.3. On average all three models over-price the options in the sample, with MOEs of 0.428, 0.455 and 1.546 for the state price, Black-Scholes and canonical values respectively. Considering puts and calls separately indicates puts are over-priced by a greater magnitude than calls for all three models. When puts and calls are categorised by moneyness the direction of MOEs indicates that the state preference approach under-prices in-the-money calls, and over-prices at-the-money and out-of-the-money calls (of course, by put-call parity, out-of-the-money puts are under-priced, and at-the-money and in-the-money puts over-priced). The Black-Scholes estimates exhibit a similar pattern of over- and under-pricing. This is consistent with the implied volatility sneer documented in previous studies. On average, canonical valuation over-prices options in all moneyness and maturity categories in the sample.

Tables 2.4, 2.5 and 2.6 report MSEs, MSOEs and MOEs respectively where 40-day historical volatility proxies for expected volatility. Consistent with the results in Tables 2.1 to 2.3, the state preference approach with historical volatility provides an overall improvement on the alternatives. MSEs for the entire sample in Table 2.4 are 3.151, 3.199 and 6.376 for state price, Black-Scholes and canonical values respectively. The overall improvement provided by the state preference approach is approximately 2% on Black-Scholes and 51% on canonical valuation. The state preference approach provides a similar improvement for call options, however when puts are considered separately, Black-Scholes provides the best estimates. MSOEs are reported in Table 2.5 and, consistent with the MSEs in Table 2.4, the state preference approach provides an overall improvement on the alternatives. Table 2.6 contains MOEs for each model. The MOEs indicate both the state preference approach and Black-Scholes formula under-price the options in the sample on average. Indeed, both puts and calls are under-priced on average across most maturity and moneyness categories. In-the-money calls are under-priced by a greater magnitude than out-of-the-money calls, and similarly out-of-the-money puts are under-priced by more than in-the-money puts.

Of interest is the relative magnitude of the MSEs in Tables 2.1 and 2.4. These indicate that VIX provides an overall better proxy for expected volatility than historical volatility. MSEs for the sample as a whole in Table 2.1 for the VIX state price values are 1.599 compared with MSEs of 3.151 for the historical volatility state price values in Table 2.4. Using the VIX to proxy expected volatility represents an improvement of almost 50%, with a similar improvement evident in the Black-Scholes values. However this improvement is not consistent across moneyness categories. While using the VIX to proxy for expected volatility produces better estimates for in-the-money calls and out-of-the-money puts, an historical volatility proxy provides better estimates for out-of-the-money calls and in-the-money puts.

Figures 2.1 and 2.2 are scatter plots of under- and over-pricing across moneyness categories for calls and puts respectively. These figures show graphically the results in Tables 2.3 and 2.6 for each model. Comparing the call option outside errors in Figure 2.1 shows the direction of over- and under-pricing. The plots on the right are the Black-Scholes and state price values where historical volatility proxies for expected volatility, and the persistent under-pricing across all moneyness categories is clearly evident. The VIX state preference and Black-Scholes outside errors are on the left. As discussed above, in-the-money calls are over-priced and out-of the-money calls under-priced when the VIX proxies for expected volatility. Overall, canonical valuation produces call option values that are too high, and there appears to be little difference in the over-pricing across moneyness categories. Figure 2.2 shows a consistent pattern to Figure 2.1 for put options (naturally, by put-call parity the pattern across moneyness categories is reversed). Similar to Figure 2.1, canonical valuation produces put option values that are too high, and historical volatility state preference and Black-Scholes estimates under-price puts across moneyness categories.

Further insight may be gained from reviewing the probability distributions associated with the state preference and canonical valuation approaches. Figures 2.3, 2.4 and 2.5 show the state price and canonical risk-neutral probability distributions for a call option with 24, 52 and 87 days to maturity respectively. The shape of the canonical probability distribution in Figure 2.3 indicates that, relative to the state price distributions, greater weight is placed on observations in the tails of the distribution. This pattern becomes more noticeable as the time to maturity increases. Figure 2.4, where T=52 days, shows an increasingly fatter left and right tail than both state price distributions, and this is even more pronounced in Figure 2.5, where T=87 days. The greater dispersion as time to maturity increases is consistent with greater probability of large price movements over longer periods, as greater weight in the tails of the distribution will place a higher probability on extreme observations. Consequently, canonical valuation will produce higher option values relative to Black-Scholes and the state preference approach. In particular, the thicker left tail and associated greater downside risk will result in higher values out-of-the-money puts, and, by put-call parity, in-the-money calls. Stutzer (1993) notes that incorporating the 1987 crash into the empirical distribution should produce canonical values for in-the-money call options on the S&P 500 index with less than 6 months to maturity which are "substantially higher" than Black-Scholes values. While the canonical values for in-the-money calls are found to be higher than the state price and Black-Scholes values, the outside errors in Figure 2.1 indicate these options are over-priced. Indeed, the results in Tables 2.3 and 2.6 indicate that canonical valuation generally over-prices both puts and calls across all moneyness and maturity categories. This is consistent with too much weight being placed on extreme observations in both the left and right tails of the distribution.

Figures 2.3, 2.4 and 2.5 also provide an insight into appropriateness of the alternative volatility measures. When historical volatility is used to proxy expected volatility the state price distribution has less weight in the tails of the distribution, and greater weighting around the mean compared to the VIX state price distribution. This is most pronounced in Figure 2.3, but is also evident in Figures 2.4 and 2.5. Comparing the MOEs in Tables 2.3 and 2.6 indicates that the historical volatility values place too little weight in the left tail of the distribution, under-pricing out-of-the-money puts and in-the-money calls. On this basis the VIX index appears to be a better measure of downside risk than historical volatility. However, historical volatility provides better estimates of out-of-the-money calls and in-the-money puts. Out-of-the money calls and in-the-money puts are over-priced when the VIX is used to proxy expected volatility, indicating that the VIX over-prices upside risk, placing too much weight in the right tail of the distribution. This has interesting implications for the valuation of index options, particularly for out-of-the-money index puts, which are used as portfolio insurance by institutional investors, and for which there is no natural counter-party (see Bollen, and Whaley (2004)).

2.6 Conclusion

The aim of this thesis is to investigate the application of state preference theory to pricing exchange-traded financial assets, and in this chapter the state preference approach is applied to pricing S&P 500 index options. Breeden and Litzenberger's (1978) approach is adopted to estimate the price of state-contingent claims on the S&P 500 index. Once state prices have been determined, these may be used to price any asset where the payoff depends on the value of the index. Values for S&P 500 index options from January 1990 through December 1993 are determined with the state preference approach, the accuracy of which is assessed by comparing model values to alternative option pricing methods, in particular, the Black-Scholes (1973) option pricing formula and Stutzer's (1996) canonical valuation method. These alternative methods provide an interesting contrast to the state preference approach. If state prices are estimated using the closed form solution of the second derivative of the Black-Scholes call option price then, as increments in the price of the underlying asset approach zero, option values should approach the Black-Scholes values. In practice however, price movements in the underlying asset are discrete rather than continuous, and one advantage of the state preference approach is that discrete price movements may be explicitly incorporated into the model. A further advantage of the state preference approach when compared to Black-Scholes is that the range of underlying asset values may be restricted. Whereas the Black-Scholes formula will price very small and very large underlying asset values, albeit with low probability, these extreme values may be excluded altogether from the state price calculation. This has the additional practical advantage of reducing the computational burden.

Stutzer's canonical valuation method determines risk-neutral probabilities from the empirical distribution of returns on the underlying asset. Canonical valuation is a non-parametric method that, unlike the Black-Scholes formula, places no restrictions on the stochastic process governing the underlying asset price. The canonical valuation method has some similarities to the state preference approach. Both methods estimate subjective probabilities associated with possible values of the underlying asset, which are applied to expected payoffs to determine the value of the asset. As such, canonical valuation may be viewed as a non-parametric alternative to the state preference approach. An additional motivation for selecting canonical valuation is that this method has been subject to little empirical testing. Accordingly, this chapter also represents a test of the canonical valuation approach.

The state preference approach to option valuation is shown to perform well, providing an overall improvement on alternative models. The state preference approach outperforms both the Black-Scholes formula and Stutzer's canonical valuation method. Canonical valuation does not perform well in comparison to either Black-Scholes or state prices, producing option values that are too high on average across moneyness and maturity categories. Comparing the results for alternative volatility measures provides an interesting insight into the appropriateness of historical volatility and the VIX index as proxies for expected volatility. In general, the VIX is a better measure for expected volatility, providing more accurate Black-Scholes and state price values. In particular, the VIX produces better estimates for in-the-money calls and out-of-the-money puts indicating that the VIX provides a better estimate of downside risk than historical volatility. The results of this study are promising for future research into the application of state-contingent claims to the valuation of other exchange-traded financial assets.

Appendix A Charts and Tables

Table 2.1 Comparison of Mean Squared Errors on S&P 500 Index Options Valued by Alternative Option Pricing Models where the VIX proxies for expected volatility

Mean squared error is measured as the squared deviation of model values from the bid-ask midpoint averaged across all days in the sample period 3 January 1990 through 29 December 1993. Implied volatility for Black-Scholes and state price estimation is proxied by the previous day's VIX index value. Moneyness is defined as the ratio of the exercise to the index value less one. Moneyness categories are based on those of Dumas et al (1998).

A: Aggregate Results

Model	All Options	Call Options	Put Options
State Price	1.599	1.475	1.731
Black-Scholes	1.712	1.590	1.843
Stutzer	6.376	6.053	6.719

B: Call Options

Moneyness	Model	Total	T ≤10	11 ≤T ≤20	21 ≤T ≤30	31 ≤T ≤40	41 ≤T ≤50	51 ≤T ≤60	61 ≤T ≤70	71 ≤T ≤80	81 ≤T ≤90	91 ≤T ≤100
Total	State Price	1.475	0.359	0.672	0.935	1.289	1.897	2.088	1.778	2.171	2.556	2.382
	Black-Scholes	1.590	0.374	0.676	0.987	1.414	2.066	2.251	1.853	2.333	2.715	2.781
	Stutzer	6.053	0.482	1.672	2.928	3.509	5.902	8.353	10.954	11.963	13.058	12.636
In-the-Money	State Price	1.063	0.264	0.483	0.760	0.831	0.620	1.587	1.396	1.729	2.501	2.969
(-0.1 ≤X/S-1 ≤-0.05)	Black-Scholes	1.068	0.291	0.504	0.843	0.618	0.749	1.663	1.493	1.640	2.540	2.868
	Stutzer	2.994	0.259	0.719	0.834	1.608	3.417	4.601	6.511	7.645	5.786	6.660
At-the-Money	State Price	1.453	0.425	0.798	1.053	1.459	2.290	1.919	1.565	1.910	2.100	1.694
(-0.05 ≤X/S-1 ≤0.05)	Black-Scholes	1.567	0.432	0.791	1.084	1.657	2.472	2.046	1.616	2.076	2.275	2.054
	Stutzer	7.778	0.639	2.305	4.268	4.800	7.634	10.627	13.746	14.463	16.024	16.157
Out-of-the-Money	State Price	3.393	0.157	0.364	0.656	1.584	3.607	4.826	4.133	4.832	6.133	4.551
(0.05 ≤X/S-1 ≤0.1)	Black-Scholes	3.973	0.197	0.398	0.753	2.257	3.814	5.508	4.294	5.608	6.529	6.805
	Stutzer	4.526	0.031	0.142	0.462	1.071	2.005	3.956	6.943	8.438	12.732	11.152

