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Stochastic analysis of response functions of nitrogen in stream water

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University of Alaska Fairbanks, 1992

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STOCHASTIC ANALYSIS OF RESPONSE FUNCTIONS OF NITROGEN IN STREAM WATER

A

THESIS

Presented to the Faculty
of the University of Alaska Fairbanks
in Partial Fulfillment of the Requirements
for the Degree of

MASTER OF SCIENCE

By

Hyun Ae Park, B.S.

Fairbanks, Alaska
December 1992

STOCHASTIC ANALYSIS OF RESPONSE FUNCTIONS OF NITROGEN IN STREAM WATER

By

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15 December 1992

Date

STOCHASTIC ANALYSIS OF RESPONSE FUNCTIONS OF NITROGEN IN STREAM WATER

Hyun A. Park

ABSTRACT

In the present study, a stochastic model of nitrogen in streams is created using a new mathematical technique, Probability Density Function/Moment technique (PDF/M). The model is based on a set of four highly non-linear second order equations for nitrogen species in streams (NH_3, NO_2, NO_3, and organic-N). The purpose of the PDF/M technique is to include occurrence of natural variability. The first step is to separate the stochastic terms from the non-stochastic terms and solve the resulting set of equations simultaneously. The moments of the output variables then are obtained using expectation mathematics. The moments are used in a solution of the Fokker-Planck equation to produce an analytical solution for the probability density functions of the dependent variables. Comparison of the present study to the results of the Monte Carlo method showed the application of PDF/M technique to nitrogen cycle simulation valid.

December 1992
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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

A : Algal biomass concentration (mg-A/L)
A_o : Initial value of algal biomass concentration (mg-A/L)
α : Defined as $\mu_{max}-\rho-\sigma_1/d$ (day^{-1})
α_1 : Fracction of Algal biomass that is notorgen (mg-N/mg-A)
B_1 : Rate constant for the biological oxidation of NH_3 to NO_2 (day^{-1})
B_2 : Rate constant for the biological oxidation of NO_2 to NO_3 (day^{-1})
B_3 : Rate constant for the hydrolysis of organic -N to NH_3 (day^{-1})
d : Depth (ft)
F_1 : Fraction of algal nitrogen uptake ammonia pool
μ_{max} : Maximum specific growth rate of algae (day^{-1})
N_1 : Concentration of NH_3-nitrogen (mg-N/L)
N_2 : Concentration of NO_2-nitrogen (mg-N/L)
N_3 : Concentration of NO_3-nitrogen (mg-N/L)
N_4 : Concentration of organic-nitrogen (mg-N/L)
P : Preference factor for ammonia nitrogen
ρ : The local respiration rate of algae (day^{-1})
S_1 : Defined as σ_1/d
σ_x : Standard deviation of x
σ_1 : The local settling rate for algae (ft/day)
S_3 : The benthos source rate for ammonia nitrogen ($mg\text{-}N/ft^2\text{-}day$)
S_4 : Rate coefficient for organic nitrogen settling (day^{-1})
t : Time (day)
u : Local specific growth rate of algae (day^{-1})
$\overline{x}$: Average of x
x' : Variation of x

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CHAPTER 1

INTRODUCTION

Mathematical modeling of our environment has continuously grown over the past few decades. Environmental quality management requires more efficient management tools based on greater knowledge of environmental phenomena. Management of environmental controls has become more costly to implement and judgement errors are resulting in more severe penalties (Zielinski, 1988). As part of the search for more efficient management tools, computer modeling has become a popular aid to the decision-maker in balancing the complex factors which must be considered in environmental management and pollution control (Tumeo, 1988). In today's high-tech, computer-oriented, hardware/software-focused world, this trend has been greatly enhanced (Heidtke et al., 1986).

A number of stochastic models for water quality processes have been proposed in recent years. The most common stochastic modeling approach is Monte Carlo simulation. Unfortunately the technique may be limited because of the time required for the computations. Another common approach involves transforming the differential equations representing water quality from deterministic to random or stochastic differential equations.

The Fokker-Planck equation is commonly used to obtain the probability density function (pdf). The moment equations are used to obtain the expectation and variance (Zielinski, 1988).

In this study, a set of dependent nitrogen equations are solved using PDF/M technique. The model is developed and applied to the Withlacoochee River in Florida, and the results are compared to QUAL2E-UNCAS.

1.1 A MATHEMATICAL MODELING TREND

The modeling of natural phenomena and the solutions of differential equations were based on deterministic solutions of differential equations representing environmental processes up to the end of the nineteenth century. It was commonly thought that if all initial data could be collected, one would be able to predict the future with certainty (Gardiner, 1985). It is still true that most available environmental models are deterministic in nature. Selection of specific values of input parameters such as reaction rates, diffusion coefficients, or growth rates is required. The fact that input parameters actually have a stochastic component is ignored (Tumeo, 1988).

The idea of including stochasticity in mathematical models used for environmental management has been evolving for quite some time (Tumeo, 1988). Unfortunately, the application of stochastic method to environmental models is difficult, if not impossible. The difficulty of including stochasticity in mathematical models for environmental management restrains modelers from more vigorous development.

The advantage in stochastic equations is that uncertainties in model parameters are considered implicitly in the solution. Therefore, it is possible to use field measurements that are imprecise as input parameters to the model. The disadvantage of stochastic models is that extensive numerical computations are usually required (Zielinski, 1988).

1.2 OBJECTIVES OF STUDY

The main objective of this study is to check the validity and feasibility of the Probability Density Function/Moment (PDF/M) technique (Tumeo and Orlob, 1989) as applied to non-linear simultaneous equations. A set of four dependent, highly non-linear second order equations modeling the nitrogen cycle in a body of stream water is used to test the technique.

A major goal of the study is to obtain an analytical solution for the probability density functions using the PDF/M technique, and to compare the results with the results of QUAL2E-UNCAS, a widely used stream water model utilizing the Monte Carlo technique (Brown and Barnwell, 1987).

1.3 ORGANIZATION OF THESIS

This study was aimed at developing a stochastic model of the nitrogen cycle in streamwater using PDF/M technique and evaluating the model by comparing the results with a Monte Carlo model.

Chapter 2 provides an overview of the selected stochastic methods currently used in environmental modeling. In Chapter 3, the technical approach to the new method is discussed, and Chapter 4 presents the application of the new technique using equations modeling the nitrogen cycle in streams. Results are compared with the output from QUAL2E-UNCAS. Conclusions and recommendations for future study are included in Chapter 5.

CHAPTER 2

LITERATURE REVIEW

2.1 Nitrogen Cycle Modeling in Streams

Nitrogen transformations in streams have been modeled to show either the direct variation of nitrogen species concentration or the indirect effect on the dissolved oxygen level (Warwick and McDonnell, 1985). Few attempts have been made to describe nitrogen progression by deterministic mathematical models, and over the last few decades, only a limited number of nitrogen cycle analysis in streams were done (Najarian and Harleman, 1977). A model developed by Warwick and McDonnell (1985) uses nitrogen mass balance to include complex nitrogen transformations in streams such as chemical and biological activity, which can affect the concentrations of the various nitrogen species. The model calculates nitrification rates from observed in-stream variations in organic-N, ammonia-N, nitrite-N, and dissolved oxygen.

Najarian and Harleman (1977) developed a real time simulation of the nitrogen cycle in an estuary. The simulation illustrates dynamic coupling between hydrodynamic transport processes and biochemical water quality transformation processes such as advective transport, conservation of mass, transformation of CBOD-DO and nutrients, and chemostat system (Najarian and Harleman, 1977). In 1981, Najarian and Taft evaluated improvements in Najarian's biological model based on the nitrogen-cycle to complement the added capabilities of the existing two-dimensional estuarine circulation models (Najarian and Taft, 1981). The study examines the sensitivity of the model to transient changes in key parameters such as phytoplankton uptake rates, added complexity in zooplankton grazing, phytoplankton settling, and phytoplankton preferences for nutrients.

There have been a few other models developed to simulate nitrogen cycle in streams and estuaries (Najarian and Harleman, 1977). They commonly incorporate nitrification in both nitrogen and dissolved oxygen mass balance. The selected input values of nitrification do not necessarily satisfy both the observed changes in nitrogen species and DO concentrations.

EPA developed a model addressing stochasticity of the nitrogen cycle in streams. QUAL2E-UNCAS developed in 1987 as an enhancement to QUAL2E released in 1985, allows the modeler to perform uncertainty analysis on the steady state water quality simulations (Brown and Barnwell, 1987). Three uncertainty options are available: sensitivity analysis; first order error analysis; and Monte Carlo simulations. The uncertainty analysis enables the user to assess the effects of model sensitivities and of uncertain input data on model forecasts. The modeler can assess the risk of imprecise forecasts and recommend measures for reducing the magnitude of that imprecision.

QUAL2E-UNCAS is a widely used water quality model that incorporates all four forms of nitrogen in streams: organic nitrogen; ammonia; nitrite; and nitrate. The Monte Carlo simulation in QUAL2E-UNCAS uses input variables generated randomly from pre-determined probability distributions. Input requirements for the Monte Carlo simulation include the variance and probability density function of the input variable, and the number of the simulations to be performed (Brown and Barnwell, 1987). The model provides summary statistics and frequency distributions for the state variables at specific locations in the system.

2.2 Probability Density Function/Moment Technique

The probability density function/moment (PDF/M) technique is based on the expansion of basic governing equations to include stochastic terms. The stochastic terms are separated from the non-stochastic terms and the resulting set of equations solved simultaneously. The moments of the output variables are obtained using expectation mathematics applied to the solution set. The moments are then used in conjunction with the Fokker-Planck equation to produce an analytical solution for the probability density functions of the dependent variables (Tumeo and Orlob, 1989).

The method may provide analytical solutions for the probability density function and associated moments of the variables, and the technique gives the modeler the ability to perform a detailed quantitative examination of the sources and magnitude of uncertainty. Furthermore, the technique may provide a means by which an optimum model could be selected, given a specific purpose for the model. This could extend the capabilities of computer models in management and decision analysis (Tumeo, 1988). The method is not presented as a replacement for existing methods, but rather, as a supplement.

It will be most useful in the cases which are beyond the current scope of stochastic differential equations, but are not so complex as to present a set of non-tractable differential equations. The in-depth theoretical background of the PDF/M technique is presented in Chapter 3.

2.3 Monte Carlo Method

The Monte Carlo technique is a method for numerically operating a complex system that has random components. It is a procedure for solving non probabilistic-type problems (problems whose outcomes do not depend on chance) by probabilistic-type methods (methods where outcomes depend on chance) (Farlow, 1982). Often a complex system is modeled into a simpler form that is more convenient to use without some of the troublesome side effects that confuse the original situation.

A random event is one which the time of occurrence is unpredictable, except in a probabilistic sense. Probability is used to indicate the chance the a particular event will or will not occur. A probability distribution or probability density function describes the probability associated with selecting a particular event from a number of possible events. The two most basic traits of Monte Carlo technique include random sampling from a specified distribution and the use of that sample in a specified equation.

To illustrate the method, the following integral is evaluated:

$$I=\int_a^b f(x)\ dx \qquad \text{Eq. 2-1}$$

To employ the Monte Carlo technique, one may devise a game of chance whose outcome is the value of the integral (or approximates the integral) (Farlow, 1982). A simplistic game to evaluate the integral with relative accuracy is throwing darts at a rectangle board, R, where R = $\{(x,y):a<x<b,\ 0<y<\max f(x)\}$ (Figure 2-1). The dart has equal probability to hit anywhere on the rectangle board. If 100 or so darts are randomly tossed at the rectangle enclosing the graph (Figure 2-1), the faction of darts hitting below the curve times the area of the board will estimate the value of the integral. The outcome of the game I = {fraction of tosses under f(x)} x (area of R) is used to estimate the true value of the integral I.

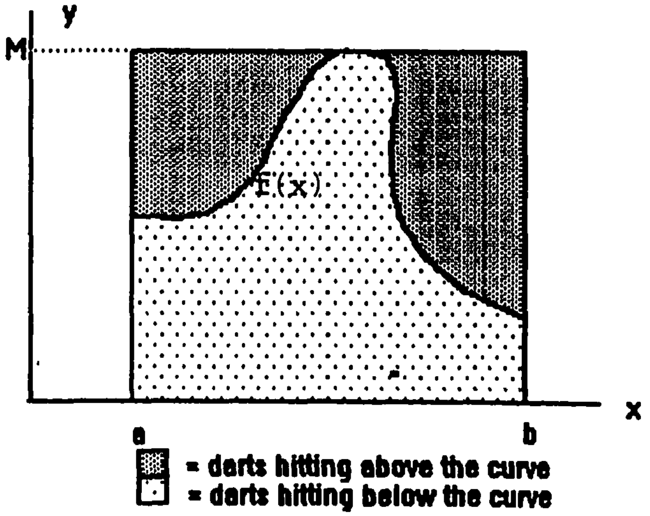


Fig. 2-1 Evaluation of an integral by Monte Carlo Technique

In environmental modeling, the Monte Carlo methods are used to randomly select input values or equation parameters. To illustrate this application of the method, consider the following equation.

$$K(T) = Kr*A^{T-Tr} \qquad \text{Eq. 2-2}$$

where T=Temperature of interest
K(T)=Reaction Rate at Temperature T
Tr=Reference Temperature (usually 20°C)
Kr=Reaction rate at reference temperature T
A=Constant

The equation is a modified form of the Arrhenius relationship, which gives a reaction rate as a function of temperature. Assuming temperature is a stochastic variable and varies normally around a mean (T) with a standard deviation of σ, the determination of the distribution of possible K values using a Monte Carlo technique involves three steps.

1) A random number (ξ) is generated. The random numbers must be normally distributed with a mean of zero and a standard deviation of one;
2) An input temperature (Ti) is selected using the generated random numbers and the following equation.

$$Ti = \overline{T}+\xi\sigma \qquad \text{Eq. 2-3}$$

where: T = Mean of T
ξ = Random number, normally distributed with a mean of zero and a standard deviation of one
σ = Standard deviation

3) The temperature found using Equation 2-3 is used to calculate a reaction rate K(T) using Equation 2-2.

Monte Carlo methods are most commonly used in environmental modeling where the problem has a stochastic component (Esen and Bennet 1971; and Dewey 1984). However, the method is limited in application. Usually, enough number of runs are required to obtain a statistically valid sample, and often it is not clear as to how many runs would be sufficient.

2.4 Other Stochastic Methods

There are other stochastic methods applied in environmental modeling, including first-order uncertainty analysis (Reckhow, 1973 and Lettenmaier and Richey, 1979), and stochastic differential equations (Soong, 1973). First-order uncertainty analysis involves the use of an assumed function to approximate the mean and variance of an output variable as a function of one or more input variables. It involves a truncation of terms which represents the functional relationships of the state variable, and includes only the first order term in a Taylor Series Expansion. Scavia et al. (1981) reported a study on the comparison of first-order uncertainty analysis and Monte Carlo simulation in time dependent lake eutrophication models. The results showed that estimates of variance for a nutrient cycle eutrophication model calculated by first-order uncertainty analysis and Monte Carlo analysis do not always agree. There are four causes for the differences: (1) deterministic and stochastic mean trajectories are different; (2) first-order uncertainty analysis employs a first-order linearization of the model while Monte Carlo analysis uses the fully nonlinear model; (3) ambiguous statistics were generated from skewed Monte Carlo populations; and (4) there is a difference in the interpretation of variances from the two analysis. A comparison of estimates of state variables indicated that Monte Carlo means are most like the measurements, and medians are most like the deterministic model output. Best agreement occurred between Monte Carlo and first-order estimates of both state variable values and their variances when Monte Carlo output distributions are symmetric. The study concluded that the Monte Carlo analysis has the advantages of estimating variability about the quantity of interest, the mean population, and of providing output frequency distributions. The disadvantage was a high computational burden for both long-term predictions and examination of relative error sources in relatively complex models. The first-order error analysis could provide error estimates and direct estimates of model sensitivity.

The limitation of first-order uncertainty analysis is that only the mean and variance of the output variables are generated. Higher order moments such as skew are not described using the analysis, thus a complete description of distribution characteristics of a function is lacking. First-order analysis can be extended by truncating the Taylor Series expansion after higher order terms, but this extensively complicates the procedure.

Stochastic differential equations originated with work on the well known differential equation for diffusion developed by Einstein in 1905 (Gardiner, 1985). The major concepts in Einstein's description of the differential equation of diffusion have been developed more generally and rigorously since then. For example, the Chapman-Kolmogorov Equation, the Fokker-Planck Equation and Kramers-Moyal expansions can be identified as more general and rigorous development of Einstein's equation (Gardiner, 1985). Handbook of Stochastic Methods by C.W. Gardiner (1985) is dedicated to a whole range of stochastic methods branched out from Brownian motion and the stochastic representation of diffusion.

CHAPTER 3

TECHNICAL APPROACH

The development of the PDF/M technique is briefly summarized in this section. Details of the development are taken from Tumeo and Orlob (1989).

Development of Basic Equations for PDF/M

The fundamental basis of the derivation of the PDF/M technique is the realization that the parameters of the equations have stochastic properties which must be carried forward in the solution process. A simple linear differential equation is considered to demonstrate the application of the technique.

$$\frac{dC(t)}{dt} = k \qquad \text{Eq. 3-1}$$

Solution of the above equation yields:

$$C(t) = k*(t-t_0) + C(t_0) \qquad \text{Eq. 3-2}$$

where: $C(t)$ = concentration at time t

$C(t_0)$ = initial concentration at time t_0

k = constant reaction rate

To apply the PDF/M technique, the reaction rate (k) is treated as a stochastic variable. Therefore C(t) is also a stochastic variable. With mean values ($\bar{k}$, $\bar{C}(t)$), and deviations about their respective means (k', $C'(t)$), equation 3-1 can then be written as

$$\frac{d[\bar{C}(t) + C'(t)]}{dt} = \bar{k} + k' \qquad \text{Eq. 3-3}$$

Equation 3-3 may be separated into two terms involving mean values and "variation" terms.

$$\frac{d\bar{C}(t)}{dt} = \bar{k} \qquad \text{Eq. 3-4}$$

$$\frac{C'(t)}{dt} = k' \qquad \text{Eq. 3-5}$$

Solving equations 3-4 and 3-5 yields:

$$\bar{C}(t) = \bar{k}*(t-t_0)+\bar{C}(t_0) \qquad \text{Eq. 3-6}$$

$$C'(t) = k' * (t-t_0) + C'(t_0) \qquad \text{Eq. 3-7}$$

where: $C'(t)$ = Fluctuation in concentration at time t

$C'(t_0)$= Random variation in initial concentration

k' = Random variation in reaction rate

Consequently the deviation of the variable [$C'(t)$] around the mean [$C(t)$] as a function of the random variations of the input parameters is represented by Eq. 3-7.

To determine the moments of the distribution of the output variables, expectation mathematics are applied to the above equations. By definition the first moment is the mean of the variable and can be found by:

$$First\ Moment = E[C(t)] = E[\overline{C}(t)] + E[C'(t)] \qquad \text{Eq. 3-8}$$

If it is assumed that k' and $C'(t_0)$ have normal distributions with a means of zero, the expectation of $C'(t)$ by definition is zero, therefore the mean of the variable can be found directly from Eq. 3-6.

The second moment, the variance is (Bendat and Piersol, 1971):

$$V[f(x)] = \sigma^2 = E\{f(x)^2 - E[f(x)]^2\} \qquad \text{Eq. 3-9}$$

where E, the expectation operator, is the mathematical operator operating on $f(x)^2$ producing the expectation (mean) of the function $f(x)^2$. For example, using the variation equation, Eq. 3-7, the operation can be performed as follows:

$$V[C'(t)^2] = (t-t_0)^2 * E[k^2] + 2*(t-t_0)*E[k'*C'(t_0)] + E[C'(t_0)^2] - \{(t-t_0)E[k'] + E[C'(t_0)]\}^2 \qquad \text{Eq. 3-10}$$

The actual expectation value of a variable depends on the type of distribution the variable represents. If it is assumed that k' and $C'(t_0)$ have normal distributions with means of zero, Eq. 3-7 yields:

$$V[C'(t)^2] = \sigma_{k^2} * (t-t_0)^2 + \sigma_{C_0^2} \qquad \text{Eq. 3-11}$$

where: $\sigma_k 2$ = standard deviation of k (mg/l/day)

$\sigma_{C0} 2$ = standard deviation of initial concentration (mg/l)

At this point probability density functions can be calculated using the Fokker-Planck Equation. The Fokker-Planck Equation was proved by Komogoroff (1931) to be a valid representation of the conditional probability of a process $P(x,t|x_0,t_0)$. The Equation in differentiated form is:

$$\frac{\delta P(\overline{x}, t|\overline{x_0}, t_0)}{\delta t} = -\sum_{i=1}^{\infty} \frac{\delta}{\delta x}\{A_i(\overline{x}, t)*P(\overline{x}, t|\overline{x_0}, t_0)\} + \frac{1}{2}*\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \frac{\delta^2}{\delta x^2}\{B_{ij}(\overline{x}, t) * P(\overline{x}, t|\overline{x_0}, t_0)\} \qquad \text{Eq. 3-12}$$

Where x is a vector set of state variables and $P(x,t|x_0,t_0)$ is the probability of x at time t given the vector of initial conditions x_0 at time t_0. $A_i(x,t)$ has a physical interpretation of the Drift Coefficient. The Drift Coefficient is the rate of change in the general trend or mean of the process. $B_{ij}(x,t)$ is the Diffusion Coefficient, the rate of change of the variation of the process around the trend. Because the coefficients are a function of time only in the current example, Equation 3-12 reduces to:

$$\frac{\delta P(C,t|x_0,t_0)}{\delta t} = -A(t)\frac{\delta P(C,t|x_0,t_0)}{\delta x} + \frac{1}{2}B(t)\frac{\delta^2 P(C,t|x_0,t_0)}{\delta x^2} \qquad \text{Eq. 3-13}$$

where x_0 is the initial condition of the state variables affecting the concentration (C_0, and k).

Solution of the Fokker-Planck Equation under the appropriate initial and boundary conditions yields the conditional probability density function for the variables in question. The following conditions apply to a probabilistic function:

$$P(\bar{x},t|\bar{x},t_0) = f(x,0) \quad at \quad t_0 = 0 \qquad \text{Eq. 3-14}$$

The one time probability of any concentration [P(x,t)] by definition is:

$$P(\overline{x},t)=\int_{\overline{x}_0} P(\overline{x},t|\overline{x_0},t_0)\,dx_0 \qquad \text{Eq. 3-15}$$

P(x,t) must satisfy the normalization requirement:

$$\int_{-\infty}^{\infty} P(\overline{x},t)\,\delta\overline{x}=1 \qquad \text{Eq. 3-16}$$

With the initial conditions given in Equation 3-14, the solution to Equation 3-13 becomes (Gardiner, 1985):

$$P(C,t|x_0,t_0)=\frac{1}{\sqrt{4\pi Z_2}}*\exp\left(\frac{-(C-Z_1)^2}{4Z_2}\right)$$

$$\textit{where: } Z_1=\int_0^{\infty} A(t)\,dt=\textit{mean of process} \qquad \text{Eq. 3-17}$$

$$Z_2=\frac{1}{2}*\int_0^{\infty} B(t)\,dt=\frac{1}{2}*\textit{variance of process}$$

which, from Equation 3-15 above and given the definition of x(o), becomes:

$$P(C,t)=\int_0^{C_{max}}\int_0^{k_{max}} P(C,t|\overline{x_0},t_0)\,\delta C_0\,\delta k \qquad \text{Eq. 3-18}$$

The analytical solution of Equation 3-18 yields a complicated function including error function terms. In general, a computer program is developed to obtain the conditional probabilities over specified ranges and normalization is achieved by dividing each term by the total sum of conditional probabilities as calculated over a specified range.

CHAPTER 4

MODEL DEVELOPMENT OF NITROGEN CYCLE IN PDF/M TECHNIQUE

Verification of the validity and feasibility of the method was accomplished using a set of four dependent highly non-linear second order equations that are used to model Nitrogen species in streams. Analytical expressions for the moments and probability density function of output variables are the final product of this study. The solutions were compared with the results of QUAL2E-UNCAS, a widely used streamwater model.

4.1 Nitrogen Cycle Equations

Nitrogen has a stepwise transformation cycle in a body of natural aerobic waters. The cycle starts from organic nitrogen to ammonia, to nitrite, and finally to nitrate. Figure 4-1 shows the transformation of nitrogen in stream water. The figure shows conceptualization of nitrogen interactions including DO and phosphorous (Brown and Barnwell, 1987). The terms used in the figure are defined in the nomenclature section of this report. There are four dependent, non-linear, first order equations which govern the transformation cycle of nitrogen in a body of water (Thomann and Mueller, 1987).

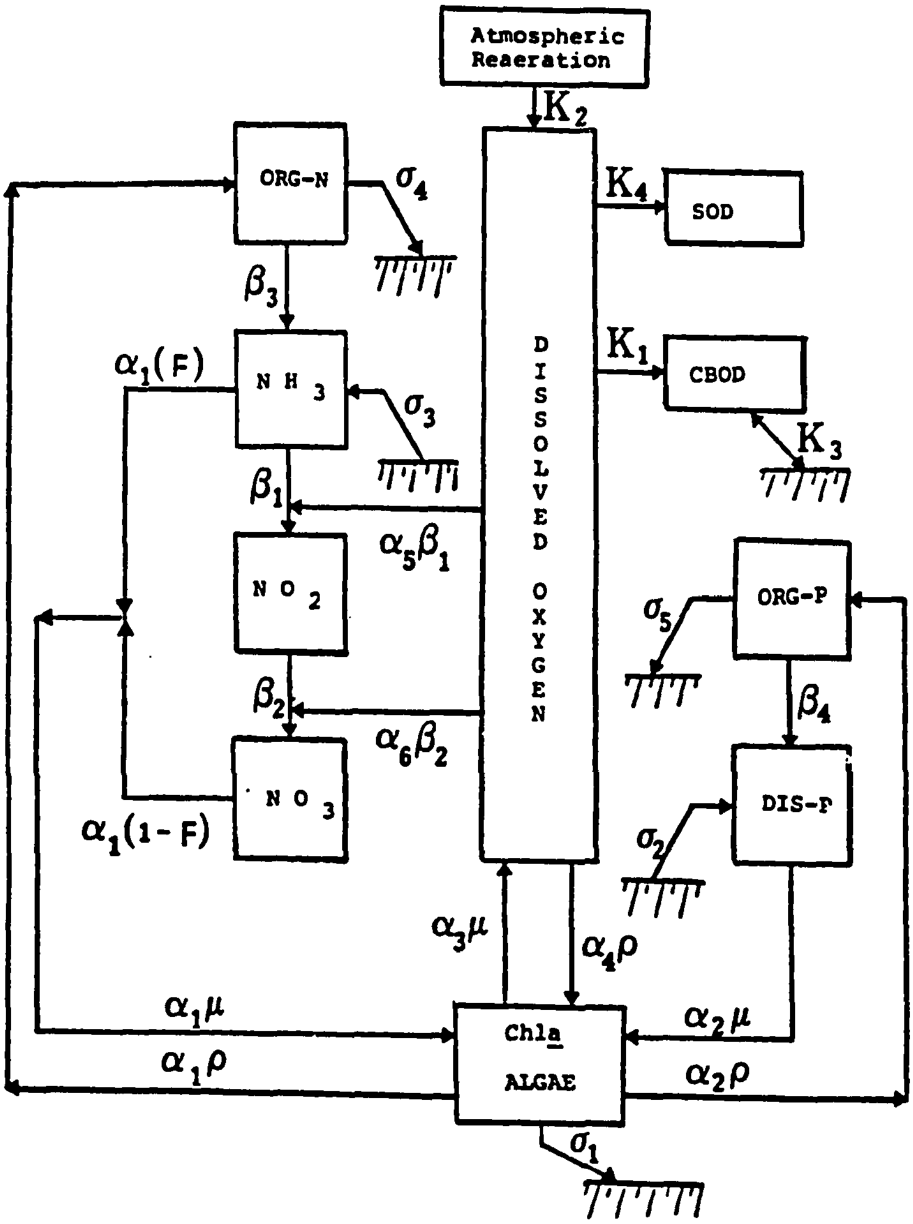


Fig. 4-1 Transformation of Nitrogen and Other Constituents in Stream Water (taken from Brown and Barnwell, 1987)

The four equations modeling the nitrogen cycle (ammonia nitrogen, nitrite nitrogen, nitrate nitrogen and organic nitrogen respectively) in differential form are shown below. (Thoman, 1987).

$$\frac{dN_1}{dt}=B_3N_4-B_1N_1+\frac{S_3}{d}-F_1\alpha_1uA \qquad \text{Eq. 4-1a}$$

$$\frac{dN_2}{dt}=B_1N_1-B_2N_2 \qquad \text{Eq. 4-1b}$$

$$\frac{dN_3}{dt}=B_2N_2-\alpha uA+(1-F_1)\alpha_1uA \qquad \text{Eq. 4-1c}$$

$$\frac{dN_4}{dt}=\rho\alpha_1A-(B_3+S_4)N_4 \qquad \text{Eq. 4-1d}$$

where,

$$F_1=\frac{PN_1}{(PN_1+N_3-PN_3)}$$

$$A=A_0e^{-\alpha t}$$

Terms in the above equations are defined in the nomenclature section at the beginning of this document. F_1 is first assumed to be constant to solve equations 4-1a through 4-1d. The assumption of F_1 to be constant is required to find an analytic solution to the four equations. The steps in solving the equations are provided in Appendix A. Solutions to the differential equations 4-1a through 4-1d are:

$$N_1 = K_{11}(e^{-\alpha t}-e^{-B_1 t}) - K_{12}(e^{-c_1 t}-e^{-B_1 t}) + K_{13}e^{-B_1 t} + K_{14} \qquad \text{Eq. 4-2a}$$

$$N_2 = K_{21}(e^{-\alpha t}-e^{-B_2 t}) - K_{22}(e^{-c_1 t}-e^{-B_2 t}) + K_{23}(1-e^{-B_2 t}) + K_{24}(e^{-B_1 t}-e^{-B_2 t}) \quad K_{25}e^{-B_2 t} \qquad \text{Eq. 4-2b}$$

$$N_3 = K_{31}(e^{-\alpha t}-1) + K_{32}(e^{c_1 t}-1) + K_{33}(e^{-B_1 t}-1) + K_{34}(e^{-B_2 t}-1) + K_{35} \qquad \text{Eq. 4-2c}$$

$$N_4 = \frac{\alpha_1 \rho A_o}{c_1-\alpha}(e^{-\alpha t}-e^{-c_1 t}) + N_4(0)e^{-c_1 t} \qquad \text{Eq. 4-2d}$$

Where the coefficients are shown as follows:

$$K_{11} = \frac{(a_1-a_2)}{(B_1-\alpha)} \qquad K_{12} = \frac{(a_1-N_4(0)B_3)}{B_1-c_1}$$

$$K_{13} = (N_1(0)-\frac{S_3}{dB_1}) \qquad K_{14} = \frac{S_3}{dB_1}$$

$$K_{21} = \frac{B_1K_{11}}{B_2-\alpha} \qquad K_{22} = \frac{B_1K_{12}}{B_2-c_1}$$

$$K_{23} = \frac{K_{14}B_1}{B_2} \qquad K_{24} = \frac{(-K_{11}+K_{12}+K_{13})B_1}{(B_2-B_1)}$$

$$K_{25} = N_2(0) \qquad K_{31} = (-\frac{B_2K_{21}}{\alpha}+\frac{B_2}{\alpha}(1-F_1)\alpha_1 u A_o)$$

$$K_{32} = \frac{B_2K_{22}}{c_1} \qquad K_{33} = -\frac{B_2K_{24}}{B_1}$$

$$K_{34} = (K_{21}-K_{22}+K_{23}+K_{24}-N_2(0)) \qquad K_{35} = K_{23}B_2t+N_3(0)$$

$$a_1 = \frac{B_2\alpha_1\rho A_o}{(c_1-\alpha)} \qquad a_2 = \frac{F_1\alpha_1 u A_o}{B_1-\alpha}$$

$$c_1 = B_3+S_4$$

4.2 Application of PDF/M Technique

4.2.1 Derivation of Basic Equations

The first step in the application of the PDF/M method, as outlined in Chapter 3 to the set of nitrogen equations (Eq. 4-1a through 4-1d), yields the following.

$$\frac{d[\overline{N_1}(t)+N_1'(t)]}{dt}=(\overline{B_3}+B_3')(\overline{N_4}(t)+N_4'(t))-(\overline{B_1}+B_1')(\overline{N_1}(t)+N_1'(t))+\frac{(\overline{S_3}+S_3')}{d}-F_1\alpha_1 uA \qquad \text{Eq. 4-3a}$$

$$\frac{d[\overline{N_2}(t)+N_2'(t)]}{dt}=(\overline{B_1}+B_1')\overline{N_1}(t)-(\overline{B_2}+B_2')(\overline{N_2}(t)+N_2'(t)) \qquad \text{Eq. 4-3b}$$

$$\frac{d[\overline{N_3}(t)+N_3'(t)]}{dt}=(\overline{B_2}+B_2')\overline{N_2}(t)-(1-F_1)\alpha_1 uA \qquad \text{Eq. 4-3c}$$

$$\frac{d[\overline{N_4}(t)+N_4'(t)]}{dt}=\rho\alpha_1 A-(\overline{B_3}+B_3'+\overline{S_4}+S_4')(\overline{N_4}(t)+N_4'(t)) \qquad \text{Eq. 4-3d}$$

Equations 4-3a through 4-3d are then expanded to yield:

$$\frac{d[\overline{N_1}(t)+N_1'(t)]}{dt}=\overline{B_3N_4}(t)+B_3'N_4'(t)+\overline{N_4}(t)B_3'+N_4'(t)\overline{B_3}$$
$$-(\overline{B_1N_1}(t)+B_1'\overline{N_1}(t)+\overline{B_1}N_1'(t)+B_1'N_1'(t))$$
$$+\frac{\overline{S_3}}{d}+\frac{S_3'}{d}-F_1\alpha_1uA \qquad \text{Eq. 4-4a}$$

$$\frac{d[\overline{N_2}(t)+N_2'(t)]}{dt}=\overline{B_1N_1}(t)+B_1'\overline{N_1}(t)$$
$$-(\overline{B_2N_2}(t)+B_2'\overline{N_2}(t)+\overline{B_2}N_2'(t)+B_2'N_2'(t)) \qquad \text{Eq. 4-4b}$$

$$\frac{d[\overline{N_3}(t)+N_3'(t)]}{dt}=\overline{B_2N_2}(t)+B_2'\overline{N_2}(t)-(1-F_1)\alpha_1uA \qquad \text{Eq. 4-4c}$$

$$\frac{d[\overline{N_4}(t)+N_4'(t)]}{dt}=\rho\alpha_1A-(\overline{B_3N_4}(t)+B_3'\overline{N_4}(t)+\overline{S_4N_4}(t)$$
$$+S_4'\overline{N_4}(t)+\overline{B_3}N_4'(t)+B_3'N_4'(t)$$
$$+\overline{S_4}N_4'(t)+S_4'N_4'(t)) \qquad \text{Eq. 4-4d}$$

Equations 4-4a through 4-4d are separated into equations with mean terms only and equations with variation and mixed mean value/variation terms, yielding:

$$\frac{d\overline{N_1}(t)}{dt}=\overline{B_3N_4}(t)-\overline{B_1N_1}(t)+\frac{\overline{S_3}}{d}-F_1\alpha_1uA \qquad \text{Eq. 4-5a}$$

$$\frac{d\overline{N_2}(t)}{dt}=\overline{B_1N_1}(t)-\overline{B_2N_2}(t) \qquad \text{Eq. 4-5b}$$

$$\frac{d\overline{N_3}(t)}{dt}=\overline{B_2N_2}(t)-(1-F_1)\alpha_1uA \qquad \text{Eq. 4-5c}$$

$$\frac{d\overline{N_4}(t)}{dt}=\rho\alpha_1A-\overline{B_3N_4}(t)-\overline{S_4N_4}(t) \qquad \text{Eq. 4-5d}$$

$$\begin{aligned}\frac{dN_1'(t)}{dt}&=B_3'N_4'(t)+\overline{N_4}(t)B_3'+N_4'(t)\overline{B_3}\\&\quad-(B_1'\overline{N_1}(t)+\overline{B_1}N_1'(t)+B_1'N_1'(t))+\frac{S_3'}{d}\end{aligned} \qquad \text{Eq. 4-6a}$$

$$\frac{dN_2'(t)}{dt}=B_1'\overline{N_1}(t)-(B_2'\overline{N_2}(t)+\overline{B_2}N_2'(t)+B_2'N_2'(t)) \qquad \text{Eq. 4-6b}$$

$$\frac{dN_3'(t)}{dt}=B_2'\overline{N_2}(t) \qquad \text{Eq. 4-6c}$$

$$\begin{aligned}\frac{dN_4'(t)}{dt}&=-(B_3'\overline{N_4}(t)+S_4'\overline{N_4}(t)+\overline{B_3}N_4'(t)\\&\quad+B_3'N_4'(t)+\overline{S_4}N_4'(t)+S_4'N_4'(t))\end{aligned} \qquad \text{Eq. 4-6d}$$

4.2.2 Analytic Solution of Basic Equations

Application of the PDF/M technique requires the development of analytic solutions for equations 4-5a through 4-5d and 4-6a through 4-5d. To solve equations 4-5a through 4-5d, F_1 is first assumed to be constant. The assumption of F_1 as constant is required to find an analytic solution. The Gauss Seidel iterative technique is used in the model to find the value of F_1. The solutions for the equations are shown in Appendix A. Equation 4-6d is solved using Eq.4-2d yielding Eq. 4-7d. Equation 4-6a is solved, substituting Eq. 4-7d, Eq. 4-2d and Eq. 4-2a. If the effect of $N_1'(t)$ to $N_2'(t)$ is ignored, Equation 4-6b may be solved yielding Equation 4-7b . Using a similar technique, Equation 4-6c is solved by ignoring the effect of $N_2'(t)$ to $N_3'(t)$ in the equation. The detailed derivations of the equations are shown in Appendix A.

