Natural Metamers

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Abstract

Given only a color camera's RGB measurement of a complete color signal spectrum, how can the spectrum be estimated? We propose and test a new method that answers this question and recovers an approximating spectrum. Although this approximation has intrinsic interest, our main focus is on using it to generate tristimulus values for color reproduction. In essence, this provides a new method of converting color camera signals to tristimulus coordinates, because a spectrum defines a unique point in tristimulus coordinates. Color reproduction is founded on producing spectra that are metamers to those appearing in the original scene. Once a spectrum's tristimulus coordinates are known, generating a metamer is a well defined problem. Unfortunately, most color cameras cannot produce the necessary tristimulus coordinates directly because their color separation filters are not related by a linear transformation to the human color-matching functions. Color cameras are more likely to reproduce colors that look correct to the camera than to a human observer. Conversion from camera RGB triples to tristimulus values will always involve some type of estimation procedure unless cameras are redesigned. We compare the accuracy of our conversion strategy to that of one based on Horn's work on the exact reproduction of colored images. Our new method relies on expressing the color signal spectrum in terms of a linear combination of basis functions. The results show that a principal component analysis in color-signal space yields the best basis for our purposes, since using it leads to the most "natural" color signal spectrum that is statistically likely to have generated a given camera signal.

1 Introduction

Two spectra that *look* the same even though the spectra themselves are different are called metamers and are said to be metameric to one another. Metamers occur because the eye, or a camera, collapses all the spectral information into a set of just three numbers—the RGB signal in a camera or a similar set of three cone excitations in a human with normal trichromatic vision. Metamers cause a problem whenever one shifts from one sensor system to another—from camera to human, say—because what is a metamer for one in general will not be for the other. This problem leads to difficulties in color reproduction systems.

As an example, consider three different objects with artificially constructed spectra (given in [29], p. 171) that would all look the same pale tangerine color to a human when viewed under a standard daylight. The CIE system quantifies this intuitive idea of what looks the same to a human observer by assigning tristimulus values X, Y, Z (or simply XYZ) to a color. These values are related to the proportion of color primaries that must be added together to make the color. From XYZ one defines the chromaticity of the color by the pair (x, y) where $x = X/(X+Y+Z)$ and $y = Y/(X+Y+Z)$. Chromaticities are usually displayed on a diagram devised by the CIE in 1931. For our tangerine color, the tristimulus values are $(129.06, 100.00, 44.20)$ and the chromaticity is $(x, y) = (0.4691, 0.3643)$.

The same objects that all looked the same color to the eye, viewed under the same daylight illumination, unfortunately all appear different from one another when viewed by a typical camera. Specifically, the values of the RGB signal produced by the camera are $(1.490, 0.901, 0.388)$ for the first, $(1.476, 0.838, 0.477)$ for the second, and $(1.189, 1.150, 0.422)$ for the last.

If these camera signals are now sent to a color display device, the "reproduced" colors of the three objects will differ from one another even though originally they were metameric. On one system we tried, passing these RGBs [1] directly to the output device resulted in the colors tangerine, salmon, and a pale green tan. The camera discerns that the spectra differ even though the eye does not. The converse situation also can occur; namely, the camera may see three different spectra as metameric (i.e., their RGB signals are the same) while the human eye does not.

Since the eye and camera do not always agree, what XYZ value should be assigned to a given RGB signal? This is the main question we seek to answer in this paper. Of course, any answer must be a compromise because of the fundamental difference between the two sets of sensor sensitivities.

[1] Throughout this paper we denote by 'RGB' the camera system signal (not three tristimulus values, as in [29]).

The problem of metamerism has a long history in color science, although it is not always recognized in computer graphics and computer vision as part of the problem in accurate color reproduction. A visible-light signal is characterized by the spectral power distribution (SPD) of relative intensities of the signal measured over many (usually evenly-spaced) wavelength intervals. The SPD can be thought of either as an analytic function of wavelength, or when the spectrum is sampled at N wavelength intervals, as a vector in an N-dimensional space. The signal that arrives at the lens (either camera or human) is composed of the light spectrum illuminating a surface multiplied by the surface spectral reflectance function. The resulting signal is variously called the *color signal* in computer vision (see, e.g., [28]) or the *object-color stimulus* in color science [29].

When filtered by color filters and viewed by camera (or eye) sensors, the color signal for each pixel is condensed into three RGB values that carry the intensity and color information for the image. (We shall sidestep the issue of just how RGB information is created in the eye by relying on the psychophysical color-matching functions [29] to characterize human sensors.) The problem of metamerism arises because the mapping from SPD to RGB is many-to-one: there are usually a great many possible color signals that could have resulted in exactly the same set of sensor responses.

In terms of color reproduction this would create no problem, if as seen in the earlier example, camera sensors were fooled by the same metamers as the human eye— i.e. if *camera metamers* were the same as *eye metamers* (*cf.* [16]).

As Horn has pointed out [12], eye metamers and camera metamers can only be made to coincide when the sensor curves of the two systems are a homogeneous linear transformation away from each other. Unfortunately, this linear relationship between response curves for machine and human vision does not in general hold.