C: Put Options

Moneyness	Model	Total	T ≤10	11 ≤T ≤20	21 ≤T ≤30	31 ≤T ≤40	41 ≤T ≤50	51 ≤T ≤60	61 ≤T ≤70	71 ≤T ≤80	81 ≤T ≤90	91 ≤T ≤100
Total	State Price	1.731	0.292	0.665	1.055	1.572	2.149	2.657	2.446	2.586	3.593	3.122
	Black-Scholes	1.843	0.311	0.710	1.129	1.666	2.254	2.861	2.598	2.710	3.795	3.421
	Stutzer	6.719	0.511	1.659	3.719	3.955	5.733	9.954	11.328	13.023	18.304	17.420
In-the-Money	State Price	2.701	0.321	0.722	1.127	1.842	3.386	4.692	4.291	4.696	5.953	6.174
(0.05 ≤X/S-1 ≤0.1)	Black-Scholes	2.927	0.328	0.778	1.242	1.991	3.593	5.116	4.672	4.983	6.455	6.997
	Stutzer	4.439	0.342	0.757	1.449	1.502	2.690	5.166	7.930	10.496	15.462	18.545
At-the-Money	State Price	1.700	0.325	0.798	1.296	1.826	2.180	2.480	2.377	2.256	3.268	2.307
(-0.05 ≤X/S-1 ≤0.05)	Black-Scholes	1.803	0.351	0.853	1.381	1.925	2.281	2.656	2.514	2.364	3.427	2.533
	Stutzer	8.383	0.654	2.315	5.197	5.472	7.558	12.600	14.119	15.534	21.602	20.564
Out-of-the-Money	State Price	0.576	0.058	0.127	0.177	0.254	0.582	0.626	0.949	1.141	1.610	2.033
(-0.1 ≤X/S-1 ≤-0.05)	Black-Scholes	0.568	0.057	0.125	0.175	0.249	0.578	0.643	0.933	1.116	1.570	1.969
	Stutzer	3.357	0.065	0.352	1.150	1.794	3.005	5.597	6.888	6.366	7.744	7.881

Table 2.2 Comparison of Mean Squared Outside Errors on S&P 500 Index Options Valued by Alternative Option Pricing Models where the VIX proxies for expected volatility

Mean squared outside error is measured as the squared deviation of model values outside of the bid-ask spread averaged across all days in the sample period 3 January 1990 through 29 December 1993. Implied volatility for Black-Scholes and state price estimation is proxied by the previous day's VIX index value. Moneyness is defined as the ratio of the exercise to the index value less one. Moneyness categories are based on those of Dumas et al (1998).

A: Aggregate Results

Model	All Options	Call Options	Put Options
State Price	1.165	1.074	1.262
Black-Scholes	1.262	1.175	1.354
Stutzer	5.464	5.126	5.823

B: Call Options

Moneyness	Model	Total	T ≤10	11 ≤T ≤20	21 ≤T ≤30	31 ≤T ≤40	41 ≤T ≤50	51 ≤T ≤60	61 ≤T ≤70	71 ≤T ≤80	81 ≤T ≤90	91 ≤T ≤100
Total	State Price	1.074	0.209	0.453	0.640	0.962	1.513	1.552	1.268	1.578	1.892	1.653
	Black-Scholes	1.175	0.215	0.452	0.673	1.086	1.660	1.697	1.325	1.729	2.030	2.012
	Stutzer	5.126	0.314	1.297	2.430	2.892	4.938	7.087	9.367	10.266	11.378	10.866
In-the-Money	State Price	0.511	0.069	0.178	0.283	0.395	0.223	0.788	0.643	0.892	1.407	1.756
(-0.1 ≤X/S-1 ≤-0.05)	Black-Scholes	0.508	0.079	0.183	0.323	0.227	0.301	0.837	0.711	0.845	1.449	1.696
	Stutzer	2.006	0.068	0.322	0.370	0.919	2.171	3.116	4.621	5.598	4.110	4.805
At-the-Money	State Price	1.123	0.301	0.615	0.834	1.163	1.906	1.485	1.167	1.418	1.599	1.191
(-0.05 ≤X/S-1 ≤0.05)	Black-Scholes	1.222	0.305	0.608	0.859	1.342	2.078	1.600	1.203	1.556	1.744	1.483
	Stutzer	6.815	0.481	1.918	3.710	4.142	6.708	9.327	12.129	12.737	14.234	14.268
Out-of-the-Money	State Price	3.037	0.131	0.323	0.583	1.393	3.270	4.375	3.651	4.274	5.529	4.055
(0.05 ≤X/S-1 ≤0.1)	Black-Scholes	3.587	0.169	0.357	0.675	2.034	3.469	5.030	3.804	5.005	5.904	6.167
	Stutzer	4.162	0.019	0.115	0.409	0.932	1.785	3.598	6.402	7.812	11.868	10.359

C: Put Options

Moneyness	Model	Total	T ≤10	11 ≤T ≤20	21 ≤T ≤30	31 ≤T ≤40	41 ≤T ≤50	51 ≤T ≤60	61 ≤T ≤70	71 ≤T ≤80	81 ≤T ≤90	91 ≤T ≤100
Total	State Price	1.262	0.175	0.442	0.736	1.117	1.602	1.984	1.792	1.870	2.737	2.358
	Black-Scholes	1.354	0.190	0.476	0.792	1.191	1.687	2.161	1.919	1.972	2.909	2.601
	Stutzer	5.823	0.363	1.321	3.169	3.308	4.907	8.662	9.882	11.404	16.233	15.469
In-the-Money	State Price	1.830	0.129	0.339	0.591	1.146	2.338	3.338	3.005	3.267	4.279	4.488
(0.05 ≤X/S-1 ≤0.1)	Black-Scholes	2.012	0.133	0.375	0.662	1.254	2.495	3.698	3.320	3.501	4.705	5.176
	Stutzer	3.330	0.134	0.331	0.783	0.862	1.729	3.681	5.995	8.326	12.666	15.530
At-the-Money	State Price	1.279	0.218	0.574	0.962	1.358	1.684	1.905	1.770	1.652	2.554	1.758
(-0.05 ≤X/S-1 ≤0.05)	Black-Scholes	1.365	0.238	0.618	1.032	1.442	1.771	2.060	1.886	1.741	2.690	1.935
	Stutzer	7.407	0.504	1.927	4.554	4.699	6.618	11.178	12.512	13.827	19.452	18.567
Out-of-the-Money	State Price	0.452	0.038	0.091	0.130	0.184	0.447	0.475	0.744	0.910	1.312	1.681
(-0.1 ≤X/S-1 ≤-0.05)	Black-Scholes	0.445	0.037	0.090	0.128	0.180	0.444	0.492	0.730	0.889	1.275	1.623
	Stutzer	3.051	0.047	0.295	1.029	1.600	2.729	5.096	6.324	5.814	7.077	7.187

Table 2.3 Comparison of Mean Outside Errors on S&P 500 Index Options Valued by Alternative Option Pricing Models where the VIX proxies for expected volatility

Mean outside error is measured as the deviation of model values from outside the bid-ask spread averaged across all days in the sample period 3 January 1990 through 29 December 1993. Implied volatility for Black-Scholes and state price estimation is proxied by the previous day's VIX index value. Moneyness is defined as the ratio of the exercise to the index value less one. Moneyness categories are based on those of Dumas et al (1998).

A: Aggregate Results

Model	All Options	Call Options	Put Options
State Price	0.428	0.321	0.543
Black-Scholes	0.455	0.336	0.582
Stutzer	1.546	1.492	1.605

B: Call Options

Moneyness	Model	Total	T ≤10	11 ≤T ≤20	21 ≤T ≤30	31 ≤T ≤40	41 ≤T ≤50	51 ≤T ≤60	61 ≤T ≤70	71 ≤T ≤80	81 ≤T ≤90	91 ≤T ≤100
Total	State Price	0.321	0.131	0.264	0.233	0.451	0.637	0.383	0.323	0.390	0.264	-0.039
	Black-Scholes	0.336	0.103	0.239	0.215	0.528	0.643	0.389	0.311	0.440	0.288	0.081
	Stutzer	1.492	0.224	0.724	1.053	1.117	1.609	1.882	2.542	2.536	2.773	2.568
In-the-Money	State Price	-0.440	-0.048	-0.148	-0.379	-0.374	-0.246	-0.662	-0.676	-0.751	-1.023	-1.154
(-0.1 ≤X/S-1 ≤-0.05)	Black-Scholes	-0.441	-0.081	-0.180	-0.413	-0.298	-0.244	-0.678	-0.712	-0.710	-1.025	-1.091
	Stutzer	0.754	0.013	0.229	0.289	0.479	0.968	1.067	1.773	1.734	1.513	1.642
At-the-Money	State Price	0.568	0.243	0.485	0.525	0.753	0.941	0.632	0.563	0.629	0.525	0.264
(-0.05 ≤X/S-1 ≤0.05)	Black-Scholes	0.580	0.217	0.462	0.511	0.817	0.946	0.637	0.558	0.668	0.555	0.371
	Stutzer	1.879	0.366	1.038	1.518	1.502	2.000	2.345	2.962	2.925	3.220	3.081
Out-of-the-Money	State Price	1.457	0.264	0.380	0.589	1.016	1.575	1.851	1.736	1.925	2.175	1.783
(0.05 ≤X/S-1 ≤0.1)	Black-Scholes	1.559	0.295	0.395	0.623	1.179	1.604	1.937	1.755	2.056	2.231	2.159
	Stutzer	1.362	0.007	0.139	0.416	0.597	1.026	1.146	2.223	2.319	3.213	2.535

C: Put Options

Moneyness	Model	Total	T ≤10	11 ≤T ≤20	21 ≤T ≤30	31 ≤T ≤40	41 ≤T ≤50	51 ≤T ≤60	61 ≤T ≤70	71 ≤T ≤80	81 ≤T ≤90	91 ≤T ≤100
Total	State Price	0.543	0.102	0.274	0.480	0.651	0.689	0.806	0.648	0.734	0.824	0.545
	Black-Scholes	0.582	0.127	0.303	0.513	0.687	0.724	0.854	0.692	0.775	0.875	0.634
	Stutzer	1.605	0.251	0.741	1.346	1.184	1.490	2.196	2.707	2.663	3.512	3.189
In-the-Money	State Price	0.904	0.014	0.241	0.499	0.780	1.154	1.479	1.456	1.611	1.810	1.922
(0.05 ≤X/S-1 ≤0.1)	Black-Scholes	0.966	0.045	0.286	0.550	0.836	1.211	1.556	1.538	1.680	1.903	2.078
	Stutzer	1.062	0.042	0.228	0.528	0.522	0.856	1.429	2.024	2.299	3.156	3.393
At-the-Money	State Price	0.691	0.183	0.434	0.713	0.874	0.859	0.912	0.914	0.840	0.908	0.642
(-0.05 ≤X/S-1 ≤0.05)	Black-Scholes	0.729	0.209	0.465	0.749	0.910	0.894	0.957	0.957	0.879	0.954	0.729
	Stutzer	1.924	0.362	1.014	1.765	1.552	1.867	2.588	3.087	3.029	3.960	3.498
Out-of-the-Money	State Price	-0.497	-0.162	-0.258	-0.312	-0.354	-0.446	-0.539	-0.807	-0.830	-0.984	-1.176
(-0.1 ≤X/S-1 ≤-0.05)	Black-Scholes	-0.488	-0.161	-0.255	-0.308	-0.347	-0.436	-0.520	-0.797	-0.817	-0.969	-1.155
	Stutzer	1.097	0.058	0.354	0.799	0.750	0.936	1.626	2.302	1.689	2.035	2.153

Table 2.4 Comparison of Mean Squared Errors on S&P 500 Index Options Valued by Alternative Option Pricing Models where 40-day historical volatility proxies for expected volatility

Mean squared error is measured as the squared deviation of model values from the bid-ask midpoint averaged across all days in the sample period 3 January 1990 through 29 December 1993. Implied volatility for Black-Scholes and state price estimation is proxied by 40-day historical volatility of daily returns on the S&P 500 index. Moneyness is defined as the ratio of the exercise to the index value less one. Moneyness categories are based on those of Dumas et al (1998).