Eq. 4-7a

$$\begin{aligned}
N_1'(t) = {} & B_3'\{\frac{\alpha_1\rho A_o}{C_1-\alpha}(\frac{e^{-\alpha t}-e^{-B_1t}}{B_1-\alpha}-\frac{e^{-C_1t}-e^{-B_1t}}{B_1-C_1})+\overline{N_4}(0)\frac{e^{-C_1t}-e^{-B_1t}}{B_1-C_1})\\
& +\frac{S_3'}{dB_1}(1-e^{-B_1t})+N_1'(0)e^{-B_1t}+\frac{C_2\alpha_1\rho A_oB_3}{(C_1-\alpha)(C_3-\alpha)}\{\frac{e^{-C_3t}-e^{-B_1t}}{B_1-C_3}\\
& -\frac{e^{-\alpha t}-e^{-B_1t}}{B_1-\alpha}\}+\{\overline{N_4}(0)-\frac{A_o\alpha_1\rho}{C_1-\alpha}\}B_3\{\frac{e^{-C_3t}-e^{-B_1t}}{B_1-C_3}-\frac{e^{-C_1t}-e^{-B_1t}}{B_1-C_1}\}\\
& +\frac{N_4'(0)B_3}{B_1-C_3}(e^{-C_3t}-e^{-B_1t})-B_1'\{\frac{a_1-a_2}{\overline{B_1}-\alpha}(\frac{e^{-\alpha t}-e^{-B_1t}}{B_1-\alpha}-\frac{e^{-\overline{B_1}t}-e^{-B_1t}}{B_1'})\\
& +(\overline{N_1}(0)-\frac{\overline{S_3}}{d\overline{B_1}})\frac{e^{-\overline{B_1}t}-e^{-B_1t}}{B_1'}-\frac{a_1-\overline{N_4}(0)\overline{B_3}}{\overline{B_1}-C_1}(\frac{e^{-C_1t}-e^{-B_1t}}{B_1-C_1}\\
& -\frac{e^{-\overline{B_1}t}-e^{-B_1t}}{B_1'})+\frac{\overline{S_3}}{d\overline{B_1}B_1}(1-e^{-B_1t})\}
\end{aligned}$$

$$\begin{aligned}
N_2'(t) = {} & \frac{C_{11}B_1-C_{21}B_2'}{B_2-\alpha}(e^{-\alpha t}-e^{-B_2t})-\frac{C_{12}B_1-C_{22}B_2'}{B_2-C_1}(e^{-C_1t}-e^{-B_2t})\\
& +\frac{C_{14}B_1-C_{23}B_2'}{B_2}(1-e^{-B_2t})+\frac{(-C_{11}+C_{12}+C_{13})B_1-C_{24}B_2'}{B_2-\overline{B_1}}\\
& (e^{-\overline{B_1}t}-e^{-B_2t})+N_2'(0)e^{-B_2t}-(C_{21}-C_{22}+C_{23}+C_{24}-\overline{N_2}(0))\\
& \frac{e^{-\overline{B_2}t}-e^{-B_2t}}{B_2'}
\end{aligned}$$

Eq. 4-7b

Eq. 4-7c

$$\begin{aligned}
N_3'(t) = {} & N_3'(0)-B_2'[\frac{C_{21}}{\alpha}(e^{-\alpha t}-1)+\frac{C_{22}}{C_1}(e^{-C_1t}-1)-\frac{1}{\overline{B_1}}C_{24}(e^{-\overline{B_1}t}-1)\\
& +\frac{1}{\overline{B_2}}(C_{21}-C_{22}+C_{23}+C_{24}-\overline{N_2}(0))(e^{\overline{B_2}t}-1)]
\end{aligned}$$

$$\begin{aligned}
N_4'(t) = {} & \frac{(B_3'+S_4')A_o\alpha\rho}{(C_1-\alpha)(B_3+S_4-\alpha)}(e^{-(B_3+S_4)t}-e^{\alpha t})+(\overline{N_4}(0)-\frac{A_o\alpha\rho}{C_1-\alpha})\\
& (e^{-(B_3+S_4)t}-e^{-C_1t})+N_4'(0)e^{-(B_3+S_4)t}
\end{aligned}$$

Eq. 4-7d

4.2.3 Derivation of Moments of the Distribution

Following the rules and definitions shown in equations B-1 through B-5 in Appendix B, the variance of process is derived. For the derivations, it is assumed that the stochastic terms, shown below, are mutually independent and normally distributed with means of zero.

$$\overline{N_1}(0),\ \overline{N_2}(0),\ \overline{N_3}(0),\ \overline{N_4}(0),\ S_3,\ S_4,\ B_1,\ B_2,\ B_3$$

The detailed derivations of the equations are shown in Appendix B.

$$\begin{aligned}
V[N_1'] &= E[N_1'^2] - (E[N_1'])^2 \\
&= \frac{1}{(\overline{B_1}-\alpha)^2+\sigma_{B_1}{}^2}\,[m_1^2\sigma_{B_3}{}^2+\frac{m_1\sigma_{C_2}{}^2}{\sigma_{\phi}{}^2}(\overline{B_3}^2+\sigma_{B_3}{}^2) \\
&\quad +U_1^2\sigma_{B_1}{}^2-2\frac{m_1^2\overline{B_3}\sigma_{B_3}{}^2}{C_1-\alpha}]\,(e^{-2\alpha t}+E2B_1-2EB_1\alpha) \\
&\quad +\frac{1}{(\overline{B_1}-C_1)^2+\sigma_{B_1}{}^2}\,[U_3^2\overline{B_3}^2+U_2^2\sigma_{B_1}{}^2] \\
&\quad *(e^{-2C_1t}+E2B_1-2EC_1B_1) \\
&\quad +\frac{1}{(\overline{B_1}-C_1)^2+\sigma_{B_1}{}^2+\sigma_{C_2}{}^2}\,(\,[m_1^2\frac{\sigma_{C_2}{}^2}{\sigma_{\phi}{}^2}+U_3^2+\sigma_{N_4(0)}{}^2] \\
&\quad *(\overline{B_3}^2+\sigma_{B_3}{}^2)+4\frac{m_1U_3\overline{B_3}\sigma_{B_3}{}^2}{C_1-\alpha})\,(E2C_3+E2B_1-2EC_3B_1) \\
&\quad +[\frac{(\sigma_{S_3}{}^2+\frac{\overline{S_3}^2\sigma_{B_1}{}^2}{\overline{B_1}^2})}{d^2(\overline{B_1}^2+\sigma_{B_1}{}^2)}+\sigma_{N_1(0)}{}^2+U_4^2]\,E2B_1 \\
&\quad +[\frac{(\sigma_{S_3}{}^2+\frac{\overline{S_3}^2\sigma_{B_1}{}^2}{\overline{B_1}^2})}{d^2(\overline{B_1}^2+\sigma_{B_1}{}^2)}+U_4^2e^{-2\overline{B_1}t}]
\end{aligned} \qquad \text{Eq. 4-8a}$$

(Eq. 4-8a ...Continued)

$$-\frac{1}{(\overline{B_1}-\alpha)(\overline{B_1}-C_1)+\sigma_{B_1}{}^2}[U_1U_2\sigma_{B_1}{}^2+m_1U_3\sigma_{B_3}{}^2+\frac{U_3m_1\overline{B_3}\sigma_{B_3}{}^2}{C_1-\alpha}]$$

$$*(E2B_1+e^{-(\alpha+C_1)t}-EB_1C_1-EB_1\alpha)$$

$$+\frac{1}{(\overline{B_1}-\alpha)(\overline{B_1}-C_1)+\sigma_{B_1}{}^2}[m_1U_3\sigma_{B_3}{}^2-\frac{m_1^2\sigma_{C_2}{}^2(\overline{B_3}^2+\sigma_{B_3}{}^2)}{\sigma_{\phi}^2}$$

$$-\frac{2*m_1U_3\overline{B_3}\sigma_{B_3}{}^2}{C_1-\alpha}+\frac{m_1^2\sigma_{B_3}{}^2\overline{B_3}}{C_1-\alpha}](E2B_1+EC_3\alpha-EB_1C_3-EB_1\alpha)$$

$$+[\frac{U_4m_1\sigma_{B_3}{}^2}{(C_1-\alpha)(\overline{B_1}-\alpha)}-\frac{U_1\overline{S_3}\sigma_{B_1}{}^2}{d\overline{B_1}(\overline{B_1}-\alpha)(\overline{B_1^2}-\overline{B_1}\alpha+\sigma_{B_1}{}^2)}](EB_1\alpha-E2B_1)$$

$$+[-\frac{U_4m_1\sigma_{B_3}{}^2}{(C_1-\alpha)(\overline{B_1}-\alpha)}e^{-\overline{B_1}t}+\frac{U_1\overline{S_3}\sigma_{B_1}{}^2}{d\overline{B_1}(\overline{B_1}-\alpha)(\overline{B_1^2}-\alpha\overline{B_1}+\sigma_{B_1}{}^2)}](e^{-\alpha t}-EB_1)$$

$$-\frac{1}{(\overline{B_1}-C_1)^2+\sigma_{B_1}{}^2}[U_3^2\overline{B_3}^2+\frac{m_1U_3\overline{B_3}\sigma_{B_3}{}^2}{C_1-\alpha}]$$

$$*(E2B_1+EC_1C_3-EC_1B_1-EC_3B_1)$$

$$+\frac{1}{\overline{B_1}-C_1}[U_4U_3\overline{B_3}+U_2U_5](EB_1C_1-E2B_1)$$

$$+\frac{1}{\overline{B_1}-C_1}[-U_4U_2e^{-\overline{B_1}t}\overline{B_3}-U_2U_5](e^{-C_1t}-EB_1)$$

$$-\frac{1}{\overline{B_1}-C_1}[U_4U_3\overline{B_3}+\frac{m_1U_4\sigma_{B_3}{}^2}{C_1-\alpha}](EB_1C_3-E2B_1)$$

$$+\frac{1}{\overline{B_1}-C_1}[U_4U_3\overline{B_3}+\frac{m_1U_4\sigma_{B_3}{}^2}{C_1-\alpha}]e^{-\overline{B_1}t}(EC_3-EB_1)$$

$$-[U_4^2e^{-\overline{B_1}t}+\frac{\sigma_{S_3}{}^2}{d^2(\overline{B_1}^2+\sigma_{B_1^2})}+\frac{\overline{S_3}U_5}{d(\overline{B_1}^2+\sigma_{B_1}{}^2)}]EB_1$$

$$-\{-\frac{m_1\sigma_{B_3}{}^2}{(C_1-\alpha)(\overline{B_1}-\alpha)}(e^{-\alpha t}-EB_1)-\frac{(\overline{N_4}(0)-m_1)\overline{B_3}}{\overline{B_1}-C_1}(e^{-C_1t}-EB_1)$$

$$+(\frac{2m_1\overline{B_3}\sigma_{B_3}{}^2}{(C_1-\alpha)(\overline{B_1}-C_1)-\sigma_{B_3}{}^2-\sigma_{S_4}{}^2}+\frac{(\overline{N_4}(0)-m_1)\overline{B_3}}{\overline{B_1}-C_1})(EC_3-EB_1)$$

$$-U_4EB_1+U_4e^{-\overline{B_1}t}\}^2$$

$$
\begin{aligned}
V[N_2'] - E[N_2'^2] - (E[N_2'])^2 \\
&- \frac{C_{11}^2 V_1 + C_{21}^2 \sigma_{B_2}{}^2}{(\overline{B_2}-\alpha)^2 + \sigma_{B_2}{}^2} (e^{-2\alpha t} + E2B_2 - 2EB_2\alpha) \\
&+ \frac{C_{12}^2 V_1 + C_{22}^2 \sigma_{B_2}{}^2}{(\overline{B_2}-C_1)^2 + \sigma_{B_2}{}^2} (e^{-2C_1 t} + E2B_2 - 2EC_1B_2) \\
&+ \frac{C_{14}^2 V_1 + C_{23}^2 \sigma_{B_2}{}^2}{V_4} (1 + E2B_2 - 2EB_2) \\
&+ \frac{V_3^2 V_1 + C_{24}^2 \sigma_{B_2}{}^2}{(\overline{B_2}-\overline{B_1})^2 + \sigma_{B_2}{}^2} (e^{-2\overline{B_1} t} + E2B_2 - 2EB_1B_2) \\
&+ \frac{V_2^2}{\sigma_{B_2}{}^2} (e^{-2\overline{B_2} t} + E2B_2 - 2E\overline{B_2}B_2) + \sigma_{N_2(0)}{}^2 E2B_2 \\
&- \frac{C_{11}C_{12}V_1 + C_{21}C_{22}\sigma_{B_2}{}^2}{(\overline{B_2}-\alpha)(\overline{B_2}-C_1) + \sigma_{B_2}{}^2} (e^{-C_1\alpha t} + E2B_2 - EB_2\alpha - EB_2C_1)
\end{aligned}
$$

Eq. 4-8b

$$
\begin{aligned}
&+ \frac{C_{11}C_{14}V_1 + C_{21}C_{23}\sigma_{B_2}{}^2}{V_4 - \alpha\overline{B_2}} (e^{-\alpha t} + E2B_2 - EB_2 - EB_2\alpha) \\
&+ \frac{C_{11}V_3V_1 + C_{21}C_{24}\sigma_{B_2}{}^2}{(\overline{B_2}-\alpha)(\overline{B_2}-\overline{B_1}) + \sigma_{B_2}{}^2} (e^{-\overline{B_1}\alpha t} + E2B_2 - EB_2\alpha - EB_2\overline{B_1}) \\
&- \frac{C_{11}V_2\overline{B_1}}{\sigma_{B_2}{}^2} (e^{(\overline{B_2}+\alpha)t} + E2B_2 - EB_2\alpha - EB_2\overline{B_2}) \\
&- \frac{C_{12}C_{14}V_1 + C_{22}C_{23}\sigma_{B_2}{}^2}{V_4 - C_1\overline{B_2}} (e^{-C_1 t} + E2B\_EB_2 - EC_1B_2) \\
&- \frac{C_{12}V_3V_1 + C_{22}C_{24}\sigma_{B_2}{}^2}{(\overline{B_2}-\overline{B_1})(\overline{B_2}-\overline{C_1}) + \sigma_{B_2}{}^2} (e^{-(C_1+\overline{B_1})t} + E2B_2 - EB_2C_1 - EB_2\overline{B_1}) \\
&+ \frac{C_{12}V_2\overline{B_1}}{\sigma_{B_2}{}^2} (e^{-(C_1+\overline{B_2})t} + E2B_2 - EB_2C_1 - EB_2\overline{B_2}) \\
&+ \frac{C_{14}V_3V_1 + C_{23}C_{24}\sigma_{B_2}{}^2}{V_4 - \overline{B_1B_2}} (e^{-\overline{B_1}t} + E2B2 - EB_2 - EB_2\overline{B_1}) \\
&- \frac{C_{14}V_2\overline{B_1}}{\sigma_{B_2}{}^2} (e^{-\overline{B_2}t} + E2B_2 - EB_2 - EB_2\overline{B_2}) \\
&- \frac{V_3V_2\overline{B_1}}{\sigma_{B_2}{}^2} (e^{-(\overline{B_1}+\overline{B_2})t} + E2B_2 - EB_2\overline{B_1} - EB_2\overline{B_2})
\end{aligned}
$$

(Eq. 4-8b...continued)

$$-\{\frac{C_{11}\overline{B_1}}{\overline{B_2}-\alpha}(e^{-\alpha t}-EB_2)-\frac{C_{12}\overline{B_1}}{\overline{B_2}-C_1}(e^{-C_1 t}-EB_2)+\frac{C_{14}\overline{B_1}}{\overline{B_2}}(1-EB_2)$$
$$+\frac{U_3\overline{B_1}}{\overline{B_2}-\overline{B_1}}(e^{-\overline{B_1}t}-EB_2)+U_2(e^{-\overline{B_2}t}-EB_2)\}^2$$

$$V[N_3'] = E[N_3'^2]-(E[N_3'])^2$$
$$= [-\frac{C_{21}}{\alpha}(e^{-\alpha t}-1)+\frac{C_{22}}{C_1}(e^{-C_1 t}-1)$$
$$-\frac{C_{24}}{\overline{B_1}}(e^{-\overline{B_1}t}-1)+\frac{1}{\overline{B_2}}C_{34}(e^{-\overline{B_2}t}-1)]^2\sigma_{B_2}{}^2+\sigma_{N_3(0)}{}^2 \qquad \text{Eq. 4-8c}$$

$$V[N_4'] = E[N_4'^2]-(E[N_4'])^2$$
$$= \frac{\sigma_{C_2}{}^2(A_o\alpha_1\rho)^2}{(C_1-\alpha)^2((C_1-\alpha)^2+E[C_2^2])}(E2C_3+e^{-2\alpha t}-2EC_3\alpha)$$
$$+(\overline{N_4}(0)-\frac{A_o\alpha_1\rho}{C_1-\alpha})^2(E2C_3+e^{-2C_1 t}-2EC_1C_1) \qquad \text{Eq. 4-8d}$$
$$+\sigma_{N_4(0)}{}^2E2C_3$$

$$-\{(\overline{N_4}(0)-\frac{A_0\alpha\rho}{C_1-\alpha})e^{-C_1 t}(e^{\sigma_{B_3}{}^2t^2/2\sigma_{S_4}{}^2t^2/2}-1)\}^2$$

where,

$U_1 = \frac{a_1 - a_2}{\overline{B_1} - \alpha}$

$U_2 = \frac{a_1 - \overline{N_4}(0)\overline{B_3}}{\overline{B_1} - C_1}$

$U_3 = \overline{N_4}(0) - m_1$

$U_5 = \frac{\overline{S_3}\sigma_{B_1}^2}{d\overline{B_1^2}}$

$U_4 = U_1 - U_2 - \overline{N_1}(0) + \frac{\overline{S_3}}{d\overline{B_1}}$

$V_1 = \overline{B_1}^2 + \sigma_{B_1}^2$

$V_2 = C_{34}$

$V_3 = -C_{11} + C_{12} + C_{13}$

$\sigma_{\psi}^2 = (C_1 - \alpha)^2 + \sigma_{B_3}^2 + \sigma_{S_4}^2$

$\sigma_{C_2}^2 = \sigma_{B_3}^2 + \sigma_{S_4}^2$

$V_4 = \overline{B_2^2} + \sigma_{B_2}^2$

$m_1 = \frac{\alpha_1 \rho A_o}{C_1 - \alpha}$

$C_1 = \overline{B_3} + \overline{S_4}$

$C_2 = B_3' + S_4'$

$C_3 = C_1 + C_2$

$EB_1 = e^{-\overline{B_1}t} e^{\sigma_{B_1}^2 \frac{t^2}{2}}$

$EB_2 = e^{-\overline{B_2}t} e^{\sigma_{B_2}^2 \frac{t^2}{2}}$

$E2B_1 = e^{-2\overline{B_1}t} e^{2\sigma_{B_1}^2 t^2}$

$E2B_2 = e^{-2\overline{B_2}t} e^{2\sigma_{B_2}^2 t^2}$

$EC_3 = e^{-C_1 t} e^{\sigma_{B_3}^2 \frac{t^2}{2}} e^{\sigma_{S_4}^2 t^2/2}$

$E2C_3 = e^{-2C_1 t} e^{2\sigma_{B_3}^2 t^2} e^{2\sigma_{S_4}^2 t^2}$

$EB_1C_1 = EB_1 * e^{-C_1 t}$

$EB_1\alpha = EB_1 * e^{-\alpha t}$

$EC_3C_1 = EC_3 * e^{-C_1 t}$

$EB_1C_3 = EB_1 * EC_3$

$EB_2C_1 = EB_2 * e^{-C_1 t}$

$EB_2\alpha = EB_2 * e^{-\alpha t}$

$EB_2\overline{B_1} = EB_2 * e^{-\overline{B_1}t}$

$EB_2\overline{B_2} = EB_2 * e^{-\overline{B_2}t}$

$EC_3\alpha = EC_3 * e^{-\alpha t}$

$EC_1B_1 = EB_1C_1$

$EC_1B_3 = EB_3C_1$

$EC_1C_3 = EC_3C_1$

4.2.4 Derivation of the Probability Density Functions

The solution to the 1-D Fokker-Planck Equation with time variant coefficients and probabilistic initial conditions can be written as follows:

$$P(C,t)=\int_{\overline{x_o}}\frac{1}{\sqrt{4\pi Z_2}}e^{\left(\frac{-(C-Z_1)^2}{4Z_2}\right)}d\overline{X_o} \qquad \text{Eq. 4-9}$$

Z_1 and Z_2 are functions of the initial conditions (x_o). Using the definitions outlined in the previous chapter with Equation 3-16, the probability density function for four Nitrogens are obtained as:

$$P(N_1,t)=\int_{N_{1_{min}}}^{N_{1_{max}}}\int_{N_{4_{min}}}^{N_{4_{max}}}\int_{\sigma_{4_{min}}}^{\sigma_{4_{max}}}\int_{\sigma_{3_{min}}}^{\sigma_{3_{max}}}\int_{B_{3_{min}}}^{B_{3_{max}}}\int_{B_{1_{min}}}^{B_{1_{max}}} \frac{1}{\sqrt{4\pi Z_{2N_1}}}e^{\left(\frac{-(C-Z_{1N_1})^2}{4Z_{2N_1}}\right)}\delta N_{1_0}\delta N_{4_0}\delta\sigma_4\delta\sigma_3\delta B_3\delta B_1 \qquad \text{Eq. 4-10a}$$

$$P(N_2,t)=\int_{N_{2_{min}}}^{N_{2_{max}}}\int_{N_{1_{min}}}^{N_{1_{max}}}\int_{N_{4_{min}}}^{N_{4_{max}}}\int_{\sigma_{4_{min}}}^{\sigma_{4_{max}}}\int_{\sigma_{3_{min}}}^{\sigma_{3_{max}}}\int_{B_{3_{min}}}^{B_{3_{max}}}\int_{B_{2_{min}}}^{B_{2_{max}}}\int_{B_{1_{min}}}^{B_{1_{max}}} \frac{1}{\sqrt{4\pi Z_{2N_2}}}e^{\left(\frac{-(C-Z_{1N_2})^2}{4Z_{2N_2}}\right)}\delta N_{2_0}\delta N_{1_0}\delta N_{4_0}\delta\sigma_4\delta\sigma_3\delta B_3\delta B_2\delta B_1 \qquad \text{Eq. 4-10b}$$

$$P(N_3,t)=\int_{N_{3_{\min}}}^{N_{3_{\max}}}\int_{N_{2_{\min}}}^{N_{2_{\max}}}\int_{N_{1_{\min}}}^{N_{1_{\max}}}\int_{N_{4_{\min}}}^{N_{4_{\max}}}\int_{\sigma_{4_{\min}}}^{\sigma_{4_{\max}}}\int_{\sigma_{3_{\min}}}^{\sigma_{3_{\max}}}\int_{B_{3_{\min}}}^{B_{3_{\max}}}\int_{B_{2_{\min}}}^{B_{2_{\max}}}\int_{B_{1_{\min}}}^{B_{1_{\max}}} \frac{1}{\sqrt{4\pi Z_{2N_3}}}e^{\left(\frac{-(C-Z_{1N_3})^2}{4Z_{2N_3}}\right)}\delta N_{3_0}\delta N_{2_0}\delta N_{1_0}\delta N_{4_0}\delta\sigma_4\delta\sigma_3\delta B_3\delta B_2\delta B_1 \quad \text{Eq. 4-10c}$$

$$P(N_4,t)=\int_{N_{4_{\min}}}^{N_{4_{\max}}}\int_{\sigma_{4_{\min}}}^{\sigma_{4_{\max}}}\int_{B_{3_{\min}}}^{B_{3_{\max}}}\frac{1}{\sqrt{4\pi Z_{2N_4}}}e^{\left(\frac{-(C-Z_{1N_4})^2}{4Z_{2N_4}}\right)}\delta N_{4_0}\delta\sigma_4\delta B_3 \quad \text{Eq. 4-10d}$$

Where:

$Z_{1N_1}=$ mean of $N_1(t)$

$Z_{2N_1}=\frac{1}{2}$ variance of $N_1(t)$

$Z_{1N_2}=$ mean of $N_2(t)$

$Z_{2N_2}=\frac{1}{2}$ variance of $N_2(t)$

$Z_{1N_3}=$ mean of $N_3(t)$

$Z_{2N_3}=\frac{1}{2}$ variance of $N_3(t)$

$Z_{1N_4}=$ mean of $N_4(t)$

$Z_{2N_4}=\frac{1}{2}$ variance of $N_4(t)$

The solutions to the above integrals were evaluated numerically.

CHAPTER 5

RESULTS AND COMPARISON OF METHODS

To demonstrate the validity and feasibility of applying the new method, the model defined by equations 4-2a through 4-2d and 4-10a through 4-10d was used to model Nitrogen in the Withlacoochee River in Florida. The input data used in QUAL2E-UNCAS is included in Appendix C.

5.1 Case Study Description

Data obtained from a USEPA survey of the Withlacoochee River during October 1984 and presented in the QUAL2E-UNCAS example application were used (Brown and Barnwell, 1987). In the original study, the results of QUAL2E-UNCAS were compared to field data and found to correlate very well. A thirty mile stretch of the river (see Fig. 5-1) was modeled from RM26 (Travel Time 0.52 days) to RM2 (Travel Time 7.91 days). The river has a uniform low slope with alternating shoals and pools. Average depths were 5.2 to 14.8 ft and widths were 90 to 140 ft with flow of 150 cfs at the headwater. The example application in the USEPA application simulated ten state variables which were temperature, dissolved oxygen, carbonaceous BOD, two phosphorus forms, (organic and dissolved), algae and four nitrogen forms, (organic, ammonia,

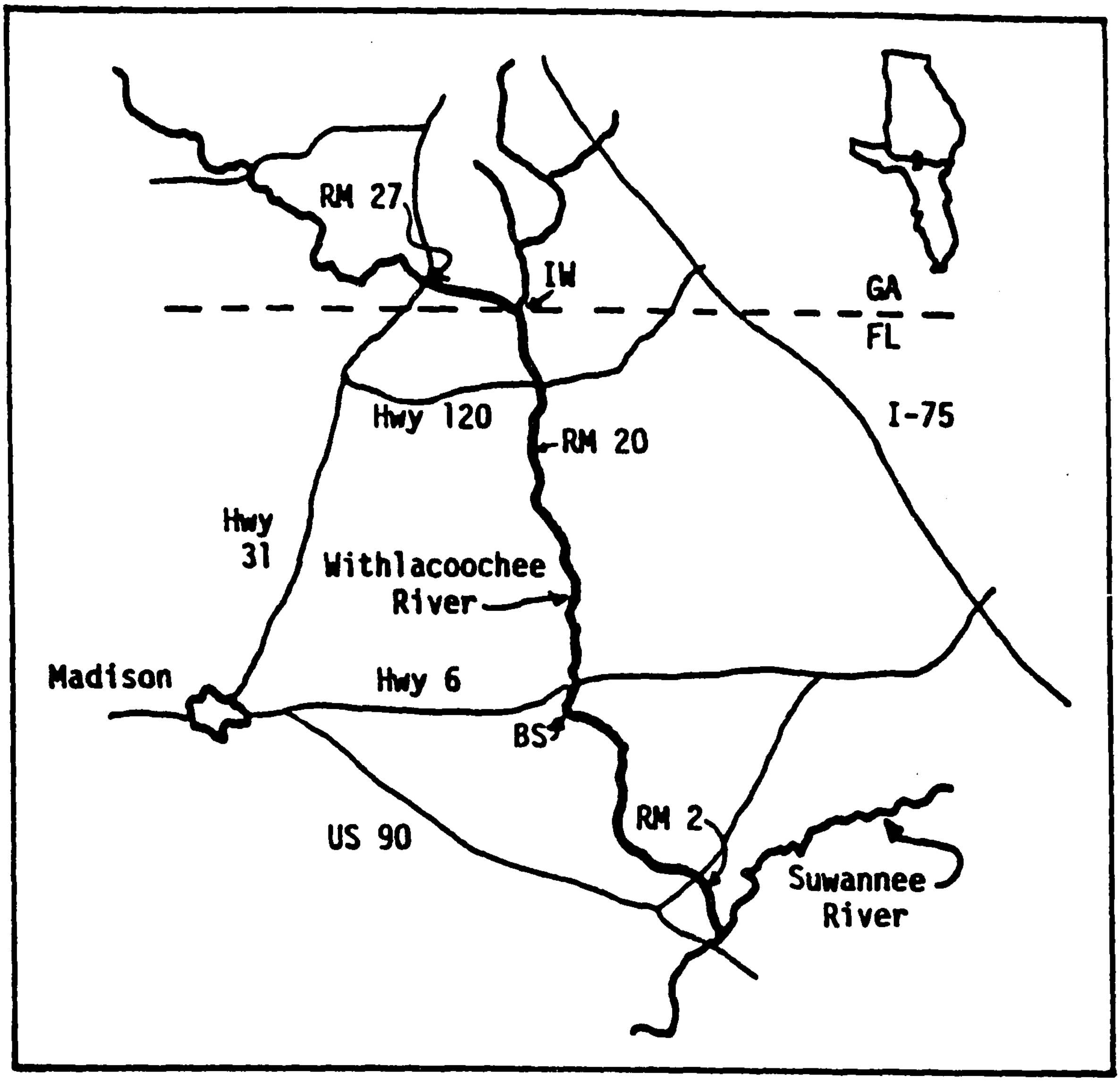


Fig. 5-1 Location Map of the Withlacooche River Basin (Taken from Brown and Barnwell, 1987)

nitrite, nitrate).

To satisfy the purpose of the present application, the state variables were reduced to the four nitrogen forms. The state variables and constants used in the present application are shown in Table 5-1 and 5-2.

In the PDF/M model development, the nine input variables of interest ($N_1(t)$, $N_2(t)$, $N_3(t)$, $N_4(t)$, B_1, B_2, B_3, σ_3, σ_4) are assumed to be independent and normally distributed. The values for these parameters are shown in Table 5-2. The mean values of the above parameters are the mean values used for the headwater in QUAL2E-UNCAS and the standard deviation of the values are calculated in QUAL2E-UNCAS using relative standard deviations.

Table 5-1 Values used for Normal Distributions

Variables	Mean	Standard Deviation
$N_1(0)$ (mg/L)	0.050	0.005
$N_2(0)$ (mg/L)	0.002	0.001
$N_3(0)$ (mg/L)	0.100	0.007
$N_4(0)$ (mg/L)	0.350	0.021
B_1 (1/day)	0.500	0.125
B_2 (1/day)	10.000	2.000
B_3 (1/day)	0.040	0.008
S_3 (mg/ft^{2}day)	0.000	0.000
S_4 (1/day)	0.000	0.000

Table 5-2 Values used for constants

Constants	Value
A_0 (mg/L)	0.030
μ_{max} (1/day)	1.307
α_1 (1/day)	0.085
S_1 (1/day)	1.000
ρ (1/day)	0.150
d (feet)	8.000
P	0.500

The Monte Carlo method in QUAL2E-UNCAS with examination of stochastic variations in the nitrogen cycle was used (Brown and Barnwell, 1987). Complete listing of the input variables required by the QUAL2EU of the values used for the modeling study are shown in Appendix C. Three hundred iterations and five hundred Monte Carlo iterations were performed in this study. There were no significant differences to the results generated by increasing the number of iterations. The result from the five hundred iteration were adopted for the comparison with PDF/M technique result. To reduce the number of state variables and simplify the problem , the original input data from QUAL2E-UNCAS example (Brown and Barnwell, 1987) application were modified by ignoring point loads along the river. The point loads were ignored for the simplicity of the modeling. Results of this simulation are included in Appendix D.

5.2 Application of the PDF/M modeling

The PDF/M computer model used the identical situation simulated using QUAL2E-UNCAS. Refer to Appendix E for the PDF/M computer model. An IBM 286(AT) compatible computer with a math coprocessor was used to run both the Monte Carlo and PDF/M models. The approximate run time was five hours for the monte carlo method and eight hours for the PDF/M method. A complete listing of the input and output from the model are included in Appendix F.

5.3 Comparison of Moments

The first moment and second moment were calculated for times 0.52 days (RM 26), 2.09 days, 3.69 days (RM 20), and 7.91 days (RM 2). As can be seen in Tables 5-3 through 5-6, the estimation of the first moments for all four nitrogens match exactly the moments of QUAL2E-UNCAS. Some deviation can be seen in estimation of second moment. The deviation is more extensive for nitrite nitrogen and ammonia nitrogen. A little less than ten percent of deviation has occurred in comparison of nitrate nitrogen. It is speculated that the deviations occurred in the process of simplifying the model. For organic nitrogen, the results of second moment matches exactly with the QUAL2E-UNCAS results.

Table 5-3 Comparison of Moments of Ammonia Nitrogen

Time of Travel (day)	First (Mean) Moment (mg/L)		Second (Variance) Moment $(mg/L)^2$	
	Monte Carlo	New Method	Monte Carlo	New Method
0.00	0.050	0.050	0.0050	0.0050
0.52	0.045	0.045	0.0050	0.0045
2.09	0.035	0.035	0.0076	0.0063
3.69	0.029	0.030	0.0092	0.0073
7.91	0.027	0.027	0.0093	0.0075

Table 5-4 Comparison of Moments of Nitrite Nitrogen

Time of Travel (day)	First Moment (Mean) (mg/L)		Second Moment (Variance) $(mg/L)^2$	
	Monte Carlo	New Method	Monte Carlo	New Method
0.00	0.002	0.0020	0.00100	0.00100
0.52	0.002	0.0023	0.00064	0.00056
2.09	0.002	0.0018	0.00050	0.00045
3.69	0.001	0.0015	0.00040	*
7.91	0.001	0.0014	0.00030	*

* Computation was stopped by Exponential Argument Range Error; Too large of a real value was encountered.

Table 5-5 Comparison of Moments of Nitrate Nitrogen

Time of Travel (day)	First Moment (Mean) (mg/L)		Second Moment (Variance) $(mg/L)^2$	
	Monte Carlo	New Method	Monte Carlo	New Method
0.00	0.10	0.10	0.007	0.0070
0.52	0.11	0.11	0.007	0.0074
2.09	0.14	0.14	0.011	0.0112
3.69	0.17	0.17	0.013	0.0156
7.91	0.18	0.18	0.015	0.0178

Table 5-6 Comparison of Moments of Organic Nitrogen

Time of Travel (day)	First Moment (Mean) (mg/L)		Second Moment (Variance) $(mg/L)^2$	
	Monte Carlo	New Method	Monte Carlo	New Method
0.00	0.35	0.35	0.021	0.021
0.52	0.34	0.34	0.020	0.021
2.09	0.32	0.32	0.020	0.020
3.69	0.30	0.30	0.020	0.020
7.91	0.29	0.29	0.021	0.020

5.4 Algal Fraction Uptake

Comparison of fraction of algal nitrogen uptake ammonia pool (F_1) was also made. An initial value of F_1 was chosen to compute the mean value of four nitrogens until F_1 converges, then the converged value of F_1 was used for the final computation of the mean value of nitrogens (See Fig. 5-6.) The result of comparison of the fraction of the algal nitrogen uptake ammonia pool is shown in Table 5-7.

Table 5-7 Comparison of F_1

Time of Travel (day)	Monte Carlo	New Method
0.00	0.333	0.333
0.52	0.279	0.290
2.09	0.195	0.200
3.69	0.149	0.150
7.91	0.130	0.133

5.5 Comparison of Probability Density Functions

The cumulative density function for ammonia nitrogen, nitrate nitrogen, and organic nitrogen at travel times of 2.09 days (Station 2) and 3.69 days (Station 4) have been calculated. The results of this analysis are shown in Figures 5-2, 5-4 and 5-5. For nitrite nitrogen, the probability density function was calculated at travel times of 0.52 days (Station 2) and 2.09 days (Station 3). Results of this analysis are shown in Fig 5-3. As can be seen in Figures 5-2, 5-3, 5-4, and 5-5, the distribution of probability density function of the new technique agrees well with QUAL2E-UNCAS analysis for all nitrogens. A Kolmogoroff-Smirnov Test, a commonly used statistical test, was performed to compare the values at the Station 3, travel time of 2.09 days. The new technique agrees with the results of the Monte Carlo simulation at the 90 to 99 % confidence level. Refer to Appendix G for Kolmogoroff-Smirnov statistical test results at station 3.

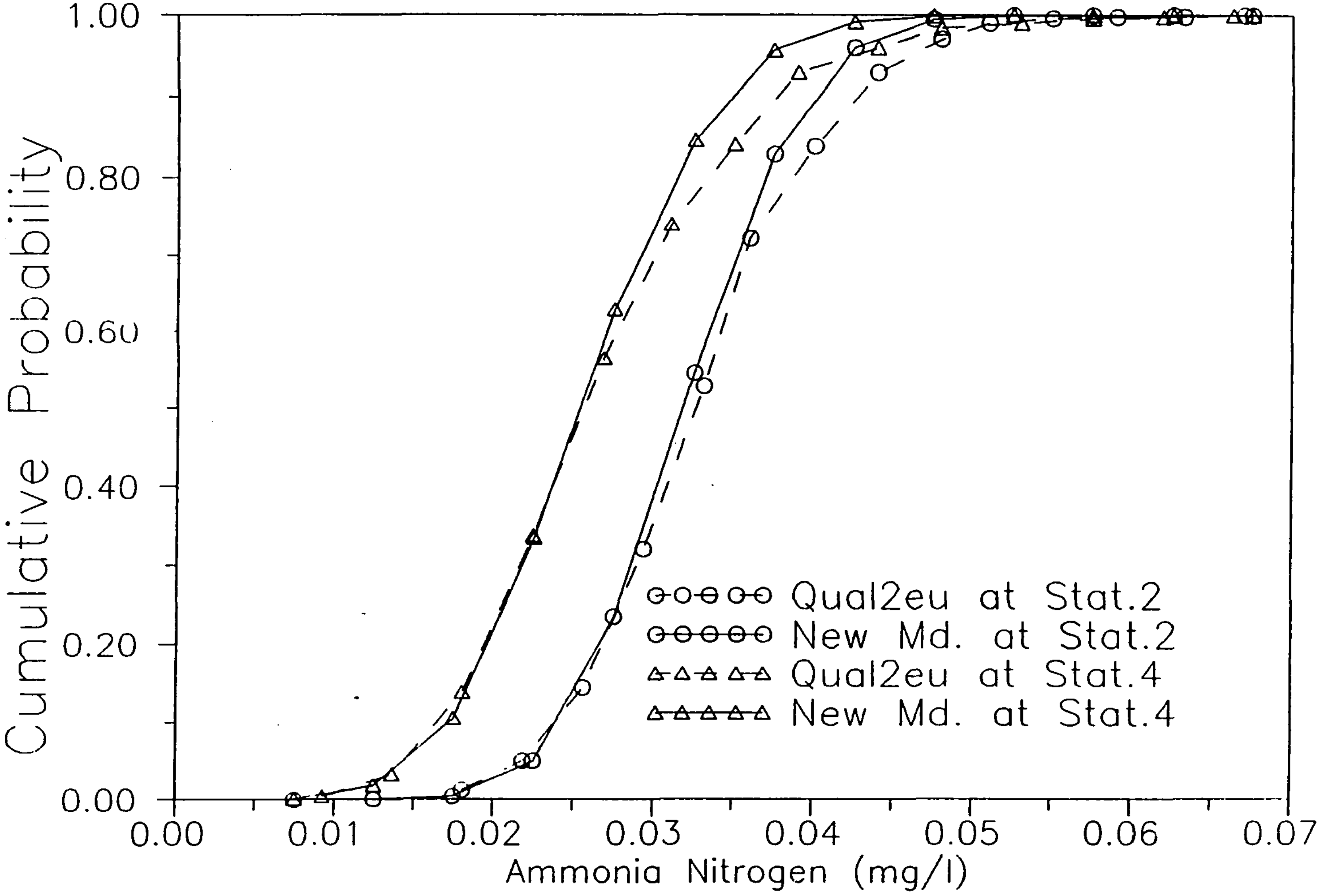


Fig. 5-2 Cumulative Density Functions for Ammonia-N

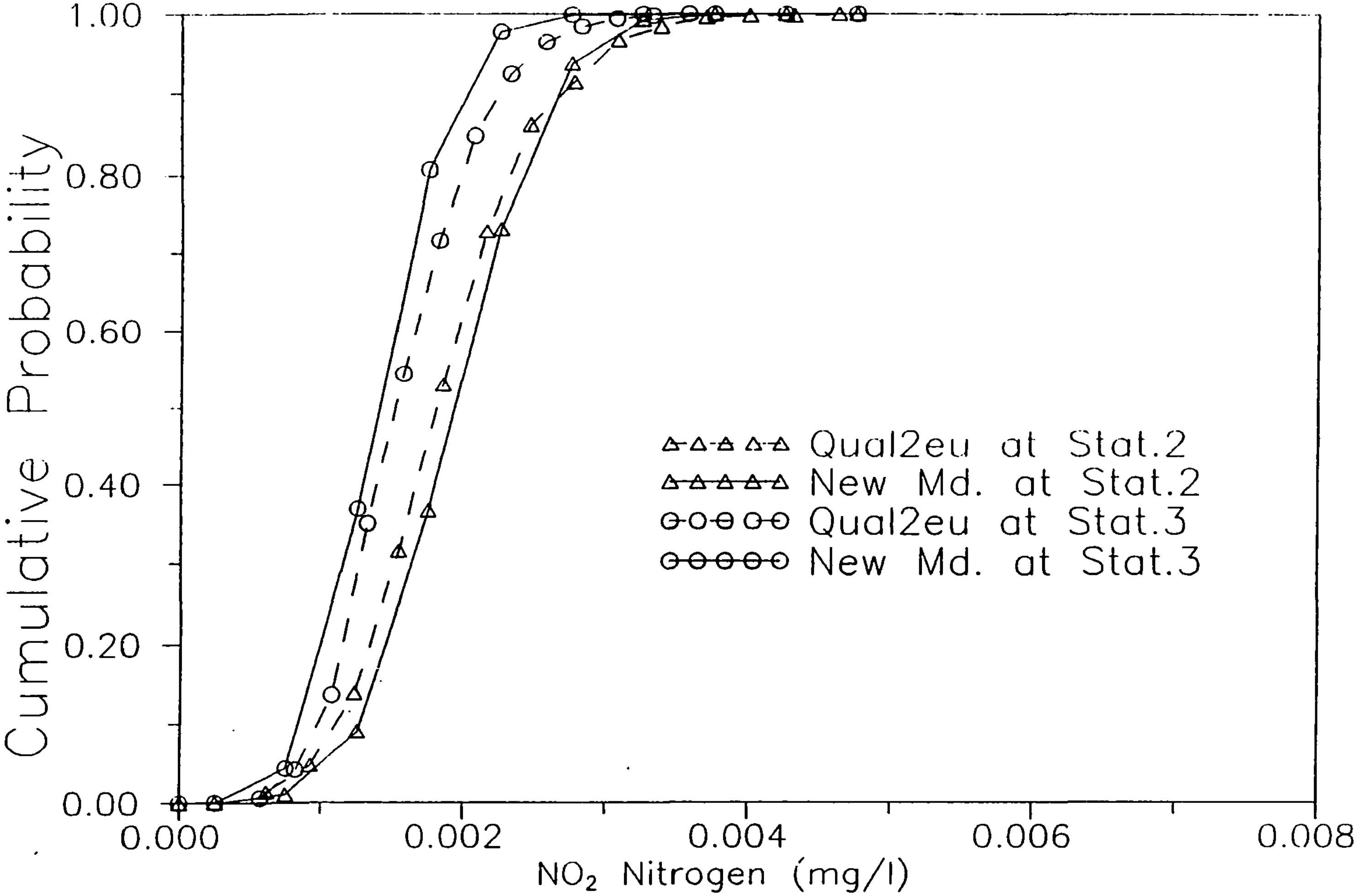


Fig. 5−3 Cumulative Density Functions for NO_2−N

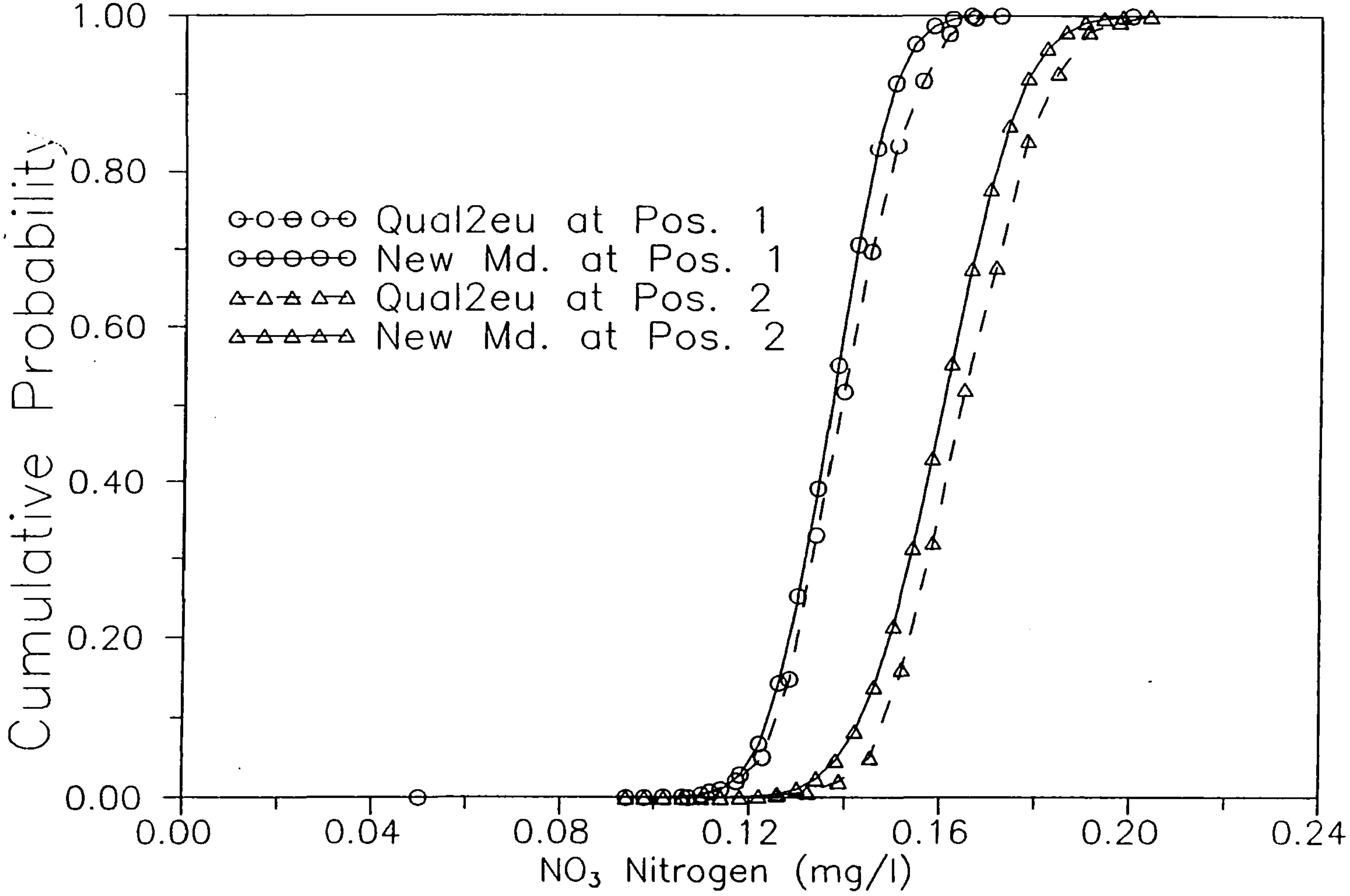


Fig. 5–4 Cumulative Density Functions for NO$_3$ N

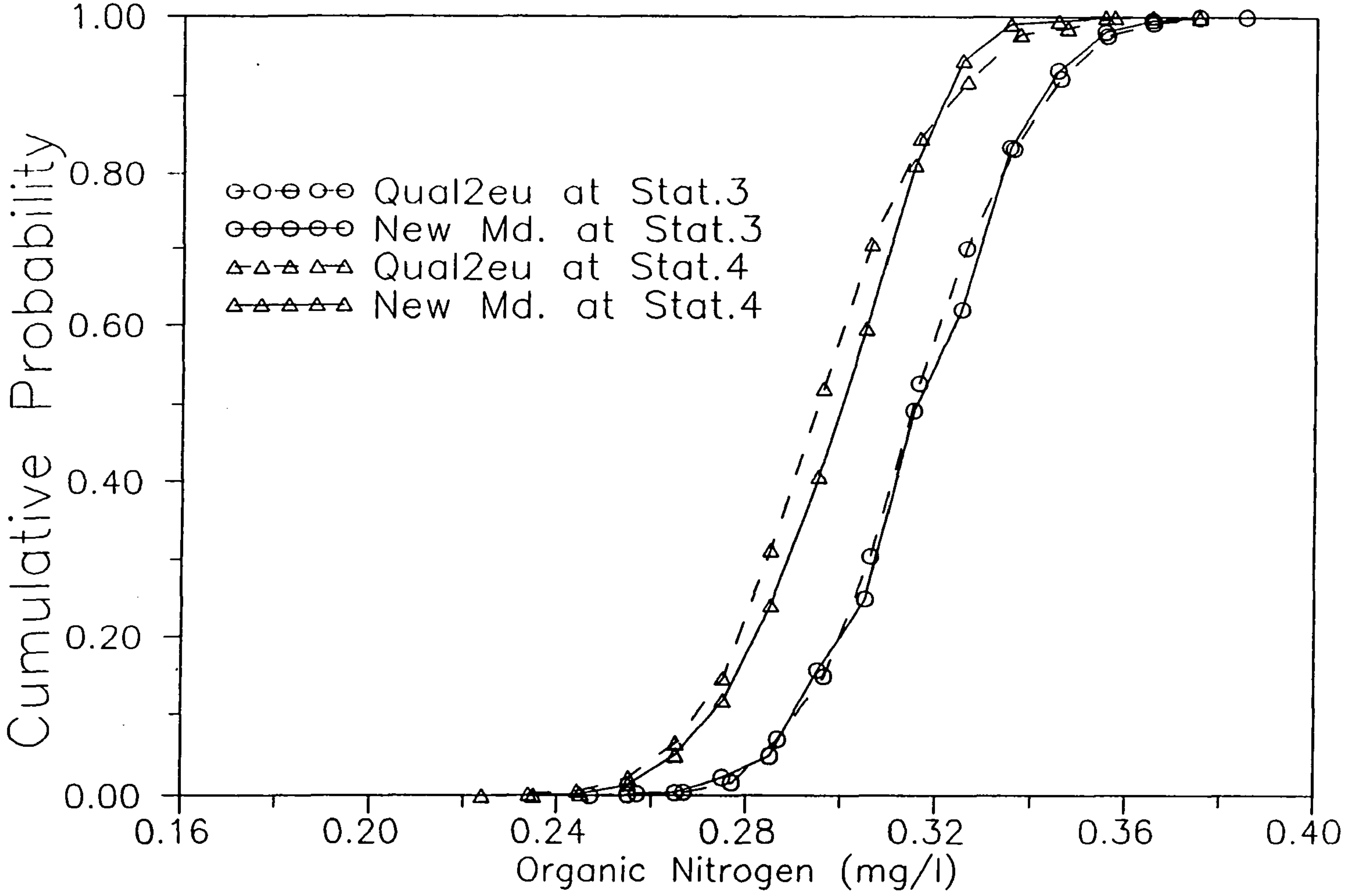
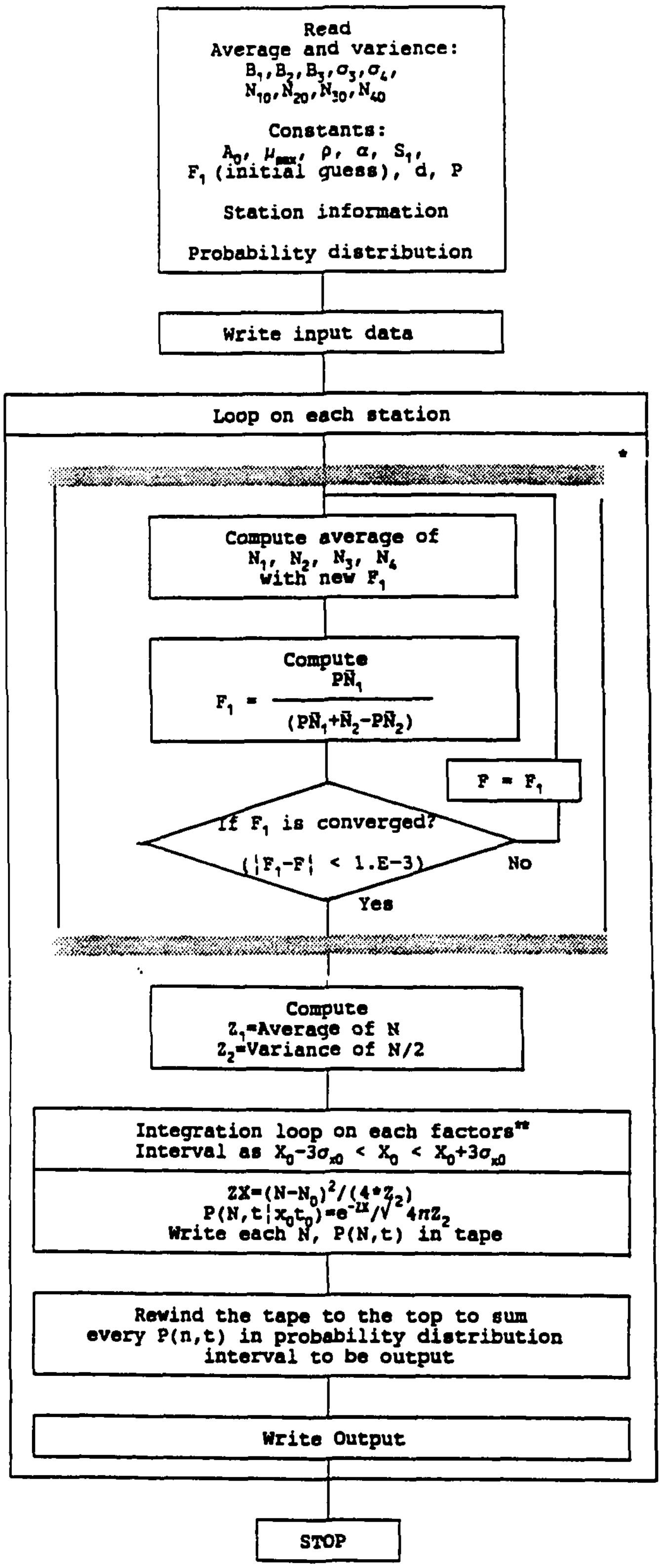


Fig. 5-5 Cumulative Density Functions for Organic N

Fig. 5-6 Flow Chart



* Skipt in Organic-Nitrogen program because Organic-Nitrogen equation isn't coupled with other nitrogens.

