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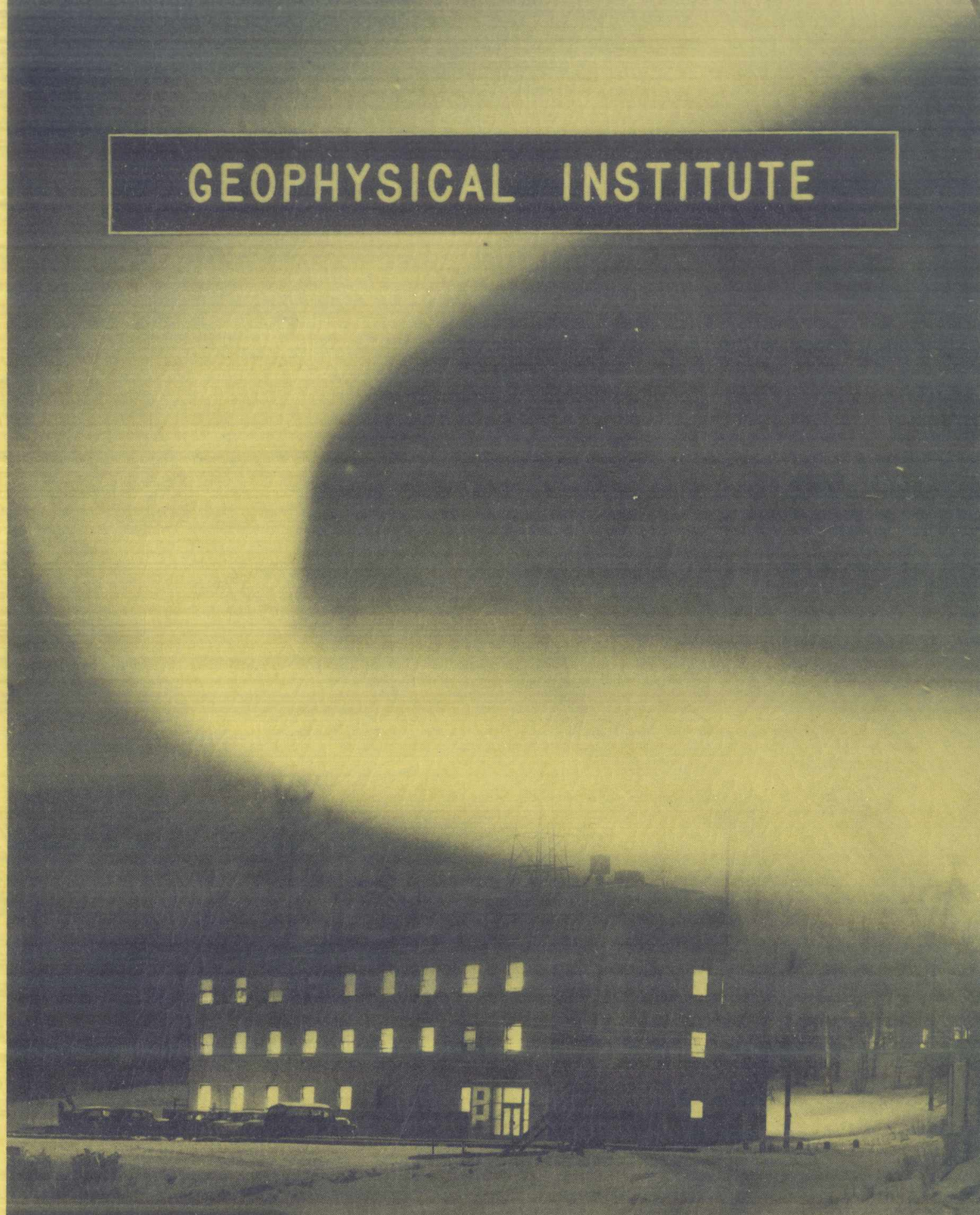
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OPTICAL EARTH SATELLITE OBSERVATIONS

Scientific Report Number 2
February 14, 1958

GEOPHYSICAL INSTITUTE

AT THE

UNIVERSITY OF ALASKA

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OPTICAL EARTH SATELLITE OBSERVATIONS

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Air Force Contract No. AF 30(635)-2887

Project No. 5535 - Task 45774

Rome Air Development Center

Griffiss Air Force Base

Rome, New York

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I. INTRODUCTION

The earlier orbits and ephemerides for the Soviet satellites were not suffic-iently accurate to be very useful in making observations in Alaska. Extrapolations from our own observations gave better predictions. This merely pointed out the fact that rough observations of meridian transits at high latitudes will give better values of the inclination of the orbit than precision observations at low latitudes. Hence, it was decided to observe visually the meridian transits estimating the altitude by noting the position with respect to the stars or using crude alidade measurements. The times of the earlier observations were observed on a watch or clock and the clock correction obtained from WWV. Later the times were determined with the aid of stop watches, taking time intervals from WWV signals.