A: Aggregate Results

Model	All Options	Call Options	Put Options
State Price	3.151	3.545	2.732
Black-Scholes	3.199	3.681	2.686
Stutzer	6.376	6.053	6.719

B: Call Options

Moneyness	Model	Total	T ≤10	11 ≤T ≤20	21 ≤T ≤30	31 ≤T ≤40	41 ≤T ≤50	51 ≤T ≤60	61 ≤T ≤70	71 ≤T ≤80	81 ≤T ≤90	91 ≤T ≤100
Total	State Price	3.545	0.337	0.621	1.372	2.134	2.759	5.058	5.800	7.151	7.801	9.501
	Black-Scholes	3.681	0.374	0.681	1.481	2.242	2.899	5.250	6.011	7.357	8.027	9.711
	Stutzer	6.053	0.482	1.672	2.928	3.509	5.902	8.353	10.954	11.963	13.058	12.636
In-the-Money	State Price	3.367	0.280	0.640	1.242	2.099	2.655	5.637	5.220	7.320	8.560	9.716
(-0.1 ≤X/S-1 ≤-0.05)	Black-Scholes	3.550	0.310	0.706	1.386	2.250	2.856	5.936	5.516	7.614	8.915	10.046
	Stutzer	2.994	0.259	0.719	0.834	1.608	3.417	4.601	6.511	7.645	5.786	6.660
At-the-Money	State Price	3.990	0.382	0.650	1.566	2.437	3.133	5.364	6.941	8.172	8.446	10.859
(-0.05 ≤X/S-1 ≤0.05)	Black-Scholes	4.120	0.423	0.711	1.666	2.541	3.263	5.536	7.146	8.373	8.648	11.046
	Stutzer	7.778	0.639	2.305	4.268	4.800	7.634	10.627	13.746	14.463	16.024	16.157
Out-of-the-Money	State Price	0.576	0.034	0.030	0.018	0.338	0.347	1.115	0.643	0.759	0.708	0.762
(0.05 ≤X/S-1 ≤0.1)	Black-Scholes	0.578	0.034	0.031	0.018	0.345	0.348	1.121	0.652	0.763	0.720	0.717
	Stutzer	4.526	0.031	0.142	0.462	1.071	2.005	3.956	6.943	8.438	12.732	11.152

C: Put Options

Moneyness	Model	Total	T ≤10	11 ≤T ≤20	21 ≤T ≤30	31 ≤T ≤40	41 ≤T ≤50	51 ≤T ≤60	61 ≤T ≤70	71 ≤T ≤80	81 ≤T ≤90	91 ≤T ≤100
Total	State Price	2.732	0.323	0.684	0.826	1.881	3.152	3.639	4.516	6.612	5.031	7.623
	Black-Scholes	2.686	0.311	0.670	0.823	1.844	3.093	3.583	4.432	6.500	4.954	7.502
	Stutzer	6.719	0.511	1.659	3.719	3.955	5.733	9.954	11.328	13.023	18.304	17.420
In-the-Money	State Price	0.638	0.317	0.599	0.520	0.377	0.697	1.010	0.470	1.056	0.845	1.039
(0.05 ≤X/S-1 ≤0.1)	Black-Scholes	0.636	0.314	0.610	0.568	0.373	0.689	1.018	0.429	1.006	0.854	1.005
	Stutzer	4.439	0.342	0.757	1.449	1.502	2.690	5.166	7.930	10.496	15.462	18.545
At-the-Money	State Price	3.299	0.363	0.798	0.979	2.463	3.851	4.246	5.843	8.005	5.862	9.740
(-0.05 ≤X/S-1 ≤0.05)	Black-Scholes	3.230	0.345	0.773	0.960	2.406	3.763	4.160	5.720	7.853	5.747	9.552
	Stutzer	8.383	0.654	2.315	5.197	5.472	7.558	12.600	14.119	15.534	21.602	20.564
Out-of-the-Money	State Price	3.316	0.121	0.372	0.633	1.840	3.627	4.714	4.619	8.342	7.409	9.023
(-0.1 ≤X/S-1 ≤-0.05)	Black-Scholes	3.303	0.121	0.371	0.631	1.831	3.611	4.696	4.602	8.310	7.376	8.986
	Stutzer	3.357	0.065	0.352	1.150	1.794	3.005	5.597	6.888	6.366	7.744	7.881

Table 2.5 Comparison of Mean Squared Outside Errors on S&P 500 Index Options Valued by Alternative Option Pricing Models where 40-day historical volatility proxies for expected volatility

Mean squared outside error is measured as the squared deviation of model values outside of the bid-ask spread averaged across all days in the sample period 3 January 1990 through 29 December 1993. Implied volatility for Black-Scholes and state price estimation is proxied by 40-day historical volatility of daily returns on the S&P 500 index. Moneyness is defined as the ratio of the exercise to the index value less one. Moneyness categories are based on those of Dumas et al (1998).

A: Aggregate Results

Model	All Options	Call Options	Put Options
State Price	2.532	2.748	2.303
Black-Scholes	2.570	2.860	2.262
Stutzer	5.464	5.126	5.823

B: Call Options

Moneyness	Model	Total	T ≤10	11 ≤T ≤20	21 ≤T ≤30	31 ≤T ≤40	41 ≤T ≤50	51 ≤T ≤60	61 ≤T ≤70	71 ≤T ≤80	81 ≤T ≤90	91 ≤T ≤100
Total	State Price	2.748	0.173	0.368	0.919	1.570	2.046	3.963	4.602	5.718	6.272	7.710
	Black-Scholes	2.860	0.195	0.410	1.000	1.655	2.159	4.125	4.783	5.897	6.471	7.896
	Stutzer	5.126	0.314	1.297	2.430	2.892	4.938	7.087	9.367	10.266	11.378	10.866
In-the-Money	State Price	2.290	0.078	0.284	0.597	1.293	1.620	3.962	3.560	5.311	6.280	7.214
(-0.1 ≤X/S-1 ≤-0.05)	Black-Scholes	2.428	0.090	0.323	0.686	1.396	1.763	4.199	3.797	5.554	6.579	7.495
	Stutzer	2.006	0.068	0.322	0.370	0.919	2.171	3.116	4.621	5.598	4.110	4.805
At-the-Money	State Price	3.253	0.239	0.438	1.174	1.908	2.494	4.393	5.788	6.780	7.039	9.189
(-0.05 ≤X/S-1 ≤0.05)	Black-Scholes	3.365	0.267	0.485	1.258	1.997	2.606	4.545	5.971	6.961	7.223	9.362
	Stutzer	6.815	0.481	1.918	3.710	4.142	6.708	9.327	12.129	12.737	14.234	14.268
Out-of-the-Money	State Price	0.456	0.022	0.020	0.009	0.278	0.258	0.945	0.496	0.562	0.524	0.558
(0.05 ≤X/S-1 ≤0.1)	Black-Scholes	0.458	0.022	0.020	0.009	0.285	0.259	0.950	0.504	0.565	0.533	0.522
	Stutzer	4.162	0.019	0.115	0.409	0.932	1.785	3.598	6.402	7.812	11.868	10.359

C: Put Options

Moneyness	Model	Total	T ≤10	11 ≤T ≤20	21 ≤T ≤30	31 ≤T ≤40	41 ≤T ≤50	51 ≤T ≤60	61 ≤T ≤70	71 ≤T ≤80	81 ≤T ≤90	91 ≤T ≤100
Total	State Price	2.303	0.214	0.490	0.637	1.540	2.661	3.071	3.892	5.727	4.282	6.671
	Black-Scholes	2.262	0.205	0.478	0.628	1.510	2.611	3.020	3.820	5.630	4.213	6.562
	Stutzer	5.823	0.363	1.321	3.169	3.308	4.907	8.662	9.882	11.404	16.233	15.469
In-the-Money	State Price	0.311	0.123	0.243	0.246	0.148	0.348	0.552	0.215	0.592	0.411	0.518
(0.05 ≤X/S-1 ≤0.1)	Black-Scholes	0.307	0.122	0.250	0.265	0.148	0.342	0.552	0.188	0.558	0.420	0.489
	Stutzer	3.330	0.134	0.331	0.783	0.862	1.729	3.681	5.995	8.326	12.666	15.530
At-the-Money	State Price	2.809	0.270	0.619	0.788	2.047	3.285	3.616	5.042	6.933	4.992	8.554
(-0.05 ≤X/S-1 ≤0.05)	Black-Scholes	2.748	0.256	0.599	0.769	1.999	3.210	3.540	4.935	6.798	4.890	8.385
	Stutzer	7.407	0.504	1.927	4.554	4.699	6.618	11.178	12.512	13.827	19.452	18.567
Out-of-the-Money	State Price	2.990	0.092	0.309	0.539	1.622	3.231	4.254	4.139	7.645	6.740	8.231
(-0.1 ≤X/S-1 ≤-0.05)	Black-Scholes	2.977	0.092	0.308	0.537	1.613	3.216	4.236	4.123	7.615	6.708	8.195
	Stutzer	3.051	0.047	0.295	1.029	1.600	2.729	5.096	6.324	5.814	7.077	7.187

Table 2.6 Comparison of Mean Outside Errors on S&P 500 Index Options Valued by Alternative Option Pricing Models where 40-day historical volatility proxies for expected volatility

Mean outside error is measured as the deviation of model values from outside the bid-ask spread averaged across all days in the sample period 3 January 1990 through 29 December 1993. Implied volatility for Black-Scholes and state price estimation is proxied by 40-day historical volatility of daily returns on the S&P 500 index. Moneyness is defined as the ratio of the exercise to the index value less one. Moneyness categories are based on those of Dumas et al (1998).