** Factors on each Nitrogen program are shown as:

N_1 ; $\sigma_3, \sigma_4, N_1, N_4$
N_2 ; $B_1, B_2, B_3, \sigma_3, \sigma_4$ N_1, N_2, N_4
N_3 ; $B_1, B_2, B_3, \sigma_3, \sigma_4$ N_1, N_2, N_3, N_4
N_4 ; σ_3, σ_4, N_4

CHAPTER 6

CONCLUSIONS

A set of four highly non-linear second order nitrogen equations were solved analytically using PDF/M method. The nonlinear part (F_1) of ammonia, nitrite and nitrate nitrogen was considered as a constant during the derivation of the model. A correct value of F_1 at each station was obtained by numerically iterating the values of F_1 and nitrogens.

Comparison of the present study to the results of the Monte Carlo method (first moment, second moment, and cumulative density functions) showed the application of PDF/M technique to nitrogen cycle simulation valid.

However, the complexity involved in the nitrogen equations has left the process of obtaining a set of analytical solutions very tedious and time consuming. As Tumeo and Orlob (1989) mentioned, PDF/M is not a replacement for existing methods but rather a supplement. Though more complexity is involved and more time is required for an analytical solution to nitrogen equations, the method is potentially a powerful tool for management and decision analysis. The method provides the modelers with an ability to perform a detailed examination of the sources of uncertainty and quantify their magnitudes.

It is the authors's hope that further studies on the PDF/M modeling of nitrogen cycle will find a direct numerical way of setting the variational values from nitrogen cycle equations. For example, at each time step of solving the stochastic numerical equations by Euler method, the solved variables will evaluate mean and variation of the variables numerically. Also, it is author's desire to find other techniques for treating the set of non-linear nitrogen cycle equation.

REFERENCES

Bendat, J. S. and Piersol, A. G., (1971), "Random Data: Analysis and Measurement Procedures," Wiley-Interscience a Division of John Wiley & Sons, Inc

Brown, L. C. and Barnwell, T. O., (1987), "The Enhanced Stream Water Quality Models QUAL2E and QUAL2E-UNCAS: Documentation and User Manual (EPA/600/3/87/007)," Environmental Research Laboratory Office Research and Development, U.S. Environmental Protection Agency

Dewey, R. J., (1984), "Application of Stochastic Dissolved Oxygen Model", Journal of Environmental Engineering, Vol. 110, No. 2, pp. 412-429

Esen, I. I. and Bennet, J. P., (1971), "Probabilistic Analysis of Dissolved Oxygen", Paper presented at the First International Symposium on Stochastic Hydraulics, Univ. of Pittsburgh, Pittsburgh, Pa.

Farlow, S. J., (1982), "Partial Differential Equations for Scientists & Engineers," John Wiley & Sons Inc.

Gardiner, C. W., (1985), "Handbook of Stochastic Methods for Physics, Chemistry and the Natural Sciences", Second edition, Springer-Verlag

Heidtke, T. M., Auer, M. T. and Canale, R. P., (1986), "Microcomputers and Water Quality Models: Access for Decision Makers", Journal WPCF, Vol. 58, No. 10, pp. 960-966

Lettenmaier, D. P. and Richey, J. E., (1979), "Theoretical Systems Ecology: Advances and Case Studies", Academic, New York Publishers, PP. 80-106

Najarian, T. O. and Harleman, D. R., (1977), "Real Time Simulation of Nitrogen Cycle in an Estuary", Journal of the Environmental Engineering, Vo. 103, No. EE4, pp. 523-538

Najarian, T. O. and Taft, J. L., (1981), "Nitrogen-Cycle Model for Aquatic Systems: Analysis", Journal of the Environmental Engineering, Vol. 107, No. EE6, pp. 1141-1156

Reckhow, K. H., (1973), "Perspectives on Lake Ecosystem Modeling", Ann Arbor Science, Ann Arbor, Mich., pp. 183-222

Scavia, D., Powers, W. F., Canale, R. P. and Moody, J. L., (1981), "Comparison of First-Order Error Analysis and Monte Carlo Simulation in Time-Dependent Lake eutrophication

Models", Water Resources Research, Vol. 17, No. 4. p. 1051-1059

Soong, T. T., (1973), "Random Differential Equations in Science and Engineering", Academic Press, New York, N.Y.

Thomann, R. V. and Mueller, J. A., (1987)," Principles of Surface Water Quality Modeling and Control," Harper & Row, Publishers, Inc.

Tumeo, M. A., (1988), "Stochastic Analysis of Response Functions in Environmental Modeling," Thesis, U.C. Davis

Tumeo, M. A. and Orlob, G. T., (1989), "An analytic Technique for Stochastic Analysis in Environmental Models," Water Resources Research, Vol. 25, No. 12, pp. 2417-2422

Warwick, J.J. and McDonnell, A. J., (1985), "Simultaneous In-Stream Nitrogen and D.O. Balancing," Journal of Environmental Engineering, Vol. 111, No. 4, pp. 401-431

Zielinski, P. A., (1988), "Stochastic Dissolved Oxygen Model," Journal of Environmental Engineering, Vol. 114, No. 1, pp. 74-90

BIBLIOGRAPHY

Beck, M. B., (1987), "Water Quality Modeling: A Review of the Analysis of Uncertainty", Water Resources Research, Vol. 23, No. 8, pp. 1393-1442

Berthouex, P. M. and Brown, L. C., (1969), "Monte Carlo Simulation of Industrial Waste Discharges," Journal of the Sanitary Engineering Division, Vol. 95, No. SA5, pp. 887-905

Camara, A. S. and Randall, C. W., (1983), "The Qual II Model", Journal of Environmental Engineering, Vol. 110, No. 5, pp. 993-996

Chadderton, R.A., Miller, A.C. and McDonnell, A.J., (1982), "Uncertainty Analysis of Dissolved Oxygen Model", Journal of the Environmental Engineering, Vol. 108, No. EE5, pp. 1003-1013

Conte, S. D. and De Boor, C., (1980), "Elementary Numerical Analysis- An Algorithmic Approach", Third edition, McGraw-Hill Co.

Curtis, E.J., Durrant, K. and Harman, M. M., (1975), "Nitrification in Rivers in the Trent Basin", Water Resources

Research, Vol. 9, pp. 255-268

Custer, S. W. and Krutchkoff, R. G., (1969), "Stochastic Model for BOD and DO in Estuaries", Journal of the Sanitary Engineering, Vol. 95, No. SA5, pp. 865-885

Di Toro, D. M., Thomann, R. V. and O'Connor, D. J., (1968), "Stochastic Model for BOD and DO in Streams," Journal of the Sanitary Engineering Division, Vol. 94, No. SA2, pp.426-431

Ellis, J. H., (1984), "Application of Stochasic Dissolved Oxygen Model", Journal of Environmental Engineering, Vol. 111, No. 3, pp. 391-393

Fujiwara, O., Puangmaha, W. and Hanaki, K., (1988), "River Basin Water Quality Management in Stochastic Environment", Journal of Environmental Engineering, Vol. 114, No. 4, pp. 864-877

Karlin, S. and Taylor, H. M., (1975), "A First Course in Stochastic Processes," Second edition, Academic Press, Inc.

Leduc, R., Unny, T. E. and McBean, E. A., (1986), "Stochastic Model First-Order BOD Kinetics", Water Resources

Research, Vol. 20, No. 5, pp. 625-632

Najarian, T. O., Kaneta, P. J., Taft, J. L. and Thatcher, M. L., (1984), "Application of Nitrogen-Cycle Model to Manasquan Estuary", Journal of Environmental Engineering, Vol. 110, No. 1, pp. 190-207

Pachner, J., (1984), "Handbook of Numerical Analysis Applications", Mc-Graw-Hill Co.

Peavy, H. S., Rowe, D. R. and Tchobanoglous, G., (1985), "Environmental Engineering," McGraw-Hill Inc.

Risken, H., (1984), "The Fokker-Planc Equation," Springer-Verlag

Stratton, F. E., (1968), "Ammonia Nitrogen Losses From Streams", Journal of Sanitary Engineering Division, Vol. 94, No. SA6, pp.1085-1092

Thayer, R. P. and Krutchkoff, R. G., (1967), "Stochastic Model for BOD and DO in Streams," Journal of Sanitary Engineering Division, Vol. 93, No. SA3, pp.59-72

Thomann, R. V., (1982), "Verification of Water Quality

Models" Journal of the Environmental Engineering Division, Vol. 108, No. EE5, pp. 923-940

Warwick, J. J. and Cale, W. G., (1986), "Effects of Parameter Uncertainty in Stream Modeling," Journal of Environmental Engineering, Vol. 112, No. 3, pp.479-489

Wezernak, C. T. and Gannon, J. J., (1968), "Evaluation of Nitrification in Streams," Journal of the Sanitary Engineering Division, Vol 94, No. SA5, pp. 883895

Williams, R.E. and Lewis, M.S., (1985), "Stream Model of Benthic Nitrification-Denitrification", Journal of Environmental Engineering, Vol. 112, No. 2, pp. 367-386

APPENDIX A

DERIVATION OF THE DIFFERENTIAL EQUATIONS

Appendix A details the derivation Equations 4-5a to 4-5d. Exactly the same approach is used in deriving Equations 4-2a to 4-2d, as they are equivalent in equation format. Equation 4-5d is solved first, as it is independent of all other equations. Then the solution to Equation 4-5d is used to solve 4-5a, the solution of which can be used in Equation 4-5b, and then Equation 4-5b is used to solve for Equation 4-5c.

Eq. 4-5a is rearranged and multiplied by $e^{\overline{B_1}t}$ yielding.

$$\left(e^{\overline{B_1}t}\frac{d\overline{N_1}}{dt}+\overline{B_1}e^{\overline{B_1}t}N_1\right)=\left(\overline{B_3N_4}+\frac{\overline{S_3}}{d}-F_1\alpha\mu A_0e^{-\alpha t}\right)e^{\overline{B_1}t} \qquad \text{Eq. A-1}$$

The both sides of the Equation A-1 are integrated as producing:

$$\int_{\overline{N_1}(0)}^{\overline{N_1}(t)}\frac{d}{dt}\left[e^{\overline{B_1}t}\overline{N_1}\right]=\int_o^t\left(\overline{B_3N_4}+\frac{\overline{S_3}}{d}-F_1\alpha\mu A_0e^{-\alpha t}\right)e^{\overline{B_1}t}dt \qquad \text{Eq. A-2}$$

Equation A-2 is then solved for $N_1(t)$ yielding:

$$\overline{N_1}(t)=e^{-\overline{B_1}t}\left[\int_o^t\left(\overline{B_3N_4}+\frac{\overline{S_3}}{d}-F_1\alpha_1\mu A_0e^{-\alpha t}\right)e^{\overline{B_1}t}dt+\overline{N_1}(0)\right] \qquad \text{Eq. A-3}$$

Assuming F_1 is constant, the solution of the Equation A-3 is:

$$\overline{N_1} = C_{11}(e^{-\alpha t}-e^{-\overline{B_1}t})-C_{12}(e^{-C_1 t}-e^{-\overline{B_1}t})+C_{13}e^{-\overline{B_1}t}+C_{14} \qquad \text{Eq. A-4}$$

Where,

$$C_{11} = \frac{(a_1-a_2)}{(\overline{B_1}-\alpha)} \qquad C_{12} = \frac{(a_1-\overline{N_4}(0)\overline{B_3})}{\overline{B_1}-C_1}$$

$$C_{13} = (\overline{N_1}(0)-\frac{\overline{S_3}}{d\overline{B_1}}) \qquad C_{14} = \frac{\overline{S_3}}{d\overline{B_1}}$$

Re-arranging equations 4-5(b,c,d) and multiplying by the integrating factors shown below each equation, the rest of the equations are developed as follows:

$$\overline{N_2}(t) = e^{-\overline{B_2}t}[\int_o^t \overline{B_1}e^{\overline{B_2}t}dt+\overline{N_2}(0)] \qquad \text{Eq. A-5}$$

Integrating factor: $e^{-\overline{B_2}t}$

$$\overline{N_3}(t) = \int_o^t(\overline{B_2N_2}-(1-F_1)\alpha_1\mu A_0e^{-\alpha t})dt+\overline{N_3}(0) \qquad \text{Eq. A-6}$$

$$\overline{N_4}(t) = e^{-(\overline{B_3}+\overline{S_4})t}\left[\int_o^t \alpha_1 \rho A_0 e^{-\alpha t} e^{(\overline{B_3}+\overline{S_4})t} dt + \overline{N_4}(0)\right] \quad \text{Eq. A-7}$$

Integrating factor: $e^{-(\overline{B_3}+\overline{S_4})t}$

The solutions of the above equations are:

$$\overline{N_2} = C_{21}(e^{-\alpha t}-e^{-\overline{B_2}t}) - C_{22}(e^{-C_1 t}-e^{-\overline{B_2}t}) + C_{23}(1-e^{-\overline{B_2}t}) + C_{24}(e^{-\overline{B_1}t}-e^{-\overline{B_2}t}) + C_{25}e^{-\overline{B_2}t} \quad \text{Eq. A-8}$$

$$\overline{N_3} = C_{31}(e^{-\alpha t}-1) + C_{32}(e^{C_1 t}-1) + C_{33}(e^{-\overline{B_1}t}-1) + C_{34}(e^{-\overline{B_2}t}-1) + C_{35} \quad \text{Eq. A-9}$$

$$\overline{N_4} = \frac{\alpha_1 \rho A_o}{C_1-\alpha}(e^{-\alpha t}-e^{-C_1 t}) + \overline{N_4}(0)e^{-C_1 t} \quad \text{Eq. A-10}$$

Where the coefficients are shown as follows:

$$C_{21} = \frac{\overline{B_1}C_{11}}{\overline{B_2}-\alpha} \quad C_{22} = \frac{\overline{B_1}C_{12}}{\overline{B_2}-C_1}$$

$$C_{23} = \frac{C_{14}\overline{B_1}}{\overline{B_2}} \qquad C_{24} = \frac{(-C_{11}+C_{12}+C_{13})\overline{B_1}}{(\overline{B_2}-\overline{B_1})}$$

$$C_{25} = \overline{N_2}(0) \qquad C_{31} = \left(-\frac{\overline{B_2}C_{21}}{\alpha}+\frac{\overline{B_2}}{\alpha}(1-F_1)\alpha_1 u A_o\right)$$

$$C_{32} = \frac{\overline{B_2}C_{22}}{C_1} \qquad C_{33} = -\frac{\overline{B_2}C_{24}}{\overline{B_1}}$$

$$C_{34} = (C_{21}-C_{22}+C_{23}+C_{24}-\overline{N_2}(0)) \qquad C_{35} = C_{23}\overline{B_2}t+\overline{N_3}(0)$$

$$a_1 = \frac{\overline{B_2}\alpha_1 \rho A_o}{(C_1-\alpha)} \qquad a_2 = \frac{F_1 \alpha_1 u A_o}{\overline{B_1}-\alpha}$$

$$C_1 = \overline{B_3}+\overline{S_4}$$

APPENDIX B
DERIVATION OF THE VARIANCE EQUATIONS FOR AMMONIA, NITRITE, NITRATE AND ORGANIC NITROGEN

Important relationships of the expectation operation used in this study may be found in Introduction to Probability and Mathematical Statistics (Bain and Engelhardt, 1987).

If any two random variables, a and b, are mutually independent, then:

$$E(a+b)=E(a)+E(b) \qquad \text{Eq. B-1}$$

$$E(a*b)=E(a)*E(b) \qquad \text{Eq. B-2}$$

If 'a' is a normally distributed random variable with a mean of zero and standard deviation of σ, then:

$$E(a)=0 \qquad \text{Eq. B-3}$$

$$E(a^2)=\sigma^2 \qquad \text{Eq. B-4}$$

$$E(e^{a})=e^{E(a^2)}=e^{\sigma^2} \qquad \text{Eq. B-5}$$

For the following derivations, it is assumed that the stochastic terms, shown below, are all independent and normally distributed with a mean of zero.

$$N_1(0),\ N_2(0),\ N_3(0),\ N_4(0),\ S_3,\ S_4,\ B_1,\ B_2,\ B_3$$

B.1 DERIVATION OF VARIANCE IN AMMONIA NITROGEN

The deviation in ammonia concentration can be expressed in terms of the process deviation from the mean as:

$$\sigma_{N_1}^2 = E[N_1'^2]-(E[N_1'])^2 \qquad \text{Eq. B-6}$$

The solution of differential equation of ammonia variance (Eq. 4-6a), is obtained from the following direct integration.

$$N_1'(t)=e^{-B_1 t}\left[\int_o^t (B_3'\overline{N_4}+B_3N_4'+\frac{S_3'}{d}-B_1'N_1)\,e^{B_1 t}dt+N_1'(0)\right] \qquad \text{Eq. B-7}$$

Each part of the integration is solved seperately as follows:

Eq. B-8

$$e^{-B_1t}\int_0^t B_3'\overline{N_4}e^{B_1t}dt=e^{-B_1t}B_3'\int_0^t [\frac{\alpha_1\rho A_o}{C_1-\alpha}(e^{(B_1-\alpha)t}-e^{(B_1-C_1)t})+N_4(0)e^{(B_1-C_1)t}]dt$$

$$=e^{-B_1t}B_3'[\frac{\alpha_1\rho A_o}{C_1-\alpha}(\frac{e^{(B_1-\alpha)t}-1}{B_1-\alpha}-\frac{e^{(B_1-C_1)t}-1}{B_1-C_1})$$

$$+\frac{\overline{N_4}(0)}{B_1-C_1}(e^{(B_1-C_1)t}-1)]$$

$$=B_3'\{\frac{\alpha_1\rho A_o}{C_1-\alpha}(\frac{e^{-\alpha t}-e^{-B_1t}}{B_1-\alpha}-\frac{e^{-C_1t}-e^{-B_1t}}{B_1-C_1})+\overline{N_4}(0)\frac{e^{-C_1t}-e^{-B_1t}}{B_1-C_1}\}$$

$$e^{-B_1t}\int_0^t B_3N_4'e^{B_1t}dt=e^{-B_1t}B_3\int_0^t [\frac{C_2\alpha_1\rho A_o}{(C_1-\alpha)(C_3-\alpha)}(e^{(B_1-C_3)t}-e^{(B_1-\alpha)t})$$

$$+(\overline{N_4}(0)-\frac{A_0\alpha_1\rho}{C-1-\alpha}(e^{(B_1-C_3)t}-e^{(B_1-C_1)t})$$

$$+N_4'(0)e^{(B_1-C_3)t}]dt$$

$$=\frac{C_2\alpha_1\rho A_oB_3}{(C_1-\alpha)(C_3-\alpha)}\{\frac{e^{-C_3t}-e^{-B_1t}}{B_1-C_3}$$

$$-\frac{e^{-\alpha t}-e^{-B_1t}}{B_1-\alpha}\}+\{\overline{N_4}(0)-\frac{A_o\alpha_1\rho}{C_1-\alpha}\}B_3\{\frac{e^{-C_3t}-e^{-B_1t}}{B_1-C_3}-\frac{e^{-C_1t}-e^{-B_1t}}{B_1-C_1}\}$$

$$+\frac{N_4'(0)B_3}{B_1-C_3}(e^{-C_3t}-e^{-B_1t})$$

$$e^{-B_1t}\int_0^t B_1'\overline{N_1}e^{B_1t}dt=e^{-B-1t}B_1'\int_0^t [C_{11}(e^{(B_1-\alpha)t})-e^{B_1't})C_{12}(e^{(B_1-C_1)t}-e^{B_1't})$$

$$+C_{13}e^{B_1't}+C_{14}e^{B_1t}]dt$$

$$=B_1'\{\frac{a_1-a_2}{\overline{B_1}-\alpha}(\frac{e^{-\alpha t}-e^{-B_1t}}{B_1-\alpha}-\frac{e^{-\overline{B_1}t}-e^{-B_1t}}{B_1'})$$

$$+(\overline{N_1}(0)-\frac{\overline{S_3}}{d\overline{B_1}})\frac{e^{-\overline{B_1}t}-e^{-B_1t}}{B_1'}-\frac{a_1-\overline{N_4}(0)\overline{B_3}}{\overline{B_1}-C_1}(\frac{e^{-C_1t}-e^{-B_1t}}{B_1-C_1}$$

$$-\frac{e^{-\overline{B_1}t}-e^{-B_1t}}{B_1'})+\frac{\overline{S_3}}{d\overline{B_1}B_1}(1-e^{-B_1t})\}$$

The variance of ammonia nitrogen (N_1) is solved as following:

Eq. B-9

$$\begin{aligned}
N_1'(t) = {} & B_3'\{\frac{\alpha_1 \rho A_o}{C_1-\alpha}(\frac{e^{-\alpha t}-e^{-B_1 t}}{B_1-\alpha}-\frac{e^{-C_1 t}-e^{-B_1 t}}{B_1-C_1})+\overline{N_4}(0)\frac{e^{-C_1 t}-e^{-B_1 t}}{B_1-C_1}\} \\
& +\frac{S_3'}{dB_1}(1-e^{-B_1 t})+N_1'(0)e^{-B_1 t}+\frac{C_2\alpha_1\rho A_o B_3}{(C_1-\alpha)(C_3-\alpha)}\{\frac{e^{-C_3 t}-e^{-B_1 t}}{B_1-C_3} \\
& -\frac{e^{-\alpha t}-e^{-B_1 t}}{B_1-\alpha}\}+\{\overline{N_4}(0)-\frac{A_o\alpha_1\rho}{C_1-\alpha}\}B_3\{\frac{e^{-C_3 t}-e^{-B_1 t}}{B_1-C_3}-\frac{e^{-C_1 t}-e^{-B_1 t}}{B_1-C_1}\} \\
& +\frac{N_4'(0)B_3}{B_1-C_3}(e^{-C_3 t}-e^{-B_1 t})-B_1'\{\frac{a_1-a_2}{\overline{B_1}-\alpha}(\frac{e^{-\alpha t}-e^{-B_1 t}}{B_1-\alpha}-\frac{e^{-\overline{B_1} t}-e^{-B_1 t}}{B_1'}) \\
& +(\overline{N_1}(0)-\frac{\overline{S_3}}{d\overline{B_1}})\frac{e^{-\overline{B_1} t}-e^{-B_1 t}}{B_1'}-\frac{a_1-\overline{N_4}(0)\overline{B_3}}{\overline{B_1}-C_1}(\frac{e^{-C_1 t}-e^{-B_1 t}}{B_1-C_1} \\
& -\frac{e^{-\overline{B_1} t}-e^{-B_1 t}}{B_1'})+\frac{\overline{S_3}}{d\overline{B_1}B_1}(1-e^{-B_1 t})\}
\end{aligned}$$

Rearraging the above equation by the terms with common exponentials, Eq. B-9 becomes:

Eq. B-10

$$\begin{aligned}
N_1'(t) = {} & P_1(e^{-\alpha t}-e^{-B_1 t})+P_2(e^{-C_1 t}-e^{-B_1 t}) \\
& +P_3(e^{-C_3 t}-e^{-B_1 t})+P_4 e^{-B_1 t}+P_5
\end{aligned}$$

, where

$$P_1 = [\frac{\alpha_1\rho A_0}{C_1-\alpha}B_3'-\frac{\alpha_1\rho A_0 C_2 B_3}{(C_1-\alpha)(C_3-\alpha)}-\frac{a_1-a_2}{\overline{B_1}-\alpha}B_1']/(B_1-\alpha)$$

$$P_2 = [-(\overline{N_4}(0)-\frac{\alpha_1\rho A_0}{C_1-\alpha})\overline{B_3}+\frac{a_1-\overline{N_4}(0)\overline{B_3}}{\overline{B_1}-C_1}B_1']/(B_1-C_1)$$

$$P_3 = [\frac{\alpha_1\rho A_0 C_2}{(C_1-\alpha)(C_3-\alpha)}+(\overline{N_4}(0)-\frac{A_0\alpha_1\rho}{C_1-\alpha})+N_4'(0)]B_3/(B_1-C_3)$$

$$P_4 = -\frac{S_3'}{dB_1}+N_1'(0)-\frac{a_1-a_2}{\overline{B_1}-\alpha}+\frac{a_1-\overline{N_4}(0)\overline{B_3}}{\overline{B_1}-C_1}+(\overline{N_1}(0)-\frac{\overline{S_3}}{d\overline{B_1}})+\frac{\overline{S_3}B_1'}{d\overline{B_1}B_1}$$

$$P_5 = \frac{S_3'}{dB_1}-\frac{\overline{S_3}B_1'}{d\overline{B_1}B_1}+(\frac{a_1-a_2}{\overline{B_1}-\alpha}-\frac{a_1-\overline{N_4}(0)\overline{B_3}}{\overline{B_1}-C_1}-(\overline{N_1}(0)-\frac{\overline{S_3}}{d\overline{B_1}}))e^{-\overline{B_1}t}$$

Then, Eq. B-10 is squared yielding:

$$\begin{aligned}
N_1'^2 = {} & P_1P_1(e^{-2\alpha t}+e^{-2B_1t}-2e^{-(\alpha+B_1)t}) \\
& +P_2P_2(e^{-2C_1t}+e^{-2B_1t}-2e^{-(C_1+B_1)t}) \\
& +P_3P_3(e^{-2C_3t}+e^{-2B_1t}-2e^{-(C_3+Hx1)t}) \\
& +P_4P_4e^{-2B_1t}+P_5P_5 \\
& +2*P_1P_2(e^{-2B_1t}+e^{-(\alpha+C_1)t}-e^{-(B_1+C_1)t}-e^{-(B_1+\alpha)t}) \\
& +2*P_1P_3(e^{-2B_1t}+e^{-(\alpha+C_3)t}-e^{-(B_1+C_3)t}-e^{-(B_1+\alpha)t}) \\
& +2*P_1P_4(e^{-(\alpha+B_1)t}-e^{-2B_1t}) \\
& +2*P_1P_5(e^{-\alpha t}-e^{-B_1t}) \\
& +2*P_2P_3(e^{-2B_1t}+e^{-(C_1+C_3)t}-e^{-(C_1+B_1)t}-e^{-(C_3+B_1)t}) \\
& +2*P_2P_4(e^{-(B_1+C_1)t}-e^{-2B_1t}) \\
& +2*P_2P_5(e^{-C_1t}-e^{-B_1t}) \\
& +2*P_3P_4(e^{-(B_1+C_3)t}-e^{-2B_1t}) \\
& +2*P_3P_5(e^{-C_3t}-e^{-B_1t})+2*P_4P_5e^{-B_1t}
\end{aligned} \tag{Eq. B-11}$$

There are some repeated patterns in expectation of the above equations, such as:

$$\begin{aligned}
E[V^2] &= \overline{V^2}+\sigma_V{}^2 \\
E[e^{-V't}] &= e^{-\sigma_V^2t^2/2} \\
E[e^{-2V't}] &= e^{-2\sigma_V^2t^2} \\
E[e^{-Vt}] &= e^{-\overline{V}t}E[e^{-V't}] \\
E[(V-k_1)(V-k_2)] &= (\overline{V}-k_1)(\overline{V}-k_2)+\sigma_V^2 \\
E[C_3-k_1] &= C_1-k_1 \\
E[C_2B_3] &= \sigma_{B_3}{}^2 \\
E[C_2^2B_3] &= \overline{B_3}(\sigma_{B_3}{}^2+\sigma_{S_4}{}^2) \\
E[C_2B_3^2] &= 2*\overline{B_3}\sigma_{B_3}{}^2 \\
E[C_2^2B_3^2] &= (\sigma_{B_3}{}^2+\sigma_{S_4}{}^2)(\overline{B_3^2}+\sigma_{B_3}{}^2) \\
E[(C_3-k_1)^2] &= (C_1-k_1)^2+\sigma_{B_3}{}^2+\sigma_{S_4}{}^2 \\
E[C_2B_3'B_3] &= \overline{B_3}\sigma_{B_3}{}^2 \\
E[(V-C_3)^2] &= (\overline{V}-\overline{C_3})^2+V'^2+C_3'^2
\end{aligned} \tag{Eq. B-12}$$

Where, k_1, k_2 are constants and V is stochastic variable defined as:

$$V = \overline{V}+V' \tag{B-13}$$

The expectations of the constants of each exponantial terms are:

$$\begin{aligned}
E[P_1^2] &= \frac{1}{(\overline{B_1}-\alpha)^2+\sigma_{B_1}^{\;2}}\,[m_1^2\sigma_{B_3}^{\;2}+\frac{m_1^2\sigma_{C_2}^{\;2}}{\sigma_{\phi'}^{2}}\,(\overline{B_3}^2+\sigma_{B_3}^{\;2}) \\
&\quad +U_1^2\sigma_{B_1}^{\;2}-2\frac{m_1^2\overline{B_3}\sigma_{B_3}^{\;2}}{C_1-\alpha}] \\
E[P_2^2] &= \frac{1}{(\overline{B_1}-C_1)^2+\sigma_{B_1}^{\;2}}\,[U_3^2\overline{B_3}^2+U_2^2\sigma_{B_1}^{\;2}] \\
E[P_3^2] &= \frac{1}{(\overline{B_1}-C_1)^2+\sigma_{B_1}^{\;2}+\sigma_{C_2}^{\;2}}\,(\,[m_1^2\frac{\sigma_{C_2}^{\;2}}{\sigma_{\phi'}^{2}}+U_3^2+\sigma_{N_4(0)}^{\;2}]\,(\overline{B_3}^2+\sigma_{B_3}^{\;2}) \\
&\quad +4*\frac{m_1U_3\overline{B_3}\sigma_{B_3}^{\;2}}{C_1-\alpha}) \\
E[P_4^2] &= [\frac{(\sigma_{S_3}^{\;2}+\frac{\overline{S_3}^2\sigma_{B_1}^{\;2}}{\overline{B_1}^2})}{d^2(\overline{B_1}^2+\sigma_{B_1}^{\;2})}+\sigma_{N_1(0)}^{\;2}+U_4^2] \\
E[P_5^2] &= [\frac{(\sigma_{S_3}^{\;2}+\frac{\overline{S_3}^2\sigma_{B_1}^{\;2}}{\overline{B_1}^2})}{d^2(\overline{B_1}^2+\sigma_{B_1}^{\;2})}+U_4^2e^{-2\overline{B_1}t}]
\end{aligned} \tag{B-14}$$

(Continued...)

(...Continued)

$$E[P_1P_2] = -\frac{1}{(\overline{B_1}-\alpha)(\overline{B_1}-C_{Hx}+\sigma_{B_1}^{\ 2}}[U_1U_2\sigma_{B_1}^{\ 2}+m_1U_3\sigma_{B_3}^{\ 2}+\frac{U_3m_1\overline{B_3}\sigma_{B_3}^{\ 2}}{C_1-\alpha}]$$

$$E[P_1P_3] = \frac{1}{(\overline{B_1}-\alpha)(\overline{B_1}-C_1)+\sigma_{B_1}^{\ 2}}[m_1U_3\sigma_{B_3}^{\ 2}-\frac{m_1^2\sigma_{C_2}^{\ 2}(\overline{B_3}^2+\sigma_{B_3}^{\ 2})}{\sigma_{\phi}^{\ 2}}$$

$$-\frac{2*m_1U_3\overline{B_3}\sigma_{B_3}^{\ 2}}{C_1-\alpha}+\frac{m_1^2\sigma_{B_3}^{\ 2}\overline{B_3}}{C_1-\alpha}]$$

$$E[P_1P_4] = [\frac{U_4m_1\sigma_{B_3}^{\ 2}}{(C_1-\alpha)(\overline{B_1}-\alpha)}-\frac{U_1\overline{S_3}\sigma_{B_1}^{\ 2}}{d\overline{B_1}(\overline{B_1}-\alpha)(\overline{B_1^2}-\overline{B_1}\alpha+\sigma_{B_1}^{\ 2})}]$$

$$E[P_1P_5] = [-\frac{U_4m_1\sigma_{B_3}^{\ 2}}{(C_1-\alpha)(\overline{B_1}-\alpha)}e^{-\overline{B_1}t}+\frac{U_1\overline{S_3}\sigma_{B_1}^{\ 2}}{d\overline{B_1}(\overline{B_1}-\alpha)(\overline{B_1^2}-\alpha\overline{B_1}+\sigma_{B_1}^{\ 2})}]$$

$$E[P_2P_3] = -\frac{1}{(\overline{B_1}-C_1)^2+\sigma_{B_1}^{\ 2}}[U_3^2\overline{B_3}^2+\frac{m_1U_3\overline{B_3}\sigma_{B_3}^{\ 2}}{C_1-\alpha}]$$

$$E[P_2P_4] = \frac{1}{\overline{B_1}-C_1}[U_4U_3\overline{B_3}+U_2U_5]$$

$$E[P_2P_5] = \frac{1}{\overline{B_1}-C_1}[-U_4U_3e^{-\overline{B_1}t}\overline{B_3}-U_2U_5]$$

$$E[P_3P_4] = -\frac{1}{\overline{B_1}-C_1}[U_4U_3\overline{B_3}+\frac{m_1U_4\sigma_{B_3}^{\ 2}}{C_1-\alpha}]$$

$$E[P_3P_5] = \frac{1}{\overline{B_1}-C_1}[U_4U_3\overline{B_3}+\frac{m_1U_4\sigma_{B_3}^{\ 2}}{C_1-\alpha}]e^{-\overline{B_1}t}$$

$$E[P_4P_5] = -[U_4^2e^{-\overline{B_1}t}+\frac{\sigma_{S_3}^{\ 2}}{d^2(\overline{B_1}^2+\sigma_{B_1^2})}+\frac{\overline{S_3}U_5}{d(\overline{B_1}^2+\sigma_{B_1}^{\ 2})}]$$

Thus, the expectation of Eq. B-11 is:

$$
\begin{aligned}
E[N_1'^2] = {} & \frac{1}{(\overline{B_1}-\alpha)^2+\sigma_{B_1}^{\;2}}[m_1^2\sigma_{B_3}^{\;2}+\frac{m_1\sigma_{C_2}^{\;2}}{\sigma_{\psi}^{2}}(\overline{B_3}^2+\sigma_{B_3}^{\;2}) \\
& +U_1^2\sigma_{B_1}^{\;2}-2\frac{m_1^2\overline{B_3}\sigma_{B_3}^{\;2}}{C_1-\alpha}](e^{-2\alpha t}+E2B_1-2EB_1\alpha) \\
& +\frac{1}{(\overline{B_1}-C_1)^2+\sigma_{B_1}^{\;2}}[U_3^2\overline{B_3}^2+U_2^2\sigma_{B_1}^{\;2}] \\
& *(e^{-2C_1t}+E2B_1-2EC_1B_1) \\
& +\frac{1}{(\overline{B_1}-C_1)^2+\sigma_{B_1}^{\;2}+\sigma_{C_2}^{\;2}}([m_1^2\frac{\sigma_{C_2}^{\;2}}{\sigma_{\psi}^{2}}+U_3^2+\sigma_{N_4(0)}^{\;2}] \\
& *(\overline{B_3}^2+\sigma_{B_3}^{\;2})+4\frac{m_1U_3\overline{B_3}\sigma_{B_3}^{\;2}}{C_1-\alpha})(E2C_3+E2B_1-2EC_3B_1) \\
& +[\frac{(\sigma_{S_3}^{\;2}+\frac{\overline{S_3}^2\sigma_{B_1}^{\;2}}{\overline{B_1}^2})}{d^2(\overline{B_1}^2+\sigma_{B_1}^{\;2})}+\sigma_{N_1(0)}^{\;2}+U_4^2]E2B_1 \\
& +[\frac{(\sigma_{S_3}^{\;2}+\frac{\overline{S_3}^2\sigma_{B_1}^{\;2}}{\overline{B_1}^2})}{d^2(\overline{B_1}^2+\sigma_{B_1}^{\;2})}+U_4^2e^{-2\overline{B_1}t}]
\end{aligned}
\qquad \text{Eq. B-15}
$$

(Continued...)

