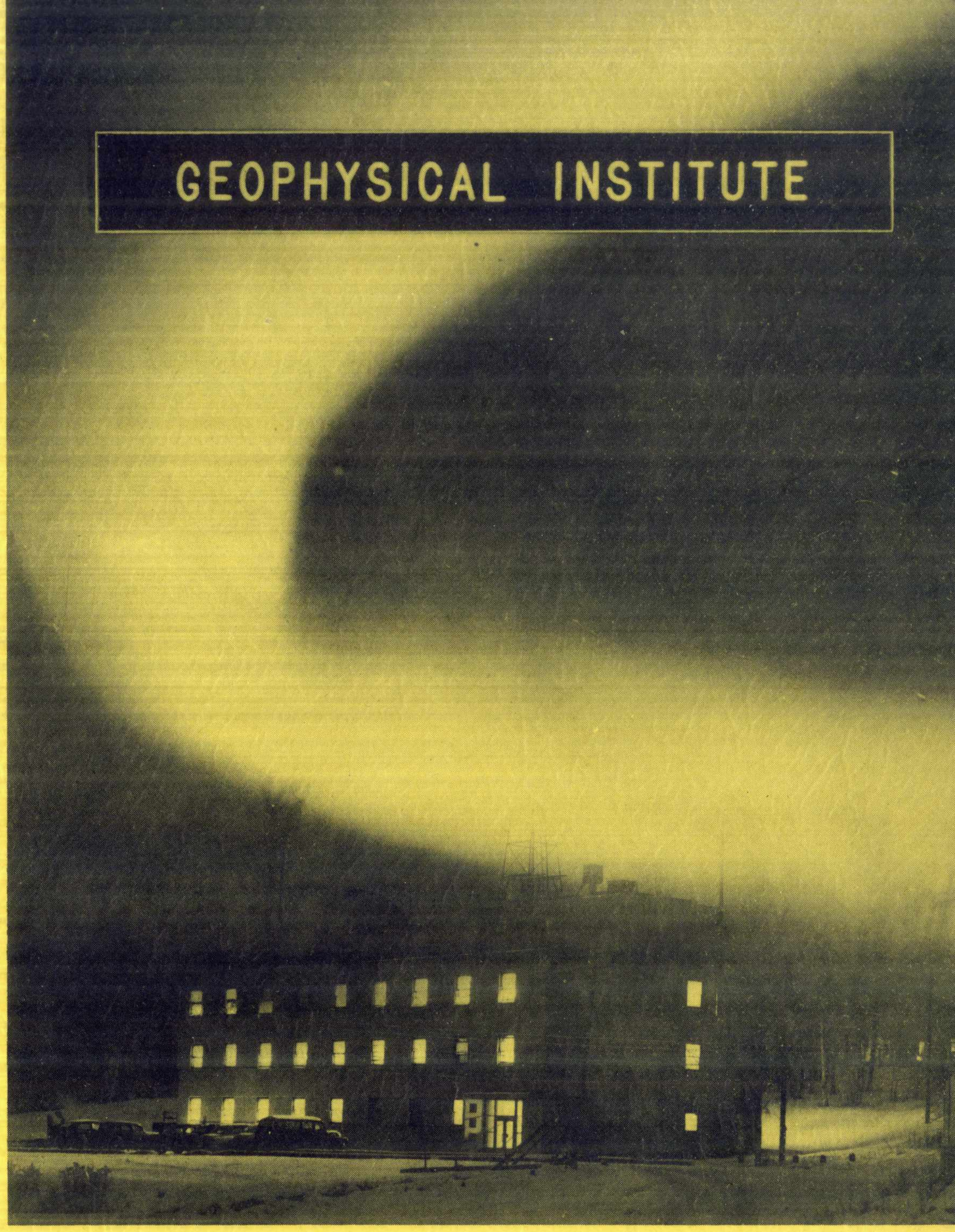

UNIVERSITY
OF ALASKA

COLLEGE
ALASKA

EXPERIMENT LUXEMBOURG

Scientific Report No. 1
Contract No. AF 19(604)-3880

by

G. C. Rumi and C. G. Little

December 1958

The research reported in this document has been sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Center, Air Research and Development Command.

AFCRC-TN-59-400
ASTIA Doc. No. AD 216 316

GEOPHYSICAL INSTITUTE

OF THE

UNIVERSITY OF ALASKA

EXPERIMENT LUXEMBOURG

Scientific Report No. 1
Contract AF 19(604)-3880

The research reported in this document has been sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Center, Air Research and Development Command, under Contract AF 19(604)-3880.

Report prepared by:

G. C. Rumi
C. G. Little

Report approved by:

C. T. Elvey, Director

December 1958

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF FIGURES CONTINUED

"Experiment Luxembourg"

Scientific Report No. 1

Introduction

This report discusses the theoretical aspects of a radio wave interaction experiment designed to determine ν (the electron collision frequency), and N (the electron density), as a function of height in the arctic D region. The proposed high latitude location of the observations plays an important role in the design of the experiment, which should be capable of providing continuous information on the state of the lower ionosphere in the auroral zone.

Present knowledge of the electron densities and collision frequencies in the D region of the ionosphere.

The D region of the ionosphere is the nearest of the ionospheric layers, and it might therefore be thought that our knowledge of its parameters would be more complete than for the ionospheric layers above it. In fact, however, this is not the case. Almost all the ionospheric information currently available has been obtained by studying, at various frequencies, the characteristics of signals reflected from the ionosphere. This technique is not readily applicable to the D region, owing to the excessive absorption experienced by radio waves undergoing reflection in a medium in which the plasma frequency is comparable with the electron collision frequency. (At greater heights, the collision frequency is less, while the electron densities and therefore the plasma frequencies are normally greater. The absorption experienced by radio waves being reflected in the E and F regions is therefore relatively small). It is convenient first to summarize the current state of knowledge of the lower ionosphere.

Most of the information available is deduced from absorption studies. These may be of two types; for frequencies below the penetration frequency of the ionosphere, the absorption of radio waves reflected by an ionospheric layer above the D region may be studied; alternatively, the absorption of radio waves of extraterrestrial origin may be used at frequencies above the penetration frequency.

The absorption experienced by a plane radio wave traversing a uniformly ionized gas is indicated by

$$E = E_o \cdot \exp\left(-\int K ds \right) \text{ with } K = \frac{60 \pi e^2}{m} \cdot \frac{N \nu}{\nu^2 + (\omega \pm \omega_L)^2} \quad (1)$$

where

E_o = amplitude of the wave entering the ionized region

E = amplitude of the wave leaving the ionized region

ds = element of path of the radio wave in the ionized region

K = absorption coefficient

e = electron charge

m = electron mass

N = electron density

ν = electron collision frequency

ω = operating angular frequency

ω_L = longitudinal component of gyromagnetic angular frequency

the + sign holds for the ordinary component of the radio wave and the - sign holds for the extraordinary component. The index of refraction is here assumed to be unity. M K S units are used.

In general, absorption measurements give an indication of the product of N and ν, but do not give specific information about the distribution of electrons and ν with height. Nevertheless one can deduce something about N and ν as function of height in the D region by considering the trends of the absorption, that is its daily, seasonal, and solar cycle variations. Mitra and Shain (1953) showed, using the cosmic noise technique, that the absorption takes place mainly in the D region - below 90 kilometer height. Appleton and Piggott (1954) deduced that the D region lies above 70 kilometer height and that during the day, it has the shape of a Chapman layer produced by photoionization. Chapman and Little (1957) suggested that in auroral latitudes the D region extends down to about 40 kilometers during periods of corpuscular bombardment.

The D region may also be studied by partial reflection techniques. Using these methods, Gardner and Pawsey (1953) concluded that the D region is split into two distinct layers, one about 70 kilometers in height, and the other 90 kilometers high, the first with a maximum intensity around noon and the second around midnight. The 70 kilometer layer apparently sinks in the atmosphere at midday. Some information about the distribution of ν and of N was inferred from the experimental data.

The characteristics of the D region can be studied also by the use of total reflection techniques, on very low frequencies. Bracewell and Bain (1952) deduced from investigations of this type that the D region comprises two layers, one around 90 kilometer height and the other around 70 kilometer height. Both layers have a diurnal height variation; neither behaves as a Chapman layer.

Polarization of vertically propagating long waves has also been used to determine the characteristics of the D region. Kelso et alii (1951), after assuming a Chapman-like electron distribution with maximum at 74 km and an associated exponential distribution of ν, derived information about N_{max}.

Several experiments have been performed in recent years on forward scatter of radio waves. Extensive work in this field has been described by Bailey, Bateman, and Kirby (1955). They found that the D region contributes considerably to that type of radio wave propagation. In relation to the work of Gardner and Pawsey (1953) they state: "Their height findings are in some respects so similar to those reported above as to suggest that the same part of the ionosphere is involved."

Some attempts have been made to correlate upper D region ionization and meteoric activity (see Dubin - 1955). Details about theoretical aspects of the problem and experimental results obtained at College will be illustrated in a later Scientific Report.

In recent years, rocket exploration has furnished data on electron densities in the D region (see Seddon - 1954; Seddon and Jackson - 1958; Lien et alii - 1954; and Berning - 1954). Differing electron densities have been recorded at different latitudes and during quiet and disturbed periods.

Abnormal absorption has been observed in connection with aurorae and solar flares. The enhancement of absorption during these events has been primarily attributed to an increase of N, although there is some evidence that ν may also be increased. This possibility and the presence of inhomogeneities in the D region will be considered in a later Scientific Report.

A technique different from those indicated above, and very suitable to the analysis of the D region is described in the following paragraphs.

<u>The theory of radio wave interaction</u>.

Radio wave interaction studies use two transmitters. One is called the wanted transmitter; its signal is recorded at a receiving site, after reflection in the higher layers of the ionosphere. The second ("disturbing") transmitter increases the electron collision frequency in the D region and hence modifies the absorption of the wanted wave as it traverses the same region.

Such a cross-modulation of radio waves was noticed for the first time at Eindhoven (Holland), where the program from Beromünster (Switzerland) was monitored. The program from the Luxembourg station, located approximately at the midpoint of the path and operating on a different frequency,was found to be superimposed on the Beromünster program. Hence radio wave interaction is often called the Luxembourg effect.

The first detailed theoretical interpretation was given by Bailey and Martyn (1934). Bailey in 1937 refined the theory and concluded that a resonance-like effect should occur under appropriate conditions at gyro-frequency. Bailey's investigation explains all the details of the problem, but is needlessly involved. Simpler treatments of the problem have been given by Ratcliffe and Shaw (1948) and by Huxley and Ratcliffe (1949). Following these latter authors, we describe the cross-modulation of a wave passing through the ionosphere as follows. The disturbing transmitter feeds energy to the free electrons of the ionized region, increasing their mean energy to a value Q larger than Q_o, the kinetic energy of agitation of a molecule of the gas. In the steady state, if ($Q-Q_o$) is small compared with Q_o, it may be assumed that the average energy ΔQ lost by an electron in each collision if given by

$$\Delta Q = G (Q - Q_o) \qquad \text{(A)}$$

where G is a constant.

This hypothesis was criticized by Huxley, who proposed an alternative formula

$$\Delta Q = 4.13 \times 10^{-38} n \frac{(Q - Q_o)^2}{Q Q_o^2} \qquad \text{(B)}$$

where n is the number of molecules per cc. Huxley therefore modified the theory of radio wave interaction using this relation. The unmodified and modified theories lead to different conclusions; different amplitude and phase cross-modulation are to be expected. It seemed to Huxley in 1953 that the experimental results supported his formula (B), but in 1955 he acknowledged that the unmodified theory better fitted the magneto-ionic theory predictions. Fejer in 1955 raised other objections to Huxley's relation (B) and supported them with experimental evidence. Prevailing opinion at present favors the formula (A).