A: Aggregate Results

Model	All Options	Call Options	Put Options
State Price	-1.022	-1.129	-0.908
Black-Scholes	-1.029	-1.168	-0.881
Stutzer	1.546	1.492	1.605

B: Call Options

Moneyness	Model	Total	T ≤10	11 ≤T ≤20	21 ≤T ≤30	31 ≤T ≤40	41 ≤T ≤50	51 ≤T ≤60	61 ≤T ≤70	71 ≤T ≤80	81 ≤T ≤90	91 ≤T ≤100
Total	State Price	-1.129	-0.182	-0.354	-0.685	-0.856	-1.054	-1.505	-1.790	-2.024	-2.135	-2.422
	Black-Scholes	-1.168	-0.218	-0.391	-0.729	-0.892	-1.096	-1.549	-1.834	-2.061	-2.173	-2.453
	Stutzer	1.492	0.224	0.724	1.053	1.117	1.609	1.882	2.542	2.536	2.773	2.568
In-the-Money	State Price	-1.054	-0.062	-0.249	-0.581	-0.857	-1.014	-1.663	-1.724	-2.079	-2.368	-2.590
(-0.1 ≤X/S-1 ≤-0.05)	Black-Scholes	-1.110	-0.099	-0.292	-0.643	-0.913	-1.078	-1.733	-1.793	-2.138	-2.430	-2.646
	Stutzer	0.754	0.013	0.229	0.289	0.479	0.968	1.067	1.773	1.734	1.513	1.642
At-the-Money	State Price	-1.270	-0.261	-0.432	-0.803	-0.969	-1.202	-1.608	-2.048	-2.263	-2.270	-2.665
(-0.05 ≤X/S-1 ≤0.05)	Black-Scholes	-1.306	-0.297	-0.468	-0.841	-1.001	-1.239	-1.646	-2.086	-2.296	-2.302	-2.691
	Stutzer	1.879	0.366	1.038	1.518	1.502	2.000	2.345	2.962	2.925	3.220	3.081
Out-of-the-Money	State Price	-0.262	-0.107	-0.063	-0.008	-0.143	-0.094	-0.295	-0.438	-0.487	-0.433	-0.437
(0.05 ≤X/S-1 ≤0.1)	Black-Scholes	-0.264	-0.108	-0.065	-0.010	-0.141	-0.099	-0.302	-0.443	-0.486	-0.437	-0.419
	Stutzer	1.362	0.007	0.139	0.416	0.597	1.026	1.146	2.223	2.319	3.213	2.535

C: Put Options

Moneyness	Model	Total	T ≤10	11 ≤T ≤20	21 ≤T ≤30	31 ≤T ≤40	41 ≤T ≤50	51 ≤T ≤60	61 ≤T ≤70	71 ≤T ≤80	81 ≤T ≤90	91 ≤T ≤100
Total	State Price	-0.908	-0.225	-0.374	-0.442	-0.752	-1.116	-1.144	-1.476	-1.845	-1.446	-2.005
	Black-Scholes	-0.881	-0.201	-0.348	-0.419	-0.725	-1.087	-1.115	-1.450	-1.814	-1.414	-1.971
	Stutzer	1.605	0.251	0.741	1.346	1.184	1.490	2.196	2.707	2.663	3.512	3.189
In-the-Money	State Price	-0.046	-0.022	0.024	0.110	-0.016	-0.064	-0.025	-0.218	-0.292	0.040	-0.216
(0.05 ≤X/S-1 ≤0.1)	Black-Scholes	-0.008	0.011	0.070	0.145	0.014	-0.023	0.014	-0.187	-0.250	0.079	-0.162
	Stutzer	1.062	0.042	0.228	0.528	0.522	0.856	1.429	2.024	2.299	3.156	3.393
At-the-Money	State Price	-1.094	-0.298	-0.479	-0.560	-0.942	-1.339	-1.362	-1.749	-2.184	-1.720	-2.453
(-0.05 ≤X/S-1 ≤0.05)	Black-Scholes	-1.064	-0.273	-0.453	-0.534	-0.911	-1.307	-1.331	-1.716	-2.150	-1.685	-2.418
	Stutzer	1.924	0.362	1.014	1.765	1.552	1.867	2.588	3.087	3.029	3.960	3.498
Out-of-the-Money	State Price	-1.330	-0.229	-0.451	-0.622	-1.082	-1.585	-1.766	-1.888	-2.526	-2.383	-2.720
(-0.1 ≤X/S-1 ≤-0.05)	Black-Scholes	-1.327	-0.229	-0.450	-0.621	-1.079	-1.581	-1.762	-1.884	-2.521	-2.377	-2.713
	Stutzer	1.097	0.058	0.354	0.799	0.750	0.936	1.626	2.302	1.689	2.035	2.153

Figure 2.1 Scatter Plots of Call Option Outside Errors by Moneyness

Outside error is measured as the difference between the estimated model value and the option's ask (bid) price if the model value is greater (less) than the ask (bid) price. Scatter plots on the left are outside errors on state price and Black-Scholes call option values where expected volatility is proxied by the previous day's VIX index value. Scatter plots on the right are outside errors on state price and Black-Scholes call option values where expected volatility is proxied by 40-day historical volatility of daily returns on the S&P 500 index. The plot below contains outside errors on canonical call option values.

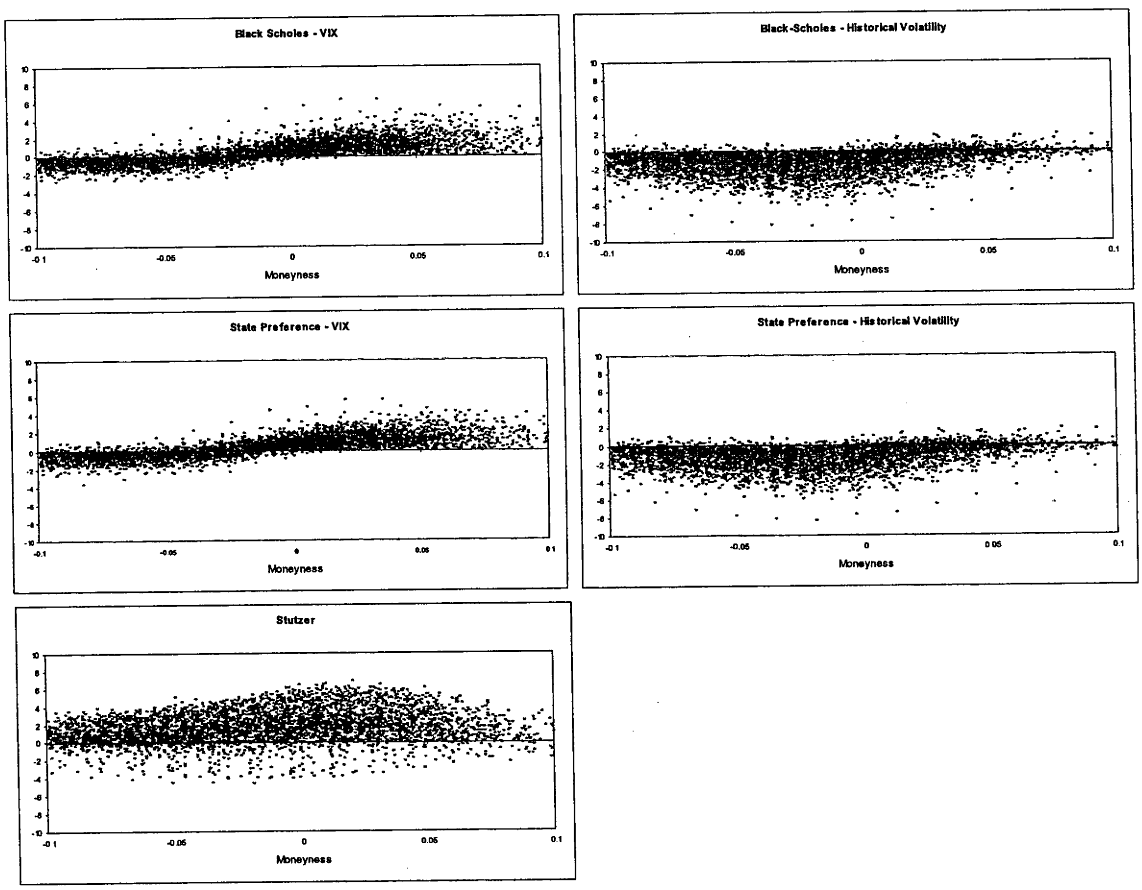


Figure 2.2 Scatter Plots of Put Option Outside Errors by Moneyness

Outside error is measured as the difference between the estimated model value and the option's ask (bid) price if the model value is greater (less) than the ask (bid) price. Scatter plots on the left are outside errors on state price and Black-Scholes put option values where expected volatility is proxied by the previous day's VIX index value. Scatter plots on the right are outside errors on state price and Black-Scholes put option values where expected volatility is proxied by 40-day historical volatility of daily returns on the S&P 500 index. The plot below contains outside errors on canonical put option values.

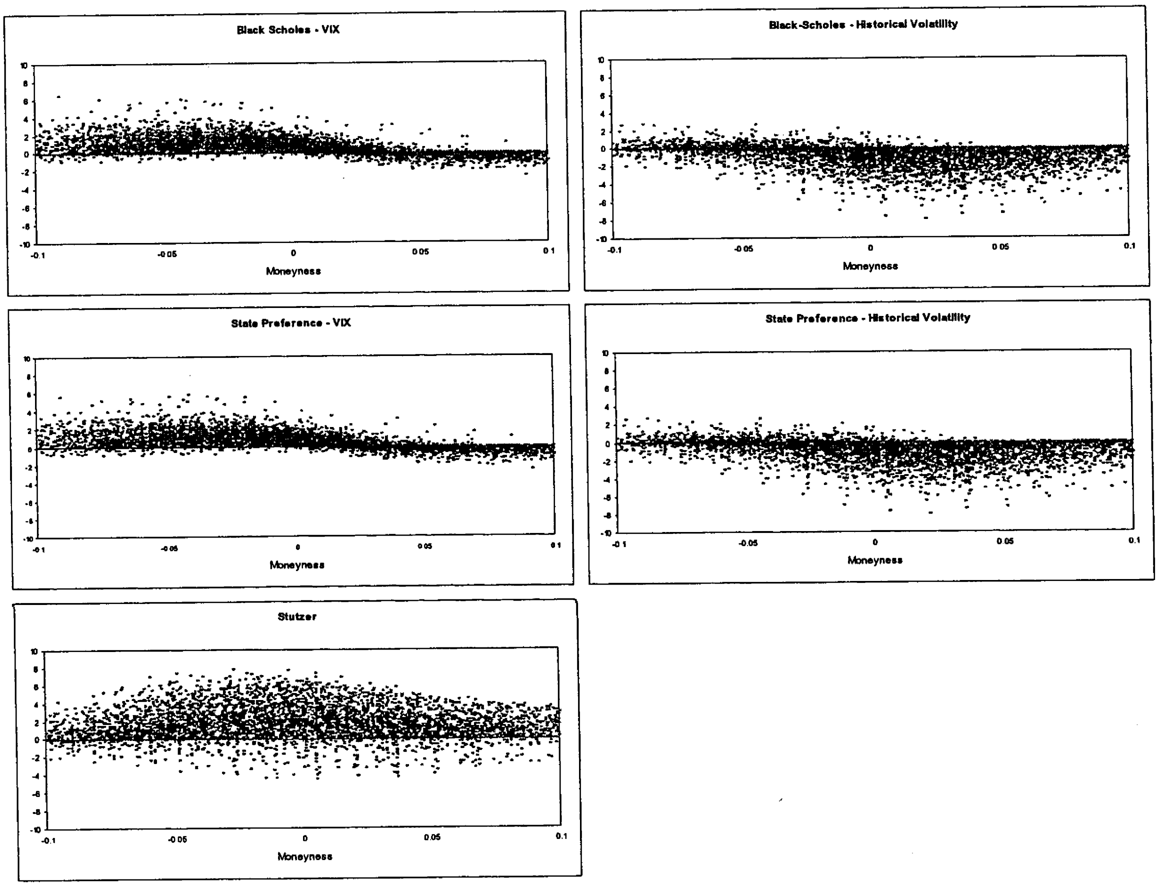


Figure 2.3 Histogram of state price and canonical risk-neutral probabilities for a call option on the S&P 500 index with T=24 days to expiry on 27 May 1992.