Horn [12] gives a straightforward method for determining the linear transformation of the color-matching functions that "best" corresponds to the actual sensor response curves. His prescription implies, as well, a simple linear transformation of the camera RGB signal into corresponding tristimulus values XYZ. (Here, we call this linear transformation "Method A".) It is the tristimulus values that form the input to the chain of transformations leading to color reproductions, either video or printed [12, 26]. Horn's approach is an attempt to be algorithmic rather than to rely on the experience and expertise of practitioners in color reproduction to apply appropriate rules of thumb to the color correction problem.

Our interest in this topic stems from our work on the computational vision problem of separating a color signal into its component factors, illumination and reflectance [11]. Using the statistical properties of typical daylights and naturally occurring reflectances, we showed that it is indeed possible to disentangle the illumination from the reflectance, provided that one has available the entire SPD color signal function, not just an RGB signal. This information turns out to be available from a lens with nonvanishing chromatic aberration [9], but is generally not available from standard video systems. We were led, therefore, to the problem of producing an estimate of the color signal spectrum from just the sensor RGB inputs, as a front end to our color-signal-separation algorithm.

It is precisely here that the problem of metamerism crops up: Which of the many available metamers, by definition all having the same sensor RGB, is the actual spectrum producing that RGB? Of course, we cannot expect to recover the precise spectrum from just three samples; however, we do claim to recover the most likely color signal spectrum.

Following the work of others [19, 6, 14, 23], we again use finite dimensional linear models as we did in our color-signal-separation algorithm [11], to reduce the amount of information required to characterize an illumination spectrum or a surface spectral reflectance function by capturing all the necessary information in a few weights of a small number of terms in a linear expansion in terms of a set of basis vectors. In the present case, however, the finite dimensional models model color signals instead of illuminants and reflectances. Color signals are approximated as a weighted sum of basis vectors. Truncating the expansion to three terms produces a set of equations that can be solved for the basis vector weights given the sensor RGB.

By generating an approximate, complete color-signal spectrum corresponding to a given RGB, we also immediately obtain a corresponding XYZ tristimulus triple. Calculating an XYZ for an RGB via the indirect step of an intermediate spectrum is in fact similar to Method A in that the XYZ values are then a simple linear transformation away from the input sensor RGB values. It turns out, though, that for a good choice of basis vectors (we examine three different alternatives) this new scheme (called "Method B") produces XYZ values that are much closer to those of the original color signal. In fact, while Method A works reasonably well for sensor response curves that are nearly a linear transformation away from the color-matching functions—although still only 40% as well as Method B—it does not work at all well for typical camera bandpass filters. The accuracy of Method B, however, remains undiminished in these cases.

As well, since we are producing XYZ values by way of generating a complete color-signal SPD, we can examine the SPD produced for appropriate physical characteristics. In particular, we find that it is simplest to eliminate any spectra having one or more negative components. This turns out to be 0.3% of the large set of sample RGBs tested, and can be accommodated easily by interpolation. The intermediate step of generating a complete SPD thus amounts to a useful tool for discarding those XYZs generated that would result from impossible spectra. This validation step is not available to Method A.

Basis functions were calculated from 1710 synthetic color signals. Tests on 2052 synthetic color signals including these 1710 color signals yielded a median error of 3.4 CIELUV units between the tristimulus coordinates of the actual color signal and those of the color signal estimated from the RGB data alone. The errors increase substantially for color signals synthesized from illuminants and reflectances far from the original data set, which points out the desirability of incorporating as many spectra from as many diverse materials and lights as possible in statistically-based modeling techniques of this sort. These tests, which are described in detail below, show that our new method does not produce perfect results. We claim only that it performs much better than existing methods.

2 Metamerism and Tristimulus Values

In reference [12], Horn is concerned with ascertaining the conditions for each step of the process of color image reproduction that guarantee that the resulting colors cannot be distinguished by humans, as well as with developing some indicators of the accuracy of reproduction when the exactness conditions are not met. Here we address just the first step of the reproduction process, that of converting the input sensor RGB responses into a best version of tristimulus values XYZ. The measure of accuracy we shall use is the CIELUV unit in uniform color space (see [29, p. 828]); this unit accounts for both the chromaticity difference between spectra and their luminance difference.

If we generate a spectrum $C(\lambda)$ from the sensor RGB that matches the RGB when filtered by the camera filters, then we have generated a metamer with respect to the color filters that produced the RGB. Our first problem consists in regenerating the one metamer among all the choices available that best matches (in CIELUV space) the XYZ of the actual color signal that produced the RGB. All the candidate metamers will have the same RGB (i.e., they will be camera metamers), but in general each will have a different XYZ (i.e. they will not be eye metamers).