The change of energy of the electrons caused by the disturbing wave changes their collision frequency ν, in a manner depending on the modulation frequency ω of the disturbing wave. This in turn affects the propagation of the wanted wave. Using formula (A), Huxley and Ratcliffe (1949) infer that ν varies as follows:

$$\nu(t) = \bar{\nu} \left[1 + m_{\omega} \cos (\omega t - \varphi_1) + m_{2\omega} \cos (2 \omega t - \varphi_2) \right] \qquad (2)$$

where

$$\tan \varphi_1 = \frac{1}{2} \tan \varphi_2 = \omega/G\bar{\nu} \quad (3)$$

$$m_\omega = \frac{M P_1}{\bar{Q} G} \left[1 + (\omega/G\bar{\nu})^2 \right]^{-\frac{1}{2}} \quad (4)$$

$$m_{2\omega} = \frac{M^2 P_1}{4 \bar{Q} G} \left[1 + (2\omega/G\bar{\nu})^2 \right]^{-\frac{1}{2}} \quad (5)$$

$$T_o = \frac{\alpha m_\omega}{\nu} \left[1 + \left|\omega/G\bar{\nu}\right|^2 \right]^{\frac{1}{2}} = \alpha M \left(\frac{e^2}{m}\right) \left(\frac{E_D^2}{p_D^2}\right) \left|3 G k \theta\right|^{-1} \quad (6)$$

where

ν = electron collision frequency

ω = angular frequency of modulation

m_ω = modulation coefficient of ν at frequency ω

$T_{\omega,o}$ = coefficient of transferred modulation at angular frequency ω, o

G = constant

α = absorption in nepers

P_1 = power supplied to each electron

M = modulation coefficient of disturbing wave

e = electron charge

m = electron mass

E_D = disturbing electric field strength

p_D = angular frequency of the disturbing wave

k = Boltzmann's constant

θ = temperature of the medium, deg K

M K S units are used.

The expression (6) for T_o is obtained for the particular case when $p_D > \nu$ and $MP_1 \ll \bar{Q}G$ over a finite path s. A more general expression for T_o is the one given by Fejer (1955) in differential form, as

$$dT_o = \frac{2\nu}{3N\,k\theta} \frac{\partial K_2}{\partial \nu} \frac{E\,dF}{4\pi h^2} \times \text{exponential decay} \qquad (7)$$

(K_2 is the absorption coefficient for the wanted wave, and EdF is the disturbing energy absorbed at the height h; in the present report the subscript 2 has to be considered interchangeable with the subscript W for the wanted wave, and 1 with D for the disturbing wave).

Expression (7) can be reduced to (6); it is not difficult to see that when $p_D > \nu$

$$\int \frac{EdF}{4\pi h^2} = \frac{NP_1}{G\nu} = \frac{N}{2G} \frac{e^2}{m} \frac{E_D^2}{p_D^2} \;;\quad \frac{\partial K_W}{\partial \nu} = \frac{K_W}{\nu} = \beta = \frac{\alpha}{\nu} \;;$$

by substitution into Fejer's formula we obtain (6). But Fejer's formula is preferable because: a) it does not imply that the coefficient of attenuation is always quasi-proportional to ν (this is not always true in high latitudes); b) it does not presume that all the power loss of the disturbing wave is efficient in producing cross-modulation (only the part spent over the path of the wanted wave is efficient); and c) it is in a differential form explicitly connected to the independent variable h.

At this point we stress the fact that equation (7) gives the coefficient of transferred modulation to be used in connection with equations (2), (3), (4), (5), and (6); it is erroneous to interpret dT_o directly as the coefficient of cross-modulation for pulses. This statement will become clear in the course of the following analysis.

We want to determine the cross-modulation for a pulsed disturbing wave. Equation (2) indicates that the modulation transferred to ν from a disturbing wave at an angular frequency ω, namely m_ω, undergoes a phase shift given by $\varphi_1 = \tan^{-1} \omega/G\bar{\nu}$. Equation (4) indicates that the amplitude of the transferred modulation varies according to the expression

$$\left[1 + (\omega/G\bar{\nu})^2\right]^{-1/2}$$

Our problem is to find the change in ν produced by a pulse of the disturbing wave, that is by a disturbance equivalent to a large spectrum of frequencies. One could proceed by considering the different components of the spectrum, the final response being the summation of the elementary responses due to each component. But a more synthetic approach is possible through the use of an analogy.

Let us consider the circuit shown in Fig. 1, where i(t) is a current generator, C is a capacity, 1/R is a conductance and v(t) is the voltage measured across the conductance. When i(t) is sinusoidal we know that the voltage v(t) is also sinusoidal and has a phase lag given by $\tan^{-1} \omega CR$. Furthermore, the moduli of i(t) and v(t) are related according to the following expression $V = IR\left[1 + (\omega CR)^2\right]^{-1/2}$. Thus the frequency response of v(t) to i(t) in the circuit depicted in Fig. 1 is analytically the same as the response of $\frac{\nu - \bar{\nu}}{\bar{\nu}}$ to the disturbing wave intensity, provided CR is put equal to $1/G\bar{\nu}$. It is quite easy to derive the pulse response for the circuit in Fig. 1; this response will also represent the solution of our problem in radio wave interaction. The Heaviside-Jeffreys operational calculus will be used in dealing with the circuit of Fig. 1. For reference see Weber - 1954.

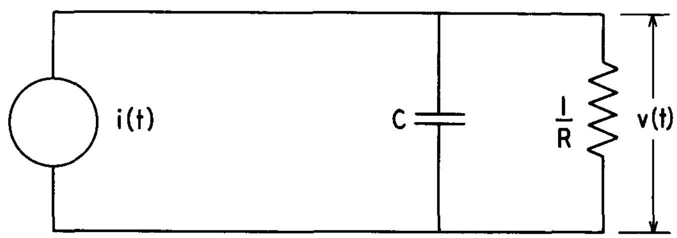

Fig. 1. Equivalent circuit.

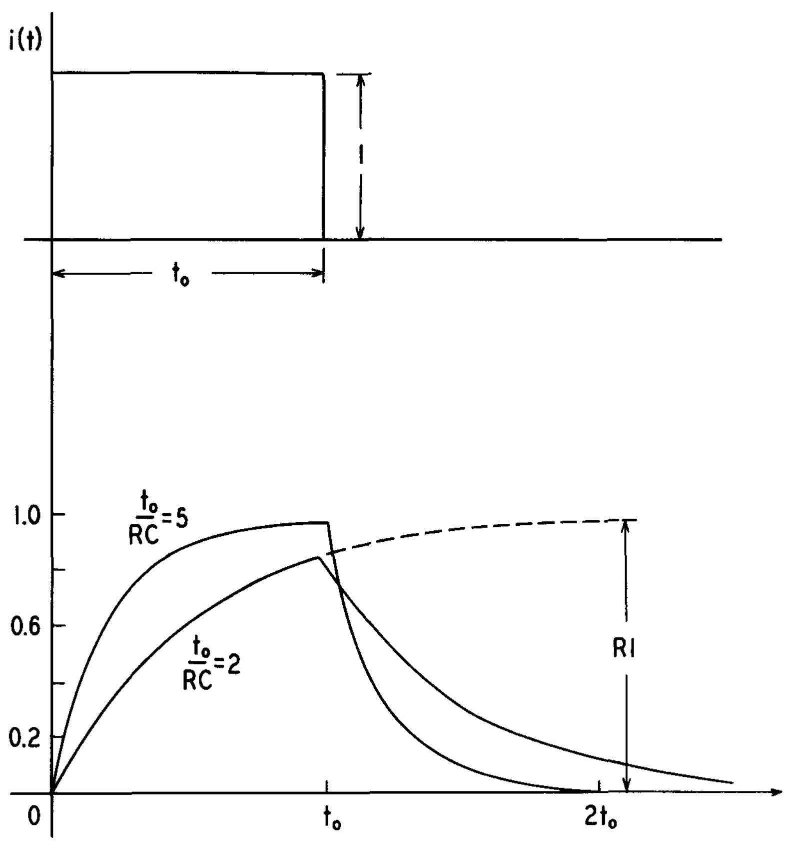

Fig. 2. Pulse of current and voltage response.

Let

$$\frac{1}{p}\; i\,(t) = \int_{t=o}^{t} i\,(t)\,dt, \text{ then the circuit equation}$$

$$C\,\frac{dv}{dt} + \frac{v}{R} = i\,(t) \quad \text{can be written as}$$

$$C\,v + \frac{v}{R\,p} = \frac{1}{p}\; i\,(t) + C\,v\,(0^-)$$

$$v(t) = \frac{1}{C + 1/Rp}\left[\frac{1}{p}\; i\,(t) + C\,v\,(0^-)\right] ;$$

in our case $Cv(0^-) = 0$, then

$$v\,(t) = \frac{1}{Cp}\;\frac{p}{p + 1/CR}\; i\,(t);$$

in our case $i\,(t) = I(1 - 1_{t-t_o})$ where

$$1 = \begin{cases} 0 & \text{for } t < 0 \\ 1/2 & \text{for } t = 0 \\ 1 & \text{for } t > 0 \end{cases}$$

$$1_{t-t_o} = \begin{cases} 0 & \text{for } t < t_o \\ 1/2 & \text{for } t = t_o \\ 1 & \text{for } t > t_o \end{cases}$$

It has been shown (Weber - 1954) that

$$\frac{p}{p + 1/RC} \; I \, (1 - 1_{t-t_o}) = I e^{-t/RC} (1 - 1_{t-t_o})$$

and

$$\frac{1}{Cp} \; \frac{p}{p + 1/RC} \; I \, 1 = RI \, (1 - e^{-t/RC}) \, 1$$

$$\therefore \; v(t) = RI \, (1 - e^{-t/RC}) \, 1 - RI \, (1 - e^{-(t-t_o)/RC}) \, 1_{t-t_o}$$

For two different values of RC, v(t) is sketched in Fig. 2. The same behavior has to be expected for $\frac{\nu - \overline{\nu}}{\overline{\nu}}$ as $1/G\overline{\nu}$ changes. That means that the increase of collision frequency due to a pulsed disturbing wave is not instantaneous, but is delayed by a relaxation time $\tau = 1/G\overline{\nu}$ the transient never being longer than the pulse length. In the D region, for heights around 80 kilometers $1/G\overline{\nu}$ is of the order of one millisecond.

The interaction between the disturbing pulse and a pulse of the wanted wave is an integrated effect over an interval of heights as can be seen from Fig. 3, where a typical situation is sketched.

The sketch illustrates the increase of ν , $\frac{\nu - \overline{\nu}}{\overline{\nu}}$, due to a disturbing pulse the front end of which has reached the height h_o; such an increase is plotted as a function of the height and, consequently, of the time, the two being related by the equation $t = 2(h - h_o)/c$, where c is the speed of light.

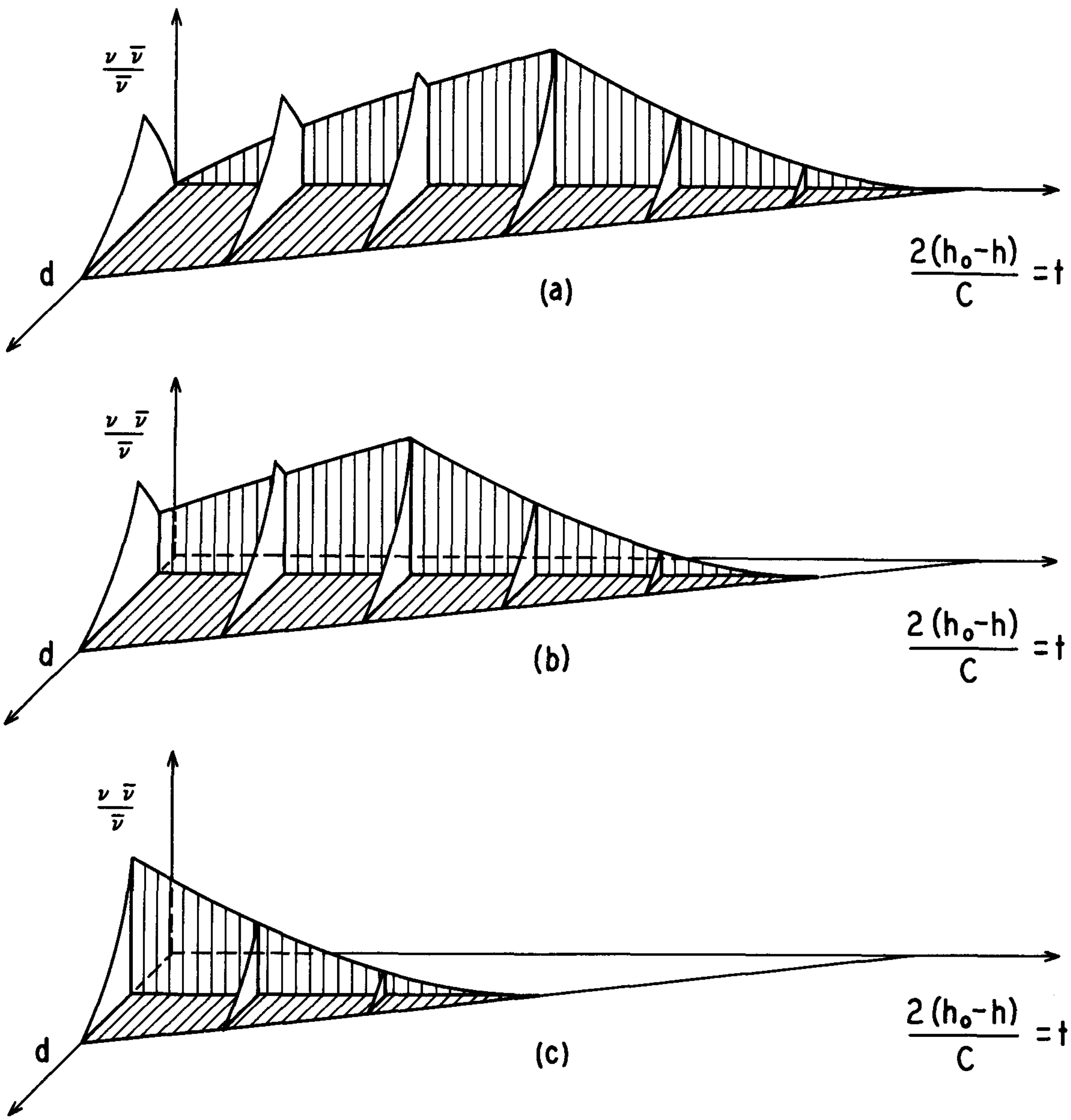

Fig. 3. Increase of collision frequency encountered by
a) the front end of the wanted pulse
b) the median portion of the wanted pulse
c) the tail end of the wanted pulse

The transverse blades depicted in the same sketch illustrate the history of the increase of ν for a given height. Then, if the front end of the wanted pulse undergoes a disturbance similar to the one drawn in the plane $d = 0$ (see Fig. 3) the successive parts of the wanted pulse undergo a disturbance that is the intersection with the transverse blades, of planes parallel to $d = 0$ and successively removed toward the observer. Note that in the equation $t = 2(h - h_o)/c$ a speed one half the speed of light has to be used to take into account the fact that the disturbing and the wanted pulse move in opposite directions, each with the velocity of light.