The solid line represents the probability distribution associated with state prices where the previous day's closing value of the VIX index was used as the proxy for implied volatility. The dashed line is where state prices are estimated with historical volatility. Input parameters are VIX=14.92, σ=11.97, r_f=3.65 and PVD=0.7. The value of the S&P 500 index was 412.17.

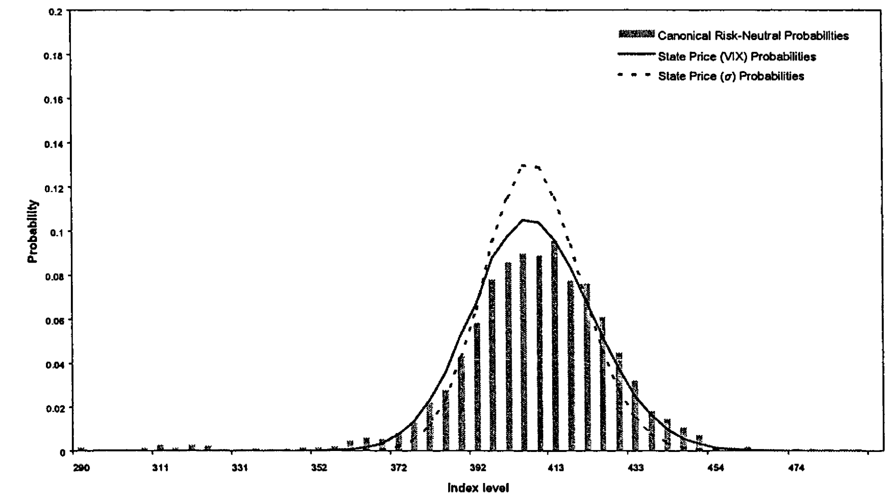


Figure 2.4 Histogram of state price and canonical risk-neutral probabilities for call option on the S&P 500 index with T=52 days to expiry on 27 May 1992.

The solid line represents the probability distribution associated with state prices where the previous day's closing value of the VIX index was used as the proxy for implied volatility. The dashed line is where state prices are estimated with historical volatility. Input parameters are VIX=14.92, σ=11.97, r_f=3.69 and PVD=1.41. The value of the S&P 500 index was 412.17.

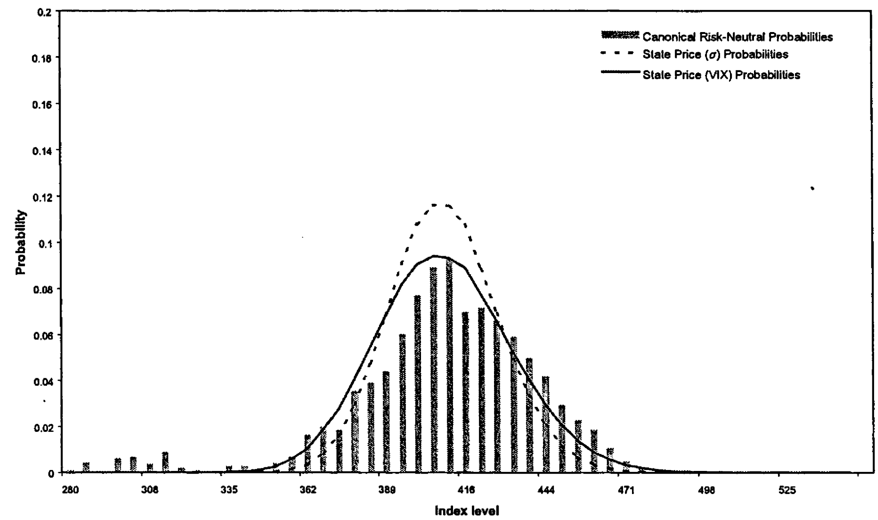
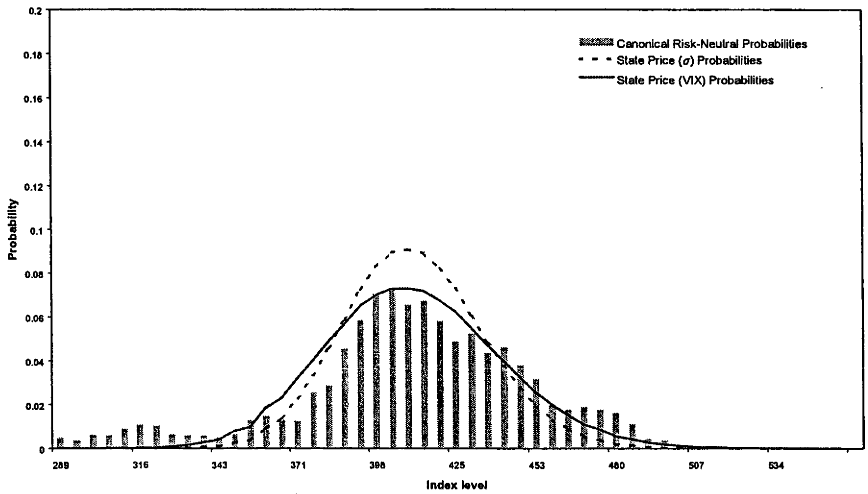


Figure 2.5 Histogram of State Price and canonical risk-neutral probabilities for a call option on the S&P 500 index with T=87 days to expiry on 27 May 1992.

The solid line represents the probability distribution associated with state prices where the previous day's closing value of the VIX index was used as the proxy for implied volatility. The dashed line is where state prices are estimated with historical volatility. Input parameters are VIX=14.92, σ=11.97, r_f=3.74 and PVD=2.82. The value of the S&P 500 index was 412.17.



Chapter 3

A State-Contingent Claim Approach to Equity Valuation

3.1 Introduction

The price of any asset is widely understood to be a function of its expected future payoffs, as expressed in the general form of the asset pricing equation:

$$P_{i0} = E\left(M_t X_{it}\right) \qquad \forall i,t \tag{21}$$

where P_{it} is the price of asset i in the current period, time 0, E is an expectations operator conditioning on information available at time t, X_{it} is the payoff of asset i at time t, and M_t is the stochastic discount factor.

While this general expression provides a simple representation for the price of an asset, there are a number of issues, both theoretical and practical, to be resolved for its application. These range from the seemingly simple – how to determine asset payoff and what time period does t refer to; to the economically complex – how to determine investor expectations and what form the stochastic discount factor should take. These questions have received significant attention in the literature, and have lead to the development of numerous alternative specifications of equation (21).

In this thesis the question of the stochastic discount factor is addressed. In a world of uncertainty the stochastic discount factor represents investors' consumption preferences across both time and uncertain outcomes, or states of nature. In a two period context, the stochastic discount factor will reflect the marginal rate of substitution between consumption in the current state and time, and state s in the next period, for the set of S possible outcomes in the next period. The stochastic discount factor may be represented by the set of state-contingent claim prices. The price of a state-contingent claim, or state price, is the price today of one unit of consumption in state s, time t. Following Ross's (1976) observation that in a complete market simple options will span the state space, and therefore state prices will be implicit in the price of any exchange-traded asset, Breeden and Litzenberger (1978) and Banz and Miller (1978) determined that the price of a state-contingent claim may be obtained from the second derivative of a call option price. The Black-Scholes (1973) option pricing formula provides a closed form solution.

In the previous chapter the state preference approach was successfully applied to pricing S&P 500 index options. State prices were calculated on the S&P 500 index and applied to possible payoffs on the option to determine option price. This was shown to provide an overall improvement on alternative option pricing methods, including the Black-Scholes formula and Stutzer's (1996) canonical valuation approach. However the general form of the asset pricing equation (21) provides that any asset may be priced in terms of its payoff and the stochastic discount factor. This implies that the state preference approach should be applicable to pricing exchange-traded financial assets more generally. Indeed, the state prices on the S&P 500 index calculated in the previous chapter may be used to value any asset where the payoff is dependant on the index.

In this chapter the state preference approach is applied to valuing equities. The state prices determined on the S&P 500 index in the previous chapter are applied to valuing stocks traded on US exchanges over the period December 1996 through July 2004. A similar test approach to the previous chapter is adopted. The accuracy of values obtained under the state preference approach is assessed relative to values determined using alternative equity valuation techniques; specifically, the residual income model popularised in the accounting literature (see for example Ohlson (1995), Feltham and Ohlson (1995) and Myers (1999)). The residual income model respecifies equation (21) in terms of accounting variables, where stock price is expressed as a function of book value per share and contemporaneous and expected earnings per share.

In the previous chapter S&P 500 index options were also valued using Stutzer's canonical valuation approach, which provides a non-parametric alternative to the state preference approach. Risk-neutral probabilities are obtained from the historical distribution of returns on the underlying asset and applied to possible future payoffs to determine option price. In the same sense that state prices may be used to value any asset, Stutzer's risk neutral probabilities should be applicable to the valuation of an asset whose payoff is a function of the index. As such, the canonical valuation approach is also applied to pricing equities in this chapter, and is shown to provide a similar level of accuracy to the state preference approach. Finally, for completeness, equities are valued with the Sharpe (1964) and Lintner (1965) Capital Asset Pricing Model (CAPM). Perhaps the most widely understood of the asset pricing formulae, the

CAPM expresses stock return as a function of the risk free rate and the return on the market portfolio.

This chapter is organised as follows. Section 3.2 reviews alternative approaches to equity valuation, and provides a derivation of the state preference approach, the residual income model and Stutzer's canonical valuation. The CAPM is briefly reviewed. Section 3.3 describes the data sources and sampling technique, Section 3.4 discusses the results and Section 3.5 concludes.

3.2 Equity Valuation

This section outlines the alternative equity valuation approaches considered in this chapter. First, the state preference approach is detailed, and a function for valuing equities is provided. Second, earnings capitalisation models are discussed, with the focus on Ohlson's (1995) residual income model. Stutzer's (1996) canonical valuation approach is then reviewed, and shown to be applicable to valuing equities as well as options. Finally, the Sharpe (1964) and Lintner (1965) Capital Asset Pricing Model (CAPM), and subsequent related literature, is briefly reviewed.

3.2.1. The state preference approach

Defining a state price as the individual's subjective probability assessment of the likelihood of a particular state s occurring, the price of a state-contingent claim will be given by investor k's true probability assessment of the occurrence of state s, π_{ts}^{k}, multiplied by the marginal rate of substitution between consumption in the current period, and consumption in state s, time t. This is determined by solving the representative investor's two-period constrained optimisation problem, providing the following expression for the state price at time t, state s:

$$\Phi_{ts}^{k} = \frac{\pi_{ts}^{k} U_{ts}'^{k}\left(C_{ts}^{k}\right)}{U_{0}'^{k}\left(C_{0}^{k}\right)} \tag{22}$$

In this respect a state price can be interpreted as the price today of one unit of consumption in state s, time t. Equation (22) indicates state prices will exhibit probability-like characteristics – they are non-negative and, in the next period, will sum to one – and therefore constitute a legitimate probability distribution. However, since state prices are implicit in exchange-traded financial assets, and therefore influenced by supply and demand, they will be subjective probabilities incorporating investor risk preferences. This implies that, relative to a true probability distribution, greater weight will be placed on outcomes with higher marginal utility of consumption, where wealth is more valuable to consumers. This provides a simple but powerful result: both outcomes that occur with greater frequency and outcomes that are improbable but undesirable attract a greater weighting, a result entirely consistent with our understanding of investor risk aversion.