Suppose that the camera has color filters $R_k(\lambda)$, so that the RGB sensor responses, denoted ρ_k, are given by

$$\rho_k = \int C(\lambda)\, R_k(\lambda)\, d\lambda \text{ , k=1} \ldots 3 \text{ .} \tag{1}$$

(For convenience, (1) employs function notation rather than vector notation.) Then any $C(\lambda)$ spectrum is a camera metamer provided it has the same values for ρ_k. [2].

Tristimulus values XYZ are determined in an analogous manner, but with the 1931 CIE standard observer color-matching functions instead of the sensor response curves R_k. These

[2]Note that to determine the correct R_k it is necessary to calibrate the camera system carefully for contributions from the lens system, the digitizer, etc. (*cf.* [16])

curves are commonly called $\bar{x}$, $\bar{y}$, $\bar{z}$, but for convenience we shall denote the collection of three curves by $\bar{x}_k$ and the collection X,Y,Z by X_k:

$$X_k \;=\; \int C(\lambda)\,\bar{x}_k(\lambda)\; d\lambda \;. \tag{2}$$

Our task is to generate from the ρ_k those values X_k that best match the X_k that would have been produced from the actual color signal. For computational vision applications, it would also be of use to know just what $C(\lambda)$ produced the measured ρ_k. We look at the second issue in the next section; here we set out a method for developing a set of X_k from the ρ_k that is based on Horn's work.

Reference [12] is concerned with determining strictures on the image reproduction system for the exact reproduction of colors. Horn shows that if the image sensor curves R_k are a linear transform of the color-matching functions $\bar{x}_k$, then camera metamers are the same as eye metamers. As well, he develops the linear transform of the $\bar{x}_k$ that "best" fits the sensor curves R_k. To do so, he assumes that the actual response curves R_k can be approximated by a linear transformation of the $\bar{x}_k$:

$$R_i(\lambda) \;\simeq\; \sum_{k=1}^{3} a_{ik}\,\bar{x}_k(\lambda) \;. \tag{3}$$

Then by minimizing the squared differences between the actual response curves R_k and the linear transform curves, the weights a_{ik} can be solved for:

$$\sum_{k=1}^{3} a_{ik}\,q_{kj} \;=\; v_{ij} \;, \tag{4}$$

where

$$q_{kj} \;=\; \int \bar{x}_k(\lambda)\,\bar{x}_j(\lambda)\; d\lambda$$

$$v_{ij} \;=\; \int R_i(\lambda)\,\bar{x}_j(\lambda)\; d\lambda$$

Or in matrix notation:

$$A \;=\; V\,Q^{-1} \tag{5}$$

provided Q is nonsingular. The approximation, equation (3), amounts to a projection of the sensor curves onto the space spanned by the functions $\bar{x}_k$.

This approximation also entails a scheme for generating the X_k triples from the input ρ_k. We have

$$\begin{aligned} \rho_i &= \int R_i(\lambda)\, C(\lambda)\; d\lambda \\ &\simeq \sum_{k=1}^{3} a_{ik} \int \bar{x}_k(\lambda)\, C(\lambda)\; d\lambda \\ &= \sum_{k=1}^{3} a_{ik} X_k \end{aligned} \tag{6}$$

Therefore, the input ρ_k are approximately given by a linear transformation of the X_k, and vice versa:

$$\boldsymbol{X} \simeq Q V^{-1} \boldsymbol{\rho}\ . \tag{7}$$

In section 5 we explore how well this approximation does in reproducing the exact X_k.

3 Metamerism and Color Signal Reconstruction

The problem with the above formulation is that while it does generate the optimal *sensor functions* from the actual R_k functions, there is no guarantee that it generates the best tristimulus values X_k. In search of a better method, we consider instead approximating the original color signal by a three-dimensional linear model and ask how well the tristimulus values match those of the original.

Suppose that $C(\lambda)$ is approximated by

$$C(\lambda) \simeq \sum_{i=1}^{3} c_i\, C_i(\lambda) \tag{8}$$

where $C_i(\lambda)$ are basis functions that do a good job of describing most color signals in a training dataset. The sensor response to this approximate color signal calculated according to equation (1) approximates the ρ_k developed from the exact $C(\lambda)$.

$$\begin{aligned} \rho_k &\simeq \sum_{i=1}^{3} c_i \int C_i(\lambda)\, R_k(\lambda)\; d\lambda \\ &\equiv \sum_{i=1}^{3} c_i b_{ik} \end{aligned} \tag{9}$$

Since we have chosen the functions $C_i(\lambda)$ and they are fixed, we know the matrix b_{ik} and it can be precalculated. The above system of equations is linear and their solution yields the weights c_i, which turn out to be simply a linear transformation of the ρ_k.

In terms of minimizations, the best available approximation in equation (8) amounts to a minimization

$$\text{Min} \int [\, C(\lambda) \; - \; \sum_{i=1}^{3} c_i \, C_i(\lambda) \,]^2 \; d\lambda. \tag{10}$$

The linear transformation (9) can be thought of as a minimization

$$\text{Min} \sum_{k=1}^{3} (\rho_k \; - \; \sum_{i=1}^{3} c_i \, b_{ik})^2 \tag{11}$$

provided the matrix b_{ik} is nonsingular.