The detailed construction of the interaction between the two pulses will be left, on the basis of Fig. 3, to the reader's imagination, since it seems that an analytical description of the interaction will be more confusing than helpful. We conclude that the disturbed wanted pulse will be shaped as shown in Fig. 4.

The preceding considerations are based upon a uniform ionosphere. In practice ν and N change with height. The end result is that the curves illustrated in Fig. 3 will be more or less distorted according to the rapid or slow changes of ν and N with height. This fact has to be taken into account when measurements of ν and N versus height are attempted.

Sometimes it is possible to obtain interaction at gyrofrequency; in this particular case the analytical results shown above are not always valid. A specific formulation of the theory will be indicated later on, during the discussion of some experimental observations that may be due to gyrointeraction.

Fig. 4.

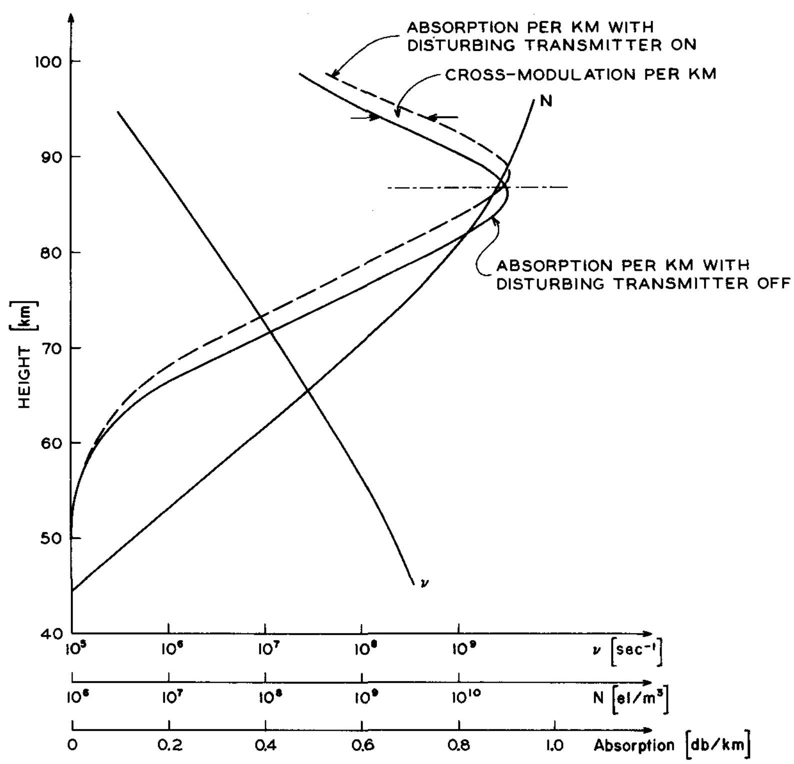

Fig. 5. Diagram illustrating expected cross-modulation. The difference in absorption of the wanted wave - 4 Mc/s, ordinary component - occurring at a given height with the transmitter ON and OFF is a direct measure of the cross-modulation. This difference in absorption is here considerably exaggerated; typical values might be 10^{-4} to 10^{-3} db per km.

It is of some interest to give a physical interpretation of the mathematical results outlined above. The situation is described in Fig. 5. The continuous line represents the absorption for the undisturbed conditions, characterized by the N and ν distributions versus height as shown on the same figure. The dotted line represents the absorption in the medium after the disturbance has gone through. The difference of absorption for a given height represents the amount of cross-modulation produced by the disturbing wave upon the wanted wave.

It is clear that at heights such that $\nu = (\omega \pm \omega_L)$, the cross-modulation for that particular polarization will be zero.

<u>The outline of the planned experiment</u>.

The experiment we plan to perform is based on Fejer's experiment (1955) as far as the block diagram is concerned. The block diagram we propose to use is shown on Fig. 6.

The mode of operation may be described briefly as follows. For each second pulse of the wanted transmitter (which operates at a pulse repetition rate twice that of the disturbing transmitter), the disturbing pulse increases ν and, consequently, changes the absorption coefficient in the D region so that alternate pulses from the wanted transmitter will be affected in amplitude or, in other words, cross-modulated. If the pulses of the wanted transmitter and of the disturbing transmitter are short, one can explore narrow sheets in the D region.

Cross-modulation and, together, the height at which it occurs will be measured. Then the distributions of ν and N versus height will be deduced.

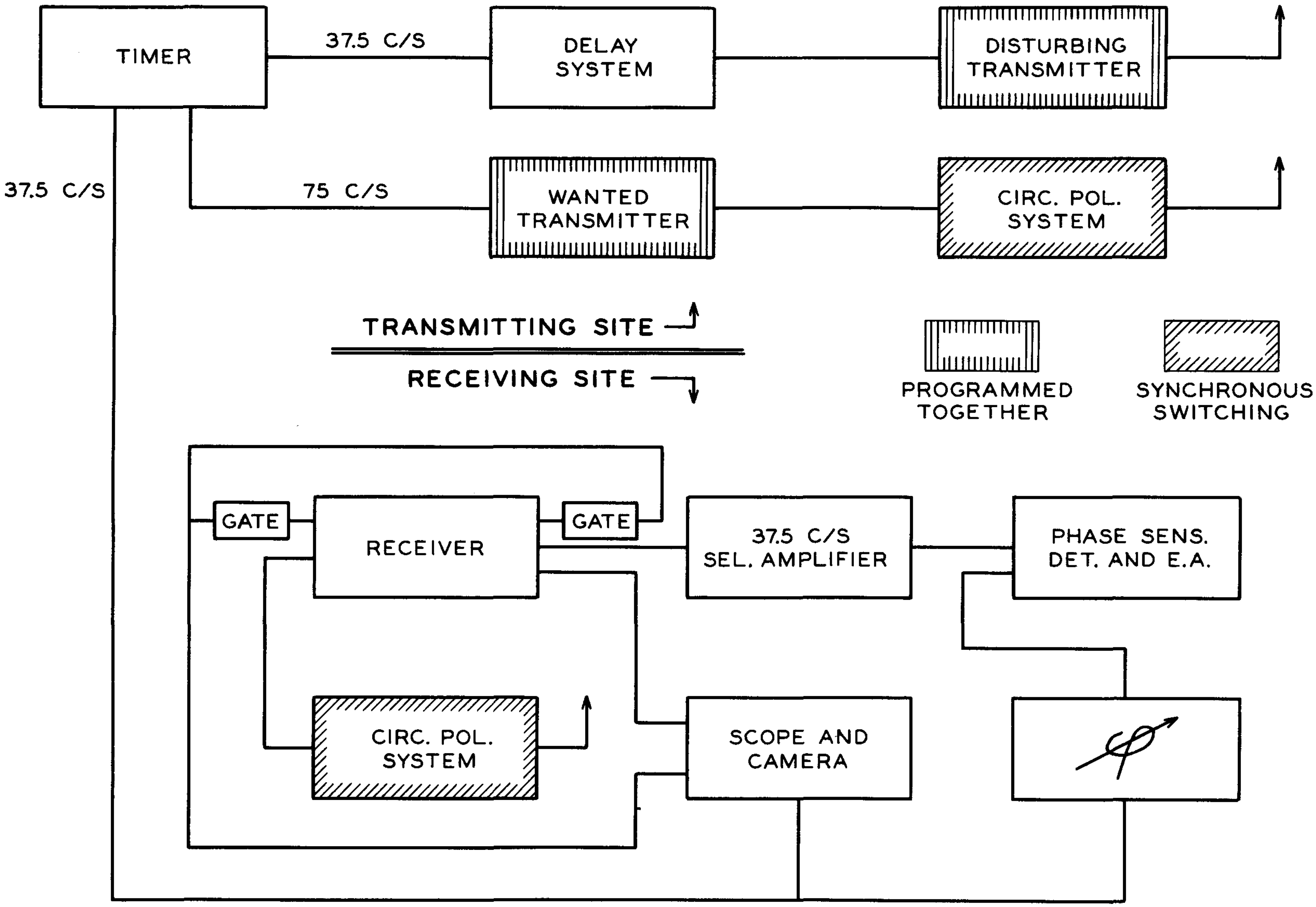

Fig. 6. Block diagram.

The characteristics of our experiment, however, will be different from Fejer's. As shown below, the wanted transmitter must operate above the minimum usable frequency and below f_oF2, and the frequency of the disturbing transmitter should be around 15 Mc/s for our high latitude location.

Thus a first difference between Fejer's operation and the proposed one at College is a major change in the range of frequencies. Furthermore, the disturbing frequency is larger than the wanted frequency. Also, the discrimination between the x-component and the o-component is performed on the wanted wave, rather than on the disturbing wave. Lastly it is worth pointing out that the present experiment is planned to operate at relatively high frequencies, with the advantage that relatively high antenna gains are obtainable. Other specifications of our transmitters and receiver are listed below.

Disturbing transmitter:

peak radiated power	200 kw
frequency (f_D or f_1)	15 Mc/s
pulse length	50 and 25 μ sec
repetition rate	37.5 c/s
antenna	16 x (3 element) Yagi array

Wanted transmitter:

peak radiated power	10 kw
frequency (f_w or f_2)	4 Mc/s
pulse length	50 and 25 μ sec
repetition rate	75 c/s

antenna: 4 horizontal half wave dipoles forming the side of a square and producing circular polarization.

Receiver:

bandwidth	25/50 kc/s
time constant	10 sec

antenna: 4 horizontal half wave dipoles forming the side of a square and producing circular polarization.

With the scheme of operation illustrated above, Fejer's formula for the determination of cross-modulation between pulses can be used, provided special care is taken in selecting the interval of integration and the required corrections are used in the determination of height; both points will be discussed in the following paragraph.

We arrived at the specifications listed above after careful consideration of the following criteria:

a) the frequency of the disturbing transmitter was selected according to the criterion that we want a cross-modulation as spread as possible along the vertical line. That means f_D high enough to avoid heavy absorption at the lowest levels, during partial blackout conditions.

b) the frequency of the wanted wave was chosen with two points in mind; namely we want to operate above the range of extremely high absorption and below f_oF2. 4 Mc/s represents a good compromise.

c) a change in f_W does not affect the amplitude of cross-modulation at the lowest heights because $\frac{\partial K_W}{\partial \nu}$ is independent of f_W where ν is large.

d) "the largest transferred modulation of both signs occurs near the height for which $\nu = (\omega_2 \pm \omega_L)$, although at this height itself there is no interaction." (ω_L = longitudinal component of ang.gyrofrequency).