(...Continued)

$$-\frac{1}{(\overline{B_1}-\alpha)(\overline{B_1}-C_1)+\sigma_{B_1}{}^2}[U_1U_2\sigma_{B_1}{}^2+m_1U_3\sigma_{B_3}{}^2+\frac{U_3m_1\overline{B_3}\sigma_{B_3}{}^2}{C_1-\alpha}]$$

$$*(E2B_1+e^{-(\alpha+C_1)t}-EB_1C_1-EB_1\alpha)$$

$$+\frac{1}{(\overline{B_1}-\alpha)(\overline{B_1}-C_1)+\sigma_{B_1}{}^2}[m_1U_3\sigma_{B_3}{}^2-\frac{m_1^2\sigma_{C_2}{}^2(\overline{B_3}^2+\sigma_{B_3}{}^2)}{\sigma_{\phi}{}^2}$$

$$-\frac{2*m_1U_3\overline{B_3}\sigma_{B_3}{}^2}{C_1-\alpha}+\frac{m_1^2\sigma_{B_3}{}^2\overline{B_3}}{C_1-\alpha}](E2B_1+EC_3\alpha-EB_1C_3-EB_1\alpha)$$

$$+[\frac{U_4m_1\sigma_{B_3}{}^2}{(C_1-\alpha)(\overline{B_1}-\alpha)}-\frac{U_1\overline{S_3}\sigma_{B_1}{}^2}{d\overline{B_1}(\overline{B_1}-\alpha)(\overline{B_1^2}-\overline{B_1}\alpha+\sigma_{B_1}{}^2)}](EB_1\alpha-E2B_1)$$

$$+[-\frac{U_4m_1\sigma_{B_3}{}^2}{(C_1-\alpha)(\overline{B_1}-\alpha)}e^{-\overline{B_1}t}+\frac{U_1\overline{S_3}\sigma_{B_1}{}^2}{d\overline{B_1}(\overline{B_1}-\alpha)(\overline{B_1^2}-\alpha\overline{B_1}+\sigma_{B_1}{}^2)}](e^{-\alpha t}-EB_1)$$

$$-\frac{1}{(\overline{B_1}-C_1)^2+\sigma_{B_1}{}^2}[U_3^2\overline{B_3}^2+\frac{m_1U_3\overline{B_3}\sigma_{B_3}{}^2}{C_1-\alpha}]$$

$$*(E2B_1+EC_1C_3-EC_1B_1-EC_3B_1)$$

$$+\frac{1}{\overline{B_1}-C_1}[U_4U_3\overline{B_3}+U_2U_5](EB_1C_1-E2B_1)$$

$$+\frac{1}{\overline{B_1}-C_1}[-U_4U_3e^{-\overline{B_1}t}\overline{B_3}-U_2U_5](e^{-C_1t}-EB_1)$$

$$-\frac{1}{\overline{B_1}-C_1}[U_4U_3\overline{B_3}+\frac{m_1U_4\sigma_{B_3}{}^2}{C_1-\alpha}](EB_1C_3-E2B_1)$$

$$+\frac{1}{\overline{B_1}-C_1}[U_4U_3\overline{B_3}+\frac{m_1U_4\sigma_{B_3}{}^2}{C_1-\alpha}]e^{-\overline{B_1}t}(EC_3-EB_1)$$

$$-[U_4^2e^{-\overline{B_1}t}+\frac{\sigma_{S_3}{}^2}{d^2(\overline{B_1}^2+\sigma_{B_1^2})}+\frac{\overline{S_3}U_5}{d(\overline{B_1}^2+\sigma_{B_1}{}^2)}]EB_1$$

The expecation of Eq. B-9 is solved as following:

Eq. B-16

$$
\begin{aligned}
E[N_1'] = & E[P_1(e^{-\alpha t}-e^{-B_1 t})+P_2(e^{-C_1 t}-e^{-B_1 t}) \\
& +P_3(e^{-C_3 t}-e^{-B_1 t})+P_4 e^{-B_1 t}+P_5] \\
& =-\frac{m_1 \sigma_{B_3}{}^2}{(C_1-\alpha)(\overline{B_1}-\alpha)}(e^{-\alpha t}-EB_1)-\frac{(\overline{N_4}(0)-m_1)\overline{B_3}}{\overline{B_1}-C_1}(e^{-C_1 t}-EB_1) \\
& +(\frac{2m_1\overline{B_3}\sigma_{B_3}{}^2}{(C_1-\alpha)(\overline{B_1}-C_1)-\sigma_{B_3}{}^2-\sigma_{S_4}{}^2}+\frac{(\overline{N_4}(0)-m_1)\overline{B_3}}{\overline{B_1}-C_1})(EC_3-EB_1) \\
& -U_4 EB_1+U_4 e^{-\overline{B_1}t}
\end{aligned}
$$

B.2 DERIVATION OF NITRITE NITROGEN EQUATION

For the differential equation of nitrite variance shown in Eq. 4-6b, the solution is obtained from the following integration.

$$N_2'(t)=e^{-B_2t}[\int_0^t(B_1\overline{N_1}-B_2'\overline{N_2})\,e^{B_2t}dt+N_2'(0)] \qquad \text{Eq. B-17}$$

Where,

Eq. B-18

$$\begin{aligned}e^{-B_2t}\int_0^{tB_1}\overline{N_1}e^{B_2t}dt=e^{-B_2t}\int_0^t B_1(C_{11}(e^{-\alpha t}-e^{-\overline{B_1}t})-C_{12}(e^{-C_1t}-e^{-\overline{B_1}t})\\ +C_{13}e^{-\overline{B_1}t}+C_{14})\,e^{B_2t}dt\\ =B_1[C_{11}\frac{e^{-\alpha t}-e^{-B_2t}}{B_2-\alpha}-C_{12}\frac{e^{-C_1t}-e^{-B_2t}}{B_2-C_1}\\ +C_{14}\frac{1-e^{-B_2t}}{B_2}+(-C_{11}+C_{12}+C_{13})\,\frac{e^{-\overline{B_1}t}-e^{-B_2t}}{B_2-\overline{B_1}}]\end{aligned}$$

$$\begin{aligned}e^{-B_2t}\int_0^t B_2'\overline{N_2}e^{B_2t}dt=e^{-B_2t}\int_0^t B_2'(C_{21}(e^{-\alpha t}-e^{-\overline{B_2}t})-C_{22}(e^{-C_1t}-e^{-\overline{B_2}t})\\ +C_{23}(1-e^{-\overline{B_2}t})+C_{24}(e^{-\overline{B_1}t}-e^{-\overline{B_2}t})+C_{25}e^{-\overline{B_2}t})\,e^{B_2t}dt\\ =B_2'[C_{21}\frac{e^{-\alpha t}-e^{-B_2t}}{B_2-\alpha}-C_{22}\frac{e^{-C_1t}-e^{-B_2t}}{B_2-C_1}\\ +C_{23}\frac{1-e^{-B_2t}}{B_2}+C_{24}\frac{e^{-\overline{B_1}t}-e^{-B_2t}}{B_2-\overline{B_1}}\\ -(C_{21}-C_{22}+C_{23}+C_{24}-C_{25})\,\frac{e^{-\overline{B_2}t}-e^{-B_2t}}{B_2'}]\end{aligned}$$

The variance of nitrite nitrogen (N_2) is solved as follows:

$$\begin{aligned} N_2'(t) = {} & \frac{C_{11}B_1-C_{21}B_2'}{B_2-\alpha}(e^{-\alpha t}-e^{-B_2 t})-\frac{C_{12}B_1-C_{22}B_2'}{B_2-C_1}(e^{-C_1 t}-e^{-B_2 t}) \\ & +\frac{C_{14}B_1-C_{23}B_2'}{B_2}(1-e^{-B_2 t})+\frac{(-C_{11}+C_{12}+C_{13})B_1-C_{24}B_2'}{B_2-\overline{B_1}} \\ & (e^{-\overline{B_1}t}-e^{-B_2 t})+N_2'(0)e^{-B_2 t}-(C_{21}-C_{22}+C_{23}+C_{24}-\overline{N_2}(0)) \\ & \frac{e^{-\overline{B_2}t}-e^{-B_2 t}}{B_2'} \end{aligned} \qquad \text{Eq. B-19}$$

Re-arranging the equation B-19 yields:

$$\begin{aligned} N_2'(t) = {} & P_1(e^{-\alpha t}-e^{-B_2 t})+P_2(e^{-C_1 t}-e^{-B_2 t}) \\ & +P_3(1-e^{-B_2 t})+P_4(e^{-\overline{B_1}t}-e^{-B_2 t})+P_6 e^{-B_2 t} \\ & +P_5(e^{-\overline{B_2}t}-e^{-B_2 t}) \end{aligned} \qquad \text{Eq. B-20}$$

where,

$$\begin{aligned} & P_1=\frac{C_{11}B_1-C_{21}B_2'}{B_2-\alpha} \\ & P_2=-\frac{C_{12}B_1-C_{22}B_2'}{B_2-C_1} \\ & P_3=\frac{C_{14}B_1-C_{23}B_2'}{B_2} \\ & P_4=\frac{(-C_{11}+C_{12}+C_{13})B_1-C_{24}B_2'}{B_2-\overline{B_1}} \\ & P_6=N_2'(0) \\ & P_5=-(C_{21}-C_{22}+C_{23}+C_{24}-C_{25})/B_2' \end{aligned} \qquad \text{Eq. B-21}$$

Eq. B-20 is then squared yielding:

$$
\begin{aligned}
N_1'^2 = & P_1P_1(e^{-2\alpha t}+e^{-2B_2t}-2e^{-(\alpha+B_2)t}) \\
& +P_2P_2(e^{-2C_1t}+e^{-2B_2t}-2e^{-(C_1+B_2)t}) \\
& +P_3P_3(1+e^{-2B_2t}-2e^{-B_2t}) \\
& +P_4P_4(e^{-2\overline{B_1}t}+e^{-2B_2t}-2e^{-(\overline{B_1}+B_2)t}) \\
& +P_5P_5(e^{-2\overline{B_2}t}+e^{-2B_2t}-2e^{-(\overline{B_2}+B_2)t}) \\
& +P_6P_6e^{-2B_2t} \\
& +2*P_1P_2(e^{-2B_2t}+e^{-(\alpha+C_1)t}-e^{-(B_2+C_1)t}-e^{-(B_2+\alpha)t}) \\
& +2*P_1P_3(e^{-\alpha t}+e^{-2B_2t}-e^{-B_2t}-e^{-(\alpha+B_2)t} \\
& +2*P_1P_4(e^{-2B_2t}+e^{-(\alpha+\overline{B_1})t}-e^{-(B_2+\overline{B_1})t}-e^{-(B_2+\alpha)t}) \\
& +2*P_1P_5(e^{-(\alpha+\overline{B_2})t}+e^{-2B_2t}-(e^{-\alpha t}+e^{-\overline{B_2}t})e^{-B_2t}) \\
& +2*P_1P_6(e^{-(\alpha+\overline{B_2})t}-e^{-2B_2t}) \\
& +2*P_2P_3(e^{-2B_2t}+e^{-C_1t}-e^{-B_2t}-e^{-(C_1+B_2)t}) \\
& +2*P_2P_4(e^{-(\overline{B_1}+C_1)t}+e^{-2B_2t}-(e^{-C_1t}+e^{-\overline{B_1}t})e^{-B_2t}) \\
& +2*P_2P_5(e^{-(C_1+\overline{B_2})t}+e^{-2B_2t}-(e^{-C_1t}+e^{-\overline{B_2}t})e^{-B_2t}) \\
& +2*P_2P_6(e^{-(C_1+B_2)t}-e^{-2B_2t}) \\
& +2*P_3P_4(e^{-\overline{B_1}t}-e^{-(\overline{B_1}+B_2)t}+e^{-2B_2t}-e^{-B_2t}) \\
& +2*P_3P_5(e^{-\overline{B_2}t}-e^{(\overline{B_2}+B_2)t}+e^{-2B_2t}-e^{-B_2t}) \\
& +2*P_3P_6(e^{-B_2t}-e^{-2B_2t}) \\
& +2*P_4P_5(e^{-(\overline{B_1}+\overline{B_2})t}+e^{-2B_2t}-(e^{-\overline{B_1}t}+e^{-\overline{B_2}t})e^{-BHxt}) \\
& +2*P_4P_6(e^{-(\overline{B_1}+B_2)t}-e^{-2B_2t}) \\
& +2*P_5P_6(e^{-(B_2+\overline{B_2})t}-e^{-2B_2t})
\end{aligned}
\tag{Eq. B-22}
$$

The expectations of the constants of each exponantial terms are:

$$
\begin{aligned}
E[P_1^2] &= \frac{C_{11}^2 V_1 + C_{21}^2 \sigma_{B_2}{}^2}{(\overline{B_2}-\alpha)^2 + \sigma_{B_2}{}^2} \\
E[P_2^2] &= \frac{C_{12}^2 V_1 + C_{22}^2 \sigma_{B_2}{}^2}{(\overline{B_2}-C_1)^2 + \sigma_{B_2}{}^2} \\
E[P_3^2] &= \frac{C_{14}^2 V_1 + C_{23}^2 \sigma_{B_2}{}^2}{V_4} \\
E[P_4^2] &= \frac{V_3^2 V_1 + C_{24}^2 \sigma_{B_2}{}^2}{(\overline{B_2}-\overline{B_1})^2 + \sigma_{B_2}{}^2} \\
E[P_5^2] &= \frac{V_2^2}{\sigma_{B_2}{}^2} \\
E[P_6^2] &= \sigma_{N_2(0)}{}^2 \\
E[P_1 P_2] &= -\frac{C_{11} C_{12} V_1 + C_{21} C_{22} \sigma_{B_2}{}^2}{(\overline{B_2}-\alpha)(\overline{B_2}-C_1) + \sigma_{B_2}{}^2}
\end{aligned}
\qquad \text{Eq. B-23}
$$

$$
\begin{aligned}
E[P_1 P_3] &= \frac{C_{11} C_{14} V_1 + C_{21} C_{23} \sigma_{B_2}{}^2}{V_4 - \alpha \overline{B_2}} \\
E[P_1 P_4] &= \frac{C_{11} V_3 V_1 + C_{21} C_{24} \sigma_{B_2}{}^2}{(\overline{B_2}-\alpha)(\overline{B_2}-\overline{B_1}) + \sigma_{B_2}{}^2} \\
E[P_1 P_5] &= -\frac{C_{11} V_2 \overline{B_1}}{\sigma_{B_2}{}^2} \\
E[P_2 P_3] &= -\frac{C_{12} C_{14} V_1 + C_{22} C_{23} \sigma_{B_2}{}^2}{V_4 - C_1 \overline{B_2}} \\
E[P_2 P_4] &= -\frac{C_{12} V_3 V_1 + C_{22} C_{24} \sigma_{B_2}{}^2}{(\overline{B_2}-\overline{B_1})(\overline{B_2}-\overline{C_1}) + \sigma_{B_2}{}^2} \\
E[P_2 P_5] &= \frac{C_{12} V_2 \overline{B_1}}{\sigma_{B_2}{}^2} \\
E[P_3 P_4] &= \frac{C_{14} V_3 V_1 + C_{23} C_{24} \sigma_{B_2}{}^2}{V_4 - \overline{B_1 B_2}} \\
E[P_3 P_5] &= -\frac{C_{14} V_2 \overline{B_1}}{\sigma_{B_2}{}^2} \\
E[P_4 P_5] &= -\frac{V_3 V_2 \overline{B_1}}{\sigma_{B_2}{}^2} \\
E[P_1 P_6] &= E[P_2 P_6] = E[P_3 P_6] = E[P_4 P_6] = E[P_5 P_6] = 0
\end{aligned}
$$

Thus, the expectation of Eq. B-22 is:

$$
\begin{aligned}
E[N_2^{\prime 2}] = {} & \frac{C_{11}^2 V_1 + C_{21}^2 \sigma_{B_2}{}^2}{(\overline{B_2}-\alpha)^2 + \sigma_{B_2}{}^2} (e^{-2\alpha t} + E2B_2 - 2EB_2\alpha) \\
& + \frac{C_{12}^2 V_1 + C_{22}^2 \sigma_{B_2}{}^2}{(\overline{B_2}-C_1)^2 + \sigma_{B_2}{}^2} (e^{-2C_1 t} + E2B_2 - 2EC_1B_2) \\
& + \frac{C_{14}^2 V_1 + C_{23}^2 \sigma_{B_2}{}^2}{V_4} (1 + E2B_2 - 2EB_2) \\
& + \frac{V_3^2 V_1 + C_{24}^2 \sigma_{B_2}{}^2}{(\overline{B_2}-\overline{B_1})^2 + \sigma_{B_2}{}^2} (e^{-2\overline{B_1} t} + E2B_2 - 2EB_1B_2) \\
& + \frac{V_2^2}{\sigma_{B_2}{}^2} (e^{-2\overline{B_2} t} + E2B_2 - 2E\overline{B_2}B_2) + \sigma_{N_2(0)}{}^2 E2B_2 \\
& - \frac{C_{11}C_{12}V_1 + C_{21}C_{22}\sigma_{B_2}{}^2}{(\overline{B_2}-\alpha)(\overline{B_2}-C_1) + \sigma_{B_2}{}^2} (e^{-C_1\alpha t} + E2B_2 - EB_2\alpha - EB_2C_1)
\end{aligned}
\tag*{Eq. B-24}
$$

$$
\begin{aligned}
& + \frac{C_{11}C_{14}V_1 + C_{21}C_{23}\sigma_{B_2}{}^2}{V_4 - \alpha\overline{B_2}} (e^{-\alpha t} + E2B_2 - EB_2 - EB_2\alpha) \\
& + \frac{C_{11}V_3V_1 + C_{21}C_{24}\sigma_{B_2}{}^2}{(\overline{B_2}-\alpha)(\overline{B_2}-\overline{B_1}) + \sigma_{B_2}{}^2} (e^{-\overline{B_1}\alpha t} + E2B_2 - EB_2\alpha - EB_2\overline{B_1}) \\
& - \frac{C_{11}V_2\overline{B_1}}{\sigma_{B_2}{}^2} (e^{(\overline{B_2}+\alpha)t} + E2B_2 - EB_2\alpha - EB_2\overline{B_2}) \\
& - \frac{C_{12}C_{14}V_1 + C_{22}C_{23}\sigma_{B_2}{}^2}{V_4 - C_1\overline{B_2}} (e^{-C_1 t} + E2B\_EB_2 - EC_1B_2) \\
& - \frac{C_{12}V_3V_1 + C_{22}C_{24}\sigma_{B_2}{}^2}{(\overline{B_2}-\overline{B_1})(\overline{B_2}-\overline{C_1}) + \sigma_{B_2}{}^2} (e^{-(C_1+\overline{B_1})t} + E2B_2 - EB_2C_1 - EB_2\overline{B_1}) \\
& + \frac{C_{12}V_2\overline{B_1}}{\sigma_{B_2}{}^2} (e^{-(C_1+\overline{B_2})t} + E2B_2 - EB_2C_1 - EB_2\overline{B_2}) \\
& + \frac{C_{14}V_3V_1 + C_{23}C_{24}\sigma_{B_2}{}^2}{V_4 - \overline{B_1B_2}} (e^{-\overline{B_1}t} + E2B2 - EB_2 - EB_2\overline{B_1}) \\
& - \frac{C_{14}V_2\overline{B_1}}{\sigma_{B_2}{}^2} (e^{-\overline{B_2}t} + E2B_2 - EB_2 - EB_2\overline{B_2}) \\
& - \frac{V_3V_2\overline{B_1}}{\sigma_{B_2}{}^2} (e^{-(\overline{B_1}+\overline{B_2})t} + E2B_2 - EB_2\overline{B_1} - EB_2\overline{B_2})
\end{aligned}
$$

The expectation of Eq. B-20 is solved as following:

$$\begin{aligned}
E[N_2'] = E[&P_1(e^{-\alpha t}-e^{-B_2 t})+P_2(e^{-C_1 t}-e^{-B_2 t}) \\
&+P_3(1-e^{-B_2 t})+P_4(e^{-\overline{B_1}t}-e^{-B_2 t})+P_6 e^{-B_2 t} \\
&+P_5(e^{-\overline{B_2}t}-e^{-B_2 t})] \\
&-\frac{C_{11}\overline{B_1}}{\overline{B_2}-\alpha}(e^{-\alpha t}-EB_2)-\frac{C_{12}\overline{B_1}}{\overline{B_2}-C_1}(e^{-C_1 t}-EB_2) \\
&+\frac{C_{14}\overline{B_1}}{\overline{B_2}}(1-EB_2)+\frac{U_3\overline{B_1}}{\overline{B_2}-\overline{B_1}}(e^{-\overline{B_1}t}-EB_2) \\
&+U_2(e^{-\overline{B_2}t}-EB_2)
\end{aligned} \qquad \text{Eq. B-25}$$

B.3 DERIVATION OF NITRATE NITROGEN EQUATION

For the differential equation of nitrate variance shown in Eq. 4-6c, the solution is obtained from the following integration.

$$N_3'(t)=\int_0^t B_2'\overline{N_2}dt+N_3'(0) \qquad \text{Eq. B-26}$$

The variance of Nitrate nitrogen (N_3) is solved as follows:

$$\begin{aligned} N_3'(t)=N_3'(0)-B_2'[&\frac{C_{21}}{\alpha}(e^{-\alpha t}-1)+\frac{C_{22}}{C_1}(e^{-C_1 t}-1)-\frac{1}{\overline{B_1}}C_{24}(e^{-\overline{B_1}t}-1)\\ &+\frac{1}{\overline{B_2}}(C_{21}-C_{22}+C_{23}+C_{24}-\overline{N_2}(0))(e^{\overline{B_2}t}-1)] \end{aligned} \qquad \text{Eq. B-27}$$

The expectation of the squared variance of N_3 is:

$$\begin{aligned} E[N_3'^2]=[&-\frac{C_{21}}{\alpha}(e^{-\alpha t}-1)+\frac{C_{22}}{C_1}(e^{-C_1 t}-1)\\ &-\frac{C_{24}}{\overline{B_1}}(e^{-\overline{B_1}t}-1)+\frac{1}{\overline{B_2}}C_{34}(e^{-\overline{B_2}t}-1)]^2\sigma_{B_2}{}^2+\sigma_{N_3(0)}{}^2 \end{aligned} \qquad \text{Eq. B-28}$$

The expectation of Eq. B-27 is zero.

B.4 DERIVATION OF ORGANIC NITROGEN EQUATION

For the differential equation of organic nitrogen variance shown in Eq. 4-6d, the solution is obtained from the following integration.

$$N_4'(t) = C_2 C_3 e^{-C_3 t}\left[\int_o^t \overline{N_4} e^{C_3 t} dt + N_4'(0)\right] \qquad \text{Eq. B-29}$$

$$\text{where, } C_2 = (B_3' + S_3')$$

$$C_3 = (\overline{B_3} + \overline{S_4} + B_3' + S_4')$$

The variance of organic nitrogen (N_4) is solved as follows:

$$N_4'(t) = \frac{(B_3' + S_4') A_o \alpha \rho}{(C_1 - \alpha)(B_3 + S_4 - \alpha)} (e^{-(B_3+S_4)t} - e^{\alpha t}) + (\overline{N_4}(0) - \frac{A_o \alpha \rho}{C_1 - \alpha}) (e^{-(B_3+S_4)t} - e^{-C_1 t}) + N_4'(0) e^{-(B_3+S_4)t} \qquad \text{Eq. B-30}$$

Equation B-30 is squared yielding:

$$\begin{aligned} N_4'^2 = {} & \frac{C_2^2 \psi^2}{(\phi\phi')^2} (e^{-C_3 t} - e^{-\alpha t})^2 \\ & + (\overline{N_4}(0) - \frac{\psi}{\phi})^2 (e^{-C_3 t} - e^{-C_1 t})^2 \\ & + (N_4'(0) e^{-C_3 t})^2 \\ & + Hx \frac{C_2 \psi}{\phi\phi'} (e^{-C_3 t} - e^{-\alpha t})] [\overline{N_4}(0) - \frac{\psi}{\phi} (e^{-C_3 t} - e^{-C_1 t})] \\ & + 2 [\frac{C_2 \psi}{\phi\phi'} (e^{-C_3 t} - e^{-\alpha t})] (N_4'(0) e^{-C_3 t}) \\ & + 2 [(\overline{N_4}(0) - \frac{\psi}{\phi}) (e^{-C_3 t} - e^{-C_1 t})] N_4'(0) e^{-C_3 t} \end{aligned} \qquad \text{Eq. B-31}$$

where,

$$\psi = A_0 \alpha \rho$$
$$\phi = C_1 - \alpha$$
$$\phi' = C_3 - \alpha$$

The expectation of Eq. B-31 is:

$$\begin{aligned} E[N_4'^2] = & \frac{\sigma_{C_2}{}^2 (A_o \alpha_1 \rho)^2}{(C_1-\alpha)^2((C_1-\alpha)^2+E[C_2^2])} (E2C_3+e^{-2\alpha t}-2EC_3\alpha) \\ & +(\overline{N_4}(0)-\frac{A_o\alpha_1\rho}{C_1-\alpha})^2 (E2C_3+e^{-2C_1 t}-2EC_3C_1) \\ & +\sigma_{N_4(0)}{}^2 E2C_3 \end{aligned} \qquad \text{Eq. B-32}$$

The expectation of Eq. B-30 is:

$$\begin{aligned} E[N_4'] = & (\overline{N_4}(0)-\frac{A_0\alpha\rho}{C_1-\alpha})(EC_3-e^{-C_1 t}) \\ & -(\overline{N_4}(0)-\frac{A_0\alpha\rho}{C_1-\alpha}) e^{-C_1 t}(e^{\sigma_{B_3}{}^2 t^2/2 \sigma_{S_4}{}^2 t^2/2}-1) \end{aligned} \qquad \text{Eq. B-33}$$