Equation (2) yields values of X_k produced from the approximated color signal, so the actual X_k are approximated as:

$$X_k \; \simeq \; \sum_{i=1}^{3} c_i \int C_i(\lambda) \; \bar{x}_k \; d\lambda$$

$$\equiv \; \sum_{i=1}^{3} c_i e_{ik} \tag{12}$$

Or in matrix form:

$$\boldsymbol{X}^T \; \simeq \; \boldsymbol{c}^T E \tag{13}$$

Writing equation (9) as

$$\boldsymbol{c}^T \; = \; \boldsymbol{\rho}^T B^{-1} \tag{14}$$

and combining with the above, we have as our final approximation of the set of X_k,

$$\boldsymbol{X}^T \; \simeq \; \boldsymbol{\rho}^T B^{-1} \, E \; . \tag{15}$$

Since matrices B and E are fixed by the choice of sensor filters and color-signal basis functions, we are left with a simple linear transformation from the ρ_k to the X_k that is fixed and independent of the values of ρ_k. A fixed linear transformation is precisely the situation that obtains for Method A described in the last section. The difference lies in the definitions of B and E, which both incorporate the choice of the color signal basis $C_i(\lambda)$. With Method B, a judicious choice of basis set greatly improves on the values of X_k derived using Method A. Method B is equivalent to the minimization (11), which determines the best basis function weights c_i given sensor responses ρ_k.

Method A invokes no assumptions about the color signals, whereas the choice of a basis set $C_i(\lambda)$ entails an assumption about what color signals can be expected. Method B performs best when that assumption is met.

4 Color Signal Basis Sets

Method B requires a basis set $C_i(\lambda)$. Following on the work of others, we consider a Fourier basis and a basis formed from products of basis functions for illumination and reflectance. We then also introduce a third basis derived from a principal component analysis of color signals. We compare Method B's performance on all three basis sets with that of Method A.

4.1 Fourier basis

As advocated in [25, 5], a set of three frequency-limited functions of wavelength can be used as a basis set for modeling spectra. Wandell [28] suggests using the first three Fourier functions, as follows:

$$\begin{aligned} F_1(\lambda) &= 1 \\ F_2(\lambda) &= \sin[2\pi(\lambda - \lambda_{min})/(\lambda_{max} - \lambda_{min})] \\ F_3(\lambda) &= \cos[2\pi(\lambda - \lambda_{min})/(\lambda_{max} - \lambda_{min})] \qquad (16). \end{aligned}$$

Weighted sums of this set of functions can be expected to generate many physical chromaticities [5], but it is not clear whether such spectra correspond to natural color signals or are simply camera metamers that may be useful in generating RGB signals for use in graphics In section 5 we test how closely weighted sums of the above functions correspond to actual color signals. [3]

[3]The Fourier basis was also used by Glassner [10] for the related problem of generating some instance of a spectrum that forms a monitor-metamer giving a particular screen RGB. Such a metameric spectrum is useful for full-spectrum-based antialiasing. Glassner's method relies on a transformation matrix M for converting monitor values to XYZ equivalents such that M is fixed, for a given monitor, and is found by calibrating to a monitor white spot (see, e.g., [22]). Glassner uses the set (16) and generates XYZ values via $X_{(i)k} = \int F_i \bar{x}_k d\lambda$. These $X_{(i)k}$ are transformed to sets $\rho_{(i)k}$ by matrix M, and weights c_i are found for any screen RGB from these $\rho_{(i)k}$ via $\rho_k = \sum_{i=1}^{3} c_i \rho_{(i)k}$. This method for generating a spectrum differs from equation (15) in that the matrix M is unconnected with the basis F_i.

4.2 Basis function products

A second set of basis functions for color signals that has been used before (see Brainard *et al.* [3] and Ho *et al.* [11]) consists of functions formed as product pairs taken from two separate basis sets, one for illumination and another for reflectance. For example, let the basis set for illumination be Judd*et al.*'s derived from a principal component analysis of many daylight SPDs [14]. Denote by $E_i(\lambda)$ these illumination basis functions. In addition, let $S_j(\lambda)$ be a basis set for reflectance; for these Brainard *et al.* [3] (see also [18]) use either Cohen's [6] Munsell chip reflectance basis or their own basis set developed using a principal component analysis of a large set of Munsell chips (*cf.* Maloney's analysis [17] of the large sample of natural reflectances obtained by Krinov [15]). Judd *et al.* modeled most daylights using just three to five illumination basis vectors; Cohen concluded that between three and six basis vectors were sufficient for modeling reflectance.