Consider T_h: coefficient of transferred modulation in the case of complete absorption of the disturbing wave in a thin layer. Both maxima are increased as the frequency of the wanted wave is decreased, as can be deduced from

$$T_{h\ max} = \frac{e^2 E}{12\ mck\ \theta\ h^2\ (\omega_2 \pm \omega_L)}$$

($T_{h\ max}$ is inversely proportional to the frequency and not to the square of the frequency; the misprint in Fejer's paper has been corrected).

e) at the highest levels the amplitude of cross-modulation is inversely proportional to $(\omega_2 \pm \omega_L)^2$.

f) changes in f_D have two effects. First, an increase in f_D permits more power to reach the higher levels. Second, an increase in f_D results in a decrease of the absorption coefficient of the disturbing wave. These two effects tend to compensate for each other, as far as the cross-modulation is concerned, at the highest levels.

g) cross-modulation is always proportional to N.

h) we notice a reversal of cross-modulation at a certain height. The zero point is determined only by the difference $(\omega_2 \pm \omega_L)^2 - \nu^2 = 0$.

i) the ratio

$$R = \frac{\text{cross-modulation for x-component of wanted wave}}{\text{cross-modulation for o-compcnent of wanted wave}}$$

is equal to

$$= \frac{(\omega_2 - \omega_L)^2 - \nu^2}{\left[(\omega_2 - \omega_L)^2 + \nu^2\right]^2} \cdot \frac{\left[(\omega_2 + \omega_L)^2 + \nu^2\right]^2}{(\omega_2 + \omega_L)^2 - \nu^2} .$$

ν, through ω_2 and ω_L, is a single valued function of R.

The application of the criteria from a) to i) requires some knowledge of the order of magnitude of ν and N in the D region. Daytime and nighttime distributions of ν and N during major absorption events have been given by Chapman and Little (1957). In our experiment we should consider days of significant, but less absorption when it is still possible to have echoes on standard ionospheric sounders. Under such conditions we can expect lower electron densities in the D and lower E region. For the purpose of this analysis we have selected daytime values of N(h) equal to 1/10 of Chapman and Little's data for noon, and nighttime values 1/3 of Chapman and Little's data for midnight. See Table I.

TABLE I

Layer (km)	40-50	50-60	60-70	70-80	80-90	90-100
ν	4×10^8	1.2×10^8	3.3×10^7	7.6×10^6	1.5×10^6	3.1×10^5
N_{noon}(el/m3)	1×10^7	8.8×10^7	6.9×10^8	4.3×10^9	1.45×10^{10}	2.46×10^{10}
$N_{midnight}$(el/m3)	6×10^5	1.49×10^7	2.7×10^8	3.5×10^9	2.13×10^{10}	6.9×10^{10}

These values of N would lead to wanted signals of adequate strength for analysis. The calculations are reported in the following tables. The data of Table 1 and the cross-modulation profiles expected are shown on Figs. 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, and 19 at the end of the tables.

Absorption at 15 Mc/s - Ordinary Component f_c - Midnight - M K S System

Δh [m]	40 - 50 x 10^3	50 - 60 x 10^3	60 - 70 x 10^3	70 - 80 x 10^3	80 - 90 x 10^3	90 - 100 x 10^3
ν [sec^{-1}]	4 x 10^8	1.2 x 10^8	3.3 x 10^7	7.6 x 10^6	1.5 x 10^6	3.1 x 10^5
N [el/m^3]	0.6 x 10^6	1.49 x 10^7	2.7 x 10^8	3.5 x 10^9	2.13 x 10^{10}	6.9 x 10^{10}
$N \nu \Delta h$	2.4 x 10^{18}	1.78 x 10^{19}	9.0 x 10^{19}	2.64 x 10^{20}	3.2 x 10^{20}	2.15 x 10^{20}
$\left[1 + \frac{\nu^2}{4\pi^2 f_c^2}\right]^{-1} = g$	0.064	0.43	0.9	1	1	1
$\Delta I = N \nu g \Delta h$	1.53 x 10^{17}	7.65 x 10^{18}	8.10 x 10^{19}	2.64 x 10^{20}	3.2 x 10^{20}	2.15 x 10^{20}
h	45 x 10^3	55 x 10^3	65 x 10^3	75 x 10^3	85 x 10^3	95 x 10^3
Absorption in db up to h	0	0.0172	0.207	0.952	2.207	3.352
Power at h	2 x 10^5	2 x 10^5	1.9 x 10^5	1.60 x 10^5	1.2 x 10^5	0.93 x 10^5

Absorption at 15 Mc/s - Ordinary Component f_c - Noon - M K S System

Δh [m]	40 - 50 x 10^3	50 - 60 x 10^3	60 - 70 x 10^3	70 - 80 x 10^3	80 - 90 x 10^3	90 - 100 x 10^3
ν [sec^{-1}]	4×10^8	1.2×10^8	3.3×10^7	7.6×10^6	1.5×10^6	3.1×10^5
N [el/m^3]	1×10^7	8.8×10^7	6.9×10^8	4.3×10^9	1.45×10^{10}	2.46×10^{10}
$N \nu \Delta h$	4×10^{19}	1.06×10^{20}	2.28×10^{20}	3.26×10^{20}	2.18×10^{20}	7.6×10^{19}
$\left[1 + \frac{\nu^2}{4\pi^2 f_c^2}\right]^{-1} = g$	0.064	0.43	0.9	1	1	1
$\Delta I = N \nu g \Delta h$	2.54×10^{18}	4.56×10^{19}	2.05×10^{20}	3.26×10^{20}	2.18×10^{20}	7.6×10^{19}
h	45 x 10^3	55 x 10^3	65 x 10^3	75 x 10^3	85 x 10^3	95 x 10^3
Absorption in db up to h	0	0.103	0.647	1.79	2.95	3.58
Power at h	2×10^5	1.94×10^5	1.72×10^5	1.32×10^5	1.02×10^5	0.88×10^5

Absorption at 30 Mc/s - Ordinary Component f_c - Midnight - M K S System

Δh [m]	40 - 50 x 10^3	50 - 60 x 10^3	60 - 70 x 10^3	70 - 80 x 10^3	80 - 90 x 10^3	90 - 100 x 10^3
ν [sec^{-1}]	4×10^8	1.2×10^8	3.3×10^7	7.6×10^6	1.5×10^6	3.1×10^5
N [el/m^3]	0.6×10^6	1.49×10^7	2.7×10^8	3.5×10^9	2.13×10^{10}	6.9×10^{10}
$N \nu \Delta h$	2.4×10^{18}	1.78×10^{19}	9.0×10^{19}	2.64×10^{20}	3.2×10^{20}	2.15×10^{20}
$\left[1 + \frac{\nu^2}{4\pi^2 f_c^2}\right]^{-1} = g$	0.2	0.736	0.985	1	1	1
$\Delta I = N \nu g \Delta h$	0.48×10^{18}	1.31×10^{19}	8.9×10^{19}	2.64×10^{20}	3.2×10^{20}	2.15×10^{20}
h	45 x 10^3	55 x 10^3	65 x 10^3	75 x 10^3	85 x 10^3	95 x 10^3
Absorption in db up to h	0.0003	0.0082	0.066	0.271	0.614	0.927
Power at h	2×10^5	2×10^5	1.97×10^5	1.88×10^5	1.74×10^5	1.61×10^5

Absorption at 30 Mc/s - Ordinary Component f_c - Noon - M K S System

Δh [m]	40 - 50 x 10^3	50 - 60 x 10^3	60 - 70 x 10^3	70 - 80 x 10^3	80 - 90 x 10^3	90 - 100 x 10^3
ν [sec^{-1}]	4×10^8	1.2×10^8	3.3×10^7	7.6×10^6	1.5×10^6	3.1×10^5
N [el/m^3]	1×10^7	8.8×10^7	6.9×10^8	4.3×10^9	1.45×10^{10}	$2.46 \times 10^{1[illegible]}$
$N \nu \Delta h$	4×10^{19}	1.06×10^{20}	2.28×10^{20}	3.26×10^{20}	2.18×10^{20}	7.6×10^{19}
$\left[1 + \frac{\nu^2}{4\pi^2 f_c^2}\right]^{-1} = g$	0.2	0.736	0.985	1	1	1
$\Delta I = N \nu g \Delta h$	0.8×10^{19}	0.78×10^{20}	2.24×10^{20}	3.26×10^{20}	2.18×10^{20}	7.6×10^{19}
h	45 x 10^3	55 x 10^3	65 x 10^3	75 x 10^3	85 x 10^3	95 x 10^3
Absorption in db up to h	0	0.055	0.232	0.554	0.873	1.04
Power at h	2×10^5	1.98×10^5	1.88×10^5	1.76×10^5	1.64×10^5	1.57×10^5

Absorption at 10 Mc/s - Ordinary Component f_c - Midnight - M K S System

Δh $[m]$	40 - 50 $\times 10^3$	50 - 60 $\times 10^3$	60 - 70 $\times 10^3$	70 - 80 $\times 10^3$	80 - 90 $\times 10^3$	90 - 100 $\times 10^3$
ν $[sec^{-1}]$	4×10^8	1.2×10^8	3.3×10^7	7.6×10^6	1.5×10^6	3.1×10^5
N $[el/m^3]$	0.6×10^6	1.49×10^7	2.7×10^8	3.5×10^9	2.13×10^{10}	6.9×10^{10}
$N \nu \Delta h$	2.4×10^{18}	1.78×10^{19}	9.0×10^{19}	2.64×10^{20}	3.2×10^{20}	2.15×10^{20}
$\left[1 + \frac{\nu^2}{4\pi^2 f_c^2}\right]^{-1} = g$	0.0322	0.27	0.83	1	1	1
$\Delta I = N \nu g \Delta h$	0.765×10^{17}	4.8×10^{18}	7.5×10^{19}	2.64×10^{20}	3.2×10^{20}	2.15×10^{20}
h	45 $\times 10^3$	55 $\times 10^3$	65 $\times 10^3$	75 $\times 10^3$	85 $\times 10^3$	95 $\times 10^3$
Absorption in db up to h	0.00034	0.022	0.376	1.703	3.69	6.16
Power at h	2×10^5	2×10^5	1.85×10^5	1.30×10^5	7.1×10^4	4.10×10^4

Absorption at 10 Mc/s - Ordinary Component f_c - Noon - M K S System

Δh [m]	40 - 50 x 10^3	50 - 60 x 10^3	60 - 70 x 10^3	70 - 80 x 10^3	80 - 90 x 10^3	90 - 100 x 10^3
ν [sec^{-1}]	4 x 10^8	1.2 x 10^8	3.3 x 10^7	7.6 x 10^6	1.5 x 10^6	3.1 x 10^5
N [el/m^3]	1 x 10^7	8.8 x 10^7	6.9 x 10^8	4.3 x 10^9	1.45 x 10^{10}	2.46 x 10^{10}
$N \nu \Delta h$	4 x 10^{19}	1.06 x 10^{20}	2.28 x 10^{20}	3.26 x 10^{20}	2.18 x 10^{20}	7.6 x 10^{19}
$\left[1 + \frac{\nu^2}{4\pi^2 f_c^2}\right]^{-1} = g$	0.0322	0.27	0.83	1	1	1
$\Delta I = N \nu g \Delta h$	1.29 x 10^{18}	2.86 x 10^{19}	1.88 x 10^{20}	3.26 x 10^{20}	2.18 x 10^{20}	7.6 x 10^{19}
h	45 x 10^3	55 x 10^3	65 x 10^3	75 x 10^3	85 x 10^3	95 x 10^3
Absorption in db up to h	0.0057	0.138	1.10	3.13	6.00	6.85
Power at h	2 x 10^5	1.94 x 10^5	1.55 x 10^5	9.14 x 10^4	5.25 x 10^4	3.88 x 10^4