Summing the set of state prices across the state space provides the price of an asset with a certain unit payoff in the next period, $\sum_{s=1}^{S} \Phi_{ts}^{k} = e^{-rt}$, where r is the risk-free rate of return. This is consistent with the conditional mean of the stochastic discount factor above[6]. The price of a risky asset *j* is expressed as the payoff in state *s* multiplied by the state price, summed over all possible *S* states:

$$P_0^j = \sum_{s=1}^{S} \Phi_{1s} d_{1s}^j \qquad \forall j = 1 \ldots N \tag{23}$$

Breeden and Litzenberger (1978) and Banz and Miller (1978) first established that the price of an elementary security may be modelled as the second derivative of a call option price. This proposition is based on the construction of a simple butterfly spread with a normalised unit payoff. If the current value of the asset is given by *M*, and ΔM is the step size between potential next period values, then the option portfolio replicating an elementary claim is created by purchasing two calls with strike prices $M - \Delta M$ and $M + \Delta M$ respectively, and selling two calls with a strike price of *M*. If the value of the underlying asset is *M* next period, then the payoff on the option portfolio will be ΔM, and zero otherwise. Dividing through by ΔM obtains a unit payoff:

$$1 = \frac{\Delta M}{\Delta M} = \frac{C(M - \Delta M, T) - 2C(M, T) + C(M + \Delta M, T)}{\Delta M} \tag{24}$$

With a step size of ΔM between possible next period values, and if *M* occurs in *T* periods then the cost of the call portfolio is $P(M, T; \Delta M)$. Dividing through by ΔM obtains the price of the call portfolio paying one unit:

$$\frac{P(M, T; \Delta M)}{\Delta M} = \frac{C(M - \Delta M, T) - 2C(M, T) + C(M + \Delta M, T)}{\Delta M^2} \tag{25}$$

[6] The conditional moments of the stochastic discount factor are easily determined. The conditional mean of the stochastic discount factor is given by price today of a riskless real asset with a certain unit payoff next period: $P = E(\mathrm{M}_t) = e^{-r_f t}$.

Taking the limit as the step size tends to zero provides the price of an elementary security evaluated at $X=M$:

$$\lim_{\Delta M \to 0} \frac{P(M,T;\Delta M)}{\Delta M} = \left.\frac{\partial^2 C(X,T)}{\partial X^2}\right|_{X=M} \tag{26}$$

Aside from the assumption of complete markets, Breeden and Litzenberger's approach places few restrictions on the option price. It is not necessary for the option pricing function to be twice differentiable as a discrete solution may be obtained from equation (25), and no restriction is placed on the stochastic process governing the time series behaviour of asset returns. With some additional assumptions[7], evaluating the second derivative of the Black-Scholes (1973) call option pricing formula provides the following explicit closed form solution for the price of an elementary claim:

$$\left.\frac{\partial^2 C}{\partial X^2}\right|_{X=M_T} = \frac{e^{-rT}}{M_T \sigma \sqrt{T}} n\left[d_2\left(X = M_T\right)\right] \tag{27}$$

where

$$d_2 = \frac{\ln\left[(M_0 - PVD)/M_T\right] + \left[r - 1/2\sigma^2\right]T}{\sigma\sqrt{T}}$$

M_T is the value of the underlying asset in T periods, M_0 its value today, PVD is the present value of dividends, and $n(d_2) = \frac{1}{\sqrt{2\pi}} e^{-d_2/2}$ is the standard normal probability density function.

[7] The assumptions of the Black-Scholes formula are well known. Namely, asset returns are assumed to follow geometric Brownian motion with constant volatility.

Describing state prices as the second derivative of a call option with respect to the strike price makes the assumption that increments in the value of the underlying stock ΔM tend to zero. However, in reality price movements will be greater than zero and it will not be possible to obtain a limiting value for ΔM. Evaluating equation (27) at discrete intervals will not capture the range of state prices between each possible level of the strike price, X. Breeden and Litzenberger's "delta security" method provides a solution to this problem. The price of a state-contingent claim with a unit payoff if the value of the underlying asset at time t is greater than or equal to a pre-specified level Y, and zero otherwise, will be simply the sum of the state prices for each asset value greater then Y. This is given by the cumulative pricing function:

$$G(Y) = \int_Y^\infty \frac{e^{-rT} n(d_2)}{X\sigma\sqrt{T}} dX = e^{-rT} N\left[d_2\left(X = Y\right)\right] \qquad (28)$$

The cost of a security with a unit payoff if the value of the underlying asset is between two predetermined levels, say Y_i and Y_{i+1}, will be the difference between the cumulative pricing function at these levels:

$$\phi\left(Y_i, Y_{i+1}\right) = e^{-rT}\left\{N\left[d_2\left(X = Y_i\right)\right] - N\left[d_2\left(X = Y_{i+1}\right)\right]\right\} \qquad (29)$$

This approach provides the flexibility to select suitable increments in the value of the underlying asset for Y_i and Y_{i+1}, allowing market microstructure aspects such as minimum price movements and price limits to be incorporated into the model. To illustrate, the minimum increment on the S&P 500 index is 0.01, so this is used for increments of Y_i. The maximum and minimum values of Y are obtained from the historical maximum and minimum monthly returns on the index over the period from July 1926 through to the valuation date. This not only reduces the computational burden, but places limits on the range of expected index values priced by investors. In comparison, the Black-Scholes option pricing formula will price very large and very small values of the index, albeit with a very low probability.

The return on the stock is calculated with reference to the index via the linear projection:

$$R_t^j = \alpha + \beta R_t^m + \varepsilon_t \qquad \forall j \tag{30}$$

where R^j is the return on asset j and R^m is the return on the S&P 500 index. This provides the following valuation expression for stock payoff:

$$d_t^j = P_0^j \exp\left(R_t^j\right) = P_0^j \exp\left(\alpha^j + \beta^j R_t^m\right) \tag{31}$$

where P_0^j is the current price of asset j. Stock values for the following period are then determined as:

$$P_t^j = \sum_{s=1}^{S} \phi_s d_s^j = \sum_{s=1}^{S} \phi_s \left[P_0^j \exp\left(\alpha^j + \beta^j R_s^m\right)\right] \tag{32}$$

where ϕ_s is the state price obtained from the S&P 500 index and P_t^j is next period's price.

As noted earlier, the performance of the state preference approach outlined in this section is assessed relative to alternative equity valuation techniques, including the residual income model, Stutzer's canonical valuation and the Sharpe-Lintner CAPM. These alternative equity valuation methods are briefly described in the following sub-sections.

3.2.2. Earnings capitalisation models

One of the earliest and most widely understood representations of the basic asset pricing equation (21) is the dividend discount model, where price is expressed as the present value of the expected dividend stream over the (indefinite) life of the firm:

$$P_t^j = \sum_{i=1}^{\infty} \frac{E_t\left(D_{t+i}^j\right)}{\left(1+r_e\right)^i} \tag{33}$$

Allowing for a number of simplifying assumptions[8], the dividend discount model is broadly consistent with the basic asset pricing expression of equation (21); however the use of dividends in valuation models presents a number of empirical problems. First, dividends are not the only means of distributing value to shareholders. Alternatives include share repurchases, acquisition by another firm, and the exercise of executive stock options, and the timing and value of these distributions complicate the application of equation (33). Second, companies may delay dividend payments until later in their life cycle. Of course, firms must eventually make a distribution to owners, however in the extreme case, firms may make only one dividend payment – a final distribution on liquidation. Under these circumstances, modelling price as a function of a stable dividend stream is simplistic at best.

In an attempt to circumvent the shortcomings surrounding the assumption of a stable dividend stream, earnings capitalisation models have been proposed as an alternative to the dividend discount approach. Earnings capitalisation models have received extensive attention in the accounting literature as these models attempt to quantify the value relevance of accounting information. Perhaps the most widely

[8] Among other things, a flat term structure of interest rates, risk-neutrality, and linear investor preferences.

adopted approach is the residual income model, which specifies a relationship between a firm's market value, its book value, and contemporaneous and future earnings (see, for example, Ohlson (1995), Feltham and Ohlson (1995) and Myers (1999)).

The residual income model relies on a number of assumptions surrounding both the structure of company accounts and investor expectations of future earnings activity. In particular, the residual income model assumes a clean surplus accounting relation, where all changes in assets and liabilities, except those related to dividends, are reflected in the balance sheet. This provides a means to re-express equation (33) in terms of company earnings. The underlying idea of the clean surplus relation is to reconcile changes in stocks with the creation and distribution of wealth. This assumes a clear distinction between value creation and value distribution activities, and that value distribution does not affect current earnings. The clean surplus relation provides for changes in book value to be expressed as equal to earnings less dividends:

$$b_t^j = b_{t+1}^j + x_{t+1}^j - d_{t+1}^j \qquad (34)$$

where b_t^j is the current period book value, b_{t+1}^j is book value in the next period, x_{t+1}^j is next period earnings, and d_{t+1}^j is next period dividend payments. Combining the present value of dividends in equation (33) and the clean surplus relation of equation (34) provides for the following expression for stock price:

$$\begin{aligned} P_t^j &= \sum_{\tau=1}^{\infty} \frac{E_t\left(b_{t+\tau-1}^j + x_{t+\tau}^j - b_{t+\tau}^j\right)}{\left(1+r_e\right)^{\tau}} \\ &= b_t^j + \sum_{\tau=1}^{\infty} \frac{E_t\left(x_{t+\tau}^j - r b_{t+\tau-1}^j\right)}{\left(1+r_e\right)^{\tau}} - \frac{E_t\left(b_{t+\infty}^j\right)}{\left(1+r_e\right)^{\infty}} \end{aligned} \qquad (35)$$

Assuming book value of equity grows at a lesser rate than the discount rate then

$\frac{E_t\left(b^j_{t+\infty}\right)}{\left(1+r_e\right)^\infty} \xrightarrow{\infty} 0$, and the final term is assumed to be zero. "Residual income" or abnormal earnings is defined as $x^a_t = x_t - rb_{t-1}$ and this provides the final expression where price is expressed as the sum of book value and the present value of future abnormal earnings:

$$P^j_t = b^j_t + \sum_{i=1}^{\infty} \frac{E_t\left(x^j_{t+i} - rb^j_{t+i-1}\right)}{\left(1+r_e\right)^i} \tag{36}$$

where "residual income" or abnormal earnings is defined as $x^a_t = x_t - rb_{t-1}$.

Generally the residual income model is tested empirically using a cross-sectional regression of prices against book values, earnings and earnings forecasts (see for example Dechow, Hutton and Sloan (1999)):

$$P_t = \beta_0 + \beta_1 b_t + \beta_2 x_t + \beta_3 f_t + \varepsilon_t \tag{37}$$

where b_t is book value per share, x_t is current earnings per share and f_t is forecast next period earnings per share. In this chapter this model is used for comparison with the state preference approach to equity valuation.

3.2.3. Canonical valuation

Stutzer's canonical valuation approach provides a non-parametric alternative to the computation of state prices using the second derivative of the Black-Scholes option-pricing formula described above. In the same sense that state prices may be used to value any asset, then the Stutzer's risk neutral probabilities should also be applicable to pricing any asset whose value depends on the index. Stutzer's canonical valuation method is computationally simple, and the only data requirements are an historical time series of underlying asset values. Risk-neutral probabilities are calculated from the historical distribution of returns and applied to the range of possible future index values to determine the expected payoff on the asset as follows.