The full set of product functions consists of all the $E_i(\lambda)S_j(\lambda)$ pairs, but for our purposes we must choose just three product basis functions $P(\lambda)$ for modeling color signals. We select pairs:

$$
\begin{aligned}
P_1(\lambda) &= E_1(\lambda)\, S_1(\lambda) \\
P_2(\lambda) &= E_1(\lambda)\, S_2(\lambda) \\
P_3(\lambda) &= E_2(\lambda)\, S_1(\lambda)
\end{aligned}
\qquad (17)
$$

Our tests employ Judd's illumination basis functions $E_i(\lambda)$. For the reflectance basis set, we follow Maloney [17] and carry out a principal component analysis on the Krinov catalogue of 370 natural reflectances [15]. [4] Since these reflectances are available in a limited range in the visible—400nm through 650nm in steps of 10nm—we keep the analysis to only 26 samples over wavelength. In fact, since we use the Krinov reflectances to generate test color signals we use 26-component vectors throughout.

Judd *et al.*'s analysis of daylight illumination was a standard principal component analysis [13]. In this type of analysis, one views the spectra as vectors in an N-dimensional vector space. The first step translates each vector to a new origin, the mean vector of all the spectra. The second step forms the variance-covariance matrix of all the mean-subtracted vector components with each other over all the cases studied and diagonalizes this matrix. The resulting vectors are ordered such that the first vector is in the direction that accounts for the maximum variability in the whole set of samples. The second vector is perpendicular to the first and accounts for the direction showing the next most important source of variability,

[4] Actually we used only 342 of the Krinov spectra, omitting those that were incomplete in the limited wavelength range utilized by Krinov and three others (snows) that had reflectance values above 1.0.

and so on. The principal component vectors are not necessarily orthogonal to the mean vector, however.

In modeling color signals as the product of a linear series in an illumination basis times a similar series in a reflectance basis, there can be uniqueness in the illumination and reflectance weights only up to an overall multiplicative factor because the color signal is formed by multiplication [11]. It then makes sense to keep to a weight of 1 for the first illumination basis function, since an overall choice of magnitude makes no difference. Therefore, using the mean vector plus a linear series for illumination, as provided by a standard principal component analysis, is just what is required.

For reflectance, however, it makes more sense to derive basis vectors that are all orthogonal, since we are not setting the first weight to a special value and more importantly because for any new reflectance we do not have available a sample mean by which to translate the origin. Therefore, instead of a standard principal component analysis, we use a Karhunen-Loève analysis [27, p. 275], which does not translate by the mean and yields basis vectors that are all orthonormal. In its simplest form, the Karhunen-Loève transformation diagonalizes the raw component crossproduct matrix (the autocorrelation matrix) rather than the variance-covariance matrix (although the term can also refer to the standard principal component analysis). This amounts to viewing the origin, rather than the mean of the reflectance set, as a distinguished point. [5]

For the recovery of color signals from ρ_k values, the product basis set P_i turns out to work quite well, better than the Fourier basis, as is shown in the section 5. This is due to the fact that the P_i incorporate a good deal of statistical information on how the expected color signals are formed.

4.3 Color signal space basis

The third basis set considered, and the one that tests show works the best, is similar to the product basis set, but developed from a statistical analysis of color signals themselves instead of their illumination and reflectance components. By forming a large number of synthetic color signals, as products of typical illumination spectra with natural reflectance spectra, we created a large data set to examine. We used the five standard Judd daylights [14] for correlated color temperatures between $4800^o K$ and $10000^o K$, multiplied by 342 reflectance spectra from the Krinov catalogue as 1710 different synthetic, yet natural, color signals. We performed a Karhunen-Loève analysis on these spectra and derived a set of basis vectors,

[5] Alternatively, one can engineer the sample set such that the mean vector is zero [17].

the first five of which are shown in Fig. 1. [6] [7] Note that although it would be desirable to use as many vectors as possible, equation (11) limits us to using only the first three of them.

Denote by $C_i(\lambda),\ i = 1 \ldots 3$ the color-signal-space basis set to be used in equation (8). We show in the next section that these C_i perform better than either the Fourier or the product basis sets in mapping an input set of ρ_k back to a color signal.

It should be noted that each of the three basis sets entails an unrestricted gamut of XYZs entirely covering the 1931 CIE chromaticity diagram. The three basis vectors C_1, C_2, C_3 can be viewed simply as three primary colors. These primaries can be combined in any linear combination, including combinations with negative intensity weights. Only when we filter out RGB-to-XYZ mappings because the $C(\lambda)$ constructed during the intermediate step contain negative components do we restrict the gamut at all.

In the next section we look at results for Method A and the results for Method B using each of the three basis sets. For convenience, we refer to the methods as shown below:

Method A	method derived from Horn's analysis
Method B.F	basis function method using Fourier basis
Method B.P	using $E_i S_j$ product basis
Method B.C	using color signal space basis

5 Results

We compare results for the various methods with two different sets of color filters, first using reflectances drawn from the original set used in deriving the color signal basis vectors, and then using reflectances drawn from other sources.

5.1 Spectra formed from original reflectance data set

For the two different sensor response functions, we use the human cone responses given by Bowmaker and Dartnell [2], and as typical camera sensors, the transmittances of Kodak filters #25 (red), #58 (green) and #47B (blue) [8]. Both sets are shown in Fig. 2.