Cross-modulation - Midnight - ΔS = 2 km; Θ = 322°K; gain = 150; $\mathcal{T}$ = 50μ sec;

f_L = 1.5 Mc/s; constant = 10^{10}; $\Delta T = 10^{10} \cdot \frac{N \nu^2 \text{ Ptu}}{h^2}$; M K S

h	45 x 10^3	55 x 10^3	65 x 10^3	75 x 10^3	85 x 10^3	95 x 10^3
h^2	2.03 x 10^9	3.00 x 10^9	4.20 x 10^9	5.60 x 10^9	7.2 x 10^9	9.00 x 10^9
$N \nu^2$	9.6 x 10^{22}	2.15 x 10^{23}	2.94 x 10^{23}	2.03 x 10^{23}	4.8 x 10^{22}	6.6 x 10^{21}
4 Mc/s; [o] a) t =	-6.1 x 10^{-18}	-5.5 x 10^{-17}	+2.1 x 10^{-17}	7.25 x 10^{-16}	8.35 x 10^{-16}	8.35 x 10^{-16}
4 Mc/s; [x] b) t =	-6.25 x 10^{-18}	-6.6 x 10^{-17}	-4.65 x 10^{-16}	2.03 x 10^{-15}	3.87 x 10^{-15}	4.00 x 10^{-15}
15 Mc/s; [o] Power at h	2 x 10^5	2 x 10^5	1.9 x 10^5	1.60 x 10^5	1.20 x 10^5	9.25 x 10^4
c) u =	5.86 x 10^{-18}	3.95 x 10^{-17}	8.35 x 10^{-17}	9.15 x 10^{-17}	9.20 x 10^{-17}	9.2 x 10^{-17}
Case I ΔT [o]	-3.4 x 10^{-6}	-3.12 x 10^{-4}	2.36 x 10^{-4}	3.86 x 10^{-3}	6.14 x 10^{-4}	5.22 x 10^{-5}
Case II ΔT [x]	-3.46 x 10^{-6}	-3.75 x 10^{-4}	-5.22 x 10^{-3}	1.08 x 10^{-2}	2.86 x 10^{-3}	2.49 x 10^{-4}
10 Mc/s; [o] Power at h	2 x 10^5	2 x 10^5	1.85 x 10^5	1.30 x 10^5	7.1 x 10^4	4.1 x 10^4
d) u =	6.25 x 10^{-18}	6.7 x 10^{-17}	6.18 x 10^{-16}	1.7 x 10^{-15}	1.88 x 10^{-15}	1.88 x 10^{-15}
Case VII ΔT [o]	-3.58 x 10^{-6}	-5.3 x 10^{-4}	+1.68 x 10^{-3}	5.84 x 10^{-2}	7.5 x 10^{-3}	4.74 x 10^{-4}
Case VIII ΔT [x]	-3.67 x 10^{-6}	-6.35 x 10^{-4}	-3.70 x 10^{-2}	1.62 x 10^{-1}	3.48 x 10^{-2}	2.26 x 10^{-3}

a)

$$t = \frac{4\pi^2(f_W + f_L)^2 - \nu^2}{\left[4\pi^2(f_W + f_L)^2 + \nu^2\right]^2}$$

b)

$$t = \frac{4\pi^2(f_W - f_L)^2 - \nu^2}{\left[4\pi^2(f_W - f_L)^2 + \nu^2\right]^2}$$

c.)

$$u = \frac{1}{\nu^2 + 40 f_c^2}$$

d)

$$u = \frac{1}{\nu^2 + 40 f_c^2}$$

Cross-modulation - Noon - ΔS = 2 km; θ = 322 °K; gain = 150; τ = 50 μsec;
f_L = 1.5 Mc/s; Constant = 10^{10}; $\Delta T = 10^{10} \cdot \frac{N \nu^2 P t u}{h^2}$; M K S

h	45×10^3	55×10^3	65×10^3	75×10^3	85×10^3	95×10^3
h^2	2.03×10^9	3.00×10^9	4.20×10^9	5.60×10^9	7.2×10^9	9.0×10^9
$N \nu^2$	1.6×10^{24}	1.27×10^{24}	7.5×10^{23}	2.48×10^{23}	3.28×10^{22}	2.36×10^{21}
4 Mc/s; [o] a) t =	-6.1×10^{-18}	-5.5×10^{-17}	$+2.1 \times 10^{-17}$	7.25×10^{-16}	8.35×10^{-16}	8.35×10^{-16}
4 Mc/s; [x] b) t =	-6.25×10^{-18}	-6.6×10^{-17}	-4.65×10^{-16}	2.03×10^{-15}	3.87×10^{-15}	4.00×10^{-15}
15 Mc/s; [o] Power at h	2×10^5	1.94×10^5	1.72×10^5	1.32×10^5	1.02×10^5	0.88×10^5
u = c)	5.86×10^{-18}	3.95×10^{-17}	8.35×10^{-17}	9.15×10^{-17}	9.2×10^{-17}	9.2×10^{-17}
Case III / $\Delta T_{[o]}$	-5.65×10^{-5}	-1.79×10^{-3}	$+5.4 \times 10^{-4}$	3.88×10^{-3}	3.58×10^{-4}	1.76×10^{-5}
Case IV / $\Delta T_{[x]}$	-5.8×10^{-5}	-2.15×10^{-3}	-1.2×10^{-2}	1.10×10^{-2}	1.66×10^{-3}	8.45×10^{-4}
30 Mc/s; [o] Power at h	2×10^5	1.98×10^5	1.88×10^5	1.76×10^5	1.64×10^5	1.57×10^5
u = d)	5×10^{-18}	1.84×10^{-17}	2.46×10^{-17}	2.5×10^{-17}	2.5×10^{-17}	2.5×10^{-17}
Case V / $\Delta T_{[o]}$	-4.8×10^{-5}	-8.5×10^{-4}	$+1.74 \times 10^{-4}$	1.41×10^{-3}	1.56×10^{-4}	8.64×10^{-6}
Case VI / $\Delta T_{[x]}$	-5.0×10^{-5}	-1.04×10^{-3}	-3.84×10^{-3}	3.88×10^{-3}	7.24×10^{-4}	4.12×10^{-5}

a) $$t = \frac{4\pi^2(f_W + f_L)^2 - \nu^2}{\left[4\pi^2(f_W + f_L)^2 + \nu^2\right]^2}$$

b) $$t = \frac{4\pi^2(f_W - f_L)^2 - \nu^2}{\left[4\pi^2(f_W - f_L)^2 + \nu^2\right]^2}$$

c) $$u = \frac{1}{\nu^2 + 40 f_c^2}$$

d) $$u = \frac{1}{\nu^2 + 40 f_c^2}$$

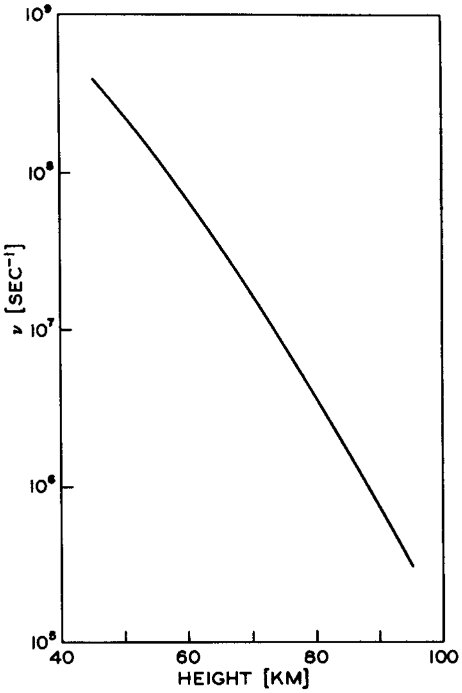

Fig. 7. Electron collision frequency in the ionosphere above College, Alaska, according to Chapman and Little (JATP 10, 29).

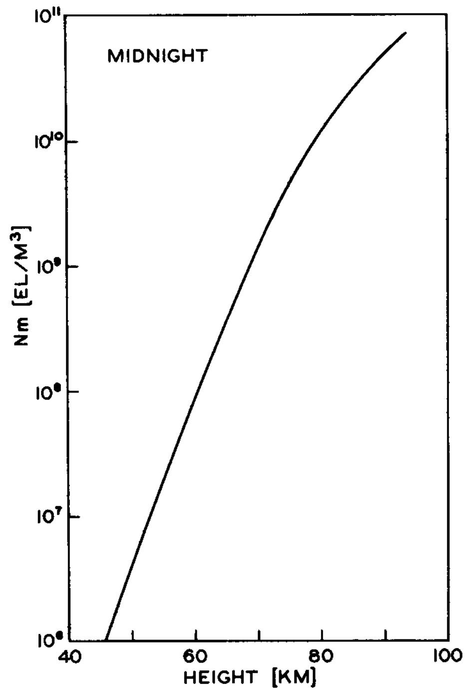

Fig. 8. Electron density versus height.

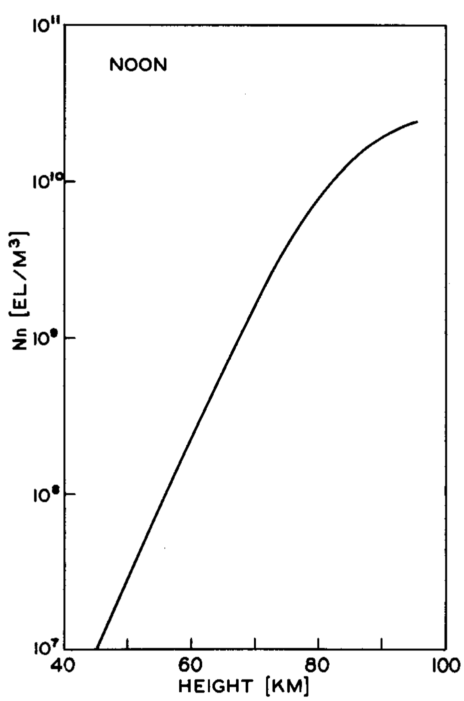

Fig. 9. Electron density versus height.

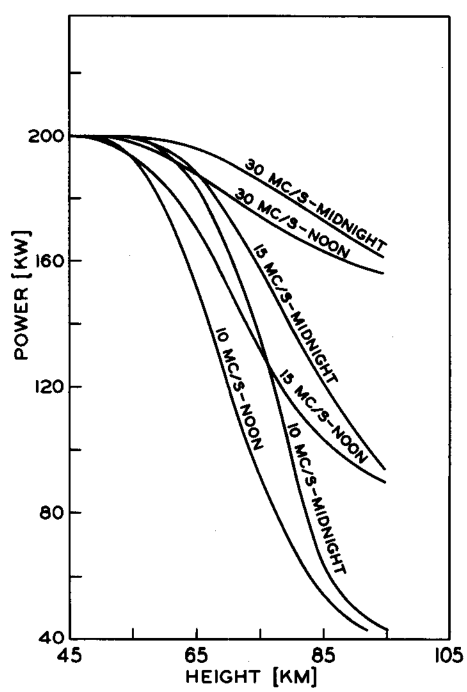

Fig. 10. Absorption in the D region according to data reported in Figs. 7, 8, and 9.

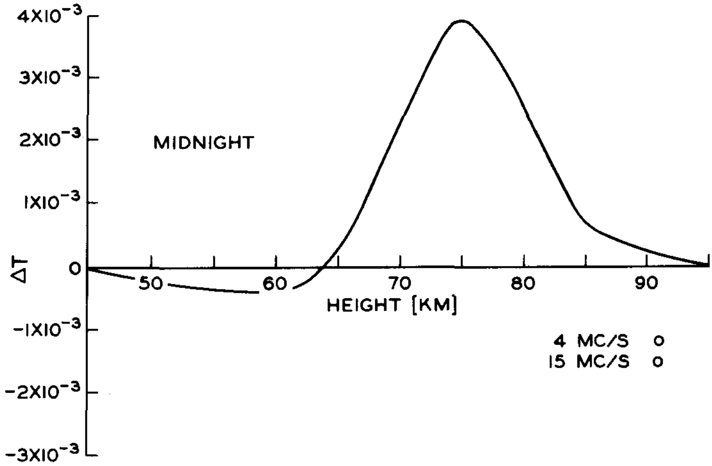

Fig. 11. Cross-modulation versus height
Case I (see page 28).

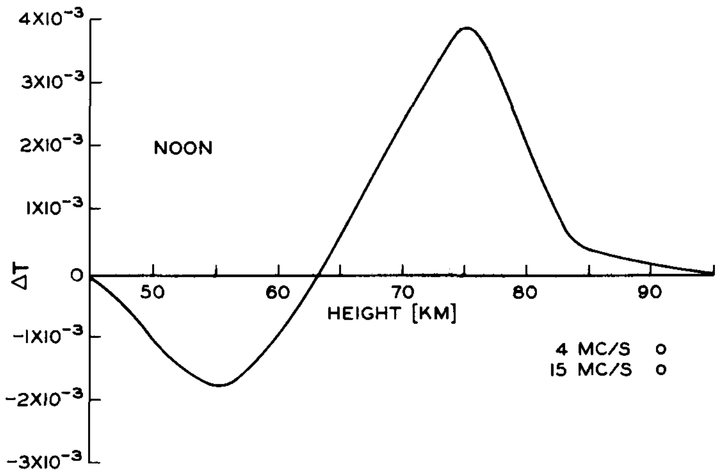

Fig. 13. Cross-modulation versus height
Case III (see page 29).

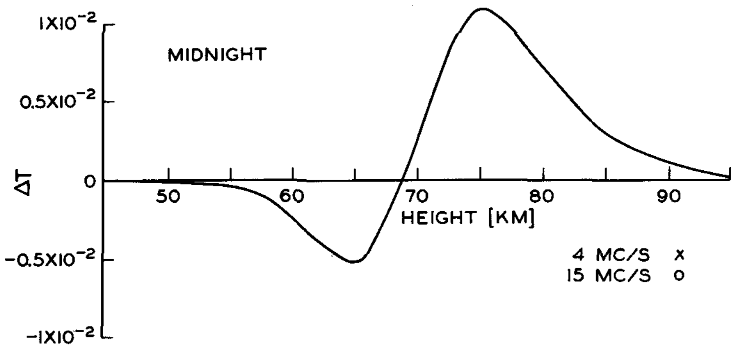

Fig. 12. Cross-modulation versus height Case II (see page 28).