APPENDIX C

QUAL2E-UNCAS INPUT DATA

MAIN INPUT DATA

```
TITLE01              FILE WEXP.DAT - WITHLACOOCHEE RIVER, 1984 DATA
TITLE02              MODIFIED QUAL2EU DEMO. DATA
TITLE03   NO         CONSERVATIVE NINERAL   I
TITLE04   NO         CONSERVATIVE MINERAL  II
TITLE05   NO         CONSERVATIVE NINERAL III
TITLE06   NO         TEMPERATURE
TITLE07   NO         BIOCHEMICAL OXYGEN DEMAND IN MG/L
TITLE08   NO         ALGAE AS CHL-A IN UG/L
TITLE09   NO         PHOSPHORUS CYCLE AS P IN MG/L
TITLE10                 (ORGANIC-P; DISSOLVED-P)
TITLE11   YES        NITROGEN CYCLE AS N IN MG/L
TITLE12                 (ORGANIC-N; AMMONIA-N; NITRITE-N; NITRATE-N)
TITLE13   NO         DISSOLVED OXYGEN IN MG/L
TITLE14   NO         FECAL COLIFORMS IN NO./100 ML
TITLE15   NO         ARBITRARY NON-CONSERVATIVE
ENDTITLE
LIST DATA INPUT
WRITE OPTIONAL SUMMARY
NO FLOW AUGNENTATION
STEADY STATE
DISCHARGE COEFFICIENTS
PRINT SOLAR/LCD DATA
PLOT DO AND BOD
FIXED DNSTRM CONC(YES=1)=       0.0          5D-ULT BOD CONV K COEF  =       0.00
INPUT METRIC (YES=1)    =       0.0          OUTPUT METRIC (YES=1)   =        0.0
NUMBER OF REACHES       =      11.0          NUMBER OF JUNCTIONS     =        0.0
NUM OF HEADWATERS       =       1.0          NUMBER OF POINT LOADS   =        0.0
TIME STEP (HOURS)       =                    LNTH. COMP. ELEMENT (DX)=       0.50
MAXIMUM ROUTE TIME (HRS)=      30.0          TIME INC. FOR RPT2 (HRS)=
LATITUDE OF BASIN (DEG) =     30.50          LONGITUDE OF BASIN (DEG)=       85.3
STANDARD MERIDIAN (DEG) =      75.0          DAY OF YEAR START TIME  =      290.0
EVAP. COEF..(AE)        = 0.0006800          EVAP. COEF..(BE)        =  0.0002700
ELEV. OF BASIN (ELEV)   =     100.0          DUST ATTENUATION COEF.  =      0.130
ENDATA1
O UPTAKE BY NH3 OXID(MG O/MG N)=  3.500  O UPTAKE BY NO2 OXID(MG O/MG N)=  1.200
O PROD  BY ALGAE (MG O/MG A)   =  1.600  O UPTAKE BY ALGAE (MG O/MG A)  =  2.000
N CONTENT OF ALGAE (MG N/MG A) =  0.085  P CONTENT OF ALGAE (MG P/MG A) =  0.012
ALG MAX SPEC GROWTH RATE(1/DAY)=  1.307  ALGAE RESPIRATION RATE (1/DAY) =  0.150
N HALF SATURATION CONST  (MG/L)=  0.200  P HALF SATURATION CONST  (MG/L)=  0.030
LIN ALG SHADE CO (1/H-UGCHA/L) = 0.0027  NLIN SHADE (1/H-(UGCHA/L)**2/3)= 0.0165
LIGHT FUNCTION OPTION (LFNOPT) =    1.0  LIGHT SATURATION COEFF(INT/MIN)=  0.030
DAILY AVERAGING OPTION(LAVOPT) =    1.0  LIGHT AVERAGING FACTOR(AFACT)  =  0.920
NUNBER OF DAYLIGHT HOURS (DLH) =   11.2  TOTAL DAILY SOLAR RADTN (INT)  =  400.0
ALGY GROWTH CALC OPTION(LGROPT)=    1.0  ALGAL PREF FOR NH3-N (PREFN)   =  0.500
ALG/TEMP SOLAR RAD FACT(TFACT) =  0.450  NITRIFICATION INHIBITION COEF  =  10.00
ENDATA1A
ENDATA1B
STREAM REACH     1.ORCH =0            FROM        27.50     TO          27.00
STREAM REACH     2.ORCH =0            FROM        27.00     TO          25.00
STREAM REACH     3.ORCH =0            FROM        25.00     TO          20.00
STREAM REACH     4.ORCH =0            FROM        20.00     TO          17.00
STREAM REACH     5.ORCH =0            FROM        17.00     TO          14.00
STREAM REACH     6.ORCH =0            FROM        14.00     TO          12.00
STREAM REACH     7.ORCH =0            FROM        12.00     TO          10.00
STREAM REACH     8.ORCH =0            FROM        10.00     TO           7.50
STREAM REACH     9.ORCH =0            FROM         7.50     TO           6.00
STREAM REACH    10.ORCH =0            FROM         6.00     TO           4.00
```

```
STREAM REACH    11.0RCH =0              FROM        4.00    TO         0.00
ENDATA2
FLOW AUGMT SOURCES    RCH=  1.0      0.0       0.00  0.0  0.0  0.0  0.0  0.0  0.0
FLOW AUGMT SOURCES    RCH=  2.0      0.0       0.00  0.0  0.0  0.0  0.0  0.0  0.0
FLOW AUGMT SOURCES    RCH=  3.0      0.0       0.00  0.0  0.0  0.0  0.0  0.0  0.0
FLOW AUGMT SOURCES    RCH=  4.0      0.0       0.00  0.0  0.0  0.0  0.0  0.0  0.0
FLOW AUGMT SOURCES    RCH=  5.0      0.0       0.00  0.0  0.0  0.0  0.0  0.0  0.0
FLOW AUGMT SOURCES    RCH=  6.0      0.0       0.00  0.0  0.0  0.0  0.0  0.0  0.0
FLOW AUGMT SOURCES    RCH=  7.0      0.0       0.00  0.0  0.0  0.0  0.0  0.0  0.0
FLOW AUGMT SOURCES    RCH=  8.0      0.0       0.00  0.0  0.0  0.0  0.0  0.0  0.0
FLOW AUGMT SOURCES    RCH=  9.0      0.0       0.00  0.0  0.0  0.0  0.0  0.0  0.0
FLOW AUGMT SOURCES    RCH= 10.0      0.0       0.00  0.0  0.0  0.0  0.0  0.0  0.0
FLOW AUGMT SOURCES    RCH= 11.0      0.0       0.00  0.0  0.0  0.0  0.0  0.0  0.0
ENDATA3
FLAG FIELD RCH=  1.0        1.0         1.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD RCH=  2.0        4.0         2.2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD RCH=  3.0       10.0         2.2.2.2.2.2.2.2.2.2.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD RCH=  4.0        6.0         2.2.2.2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD RCH=  5.0        6.0         2.2.2.2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD RCH=  6.0        4.0         2.2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD RCH=  7.0        4.0         2.2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD RCH=  8.0        5.0         2.2.2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD RCH=  9.0        3.0         2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD RCH= 10.0        4.0         2.2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD RCH= 11.0        8.0         2.2.2.2.2.2.2.5.0.0.0.0.0.0.0.0.0.0.0.0.
ENDATA4
HYDRAULICS RCH=  1.0       60.0 0.117000  0.000000  14.80000    0.00000     0.0800
HYDRAULICS RCH=  2.0       60.0 0.237000  0.000000   7.37000    0.00000     0.0800
HYDRAULICS RCH=  3.0       60.0 0.234000  0.000000   7.57000    0.00000     0.0800
HYDRAULICS RCH=  4.0       60.0 0.336500  0.000000   5.20000    0.00000     0.0800
HYDRAULICS RCH=  5.0       60.0 0.262000  0.000000   7.85000    0.00000     0.0800
HYDRAULICS RCH=  6.0       60.0 0.396800  0.000000   5.67500    0.00000     0.0800
HYDRAULICS RCH=  7.0       60.0 0.650100  0.000000   5.67500    0.00000     0.0800
HYDRAULICS RCH=  8.0       60.0 0.476900  0.000000   8.84000    0.00000     0.0800
HYDRAULICS RCH=  9.0       60.0 0.548000  0.000000   7.77000    0.00000     0.0800
HYDRAULICS RCH= 10.0       60.0 0.610100  0.000000   6.98000    0.00000     0.0800
HYDRAULICS RCH= 11.0       60.0 0.785300  0.000000   5.90000    0.00000     0.0800
ENDATA5
ENDATA5A
REACT COEF RCH=  1.0   0.098    0.00     0.132  1.    0.078  0.0540 0.00000
REACT COEF RCH=  2.0   0.098    0.00     0.132  1.    0.315  0.0540 0.00000
REACT COEF RCH=  3.0   0.068    0.00     0.123  1.    0.301  0.0540 0.00000
REACT COEF RCH=  4.0   0.053    0.00     0.112  1.    0.634  0.0540 0.00000
REACT COEF RCH=  5.0   0.059    0.00     0.112  1.    0.302  0.0540 0.00000
REACT COEF RCH=  6.0   0.042    0.00     0.112  1.    0.604  0.0540 0.00000
REACT COEF RCH=  7.0   0.042    0.00     0.041  1.    0.773  0.0540 0.00000
REACT COEF RCH=  8.0   0.042    0.00     0.041  1.    0.341  0.0540 0.00000
REACT COEF RCH=  9.0   0.042    0.00     0.041  1.    0.443  0.0540 0.00000
REACT COEF RCH= 10.0   0.042    0.00     0.041  1.    0.549  0.0540 0.00000
REACT COEF RCH= 11.0   0.069    0.00     0.041  1.    0.801  0.0540 0.00000
ENDATA6
N AND P COEF    RCH=  1.0     .04     0.000  0.50   0.000 10.000  0.250  0.000  0.0
N AND P COEF    RCH=  2.0     .04     0.000  0.50   0.000 10.000  0.250  0.000  0.000
N AND P COEF    RCH=  3.0     .04     0.000  0.50   0.000 10.000  0.250  0.000  0.000
N AND P COEF    RCH=  4.0     .04     0.000  0.50   0.000 10.000  0.250  0.000  0.000
N AND P COEF    RCH=  5.0     .04     0.000  0.50   0.000 10.000  0.250  0.000  0.000
N AND P COEF    RCH=  6.0     .04     0.000  0.50   0.000 10.000  0.250  0.000  0.000
N AND P COEF    RCH=  7.0     .04     0.000  0.50   0.000 10.000  0.250  0.000  0.000
N AND P COEF    RCH=  8.0     .04     0.000  0.50   0.000 10.000  0.250  0.000  0.000
N AND P COEF    RCH=  9.0     .04     0.000  0.50   0.000 10.000  0.250  0.000  0.000
N AND P COEF    RCH= 10.0     .04     0.000  0.50   0.000 10.000  0.250  0.000  0.000
N AND P COEF    RCH= 11.0     .04     0.000  0.50   0.000 10.000  0.250  0.000  0.000
ENDATA6A
ALG/OTHER COEF RCH=  1.0    60.0  1.000  0.107  0.000  0.000  0.000  0.000
ALG/OTHER COEF RCH=  2.0    60.0  1.000  0.107  0.000  0.000  0.000  0.000
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ALG/OTHER COEF RCH=  3.0   60.0 1.000 0.468 0.000 0.000 0.000 0.000
ALG/OTHER COEF RCH=  4.0   60.0 1.000 0.468 0.000 0.000 0.000 0.000
ALG/OTHER COEF RCH=  5.0   60.0 1.000 0.468 0.000 0.000 0.000 0.000
ALG/OTHER COEF RCH=  6.0   60.0 1.000 0.468 0.000 0.000 0.000 0.000
ALG/OTHER COEF RCH=  7.0   60.0 1.000 0.250 0.000 0.000 0.000 0.000
ALG/OTHER COEF RCH=  8.0   60.0 1.000 0.250 0.000 0.000 0.000 0.000
ALG/OTHER COEF RCH=  9.0   60.0 1.000 0.250 0.000 0.000 0.000 0.000
ALG/OTHER COEF RCH= 10.0   60.0 1.000 0.250 0.000 0.000 0.000 0.000
ALG/OTHER COEF RCH= 11.0   60.0 1.000 0.250 0.000 0.000 0.000 0.000
ENDATA6B
INITIAL COND-1 RCH=  1.0 68.00  0.00  0.00  0.00  0.00  0.00 0.000     0.
INITIAL COND-1 RCH=  2.0 68.00  0.00  0.00  0.00  0.00  0.00 0.000     0.
INITIAL COND-1 RCH=  3.0 68.00  0.00  0.00  0.00  0.00  0.00 0.000     0.
INITIAL COND-1 RCH=  4.0 68.00  0.00  0.00  0.00  0.00  0.00 0.000     0.
INITIAL COND-1 RCH=  5.0 68.00  0.00  0.00  0.00  0.00  0.00 0.000     0.
INITIAL COND-1 RCH=  6.0 68.00  0.00  0.00  0.00  0.00  0.00 0.000     0.
INITIAL COND-1 RCH=  7.0 68.00  0.00  0.00  0.00  0.00  0.00 0.000     0.
INITIAL COND-1 RCH=  8.0 68.00  0.00  0.00  0.00  0.00  0.00 0.000     0.
INITIAL COND-1 RCH=  9.0 68.00  0.00  0.00  0.00  0.00  0.00 0.000     0.
INITIAL COND-1 RCH= 10.0 68.00  0.00  0.00  0.00  0.00  0.00 0.000     0.
INITIAL COND-1 RCH= 11.0 68.00  0.00  0.00  0.00  0.00  0.00 0.000     0.
ENDATA7
INITIAL COND-2 RCH=  1.0 0.000 0.000 0.000 0.000 0.000 0.000 0.000
INITIAL COND-2 RCH=  2.0 0.000 0.000 0.000 0.000 0.000 0.000 0.000
INITIAL COND-2 RCH=  3.0 0.000 0.000 0.000 0.000 0.000 0.000 0.000
INITIAL COND-2 RCH=  4.0 0.000 0.000 0.000 0.000 0.000 0.000 0.000
INITIAL COND-2 RCH=  5.0 0.000 0.000 0.000 0.000 0.000 0.000 0.000
INITIAL COND-2 RCH=  6.0 0.000 0.000 0.000 0.000 0.000 0.000 0.000
INITIAL COND-2 RCH=  7.0 0.000 0.000 0.000 0.000 0.000 0.000 0.000
INITIAL COND-2 RCH=  8.0 0.000 0.000 0.000 0.000 0.000 0.000 0.000
INITIAL COND-2 RCH=  9.0 0.000 0.000 0.000 0.000 0.000 0.000 0.000
INITIAL COND-2 RCH= 10.0 0.000 0.000 0.000 0.000 0.000 0.000 0.000
INITIAL COND-2 RCH= 11.0 0.000 0.000 0.000 0.000 0.000 0.000 0.000
ENDATA7A
INCR INFLOW-1  RCH=  1.0       69.80  2.35  0.63  0.00  0.00  0.00 0.000    0.
INCR INFLOW-1  RCH=  2.0       69.80  2.35  0.63  0.00  0.00  0.00 0.000    0.
INCR INFLOW-1  RCH=  3.0       69.80  2.35  0.63  0.00  0.00  0.00 0.000    0.
INCR INFLOW-1  RCH=  4.0       69.80  2.35  0.63  0.00  0.00  0.00 0.000    0.
INCR INFLOW-1  RCH=  5.0       69.80  2.35  0.63  0.00  0.00  0.00 0.000    0.
INCR INFLOW-1  RCH=  6.0       69.80  2.35  0.63  0.00  0.00  0.00 0.000    0.
INCR INFLOW-1  RCH=  7.0       68.90  2.35  0.63  0.00  0.00  0.00 0.000    0.
INCR INFLOW-1  RCH=  8.0       69.80  2.35  0.63  0.00  0.00  0.00 0.000    0.
INCR INFLOW-1  RCH=  9.0       69.80  2.35  0.63  0.00  0.00  0.00 0.000    0.
INCR INFLOW-1  RCH= 10.0       69.80  2.35  0.63  0.00  0.00  0.00 0.000    0.
INCR INFLOW-1  RCH= 11.0       69.80  2.35  0.63  0.00  0.00  0.00 0.000    0.
ENDATA8
INCR INFLOW-2  RCH=  1.0 0.230                         0.010 0.040
INCR INFLOW-2  RCH=  2.0 0.230                         0.010 0.040
INCR INFLOW-2  RCH=  3.0 0.230                         0.010 0.040
INCR INFLOW-2  RCH=  4.0 0.230                         0.010 0.040
INCR INFLOW-2  RCH=  5.0 0.230                         0.010 0.040
INCR INFLOW-2  RCH=  6.0 0.230                         0.010 0.040
INCR INFLOW-2  RCH=  7.0 0.230                         0.010 0.040
INCR INFLOW-2  RCH=  8.0 0.230                         0.010 0.040
INCR INFLOW-2  RCH=  9.0 0.230                         0.010 0.040
INCR INFLOW-2  RCH= 10.0 0.230                         0.010 0.040
INCR INFLOW-2  RCH= 11.0 0.230                         0.010 0.040
ENDATA8A
ENDATA9
HEADWTR-1 HDW=  1.0WITHLACOOCHEE     400.000 72.68  6.40  2.63   0.0   0.0   0.0
ENDATA10
HEADWTR-2 HDW=  1.0 0.000    0.019.26 0.350 0.050 0.000 0.100 0.040 0.0750
ENDATA10A
ENDATA11
ENDATA11A
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ENDATA12
DOWNSTREAM BOUNDARY-1     0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
ENDATA13
DOWNSTREAM BOUNDARY-2     0.00   0.00   0.00   0.00   0.00   0.00   0.00
ENDATA13A
BEGIN  RCH    1
PLOT   RCH    1    2    3    4    5    6    7    8    9   10   11
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VARIANCE INPUT DATA

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        INPUT VARIANCE DATA FOR QUAL2E-UNCAS; WITHLACOOCHEE (1984)
     INPUT VARIABLE NAME            INPUT CODE   TYPE  COEF VAR     PDF
   EVAPORATION COEF - AE             ECOEF-AE     1     0.100        NM
   EVAPORATION COEF - BE             ECOEF-BE     1     0.100        NM
   OXYGEN UPTAKE BY NH3 OXDTN        NH3OXYUP     1A    0.100        NM
   OXYGEN UPTAKE BY NO2 OXDTN        NO2OXYUP     1A    0.100        NM
   OXYGEN PROD BY ALGAE GRWTH        AGYOXYPR     1A    0.100        NM
   OXYGEN UPTAKE BY ALGY RESP        AGYOXYUP     1A    0.100        NM
   NITROGEN CONTENT OF ALGAE         AGYNCON      1A    0.100        NM
   PHOSPHORUS CONTENT OF ALGY        AGYPCON      1A    0.100        NM
   ALGY MAX SPEC GROWTH RATE         AGYGROMX     1A    0.100        NM
   ALGAE RESPIRATION RATE            AGYRESPR     1A    0.100        NM
   NITROGEN HALF SAT'N COEF          NHALFSAT     1A    0.100        NM
   PHOSPHORUS HALF SAT'N COEF        PHALFSAT     1A    0.100        NM
   LINEAR ALG SELF SHADE COEF        AGYEXTLN     1A    0.100        NM
   NON-LIN ALG SELF SHADE CO         AGYEXTNL     1A    0.100        NM
   LIGHT SAT'N COEFFICIENT           LSATCOEF     1A    0.100        NM
   LIGHT AVERAGING FACTOR            LAVGFACT     1A    0.020        NM
   NUMBER OF DAYLIGHT HOURS          NUMBDLH      1A    0.020        NM
   TOTAL DAILY SOLAR RADT'N          TDYSOLAR     1A    0.100        NM
   ALG PREF FOR AMMONIA-N            APREFNH3     1A    0.100        NM
   ALG TO TEMP SOLAR FACTOR          A/TFACT      1A    0.010        NM
   NITRIFICATION INHIB FACT          NHIBFACT     1A    0.100        NM
   5-D TO ULT BOD CONV R-COF         5TOUBODK     1     0.100        NM
   TEMP COEF BOD DECAY               TC/BODDC     1B    0.030        NM
   TEMP COEF BOD SETTLING            TC/BODST     1B    0.030        NM
   TEMP COEF O2 REAERATION           TC/REAER     1B    0.030        NM
   TEMP COEF SED O2 DEMAND           TC/SOD       1B    0.030        NM
   TEMP COEF ORGANIC-N DECAY         TC/NH2DC     1B    0.030        NM
   TEMP COEF ORGANIC-N SET           TC/NH2ST     1B    0.030        NM
   TEMP COEF AMMONIA DECAY           TC/NH3DC     1B    0.030        NM
   TEMP COEF AMMONIA SRCE            TC/NH3SC     1B    0.030        NM
   TEMP COEF NITRITE DECAY           TC/NO2DC     1B    0.030        NM
   TEMP COEF ORGANIC-P DECAY         TC/PRGDC     1B    0.030        NM
   TEMP COEF ORGANIC-P SET           TC/PRGST     1B    0.030        NM
   TEMP COEF DISS-P SOURCE           TC/PO4SC     1B    0.030        NM
   TEMP COEF ALGY GROWTH             TC/ALGRO     1B    0.030        NM
   TEMP COEF ALGY RESPR              TC/ALRES     1B    0.030        NM
   TEMP COEF ALGY SETTLING           TC/ALSET     1B    0.030        NM
   TEMP COEF COLI DECAY              TC/CLIDC     1B    0.030        NM
   TEMP COEF ANC DECAY               TC/ANCDC     1B    0.030        NM
   TEMP COEF ANC SETTLING            TC/ANCST     1B    0.030        NM
   TEMP COEF ANC SOURCE              TC/ANCSC     1B    0.030        NM
   DAILY AVERAGING OPTION            DIURNOPT     1A    0.000        NM
   LIGHT FUNCTION OPTION             LFNOPTN      1A    0.000        NM
   ALGAE GROWTH CALC OPTION          AGYGROPT     1A    0.000        NM
   DISPERSION CORR CONSTANT          DISPSN-K     5     0.200        NM
   COEF ON FLOW FOR VELOCITY         COEFQV-A     5     0.080        NM
   EXPO ON FLOW FOR VELOCITY         EXPOQV-B     5     0.001        NM
   COEF ON FLOW FOR DEPTH            COEFQH-C     5     0.080        NM
   EXPO ON FLOW FOR DEPTH            EXPOQH-D     5     0.001        NM
   MANNING'S ROUGHNESS N             MANNINGS     5     0.100        NM
   SIDE SLOPE 1                      TRAP-SS1     5     0.050        NM
   SIDE SLOPE 2                      TRAP-SS2     5     0.050        NM
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BOTTOM WIDTH	TRAP-WTH	5	0.050	NM
SLOPE OF CHANNEL	TRAP-SLP	5	0.050	NM
MEAN ELEVATION OF REACH	ELEVATIN	5A	0.100	NM
DUST ATTENUATION COEF	DUSTATTN	5A	0.100	NM
FRACTION OF CLOUDINESS	CLOUD	5A	0.130	NM
DRY BULB AIR TEMPERATURE	DRYBULB	5A	0.020	NM
WET BULB AIR TEMPERATURE	WETBULB	5A	0.020	NM
ATMOSPHERIC PRESSURE	ATMPRES	5A	0.010	NM
WIND VELOCITY	WINDVEL	5A	0.150	NM
SOLAR RADIATION ATTN FCTR	SRADATN	5A	0.100	NM
CBOD OXIDATION RATE	BOD DECA	6	0.150	NM
CBOD SETTLING RATE	BOD SETT	6	0.150	NM
SOD UPTAKE RATE	SOD RATE	6	0.120	NM
REAERATION RATE OPTION 1	K2-OPT1	6	0.130	NM
COEF OF FLOW FOR K2 OPT-7	CQK2-OP7	6	0.100	NM
EXPO OF FLOW FOR K2 OPT-7	EQK2-OP7	6	0.100	NM
COEF FOR K2 (TSIV) OPT-8	K2COEF-8	6	0.100	NM
SLOPE FOR K2 (TSIV) OPT-8	K2SLOP-8	6	0.100	NM
ORGANIC-N HYDROLYSIS RATE	NH2 DECA	6A	0.200	NM
ORGANIC-N SETTLING RATE	NH2 SETT	6A	0.150	NM
AMMONIA-N DECAY RATE	NH3 DECA	6A	0.250	NM
AMMONIA-N BENTHAL SOURCE	NH3 SRCE	6A	0.250	NM
NITRITE-N DECAY RATE	NO2 DECA	6A	0.200	NH
ORGANIC-P HYDROLYSIS RATE	PORG DEC	6A	0.200	NM
ORGANIC-P SETTLING RATE	PORG SET	6A	0.150	NM
DISSOLVED-P BENTHAL SRCE	DISP SRC	6A	0.250	NM
CHLA TO ALGAE RATIO	CHLA/ART	6B	0.200	NM
ALGAE SETTLING RATE	ALG SETT	6B	0.150	NM
LIGHT EXT COEFFICIENT	LTEXTNCO	6B	0.050	NM
COLIFORM DECAY RATE	COLI DEC	6B	0.150	NM
ANC DECAY RATE	ANC DECA	6B	0.150	NM
ANC SETTLING RATE	ANC SETT	6B	0.150	NM
ANC BENTHAL SOURCE	ANC SRCE	6B	0.150	NM
INITIAL TEMPERATURE	INITTEMP	7A	0.030	NH
REAERATION EQUATION OPT.	K2OPTION	6	0.000	NM
INCREMENTAL FLOW	INCRFLOW	8	0.030	NM
INCR-TEMPERATURE	INCRTEMP	8	0.010	NM
INCR-DISSOLVED OXYGEN	INCRDO	8	0.030	NM
INCR-BOD	INCRBOD	8	0.150	NM
INCR-CONSV MIN 1	INCRCM1	8	0.020	NM
INCR-CONSV MIN 2	INCRCM2	8	0.020	NM
INCR-CONSV MIN 3	INCRCM3	8	0.020	NM
INCR-ARBITRARY NON-CONS	INCRANC	8	0.030	NM
INCR-COLIFORM	INCRCOLI	8	0.050	NM
INCR-ALGAE	INCRCHLA	8A	0.130	NM
INCR-ORGANIC-N	INCRNH2N	8A	0.250	NM
INCR-AMMONIA-N	INCRNH3N	8A	0.150	NM
INCR-NITRITE-N	INCRNO2N	8A	0.150	NM
INCR-NITRATE-N	INCRNO3N	8A	0.150	NM
INCR-ORGANIC-PHOS	INCRPORG	8A	0.250	NM
INCR-DISSOLVED-PHOS	INCRDISP	8A	0.200	NM
HEADWATER FLOW	HWTRFLOW	10	0.030	NM
HWTR-TEMPERATURE	HWTRTEMP	10	0.010	NM
HWTR-DISSOLVED OXYGEN	HWTRDO	10	0.030	NM
HWTR-BOD	HWTRBOD	10	0.150	NM
HWTR-CONSV MIN 1	HWTRCM1	10	0.040	NM
HWTR-CONSV MIN 2	HWTRCM2	10	0.040	NM
HWTR-CONSV MIN 3	HWTRCM3	10	0.040	NM
HWTR-ARBITRARY NON-CONS	HWTRANC	10A	0.060	NM
HWTR-COLIFORM	HWTRCOLI	10A	0.100	NM
HWTR-ALGAE	HWTRCHLA	10A	0.040	NM
HWTR-ORGANIC-N	HWTRNH2N	10A	0.060	NM
HWTR-AMMONIA-N	HWTRNH3N	10A	0.100	NM
HWTR-NITRITE-N	HWTRNO2N	10A	0.100	NM
HWTR-NITRATE-N	HWTRNO3N	10A	0.070	NM

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HWTR-ORGANIC-PHOS              MWTRPORG    10A    0.250       NM
HWTR-DISSOLVED-PHOS            HWTRDISP    10A    0.070       NM
PTLD-TRTMNT FACTOR             PTLDTFCT    11     0.020       NM
POINT LOAD FLOW                PTLDFLOW    11     0.030       NM
PTLD-TEMPERATURE               PTLDTEMP    11     0.010       NM
PTLD-DISSOLVED OXYGEN          PTLDDO      11     0.030       NM
PTLD-BOD                       PTLDBOD     11     0.150       NM
PTLD-CONSV MIN 1               PTLDCM1     11     0.040       NM
PTLD-CONSV MIN 2               PTLDCM2     11     0.040       NM
PTLD-CONSV MIN 3               PTLDCM3     11     0.040       NM
PTLD-ARBITRARY NON-CONS        PTLDANC     11A    0.060       NM
PTLD-COLIFORM                  PTLDCOLI    11A    0.100       NM
PTLD-ALGAE                     PTLDCHLA    11A    0.130       NM
PTLD-ORGANIC-N                 PTLDNH2N    11A    0.010       NM
PTLD-AMMONIA-N                 PTLDNH3N    11A    0.100       NM
PTLD-NITRITE-N                 PTLDNO2N    11A    0.100       NM
PTLD-NITRATE-N                 PTLDNO3N    11A    0.080       NM
PTLD-ORGANIC-PHOS              PTLDPORG    11A    0.250       NM
PTLD-DISOLVED-PHOS             PTLDDISP    11A    0.140       NM
DAM COEFFICIENT A              DAMSACOF    12     0.030       NM
DAM COEFFICIENT B              DAMSBCOF    12     0.030       NM
FRACTION OF FLOW OVER DAM      DAMSFRAC    12     0.050       NM
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UNCERTAINTY INPUT DATA

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UNCAS1   *HEADING             *QUAL2E UNCERTAINTY ANALYSIS; FILE WUAM.DAT
UNCAS2   *SYSTEM TITLE        *WITHLACOOCHEE RIVER (1984); UNCAS/MCS
UNCAS3   *TYPE OF ANALYSIS    *MONTE CARLO SIMULATION         600 SIMULATIONS
UNCAS4   *INPUT CONDITION     *ALL INPUTS
UNCAS5   *INTERMED OUTPUT     *NONE
UNCAS6   *OUTPUT VARIABLES    *QUALITY        INTERNAL
UNCAS7   *OUTPUT LOCATIONS    *    2  2    3 10    7  1    11  5
UNCAS8   *INPUT VARIABLES     *
UNCAS9   *ENDING              *ENDUNCERTAINTY
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APPENDIX D

QUAL2E-UNCAS OUTPUT DATA

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                                     * * * QUAL-2E  STREAM QUALITY ROUTING MODEL * * *
                                             * * * EPA/NCASI VERSION * * *


 $$$ (PROBLEM TITLES) $$$

CARD TYPE                          QUAL-2E PROGRAM TITLES
TITLE01              FILE WEXP.DAT - WITHLACOOCHEE RIVER, 1984 DATA
TITLE02              MODIFIED QUAL2EU DEMO. DATA
TITLE03   NO         CONSERVATIVE MINERAL   I
TITLE04   NO         CONSERVATIVE MINERAL  II
TITLE05   NO         CONSERVATIVE NINERAL III
TITLE06   NO         TEMPERATURE
TITLE07   NO         BIOCHEMICAL OXYGEN DEMAND IN MG/L
TITLE08   NO         ALGAE AS CHL-A IN UG/L
TITLE09   NO         PHOSPHORUS CYCLE AS P IN MG/L
TITLE10                 (ORGANIC-P; DISSOLVED-P)
TITLE11  YES         NITROGEN CYCLE AS N IN MG/L
TITLE12                 (ORGANIC-N; AMMONIA-N; NITRITE-N; NITRATE-N)
TITLE13   NO         DISSOLVED OXYGEN IN MG/L
TITLE14   NO         FECAL COLIFORMS IN NO./100 ML
TITLE15   NO         ARBITRARY NON-CONSERVATIVE
ENDTITLE

 $$$ DATA TYPE 1 (CONTROL DATA) $$$

CARD TYPE                                    CARD TYPE
LIST DATA INPUT            0.00000                                   0.00000
WRITE OPTIONAL SUMMARY     0.00000                                   0.00000
NO FLOW AUGMENTATION       0.00000                                   0.00000
STEADY STATE               0.00000                                   0.00000
DISCHARGE COEFFICIENTS     0.00000                                   0.00000
PRINT SOLAR/LCD DATA       0.00000                                   0.00000
PLOT DO AND BOD            0.00000                                   0.00000
FIXED DNSTRM CONC(YES=1)=  0.00000           5D-ULT BOD CONV K COEF  =   0.23000
INPUT METRIC (YES=1)    =  0.00000           OUTPUT METRIC (YES=1)   =   0.00000
NUMBER OF REACHES       = 11.00000           NUMBER OF JUNCTIONS     =   0.00000
NUM OF HEADWATERS       =  1.00000           NUMBER OF POINT LOADS   =   0.00000
TIME STEP (HOURS)       =  0.00000           LNTH. COMP. ELEMENT (DX)=   0.50000
MAXIMUM ROUTE TINE (HRS)= 30.00000           TIME INC. FOR RPT2 (HRS)=   0.00000
LATITUDE OF BASIN (DEG) = 30.50000           LONGITUDE OF BASIN (DEG)=  85.30000
STANDARD MERIDIAN (DEG) = 75.00000           DAY OF YEAR START TINE  = 290.00000
EVAP. COEF..(AE)        =  0.00068           EVAP. COEF..(BE)        =   0.00027
ELEV. OF BASIN (ELEV)   =100.00000           DUST ATTENUATION COEF.  =   0.13000
ENDATA1                    0.00000                                   0.00000

 $$$ DATA TYPE 1A (ALGAE PRODUCTION AND NITROGEN OXIDATION CONSTANTS) $$$

CARD TYPE                                        CARD TYPE
O UPTAKE BY NH3 OXID(MG O/MG N)=    3.5000       O UPTAKE BY NO2 OXID(MG O/MG N)=     1.2000
O PROD  BY ALGAE (MG O/MG A)   =    1.6000       O UPTAKE BY ALGAE (MG O/MG A)  =     2.0000
N CONTENT OF ALGAE (MG N/MG A) =    0.0850       P CONTENT OF ALGAE (MG P/MG A) =     0.0120
ALG MAX SPEC GROWTH RATE(1/DAY)=    1.3070       ALGAE RESPIRATION RATE (1/DAY) =     0.1500
N HALF SATURATION CONST  (MG/L)=    0.2000       P HALF SATURATION CONST  (MG/L)=     0.0300
LIN ALG SHADE CO (1/FT-UGCHA/L=)    0.0027       NLIN SHADE(1/FT-(UGCHA/L)**2/3)=     0.0165
LIGHT FUNCTION OPTION (LFNOPT) =    1.0000       LIGHT SAT'N COEF (BTU/FT2-MIN) =     0.0300
DAILY AVERAGING OPTION(LAVOPT) =    1.0000       LIGHT AVERAGING FACTOR(AFACT)  =     0.9200
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NUMBER OF DAYLIGHT HOURS (DLH) =    11.2000        TOTAL DAILY SOLR RAD (BTU/FT-2)=  400.0000
ALGY GROWTH CALC OPTION(LGROPT)=     1.0000        ALGAL PREF FOR NH3-N (PREFN)   =    0.5000
ALG/TEMP SOLAR RAD FACT(TFACT) =     0.4500        NITRIFICATION INHIBITION COEF  =   10.0000
ENDATA1A                             0.0000                                            0.0000

 $$$ DATA TYPE 1B (TEMPERATURE CORRECTION CONSTANTS FOR RATE COEFFICIENTS) $$$

CARD TYPE       RATE CODE      THETA VALUE

THETA( 1)        BOD DECA         1.047       DFLT
THETA( 2)        BOD SETT         1.024       DFLT
THETA( 3)        OXY TRAN         1.024       DFLT
THETA( 4)        SOD RATE         1.060       DFLT
THETA( 5)        ORGN DEC         1.047       DFLT
THETA( 6)        ORGN SET         1.024       DFLT
THETA( 7)        NH3 DECA         1.083       DFLT
THETA( 8)        NH3 SRCE         1.074       DFLT
THETA( 9)        NO2 DECA         1.047       DFLT
THETA(10)        PORG DEC         1.047       DFLT
THETA(11)        PORG SET         1.024       DFLT
THETA(12)        DISP SRC         1.074       DFLT
THETA(13)        ALG GROW         1.047       DFLT
THETA(14)        ALG RESP         1.047       DFLT
THETA(15)        ALG SETT         1.024       DFLT
THETA(16)        COLI DEC         1.047       DFLT
THETA(17)        ANC DECA         1.000       DFLT
THETA(18)        ANC SETT         1.024       DFLT
THETA(19)        ANC SRCE         1.000       DFLT
ENDATA1B

 $$$ DATA TYPE 2 (REACH IDENTIFICATION) $$$

CARD TYPE              REACH ORDER AND IDENT                R. MI/KM              R. MI/KM
STREAM REACH        1.0  RCH =0                 FROM        27.5      TO         27.0
STREAM REACH        2.0  RCH =0                 FROM        27.0      TO         25.0
STREAM REACH        3.0  RCH =0                 FROM        25.0      TO         20.0
STREAM REACH        4.0  RCH =0                 FROM        20.0      TO         17.0
STREAM REACH        5.0  RCH =0                 FROM        17.0      TO         14.0
STREAM REACH        6.0  RCH =0                 FROM        14.0      TO         12.0
STREAM REACH        7.0  RCH =0                 FROM        12.0      TO         10.0
STREAM REACH        8.0  RCH =0                 FROM        10.0      TO          7.5
STREAM REACH        9.0  RCH =0                 FROM         7.5      TO          6.0
STREAM REACH       10.0  RCH =0                 FROM         6.0      TO          4.0
STREAM REACH       11.0  RCH =0                 FROM         4.0      TO          0.0
ENDATA2             0.0                                      0.0                  0.0

 $$$ DATA TYPE 3 (TARGET LEVEL DO AND FLOW AUGMENTATION SOURCES) $$$

CARD TYPE                   REACH  AVAIL HDWS TARGET     ORDER OF AVAIL SOURCES
FLOW AUGMT SOURCES            1.        0.       0.0   0.   0.   0.   0.   0.   0.
FLOW AUGMT SOURCES            2.        0.       0.0   0.   0.   0.   0.   0.   0.
FLOW AUGMT SOURCES            3.        0.       0.0   0.   0.   0.   0.   0.   0.
FLOW AUGMT SOURCES            4.        0.       0.0   0.   0.   0.   0.   0.   0.
FLOW AUGMT SOURCES            5.        0.       0.0   0.   0.   0.   0.   0.   0.
FLOW AUGMT SOURCES            6.        0.       0.0   0.   0.   0.   0.   0.   0.
FLOW AUGMT SOURCES            7.        0.       0.0   0.   0.   0.   0.   0.   0.
FLOW AUGMT SOURCES            8.        0.       0.0   0.   0.   0.   0.   0.   0.
FLOW AUGHT SOURCES            9.        0.       0.0   0.   0.   0.   0.   0.   0.
FLOW AUGHT SOURCES           10.        0.       0.0   0.   0.   0.   0.   0.   0.
FLOW AUGMT SOURCES           11.        0.       0.0   0.   0.   0.   0.   0.   0.
ENDATA3                       0.        0.       0.0   0.   0.   0.   0.   0.   0.

 $$$ DATA TYPE 4 (COMPUTATIONAL REACH FLAG FIELD) $$$

CARD TYPE          REACH ELEMENTS/REACH            COMPUTATIONAL FLAGS
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FLAG FIELD      1.      1.       1.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD      2.      4.       2.2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD      3.     10.       2.2.2.2.2.2.2.2.2.2.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD      4.      6.       2.2.2.2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD      5.      6.       2.2.2.2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD      6.      4.       2.2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD      7.      4.       2.2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD      8.      5.       2.2.2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD      9.      3.       2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD     10.      4.       2.2.2.2.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.
FLAG FIELD     11.      8.       2.2.2.2.2.2.2.5.0.0.0.0.0.0.0.0.0.0.0.0.
ENDATA4         0.      0.       0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.

 $$$ DATA TYPE 5 (HYDRAULIC DATA FOR DETERMINING VELOCITY AND DEPTH) $$$

CARD TYPE       REACH  COEF-DSPN  COEFQV    EXPOQV    COEFQH   EXPOQH     CMANN
HYDRAULICS       1.      60.00    0.117     0.000    14.800    0.000      0.080
HYDRAULICS       2.      60.00    0.237     0.000     7.370    0.000      0.080
HYDRAULICS       3.      60.00    0.234     0.000     7.570    0.000      0.080
HYDRAULICS       4.      60.00    0.336     0.000     5.200    0.000      0.080
HYDRAULICS       5.      60.00    0.262     0.000     7.850    0.000      0.080
HYDRAULICS       6.      60.00    0.397     0.000     5.675    0.000      0.080
HYDRAULICS       7.      60.00    0.650     0.000     5.675    0.000      0.080
HYDRAULICS       8.      60.00    0.477     0.000     8.840    0.000      0.080
HYDRAULICS       9.      60.00    0.548     0.000     7.770    0.000      0.080
HYDRAULICS      10.      60.00    0.610     0.000     6.980    0.000      0.080
HYDRAULICS      11.      60.00    0.785     0.000     5.900    0.000      0.080
ENDATA5          0.       0.00    0.000     0.000     0.000    0.000      0.000

 $$$ DATA TYPE 5A (STEADY STATE TEMPERATURE AND CLIMATOLOGY DATA) $$$

       CARD TYPE                          DUST     CLOUD   DRY BULB  WET BULB    ATM           SOLAR
RAD
                    REACH    ELEVATION     COEF     COVER     TEMP       TEMP    PRESSURE      WIND
ATTENUATION
       ENDATA5A      0.         0.00       0.00      0.00      0.00      0.00      0.00       0.00
0.00

 $$$ DATA TYPE 6 (REACTION COEFFICIENTS FOR DEOXYGENATION AND REAERATION) $$$

       CARD TYPE    REACH     K1       K3       SOD       K2OPT     K2     COEQK2    OR   EXPQK2
                                                RATE                       TSIV COEF OR    SLOPE
                                                                           FOR OPT 8       FOR OPT
8
       REACT COEF    1.      0.10     0.00     0.132       1.      0.08     0.054         0.00000
       REACT COEF    2.      0.10     0.00     0.132       1.      0.31     0.054         0.00000
       REACT COEF    3.      0.07     0.00     0.123       1.      0.30     0.054         0.00000
       REACT COEF    4.      0.05     0.00     0.112       1.      0.63     0.054         0.00000
       REACT COEF    5.      0.06     0.00     0.112       1.      0.30     0.054         0.00000
       REACT COEF    6.      0.04     0.00     0.112       1.      0.60     0.054         0.00000
       REACT COEF    7.      0.04     0.00     0.041       1.      0.77     0.054         0.00000
       REACT COEF    8.      0.04     0.00     0.041       1.      0.34     0.054         0.00000
       REACT COEF    9.      0.04     0.00     0.041       1.      0.44     0.054         0.00000
       REACT COEF   10.      0.04     0.00     0.041       1.      0.55     0.054         0.00000
       REACT COEF   11.      0.07     0.00     0.041       1.      0.80     0.054         0.00000
       ENDATA6       0.      0.00     0.00     0.000       0.      0.00     0.000         0.00000

 $$$ DATA TYPE 6A (NITROGEN AND PHOSPHORUS CONSTANTS) $$$

       CARD TYPE          REACH    CKNH2    SETNH2    CKNH3    SNH3     CKNO2    CKPORG   SETPORG
SPO4
       N AND P COEF         1.      0.04     0.00      0.50     0.00    10.00     0.25      0.00
0.00
       N AND P COEF         2.      0.04     0.00      0.50     0.00    10.00     0.25      0.00
0.00
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        N AND P COEF          3.     0.04     0.00     0.50     0.00    10.00     0.25     0.00
0.00
        N AND P COEF          4.     0.04     0.00     0.50     0.00    10.00     0.25     0.00
0.00
        N AND P COEF          5.     0.04     0.00     0.50     0.00    10.00     0.25     0.00
0.00
        N AND P COEF          6.     0.04     0.00     0.50     0.00    10.00     0.25     0.00
0.00
        N AND P COEF          7.     0.04     0.00     0.50     0.00    10.00     0.25     0.00
0.00
        N AND P COEF          8.     0.04     0.00     0.50     0.00    10.00     0.25     0.00
0.00
        N AND P COEF          9.     0.04     0.00     0.50     0.00    10.00     0.25     0.00
0.00
        N AND P COEF         10.     0.04     0.00     0.50     0.00    10.00     0.25     0.00
0.00
        N AND P COEF         11.     0.04     0.00     0.50     0.00    10.00     0.25     0.00
0.00
        ENDATA6A              0.     0.00     0.00     0.00     0.00     0.00     0.00     0.00
0.00

         $$$ DATA TYPE 6B (ALGAE/OTHER COEFFICIENTS) $$$

        CARD TYPE           REACH   ALPHAO   ALGSET   EXCOEF     CK5     CKANC   SETANC   SRCANC
                                                               CKCOLI
        ALG/OTHER COEF        1.    60.00     1.00     0.11     0.00     0.00     0.00     0.00
        ALG/OTHER COEF        2.    60.00     1.00     0.11     0.00     0.00     0.00     0.00
        ALG/OTHER COEF        3.    60.00     1.00     0.47     0.00     0.00     0.00     0.00
        ALG/OTHER COEF        4.    60.00     1.00     0.47     0.00     0.00     0.00     0.00
        ALG/OTHER COEF        5.    60.00     1.00     0.47     0.00     0.00     0.00     0.00
        ALG/OTHER COEF        6.    60.00     1.00     0.47     0.00     0.00     0.00     0.00
        ALG/OTHER COEF        7.    60.00     1.00     0.25     0.00     0.00     0.00     0.00
        ALG/OTHER COEF        8.    60.00     1.00     0.25     0.00     0.00     0.00     0.00
        ALG/OTHER COEF        9.    60.00     1.00     0.25     0.00     0.00     0.00     0.00
        ALG/OTHER COEF       10.    60.00     1.00     0.25     0.00     0.00     0.00     0.00
        ALG/OTHER COEF       11.    60.00     1.00     0.25     0.00     0.00     0.00     0.00
        ENDATA6B              0.     0.00     0.00     0.00     0.00     0.00     0.00     0.00

         $$$ DATA TYPE 7 (INITIAL CONDITIONS) $$$

        CARD TYPE           REACH    TEMP      D.O.     BOD      CM-1     CM-2     CM-3      ANC
COLI
        INITIAL COND-1        1.    68.00     0.00     0.00     0.00     0.00     0.00     0.00
0.00
        INITIAL COND-1        2.    68.00     0.00     0.00     0.00     0.00     0.00     0.00
0.00
        INITIAL COND-1        3.    68.00     0.00     0.00     0.00     0.00     0.00     0.00
0.00
        INITIAL COND-1        4.    68.00     0.00     0.00     0.00     0.00     0.00     0.00
0.00
        INITIAL COND-1        5.    68.00     0.00     0.00     0.00     0.00     0.00     0.00
0.00
        INITIAL COND-1        6.    68.00     0.00     0.00     0.00     0.00     0.00     0.00
0.00
        INITIAL COND-1        7.    68.00     0.00     0.00     0.00     0.00     0.00     0.00
0.00
        INITIAL COND-1        8.    68.00     0.00     0.00     0.00     0.00     0.00     0.00
0.00
        INITIAL COND-1        9.    68.00     0.00     0.00     0.00     0.00     0.00     0.00
0.00
        INITIAL COND-1       10.    68.00     0.00     0.00     0.00     0.00     0.00     0.00
0.00
        INITIAL COND-1       11.    68.00     0.00     0.00     0.00     0.00     0.00     0.00
0.00
        ENDATA7               0.     0.00     0.00     0.00     0.00     0.00     0.00     0.00
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0.00

     $$$ DATA TYPE 7A (INITIAL CONDITIONS FOR CHOROPHYLL A, NITROGEN, AND PHOSPHORUS) $$$

     CARD TYPE           REACH   CHL-A    ORG-N    NH3-N    NO2-N    NO3-N    ORG-P    DIS-P
     INITIAL COND-2        1.    0.00     0.00     0.00     0.00     0.00     0.00     0.00
     INITIAL COND-2        2.    0.00     0.00     0.00     0.00     0.00     0.00     0.00
     INITIAL COND-2        3.    0.00     0.00     0.00     0.00     0.00     0.00     0.00
     INITIAL COND-2        4.    0.00     0.00     0.00     0.00     0.00     0.00     0.00
     INITIAL COND-2        5.    0.00     0.00     0.00     0.00     0.00     0.00     0.00
     INITIAL COND-2        6.    0.00     0.00     0.00     0.00     0.00     0.00     0.00
     INITIAL COND-2        7.    0.00     0.00     0.00     0.00     0.00     0.00     0.00
     INITIAL COND-2        8.    0.00     0.00     0.00     0.00     0.00     0.00     0.00
     INITIAL COND-2        9.    0.00     0.00     0.00     0.00     0.00     0.00     0.00
     INITIAL COND-2       10.    0.00     0.00     0.00     0.00     0.00     0.00     0.00
     INITIAL COND-2       11.    0.00     0.00     0.00     0.00     0.00     0.00     0.00
     ENDATA7A              0.    0.00     0.00     0.00     0.00     0.00     0.00     0.00

     $$$ DATA TYPE 8 (INCREMENTAL INFLOW CONDITIONS) $$$

     CARD TYPE           REACH   FLOW     TEMP     D.O.     BOD      CM-1     CM-2     CM-3
ANC    COLI
     INCR INFLOW-1         1.    0.000    69.80    2.35     0.63     0.00     0.00     0.00
0.00   0.00
     INCR INFLOW-1         2.    0.000    69.80    2.35     0.63     0.00     0.00     0.00
0.00   0.00
     INCR INFLOW-1         3.    0.000    69.80    2.35     0.63     0.00     0.00     0.00
0.00   0.00
     INCR INFLOW-1         4.    0.000    69.80    2.35     0.63     0.00     0.00     0.00
0.00   0.00
     INCR INFLOW-1         5.    0.000    69.80    2.35     0.63     0.00     0.00     0.00
0.00   0.00
     INCR INFLOW-1         6.    0.000    69.80    2.35     0.63     0.00     0.00     0.00
0.00   0.00
     INCR INFLOW-1         7.    0.000    68.90    2.35     0.63     0.00     0.00     0.00
0.00   0.00
     INCR INFLOW-1         8.    0.000    69.80    2.35     0.63     0.00     0.00     0.00
0.00   0.00
     INCR INFLOW-1         9.    0.000    69.80    2.35     0.63     0.00     0.00     0.00
0.00   0.00
     INCR INFLOW-1        10.    0.000    69.80    2.35     0.63     0.00     0.00     0.00
0.00   0.00
     INCR INFLOW-1        11.    0.000    69.80    2.35     0.63     0.00     0.00     0.00
0.00   0.00
     ENDATA8               0.    0.000    0.00     0.00     0.00     0.00     0.00     0.00
0.00   0.00

     $$$ DATA TYPE 8A (INCREMENTAL INFLOW CONDITIONS FOR CHLOROPHYLL A, NITROGEN, AND PHOSPHORUS) $$$

     CARD TYPE           REACH   CHL-A    ORG-N    NH3-N    NO2-N    NO3-N    ORG-P    DIS-P
     INCR INFLOW-2         1.    0.23     0.00     0.00     0.00     0.00     0.01     0.04
     INCR INFLOW-2         2.    0.23     0.00     0.00     0.00     0.00     0.01     0.04
     INCR INFLOW-2         3.    0.23     0.00     0.00     0.00     0.00     0.01     0.04
     INCR INFLOW-2         4.    0.23     0.00     0.00     0.00     0.00     0.01     0.04
     INCR INFLOW-2         5.    0.23     0.00     0.00     0.00     0.00     0.01     0.04
     INCR INFLOW-2         6.    0.23     0.00     0.00     0.00     0.00     0.01     0.04
     INCR INFLOW-2         7.    0.23     0.00     0.00     0.00     0.00     0.01     0.04
     INCR INFLOW-2         8.    0.23     0.00     0.00     0.00     0.00     0.01     0.04
     INCR INFLOW-2         9.    0.23     0.00     0.00     0.00     0.00     0.01     0.04
     INCR INFLOW-2        10.    0.23     0.00     0.00     0.00     0.00     0.01     0.04
     INCR INFLOW-2        11.    0.23     0.00     0.00     0.00     0.00     0.01     0.04
     ENDATA8A              0.    0.00     0.00     0.00     0.00     0.00     0.00     0.00

     $$$ DATA TYPE 9 (STREAM JUNCTIONS) $$$
```

```
        CARD TYPE              JUNCTION ORDER AND IDENT         UPSTRH   JUNCTION    TRIB
        ENDATA9               0.                                 0.        0.         0.

         $$$ DATA TYPE 10 (HEADWATER SOURCES) $$$

        CARD TYPE    HDWTR      NAME              FLOW      TEMP      D.O.       BOD       CM-1       CM-2
    CM-3
                     ORDER
        HEADWTR-1     1.  WITHLACOOCHEE         400.00     72.68      6.40      2.63      0.00       0.00
    0.00
        ENDATA10      0.                          0.00      0.00      0.00      0.00      0.00       0.00
    0.00

         $$$ DATA TYPE 10A (HEADWATER CONDITIONS FOR CHLOROPHYLL, NITROGEN, PHOSPHORUS,
                            COLIFORM AND SELECTED NON-CONSERVATIVE CONSTITUENT) $$$

        CARD TYPE     HDWTR     ANC      COLI     CHL-A     ORG-N     NH3-N     NO2-N     NO3-N     ORG-P     DIS-P
                      ORDER
        HEADWTR-2      1.      0.00      0.00     19.26      0.35      0.05      0.00      0.10      0.04      0.08
        ENDATA10A      0.      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00

         $$$ DATA TYPE 11 (POINT SOURCE / POINT SOURCE CHARACTERISTICS) $$$

                      POINT
        CARD TYPE     LOAD     NAME              EFF      FLOW      TEMP      D.O.       BOD       CM-1       CM-2
     CM-3
                      ORDER
        ENDATA11       0.                        0.00      0.00      0.00      0.00      0.00      0.00       0.00
     0.00

         $$$ DATA TYPE 11A (POINT SOURCE CHARACTERISTICS - CHLOROPHYLL A, NITROGEN, PHOSPHORUS,
                            COLIFORMS AND SELECTED NON-CONSERVATIVE CONSTITUENT) $$$

                      POINT
        CARD TYPE     LOAD      ANC      COLI     CHL-A     ORG-N     NH3-N     NO2-N     NO3-N     ORG-P     DIS-P
                      ORDER
        ENDATA11A      0.      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00      0.00

         $$$ DATA TYPE 12 (DAM CHARACTERISTICS) $$$

                              DAM    RCH    ELE     ADAM     BDAM     FDAM     HDAM

        ENDATA12                0.     0.     0.     0.00     0.00     0.00     0.00

         $$$ DATA TYPE 13 (DOWNSTREAM BOUNDARY CONDITIONS-1) $$$

            CARD TYPE                 TEMP       D.O.       BOD        CM-1       CM-2       CM-3        ANC
   COLI

        DOWNSTREAM BOUNDARY-1        DOWNSTREAM BOUNDARY CONCENTRATIONS ARE UNCONSTRAINED
        ENDATA13

         $$$ DATA TYPE 13A (DOWNSTREAM BOUNDARY CONDITIONS-2) $$$

            CARD TYPE                 CHL-A      ORG-N      NH3-N      NO2-N      NH3-N      ORG-P       DIS-P

        DOWNSTREAM BOUNDARY-2        DOWNSTREAM BOUNDARY CONCENTRATIONS ARE UNCONSTRAINED
        ENDATA13A
```

```
                   ORGANIC NITROGEN AS N IN MG/L                                   ITERATION  0
RCH/CL  1      2      3      4      5      6      7      8      9     10     11     12     13     14     15     16     17
 18    19     20

  1  0.35
  2  0.34   0.34   0.34   0.34
  3  0.34   0.34   0.33   0.33   0.33   0.33   0.33   0.32   0.32   0.32
  4  0.32   0.32   0.32   0.32   0.32   0.31
  5  0.31   0.31   0.31   0.31   0.31   0.31
  6  0.30   0.30   0.30   0.30
  7  0.30   0.30   0.30   0.30
  8  0.30   0.30   0.30   0.30   0.30
  9  0.29   0.29   0.29
 10  0.29   0.29   0.29   0.29
 11  0.29   0.29   0.29   0.29   0.29   0.29   0.29   0.29

                   AMMONIA AS N IN MG/L                                            ITERATION  0
RCH/CL  1      2      3      4      5      6      7      8      9     10     11     12     13     14     15     16     17
 18    19     20

  1  0.05
  2  0.05   0.04   0.04   0.04
  3  0.04   0.04   0.04   0.04   0.04   0.04   0.04   0.04   0.04   0.03
  4  0.03   0.03   0.03   0.03   0.03   0.03
  5  0.03   0.03   0.03   0.03   0.03   0.03
  6  0.03   0.03   0.03   0.03
  7  0.03   0.03   0.03   0.03
  8  0.03   0.03   0.03   0.03   0.03
  9  0.03   0.03   0.03
 10  0.03   0.03   0.03   0.03
 11  0.03   0.03   0.03   0.03   0.03   0.03   0.03   0.03

                   NITRITE AS N IN MG/L                                            ITERATION  0
RCH/CL  1      2      3      4      5      6      7      8      9     10     11     12     13     14     15     16     17
 18    19     20

  1  0.00
  2  0.00   0.00   0.00   0.00
  3  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00
  4  0.00   0.00   0.00   0.00   0.00   0.00
  5  0.00   0.00   0.00   0.00   0.00   0.00
  6  0.00   0.00   0.00   0.00
  7  0.00   0.00   0.00   0.00
  8  0.00   0.00   0.00   0.00   0.00
  9  0.00   0.00   0.00
 10  0.00   0.00   0.00   0.00
 11  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00

                   NITRATE AS N IN MG/L                                            ITERATION  0
RCH/CL  1      2      3      4      5      6      7      8      9     10     11     12     13     14     15     16     17
 18    19     20

  1  0.10
  2  0.11   0.11   0.11   0.12
  3  0.12   0.12   0.12   0.13   0.13   0.13   0.13   0.14   0.14   0.14
  4  0.14   0.15   0.15   0.15   0.15   0.15
  5  0.15   0.16   0.16   0.16   0.16   0.16
  6  0.16   0.17   0.17   0.17
  7  0.17   0.17   0.17   0.17
  8  0.17   0.17   0.17   0.17   0.17
  9  0.18   0.18   0.18
 10  0.18   0.18   0.18   0.18
 11  0.18   0.18   0.18   0.18   0.18   0.18   0.18   0.18
```

```
        STREAM QUALITY SIMULATION                                                       OUTPUT
PAGE NUMBER     1
        QUAL-2E STREAM QUALITY ROUTING MODEL
EPA/NCASI VERSION
                                      ***** STEADY STATE SIMULATION *****

                                         ** HYDRAULICS SUMMARY **

ELE RCH ELE   BEGIN    END           POINT    INCR          TRVL                                       BOTTOM
   X-SECT   DSPRSN
ORD NUM NUM    LOC     LOC    FLOW    SRCE    FLOW     VEL   TIME    DEPTH    WIDTH     VOLUME         AREA
    AREA      COEF
              MILE    MILE     CFS     CFS     CFS     FPS    DAY       FT       FT     K-FT-3       K-FT-2
     FT-2   FT-2/S

  1   1   1   27.50   27.00  400.00    0.00    0.00   0.117  0.261   14.800  231.000    9025.64       687.98
   3418.80    20.25


  2   2   1   27.00   26.50  400.00    0.00    0.00   0.237  0.129    7.370  229.005    4455.70       643.49
   1687.76    22.94
  3   2   2   26.50   26.00  400.00    0.00    0.00   0.237  0.129    7.370  229.005    4455.70       643.49
   1687.76    22.94
  4   2   3   26.00   25.50  400.00    0.00    0.00   0.237  0.129    7.370  229.005    4455.70       643.49
   1687.76    22.94
  5   2   4   25.50   25.00  400.00    0.00    0.00   0.237  0.129    7.370  229.005    4455.70       643.49
   1687.76    22.94


  6   3   1   25.00   24.50  400.00    0.00    0.00   0.234  0.131    7.570  225.813    4512.82       636.11
   1709.40    23.16
  7   3   2   24.50   24.00  400.00    0.00    0.00   0.234  0.131    7.570  225.813    4512.82       636.11
   1709.40    23.16
  8   3   3   24.00   23.50  400.00    0.00    0.00   0.234  0.131    7.570  225.813    4512.82       636.11
   1709.40    23.16
  9   3   4   23.50   23.00  400.00    0.00    0.00   0.234  0.131    7.570  225.813    4512.82       636.11
   1709.40    23.16
 10   3   5   23.00   22.50  400.00    0.00    0.00   0.234  0.131    7.570  225.813    4512.82       636.11
   1709.40    23.16
 11   3   6   22.50   22.00  400.00    0.00    0.00   0.234  0.131    7.570  225.813    4512.82       636.11
   1709.40    23.16
 12   3   7   22.00   21.50  400.00    0.00    0.00   0.234  0.131    7.570  225.813    4512.82       636.11
   1709.40    23.16
 13   3   8   21.50   21.00  400.00    0.00    0.00   0.234  0.131    7.570  225.813    4512.82       636.11
   1709.40    23.16
 14   3   9   21.00   20.50  400.00    0.00    0.00   0.234  0.131    7.570  225.813    4512.82       636.11
   1709.40    23.16
 15   3  10   20.50   20.00  400.00    0.00    0.00   0.234  0.131    7.570  225.813    4512.82       636.11
   1709.40    23.16


 16   4   1   20.00   19.50  400.00    0.00    0.00   0.336  0.091    5.200  228.598    3138.19       630.95
   1188.71    24.36
 17   4   2   19.50   19.00  400.00    0.00    0.00   0.336  0.091    5.200  228.598    3138.19       630.95
   1188.71    24.36
 18   4   3   19.00   18.50  400.00    0.00    0.00   0.336  0.091    5.200  228.598    3138.19       630.95
   1188.71    24.36
 19   4   4   18.50   18.00  400.00    0.00    0.00   0.336  0.091    5.200  228.598    3138.19       630.95
   1188.71    24.36
 20   4   5   18.00   17.50  400.00    0.00    0.00   0.336  0.091    5.200  228.598    3138.19       630.95
   1188.71    24.36
 21   4   6   17.50   17.00  400.00    0.00    0.00   0.336  0.091    5.200  228.598    3138.19       630.95
   1188.71    24.36
```

```
22   5   1   17.00   16.50  400.00    0.00    0.00   0.262   0.117     7.850  194.486      4030.53        554.89
  1526.72     26.73
23   5   2   16.50   16.00  400.00    0.00    0.00   0.262   0.117     7.850  194.486      4030.53        554.89
  1526.72     26.73
24   5   3   16.00   15.50  400.00    0.00    0.00   0.262   0.117     7.850  194.486      4030.53        554.89
  1526.72     26.73
25   5   4   15.50   15.00  400.00    0.00    0.00   0.262   0.117     7.850  194.486      4030.53        554.89
  1526.72     26.73
26   5   5   15.00   14.50  400.00    0.00    0.00   0.262   0.117     7.850  194.486      4030.53        554.89
  1526.72     26.73
27   5   6   14.50   14.00  400.00    0.00    0.00   0.262   0.117     7.850  194.486      4030.53        554.89
  1526.72     26.73

28   6   1   14.00   13.50  400.00    0.00    0.00   0.397   0.077     5.675  177.633      2661.29        498.91
  1008.06     30.90
29   6   2   13.50   13.00  400.00    0.00    0.00   0.397   0.077     5.675  177.633      2661.29        498.91
  1008.06     30.90
30   6   3   13.00   12.50  400.00    0.00    0.00   0.397   0.077     5.675  177.633      2661.29        498.91
  1008.06     30.90
31   6   4   12.50   12.00  400.00    0.00    0.00   0.397   0.077     5.675  177.633      2661.29        498.91
  1008.06     30.90

32   7   1   12.00   11.50  400.00    0.00    0.00   0.650   0.047     5.675  108.421      1624.37        316.20
   615.29     50.62
33   7   2   11.50   11.00  400.00    0.00    0.00   0.650   0.047     5.675  108.421      1624.37        316.20
   615.29     50.62
34   7   3   11.00   10.50  400.00    0.00    0.00   0.650   0.047     5.675  108.421      1624.37        316.20
   615.29     50.62
```

```
          STREAM QUALITY SIMULATION                                                           OUTPUT
PAGE NUMBER    2
          QUAL-2E STREAM QUALITY ROUTING MODEL
EPA/NCASI VERSION
                                                   ***** STEADY STATE SIMULATION *****
                                                        ** HYDRAULICS SUMMARY **

ELE RCH ELE   BEGIN    END          POINT   INCR          TRVL                                          BOTTOM
    X-SECT   DSPRSN
ORD NUM NUM    LOC     LOC    FLOW   SRCE   FLOW    VEL   TIME    DEPTH   WIDTH      VOLUME           AREA
     AREA     COEF
              MILE    MILE     CFS    CFS    CFS    FPS    DAY       FT      FT     K-FT-3          K-FT-2
      FT-2   FT-2/S

 35  7  4    10.50   10.00  400.00   0.00   0.00  0.650  0.047    5.675 108.421    1624.37          316.20
    615.29   50.62


 36  8  1    10.00    9.50  400.00   0.00   0.00  0.477  0.064    8.840  94.881    2214.30          297.16
    838.75   53.72
 37  8  2     9.50    9.00  400.00   0.00   0.00  0.477  0.064    8.840  94.881    2214.30          297.16
    838.75   53.72
 38  8  3     9.00    8.50  400.00   0.00   0.00  0.477  0.064    8.840  94.881    2214.30          297.16
    838.75   53.72
 39  8  4     8.50    8.00  400.00   0.00   0.00  0.477  0.064    8.840  94.881    2214.30          297.16
    838.75   53.72
 40  8  5     8.00    7.50  400.00   0.00   0.00  0.477  0.064    8.840  94.881    2214.30          297.16
    838.75   53.72


 41  9  1     7.50    7.00  400.00   0.00   0.00  0.548  0.056    7.770  93.942    1927.01          289.03
    729.93   55.44
 42  9  2     7.00    6.50  400.00   0.00   0.00  0.548  0.056    7.770  93.942    1927.01          289.03
    729.93   55.44
 43  9  3     6.50    6.00  400.00   0.00   0.00  0.548  0.056    7.770  93.942    1927.01          289.03
    729.93   55.44


 44 10  1     6.00    5.50  400.00   0.00   0.00  0.610  0.050    6.980  93.930    1730.86          284.83
    655.63   56.45
 45 10  2     5.50    5.00  400.00   0.00   0.00  0.610  0.050    6.980  93.930    1730.86          284.83
    655.63   56.45
 46 10  3     5.00    4.50  400.00   0.00   0.00  0.610  0.050    6.980  93.930    1730.86          284.83
    655.63   56.45
 47 10  4     4.50    4.00  400.00   0.00   0.00  0.610  0.050    6.980  93.930    1730.86          284.83
    655.63   56.45


 48 11  1     4.00    3.50  400.00   0.00   0.00  0.785  0.039    5.900  86.332    1344.71          259.07
    509.36   63.16
 49 11  2     3.50    3.00  400.00   0.00   0.00  0.785  0.039    5.900  86.332    1344.71          259.07
    509.36   63.16
 50 11  3     3.00    2.50  400.00   0.00   0.00  0.785  0.039    5.900  86.332    1344.71          259.07
    509.36   63.16
 51 11  4     2.50    2.00  400.00   0.00   0.00  0.785  0.039    5.900  86.332    1344.71          259.07
    509.36   63.16
 52 11  5     2.00    1.50  400.00   0.00   0.00  0.785  0.039    5.900  86.332    1344.71          259.07
    509.36   63.16
 53 11  6     1.50    1.00  400.00   0.00   0.00  0.785  0.039    5.900  86.332    1344.71          259.07
    509.36   63.16
 54 11  7     1.00    0.50  400.00   0.00   0.00  0.785  0.039    5.900  86.332    1344.71          259.07
    509.36   63.16
 55 11  8     0.50    0.00  400.00   0.00   0.00  0.785  0.039    5.900  86.332    1344.71          259.07
    509.36   63.16
```

```
        STREAM QUALITY SIMULATION                                                        OUTPUT
PAGE NUMBER    3
        QUAL-2E STREAM QUALITY ROUTING MODEL
EPA/NCASI VERSION
                                              ***** STEADY STATE SIMULATION *****

                                              ** REACTION COEFFICIENT SUMMARY **

RCH ELE      DO  K2  OXYGN    BOD    BOD    SOD   ORGN   ORGN    NH3    NH3    NO2   ORGP   ORGP   DISP   COLI
  ANC    ANC    ANC
NUM NUM    SAT OPT  REAIR  DECAY   SETT   RATE  DECAY   SETT  DECAY   SRCE  DECAY  DECAY   SETT   SRCE  DECAY
DECAY   SETT   SRCE
          MG/L      1/DAY  1/DAY  1/DAY  G/F2D  1/DAY  1/DAY  1/DAY  MG/F2D 1/DAY  1/DAY  1/DAY MG/F2D  1/DAY
1/DAY  1/DAY MG/F2D

  1   1   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00


  2   1   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  2   2   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  2   3   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  2   4   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00


  3   1   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  3   2   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  3   3   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  3   4   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  3   5   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  3   6   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  3   7   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  3   8   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  3   9   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  3  10   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00


  4   1   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  4   2   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  4   3   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  4   4   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  4   5   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  4   6   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
```

```
 5  1   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00
 5  2   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00
 5  3   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00
 5  4   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00
 5  5   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00
 5  6   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00

 6  1   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00
 6  2   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00
 6  3   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00
 6  4   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00

 7  1   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00
 7  2   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00
 7  3   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
0.00   0.00   0.00
```

```
        STREAM QUALITY SIMULATION                                                         OUTPUT
PAGE NUMBER     4
        QUAL-2E STREAM QUALITY ROUTING MODEL
EPA/NCASI VERSION
                                               ***** STEADY STATE SIMULATION *****

                                                ** REACTION COEFFICIENT SUMMARY **

RCH ELE      DO  K2  OXYGN    BOD    BOD    SOD   ORGN   ORGN    NH3    NH3    NO2   ORGP   ORGP   DISP   COLI
  ANC     ANC    ANC
NUM NUN     SAT OPT  REAIR  DECAY   SETT   RATE  DECAY   SETT  DECAY   SRCE  DECAY  DECAY   SETT   SRCE  DECAY
DECAY   SETT   SRCE
            MG/L     1/DAY  1/DAY  1/DAY  G/F2D  1/DAY  1/DAY  1/DAY MG/F2D  1/DAY  1/DAY  1/DAY MG/F2D  1/DAY
1/DAY  1/DAY MG/F2D

  7   4   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00


  8   1   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  8   2   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  8   3   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  8   4   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  8   5   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00


  9   1   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  9   2   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
  9   3   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00


 10   1   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
 10   2   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
 10   3   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
 10   4   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00

 11   1   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
 11   2   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
 11   3   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
 11   4   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
 11   5   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
 11   6   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
 11   7   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
 11   8   0.00   1   0.00   0.00   0.00   0.00   0.04   0.00   0.50   0.00  10.00   0.00   0.00   0.00   0.00
 0.00   0.00   0.00
```

```
        STREAM QUALITY SIMULATION                                                          OUTPUT
PAGE NUMBER     5
        QUAL-2E STREAM QUALITY ROUTING MODEL
EPA/NCASI VERSION
                                            ***** STEADY STATE SIMULATION *****

                                              ** WATER QUALITY VARIABLES **

RCH ELE              CM-1   CM-2   CM-3
         ANC
NUN NUN   TEMP                             DO    BOD   ORGN   NH3N   NO2N   NO3N  SUM-N   ORGP  DIS-P  SUM-P
 COLI      CHLA
          DEG-F                           MG/L   MG/L   MG/L   MG/L   MG/L   MG/L   MG/L   MG/L   MG/L   MG/L
#/100ML     UG/L

  1  1    68.00   0.00   0.00   0.00   0.00   0.00   0.35   0.05   0.00   0.10   0.50   0.00   0.00   0.00
 0.00   0.00   0.00


  2  1    68.00   0.00   0.00   0.00   0.00   0.00   0.34   0.05   0.00   0.11   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  2  2    68.00   0.00   0.00   0.00   0.00   0.00   0.34   0.04   0.00   0.11   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  2  3    68.00   0.00   0.00   0.00   0.00   0.00   0.34   0.04   0.00   0.11   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  2  4    68.00   0.00   0.00   0.00   0.00   0.00   0.34   0.04   0.00   0.12   0.50   0.00   0.00   0.00
 0.00   0.00   0.00


  3  1    68.00   0.00   0.00   0.00   0.00   0.00   0.34   0.04   0.00   0.12   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  3  2    68.00   0.00   0.00   0.00   0.00   0.00   0.34   0.04   0.00   0.12   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  3  3    68.00   0.00   0.00   0.00   0.00   0.00   0.33   0.04   0.00   0.12   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  3  4    68.00   0.00   0.00   0.00   0.00   0.00   0.33   0.04   0.00   0.13   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  3  5    68.00   0.00   0.00   0.00   0.00   0.00   0.33   0.04   0.00   0.13   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  3  6    68.00   0.00   0.00   0.00   0.00   0.00   0.33   0.04   0.00   0.13   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  3  7    68.00   0.00   0.00   0.00   0.00   0.00   0.33   0.04   0.00   0.13   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  3  8    68.00   0.00   0.00   0.00   0.00   0.00   0.32   0.04   0.00   0.14   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  3  9    68.00   0.00   0.00   0.00   0.00   0.00   0.32   0.04   0.00   0.14   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  3 10    68.00   0.00   0.00   0.00   0.00   0.00   0.32   0.03   0.00   0.14   0.50   0.00   0.00   0.00
 0.00   0.00   0.00


  4  1    68.00   0.00   0.00   0.00   0.00   0.00   0.32   0.03   0.00   0.14   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  4  2    68.00   0.00   0.00   0.00   0.00   0.00   0.32   0.03   0.00   0.15   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  4  3    68.00   0.00   0.00   0.00   0.00   0.00   0.32   0.03   0.00   0.15   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  4  4    68.00   0.00   0.00   0.00   0.00   0.00   0.32   0.03   0.00   0.15   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  4  5    68.00   0.00   0.00   0.00   0.00   0.00   0.32   0.03   0.00   0.15   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
  4  6    68.00   0.00   0.00   0.00   0.00   0.00   0.31   0.03   0.00   0.15   0.50   0.00   0.00   0.00
 0.00   0.00   0.00
```

```
 5  1      68.00  0.00   0.00   0.00   0.00   0.00   0.31   0.03   0.00   0.15   0.50   0.00   0.00   0.00
0.00   0.00   0.00
 5  2      68.00  0.00   0.00   0.00   0.00   0.00   0.31   0.03   0.00   0.16   0.50   0.00   0.00   0.00
0.00   0.00   0.00
 5  3      68.00  0.00   0.00   0.00   0.00   0.00   0.31   0.03   0.00   0.16   0.50   0.00   0.00   0.00
0.00   0.00   0.00
 5  4      68.00  0.00   0.00   0.00   0.00   0.00   0.31   0.03   0.00   0.16   0.50   0.00   0.00   0.00
0.00   0.00   0.00
 5  5      68.00  0.00   0.00   0.00   0.00   0.00   0.31   0.03   0.00   0.16   0.50   0.00   0.00   0.00
0.00   0.00   0.00
 5  6      68.00  0.00   0.00   0.00   0.00   0.00   0.31   0.03   0.00   0.16   0.50   0.00   0.00   0.00
0.00   0.00   0.00

 6  1      68.00  0.00   0.00   0.00   0.00   0.00   0.30   0.03   0.00   0.16   0.50   0.00   0.00   0.00
0.00   0.00   0.00
 6  2      68.00  0.00   0.00   0.00   0.00   0.00   0.30   0.03   0.00   0.17   0.50   0.00   0.00   0.00
0.00   0.00   0.00
 6  3      68.00  0.00   0.00   0.00   0.00   0.00   0.30   0.03   0.00   0.17   0.50   0.00   0.00   0.00
0.00   0.00   0.00
 6  4      68.00  0.00   0.00   0.00   0.00   0.00   0.30   0.03   0.00   0.17   0.50   0.00   0.00   0.00
0.00   0.00   0.00

 7  1      68.00  0.00   0.00   0.00   0.00   0.00   0.30   0.03   0.00   0.17   0.50   0.00   0.00   0.00
0.00   0.00   0.00
 7  2      68.00  0.00   0.00   0.00   0.00   0.00   0.30   0.03   0.00   0.17   0.50   0.00   0.00   0.00
0.00   0.00   0.00
 7  3      68.00  0.00   0.00   0.00   0.00   0.00   0.30   0.03   0.00   0.17   0.50   0.00   0.00   0.00
0.00   0.00   0.00
```

```
        STREAM QUALITY SIMULATION                                                                  OUTPUT
PAGE NUMBER     6
        QUAL-2E STREAM QUALITY ROUTING MODEL
EPA/NCASI VERSION
                                              ***** STEADY STATE SIMULATION *****

                                                 ** WATER QUALITY VARIABLES **

RCM ELE              CM-1    CM-2    CM-3
         ANC
NUM NUM   TEMP                               DO     BOD    ORGN    NH3N    NO2N    NO3N   SUM-N   ORGP   DIS-P   SUM-P
 COLI      CHLA
          DEG-F                             MG/L   MG/L    MG/L    MG/L    MG/L    MG/L    MG/L   MG/L    MG/L    MG/L
#/100ML      UG/L

 7   4     68.00   0.00    0.00    0.00    0.00    0.00    0.30    0.03    0.00    0.17    0.50    0.00    0.00    0.00
 0.00   0.00   0.00


 8   1     68.00   0.00    0.00    0.00    0.00    0.00    0.30    0.03    0.00    0.17    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
 8   2     68.00   0.00    0.00    0.00    0.00    0.00    0.30    0.03    0.00    0.17    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
 8   3     68.00   0.00    0.00    0.00    0.00    0.00    0.30    0.03    0.00    0.17    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
 8   4     68.00   0.00    0.00    0.00    0.00    0.00    0.30    0.03    0.00    0.17    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
 8   5     68.00   0.00    0.00    0.00    0.00    0.00    0.30    0.03    0.00    0.17    0.50    0.00    0.00    0.00
 0.00   0.00   0.00


 9   1     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
 9   2     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
 9   3     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00


10   1     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
10   2     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
10   3     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
10   4     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00

11   1     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
11   2     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
11   3     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
11   4     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
11   5     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
11   6     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
11   7     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
11   8     68.00   0.00    0.00    0.00    0.00    0.00    0.29    0.03    0.00    0.18    0.50    0.00    0.00    0.00
 0.00   0.00   0.00
```

```
        STREAM QUALITY SIMULATION                                                       OUTPUT
PAGE NUMBER     7
        QUAL-2E STREAM QUALITY ROUTING MODEL
EPA/NCASI VERSION
                                            ***** STEADY STATE SIMULATION *****

                                               ** DISSOLVED OXYGEN DATA **

                                                           COMPONENTS OF DISSOLVED OXYGEN MASS BALANCE
(MG/L-DAY)
ELE RCH ELE                DO             DO     DAM     NIT
ORD NUM NUM   TEMP        SAT     DO     DEF    INPUT   INHIB      F-FNCTN  OXYGN                    NET
             DEG-F       MG/L   MG/L    MG/L    MG/L    FACT       INPUT   REAIR   C-BOD     SOD     P-R
NH3-N   NO2-N

  1  1   1   68.00     0.00    0.00    0.00    0.00    1.00       24.51    0.00    0.00    0.00    0.00
-0.08   -0.02


  2  2   1   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.08   -0.02
  3  2   2   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.08   -0.03
  4  2   3   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.08   -0.03
  5  2   4   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.07   -0.03


  6  3   1   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.07   -0.03
  7  3   2   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.07   -0.02
  8  3   3   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.07   -0.02
  9  3   4   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.07   -0.02
 10  3   5   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.07   -0.02
 11  3   6   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.07   -0.02
 12  3   7   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.06   -0.02
 13  3   8   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.06   -0.02
 14  3   9   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.06   -0.02
 15  3  10   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.06   -0.02


 16  4   1   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.06   -0.02
 17  4   2   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.06   -0.02
 18  4   3   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.06   -0.02
 19  4   4   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.06   -0.02
 20  4   5   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.06   -0.02
 21  4   6   68.00     0.00    0.00    0.00    0.00    1.00        0.00    0.00    0.00    0.00    0.00
-0.06   -0.02
```

```
22   5   1  68.00   0.00   0.00   0.00   0.00   1.00      0.00   0.00   0.00   0.00   0.00
-0.06   -0.02
23   5   2  68.00   0.00   0.00   0.00   0.00   1.00      0.00   0.00   0.00   0.00   0.00
-0.06   -0.02
24   5   3  68.00   0.00   0.00   0.00   0.00   1.00      0.00   0.00   0.00   0.00   0.00
-0.06   -0.02
25   5   4  68.00   0.00   0.00   0.00   0.00   1.00      0.00   0.00   0.00   0.00   0.00
-0.05   -0.02
26   5   5  68.00   0.00   0.00   0.00   0.00   1.00      0.00   0.00   0.00   0.00   0.00
-0.05   -0.02
27   5   6  68.00   0.00   0.00   0.00   0.00   1.00      0.00   0.00   0.00   0.00   0.00
-0.05   -0.02

28   6   1  68.00   0.00   0.00   0.00   0.00   1.00      0.00   0.00   0.00   0.00   0.00
-0.05   -0.02
29   6   2  68.00   0.00   0.00   0.00   0.00   1.00      0.00   0.00   0.00   0.00   0.00
-0.05   -0.02
30   6   3  68.00   0.00   0.00   0.00   0.00   1.00      0.00   0.00   0.00   0.00   0.00
-0.05   -0.02
31   6   4  68.00   0.00   0.00   0.00   0.00   1.00      0.00   0.00   0.00   0.00   0.00
-0.05   -0.02

32   7   1  68.00   0.00   0.00   0.00   0.00   1.00      0.00   0.00   0.00   0.00   0.00
-0.05   -0.02
33   7   2  68.00   0.00   0.00   0.00   0.00   1.00      0.00   0.00   0.00   0.00   0.00
-0.05   -0.02
34   7   3  68.00   0.00   0.00   0.00   0.00   1.00      0.00   0.00   0.00   0.00   0.00
-0.05   -0.02
```

```
        STREAM QUALITY SIMULATION                                                    OUTPUT
PAGE NUMBER     8
        QUAL-2E STREAM QUALITY ROUTING MODEL
EPA/NCASI VERSION
                                   ***** STEADY STATE SIMULATION *****

                                       ** DISSOLVED OXYGEN DATA **

                                               COMPONENTS OF DISSOLVED OXYGEN MASS BALANCE
(MG/L-DAY)
ELE RCH ELE              DO              DO     DAM     NIT
ORD NUM NUM    TEMP     SAT      DO     DEF   INPUT   INHIB      F-FNCTN   OXYGN                    NET
              DEG-F    MG/L    MG/L    MG/L    MG/L    FACT        INPUT   REAIR   C-BOD     SOD     P-R
NH3-N   NO2-N

 35   7   4   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02


 36   8   1   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 37   8   2   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 38   8   3   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 39   8   4   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 40   8   5   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02


 41   9   1   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 42   9   2   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 43   9   3   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02


 44  10   1   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 45  10   2   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 46  10   3   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 47  10   4   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02

 48  11   1   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 49  11   2   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 50  11   3   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 51  11   4   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 52  11   5   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 53  11   6   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 54  11   7   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
 55  11   8   68.00    0.00    0.00    0.00    0.00    1.00         0.00    0.00    0.00    0.00    0.00
-0.05   -0.02
```