To price an asset maturing in T-periods, first construct an historical time series of T-period returns on the underlying asset:

$$R_h = M_h / M_{T-h}, \qquad h = 1, 2, \ldots, H - T \tag{38}$$

providing H-T possible values of the underlying asset's price in T-periods,

$$M_T = M_t R_h, \qquad h = 1, 2, \ldots, H - T \tag{39}$$

with an estimated actual probability of $\hat{\pi}_h = \frac{1}{H - T}$. The estimated risk-neutral probabilities derived from the empirical probabilities must be non-negative and satisfy the following constraint, where r is the one-period riskless rate:

$$\sum_{h}^{H-T} \pi_h^* \frac{R_h}{r^T} = 1 \tag{40}$$

While there are many choices for π^* that would satisfy these twin constraints, Stutzer chooses an estimate $\hat{\pi}^*$ that minimises the Kullback-Leibler Information Criterion distance between the empirical probabilities $\hat{\pi}$, and the risk neutral probabilities π^*.

$$\hat{\pi}^* = \underset{\substack{\pi_h^* > 0, \sum \pi_h^* = 1 \\ \forall h}}{\arg\min}\ I\left(\pi^*, \hat{\pi}\right) = \sum_{h=1}^{H-T} \pi_h^* \ln\left(\frac{\pi_h^*}{\hat{\pi}_h}\right) \text{ s.t. } \sum_{h}^{H-T} \pi_h^* \frac{R_h}{r^T} = 1 \tag{41}$$

The solution to the constrained maximisation problem in the equation above is obtained using the Lagrange multiplier method, providing the Gibb's canonical distribution:

$$\hat{\pi}_h^* = \frac{\hat{\pi}_h \exp\left[\gamma^*\left(R_h / r^T\right)\right]}{\sum_{\forall h} \hat{\pi}_h \exp\left[\gamma^*\left(R_h / r^T\right)\right]}, \qquad h = 1, 2, \ldots, H - T \tag{42}$$

The Lagrange multiplier, γ^*, is found by solving the unconstrained minimisation problem:

$$\gamma^* = \arg\min_{\gamma} \sum_{h} \exp\left[\gamma\left(R_h / r^T - 1\right)\right] \tag{43}$$

Possible payoffs are determined with reference to the underlying asset via the linear projection:

$$R_t^j = \alpha + \beta R_t^m + \varepsilon_t \qquad \forall j \tag{44}$$

where R^j is the return on asset j and R^m is the return on the S&P 500 index. This provides the following valuation expression for next period stock values:

$$P_t^j = \sum_h \pi_h \left[P_0^j \exp\left(\alpha^j + \beta^j R_h^m\right)\right] \tag{45}$$

where π_h is the risk-neutral probability of the return on the S&P500 index R_h^m, P_0^j is the current price and P_t^j is next period's price.

Stutzer's canonical valuation approach was originally proposed as a non-parametric option pricing model, rather than an equity valuation technique. As such, this chapter also represents an initial application of canonical valuation to stock valuation.

3.2.4. The Sharpe-Lintner Capital Asset Pricing Model

Perhaps the most widely understood of the asset pricing formulae, the Sharpe (1964) and Lintner (1965) Capital Asset Pricing Model (CAPM) provides a simple yet elegant relationship between stock return and return on the market portfolio. This relationship follows from Markowitz's (1952) portfolio selection theory, in which it is established that it is not the risk and return of an individual asset that is priced by investors, but rather the asset's contribution to portfolio risk and return. Once a portfolio of risky assets is of a sufficient size, the addition of another risky asset contributes little to the overall portfolio variance; rather, it is the covariance between individual assets that takes on greater importance. The CAPM builds on this central premise to price an individual asset in terms of the covariance of individual asset returns and returns on the market portfolio:

$$E\left(R_t^j\right)=R_t^f+\beta^j\left[E\left(R_t^m\right)-R_t^f\right] \tag{46}$$

where R_t^j is the return on asset j, R_t^f is the return on the risk-free asset, R_t^m is the return on the market portfolio and $\beta^j=\dfrac{\operatorname{cov}\left(R^j,R^m\right)}{\operatorname{var}\left(R^m\right)}$.

The CAPM has been subject to extensive debate in the literature. The results of early tests of the CAPM were mixed (see for example Black, Jensen and Scholes (1972), Fama and MacBeth (1973), Blume and Friend (1973)). These studies established a positive linear relationship between portfolio return and beta, which is consistent with the CAPM, but also found evidence that factors other than beta systematically impact returns. Overall, the results of these studies lead researchers to reject the CAPM. However, Roll (1977) identified a serious flaw in any test of the CAPM, suggesting the only testable hypothesis is that the market portfolio is mean-variance efficient. Roll observed that there will be any number of ex-post efficient portfolios, and as this will result in observed linearity between beta and returns, the relationship is not independently testable. Roll also noted that the market portfolio is effectively unobservable, as every individual asset (including human capital and other non-traded assets) must be included in its composition.

Following Roll's critique, tests of the CAPM relationship continued, however researchers tended to be more cautious in interpreting their results. One area of research focused on market anomalies or empirical contradictions to the CAPM relationship, such as the size effect first documented by Banz (1981), where small or low market value stocks exhibit higher average returns, or the momentum effect identified by Jegadeesh and Titman (1993), where stocks with high returns over the previous three to twelve months tend to exhibit higher returns in future periods. Others examined the relationship between average return and other factors, such as earnings-price ratios (Basu (1983)), the ratio of book value of equity to market value (Fama and French (1992)), and leverage (Bhandari (1988)). Direct tests of CAPM also became more sophisticated. Gibbons (1982) developed an alternative test methodology based on a multivariate approach equations based on the market model; Gibbons, Ross and Shanken (1989) developed an exact multivariate *F*-test; and MacKinlay and Richardson (1991) tested mean-variance efficiency via a generalised method of moments approach.

The CAPM has also been extended in a number of ways. Black (1972) provided a zero-beta CAPM that does not require the existence of a risk-free asset; Merton (1973) developed an intertemporal CAPM where expected return is a function of a number of state variables or hedge factors; and Breeden (1979) extended Merton's model to develop a consumption CAPM, where the market portfolio is replaced by aggregate consumption. Jagannathan and Wang (1996) investigated a conditional version of the CAPM where betas and the market risk premium vary over time. Other extensions include an allowance for the impact of differential taxation of dividends and capital gains (see for example Litzenberger and Ramaswamy (1979)), or including higher moments of the distribution of returns (see for example Kraus and Litzenberger (1976)).

While the CAPM has been subject to intense academic debate it remains one of the most theoretically tractable of the asset pricing models, and empirical support for other models is no better than that for the CAPM, an indication that a suitable alternative is yet to be found. Furthermore the CAPM remains widely used in practice. As such, the CAPM is included as a basis of comparison for the state preference approach in this paper. Before turning to the results of this study, the following section discusses the data inputs required for each equity valuation approach and describes the sample selection process.

3.3 Sample Description

Companies traded on US exchanges are valued annually over the period January 1997 through 2 January 2004. Valuing stocks with the state preference approach requires inputs for the risk free rate of interest, expected volatility of the underlying asset, and the contemporaneous index level. The CRSP files provide stock price and return, index level and return, and the risk free rate. The expected volatility of the S&P 500 index is proxied by the CBOE Market Volatility Index (VIX) level. As outlined in the previous chapter, the VIX is expected to provide a more appropriate proxy for expected volatility than an estimate based on historical returns, as it represents the market's consensus view on expected future stock market volatility, and is therefore forward-looking and market determined. The range of expected future values of the S&P 500 index is based on historical maximum and minimum returns obtained from the empirical return distribution of the S&P 500 index over the period from 2 July 1962 through to each valuation date. Similarly, the canonical valuation approach requires a time series of historical values of the S&P 500 index and a risk free rate. Regression estimates for the linear projection in equations (30) and (44) are determined from monthly return observations on individual stocks and the S&P 500 index over the previous five years from the valuation date. Beta estimates for the CAPM are also sourced from these regressions.

The residual income model requires inputs for book value of equity, current earnings and expected future earnings. Accounting variables are obtained from the Compustat database. Observations for book value of equity, earnings, number of common shares outstanding and an adjustment factor reflecting capitalisation changes over the period are collected for both active and inactive companies over the period 1993 through 2004. To proxy for expected future earnings consensus earnings forecasts are obtained from the I/B/E/S files. The I/B/E/S files also provide contemporaneous earnings data, which is used in preference to the Compustat earnings data so as to align reported earnings with analysts' forecasts. The I/B/E/S earnings per share measure excludes non-recurring or unusual accounting entries, which is also more consistent with analysts' earnings forecasts.

The initial sample obtained from Compustat contains 289,299 annual observations. Observations with missing values, observations where price or equity is less than or equal to zero, and observations where the number of shares outstanding is less than or equal to 10,000 are removed (194,428 in total). Descriptive statistics on the remaining sample of 94,871 observations indicate a correlation between book value per share and price of 0.995. This high value is driven by a small number of observations with extreme values for book value per share, and removing observations for which book value per sale is more than three standard deviations away from the mean reduces the observed correlation between book value per share and price to 0.689[9].

The remaining sample of 94,859 observations is then matched to the I/B/E/S dataset based on the date of disclosure of annual earnings information. After removing observations with missing values and repeated observations from the I/B/E/S files, the merged Compusat and I/B/E/S dataset contains 34,032 observations. Consensus forecasts are available on the I/B/E/S files from the middle of the month following release of the annual report to the market. The date on which the security is valued is the end of that month. To illustrate, if Company A's financial year-end is 31 December 1998, then book value, earnings and number of shares outstanding will be as at this balance date. If the disclosure date for financial statement information is 27 March 1998, then consensus earnings forecasts for the following fiscal year are reported in the I/B/E/S files at 16 April 1998. The stock is then valued at the end of that month, on 30 April 1998. The value determined on this date is compared to the actual price at the end of the following month on 31 May 1998. To obtain the actual price the merged Compustat and I/B/E/S dataset is matched to price data obtained from the CRSP files. The final dataset contains 13,042 observations.

[9] This process of trimming or winsorising the sample is widely adopted in the literature (see for example Dechow, Hutton and Sloan (1999), Collins, Pincus and Xie (1999), Barth, Beaver, Hand and Landsman (1999), Fama and French (1998), and Frankel and Lee (1998)).

3.4 Results

The relative performance of the alternative stock valuation approaches is assessed with three goodness of fit measures. First, mean squared error, which measures the average squared deviation of model values from the actual price, provides an overall indication of goodness of fit. Second, comparison is made of adjusted R^2 from a regression of estimated values on actual values:

$$P_t = a + b\hat{P}_t + e \tag{47}$$

where P_t is the actual realised price, $\hat{P}_t$ is the estimated value. This provides an indication of the relative explanatory power of each valuation approach. Third, the Bayesian Information Criterion (BIC) is calculated for each approach. The BIC is based on the log likelihood function, but imposes a penalty for the number of estimated model parameters so that, everything else being equal, the BIC will tend to favour more simplistic models:

$$BIC(k) = -2\ln(L) + k\ln(n) \tag{48}$$

where L is the log likelihood function, k is the number of estimated model parameters and n is the number of observations.