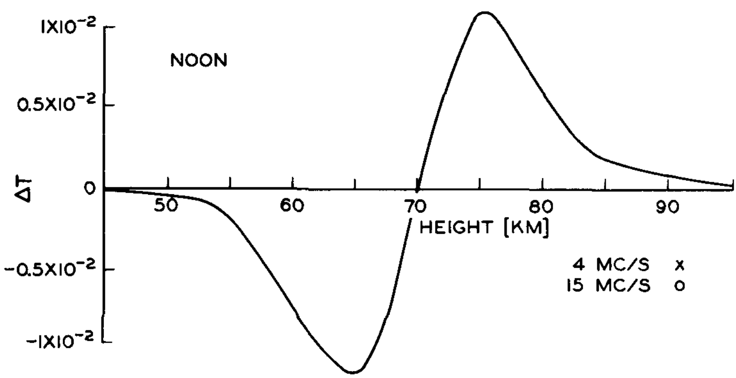

Fig. 14. Cross-modulation versus height Case IV (see page 29).

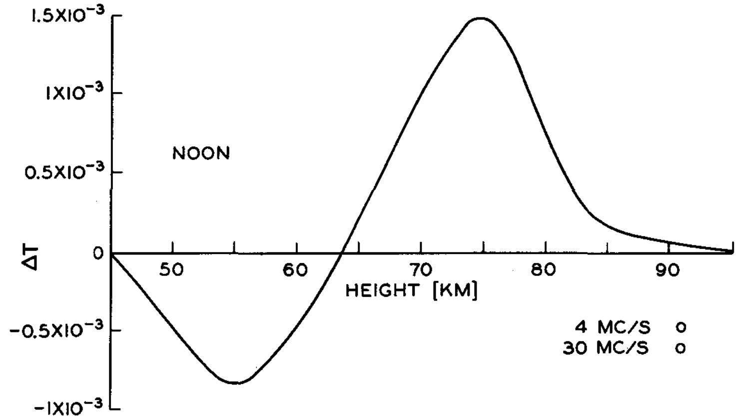

Fig. 15. Cross-modulation versus height Case V (see page 29).

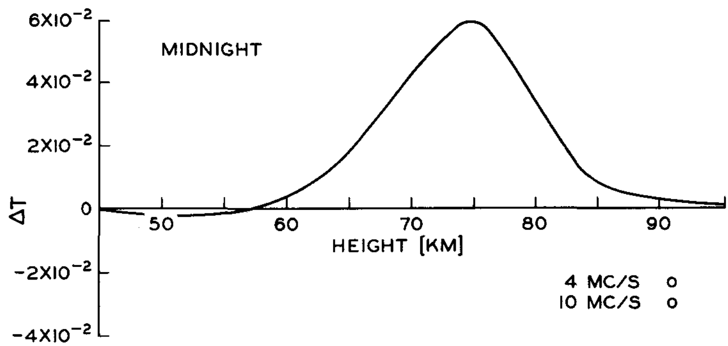

Fig. 17. Cross-modulation versus height Case VII (see page 28).

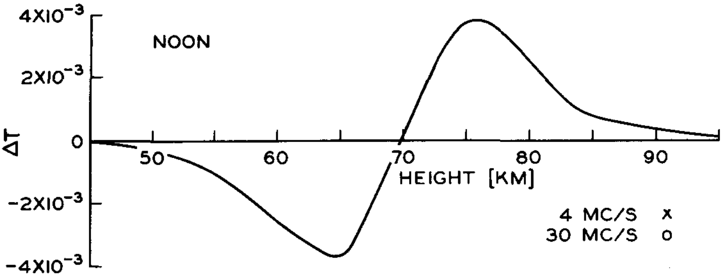

Fig. 16. Cross-modulation versus height Case VI (see page 29).

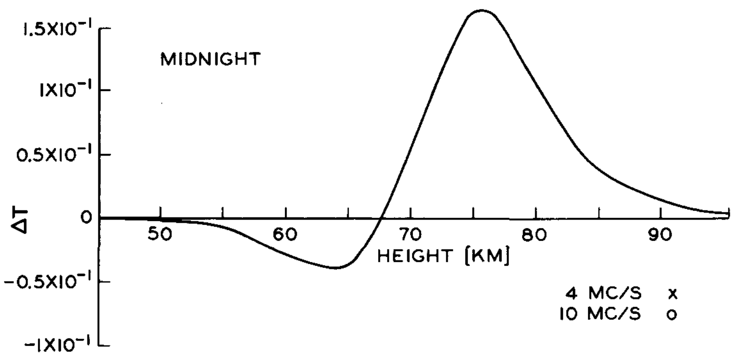

Fig. 18. Cross-modulation versus height Case VIII (see page 28).

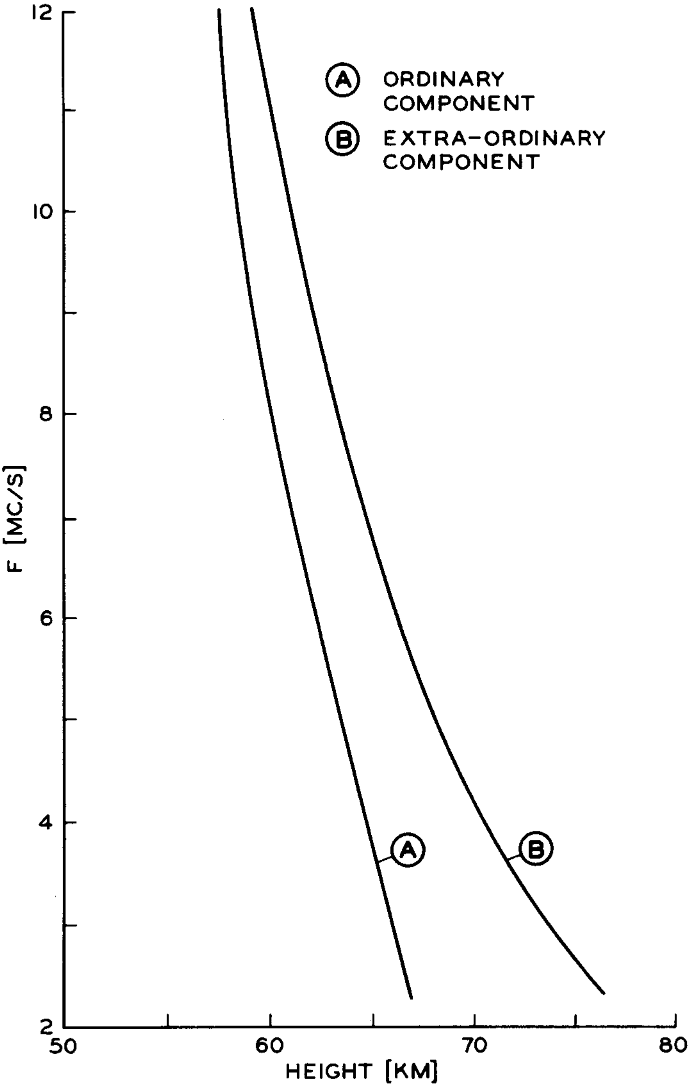

Fig. 19. Frequency of wanted wave versus height of zero cross-modulation.

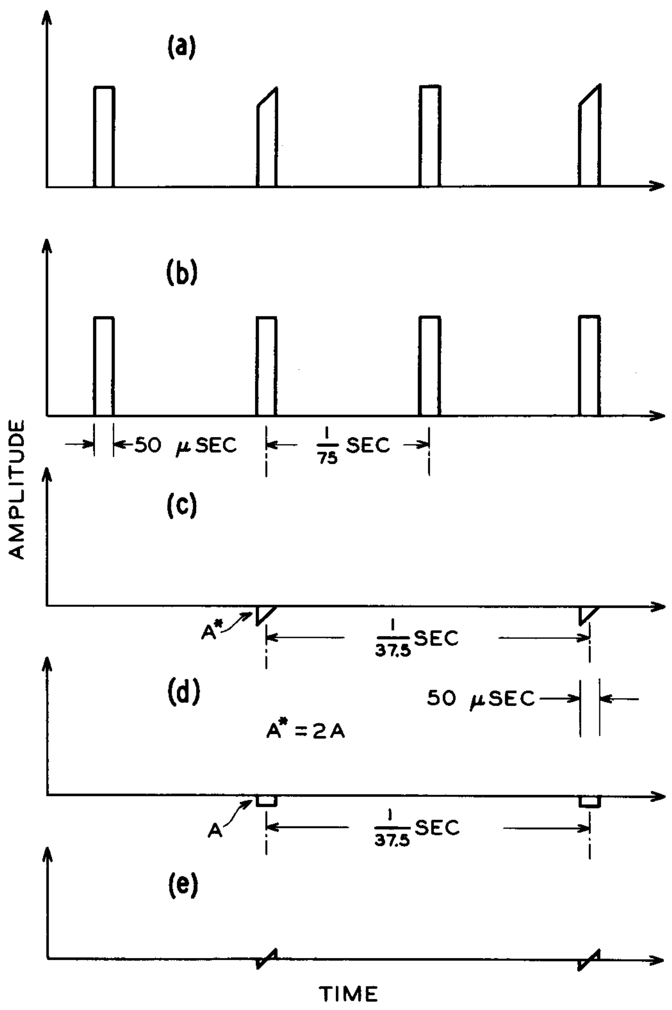

Fig. 20. Received pulses.

Note:

The values of $\triangle T$ presented above refer to cross-modulation arising in 2 kilometers interval; 2 kilometers are about 1/4 of the scale height (see Rocket Panel - 1952); the characteristics of the ionosphere are not supposed to change drastically over such a distance. Furthermore, the actual integration interval is always larger than 2 kilometers when 50 microsecond pulses are used; then the actual values of $\triangle T$ will be larger than the ones presented above.

Some comments about the planned experiment.

Some comments follow:

We need to justify the use of Fejer's formula in the previous calculations and indicate how we can rely on it. Let us refer to Fig. 20. The succession of pulses in a) will be detected by our receiver; it can be represented as the summation of two successions of pulses, as in b) and c). In our experiment we use a selective amplifier on 37.5 c/s; therefore the train of pulses in b) will not be recorded; the recorder will give an indication of the fundamental component of train c).

Train c) can be in turn considered as the summation of the train d) and the train e). Our measurement will be essentially concerned with the train d) of which e) is only a perturbation. In support to this statement we recall (Reference Data for Radio Engineers - 1956) that the amplitude of the fundamental for d) is

$$C_1 = \frac{2 A t_o}{T} \left| \frac{\sin \pi t_o / T}{\pi t_o / T} \right|$$

and the amplitude of the fundamental for c) is

$$C_1 = \frac{A^* t_o}{T} \frac{1}{a^2} \left[\sin^2 a + a(a - \sin 2a)\right]^{\frac{1}{2}} ,$$

with $A^* = 2A$ and $a = \frac{\pi t_o}{T}$.

By introducing into these formulas t_o = 50 microseconds and T = 1/37.5 cycles per second it is possible to check that the difference between the fundamental component of the train of pulses shown in c) and the fundamental for the train shown in d) is small. We will be recording the cross-modulation on the axis of the single pulses. To this portion of the wanted pulse we can apply Fejer's formula provided the relative integration interval is determined.

The experiment requires that considerable attention be given to the design of the antennas, particularly to the arrangement for circular polarization of the wanted wave. A rough discrimination of ordinary and extraordinary wave could lead to major errors.

Such a statement will become clear when the method of interpretation of our data is described. First, we plan to make use of the very fortunate situation, predictable at high latitudes, namely, the inversion of sign of the cross-modulation at a certain height. At this height ν^2 is simply equal to $(\omega_2 \pm \omega_L)^2$. This height differs for the ordinary and the extraordinary wave.

By changing the frequency of the wanted wave in the range 2.5-12 Mc/s it should be possible to obtain information about ν from approximately 57 to 77 kilometers in height by this method.

Without shifting frequency the same information can also be deduced by reading from the records

$$R = \frac{\text{cross-modulation for x-component of wanted wave}}{\text{cross-modulation for o-component of wanted wave}}$$

$$= \frac{(\omega_2 - \omega_L)^2 - \nu^2}{\left[(\omega_2 - \omega_L)^2 + \nu^2\right]^2} \cdot \frac{\left[(\omega_2 + \omega_L)^2 + \nu^2\right]^2}{(\omega_2 + \omega_L)^2 - \nu^2} .$$

ν , through ω_2 and ω_L, is a single valued function of R.