SUMMARY OF MONTE CARLO INPUT VARIANCE CONDITIONS

	INPUT VARIABLE OR PARAMETER	INPUT DATA TYPE	RELATIVE STANDARD DEVIATION (%)
1	NITROGEN CONTENT OF ALGAE	1A	10.00
2	ALGY MAX SPEC GROWTH RATE	1A	10.00
3	ALGAE RESPIRATION RATE	1A	10.00
4	NITROGEN HALF SAT'N COEF	1A	10.00
5	LINEAR ALG SELF SHADE COEF	1A	10.00
6	NON-LIN ALG SELF SHADE CO	1A	10.00
7	LIGHT SAT'N COEFFICIENT	1A	10.00
8	LIGHT AVERAGING FACTOR	1A	2.00
9	ALG PREF FOR AMMONIA-N	1A	10.00
10	ALG TO TEMP SOLAR FACTOR	1A	1.00
11	TEMP COEF ORGANIC-N DECAY	1B	3.00
12	TEMP COEF ORGANIC-N SET	1B	3.00
13	TEMP COEF AMMONIA DECAY	1B	3.00
14	TEMP COEF AMMONIA SRCE	1B	3.00
15	TEMP COEF NITRITE DECAY	1B	3.00
16	TEMP COEF ORGANIC-P DECAY	1B	3.00
17	TEMP COEF ORGANIC-P SET	1B	3.00
18	TEMP COEF DISS-P SOURCE	1B	3.00
19	TEMP COEF ALGY GROWTH	1B	3.00
20	TEMP COEF ALGY RESPR	1B	3.00
21	TEMP COEF ALGY SETTLING	1B	3.00
22	DISPERSION CORR CONSTANT	5	20.00
23	COEF ON FLOW FOR VELOCITY	5	8.00
24	COEF ON FLOW FOR DEPTH	5	8.00
25	MANNING'S ROUGHNESS N	5	10.00
26	ORGANIC-N HYDROLYSIS RATE	6A	20.00
27	AMMONIA-N DECAY RATE	6A	25.00
28	NITRITE-N DECAY RATE	6A	20.00
29	ORGANIC-P HYDROLYSIS RATE	6A	20.00
30	CHLA TO ALGAE RATIO	6B	20.00
31	ALGAE SETTLING RATE	6B	15.00
32	LIGHT EXT COEFFICIENT	6B	5.00
33	INITIAL TEMPERATURE	7A	3.00
34	INCR-TEMPERATURE	8	1.00
35	INCR-ALGAE	8A	13.00
36	INCR-ORGANIC-PHOS	8A	25.00
37	INCR-DISSOLVED-PHOS	8A	20.00
38	HEADWATER FLOW	10	3.00
39	HWTR-TEMPERATURE	10	1.00
40	HWTR-ALGAE	10A	4.00
41	HWTR-ORGANIC-N	10A	6.00
42	HWTR-AMMONIA-N	10A	10.00
43	HWTR-NITRATE-N	10A	7.00
44	HWTR-ORGANIC-PHOS	10A	25.00
45	HWTR-DISSOLVED-PHOS	10A	7.00

```
MONTE CARLO SIMULATION SUMMARY, RESPONSE NO.   1  ORGN


                                       LOCATION
    STATISTIC

                       REACH  2       REACH  3       REACH  7       REACH 11
                      ELEMENT  2     ELEMENT 10     ELEMENT  1     ELEMENT  5

     BASE MEAN          0.342          0.321          0.301          0.289

     SIM MEAN           0.343          0.322          0.301          0.289

       BIAS             0.001          0.000          0.000          0.000


     MINIMUM            0.277          0.257          0.237          0.226

     MAXIMUM            0.406          0.375          0.360          0.353

     RANGE              0.129          0.118          0.123          0.127

     STD DEV            0.020          0.020          0.020          0.021

     COEF VAR           0.059          0.061          0.066          0.071

     SKEW COEF          0.062          0.075          0.124          0.160

   FREQUENCY
       DISTRIBUTION
    (STDV FROM MEAN)

                          CUM REL        CUM REL        CUM REL        CUM REL        CUM REL
                    FREQ  FREQ     FREQ  FREQ     FREQ  FREQ     FREQ  FREQ     FREQ  FREQ

          LT -4.0    0.  0.000      0.  0.000      0.  0.000      0.  0.000
     -4.0 TO -3.5    0.  0.000      0.  0.000      0.  0.000      0.  0.000
     -3.5 TO -3.0    1.  0.002      1.  0.002      1.  0.002      1.  0.002
     -3.0 TO -2.5    1.  0.004      1.  0.004      2.  0.006      2.  0.006
     -2.5 TO -2.0    7.  0.018      6.  0.016      8.  0.022      7.  0.020
     -2.0 TO -1.5   24.  0.066     26.  0.068     21.  0.064     16.  0.052
     -1.5 TO -1.0   46.  0.158     40.  0.148     41.  0.146     43.  0.138
     -1.0 TO -0.5   74.  0.306     78.  0.304     83.  0.312     81.  0.300
     -0.5 TO  0.0  117.  0.540    111.  0.526    104.  0.520    111.  0.522
      0.0 TO  0.5   78.  0.696     87.  0.700     93.  0.706     98.  0.718
      0.5 TO  1.0   65.  0.826     66.  0.832     70.  0.846     65.  0.848
      1.0 TO  1.5   50.  0.926     45.  0.922     36.  0.918     37.  0.922
      1.5 TO  2.0   26.  0.978     27.  0.976     30.  0.978     30.  0.982
      2.0 TO  2.5    9.  0.996      8.  0.992      4.  0.986      3.  0.988
      2.5 TO  3.0    1.  0.998      4.  1.000      7.  1.000      5.  0.998
      3.0 TO  3.5    1.  1.000      0.  1.000      0.  1.000      1.  1.000
      3.5 TO  4.0    0.  1.000      0.  1.000      0.  1.000      0.  1.000
          GT +4.0    0.  1.000      0.  1.000      0.  1.000      0.  1.000
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MONTE CARLO SIMULATION SUMMARY, RESPONSE NO.   2  NH3N


                                LOCATION
   STATISTIC

                   REACH  2      REACH  3      REACH  7      REACH 11
                  ELEMENT  2    ELEMENT 10    ELEMENT  1    ELEMENT  5

    BASE MEAN        0.045         0.035         0.029         0.027

    SIM MEAN         0.044         0.035         0.030         0.028

      BIAS           0.000         0.000         0.001         0.001


    MINIMUM          0.031         0.018         0.012         0.011

    MAXIMUM          0.057         0.065         0.070         0.072

    RANGE            0.026         0.047         0.057         0.061

    STD DEV          0.005         0.007         0.009         0.009

    COEF VAR         0.109         0.211         0.286         0.314

    SKEW COEF       -0.063         0.498         0.870         1.062

  FREQUENCY
     DISTRIBUTION
   (STDV FROM MEAN)

                        CUM REL       CUM REL       CUM REL       CUM REL       CUM REL
                  FREQ   FREQ   FREQ   FREQ   FREQ   FREQ   FREQ   FREQ   FREQ   FREQ

        LT -4.0    0.  0.000    0.  0.000    0.  0.000    0.  0.000
   -4.0 TO -3.5    0.  0.000    0.  0.000    0.  0.000    0.  0.000
   -3.5 TO -3.0    0.  0.000    0.  0.000    0.  0.000    0.  0.000
   -3.0 TO -2.5    6.  0.012    0.  0.000    0.  0.000    0.  0.000
   -2.5 TO -2.0    7.  0.026    6.  0.012    2.  0.004    0.  0.000
   -2.0 TO -1.5   20.  0.066   19.  0.050   14.  0.032   11.  0.022
   -1.5 TO -1.0   42.  0.150   47.  0.144   53.  0.138   55.  0.132
   -1.0 TO -0.5   77.  0.304   88.  0.320  100.  0.338  103.  0.338
   -0.5 TO  0.0  108.  0.520  104.  0.528  113.  0.564  119.  0.576
    0.0 TO  0.5   94.  0.708   97.  0.722   86.  0.736   82.  0.740
    0.5 TO  1.0   67.  0.842   59.  0.840   53.  0.842   55.  0.850
    1.0 TO  1.5   45.  0.932   43.  0.926   41.  0.924   38.  0.926
    1.5 TO  2.0   22.  0.976   20.  0.966   17.  0.958   16.  0.958
    2.0 TO  2.5   11.  0.998   11.  0.988   13.  0.984   12.  0.982
    2.5 TO  3.0    1.  1.000    4.  0.996    3.  0.990    3.  0.988
    3.0 TO  3.5    0.  1.000    1.  0.998    3.  0.996    2.  0.992
    3.5 TO  4.0    0.  1.000    0.  0.998    1.  0.998    2.  0.996
        GT +4.0    0.  1.000    1.  1.000    1.  1.000    2.  1.000
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MONTE CARLO SIMULATION SUMMARY, RESPONSE NO.   3  NO2N


                                     LOCATION
   STATISTIC

                         REACH  2      REACH  3      REACH  7      REACH 11
                        ELEMENT  2    ELEMENT 10    ELEMENT  1    ELEMENT  5

    BASE MEAN             0.002         0.002         0.001         0.001

    SIM MEAN              0.002         0.002         0.001         0.001

      BIAS                0.000         0.000         0.000         0.000


    MINIMUM               0.001         0.001         0.001         0.001

    MAXIMUM               0.005         0.004         0.003         0.003

    RANGE                 0.004         0.003         0.002         0.002

    STD DEV               0.001         0.000         0.000         0.000

    COEF VAR              0.292         0.251         0.248         0.252

    SKEW COEF             0.605         0.621         0.643         0.646

  FREQUENCY
    DISTRIBUTION
   (STDV FROM MEAN)

                         CUM REL       CUM REL       CUM REL       CUM REL       CUM REL
                   FREQ   FREQ   FREQ   FREQ   FREQ   FREQ   FREQ   FREQ   FREQ   FREQ

          LT -4.0    0.  0.000    0.  0.000    0.  0.000    0.  0.000
    -4.0 TO -3.5     0.  0.000    0.  0.000    0.  0.000    0.  0.000
    -3.5 TO -3.0     0.  0.000    0.  0.000    0.  0.000    0.  0.000
    -3.0 TO -2.5     0.  0.000    0.  0.000    0.  0.000    0.  0.000
    -2.5 TO -2.0     7.  0.014    3.  0.006    1.  0.002    2.  0.004
    -2.0 TO -1.5    17.  0.048   18.  0.042   18.  0.038   17.  0.038
    -1.5 TO -1.0    45.  0.138   47.  0.136   59.  0.156   61.  0.160
    -1.0 TO -0.5    89.  0.316  107.  0.350   96.  0.348   92.  0.344
    -0.5 TO  0.0   107.  0.530   97.  0.544  100.  0.548  103.  0.550
     0.0 TO  0.5    99.  0.728   86.  0.716   90.  0.728   90.  0.730
     0.5 TO  1.0    67.  0.862   66.  0.848   55.  0.838   54.  0.838
     1.0 TO  1.5    26.  0.914   38.  0.924   40.  0.918   41.  0.920
     1.5 TO  2.0    26.  0.966   20.  0.964   20.  0.958   20.  0.960
     2.0 TO  2.5     9.  0.984   10.  0.984   12.  0.982   11.  0.982
     2.5 TO  3.0     6.  0.996    5.  0.994    5.  0.992    5.  0.992
     3.0 TO  3.5     1.  0.998    2.  0.998    4.  1.000    3.  0.998
     3.5 TO  4.0     0.  0.998    1.  1.000    0.  1.000    1.  1.000
          GT +4.0    1.  1.000    0.  1.000    0.  1.000    0.  1.000
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MONTE CARLO SIMULATION SUMMARY, RESPONSE NO.   4  NO3N


                                      LOCATION
  STATISTIC

                   REACH  2      REACH  3      REACH  7      REACH 11
                  ELEMENT  2    ELEMENT 10    ELEMENT  1    ELEMENT  5

   BASE MEAN        0.111         0.142         0.168         0.183

   SIM MEAN         0.112         0.142         0.168         0.182

     BIAS           0.000         0.000        -0.001        -0.001


   MINIMUM          0.091         0.111         0.128         0.137

   MAXIMUM          0.133         0.172         0.206         0.227

   RANGE            0.042         0.062         0.078         0.091

   STD DEV          0.007         0.011         0.013         0.015

   COEF VAR         0.066         0.076         0.080         0.081

   SKEW COEF        0.163         0.098         0.033         0.043

 FREQUENCY
   DISTRIBUTION
 (STDV FROM MEAN)

                      CUM REL       CUM REL       CUM REL       CUM REL       CUM REL
                 FREQ  FREQ    FREQ  FREQ    FREQ  FREQ    FREQ  FREQ    FREQ  FREQ

        LT -4.0   0.  0.000     0.  0.000     0.  0.000     0.  0.000
  -4.0 TO -3.5    0.  0.000     0.  0.000     0.  0.000     0.  0.000
  -3.5 TO -3.0    0.  0.000     0.  0.000     0.  0.000     1.  0.002
  -3.0 TO -2.5    2.  0.004     3.  0.006     3.  0.006     3.  0.008
  -2.5 TO -2.0    7.  0.018     6.  0.018     8.  0.022     7.  0.022
  -2.0 TO -1.5   17.  0.052    22.  0.062    19.  0.060    19.  0.060
  -1.5 TO -1.0   51.  0.154    44.  0.150    50.  0.160    53.  0.166
  -1.0 TO -0.5   85.  0.324    85.  0.320    74.  0.308    68.  0.302
  -0.5 TO  0.0   96.  0.516    91.  0.502   102.  0.512   102.  0.506
   0.0 TO  0.5   85.  0.686   103.  0.708    91.  0.694    89.  0.684
   0.5 TO  1.0   70.  0.826    62.  0.832    75.  0.844    79.  0.842
   1.0 TO  1.5   58.  0.942    45.  0.922    45.  0.934    46.  0.934
   1.5 TO  2.0   16.  0.974    28.  0.978    19.  0.972    21.  0.976
   2.0 TO  2.5    8.  0.990     8.  0.994    10.  0.992     7.  0.990
   2.5 TO  3.0    5.  1.000     3.  1.000     4.  1.000     3.  0.996
   3.0 TO  3.5    0.  1.000     0.  1.000     0.  1.000     2.  1.000
   3.5 TO  4.0    0.  1.000     0.  1.000     0.  1.000     0.  1.000
        GT +4.0   0.  1.000     0.  1.000     0.  1.000     0.  1.000
```

```
MONTE CARLO SIMULATION SUMMARY, RESPONSE NO.   5  SUMN


                               LOCATION
   STATISTIC

                     REACH  2      REACH  3       REACH  7       REACH 11
                    ELEMENT  2    ELEMENT 10     ELEMENT  1     ELEMENT  5

    BASE MEAN          0.500         0.500          0.500          0.500

    SIM MEAN           0.501         0.501          0.501          0.501

      BIAS             0.001         0.001          0.001          0.001


    MINIMUM            0.436         0.436          0.436          0.436

    MAXIMUM            0.561         0.561          0.561          0.561

    RANGE              0.125         0.125          0.125          0.125

    STD DEV            0.022         0.022          0.022          0.022

    COEF VAR           0.044         0.044          0.044          0.044

    SKEW COEF          0.010         0.010          0.010          0.010

  FREQUENCY
      DISTRIBUTION
   (STDV FROM MEAN)

                     CUM REL        CUM REL        CUM REL        CUM REL        CUM REL
                FREQ  FREQ     FREQ  FREQ     FREQ  FREQ     FREQ  FREQ     FREQ  FREQ

       LT -4.0    0.  0.000      0.  0.000      0.  0.000      0.  0.000
   -4.0 TO -3.5   0.  0.000      0.  0.000      0.  0.000      0.  0.000
   -3.5 TO -3.0   0.  0.000      0.  0.000      0.  0.000      0.  0.000
   -3.0 TO -2.5   4.  0.008      4.  0.008      4.  0.008      4.  0.008
   -2.5 TO -2.0   7.  0.022      7.  0.022      7.  0.022      7.  0.022
   -2.0 TO -1.5  23.  0.068     23.  0.068     23.  0.068     23.  0.068
   -1.5 TO -1.0  39.  0.146     39.  0.146     39.  0.146     39.  0.146
   -1.0 TO -0.5  78.  0.302     78.  0.302     78.  0.302     78.  0.302
   -0.5 TO  0.0 103.  0.508    103.  0.508    103.  0.508    103.  0.508
    0.0 TO  0.5  90.  0.688     90.  0.688     90.  0.688     90.  0.688
    0.5 TO  1.0  79.  0.846     79.  0.846     79.  0.846     79.  0.846
    1.0 TO  1.5  45.  0.936     45.  0.936     45.  0.936     45.  0.936
    1.5 TO  2.0  18.  0.972     18.  0.972     18.  0.972     18.  0.972
    2.0 TO  2.5  11.  0.994     11.  0.994     11.  0.994     11.  0.994
    2.5 TO  3.0   3.  1.000      3.  1.000      3.  1.000      3.  1.000
    3.0 TO  3.5   0.  1.000      0.  1.000      0.  1.000      0.  1.000
    3.5 TO  4.0   0.  1.000      0.  1.000      0.  1.000      0.  1.000
       GT +4.0    0.  1.000      0.  1.000      0.  1.000      0.  1.000
```