The results contained in Panel A of Table 3.1 indicate that the state preference approach provides a significant improvement on the residual income model. Overall MSEs on state price values are 35.179, compared to 418.943 for the residual income model. The canonical valuation approach provides a similar level of accuracy to the state preference approach, with overall MSEs of 35.214. The CAPM, however, provides the best fit with an overall MSE of 33.947. This pattern is generally consistent when the sample is split by share price. The CAPM performs better across all share price brackets except for companies with a share price of less than $1, where the state preference approach provides more accurate values. Note that the results in Panel A of Table 3.1 where the sample is broken down by share price indicate the state price, canonical valuation and CAPM perform better for lower priced stocks in comparison to higher priced stocks. To illustrate, for stocks with a price of less than $1 MSE is 0.028, 0.028 and 0.030 for the state price, canonical valuation and CAPM respectively, compared to 246.223, 247.089 and 241.379 respectively for stocks with prices of greater than $50. While this is largely due to the relative magnitude of errors for higher priced stocks, this pattern is not exhibited for the residual income model, which performs better for stocks with prices between $10 and $20 where MSE is 99.012 compared to a MSE of 132.646 for stocks priced less than $1 and a MSE of 2,845.913 for stocks priced greater than $50.

Panel B of Table 3.1 contains mean squared relative errors, which measures the average error relative to the actual price and should overcome the effect of higher priced stocks exhibiting higher errors as shown in Panel A of Table 3.1. These results indicate that all three models perform better for stocks priced between $20 and $50, followed by stocks priced greater than $50. All three stock valuation approaches perform worst with lower priced stocks, with the highest errors for stocks priced between $1 and $5, followed by stocks priced less than $1. Overall, comparing mean relative errors does not change that relative performance of the alternative valuation approaches. The CAPM provides the most accurate values, with mean squared relative errors of 0.027 across the sample. The canonical valuation and state preference approaches provide similar levels of accuracy, both with mean squared relative errors of 0.029. The residual income model performs the worst, with overall mean squared relative errors of 4.574.

Sample MSEs are split by market capitalisation in Panel A of Table 3.2. Similar to the results where MSE is split by price, the CAPM provides the most accurate values across most capitalisation levels, however the state preference approach performs better for larger companies. For companies with a market capitalisation of between $10 million and $50 million MSEs for the state price approach are 82.774 compared with 83.046, 85.404 and 1,131.134 for canonical valuation, CAPM and the residual income model respectively. For companies with a market capitalisation of greater than $50 million the state preference approach also provides the most accurate values, with MSEs of 467.123 compared with MSEs of 471.257 for the canonical valuation approach, 510.129 for the CAPM and 4,164.760 for the residual income model. Similar to the results displayed in Table 3.1 MSEs are relatively greater for companies with higher market capitalisation due to the impact of higher stock prices inducing greater errors. Panel B of Table 3.2 provides mean squared relative errors across alternative market capitalisation levels. Once again the residual income model performs poorly relative to the other valuation approaches.

Adjusted R^2 is used as an alternative goodness of fit measure to mean squared error, and is reported in Table 3.3[10]. These results are consistent with those for mean squared error reported in Tables 3.1 and 3.2. The residual income model performs poorly in comparison to the state preference approach with an adjusted R^2 of 0.417

[10] The regressions were performed across the entire dataset, however to ensure the results were robust to the potentially spurious impact of including a time-series of asset prices, cross-sectional regressions for the first and last years in the sample are also undertaken. The results of these regressions are consistent with those presented in Table 3.3.

compared to 0.951. Once again, the state preference and canonical valuation approaches provide similar levels of explanatory power: both models provide an adjusted R^2 of 0.951, however the CAPM provides the most accurate estimates with an adjusted R^2 of 0.952. The regression estimates presented in Table 3.3 are also of interest. For the models in question to be a good fit the intercept should be equal to zero and slope should be equal to one. The *t*-statistics associated with the slope coefficients provided in Table 3.3 test the hypothesis that the slope is not significantly equal to one. For the CAPM slope is equal to 1.002 with a t-statistic of 1.107, which indicates that the hypothesis of slope equal to one cannot be rejected at the 95% confidence level. This hypothesis is rejected for the other models however. The slope coefficients for the state preference and canonical valuation approaches are close to one, at 0.985 and 0.986 respectively, however the associated *t*-statistics indicate that these estimates are significantly different from one. The slope coefficient for the residual income model is 1.137, and again, the hypothesis that the beta is equal to one is rejected at the 95% confidence level. Finally, it is worth noting that the intercept estimates are all significantly different from zero, an indication of model bias.

The final goodness of fit measure is the BIC, the results for which are also presented in Table 3.3. Again, the CAPM proves to be the better model with a lower BIC when compared to the alternatives. The state preference and canonical valuation approaches provide largely consistent levels of accuracy, and the residual income model does not perform well.

3.5 Conclusion

Following the successful application of the state preference approach to pricing options this paper applies the state preference approach to pricing US stocks. The state preference approach is compared to alternative equity valuation approaches, including the residual income model and the CAPM. The residual income model is selected as a basis of comparison as it has received significant attention in the accounting literature, largely due to its high explanatory power. The state preference approach is found to provide a significant improvement on the residual income model. Little difference is found between the state preference approach, the CAPM and Stutzer's canonical valuation. However, the CAPM is found to provide the greatest overall accuracy in pricing stocks.

Appendix B Tables and Charts

Table 3.1 Comparison of Mean Squared Errors and Means Squared Relative Errors on U.S. Equities Valued by Alternative Valuation Models Categorised by Stock Price

Panel A reports mean squared errors, which are measured as squared deviations of model values from observed price averaged across all stocks in the sample. Panel B reports mean squared relative errors, which are measured as the average deviation of model values from observed prices divided by the observed price. The sample is broken into different price brackets and consists of 13,045 observations on 3,529 companies over the period December 1996 through July 2004.

A: Mean Squared Error

Model	Total	P<1	1<=P<5	5<=P<10	10<=P<20	20<=P<50	P>=50
State Price	35.179	0.028	0.761	2.618	6.025	17.259	246.223
Canonical Valuation	35.214	0.028	0.756	2.601	5.983	17.156	247.089
CAPM	33.947	0.030	0.710	2.439	5.600	15.846	241.379
Residual Income	418.943	132.646	152.240	136.482	99.012	145.967	2845.913
No. Obs	13,042	38	1,063	1,753	3,480	5,325	1,383

B: Mean Squared Relative Error

Model	Total	P<1	1<=P<5	5<=P<10	10<=P<20	20<=P<50	P>=50
State Price	0.029	0.049	0.074	0.045	0.030	0.017	0.023
Canonical Valuation	0.029	0.050	0.073	0.045	0.029	0.017	0.023
CAPM	0.027	0.052	0.070	0.042	0.027	0.016	0.020
Residual Income	4.574	669.227	24.984	2.729	0.521	0.129	0.271
No. Obs	13,042	38	1,063	1,753	3,480	5,325	1,383

Table 3.2 Comparison of Mean Squared Errors and Means Squared Relative Errors on U.S. Equities Valued by Alternative Valuation Models Categorised by Market Capitalisation

Panel A reports mean squared errors, which are measured as squared deviations of model values from observed price averaged across all stocks in the sample. Panel B reports mean squared relative errors, which are measured as the average deviation of model values from observed prices divided by the observed price. The sample is broken into different capitalisation brackets and consists of 13,045 observations on 3,529 companies over the period December 1996 through July 2004.

A: Mean Squared Error

Model	Total	K<$100,000	$100,000<=K <$500,000	$0.5m<=K <$1m	$1m<=K <$10m	$10m<=K <$50m	K>=$50m
State Price	35.179	2.358	7.349	18.498	50.936	82.774	467.123
Canonical Valuation	35.214	2.345	7.327	18.439	50.784	83.046	471.257
CAPM	33.947	2.196	6.881	17.112	44.701	85.404	510.129
Residual Income	418.943	138.336	114.618	163.437	607.029	1131.134	4164.760
No. Obs	13,042	2,079	4,044	1,896	3,916	852	255

B: Mean Squared Relative Error

Model	Total	K<$100,000	$100,000<=K <$500,000	$0.5m<=K <$1m	$1m<=K <$10m	$10m<=K <$50m	K>=$50m
State Price	0.029	0.045	0.034	0.028	0.022	0.012	0.012
Canonical Valuation	0.029	0.045	0.034	0.028	0.021	0.012	0.012
CAPM	0.027	0.043	0.032	0.026	0.019	0.012	0.011
Residual Income	4.574	23.340	2.126	0.505	0.322	0.255	0.362
No. Obs	13,042	2,079	4,044	1,896	3,916	852	255

Table 3.3 Comparison of Regression Values and Bayesian Information Criterion of Alternative Valuation Models

This table presents output from the regression of estimated prices on actual prices as given by $P_t = a + b\hat{P}_t + e$, including adjusted R^2, slope and intercept values and associated *t*-statistics. The Bayesian Information Criterion, which is based on a log likelihood and adjusted for the number of estimated parameters is also presented. The sample consists of 13,045 observations on 3,529 companies over the period December 1996 through July 2004. The *t*-statistics associated with the slope coefficients test for beta equal to 1.

Model	R^2	α	*t*-stat	β	*t*-stat	BIC
State Price	0.951	0.351	4.802	0.985	7.651	83,465.929
Canonical Valuation	0.951	0.342	4.667	0.986	6.993	83,479.053
CAPM	0.952	0.196	2.718	1.002	1.107	83,000.845
Residual Income	0.417	-2.677	7.686	1.137	11.667	115,794.556

Chapter 4

Conclusion

The state preference approach to decision making under uncertainty, initially proposed by Arrow (1964) and Debreu (1959), provides an elegant solution to the asset pricing question. State-contingent claims have long been regarded as a fundamental building block in asset pricing theory, however little work has been done on the empirical application of state preference theory. In this thesis the state-contingent claim approach is empirically applied to pricing exchange-traded financial assets. First, the prices of state-contingent claims, or state prices, are determined using Breeden and Litzenberger's (1978) and Banz and Miller's (1978) approach and applied to pricing S&P 500 index options. An advantage of this approach is that market microstructure features such as minimum tick size may be directly incorporated into the model. In addition, empirical maximum and minimum returns on the index may be used to limit the range of the distribution of expected returns. Prices determined via the state preference approach are compared to those estimated from Stutzer's (1996) canonical valuation model and the Black-Scholes formula. Overall the state preference provides an improvement on the alternative approaches.

The successful application to pricing options indicates that the state preference approach should be applicable to asset pricing more generally. As such, the state preference approach is then applied to pricing equities. Once again, values determined under the state preference approach are compared to alternative models, primarily the residual income model (see, for example, Ohlson (1995), Feltham and Ohlson (1995) and Myers (1999)) and the Sharpe (1964) and Lintner (1965) Capital Asset Pricing Model (CAPM). Equities are also valued with Stutzer's canonical valuation approach. The state preference approach provides a significant improvement over the residual income model, which performs poorly in comparison to the alternative models. While the state preference approach provides a similar level of accuracy to the CAPM and canonical valuation, the CAPM provides the greatest overall accuracy in pricing equities. The results of this thesis indicate that the state preference approach is empirically applicable to pricing exchange-traded financial assets, and are promising for future research into the application of the state-contingent claim approach to asset valuation.

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