As a measure of accuracy of the color signal reconstruction, we use the CIELUV unit of distance ΔE in uniform color space [29]. Since a color signal is already an illumination

[6] Buchsbaum [4] uses a Karhunen-Loève expansion as well, but diagonalizes in an RGB space, not with respect to the components of the color signal. See also [21].

[7] The (cumulative) variance-accounted for by the first five vectors is: 0.95819, 0.99063, 0.99724, 0.99839, 0.99893.

multiplied by a reflectance, it is not necessary to multiply by the standard light D65 to obtain tristimulus values.

To provide a common ground for comparison, we normalize the luminance of each of the synthesized color signals by setting the Y values to 100, as is done for illuminations in [29, p. 149]. The X_k given by equation (7) must also be normalized in the same way, i.e. by multiplication with $100/Y_{actual}$, where Y_{actual} is the unnormalized value of the luminance for the actual color signal. Similarly for the basis function methods, one should scale the tristimulus values calculated from equation (15) by Y_{actual}. Since both actual and approximate spectra are multiplied by the same normalization factor the ΔE produced is 3-dimensional in that it accounts for intensity change as well as chromaticity change.

The set of synthetic color signals used to test each method consists of the five Judd standard daylights as well as the mean vector for daylight in Australia [7], each multiplied by each of the Krinov reflectance functions [15]. Since the color signal basis set was derived in part from this same set of product functions, it could be argued that this sample set does not provide a stringent test for the method. However, the fact that a principal component basis accounts for the variability in the whole data set does not imply that any particular spectrum is well-modeled by a few basis functions. The chosen set provides a large sample so many of the spectra will be poorly modeled. One does, however, expect the principal component set to do relatively well overall.

While in general there is a good argument for choosing a set of basis functions that matchs as well as possible the set of color signals actually expected, to see what happens when the expectations are not met, we apply the method to spectra not drawn from this set to determine how much the results degrade. We are also interested in how a change in filter functions affects the average error of each method.

For the human cone response functions, the results for the sample set are shown in Fig. 3 and Table 1.

	Mean	Median	StdDev
Method A	16.4	16.2	6.6
Method B.F	16.9	16.5	6.9
Method B.P	11.8	9.8	9.3
Method B.C	7.4	5.4	6.6

Table 1: Statistics using human cone response functions for 2052 typical color signals.

Color differences are given in terms of 3-dimensional CIELUV ΔE.

As can be seen, from poorest to best the methods are: Method B.F; Method A; Method B.P; Method B.C. The best mean uniform color space distance is 7.4 units. For the color signal space basis (Method B.C), 2% of the approximate color signals were not included in the histogram analysis (Fig. 3) because at least one vector component turned out to be negative. Of the 2052 sample color signals examined, 37 had at least one negative component. Of these, the average number of negative components was 2, the median was 2 and the maximum was 4. Another way of dealing with reconstructed signals with negative components, rather than simply omitting them, would be by adding metameric black signals [29, p. 187], but we do not address this option here.

For each of the other basis function methods, the histograms shown also omit any signals with negative components. Signals that must be omitted strongly correlate among the three basis function methods. For Method A, it is not possible to screen unphysical color signals in this way. The fact that the cone functions are close to being just a linear transformation of the color-matching functions means that the non-basis method, Method A, works not too badly.

When applied to the typical camera filter functions, Method B continues working well, with the best ΔE average being 4.4 for Method B.C. Method A's error rises considerably—up by a factor of 2 in CIELUV units—presumably because the sensor functions are far from being a linear combination of eye functions. Table 2 tabulates the results with the camera filters and Fig. 4 histograms them. For all the methods, the standard deviations are quite wide.

	Mean	Median	StdDev
Method A	30.7	31.0	8.4
Method B.F	11.0	10.0	5.6
Method B.P	5.5	4.3	4.6
Method B.C	4.4	3.4	3.9

Table 2: Statistics using camera response functions for 2052 typical color signals, in terms of ΔE.

As a particular case, consider as an example Method B.C applied to a color signal composed of the Australian illumination spectrum multiplied by the first principal component vector in Cohen's analysis of reflectances. These spectra have no relationship to the development of the color-signal-space basis set. For this 'typical' *natural* color signal the tristimulus values are X_k = (98.55, 100.00, 95.59) so that the chromaticity is $(x, y) = (0.3350, 0.3400)$. This color is very close to an ideal white. Using the camera sensors, the camera signal is $\rho_k = (1.086, 1.012, 0.904)$ which displays as a pale pink—the camera does not see what the eye does. Nonetheless, applying equation (15) Method B.C maps these ρ_k back to tristimulus values X_k = (98.13, 99.55, 99.58), or chromaticity $(x, y) = (0.3301, 0.3349)$. In other words, Method B.C successfully reproduces the white the eye sees; the CIELUV error is only $\Delta E = 4.76$. Fig. 5 shows the original color signal and the approximation derived by the algorithm. Their agreement is striking.