APPENDIX E

PDF/M SOURCE PROGRAM

```
C*****************************************************
C     Ammonia Nitrogen Solver
C*****************************************************
      CHARACTER *32 IFILE
      REAL INIT,M1
      DIMENSION STA(50), TTRAV(50), ADD(41), SL(40)
C
C  Input, Output File Name Input
C
      WRITE(*,100)
      READ(*,105) IFILE
      OPEN(UNIT=5,FILE=IFILE,STATUS='OLD')
      WRITE(*,110)
      READ(*,105) IFILE
      OPEN(UNIT=6,FILE=IFILE,STATUS='NEW')
C
C  Station Input
C
      READ(5,*) NMS
      DO 5 I = 1,NMS
       READ(5,*) STA(I), TTRAV(I)
   5  CONTINUE
C
C  Probability Distribution Control
C
      READ(5,*) ONMIN, ONMAX, ONINT
      RANGE1 = ONMAX - ONMIN
      STEP1 = RANGE1/ONINT
      NPROBV = IFIX(STEP1)+1
C
C  Invariable Input
C
      READ(5,*) AO, UMAX, ALP1, RHO, S1,F1,D,P
C
C  Variable Input
C
      READ(5,*) AB1,SB1,AB2,SB2,AB3,SB3,AS3,SS3,AS4,SS4,
     +AN10,SN10,AN20,SN20,AN30,SN30,AN40,SN40
C
C  Write Variable Input
C
      WRITE(6,135) AB1,SB1,AB2,SB2,AB3, SB3, AS3, SS3,AS4,SS4,
     +AN10,SN10,AN20,SN20,AN30,SN30,AN40, SN40, NMS
C
C  Write Station Input
C
      IF(NMS.LT.10) THEN
        WRITE(6,140) (STA(J), J= 1,NMS)
      ELSE
        R1 = FLOAT(NMS)/10
        IR1 = IFIX(R1)
        DO 7  J1 = 1, IR1
          I1 = (J1-1)*10 + 1
          I2 =  I1 + 9
          IF(I2.GT.NMS) I2 = NMS
          WRITE(6,145) (STA(J), J= I1,I2)
   7    CONTINUE
      END IF
C
C  Write Variable Input
C
      WRITE(6,145) AO, UMAX, RHO, ALP1, S1,F1,D,P
C
C  First, the loop interates on each station
C
      DO 10 ICSTA = 1, NMS
       OPEN(3,FILE='SCRTCH.FIL',STATUS='NEW')
       TOTPROB = 0
       T = TTRAV(ICSTA)
C
C  Calculation of average Ammonia Nitrogen
C
      WRITE(*,*)' STATION :',ICSTA
 101  F=F1
      C1 = AB3 + AS4
      ALP = UMAX - RHO - S1
      M1 = AO*ALP1*RHO/(C1-ALP)
      A1=AB3*M1
      A2=F1*ALP1*UMAX*AO/(AB1-ALP)
      E1=EXP(-ALP*T)-EXP(-AB1*T)
      E2=EXP(-C1*T)-EXP(-AB1*T)
      E3=EXP(-AB1*T)
      AN1 = (A1-A2)/(AB1-ALP)*E1-(A1-AN40*AB3)/(AB1-C1)*E2+
     +(AN10-AS3/D/AB1)*E3+AS3/D/AB1

      CL1=(A1-A2)/(AB1-ALP)
      CL2=(A1-AN40*AB3)/(AB1-C1)
      CL3=(AN10-AS3/D/AB1)
      CL4=AS3/D/AB1
      CN1=AB1*CL1/(AB2-ALP)
      CN2=AB1*CL2/(AB2-C1)
      CN3=CL4*AB1/AB2
      CN4=(-CL1+CL2+CL3)*AB1/(AB2-AB1)
      E1=EXP(-ALP*T)-EXP(-AB2*T)
      E2=EXP(-C1*T)-EXP(-AB2*T)
      E3=1-EXP(-AB2*T)
```

```
      E4=EXP(-AB1*T)-EXP(-AB2*T)
      E5=EXP(-AB2*T)
      AN2=CN1*E1-CN2*E2+CN3*E3+CN4*E4+AN20*E5

      CM1=(-AB2*CN1/ALP+1./ALP*(1-F1)*ALP1*UMAX*A0)
      CM21=AB2*CN2/C1
      CM22=AB2*CN4/AB1
      CM3=CN1-CN2+CN3+CN4-AN20
      E1=EXP(-ALP*T)-1
      E21=EXP(-C1*T)-1
      E22=EXP(-AB1*T)-1
      E3=EXP(-AB2*T)-1
      AN3=CM1*E1+CM21*E21-CM22*E22+CM3*E3+CN3*AB2*T+AN30
C
C  F1 Convergency Check
C
      F1=P*AN1/(P*AN1+AN3-P*AN3)
      WRITE(*,*)' AN1,AN2,AN3,F1 LOOP:', AN1,AN2,AN3,F1
      IF(ABS(F1-F).LT.0.001) GOTO 102
      GOTO 101
  102 AVEN4=AN1
C
C  Constants for variance calculations
C
      A = A0*ALP1*RHO
      E1C1 = EXP(-C1*T)
      E2C1 = EXP(-2*C1*T)
      E1AL = EXP(-ALP*T)
      E2AL = EXP(-2*ALP*T)
      E1B1 = EXP(-AB1*T)
      E2B1 = EXP(-2*AB1*T)
      EE1S4 = EXP(SS4*SS4*T*T/2)
      EE2S4 = EXP(2*SS4*SS4*T*T)
      EE1B3 = EXP(SB3*SB3*T*T/2)
      EE2B3 = EXP(2*SB3*SB3*T*T)
      EE1S3 = EXP(SS3*SS3*T*T/2)
      EE2S3 = EXP(2*SS3*SS3*T*T)
      EE1B1 = EXP(SB1*SB1*T*T/2)
      EE2B1 = EXP(2*SB1*SB1*T*T)
      CE2B1=E2B1*EE2B1
      CE1B1=E1B1*EE1B1
      CEBC1=E1B1*E1C1*EE1B1
      CEBAL=E1B1*E1AL*EE1B1
      E1C3=E1C1*EE1B3*EE1S4
      E2C3=E2C1*EE2B3*EE2S4
      CECC=E2C1*EE1B3*EE1S4
      CEBC3=E1B1*E1C1*EE1B3*EE1S4*EE1B1
      P11=E2AL+CE2B1-2*CEBAL
      P22=E2C1+CE2B1-2*CEBC1
      P33=E2C3+CE2B1-2*CEBC3
      P44=CE2B1
      P55=1.
      P12=(CE2B1+E1AL*E1C1-CEBC1-CEBAL)
      P13=(CE2B1+E1AL*E1C3-CEBC3-CEBAL)
      P14=(-CE2B1+CEBAL)
      P15=E1AL-CE1B1
      P23=(CE2B1+CECC-CEBC1-CEBC3)
      P24=(-CE2B1+CEBC1)
      P25=E1C1-CE1B1
      P34=(-CE2B1+CEBC3)
      P35=E1C3-CE1B1
      P45=CE1B1
      PAI=(C1-ALP)
      UPAID=PAI*PAI+SB3*SB3+SS4*SS4
      UC2S=SB3*SB3+SS4*SS4
      U1=(A1-A2)/(AB1-ALP)
      U2=(A1-AN40*AB3)/(AB1-C1)
      U3=(AN40-M1)
      U4=AN10-AS3/D/AB1
      U5=-U1+U2+U4
      U6=-U5
      C11=(M1*M1*SB3*SB3+M1*M1*UC2S/UPAID*(AB3*AB3+SB3*SB3)+
     +U1*U1*SB1*SB1-2.*M1*M1*AB3*SB3*SB3/(C1-ALP))/((AB1-ALP)*
     +(AB1-ALP)+SB1*SB1)
      C22=(U3*U3*AB3*AB3+U2*U2*SB1*SB1 )/
     +((AB1-C1)*(AB1-C1)+SB1*SB1)
      C33=((M1*M1*UC2S/UPAID+U3*U3+SN40*SN40)*(AB3*AB3+SB3*SB3)
     ++4.*M1*U3*AB3*SB3*SB3/(C1-ALP))/
     +((AB1-C1)*(AB1-C1)+SB3*SB3+SS4*SS4+SB1*SB1)
      C44=(SS3*SS3+AS3*AS3*SB1*SB1/AB1/AB1)/D/D/
     +(AB1*AB1+SB1*SB1)+U5*U5+SN10*SN10
      C55=(SS3*SS3+AS3*AS3*SB1*SB1/AB1/AB1)/D/D/(AB1*AB1+SB1*SB1)+
     +U6*U6*E2B1
      C12=-(U1*U2*SB1*SB1+M1*U3*SB3*SB3+U3*M1*AB3*SB3*SB3/(C1-ALP))
     +/((AB1-ALP)*(AB1-C1)+SB1*SB1)
      C13=(M1*U3*SB3*SB3-M1*M1*UC2S/UPAID*(AB3*AB3+SB3*SB3)-
     +2.*M1*U3*AB3*SB3*SB3/(C1-ALP)+M1*M1*SB3*SB3*AB3/(C1-ALP))
     +/((AB1-ALP)*(AB1-C1)+SB1*SB1)
      C14=(-U5*M1*SB3*SB3/(C1-ALP)/(AB1-ALP)-
     +U1*AS3*SB1*SB1/(D*AB1*(AB1-ALP)*(AB1*AB1-AB1*ALP+SB1*SB1)))
      C15=(U5*M1*SB3*SB3/(C1-ALP)/(AB1-ALP)*E1B1+
     +U1*AS3*SB1*SB1/(D*AB1*(AB1-ALP)*(AB1*AB1-AB1*ALP+SB1*SB1)))
      C23=-(U3*U3*AB3*AB3+M1*U3*AB3*SB3*SB3/(C1-ALP))
     +/((AB1-C1)*(AB1-C1)+SB1*SB1)
      C24=(-U5*U3*AB3+U2*AS3/D/AB1/AB1*SB1*SB1)/(AB1-C1)
      C25=(-U2*AS3*SB1*SB1/D/AB1/AB1-U6*E1B1*U3*AB3)/(AB1-C1)
```

```
      C34=-(U6*U3*AB3+M1*U6*SB3*SB3/(C1-ALP))/(AB1-C1)
      C35=(U6*U3*AB3+M1*U4*SB3*SB3/(C1-ALP))*E1B1/(AB1-C1)
      C45=U5*U6*E1B1-SS3*SS3/D/D/(AB1*AB1+SB1*SB1)-SS3*SS3*SB1*SB1
     +/D/D/D/AB1/AB1/(AB1*AB1+SB1*SB1)
      V2N1=C11*P11+2*C12*P12+2*C13*P13+2*C14*P14+2*C15*P15+
     +C22*P22+2*C23*P23+2*C24*P24+2*C25*P25+C33*P33+
     +2*C34*P34+2*C35*P35+C44*P44+2*C45*P45+C55*P55

      V1N1=-M1*SB3*SB3/(C1-ALP)/(AB1-ALP)*(E1AL-CE1B1)-
     +U3*AB3/(AB1-C1)*(E1C1-CE1B1)+
     +(2.*M1*AB3*SB3*SB3/((C1-ALP)*(AB1-C1)-UC2S)+U3*AB3/(AB1-C1))*
     +(E1C3-CE1B1)-U6*CE1B1+U6*E1B1
      V1=V2N1-V1N1*V1N1
      Z2N4 = V1/2.
C
C  A range of values for B1,B3,S3,S4,Initial Organic Nitrogen
C  and Initial Ammonia Nitrogen will be examined and the resulting
C  Ammonia NITROGEN probability density function
C  generated at the selected point.
C
       ICC = 0
       B3 = AB3 - (4.*SB3)
       DO 15 IK = 1,6
       B3 = B3 + SB3
       S4 = AS4 - (4.*SS4)
       DO 20 IM = 1,6
       S4 = S4 + SS4
       B1=AB1-4*SB1
       DO 21 IL=1,6
       B1=B1+SB1
       S3=AS3-4*SS3
       DO 22 IZ=1,6
       S3=S3+SS3
       AN4=AN40-4*SN40
       DO 23 IU=1,6
       AN4=AN4+SN40
       ONO=AN10-4*SN10
       DO 25 IN = 1,6
       ONO = ONO + SN10
C
C  Establish variables for calculation and calculate AMMONIA NITROGEN
C
      C1 = B3 + S4
      ALP = UMAX - RHO - S1
      M1 = A0*ALP1*RHO/(C1-ALP)
      A1= B3*M1
      A2=F1*ALP1*UMAX*A0/(B1-ALP)
      E1=EXP(-ALP*T)-EXP(-B1*T)
```

```
      E2=EXP(-C1*T)-EXP(-B1*T)
      E3=EXP(-B1*T)
      ON4 = (A1-A2)/(B1-ALP)*E1-(A1-AN4*B3)/(B1-C1)*E2+
     +(ONO-S3/D/B1)*E3+S3/D/B1
C
C     Calculate associated probability
C
       ZX = ((ON4-AVEN4)*(ON4-AVEN4))/(4*Z2N4)
       PROBN = EXP(-ZX)/(2*SQRT(3.1416*Z2N4))
       TOTPROB = TOTPROB + PROBN
       ICC = ICC + 1
C
       WRITE(3,*) ON4, PROBN
C
  25   CONTINUE
  23   CONTINUE
  22   CONTINUE
  21   CONTINUE
  20   CONTINUE
  15   CONTINUE
C
C      Output file headings are printed for this pass
C
       WRITE(6,150) STA(ICSTA), TTRAV(ICSTA)
      WRITE(6,*)' ANH3 :',AVEN4,'  SN3:',SQRT(V1),'  CNVGD F1 :',F1
C
C  The total probabilities for this time (mile station) are calculated
C  First, the intervals are calculated based on user input
C
      SL(1) = ONMIN
      DO 51  ICIN = 2, NPROBV
         SL(ICIN) = SL(ICIN-1) + ONINT
 51   CONTINUE
C
C  Counters are zeroed
C
      DO 52 IZC = 1,NPROBV+1
        ADD(IZC) = 0.0
 52   CONTINUE
C
      REWIND 3
      DO 35 JJ=1,ICC
      READ(3,*) ON4, PROB
           IF(ON4.LT.SL(1)) ADD(1) = ADD(1) + PROB
           IF(ON4.GE.SL(NPROBV)) ADD(NPROBV+1) = ADD(NPROBV+1) + PROB
C
         DO 40 K=2,NPROBV
             IF(ON4.GE.SL(K-1).AND.ON4.LT.SL(K)) ADD(K) = ADD(K) + PROB
```

```
   40     CONTINUE
   35   CONTINUE
C
C  Normalized probabilities are calculated and printed
C
        WRITE(6,155)
        ADD(1) = ADD(1)/TOTPROB
        CPROBL = ADD(1)
        WRITE(6,160) SL(1), ADD(1), SL(1), CPROBL
        DO 45 K=2,NPROBV
          ADD(K) = ADD(K)/TOTPROB
          CPROBL = CPROBL + ADD(K)
          WRITE(6,165) SL(K-1),SL(K),ADD(K),SL(K), CPROBL
   45   CONTINUE
        CPROBL = CPROBL + ADD(NPROBV+1)/TOTPROB
        WRITE(6,170) SL(NPROBV),ADD(NPROBV+1)/TOTPROB, CPROBL
   10   CONTINUE
C
        CLOSE(UNIT=6)
        CLOSE(UNIT=5)
        STOP
C
C   **********  FORMAT STATEMENTS ************
C
  100   FORMAT(' What is the name of the input file ? :',$)
  105   FORMAT(A32)
  110   FORMAT(' What do you wish to name the output file ? :',$)
  135   FORMAT(5X,'The Input Values for this run are:',//
     +' B1 AVE :',F16.5,' STD. :',F16.5/
     +' B2 AVE :',F16.5,' STD. :',F16.5/
     +' B3 AVE :',F16.5,' STD. :',F16.5/
     +' S3 AVE :',F16.5,' STD. :',F16.5/
     +' S4 AVE :',F16.5,' STD. :',F16.5/
     +' N1 AVE :',F16.5,' STD. :',F16.5/
     +' N2 AVE :',F16.5,' STD. :',F16.5/
     +' N3 AVE :',F16.5,' STD. :',F16.5/
     +' N4 AVE :',F16.5,' STD. :',F16.5/
     +' NUMBER OF REACH :',I5)
  140   FORMAT(10(5X,F6.2))
  145   FORMAT(//,10X,'The Following Algae Constants Have Been Entered:',
     1/,2X,'Ao = ',F6.2,'  u max = ',F6.3,'  p = ',F6.3,'  Alpha = ',
     3F6.3,'  Sed = ',F6.2,' F1 = ',F6.2,' D = ',F6.2,' P = ',F6.2//)
  150   FORMAT(//' This is the Probability Density Function at River Mile',
     12F10.2,/)
  155   FORMAT(//,30X,'AMMONIA NITROGEN',/,5X,'PROBABILITY DENSITY CURVE',
     118X,'CUMMULATIVE DENSITY CURVE',/,5X,'RANGE',17X,'PROBABILITY',
     210X,'RANGE',9X,'PROBABILITY',/,35('='),10X,30('='))
  160   FORMAT(10X,'O-N < ',F9.4,6X,F8.5,9X,'< ',F9.4,7X,F8.5)
```

```
165  FORMAT(5X,F9.4,'-',F9.4,7X,F8.5,9X,'< ',F9.4,7X,F8.5)
170  FORMAT(9X,'0-N > ',F9.4,7X,F8.5,9X,'Total for Run',7X,F8.5)
     END
```

```
C*************************************************
C     Nitrite Nitrogen Solver
C*************************************************
      CHARACTER *32 IFILE
      REAL INIT,M1
      DIMENSION STA(50), TTRAV(50), ADD(41), SL(40)
C
C  Input, Output File Name Input
C
      WRITE(*,100)
      READ(*,105) IFILE
      OPEN(UNIT=5,FILE=IFILE,STATUS='OLD')
      WRITE(*,110)
      READ(*,105) IFILE
      OPEN(UNIT=6,FILE=IFILE,STATUS='NEW')
C
C  Station Input
C
      READ(5,*) NMS
      DO 5 I = 1,NMS
       READ(5,*) STA(I), TTRAV(I)
    5 CONTINUE
C
C  Probability Distribution Control
C
      READ(5,*) ONMIN, ONMAX, ONINT
      RANGE1 = ONMAX - ONMIN
      STEP1 = RANGE1/ONINT
      NPROBV = IFIX(STEP1)+1
C
C  Invariable Input
C
      READ(5,*) AO, WMAX, ALP1, RHO, S1,F1,D,P
C
C  Variable Input
C
      READ(5,*) AB1,SB1,AB2,SB2,AB3,SB3,AS3,SS3,AS4,SS4,
     +AN10,SN10,AN20,SN20,AN30,SN30,AN40,SN40
C
C  Write Variable Input
C
      WRITE(6,135) AB1,SB1,AB2,SB2,AB3, SB3, AS3, SS3,AS4,SS4,
     +AN10,SN10,AN20,SN20,AN30,SN30,AN40, SN40, NMS
C
C  Write Station Input
C
      IF(NMS.LT.10) THEN
        WRITE(6,140) (STA(J), J= 1,NMS)
      ELSE
        R1 = FLOAT(NMS)/10
        IR1 = IFIX(R1)
        DO 7  J1 = 1, IR1
          I1 = (J1-1)*10 + 1
          I2 =  I1 + 9
          IF(I2.GT.NMS) I2 = NMS
          WRITE(6,145) (STA(J), J= I1,I2)
   7    CONTINUE
      END IF
C
C  Write Variable Input
C
      WRITE(6,145) AO, UMAX, RHO, ALP1, S1,F1,D,P
C
C  First, the loop interates on each station
C
      DO 10 ICSTA = 1, NMS
       OPEN(3,FILE='SCRTCH.FIL',STATUS='NEW')
       TOTPROB = 0
       T = TTRAV(ICSTA)
C
C  Calculation of average Nitrite Nitrogen
C
      WRITE(*,*)' STATION :',ICSTA
  101 F=F1
      C1 = AB3 + AS4
      ALP = WMAX - RHO - S1
      M1 = AO*ALP1*RHO/(C1-ALP)
      A1=AB3*M1
      A2=F1*ALP1*UMAX*AO/(AB1-ALP)
      E1=EXP(-ALP*T)-EXP(-AB1*T)
      E2=EXP(-C1*T)-EXP(-AB1*T)
      E3=EXP(-AB1*T)
      AN1 = (A1-A2)/(AB1-ALP)*E1-(A1-AN40*AB3)/(AB1-C1)*E2+
     +(AN10-AS3/D/AB1)*E3+AS3/D/AB1

      CL1=(A1-A2)/(AB1-ALP)
      CL2=(A1-AN40*AB3)/(AB1-C1)
      CL3=(AN10-AS3/D/AB1)
      CL4=AS3/D/AB1
      CN1=AB1*CL1/(AB2-ALP)
      CN2=AB1*CL2/(AB2-C1)
      CN3=CL4*AB1/AB2
      CN4=(-CL1+CL2+CL3)*AB1/(AB2-AB1)
      E1=EXP(-ALP*T)-EXP(-AB2*T)
      E2=EXP(-C1*T)-EXP(-AB2*T)
      E3=1-EXP(-AB2*T)
```

```
      E4=EXP(-AB1*T)-EXP(-AB2*T)
      E5=EXP(-AB2*T)
      AN2=CN1*E1-CN2*E2+CN3*E3+CN4*E4+AN20*E5

      CM1=(-AB2*CN1/ALP+1./ALP*(1-F1)*ALP1*UMAX*A0)
      CM21=AB2*CN2/C1
      CM22=AB2*CN4/AB1
      CM3=CN1-CN2+CN3+CN4-AN20
      E1=EXP(-ALP*T)-1
      E21=EXP(-C1*T)-1
      E22=EXP(-AB1*T)-1
      E3=EXP(-AB2*T)-1
      AN3=CM1*E1+CM21*E21-CM22*E22+CM3*E3+CN3*AB2*T+AN30
C
C  F1 Convergency Check
C
      F1=P*AN1/(P*AN1+AN3-P*AN3)
      WRITE(*,*)' AN1,AN2,AN3,F1 LOOP:', AN1,AN2,AN3,F1
      IF(ABS(F1-F).LT.0.001) GOTO 102
      GOTO 101
  102 AVEN4=AN2
C
C  Constants for variance calculations
C
      A = A0*ALP1*RHO
      E1C1 = EXP(-C1*T)
      E2C1 = EXP(-2*C1*T)
      E1AL = EXP(-ALP*T)
      E2AL = EXP(-2*ALP*T)
      E1B1 = EXP(-AB1*T)
      E2B1 = EXP(-2*AB1*T)
      E1B2 = EXP(-AB2*T)
      E2B2 = EXP(-2*AB2*T)
      DON1=SB2*SB2*T*T/2
      DON2=DON1*4
      EE1B2 = EXP(DON1)
      EE2B2 = EXP(DON2)
      EE1B1 = EXP(SB1*SB1*T*T/2)
      EE2B1 = EXP(2*SB1*SB1*T*T)
      CE2B2=E2B2*EE2B2
      CE1B2=E1B2*EE1B2
      CEBC1=E1B2*E1C1*EE1B2
      CEBAL=E1B2*E1AL*EE1B2
      CEB1B=E1B1*CE1B2
      CEB2B=E1B2*CE1B2
      P11=E2AL+CE2B2-2*CEBAL
      P22=E2C1+CE2B2-2*CEBC1
      P33=1+CE2B2-2*CE1B2
      P44=E2B1+CE2B2-2*CEB1B
      P55=E2B2+CE2B2-2*CEB2B
      P66=CE2B2
      P12=(CE2B2+E1AL*E1C1-CEBC1-CEBAL)
      P13=(CE2B2+E1AL-CE1B2-CEBAL)
      P14=(CE2B2+E1AL*E1B1-CEBC1-CEB1B)
      P15=(CE2B2+E1AL*E1B2-CEB2B-CEBAL)
      P16=CEBAL-CE2B2
      P23=(CE2B2+E1C1-CEBC1-CE1B2)
      P24=(CE2B2+E1C1*E1B1-CEBC1-CEB1B)
      P25=(CE2B2+E1C1*E1B2-CEBC1-CEB2B)
      P26=(-CE2B2+CEBC1)
      P34=(CE2B2+E2B1-CE1B2-CEB1B)
      P35=(CE2B2+E2B2-CE1B2-CEB2B)
      P36=CE1B2-CE2B2
      P45=(CE2B2+E1B1*E1B2-CEB2B-CEB1B)
      P46=CEB1B-CE2B2
      P56=CEB2B-CE2B2
      US1=AB1*AB1+SB1*SB1
      US2=AB2*AB2+SB2*SB2
      U2=CN1-CN2+CN3+CN4-AN20
      U3=-CL1+CL2+CL3
      C11=(CL1*CL1*US1+CN1*CN1*SB2*SB2)/((AB2-ALP)*(AB2-ALP)+SB2*SB2)
      C22=(CL2*CL2*US1+CN2*CN2*SB2*SB2)/((AB2-C1)*(AB2-C1)+SB2*SB2)
      C33=(CL4*CL4*US1+CN3*CN3*SB2*SB2)/(US2)
      C44=(U3*U3*US1+CN4*CN4*SB2*SB2)/((AB2-AB1)*(AB2-AB1)+SB2*SB2)
      C55=U2*U2
      C66=SN20*SN20
      C12=-(CL1*CL2*US1+CN1*CN2*SB2*SB2)/((AB2-ALP)*(AB2-C1)+SB2*SB2)
      C13=(CL1*CL4*US1+CN1*CN3*SB2*SB2)/(US2-ALP*AB2)
      C14=(CL1*U3*US1+CN1*CN4*SB2*SB2)/((AB2-ALP)*(AB2-AB1)+SB2*SB2)
      C15=CL1*U2*AB1/(AB2-ALP)
      C16=0
      C23=-(CL2*CL4*US1+CN2*CN3*SB2*SB2)/(US2-C1*AB2)
      C24=-(U3*CL2*US1+CN2*CN4*SB2*SB2)/((AB2-C1)*(AB2-AB1)+SB2*SB2)
      C25=-CL2*U2*AB1/(AB2-C1)
      C26=0
      C34=(CL4*U3*US1+CN3*CN4*SB2*SB2)/(US2-AB1*AB2)
      C35=CL4*U2*AB1/(AB2)
      C36=0
      C45=U2*U3*AB1/(AB2-AB1)
      C46=0
      C56=0
      V2N2=C11*P11+2*C12*P12+2*C13*P13+2*C14*P14+2*C15*P15+2*C16*P16
     ++C22*P22+2*C23*P23+2*C24*P24+2*C25*P25+2*C26*P26
     ++C33*P33+2*C34*P34+2*C35*P35+2*C36*P36
     ++C44*P44+2*C45*P45+2*C46*P46
     ++C55*P55+2*C56*P56
```

```
     ++C66*P66
      V1N2=CL1*AB1/(AB2-ALP)*(E1AL-CE1B2)-CL2*AB1/(AB2-C1)*(E1C1-CE1B2)
     ++CL4*AB1/AB2*(1-CE1B2)+U3*AB1/(AB2-AB1)*(E1B1-CE1B2)
     ++U2*(E1B2-CE1B2)
      V2=V2N2-V1N2*V1N2
      Z2N4 = V2/2.
C
C  A range of values for B1,B2,B3,S3,S4,Initial Organic Nitrogen,
C  Initial Ammonia Nitrogen and Initial Nitrite Nitrogen will be
C  examined and the resulting Nitrite Nitrogen probability density
C  function generated at the selected point.
C
       ICC = 0
       B2 = AB2-(4.*SB2)
       DO 14 IA=1,6
       B2=B2+SB2
       B3 = AB3 - (4.*SB3)
       DO 15 IK = 1,6
       B3 = B3 + SB3
       S4 = AS4 - (4.*SS4)
       DO 20 IN = 1,6
       S4 = S4 + SS4
       B1=AB1-4*SB1
       DO 21 IL=1,6
       B1=B1+SB1
       S3=AS3-4*SS3
       DO 22 IZ=1,6
       S3=S3+SS3
       AN4=AN40-4.*SN40
       DO 23 IU=1,6
       AN4=AN4+SN40
       AN1=AN10-4.*SN10
       DO 24 IV=1,6
       AN1=AN1+SN10
       ONO=AN20-4*SN20
       DO 25 IN = 1,6
       ONO = ONO + SN20
C
C  Establish variables for calculation and calculate O-NITROGEN
C
      C1 = B3 + S4
      ALP = UMAX - RHO - S1
      M1 = AO*ALP1*RHO/(C1-ALP)
      A1= B3*M1
      A2=F1*ALP1*UMAX*AO/(B1-ALP)
      CL1=(A1-A2)/(B1-ALP)
      CL2=(A1-AN4*B3)/(B1-C1)
      CL3=(AN1-S3/D/B1)
```

```
      CL4=S3/D/B1
      CN1=B1*CL1/(B2-ALP)
      CN2=B1*CL2/(B2-C1)
      CN3=CL4*B1/B2
      CN4=(-CL1+CL2+CL3)*B1/(B2-B1)
      E1=EXP(-ALP*T)-EXP(-B2*T)
      E2=EXP(-C1*T)-EXP(-B2*T)
      E3=1-EXP(-B2*T)
      E4=EXP(-B1*T)-EXP(-B2*T)
      E5=EXP(-B2*T)
      AN2=CN1*E1-CN2*E2+CN3*E3+CN4*E4+ONO*E5
      ON4 =AN2
C
C    Calculate associated probability
C
       IF(ABS(Z2N4).LT.1.E-20) Z2N4=1.E-20
       ZX = ((ON4-AVEN4)*(ON4-AVEN4))/(4*Z2N4)
       PROBN = EXP(-ZX)/(2*SQRT(3.1416*Z2N4))
       TOTPROB = TOTPROB + PROBN
       ICC = ICC + 1
C
       WRITE(3,*) ON4, PROBN
C
  25   CONTINUE
  24   CONTINUE
  23   CONTINUE
  21   CONTINUE
  15   CONTINUE
  14   CONTINUE
C
C     Output file headings are printed for this pass
C
       WRITE(6,150) STA(ICSTA), TTRAV(ICSTA)
      WRITE(6,*)' ANO2 :',AVEN4,'  SNO2: ',SQRT(V2),'  CNVGD F1 :',F1
C
C  The total probabilities for this time (mile station) are calculated
C  First, the intervals are calculated based on user input
C
      SL(1) = ONMIN
      DO 51  ICIN = 2, NPROBV
         SL(ICIN) = SL(ICIN-1) + ONINT
 51   CONTINUE
C
C  Counters are zeroed
C
      DO 52 IZC = 1,NPROBV+1
        ADD(IZC) = 0.0
 52   CONTINUE
```

```
C
      REWIND 3
      DO 35 JJ=1,ICC
      READ(3,*) ON4, PROB
           IF(ON4.LT.SL(1)) ADD(1) = ADD(1) + PROB
           IF(ON4.GE.SL(NPROBV)) ADD(NPROBV+1) = ADD(NPROBV+1) + PROB
C
         DO 40 K=2,NPROBV
             IF(ON4.GE.SL(K-1).AND.ON4.LT.SL(K)) ADD(K) = ADD(K) + PROB
  40     CONTINUE
  35  CONTINUE
C
C  Normalized probabilities are calculated and printed
C
      WRITE(6,155)
      ADD(1) = ADD(1)/TOTPROB
      CPROBL = ADD(1)
      WRITE(6,160) SL(1), ADD(1), SL(1), CPROBL
      DO 45 K=2,NPROBV
        ADD(K) = ADD(K)/TOTPROB
        CPROBL = CPROBL + ADD(K)
        WRITE(6,165) SL(K-1),SL(K),ADD(K),SL(K), CPROBL
  45  CONTINUE
      CPROBL = CPROBL + ADD(NPROBV+1)/TOTPROB
      WRITE(6,170) SL(NPROBV),ADD(NPROBV+1)/TOTPROB, CPROBL
  10  CONTINUE
C
      CLOSE(UNIT=6)
      CLOSE(UNIT=5)
      STOP
C
C   **********  FORMAT STATEMENTS ***********
C
 100  FORMAT(' What is the name of the input file ? :',$)
 105  FORMAT(A32)
 110  FORMAT(' What do you wish to name the output file ? :',$)
 135  FORMAT(5X,'The Input Values for this run are:',//
     +' B1 AVE :',F16.5,' STD. :',F16.5/
     +' B2 AVE :',F16.5,' STD. :',F16.5/
     +' B3 AVE :',F16.5,' STD. :',F16.5/
     +' S3 AVE :',F16.5,' STD. :',F16.5/
     +' S4 AVE :',F16.5,' STD. :',F16.5/
     +' N1 AVE :',F16.5,' STD. :',F16.5/
     +' N2 AVE :',F16.5,' STD. :',F16.5/
     +' N3 AVE :',F16.5,' STD. :',F16.5/
     +' N4 AVE :',F16.5,' STD. :',F16.5/
     +' NUMBER OF REACH :',I5)
 140  FORMAT(10(5X,F6.2))
```

```
145  FORMAT(//,10X,'The Following Algae Constants Have Been Entered:',
    1/,2X,'Ao = ',F6.2,'  u max = ',F6.3,'  p = ',F6.3,'  Alpha = ',
    3F6.3,'  Sed = ',F6.2,' F1 = ',F6.2,' D = ',F6.2,' P = ',F6.2//)
150  FORMAT(//' This is the Probability Density Function at River Mile',
    12F10.2,/)
155  FORMAT(//,30X,'NITRITE NITROGEN',/,5X,'PROBABILITY DENSITY CURVE',
    118X,'CUMMULATIVE DENSITY CURVE',/,5X,'RANGE',17X,'PROBABILITY',
    210X,'RANGE',9X,'PROBABILITY',/,35('='),10X,30('='))
160  FORMAT(10X,'O-N < ',F9.4,6X,F8.5,9X,'< ',F9.4,7X,F8.5)
165  FORMAT(5X,F9.4,'-',F9.4,7X,F8.5,9X,'< ',F9.4,7X,F8.5)
170  FORMAT(9X,'O-N > ',F9.4,7X,F8.5,9X,'Total for Run',7X,F8.5)
     END
```

```
C*****************************************************
C     Nitrate Nitrogen Solver
C*****************************************************
      CHARACTER *32 IFILE
      REAL INIT,M1
      DIMENSION STA(50), TTRAV(50), ADD(41), SL(40)
C
C  Input, Output File Name Input
C
      WRITE(*,100)
      READ(*,105) IFILE
      OPEN(UNIT=5,FILE=IFILE,STATUS='OLD')
      WRITE(*,110)
      READ(*,105) IFILE
      OPEN(UNIT=6,FILE=IFILE,STATUS='NEW')
C
C  Station Input
C
      READ(5,*) NMS
      DO 5 I = 1,NMS
       READ(5,*) STA(I), TTRAV(I)
    5 CONTINUE
C
C  Probability Distribution Control
C
      READ(5,*) ONMIN, ONMAX, ONINT
      RANGE1 = ONMAX - ONMIN
      STEP1 = RANGE1/ONINT
      NPROBV = IFIX(STEP1)+1
C
C  Invariable Input
C
      READ(5,*) AO, UMAX, ALP1, RHO, S1,F1,D,P
C
C  Variable Input
C
      READ(5,*) AB1,SB1,AB2,SB2,AB3,SB3,AS3,SS3,AS4,SS4,
     +AN10,SN10,AN20,SN20,AN30,SN30,AN40,SN40
C
C  Write Variable Input
C
      WRITE(6,135) AB1,SB1,AB2,SB2,AB3, SB3, AS3, SS3,AS4,SS4,
     +AN10,SN10,AN20,SN20,AN30,SN30,AN40, SN40, NMS
C
C  Write Station Input
C
      IF(NMS.LT.10) THEN
        WRITE(6,140) (STA(J), J= 1,NMS)
```

```
    ELSE
      R1 = FLOAT(NMS)/10
      IR1 = IFIX(R1)
      DO 7  J1 = 1, IR1
        I1 = (J1-1)*10 + 1
        I2 =  I1 + 9
        IF(I2.GT.NMS) I2 = NMS
        WRITE(6,145) (STA(J), J= I1,I2)
 7    CONTINUE
    END IF
C
C  Write Variable Input
C
    WRITE(6,145) A0, UMAX, RHO, ALP1, S1,F1,D,P
C
C  First, the loop interates on each station
C
    DO 10 ICSTA = 1, NMS
     OPEN(3,FILE='SCRTCH.FIL',STATUS='NEW')
     TOTPROB = 0
     T = TTRAV(ICSTA)
C
C  Calculation of average Nitrate Nitrogen
C
    WRITE(*,*)' STATION :',ICSTA
 101 F=F1
    C1 = AB3 + AS4
    ALP = UMAX - RHO - S1
    M1 = A0*ALP1*RHO/(C1-ALP)
    A1=AB3*M1
    A2=F1*ALP1*UMAX*A0/(AB1-ALP)
    E1=EXP(-ALP*T)-EXP(-AB1*T)
    E2=EXP(-C1*T)-EXP(-AB1*T)
    E3=EXP(-AB1*T)
    AN1 = (A1-A2)/(AB1-ALP)*E1-(A1-AN40*AB3)/(AB1-C1)*E2+
   +(AN10-AS3/D/AB1)*E3+AS3/D/AB1

    CL1=(A1-A2)/(AB1-ALP)
    CL2=(A1-AN40*AB3)/(AB1-C1)
    CL3=(AN10-AS3/D/AB1)
    CL4=AS3/D/AB1
    CN1=AB1*CL1/(AB2-ALP)
    CN2=AB1*CL2/(AB2-C1)
    CN3=CL4*AB1/AB2
    CN4=(-CL1+CL2+CL3)*AB1/(AB2-AB1)
    E1=EXP(-ALP*T)-EXP(-AB2*T)
    E2=EXP(-C1*T)-EXP(-AB2*T)
    E3=1-EXP(-AB2*T)
```

```
      E4=EXP(-AB1*T)-EXP(-AB2*T)
      E5=EXP(-AB2*T)
      AN2=CN1*E1-CN2*E2+CN3*E3+CN4*E4+AN20*E5

      CM1=(-AB2*CN1/ALP+1./ALP*(1-F1)*ALP1*UMAX*AO)
      CM21=AB2*CN2/C1
      CM22=AB2*CN4/AB1
      CM3=CN1-CN2+CN3+CN4-AN20
      E1=EXP(-ALP*T)-1
      E21=EXP(-C1*T)-1
      E22=EXP(-AB1*T)-1
      E3=EXP(-AB2*T)-1
      AN3=CM1*E1+CM21*E21-CM22*E22+CM3*E3+CN3*AB2*T+AN30
C
C  F1 Convergency Check
C
      F1=P*AN1/(P*AN1+AN3-P*AN3)
      WRITE(*,*)' AN1,AN2,AN3,F1 LOOP:', AN1,AN2,AN3,F1
      IF(ABS(F1-F).LT.0.001) GOTO 102
      GOTO 101
  102 AVEN4=AN3
C
C  Constants for variance calculations
C
      A = AO*ALP1*RHO
      E1C1 = EXP(-C1*T)-1
      E1AL = EXP(-ALP*T)-1
      E1B1 = EXP(-AB1*T)-1
      E1B2 = EXP(-AB2*T)-1
      write(6,*)'c1,al,b1,b2',e1c1,e1al,e1b1,e1b2
      D1=-CN1/ALP
      D2=CN1-CN2+CN3+CN4-AN20
      U=D1*E1AL+CN2/C1*E1C1-CN4/AB1*E1B1+D2/AB2*E1B2
      V3=U*U*SB2*SB2+SN30*SN30
      Z2N4 = V3/2.
C
C  A range of values for B1,B2,B3,S3,S4,Initial Organic Nitrogen,
C  Initial Ammonia Nitrogen, Initial Nitrite Nitrogen and Initial
C  Nitrate Nitrogen will be examined and the resulting Nitrate
C  NITROGEN probability density function
C  generated at the selected point.
C
      ICC = 0
      B2 = AB2-(4.*SB2)
      DO 14 IA=1,6
      B2=B2+SB2
      B3 = AB3 - (4.*SB3)
      DO 15 IK = 1,6
```

```
       B3 = B3 + SB3
       S4 = AS4 - (4.*SS4)
       DO 20 IM = 1,6
       S4 = S4 + SS4
       B1=AB1-4*SB1
       DO 21 IL=1,6
       B1=B1+SB1
       S3=AS3-4*SS3
       DO 22 IZ=1,6
       S3=S3+SS3
       AN4=AN40-4.*SN40
       DO 23 IU=1,6
       AN4=AN4+SN40
       AN1=AN10-4.*SN10
       DO 24 IV=1,6
       AN1=AN1+SN10
       AN2=AN20-4.*SN20
       DO 26 IW=1,6
       AN2=AN2+SN20
       ONO=AN30-4*SN30
       DO 25 IN = 1,6
       ONO = ONO + SN30
C
C  Establish variables for calculation and calculate O-NITROGEN
C
      C1 = B3 + S4
      ALP = UMAX - RHO - S1
      M1 = AO*ALP1*RHO/(C1-ALP)
      A1= B3*M1
      A2=F1*ALP1*UMAX*AO/(B1-ALP)
      CL1=(A1-A2)/(B1-ALP)
      CL2=(A1-AN4*B3)/(B1-C1)
      CL3=(AN1-S3/D/B1)
      CL4=S3/D/B1
      CN1=B1*CL1/(B2-ALP)
      CN2=B1*CL2/(B2-C1)
      CN3=CL4*B1/B2
      CN4=(-CL1+CL2+CL3)*B1/(B2-B1)
      CM1=(-B2*CN1/ALP+1./ALP*(1-F1)*ALP1*UMAX*AO)
      CM21=B2*CN2/C1
      CM22=B2*CN4/B1
      CM3=CN1-CN2+CN3+CN4-AN2
      E1=EXP(-ALP*T)-1
      E21=EXP(-C1*T)-1
      E22=EXP(-B1*T)-1
      E3=EXP(-B2*T)-1
      AN3=CM1*E1+CM21*E21-CM22*E22+CM3*E3+CN3*AB2*T+ONO
      ON4 =AN3
```

```
C
C    Calculate associated probability
C
       IF(ABS(Z2N4).LT.1.E-20) Z2N4=1.E-20
       ZX = ((ON4-AVEN4)*(ON4-AVEN4))/(4*Z2N4)
       PROBN = EXP(-ZX)/(2*SQRT(3.1416*Z2N4))
       TOTPROB = TOTPROB + PROBN
       ICC = ICC + 1
C
       WRITE(3,*) ON4, PROBN
C
  25   CONTINUE
  26   CONTINUE
  24   CONTINUE
  23   CONTINUE
  21   CONTINUE
  15   CONTINUE
  14   CONTINUE
C
C     Output file headings are printed for this pass
C
       WRITE(6,150) STA(ICSTA), TTRAV(ICSTA)
      WRITE(6,*)' ANO3 :',AVEN4,'  SNO3: ',SQRT(V3),' CNVGD F1 :',F1
C
C  The total probabilities for this time (mile station) are calculated
C  First, the intervals are calculated based on user input
C
      SL(1) = ONMIN
      DO 51  ICIN = 2, NPROBV
         SL(ICIN) = SL(ICIN-1) + ONINT
 51   CONTINUE
C
C  Counters are zeroed
C
      DO 52 IZC = 1,NPROBV+1
        ADD(IZC) = 0.0
 52   CONTINUE
C
      REWIND 3
      DO 35 JJ=1,ICC
      READ(3,*) ON4, PROB
           IF(ON4.LT.SL(1)) ADD(1) = ADD(1) + PROB
           IF(ON4.GE.SL(NPROBV)) ADD(NPROBV+1) = ADD(NPROBV+1) + PROB
C
         DO 40 K=2,NPROBV
             IF(ON4.GE.SL(K-1).AND.ON4.LT.SL(K)) ADD(K) = ADD(K) + PROB
  40     CONTINUE
  35  CONTINUE
```

```
C
C  Normalized probabilities are calculated and printed
C
      WRITE(6,155)
      ADD(1) = ADD(1)/TOTPROB
      CPROBL = ADD(1)
      URITE(6,160) SL(1), ADD(1), SL(1), CPROBL
      DO 45 K=2,NPROBV
        ADD(K) = ADD(K)/TOTPROB
        CPROBL = CPROBL + ADD(K)
        WRITE(6,165) SL(K-1),SL(K),ADD(K),SL(K), CPROBL
  45  CONTINUE
      CPROBL = CPROBL + ADD(NPROBV+1)/TOTPROB
      WRITE(6,170) SL(NPROBV),ADD(NPROBV+1)/TOTPROB, CPROBL
  10  CONTINUE
C
      CLOSE(UNIT=6)
      CLOSE(UNIT=5)
      STOP
C
C   **********  FORMAT STATEMENTS ************
C
 100  FORMAT(' What is the name of the input file ? :',$)
 105  FORMAT(A32)
 110  FORMAT(' What do you wish to name the output file ? :',$)
 135  FORMAT(5X,'The Input Values for this run are:',//
     +' B1 AVE :',F16.5,' STD. :',F16.5/
     +' B2 AVE :',F16.5,' STD. :',F16.5/
     +' B3 AVE :',F16.5,' STD. :',F16.5/
     +' S3 AVE :',F16.5,' STD. :',F16.5/
     +' S4 AVE :',F16.5,' STD. :',F16.5/
     +' N1 AVE :',F16.5,' STD. :',F16.5/
     +' N2 AVE :',F16.5,' STD. :',F16.5/
     +' N3 AVE :',F16.5,' STD. :',F16.5/
     +' N4 AVE :',F16.5,' STD. :',F16.5/
     +' NUMBER OF REACH :',I5)
 140  FORMAT(10(5X,F6.2))
 145  FORMAT(//,10X,'The Following Algae Constants Have Been Entered:',
     1/,2X,'Ao = ',F6.2,'  u max = ',F6.3,'  p = ',F6.3,'  Alpha = ',
     3F6.3,'  Sed = ',F6.2,' F1 = ',F6.2,' D = ',F6.2,' P = ',F6.2//)
 150  FORMAT(//' This is the Probability Density Function at River Nile',
     12F10.2,/)
 155  FORMAT(//,30X,'NITRATE NITROGEN',/,5X,'PROBABILITY DENSITY CURVE',
     118X,'CUMMULATIVE DENSITY CURVE',/,5X,'RANGE',17X,'PROBABILITY',
     210X,'RANGE',9X,'PROBABILITY',/,35('='),10X,30('='))
 160  FORMAT(10X,'O-N < ',F9.4,6X,F8.5,9X,'< ',F9.4,7X,F8.5)
 165  FORMAT(5X,F9.4,'-',F9.4,7X,F8.5,9X,'< ',F9.4,7X,F8.5)
 170  FORMAT(9X,'O-N > ',F9.4,7X,F8.5,9X,'Total for Run',7X,F8.5)
```

```
C
      END
```

```
C*****************************************************
C     Organic Nitrogen Solver
C*****************************************************
      CHARACTER *32 IFILE
      REAL INIT
      DIMENSION STA(50), TTRAV(50), ADD(41), SL(40)
C
C  Input, Output File Name Input
C
      WRITE(*,100)
      READ(*,105) IFILE
      OPEN(UNIT=5,FILE=IFILE,STATUS='OLD')
      WRITE(*,110)
      READ(*,105) IFILE
      OPEN(UNIT=6,FILE=IFILE,STATUS='NEW')
C
C  Station Input
C
      READ(5,*) NMS
      DO 5 I = 1,NMS
       READ(5,*) STA(I), TTRAV(I)
   5  CONTINUE
C
C  Probability Distribution Control
C
      READ(5,*) ONMIN, ONMAX, ONINT
      RANGE1 = ONMAX - ONMIN
      STEP1 = RANGE1/ONINT
      NPROBV = IFIX(STEP1)+1
C
C  Invariable Input
C
      READ(5,*) A0, UMAX, ALP1, RHO, S1,F1,D,P
C
C  Variable Input
C
      READ(5,*) AB1,SB1,AB2,SB2,AB3,SB3,AS3,SS3,AS4,SS4,
     +AN10,SN10,AN20,SN20,AN30,SN30,AN40,SN40
C
C  Write Variable Input
C
      WRITE(6,135) AB1,SB1,AB2,SB2,AB3, SB3, AS3, SS3,AS4,SS4,
     +AN10,SN10,AN20,SN20,AN30,SN30,AN40, SN40, NMS
C
C  Write Station Input
C
      IF(NMS.LT.10) THEN
```

```
        WRITE(6,140) (STA(J), J= 1,NMS)
      ELSE
        R1 = FLOAT(NMS)/10
        IR1 = IFIX(R1)
        DO 7  J1 = 1, IR1
          I1 = (J1-1)*10 + 1
          I2 =  I1 + 9
          IF(I2.GT.NMS) I2 = NMS
          WRITE(6,145) (STA(J), J= I1,I2)
   7    CONTINUE
      END IF
C
C  Write Variable Input
C
      WRITE(6,145) AO, UMAX, RHO, ALP1, S1,F1,D,P

C
C  First, the loop interates on each station
C
      DO 10 ICSTA = 1, NMS
       OPEN(3,FILE='SCRTCH.FIL',STATUS='NEW')
       TOTPROB = 0
       T = TTRAV(ICSTA)
C
C  Calculation of average Organic Nitrogen
C
      C1 = AB3 + AS4
      ALP = UMAX - RHO - S1
      CSTE = AO*ALP1*RHO/(C1-ALP)
      EXPONT = EXP(-ALP*T) - EXP(-C1*T)
      INIT = AN40*EXP(-C1*T)
      AVEN4 = CSTE*EXPONT + INIT
C
C  Constants for variance calculations
C
      A = AO*ALP1*RHO
      EOA = EXP(-C1*T)
      EOB = EXP(-2*C1*T)
      E1 = EXP(SS4*SS4*T*T/2)
      E2 = EXP(2*SS4*SS4*T*T)
      E3 = EXP(SB3*SB3*T*T/2)
      E4 = EXP(2*SB3*SB3*T*T)
      E5A = EXP(-ALP*T)
      E5B = EXP(-2*ALP*T)
      CSTEN = A*A*(SB3*SB3 + SS4*SS4)
      CSTED = (C1-ALP)*(C1-ALP)*(C1*C1 + SB3*SB3 + SS4*SS4)
      CSTE3 = (AN40 - CSTE)*(AVEN4 - CSTE)
C
```

```
C  Calculation of variance, integrated over initial conditions
C
      TERM1 = CSTEN * (EOB*E2*E4 - 2*EOA*E5A*E1*E3 + E5B)/ CSTED
      TERM2 = SN40*SN40*EOB*E2*E4
      TERM3 = CSTE3*EOB*(E2*E4 - 2*E1*E3 + 1)

      ec2=sb3*sb3+ss4*ss4
      EE2C3=EOB*E4*E2
      EEC3=EOA*E3*E1
      PAI=C1-ALP
      TRM1=EC2*A*A/PAI/PAI/(PAI*PAI+EC2)
      TRM1=TRM1*(EE2C3+E5B-2*EEC3*E5A)
      TRM2=(AN40-A/PAI)*(AN40-A/PAI)*(EE2C3+EOB-2*EEC3*EOA)
      TRM3=SN40*SN40*EE2C3
      V1N4 = (AN40-A/PAI)*(EEC3-EOA)
      V2N4 = TERM1 + TERM2 + TERM3
      V4=V2N4-V1N4*V1N4
      Z2N4 = V4/2.
C
C  A range of values for B3, S4, and Initial Organic Nitrogen will be
C  examined and the resulting O-NITROGEN probability density function
C  generated at the selected point.
C
      ICC = 0
      B3 = AB3 - (4.*SB3)
      DO 15 IK = 1,6
      B3 = B3 + SB3
      IF(B3.LT.0) GO TO 15
      S4 = AS4 - (4.*SS4)
      DO 20 IM = 1,6
      S4 = S4 + SS4
      IF(S4.LT.0) GO TO 20
      ONO = AN40 - (4.*SN40)
      DO 25 IN = 1,6
      ONO = ONO + SN40
      IF(ONO.LT.0) GO TO 25
C
C  Establish variables for calculation and calculate O-NITROGEN
C
      C1 = B3 + S4
      CSTE = AO*ALP1*RHO/(C1-ALP)
      EXPONT = EXP(-ALP*T) - EXP(-C1*T)
      INIT = ONO*EXP(-C1*T)
      ON4 = CSTE*EXPONT + INIT
C
C  Calculate associated probability
C
      ZX = ((ON4-AVEN4)*(ON4-AVEN4))/(4*Z2N4)
```

```
      PROBN = EXP(-ZX)/(2*SQRT(3.1416*Z2N4))
      TOTPROB = TOTPROB + PROBN
      ICC = ICC + 1
C
      WRITE(3,*) ON4, PROBN
C
 25   CONTINUE
 20   CONTINUE
 15   CONTINUE
C
C    Output file headings are printed for this pass
C
      WRITE(6,150) STA(ICSTA), TTRAV(ICSTA)
     WRITE(6,*)' AN4 :',AVEN4,'  VN4 :',SQRT(V4)
C
C  The total probabilities for this time (mile station) are calculated
C  First, the intervals are calculated based on user input
C
     SL(1) = ONMIN
     DO 21  ICIN = 2, NPROBV
        SL(ICIN) = SL(ICIN-1) + ONINT
 21  CONTINUE
C
C  Counters are zeroed
C
     DO 22 IZC = 1,NPROBV+1
       ADD(IZC) = 0.0
 22  CONTINUE
C  Acumulate the probability in each distribution
     REWIND 3
     DO 35 JJ=1,ICC
     READ(3,*) ON4, PROB
          IF(ON4.LT.SL(1)) ADD(1) = ADD(1) + PROB
          IF(ON4.GE.SL(NPROBV)) ADD(NPROBV+1) = ADD(NPROBV+1) + PROB
        DO 40 K=2,NPROBV
            IF(ON4.GE.SL(K-1).AND.ON4.LT.SL(K)) ADD(K) = ADD(K) + PROB
  40    CONTINUE
  35 CONTINUE
C
C  Normalized probabilities are calculated and printed
C
     WRITE(6,155)
     ADD(1) = ADD(1)/TOTPROB
     CPROBL = ADD(1)
     WRITE(6,160) SL(1), ADD(1), SL(1), CPROBL
     DO 45 K=2,NPROBV
       ADD(K) = ADD(K)/TOTPROB
       CPROBL = CPROBL + ADD(K)
```

```
        WRITE(6,165) SL(K-1),SL(K),ADD(K),SL(K), CPROBL
  45  CONTINUE
      CPROBL = CPROBL + ADD(NPROBV+1)/TOTPROB
      WRITE(6,170) SL(NPROBV),ADD(NPROBV+1)/TOTPROB, CPROBL
  10  CONTINUE
C
      CLOSE(UNIT=6)
      CLOSE(UNIT=5)
      STOP
C
C   **********  FORMAT STATEMENTS ************
C
 100  FORMAT(' What is the name of the input file ? :',$)
 105  FORMAT(A32)
 110  FORMAT(' What do you wish to name the output file ? :',$)
 135  FORMAT(5X,'The Input Values for this run are:',//
     +' B1 AVE :',F16.5,' STD. :',F16.5/
     +' B2 AVE :',F16.5,' STD. :',F16.5/
     +' B3 AVE :',F16.5,' STD. :',F16.5/
     +' S3 AVE :',F16.5,' STD. :',F16.5/
     +' S4 AVE :',F16.5,' STD. :',F16.5/
     +' N1 AVE :',F16.5,' STD. :',F16.5/
     +' N2 AVE :',F16.5,' STD. :',F16.5/
     +' N3 AVE :',F16.5,' STD. :',F16.5/
     +' N4 AVE :',F16.5,' STD. :',F16.5/
     +' NUMBER OF REACH :',I5)
 140  FORMAT(10(5X,F6.2))
 145  FORMAT(//,10X,'The Following Algae Constants Have Been Entered:',
     1/,2X,'Ao = ',F6.2,'  u max = ',F6.3,'  p = ',F6.3,'  Alpha = ',
     3F6.3,'  Sed = ',F6.2,' F1 = ',F6.2,' D = ',F6.2,' P = ',F6.2//)
 150  FORMAT(/' This is the Probability Density Function at River Mile',
     12F10.2,/)
 155  FORMAT(//,30X,'ORGANIC NITROGEN',/,5X,'PROBABILITY DENSITY CURVE',
     118X,'CUMMULATIVE DENSITY CURVE',/,5X,'RANGE',17X,'PROBABILITY',
     210X,'RANGE',9X,'PROBABILITY',/,35('='),10X,30('='))
 160  FORMAT(10X,'O-N < ',F9.4,6X,F8.5,9X,'< ',F9.4,7X,F8.5)
 165  FORMAT(5X,F9.4,'-',F9.4,7X,F8.5,9X,'< ',F9.4,7X,F8.5)
 170  FORMAT(9X,'O-N > ',F9.4,7X,F8.5,9X,'Total for Run',7X,F8.5)
      END
```