The reliability of this method is now analyzed. Let $(\omega_2 + \omega_L)^2 = b^2$ and $(\omega_2 - \omega_L)^2 = a^2$;

$$R = \frac{a^2 - \nu^2}{b^2 - \nu^2} \cdot \frac{(b^2 + \nu^2)^2}{(a^2 + \nu^2)^2}$$

$$\log R = 2 \log (b^2 + \nu^2) - 2 \log (a^2 + \nu^2) + \log (a^2 - \nu^2) - \log (b^2 - \nu^2)$$

$$\frac{dR}{R} = \nu^2 \left(\frac{4}{b^2 + \nu^2} - \frac{4}{a^2 + \nu^2} - \frac{2}{a^2 - \nu^2} + \frac{2}{b^2 - \nu^2} \right) \cdot \frac{d\nu}{\nu}$$

For $a^2 \sim \nu^2$ and $b^2 \sim \nu^2$ any error in R is reflected in a small error in ν . For $b^2 > a^2 \gg \nu^2$ (heights greater than 75 kilometers for 4 Mc/s) or $a^2 < b^2 \ll \nu^2$ (heights less than 60 kilometers for 4 Mc/s) R tends to unity and small errors in determining the ratio of the cross-modulation of the two components could lead to major errors in determining ν. By changing the frequency of the wanted transmitter in the range 2.5 - 12 Mc/s it would be possible to extend the range of operation, namely from about 60-75 kilometers to about 50-80 kilometers.

It should be noted that θ and G do not play any role in the outlined method of determining ν. Once ν is established, we can find the distribution of N by introducing tentative values of N in Fejer's formula, until we come out with a diagram of calculated cross-modulation that matches the experimental results.

Unfortunately this procedure is cumbersome. A simpler method is described in the following lines. The elementary cross-modulation can be written as

$$\Delta T = \text{constant} \cdot \frac{N \nu^2 t \, u \, P_o \exp\left(-\int 2K_1 ds\right)}{h^2}$$

(see preceding tables). Once ν and ΔT are determined, N is function only of $\exp(-\int 2K_1 dh)$. In our cases I, II, III, and IV illustrated in the tables, $\exp(-\int 2K_1 dh)$ varies from 1 to about 1/2 as the wave moves from the bottom to the top of the D region. Thus if we attribute to $\exp(-\int 2K_1 dh)$, in first approximation, the value 1 and introduce it into the formula for ΔT, we can deduce directly the value of N. The largest error could be of the order of 50% for the highest heights. It will always be possible to correct this error through successive approximations; namely ν and approximately N are deduced for the D region from the bottom to the top through the steps outlined above; then $\exp(-\int 2K_1 dh)$ is calculated accordingly. A new set of values for N is in turn deduced, in accordance with the figures newly found for $\exp(-\int 2K_1 dh)$ and so on. The process suggested converges fast on account of the exponential law that is involved.

Another aspect of the problem that must be discussed is the measurement of the height of cross-modulation. The measurement of the delay between the disturbing and the disturbed pulse, Δt, gives the height where the two pulses first meet, $h_o = c\Delta t/2$. We know that cross-modulation is spread over an interval of heights that can extend appreciably below h_o. The actual distribution of cross-modulation versus height for two pulses, disturbing and wanted, that meet at h_o is the product of a curve similar to the ones sketched in Fig. 3 and the curves drawn in Figs. 10 to 18. When our detector will indicate zero cross-modulation, the null will take place at $h_o - \delta$; actually the detector indicates that the integrated cross-modulation of one sign balances the integrated cross-modulation of the opposite sign. Reasonable hypotheses about the actual distribution with height of cross-modulation can be formulated and then approximated values of δ predetermined. The height correction term is function of $G\nu t_o$, t_o being as always the pulse length. When $G\nu t_o$ is larger than 5, δ is of the order of 2 kilometers; when $G\nu t_o$ is between 5 and 2, δ may be taken as 3 kilometers; for $G\nu t_o$ between 2 and 1, $\delta \sim 4$ kilometers. When $G\nu t_o < 1$ the measurements become meaningless on account of the large spread of the interaction.

When the measurement of ν is performed by taking the ratio of ordinary and extraordinary component of cross-modulation the problem of determining the effective height or, in other words, the height where cross-modulation should be concentrated to give the same ν is not simple. An approximate solution is to take the height of the center of gravity of the two areas that represent the distribution of the cross-modulation versus height. These area are again obtained by the multiplication of one curve like in Fig. 3 and curves in Figs. 10 to 18. According to this criterion the correction terms for the determination of height, δ , can be derived. The figures quoted above relating to different values of $G \nu t_o$ seem reasonable even in the case of measurements that are based upon determination of the ordinary and extraordinary cross-modulation. Again the condition $G \nu t_o < 1$ puts an upper limit to the usefulness of radio wave interaction technique for the study of ν and N in the D region. Such an upper limit for t_o = 50 microseconds is about 75 kilometers. Above 75 kilometers other techniques should be used.

The correction terms δ proposed above have been determined in a crude way, by figuring out the dispersion of cross-modulation versus height on uncertain theoretical grounds. A more careful approach will require a successive approximation method, by which we start with $\delta = 0$, determine $\nu(h)$ and N (h); on the basis of these experimental approximated $\nu(h)$ and N (h), we derive as outlined above the δ's and so on.

At the end of these comments it appears evident that the technique of radio wave interaction will furnish a very reliable information only when the general trends of ν versus height and N versus height will be considered, instead of the single values of ν and N per se.

Some comments about gyrointeraction.

The subject of gyrointeraction or interaction with f_D = gyrofrequency is a matter of controversy. Bailey in 1937 showed the possibility of an enhanced interaction effect; under special conditions a small disturbing power is able to produce a detectable cross-modulation. In Bailey's paper one finds described the alternative dromedarian or bactrian curve of resonance, the curve being always centered around the gyrofrequency. Experimental results that confirm the predictions of Bailey were obtained by Cutolo (1950) and by Bailey et alii (1952). On the other side Shaw (1951) in the course of several attempts was not able to detect such a resonance; he even finds difficult to predict gyrointeraction. Anyway it seems that to argue, as Shaw does, only about the amount of disturbing power that is absorbed in the ionosphere and not about the efficiency of such an absorption, is misleading. Indeed it is possible that only a fraction of the disturbing power is dissipated along the path of the wanted wave.

When dealing with gyrointeraction the absorption of the disturbing wave takes place in a very thin layer; therefore all the considerations we have developed about integration intervals must be correspondingly modified. In particular pulsed gyrointeraction can be used for the determination of ν and N above 75 kilometers because of the thinness of the absorbing layer. Height measurements are still affected by the relaxation time $1/G\nu$ and the determination of the correction term is required.

Gyrointeraction can easily be too intense such that the condition $MP_1 << \bar{Q}G$ does not hold any longer. In such a case the analytical treatment represented by equations (2), (3), (4), (5), (6), and (7) is not valid. Only when $MP_1 << \bar{Q}G$ we can derive from (7) the expression

$$T_h = \frac{e^2 E\nu}{3mck\theta h^2} \quad \frac{(\omega_2 \pm \omega_L)^2 - \nu^2}{\left[(\omega_2 \pm \omega_L)^2 + \nu^2\right]^2}$$

and use it for gyrointeraction; where T_h is the coefficient of transferred modulation in the case of complete absorption of the disturbing wave in a thin layer at the height h, E is the disturbing energy, and the other symbols are the same as in (7). T_h has two maxima at the two heights defined by $\nu(h) = (\sqrt{2} - 1) \cdot (\omega_2 \pm \omega_L)$ and $\nu(h) = (\sqrt{2} + 1) \cdot (\omega_2 \pm \omega_L)$. The absolute value of these maxima is $T_{hmax} = \dfrac{e^2 E}{12\, mck\, \vartheta h^2\, (\omega_2 \pm \omega_L)}$ (the square on $(\omega_2 \pm \omega_L)$ in Fejer's paper is a misprint). It is worth pointing out that the lowest of the maxima, in practice, does not exist; indeed K_1 is too small at the height defined by $\nu(h) = (\sqrt{2} + 1) \cdot (\omega_2 \pm \omega_L)$, so that the disturbing energy could not be absorbed all together. It seems that gyrointeraction requires the coexistance of the following conditions:

a) the disturbing frequency is close to the gyrofrequency.

b) the absorbing layer is quite high and very thin.

c) disturbing and wanted wave do cross in this layer.

This is not always attainable, then Shaw's negative result may be explained.

In what follows we will be concerned with a strong gyrointeraction where the condition $MP_1 << \bar{Q}G$ does not hold any longer.

Four records obtained during satellite radio tracking at College, Alaska, present some features that probably have something to do with gyrointeraction. The equipment located at College, Ballaines Lake site, is essentially a receiver connected to a Sanborn recorder; it records the signal strength received from the satellite 1958 δ_2 on 20 Mc/s. The recordings we are discussing now were obtained on May 25, 1958 - 1123, 1311, 1459, and on July 17, 1958 - 2309 AST. Thanks are due to Roy Basler, Ron DeWitt, and Nate Warman, the team which operated the satellite equipment, for singling out and commenting about them. Fig. 21 shows the record obtained on July 17; on the left side is the calibration of the equipment and in the lower part the satellite signal; the chart had the speed of 25 small divisions per second.

Fig. 22 shows the f plots obtained at College on July 17, 1958.

We interpret the quasi-sinusoidal oscillation superimposed upon the satellite pulses as possibly due to gyrointeraction of the satellite signal with a disturbing broadcasting station. The examination of many satellite records leads to exclude that the modulation was produced by the satellite itself. Multipath propagation would not be a satisfactory explanation; it would require either an extremely fast moving reflector, near the receiving site (acting for the case of May 25 on three successive passages), or an extremely large fixed reflecting surface far removed from the receiver; ionospheric reflection being excluded by the fact that the satellite was, at least on one of the four occasions, above the F region.

We propose the following interpretation. The extraordinary component of the transmitter, operating near the gyromagnetic frequency, was absorbed in a thin layer in the upper D region where $\nu^2 \gg (\omega - \omega_L)^2$. Energy is absorbed according to the absorption coefficient K; $1/G\nu$ seconds after the absorption has taken place an increase of ν follows. For the extraordinary component of the gyrowave the absorption coefficient is inversely proportional to ν ; therefore an increase of ν results in a decrease of K and vice versa. This inverse proportionality and the delay between the absorption of energy from the radio wave and the transformation of it into heat bring about an oscillation of ν. The succession of the events can be reported in the

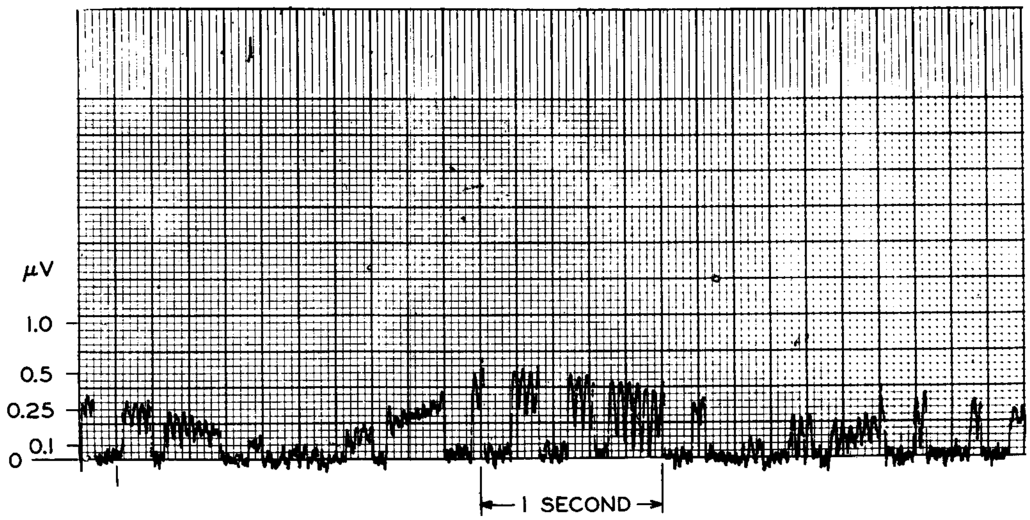

Fig. 21. Signal strength from 1958 δ_2 - 20 Mc/s College, Alaska, July 17, 1958 - 2309 AST.