For comparison, the results derived using Method A are $X_k = (\ 133.6,\ 124.4,\ 102.0\ )$ with chromaticity $(x, y) = (\ 0.3712,\ 0.3456)$, giving a ΔE of 37.2. The results for Method B.F are $X_k = (\ 86.5,\ 95.7,\ 92.9)$ and $(x, y) = (0.3145,\ 0.3478)$, yielding $\Delta E = 22.0$. For method B.P, the results are $X_k = (\ 97.3,\ 98.9,\ 100.1)$, $(x, y) = (0.3284,\ 0.3338)$ and $\Delta E = 6.2$.

5.2 Spectra formed from other reflectances

For a more stringent test, we apply the various methods to color signals that are modeled much less well by the color signal basis set by composing natural color signals as products of standard illuminant A, representing a full-radiator approximation of an incandescent source [29], and the spectra from two sets of published reflectance data.

First, we used the set of reflectance patches on the Macbeth Color Checker chart [20] (and see also [21]). This set consists of twelve custom-made patches and twelve standard Munsell chips. The last 6 patches are neutrals. Data was drawn from a digitization of the curves in [20]; the data was also checked in part using a 0/45 spectroradiometer system.[8] As expected, the method does not perform as well as when the color signal basis set is tailored to the expected illuminants and reflectance functions. The results are as in Table 3. In this Table we show the number of negative components, if any, in the reconstructed color signal. The existence of negative components is consistent across all the methods for reflectances with a large number of negative components.

The best results are given by Method B.C, with average errors for the four methods being $\Delta E = 54.8,\ 32.7,\ 32.5,\ 21.8$, for Methods A, B.F, B.P, and B.C respectively.

[8]Data kindly supplied in part by Pthalo Systems, Inc., 8500 Baxter Pl., Burnaby, B.C., Canada V5A 4T8., using their in-house spectroradiometer.

Sample	Method A	Method B.F	Negs	Method B.P	Negs	Method B.C	Negs
1	61.27	17.02	9	47.01	4	30.52	3
2	62.99	16.01	6	37.16	4	22.34	3
3	44.32	11.51		3.62		1.19	
4	36.15	25.05		11.84	3	10.36	3
5	62.48	10.08		11.76		5.38	
6	37.18	8.95		14.87		12.31	
7	53.43	41.23	10	72.39	7	49.20	6
8	60.96	23.28		11.38		14.65	
9	76.77	75.99	11	73.89	7	44.44	7
10	88.35	20.72	3	37.06	3	21.57	
11	32.18	31.76	3	16.89	4	14.49	4
12	46.18	22.77	9	58.60	4	41.81	4
13	57.70	37.49		11.99		14.67	
14	17.38	20.54		12.99	1	7.48	2
15	96.66	187.7	11	86.28	9	48.69	8
16	44.72	26.12	8	39.51	4	28.84	4
17	89.95	59.96	9	63.69	6	34.60	5
18	54.23	33.43		54.19		47.56	
19	48.44	19.10	1	18.36		11.74	
20	48.37	19.01	1	18.90		12.26	
21	48.29	19.30	1	18.95		12.22	
22	48.25	19.18	1	18.57		11.97	
23	48.59	19.24	1	19.66		12.72	
24	49.91	18.35	1	20.43		13.22	
Mean	54.8	32.7	3.5	32.5	2.3	21.8	2.0

Table 3: ΔE for illuminant A and Macbeth patches using the camera filters given in Fig 2.

1=dark skin, 2=light skin, 3=blue sky, 4=foliage, 5=blue flower, 6=bluish green, 7=orange, 8=purplish blue, 9=moderate red, 10=purple, 11=yellow green, 12=orange yellow, 13=Blue, 14=Green, 15=Red (primary), 16=Yellow, 17=Magenta, 18=Cyan, 19=white, 20=neutral 8, 21=neutral 6.5, 22=neutral 5, 23=neutral 3.5, 24=black.

Another data set used was the set of reflectance spectra for ceramic tiles given in [1]. Again, negatives appeared for all the basis sets for these samples. Results are given in Table 4.

Sample	Method A	Method B.F	Negs	Method B.P	Negs	Method B.C	Negs
1	51.48	18.22	2	20.86		13.31	1
2	39.61	14.85		25.57		19.85	
3	50.69	19.47		27.50		25.35	
4	49.78	18.72	1	19.93		12.87	
5	44.71	6.59		20.24	1	17.96	
6	49.45	18.56	1	18.47		11.75	
7	47.37	37.69	10	73.87	7	51.77	6
8	48.94	18.81	1	18.58		11.91	
9	75.62	31.51	8	53.89	7	32.50	5
10	91.12	140.0	11	80.23	9	45.80	8
11	48.98	19.55	1	20.53		13.38	1
12	44.75	26.54	8	34.79	4	25.40	4
Mean	53.5	30.9	3.6	34.5	2.3	23.5	2.0

Table 4: ΔE for illuminant A and ceramic tile reflectances for camera response functions.

1=black, 2=blue, 3=cyan, 4=deep gray, 5=green, 6=mid gray, 7=orange, 8=pale gray, 9=pink, 10=red, 11=white, 12=yellow.