APPENDIX F

PDF/M INPUT AND OUTPUT DATA

INPUT FOR AMMONIA NITROGEN

```
5
1 0
2 .519
3 2.087
4 3.69
5 4.558
0. 0.1 .005
.03 1.307 .085 .15 0 1. 8. 0.5
.5 .125 10. 2. .04 .008 0 0 0 0 .05 .005 0.002 0.001 0.1 .007 .35 .021
```

OUTPUT FOR AMMONIA NITROGEN

```
    The Input Values for this run are:

B1 AVE :          .50000 STD. :          .12500
B2 AVE :        10.00000 STD. :         2.00000
B3 AVE :          .04000 STD. :          .00800
S3 AVE :          .00000 STD. :          .00000
S4 AVE :          .00000 STD. :          .00000
N1 AVE :          .05000 STD. :          .00500
N2 AVE :          .00200 STD. :          .00100
N3 AVE :          .10000 STD. :          .00700
N4 AVE :          .35000 STD. :          .02100
NUMBER OF REACH :    5
      1.00       2.00       3.00       4.00       5.00


          The Following Algae Constants Have Been Entered:
 Ao =     .03  u max =  1.307  p =   .150  Alpha =   .085  Sed =    .00 F1 =   1.00 D =   8.00 P =
.50



This is the Probability Density Function at River Nile      1.00       .00

 ANH3 :  5.000000E-002  SN3:  4.999998E-003  CNVGD F1 :  3.333333E-001


                              AMMONIA NITROGEN
    PROBABILITY DENSITY CURVE                     CUMMULATIVE DENSITY CURVE
    RANGE                   PROBABILITY           RANGE          PROBABILITY
================================                ======================
           O-N <     .0000       .00000           <    .0000       .00000
          .0000-    .0050        .00000           <    .0050       .00000
          .0050-    .0100        .00000           <    .0100       .00000
          .0100-    .0150        .00000           <    .0150       .00000
          .0150-    .0200        .00000           <    .0200       .00000
          .0200-    .0250        .00000           <    .0250       .00000
          .0250-    .0300        .00000           <    .0300       .00000
          .0300-    .0350        .00000           <    .0350       .00000
          .0350-    .0400        .00445           <    .0400       .00445
          .0400-    .0450        .05425           <    .0450       .05870
          .0450-    .0500        .24311           <    .0500       .30181
          .0500-    .0550        .40082           <    .0550       .70264
          .0550-    .0600        .24311           <    .0600       .94575
          .0600-    .0650        .05425           <    .0650       .99999
          .0650-    .0700        .00000           <    .0700       .99999
          .0700-    .0750        .00000           <    .0750       .99999
          .0750-    .0800        .00000           <    .0800       .99999
          .0800-    .0850        .00000           <    .0850       .99999
          .0850-    .0900        .00000           <    .0900       .99999
          .0900-    .0950        .00000           <    .0950       .99999
          .0950-    .1000        .00000           <    .1000       .99999
```

```
        O-N >     .1000        .00000        Total for Run        .99999

This is the Probability Density Function at River Mile     2.00      .52

 ANH3 :  4.540391E-002  SN3:  4.524591E-003  CNVGD F1 :  2.900558E-001

                              AMMONIA NITROGEN
    PROBABILITY DENSITY CURVE                    CUMMULATIVE DENSITY CURVE
    RANGE                  PROBABILITY           RANGE          PROBABILITY
=============================              ==========================
        O-N <     .0000        .00000        <     .0000         .00000
       .0000-    .0050        .00000        <     .0050         .00000
       .0050-    .0100        .00000        <     .0100         .00000
       .0100-    .0150        .00000        <     .0150         .00000
       .0150-    .0200        .00000        <     .0200         .00000
       .0200-    .0250        .00000        <     .0250         .00000
       .0250-    .0300        .00009        <     .0300         .00009
       .0300-    .0350        .00765        <     .0350         .00774
       .0350-    .0400        .10843        <     .0400         .11617
       .0400-    .0450        .38628        <     .0450         .50245
       .0450-    .0500        .38446        <     .0500         .88691
       .0500-    .0550        .10461        <     .0550         .99152
       .0550-    .0600        .00832        <     .0600         .99984
       .0600-    .0650        .00016        <     .0650        1.00000
       .0650-    .0700        .00000        <     .0700        1.00000
       .0700-    .0750        .00000        <     .0750        1.00000
       .0750-    .0800        .00000        <     .0800        1.00000
       .0800-    .0850        .00000        <     .0850        1.00000
       .0850-    .0900        .00000        <     .0900        1.00000
       .0900-    .0950        .00000        <     .0950        1.00000
       .0950-    .1000        .00000        <     .1000        1.00000
        O-N >     .1000        .00000        Total for Run        1.00000

This is the Probability Density Function at River Mile     3.00      2.09

 ANH3 :  3.530555E-002  SN3:  6.279240E-003  CNVGD F1 :  1.995137E-001

                              AMMONIA NITROGEN
    PROBABILITY DENSITY CURVE                    CUMMULATIVE DENSITY CURVE
    RANGE                  PROBABILITY           RANGE          PROBABILITY
=============================              ==========================
        O-N <     .0000        .00000        <     .0000         .00000
       .0000-    .0050        .00000        <     .0050         .00000
       .0050-    .0100        .00000        <     .0100         .00000
       .0100-    .0150        .00010        <     .0150         .00010
       .0150-    .0200        .00418        <     .0200         .00428
       .0200-    .0250        .04550        <     .0250         .04978
       .0250-    .0300        .18486        <     .0300         .23464
       .0300-    .0350        .30998        <     .0350         .54461
       .0350-    .0400        .28156        <     .0400         .82617
       .0400-    .0450        .13563        <     .0450         .96180
       .0450-    .0500        .03296        <     .0500         .99476
       .0500-    .0550        .00486        <     .0550         .99962
       .0550-    .0600        .00037        <     .0600         .99998
       .0600-    .0650        .00001        <     .0650        1.00000
       .0650-    .0700        .00000        <     .0700        1.00000
       .0700-    .0750        .00000        <     .0750        1.00000
       .0750-    .0800        .00000        <     .0800        1.00000
       .0800-    .0850        .00000        <     .0850        1.00000
       .0850-    .0900        .00000        <     .0900        1.00000
       .0900-    .0950        .00000        <     .0950        1.00000
```

```
         .0950-    .1000          .00000          <    .1000         1.00000
          0-N >    .1000          .00000          Total for Run       1.00000

 This is the Probability Density Function at River Mile       4.00       3.69

  ANH3 :  2.953509E-002  SN3:  7.272332E-003  CNVGD F1 :  1.500706E-001


                                    AMMONIA NITROGEN
    PROBABILITY DENSITY CURVE                         CUMMULATIVE DENSITY CURVE
    RANGE                     PROBABILITY             RANGE          PROBABILITY
 ===================================               ============================
          0-N <    .0000          .00000          <    .0000          .00000
         .0000-    .0050          .00000          <    .0050          .00000
         .0050-    .0100          .00090          <    .0100          .00090
         .0100-    .0150          .01697          <    .0150          .01786
         .0150-    .0200          .08694          <    .0200          .10481
         .0200-    .0250          .23049          <    .0250          .33529
         .0250-    .0300          .29330          <    .0300          .62859
         .0300-    .0350          .21864          <    .0350          .84723
         .0350-    .0400          .10977          <    .0400          .95700
         .0400-    .0450          .03451          <    .0450          .99151
         .0450-    .0500          .00736          <    .0500          .99888
         .0500-    .0550          .00104          <    .0550          .99991
         .0550-    .0600          .00008          <    .0600          .99999
         .0600-    .0650          .00000          <    .0650         1.00000
         .0650-    .0700          .00000          <    .0700         1.00000
         .0700-    .0750          .00000          <    .0750         1.00000
         .0750-    .0800          .00000          <    .0800         1.00000
         .0800-    .0850          .00000          <    .0850         1.00000
         .0850-    .0900          .00000          <    .0900         1.00000
         .0900-    .0950          .00000          <    .0950         1.00000
         .0950-    .1000          .00000          <    .1000         1.00000
          0-N >    .1000          .00000          Total for Run       1.00000

 This is the Probability Density Function at River Mile       5.00       4.56

  ANH3 :  2.748497E-002  SN3:  7.452490E-003  CNVGD F1 :  1.327220E-001


                                    AMMONIA NITROGEN
    PROBABILITY DENSITY CURVE                         CUMMULATIVE DENSITY CURVE
    RANGE                     PROBABILITY             RANGE          PROBABILITY
 ===================================               ============================
          0-N <    .0000          .00000          <    .0000          .00000
         .0000-    .0050          .00000          <    .0050          .00000
         .0050-    .0100          .00404          <    .0100          .00404
         .0100-    .0150          .03992          <    .0150          .04397
         .0150-    .0200          .15218          <    .0200          .19614
         .0200-    .0250          .26991          <    .0250          .46605
         .0250-    .0300          .26985          <    .0300          .73591
         .0300-    .0350          .16793          <    .0350          .90383
         .0350-    .0400          .07162          <    .0400          .97545
         .0400-    .0450          .02020          <    .0450          .99564
         .0450-    .0500          .00387          <    .0500          .99951
         .0500-    .0550          .00045          <    .0550          .99996
         .0550-    .0600          .00004          <    .0600         1.00000
         .0600-    .0650          .00000          <    .0650         1.00000
         .0650-    .0700          .00000          <    .0700         1.00000
         .0700-    .0750          .00000          <    .0750         1.00000
         .0750-    .0800          .00000          <    .0800         1.00000
         .0800-    .0850          .00000          <    .0850         1.00000
         .0850-    .0900          .00000          <    .0900         1.00000
```

```
.0900-    .0950       .00000       <     .0950       1.00000
.0950-    .1000       .00000       <     .1000       1.00000
O-N >     .1000       .00000       Total for Run       1.00000
```

INPUT FOR NITRITE NITROGEN

```
3
1 0
2 .519
3 2.087
0. 0.01 .0005
.03 1.307 .085 .15 0 1. 8. 0.5
.5 .125 10. 2. .04 .008 0 0 0 0 .05 .005 0.002 0.001 0.1 .007 .35 .021
```

OUTPUT FOR NITRITE NITROGEN

```
    The Input Values for this run are:

 B1 AVE :          .50000 STD. :          .12500
 B2 AVE :        10.00000 STD. :         2.00000
 B3 AVE :          .04000 STD. :          .00800
 S3 AVE :          .00000 STD. :          .00000
 S4 AVE :          .00000 STD. :          .00000
 N1 AVE :          .05000 STD. :          .00500
 N2 AVE :          .00200 STD. :          .00100
 N3 AVE :          .10000 STD. :          .00700
 N4 AVE :          .35000 STD. :          .02100
 NUMBER OF REACH :    3
      1.00       2.00       3.00


         The Following Algae Constants Have Been Entered:
  Ao =     .03  u max =  1.307  p =   .150  Alpha =    .085  Sed =     .00 F1 =   1.00 D =    8.00 P =
.50



 This is the Probability Density Function at River Mile      1.00       .00

  ANO2 :  2.000000E-003  SNO2:   1.000000E-003  CNVGD F1 :  3.333333E-001


                                 NITRITE NITROGEN
    PROBABILITY DENSITY CURVE                     CUMMULATIVE DENSITY CURVE
    RANGE                  PROBABILITY            RANGE          PROBABILITY
==================================            ===========================
          O-N <     .0000         .00445           <     .0000          .00445
          .0000-    .0005         .05424           <     .0005          .05869
          .0005-    .0010         .00000           <     .0010          .05869
          .0010-    .0015         .24309           <     .0015          .30179
          .0015-    .0020         .00000           <     .0020          .30179
          .0020-    .0025         .40080           <     .0025          .70258
          .0025-    .0030         .24309           <     .0030          .94568
          .0030-    .0035         .00000           <     .0035          .94568
          .0035-    .0040         .00000           <     .0040          .94568
          .0040-    .0045         .05424           <     .0045          .99992
          .0045-    .0050         .00000           <     .0050          .99992
          .0050-    .0055         .00000           <     .0055          .99992
          .0055-    .0060         .00000           <     .0060          .99992
          .0060-    .0065         .00000           <     .0065          .99992
          .0065-    .0070         .00000           <     .0070          .99992
          .0070-    .0075         .00000           <     .0075          .99992
          .0075-    .0080         .00000           <     .0080          .99992
          .0080-    .0085         .00000           <     .0085          .99992
          .0085-    .0090         .00000           <     .0090          .99992
          .0090-    .0095         .00000           <     .0095          .99992
          O-N >     .0095         .00000           Total for Run          .99992
```

```
This is the Probability Density Function at River Mile      2.00       .52

 ANO2 :  2.309327E-003  SNO2:   5.634175E-004  CNVGD F1 :  2.900558E-001


                                   NITRITE NITROGEN
    PROBABILITY DENSITY CURVE                       CUMMULATIVE DENSITY CURVE
     RANGE                    PROBABILITY            RANGE         PROBABILITY
==============================                 ==========================
          O-N <      .0000          .00000          <      .0000        .00000
         .0000-     .0005           .00037          <      .0005        .00037
         .0005-     .0010           .01015          <      .0010        .01052
         .0010-     .0015           .07939          <      .0015        .08991
         .0015-     .0020           .27651          <      .0020        .36643
         .0020-     .0025           .36489          <      .0025        .73132
         .0025-     .0030           .20542          <      .0030        .93674
         .0030-     .0035           .05609          <      .0035        .99282
         .0035-     .0040           .00675          <      .0040        .99957
         .0040-     .0045           .00042          <      .0045        .99999
         .0045-     .0050           .00001          <      .0050       1.00000
         .0050-     .0055           .00000          <      .0055       1.00000
         .0055-     .0060           .00000          <      .0060       1.00000
         .0060-     .0065           .00000          <      .0065       1.00000
         .0065-     .0070           .00000          <      .0070       1.00000
         .0070-     .0075           .00000          <      .0075       1.00000
         .0075-     .0080           .00000          <      .0080       1.00000
         .0080-     .0085           .00000          <      .0085       1.00000
         .0085-     .0090           .00000          <      .0090       1.00000
         .0090-     .0095           .00000          <      .0095       1.00000
          O-N >     .0095           .00000          Total for Run        1.00000

This is the Probability Density Function at River Mile      3.00       2.09

 ANO2 :  1.789533E-003  SNO2:   4.509979E-004  CNVGD F1 :  1.995137E-001


                                   NITRITE NITROGEN
    PROBABILITY DENSITY CURVE                       CUMMULATIVE DENSITY CURVE
     RANGE                    PROBABILITY            RANGE         PROBABILITY
==============================                 ==========================
          O-N <      .0000          .00000          <      .0000        .00000
         .0000-     .0005           .00050          <      .0005        .00050
         .0005-     .0010           .04307          <      .0010        .04357
         .0010-     .0015           .32461          <      .0015        .36818
         .0015-     .0020           .43867          <      .0020        .80686
         .0020-     .0025           .17016          <      .0025        .97702
         .0025-     .0030           .02188          <      .0030        .99891
         .0030-     .0035           .00107          <      .0035        .99998
         .0035-     .0040           .00002          <      .0040       1.00000
         .0040-     .0045           .00000          <      .0045       1.00000
         .0045-     .0050           .00000          <      .0050       1.00000
         .0050-     .0055           .00000          <      .0055       1.00000
         .0055-     .0060           .00000          <      .0060       1.00000
         .0060-     .0065           .00000          <      .0065       1.00000
         .0065-     .0070           .00000          <      .0070       1.00000
         .0070-     .0075           .00000          <      .0075       1.00000
         .0075-     .0080           .00000          <      .0080       1.00000
         .0080-     .0085           .00000          <      .0085       1.00000
         .0085-     .0090           .00000          <      .0090       1.00000
         .0090-     .0095           .00000          <      .0095       1.00000
          O-N >     .0095           .00000          Total for Run        1.00000
```

INPUT FOR NITRATE NITROGEN

```
5
1 0
2 .519
3 2.087
4 3.69
5 4.558
0.1 0.2 .004
.03 1.307 .085 .15 0 1. 8. 0.5
.5 .125 10. 2. .04 .008 0 0 0 0 .05 .005 0.002 0.001 0.1 .007 .35 .021
```

OUTPUT FOR NITRATE NITROGEN

```
    The Input Values for this run are:

B1 AVE :           .50000 STD. :           .12500
B2 AVE :         10.00000 STD. :          2.00000
B3 AVE :           .04000 STD. :           .00800
S3 AVE :           .00000 STD. :           .00000
S4 AVE :           .00000 STD. :           .00000
N1 AVE :           .05000 STD. :           .00500
N2 AVE :           .00200 STD. :           .00100
N3 AVE :           .10000 STD. :           .00700
N4 AVE :           .35000 STD. :           .02100
NUMBER OF REACH :    5
     1.00       2.00       3.00       4.00       5.00


          The Following Algae Constants Have Been Entered:
  Ao =    .03  u max =  1.307  p =   .150  Alpha =   .085  Sed =    .00 F1 =   1.00 D =   8.00 P =
.50



This is the Probability Density Function at River Mile      1.00       .00

  ANO3 :  1.000000E-001  SNO3:   7.000000E-003 CNVGD F1 :  3.333333E-001


                             NITRATE NITROGEN
    PROBABILITY DENSITY CURVE                     CUMMULATIVE DENSITY CURVE
    RANGE                    PROBABILITY          RANGE        PROBABILITY
===============================              =========================
        O-N <     .1000        .70266         <     .1000         .70266
       .1000-    .1040         .00000         <     .1040         .70266
       .1040-    .1080         .24313         <     .1080         .94580
       .1080-    .1120         .00000         <     .1120         .94580
       .1120-    .1160         .05426         <     .1160        1.00006
       .1160-    .1200         .00000         <     .1200        1.00006
       .1200-    .1240         .00000         <     .1240        1.00006
       .1240-    .1280         .00000         <     .1280        1.00006
       .1280-    .1320         .00000         <     .1320        1.00006
       .1320-    .1360         .00000         <     .1360        1.00006
       .1360-    .1400         .00000         <     .1400        1.00006
       .1400-    .1440         .00000         <     .1440        1.00006
       .1440-    .1480         .00000         <     .1480        1.00006
       .1480-    .1520         .00000         <     .1520        1.00006
       .1520-    .1560         .00000         <     .1560        1.00006
       .1560-    .1600         .00000         <     .1600        1.00006
       .1600-    .1640         .00000         <     .1640        1.00006
       .1640-    .1680         .00000         <     .1680        1.00006
       .1680-    .1720         .00000         <     .1720        1.00006
```

```
          .1720-     .1760           .00000         <      .1760         1.00006
          .1760-     .1800           .00000         <      .1800         1.00006
          .1800-     .1840           .00000         <      .1840         1.00006
          .1840-     .1880           .00000         <      .1880         1.00006
          .1880-     .1920           .00000         <      .1920         1.00006
          .1920-     .1960           .00000         <      .1960         1.00006
          .1960-     .2000           .00000         <      .2000         1.00006
          O-N >      .2000           .00000         Total for Run          1.00006
                     .

 This is the Probability Density Function at River Mile        2.00         .52

  ANO3 :   1.111311E-001   SNO3:    7.403512E-003 CNVGD F1 :   2.900490E-001


                                 NITRATE NITROGEN
     PROBABILITY DENSITY CURVE                      CUMMULATIVE DENSITY CURVE
     RANGE                     PROBABILITY          RANGE          PROBABILITY
 ================================               ==========================

           O-N <      .1000          .06909         <      .1000          .06909
          .1000-     .1040           .10591         <      .1040          .17500
          .1040-     .1080           .17050         <      .1080          .34550
          .1080-     .1120           .22833         <      .1120          .57384
          .1120-     .1160           .18297         <      .1160          .75681
          .1160-     .1200           .15749         <      .1200          .91430
          .1200-     .1240           .06628         <      .1240          .98058
          .1240-     .1280           .01740         <      .1280          .99798
          .1280-     .1320           .00194         <      .1320          .99991
          .1320-     .1360           .00009         <      .1360         1.00000
          .1360-     .1400           .00000         <      .1400         1.00000
          .1400-     .1440           .00000         <      .1440         1.00000
          .1440-     .1480           .00000         <      .1480         1.00000
          .1480-     .1520           .00000         <      .1520         1.00000
          .1520-     .1560           .00000         <      .1560         1.00000
          .1560-     .1600           .00000         <      .1600         1.00000
          .1600-     .1640           .00000         <      .1640         1.00000
          .1640-     .1680           .00000         <      .1680         1.00000
          .1680-     .1720           .00000         <      .1720         1.00000
          .1720-     .1760           .00000         <      .1760         1.00000
          .1760-     .1800           .00000         <      .1800         1.00000
          .1800-     .1840           .00000         <      .1840         1.00000
          .1840-     .1880           .00000         <      .1880         1.00000
          .1880-     .1920           .00000         <      .1920         1.00000
          .1920-     .1960           .00000         <      .1960         1.00000
          .1960-     .2000           .00000         <      .2000         1.00000
          O-N >      .2000           .00000         Total for Run          1.00000


 This is the Probability Density Function at River Nile        3.00        2.09

  ANO3 :   1.416476E-001   SNO3:    1.120492E-002 CNVGD F1 :   1.994670E-001


                                 NITRATE NITROGEN
     PROBABILITY DENSITY CURVE                      CUMMULATIVE DENSITY CURVE
     RANGE                     PROBABILITY          RANGE          PROBABILITY
 ================================               ==========================

           O-N <      .1000          .00004         <      .1000          .00004
          .1000-     .1040           .00015         <      .1040          .00019
          .1040-     .1080           .00065         <      .1080          .00083
          .1080-     .1120           .00224         <      .1120          .00307
          .1120-     .1160           .00687         <      .1160          .00994
          .1160-     .1200           .01803         <      .1200          .02797
          .1200-     .1240           .03889         <      .1240          .06686
          .1240-     .1280           .07526         <      .1280          .14212
```

```
        .1280-    .1320          .10901         <     .1320         .25113
        .1320-    .1360          .14010         <     .1360         .39124
        .1360-    .1400          .16003         <     .1400         .55127
        .1400-    .1440          .15459         <     .1440         .70585
        .1440-    .1480          .12271         <     .1480         .82856
        .1480-    .1520          .08582         <     .1520         .91439
        .1520-    .1560          .04955         <     .1560         .96394
        .1560-    .1600          .02341         <     .1600         .98735
        .1600-    .1640          .00906         <     .1640         .99641
        .1640-    .1680          .00282         <     .1680         .99922
        .1680-    .1720          .00066         <     .1720         .99989
        .1720-    .1760          .00011         <     .1760        1.00000
        .1760-    .1800          .00001         <     .1800        1.00001
        .1800-    .1840          .00000         <     .1840        1.00001
        .1840-    .1880          .00000         <     .1880        1.00001
        .1880-    .1920          .00000         <     .1920        1.00001
        .1920-    .1960          .00000         <     .1960        1.00001
        .1960-    .2000          .00000         <     .2000        1.00001
        0-N >     .2000          .00000         Total for Run         1.00001

This is the Probability Density Function at River Mile      4.00       3.69

 ANO3 :  1.672560E-001  SNO3:   1.559351E-002 CNVGD F1 :  1.500055E-001


                                 NITRATE NITROGEN
   PROBABILITY DENSITY CURVE                          CUMMULATIVE DENSITY CURVE
    RANGE                 PROBABILITY                 RANGE         PROBABILITY
==============================                     ========================
        0-N <      .1000          .00000        <     .1000         .00000
        .1000-    .1040          .00000         <     .1040         .00001
        .1040-    .1080          .00002         <     .1080         .00002
        .1080-    .1120          .00005         <     .1120         .00007
        .1120-    .1160          .00015         <     .1160         .00022
        .1160-    .1200          .00044         <     .1200         .00066
        .1200-    .1240          .00113         <     .1240         .00180
        .1240-    .1280          .00279         <     .1280         .00459
        .1280-    .1320          .00631         <     .1320         .01090
        .1320-    .1360          .01262         <     .1360         .02351
        .1360-    .1400          .02304         <     .1400         .04655
        .1400-    .1440          .03664         <     .1440         .08319
        .1440-    .1480          .05506         <     .1480         .13825
        .1480-    .1520          .07642         <     .1520         .21466
        .1520-    .1560          .09889         <     .1560         .31355
        .1560-    .1600          .11692         <     .1600         .43047
        .1600-    .1640          .12366         <     .1640         .55413
        .1640-    .1680          .12072         <     .1680         .67485
        .1680-    .1720          .10255         <     .1720         .77741
        .1720-    .1760          .08297         <     .1760         .86037
        .1760-    .1800          .06011         <     .1800         .92049
        .1800-    .1840          .03812         <     .1840         .95861
        .1840-    .1880          .02165         <     .1880         .98026
        .1880-    .1920          .01138         <     .1920         .99164
        .1920-    .1960          .00530         <     .1960         .99694
        .1960-    .2000          .00205         <     .2000         .99899
        0-N >     .2000          .00102         Total for Run         1.00001

This is the Probability Density Function at River Mile      5.00       4.56

 ANO3 :  1.795767E-001  SNO3:   1.784291E-002 CNVGD F1 :  1.326552E-001


                                 NITRATE NITROGEN
```

```
      PROBABILITY DENSITY CURVE                    CUMMULATIVE DENSITY CURVE
      RANGE                    PROBABILITY         RANGE         PROBABILITY
====================================               ==========================
         O-N <      .1000        .00000            <     .1000        .00000
        .1000-     .1040         .00000            <     .1040        .00000
        .1040-     .1080         .00000            <     .1080        .00000
        .1080-     .1120         .00001            <     .1120        .00002
        .1120-     .1160         .00004            <     .1160        .00006
        .1160-     .1200         .00011            <     .1200        .00017
        .1200-     .1240         .00028            <     .1240        .00045
        .1240-     .1280         .00067            <     .1280        .00112
        .1280-     .1320         .00152            <     .1320        .00264
        .1320-     .1360         .00327            <     .1360        .00591
        .1360-     .1400         .00655            <     .1400        .01246
        .1400-     .1440         .01185            <     .1440        .02432
        .1440-     .1480         .01993            <     .1480        .04424
        .1480-     .1520         .03038            <     .1520        .07462
        .1520-     .1560         .04441            <     .1560        .11902
        .1560-     .1600         .06062            <     .1600        .17965
        .1600-     .1640         .07976            <     .1640        .25941
        .1640-     .1680         .09618            <     .1680        .35560
        .1680-     .1720         .10659            <     .1720        .46219
        .1720-     .1760         .11140            <     .1760        .57359
        .1760-     .1800         .10444            <     .1800        .67803
        .1800-     .1840         .09301            <     .1840        .77105
        .1840-     .1880         .07661            <     .1880        .84766
        .1880-     .1920         .05655            <     .1920        .90422
        .1920-     .1960         .03986            <     .1960        .94408
        .1960-     .2000         .02589            <     .2000        .96997
        O-N >      .2000         .03004            Total for Run     1.00001
```

INPUT FOR ORGANIC NITROGEN

```
5
1 0
2 .519
3 2.087
4 3.69
5 4.558
0.2 .4  .01
.03 1.307 .085 .15 0 1. 8. 0.5
.5 .125 10. 2. .04 .008 0 0 0 0 .05 .005 0.002 0.001 0.1 .007 .35 .021
```

OUTPUT FOR ORGANIC NITROGEN

```
    The Input Values for this run are:

 B1 AVE :          .50000 STD. :          .12500
 B2 AVE :        10.00000 STD. :         2.00000
 B3 AVE :          .04000 STD. :          .00800
 S3 AVE :          .00000 STD. :          .00000
 S4 AVE :          .00000 STD. :          .00000
 N1 AVE :          .05000 STD. :          .00500
 N2 AVE :          .00200 STD. :          .00100
 N3 AVE :          .10000 STD. :          .00700
 N4 AVE :          .35000 STD. :          .02100
 NUMBER OF REACH :     5
       1.00       2.00       3.00       4.00       5.00


        The Following Algae Constants Have Been Entered:
  Ao =     .03  u max =  1.307  p =   .150  Alpha =   .085  Sed =     .00 F1 =   1.00 D =   8.00 P =
.50


 This is the Probability Density Function at River Mile      1.00       .00

  AN4 :  3.500000E-001  VN4 :  2.100000E-002


                                 ORGANIC NITROGEN
    PROBABILITY DENSITY CURVE                      CUMMULATIVE DENSITY CURVE
    RANGE                    PROBABILITY           RANGE        PROBABILITY
 ===============================               =========================
         O-N <     .2000          .00000          <     .2000         .00000
         .2000-    .2100          .00000          <     .2100         .00000
         .2100-    .2200          .00000          <     .2200         .00000
         .2200-    .2300          .00000          <     .2300         .00000
         .2300-    .2400          .00000          <     .2400         .00000
         .2400-    .2500          .00000          <     .2500         .00000
         .2500-    .2600          .00000          <     .2600         .00000
         .2600-    .2700          .00000          <     .2700         .00000
         .2700-    .2800          .00000          <     .2800         .00000
         .2800-    .2900          .00445          <     .2900         .00445
         .2900-    .3000          .00000          <     .3000         .00445
         .3000-    .3100          .05425          <     .3100         .05870
         .3100-    .3200          .00000          <     .3200         .05870
         .3200-    .3300          .24311          <     .3300         .30181
         .3300-    .3400          .00000          <     .3400         .30181
         .3400-    .3500          .00000          <     .3500         .30181
         .3500-    .3600          .40083          <     .3600         .70264
         .3600-    .3700          .00000          <     .3700         .70264
         .3700-    .3800          .24311          <     .3800         .94575
         .3800-    .3900          .00000          <     .3900         .94575
```

```
         .3900-     .4000          .05425          <     .4000          1.00000
         O-N >      .4000          .00000          Total for Run         1.00000

This is the Probability Density Function at River Mile     2.00      .52

 AN4 :  3.429565E-001  VN4 :  2.061733E-002


                               ORGANIC NITROGEN
   PROBABILITY DENSITY CURVE                        CUMMULATIVE DENSITY CURVE
   RANGE               PROBABILITY                  RANGE         PROBABILITY
===============================                  ==========================
         O-N <      .2000          .00000          <     .2000          .00000
         .2000-     .2100          .00000          <     .2100          .00000
         .2100-     .2200          .00000          <     .2200          .00000
         .2200-     .2300          .00000          <     .2300          .00000
         .2300-     .2400          .00000          <     .2400          .00000
         .2400-     .2500          .00000          <     .2500          .00000
         .2500-     .2600          .00000          <     .2600          .00000
         .2600-     .2700          .00000          <     .2700          .00000
         .2700-     .2800          .00054          <     .2800          .00054
         .2800-     .2900          .00460          <     .2900          .00513
         .2900-     .3000          .00711          <     .3000          .01225
         .3000-     .3100          .05196          <     .3100          .06421
         .3100-     .3200          .03539          <     .3200          .09960
         .3200-     .3300          .21608          <     .3300          .31568
         .3300-     .3400          .00000          <     .3400          .31568
         .3400-     .3500          .39765          <     .3500          .71332
         .3500-     .3600          .00000          <     .3600          .71332
         .3600-     .3700          .23471          <     .3700          .94803
         .3700-     .3800          .00000          <     .3800          .94803
         .3800-     .3900          .05197          <     .3900         1.00000
         .3900-     .4000          .00000          <     .4000         1.00000
         O-N >      .4000          .00000          Total for Run         1.00000

This is the Probability Density Function at River Mile     3.00      2.09

 AN4 :  3.222527E-001  VN4 :  2.000193E-002


                               ORGANIC NITROGEN
   PROBABILITY DENSITY CURVE                        CUMMULATIVE DENSITY CURVE
   RANGE               PROBABILITY                  RANGE         PROBABILITY
===============================                  ==========================
         O-N <      .2000          .00000          <     .2000          .00000
         .2000-     .2100          .00000          <     .2100          .00000
         .2100-     .2200          .00000          <     .2200          .00000
         .2200-     .2300          .00000          <     .2300          .00000
         .2300-     .2400          .00000          <     .2400          .00000
         .2400-     .2500          .00000          <     .2500          .00000
         .2500-     .2600          .00076          <     .2600          .00076
         .2600-     .2700          .00281          <     .2700          .00357
         .2700-     .2800          .01879          <     .2800          .02236
         .2800-     .2900          .02572          <     .2900          .04808
         .2900-     .3000          .10814          <     .3000          .15622
         .3000-     .3100          .09179          <     .3100          .24801
         .3100-     .3200          .24321          <     .3200          .49121
         .3200-     .3300          .12840          <     .3300          .61961
         .3300-     .3400          .21473          <     .3400          .83434
         .3400-     .3500          .09749          <     .3500          .93182
         .3500-     .3600          .04901          <     .3600          .98083
         .3600-     .3700          .01550          <     .3700          .99633
         .3700-     .3800          .00367          <     .3800         1.00000
         .3800-     .3900          .00000          <     .3900         1.00000
         .3900-     .4000          .00000          <     .4000         1.00000
```

```
      O-N >     .4000         .00000        Total for Run        1.00000

This is the Probability Density Function at River Mile       4.00       3.69

 AN4 :  3.022623E-001  VN4 :  1.994251E-002


                                 ORGANIC NITROGEN
    PROBABILITY DENSITY CURVE                     CUMMULATIVE DENSITY CURVE
    RANGE                   PROBABILITY           RANGE         PROBABILITY
==============================================   ===========================
      O-N <      .2000        .00000             <     .2000         .00000
     .2000-     .2100         .00000             <     .2100         .00000
     .2100-     .2200         .00000             <     .2200         .00000
     .2200-     .2300         .00000             <     .2300         .00000
     .2300-     .2400         .00017             <     .2400         .00017
     .2400-     .2500         .00207             <     .2500         .00224
     .2500-     .2600         .01175             <     .2600         .01399
     .2600-     .2700         .03538             <     .2700         .04937
     .2700-     .2800         .06758             <     .2800         .11695
     .2800-     .2900         .12292             <     .2900         .23987
     .2900-     .3000         .16650             <     .3000         .40637
     .3000-     .3100         .18900             <     .3100         .59537
     .3100-     .3200         .21608             <     .3200         .81145
     .3200-     .3300         .13317             <     .3300         .94462
     .3300-     .3400         .04630             <     .3400         .99092
     .3400-     .3500         .00422             <     .3500         .99514
     .3500-     .3600         .00466             <     .3600         .99980
     .3600-     .3700         .00020             <     .3700        1.00000
     .3700-     .3800         .00000             <     .3800        1.00000
     .3800-     .3900         .00000             <     .3900        1.00000
     .3900-     .4000         .00000             <     .4000        1.00000
      O-N >     .4000         .00000        Total for Run          1.00000

This is the Probability Density Function at River Mile       5.00       4.56

 AN4 :  2.919507E-001  VN4 :  2.004495E-002


                                 ORGANIC NITROGEN
    PROBABILITY DENSITY CURVE                     CUMMULATIVE DENSITY CURVE
    RANGE                   PROBABILITY           RANGE         PROBABILITY
==============================================   ===========================
      O-N <      .2000        .00000             <     .2000         .00000
     .2000-     .2100         .00000             <     .2100         .00000
     .2100-     .2200         .00000             <     .2200         .00000
     .2200-     .2300         .00016             <     .2300         .00016
     .2300-     .2400         .00446             <     .2400         .00462
     .2400-     .2500         .01125             <     .2500         .01587
     .2500-     .2600         .03917             <     .2600         .05504
     .2600-     .2700         .08083             <     .2700         .13586
     .2700-     .2800         .12472             <     .2800         .26058
     .2800-     .2900         .23225             <     .2900         .49283
     .2900-     .3000         .18180             <     .3000         .67463
     .3000-     .3100         .19564             <     .3100         .87027
     .3100-     .3200         .06525             <     .3200         .93552
     .3200-     .3300         .05024             <     .3300         .98576
     .3300-     .3400         .01161             <     .3400         .99737
     .3400-     .3500         .00181             <     .3500         .99919
     .3500-     .3600         .00073             <     .3600         .99992
     .3600-     .3700         .00008             <     .3700        1.00000
     .3700-     .3800         .00000             <     .3800        1.00000
     .3800-     .3900         .00000             <     .3900        1.00000
     .3900-     .4000         .00000             <     .4000        1.00000
      O-N >     .4000         .00000        Total for Run          1.00000
```

APPENDIX G

Kolmogoroff-Smirnov Test Statistics

Station 3 : 2.09 day (Travel Time)
Monte Carlo 600 Simulation

Range	Par. Organic-N	Range	Par. NO2-N	Range	Par. NO3-N	Range	Par. NH3-N
0.24725		-0.00018		0.099727		0.0068	
0.257083	0.00124	7.5E-05		0.105364		0.01056	
0.266917	0.00043	0.000325	-0.0002	0.111	0.00064	0.01432	-0.00028
0.27675	-0.00636	0.000575	0.00105	0.116636	-0.00644	0.01808	0.00772
0.286583	0.01992	0.000825	0.01007	0.122273	-0.02082	0.02184	0.01951
0.296417	-0.00822	0.001075	0.00972	0.127909	-0.06489	0.0256	0.02821
0.30625	0.05599	0.001325	0.03006	0.133545	-0.08649	0.02936	0.01315
0.316083	0.03479	0.001575	-0.01872	0.139182	-0.12881	0.03312	-0.01666
0.325917	0.02016	0.001825	-0.05914	0.144818	-0.11062	0.03688	-0.04438
0.33575	-0.00234	0.002075	-0.05886	0.150455	-0.1016	0.04064	-0.06803
0.345583	-0.00982	0.002325	-0.04519	0.156091	-0.06099	0.0444	-0.04904
0.355417	-0.00483	0.002575	-0.02839	0.161727	-0.01904	0.04816	-0.02876
0.36525	-0.00433	0.002825	-0.01446	0.167364	-0.00571	0.05192	-0.01127
0.375083	0	0.003075	-0.00576	0.173	0	0.05568	-0.00393
0.384917		0.003325	-0.00198	0.178636		0.05944	-0.002
0.39475		0.003575	0	0.184273		0.0632	-0.002
0.404583		0.003825		0.189909		0.06696	0