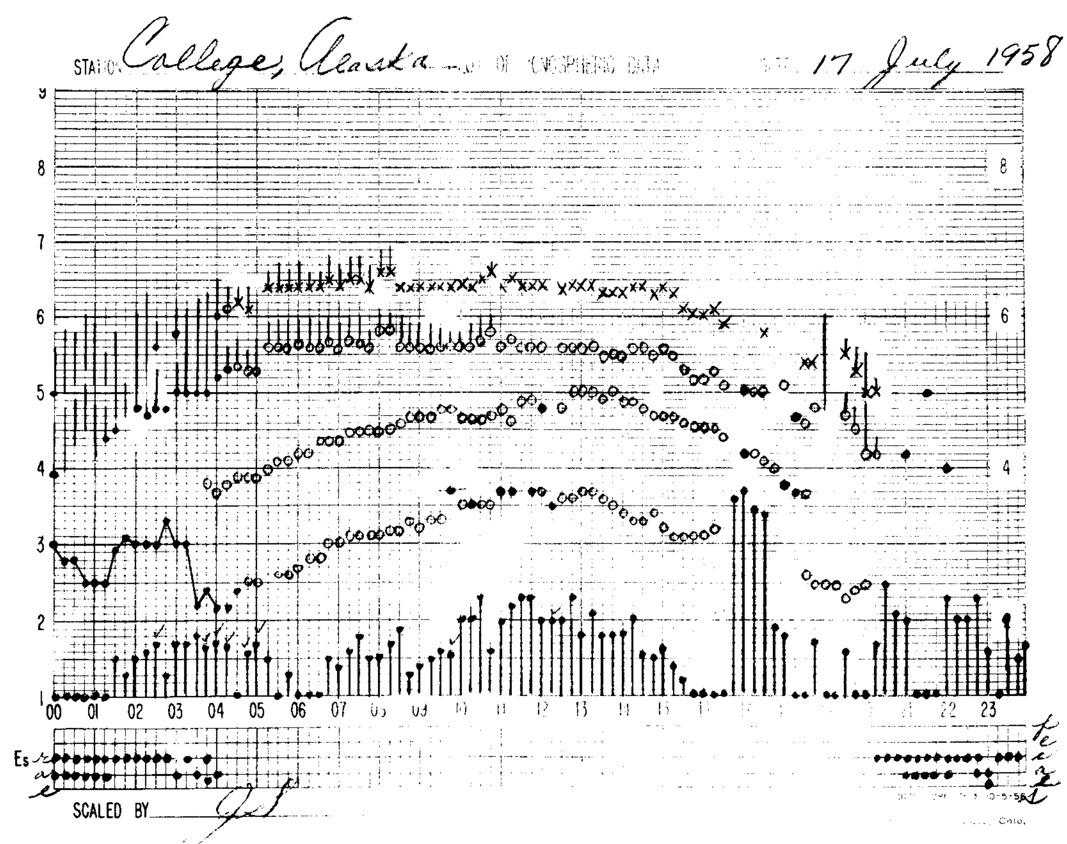

Fig. 22. F plot - College, Alaska, July 17, 1958.

following manner: absorption according to K - delay - increase of ν and decrease of K - delay - decrease of ν and increase of K - delay - increase of ν and so on.

The absorption coefficient for 20 Mc/s, frequency of the satellite signal, is proportional to ν (in the upper part of the D region); hence to an oscillation of ν will correspond an oscillation of the absorption coefficient and of the received signal. This is the mechanism we propose for the interpretation of the record under discussion. The time constant of this recurrent process should be of the order of $1/\nu G$ with $\nu = 10^5$ and $G = 10^{-3}$, or about 10 milli-seconds; that fits the results under discussion.

Furthermore let us think that the extraordinary component of the disturbing power is of the order of magnitude of 10 kw. At 100 kilometer height the field (square) would be $E_D^2 = 3 \times 10^{-5}$ (volts per meter)^2 and $P_{1/\nu} = 1/2 \times e^2/m \times E_D^2 / \nu^2 = 4 \times 10^{-23}$ J/collision. According to Huxley and Ratcliffe (1949 - bottom of page 434) an energy of 5.5×10^{-24} J/collision should be able to produce a 33% increase of the collision frequency. With our calculation we are about seven times better off, and the interpretation of the record does not seem unreasonable even from a quantitative point of view. It seems then possible that gyrointeraction has been the cause that produced our peculiar records.

This possibility indicates that we can extend our experiment and enlarge the field of our investigation without any supplementary equipment. If a narrow beam antenna is used in connection with a receiver, the signal from the satellite can be replaced by cosmic noise; that is, cosmic noise becomes our wanted wave. The difficulty in this substitution lies in the fact that while the signal from the satellite has a steady amplitude, or a well defined modulation, the cosmic noise is affected by random fluctuations that tend to mask the cross-modulation we are looking for.

The expression we have to use for the determination of noise fluctuation is $\Delta P_{rms}/P \approx 1/\sqrt{BT}$ where B is the input bandwidth of the receiver as far as the detector and T is the output time constant. This leads for typical operating conditions to a value of the order of magnitude of 10^{-2}. This would exclude any detectability of cross-modulation less than 5%, but the satellite records seem to indicate that we should expect at least in the most fortunate cases a cross-modulation as large as 50%. If this is true, then the effect would be detectable even with cosmic noise. The technique of using cosmic noise as a wanted wave has noteworthy features; namely not only does it permit the elimination of one transmitter with great advantage of simplicity, but also it permits the simultaneous registration of cross-modulation on two or more different frequencies by duplicating the receiver.

As a consequence of the preceding considerations we plan to connect a receiver to the antenna of the disturbing transmitter-15 Mc/s- when this one is not in operation and look for the effects produced on the received cosmic noise signal when the swept frequency ionospheric sounder C4 is turned on. The C4, already in operation at our transmitting site, has characteristics that fit very well the requirements for an attempt to detect gyrointeraction. They are: Frequency range 1 - 25 Mc/s; - Peak Pulse Output Power 10 kw (nominal); - Sweep time 15, 30, and 120 seconds; - Manual sweep control available. The operation of the C4 is programmed such that it goes on 4 times every hour.

A recorder connected to the output of the receiver on 15 Mc/s and gated with the C4 will furnish the information on the gyrointeraction.

Before we conclude the present discussion it is worth pointing out that gyrointeraction can furnish information, not only about the D region, but also probably about the F region. Whenever the ionization in the D region and E region is very low the extraordinary gyrowave must be very intensely absorbed in the F region, the coefficient of absorption being proportional to N/ν . In the F region N is quite large and ν quite small. It is true that in the F region the absorption of the wanted cosmic noise is small; however, we have to take into account the fact that in the F region the relaxation time of the process of cross-modulation is quite large and the effect will be the result of a long integration over time.

Conclusion

In the present Scientific Report on "Experiment Luxembourg" it has been shown how we plan to utilize the interaction technique in order to explore the lower part of the ionosphere and to obtain in a routine manner and with accuracy the distribution of ν and N versus height in the range between 50 and 75 kilometers. As a by-product we hope to be able to obtain, through gyrointeraction, some information about E region and F region as well.

The data that will be furnished by the experiment have a noteworthy significanee. We plan to study these data in connection with the problem of the apparent increase of electron collision frequency during periods of auroral activity. As it will be shown in a Scientific Report to be published in the near future, there is some indication that at high latitudes ν increases during disturbed periods. Excluding rocket experiments there are no techniques other than radio wave interaction able to tell us whether: a) ν in itself is remaining constant at any given height, but the layer in the D region that is responsible for the absorption appears at lower heights than normal during auroral disturbances; b) the layer under observation forms always at the same height, but ν increases. Of course, intermediate cases are also possible, and it will be part of the experiment to throw light on this problem.

Together with information of interest for radio propagation, we expect to obtain data that will be useful in the general field of gaseous plasmas. We recall that the technique of radio wave interaction has already been used with success on a laboratory scale by Goldstein, Anderson, and Clark (1953a and 1953b) and by Anderson and Goldstein (1955). In general we may hope that our experiment will contribute to the advancement of knowledge in the fields of the physics of gases, arctic radio wave propagation and auroral theory.

Acknowledgements.

The authors are indebted to Professor S. Chapman for his comments on the draft manuscript.

References

ANDERSON, J. M. and GOLDSTEIN, L. Interaction of electromagnetic waves of radio-frequency in isothermal plasmas; collision cross section of helium atoms and ions for electrons.	1955	Phys. Rev. 100, 1037
APPLETON SIR E. and PIGGOTT, W. R. Ionospheric absorption measurements during a sunspot cycle.	1954	JATP 5, 141
BAILEY, D. K., BATEMAN, R. and KIRBY, R. C. Radio transmission at UHF by scattering and other processes in the lower ionosphere.	1955	Proc. IRE 43, 1181
BAILEY, V. A. and MARTYN, D. F. The influence of electric waves on the ionosphere.	1934	Phil. Mag. 18, 369
BAILEY, V. A. The motion of electrons in a gas in the presence of variable electric fields and a constant magnetic field.	1937	Phil. Mag. 23, 774
BAILEY, V. A. On some effects caused in the ionosphere by electric waves. Part I.	1937	Phil. Mag. 23, 929
BAILEY, V. A., SMITH, R. A., LANDECKER, K., HIGGS, A. J. and HIBBERD, F. H. Resonance in gyrointeraction of radio waves.	1952	Nature 169, 911
BERNING, W. W. The determination of charge density in the ionosphere by radio doppler techniques.	1954	Rocket Exploration of the U. A. - Interscience Publisher Inc. N. Y. - p. 261

BRACEWELL, R. N., and BAIN, W. C. An explanation of radio propagation at 16 kc/sec in terms of two layers below E layer.	1952	JATP 2, 216
CHAPMAN, S., and LITTLE, C. G. The nondeviative absorption of high-frequency radio waves in auroral latitudes.	1957	JATP 10, 20
CUTOLO, M. Effects of gyrointeraction and their interpretation.	1950	Nature 166, 98
DUBIN, M. Meteor ionization in the E-region.	1955	Meteors - Spec. Suppl. (Vol. II) to JATP
FEJER, J. A. The interaction of pulsed radio waves in the ionosphere.	1955	JATP 7, 322
GARDNER, F. F., and PAWSEY, J. L. Study of the ionospheric D region using partial reflections.	1953	JATP 3, 321
GOLDSTEIN, L., ANDERSON, J. M., and CLARK, G. L. Interaction of microwaves propagated through a gaseous discharge plasma.	1953	Phys. Rev. 90, 151
GOLDSTEIN, L., ANDERSON, J. M., and CLARK, G. L. Quenching of afterglow in gaseous discharge plasmas by low power micro-waves.	1953	Phys. Rev. 90, 487
HUXLEY, L. G. H., and RATCLIFFE, J. A. A survey of ionospheric cross-modulation (wave interaction or Luxembourg effect).	1949	Proc. IEE 96p3, 433

HUXLEY, L. G. H. Alternative developments of the theory of radio wave interaction.	1953	Proc. Roy. Soc. 218, 520
HUXLEY, L. G. H. Comment on theory of radio-wave interaction.	1955	Proc. Roy. Soc. A 229, 405
KELSO, J. M., NEARHOOF, H. J., NERTNEY, R. J., and WAYNICK, A. H. The polarization of vertically incident long radio waves.	1951	Annales de Geophysique 7, 215
LIEN, J. R., MARCOU, R. J., ULWICK, J. C., AARONS, J., and MCMORROW, D. R. Ionosphere research with rocket-borne instruments.	1954	Rocket Exploration of the U. A. - Interscience Publisher Inc. N. Y. - p. 223
MITRA, A. P., and SHAIN, C. A. The measurement of ionospheric absorption using observations of 18.3 Mc/s cosmic radio noise.	1953	JATP 4, 204
RATCLIFFE, J. A., and SHAW, I. J. A study of the interaction of radio waves.	1948	Proc. Roy. Soc. 193, 311
REFERENCE DATA FOR RADIO ENGINEERS	1956	International Telephone and Telegraph Co. pp. 1019-1020
THE ROCKET PANEL Pressures, densities, and temperatures in the upper atmosphere.	1952	Phys. Rev. 88, 1027
SEDDON, J. C. Propagation measurements in the ionosphere with the aid of rockets.	1954	Rocket Exploration of the U. A. - Interscience Publisher Inc. N. Y. - p. 214

SEDDON, J. C. and JACKSON, J. E. Rocket arctic ionospheric measurements - Ionospheric electron densities and differential absorption.	1958	IGY Rocket Report Series No. 1 - 140, 149
SHAW, I. J. Some further investigations of ionospheric cross-modulation.	1951	Proc. Phys. Soc. B64, 1
WEBER, E.	1954	Linear Transient Analysis John Wiley & Sons, Inc. N. Y.