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SOME RESULTS ON THE CHOICE OF RUN ORDER  
FOR  
EXPERIMENTAL DESIGNS WITH CORRELATED ERRORS

A THESIS PRESENTED IN PARTIAL FULFILMENT OF THE  
REQUIREMENTS FOR THE DEGREE OF  
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## ABSTRACT

This thesis examines the efficiency of some commonly used experimental designs in situations where the assumption of independent errors is violated. In particular this research mainly involves finding efficient run orders for various models of two level factorial experiments, three level factorial experiments and response surface designs when errors are assumed to follow either first order moving average model or first order autoregressive model. In this thesis, attention is given to systematic methods of allocating treatments based on various algorithms which provide more efficient designs and lead to good estimates of the parameters.

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