Again, Method B.C generates the best results with results being ΔE =53.5, 30.9, 34.5, 23.5.

5.3 Discussion

The average error of $\Delta E = 4.4$ reported in Table 2 is really quite good. Although printing a color consistently is quite a different problem from reproducing a color, Stamm [24] reports results that can be used for comparison. Stamm's results show that the average allowable color variation in typical printing applications is $\Delta E = 6$ units, with standard deviations $\sim$3-4 (in the related CIELAB system). For another related problem, that of duplicating a color on a different device, Stone *et al.* report results of $8 - 14$ CIELUV units [26].

For the cases of the Macbeth patches and the ceramic tile reflectances, the errors shown in Tables 3 and 4 grow substantially. These are color signals that are not modeled well by the color signal basis set. Even though these errors are quite high, it remains the case that Method B.C does better than the others—better than Method A in particular.

The best case is Illuminant A multiplied by Macbeth patch #3. The worst case for a spectrum recovered with all positive components is given by the same illuminant multiplied by Macbeth patch #18. These cases are shown in Figs. 6 and 7. While the worst case is clearly very poorly modeled, Method B.C still does better than the other methods. Fig. 8

provides some intuition as to how a curve reconstructed with some negative components fits the original color signal.

Similar results are found using other sets of camera filters. We carried out the same tests using another set with somewhat narrower spectral response curves and slightly shifted peaks. The results were substantially the same.

6 Conclusions

Each of the methods provides a homogeneous linear transformation from sensor RGB responses to XYZ coordinates. Method A makes no assumptions about the type of color signals expected at the sensor, but optimizes a given set of sensor sensitivities to the human cone sensitivities. Horn's work, on which Method A is based, points to the need for cameras with spectral sensitivities that are appropriately matched to those of the human visual system.

We have taken a typical *existing* camera as a starting point and found that a better linear mapping from RGB to XYZ can be derived via the step of constructing an intermediate spectrum. The intermediate spectrum involves an assumption about what color signals are statistically likely to be seen by the camera. A principal component analysis in color signal space determines which these are and generates a set of basis functions. The equations restrict us to using only the first three of these basis functions, but within the limits of this constraint, the algorithm recovers the most "natural" color signal that is also a camera metamer to the input RGB. The RGB is then mapped to the XYZ of that metameric color signal. The new method of RGB-to-XYZ mapping works well when either cone response functions or camera response functions are used.

Since Method B amounts to a simple linear transformation that is independent of the actual values of ρ_k, the method achieves exact reproduction of XYZ when the camera response curves are exactly a linear transform of the color-matching functions, just as Method A does. It is superior only when the linear-transform condition does not hold. If better cameras become available, that more closely approximate a linear transform of the color-matching functions, Method B.C will still be upward-compatible.

We found the color-signal basis derived from the statistical properties of natural color signals to be the one least likely to generate intermediate color-signal spectra containing negative components. The approximate color signal SPD can be examined for unphysical components, and is of interest in its own right for other applications. Even when the method produces a few negative components, the non-negative part of the spectrum is probably still usable. The tests with the color-standard patches indicate that the estimated spectrum is not too bad even in cases where the input color signal differs substantially from those in the data set used to construct the basis functions.

Method A performs well when applied to sensor RGBs that result from sensor sensitivities close to the eye's; however, as we have shown, it generates unreliable results when the response curves are far from being linear transforms of cone responses. Method B, on the other hand, works reasonably well in both cases. We claim not that it is perfect, only that it does better than existing methods for existing cameras.

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8 Figure captions

Figure 1. The first five color signal space basis vectors derived from a Karhunen-Loève analysis of 1710 synthetic natural color signals.

Figure 2. Sensor response functions for human cones and typical camera sensors. Human cones: solid lines; Filters: dashed lines.

Figure 3. Histograms showing frequency over ΔE for recovered color signal compared to actual color signal for each method, using human cone sensor response functions. The number of synthetic spectra tested was 2052. For the basis function methods the percentages do not add up to 100% because cases were omitted if one or more components of the recovered color signal were negative. Percentages retained were: Fourier: 88.7%, Product: 90.4%, Color signal space basis: 98.2%.

Figure 4. Histograms showing frequency over ΔE for recovered color signal compared to actual color signal for each method, using camera sensor response functions and 2052 synthetic spectra. For the basis function methods the percentages of cases retained were: Fourier:

99.85%, Product: 99.27%, Color signal space basis: 99.66%.

Figure 5. Sample test color signal: mean vector for Australian daylight multiplied by first principal component vector for Munsell chip reflectances, as given by Cohen.

Figure 6. Best case: actual and approximate color signal spectrum for standard Illuminant A reflected from Macbeth patch #3 (blue sky), using color signal basis set derived from Judd illuminants and Krinov reflectances.

Figure 7. Worst case: color signal recovery for illuminant A multiplying Macbeth patch #18 (Cyan).

Figure 8. Negative components: illuminant A multiplying Macbeth patch #4 (foliage).

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