This rather meager program of optical observations of the Soviet satellites was undertaken to give supplementary data for use of the radio observations, and partic-ularly to assist in the prediction of position of the satellite so that the 61-foot radar of Stanford Research Institute could be set accurately enough to observe it (the beam width at the half-power points is about 3°).

This report contains primarily the visual observations made at the Geophysical Institute by various members of the staff, and a series of observations by Olaf Halverson at Nome, Alaska. In addition there is a short discussion of the geometry of the trajectory, the illumination of a circumpolar satellite, and a note on the evaluation of Brouwer's moment factors.

II. VISUAL OBSERVATIONS

Visual Observations of 1957-Beta.

Two series of visual observations of meridian crossings of 1957-Beta, one, dur-
ing the morning passages of November-December 1957 and one, during the evening pas-
sages of January 1958, present an almost uninterrupted continuity. The morning pas-
sages were observed as from November 26 to December 8 with one interruption due to
overcast. Meridian crossings were timed and elevations estimated as to surrounding
stars. The second series, or evening series, present likewise an extremely satis-
factory continuity due to the exceptional conditions of visibility and to the lati-
tude of this station. Observations were begun on January 9 and continued almost with
out interruption until January 27. There has been one day of complete overcast, and
three days of broken nebulosity which prevented timing of meridian crossings.

Timing of meridian crossings was made by the stop watch and chronometer method,
generally by more than one observer, and the mean error does not exceed $\pm$ 0.1 min.
Likewise elevations were estimated by more than one observer, and their accuracy was
of the order of plus or minus one degree, topocentric, or 0.10 geocentric.

Inclination of the Orbit.

Our observation station being at a latitude of 64°   51.3' , i.e. very close to
the summit subpoint of the orbit, it was possible to obtain a fairly correct esti-
mate of the inclination of the orbit. On several occasions, during the so-called
overhead passages, the satellite was sighted north of zenith as far as bout 10°
viewed from College, and on one occasion as far as 30° north of zenith viewed from
Nome (height of satellite = 280 km). No accurate measurements were made but the error
being of the order of plus or minus one degree, as above stated, we may safely esti-
mate the topocentric angle of the summit point, from our station, as being 10° north

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assuming that the height was of the order of 600 to 700 km. The geocentric angle for this height would be of the order of one degree. We may safely state therefore, that the inclination of the orbit was 65.7°.

Light-fluctuations.

During all passages and independently of the angle of illumination, angle of sight and angle of the orbit, or the orbit's plane, the light of the satellite presented periodic fluctuations of a range of about seven stellar magnitudes. This would give a range for the actual intensity of one to a thousand. During the overhead passages the period of fluctuation was of the order of a minute, during the later passages (westward) the period seemed to be longer. Sudden flare-ups of reflected light were actually observed by more than one observer. These variations of light are composite effects depending, a) on the reflectivity of the surface-material, b) The relative angles of sun and the observer, or phase, c) the period or rotation of the satellite around its axis, d) the angle of this axis with the normal to the orbit's plane, and e) the actual shape of the satellite body, which in the case of 1957-Beta was very irregular (see Sky and Telescope, January 1958 p. 130).

Visibility.

Conditions of visibility were, as a rule exceptional; on January 26, not withstanding the moon, then at its first quarter, the satellite could be seen as a star of second magnitude, at ten degrees above the horizon, the slant-distance being of the order of two thousand kilometres.

Height Measurements.

Extinction angle was measured on certain favorable occasions and the height derived therefrom. These ranged from 500 to 700 km. These estimates were confirmed by measurements made by other agencies. Actually at the last passages observed the visibility of the satellite at great distances and at the angles observed would suggest a height of 600-700 km, for the N-S 40 parallel crossing.

Photography.

Amateur photographs were secured of the various passages mainly by the use of amateur equipment. Unfortunately these photographs are either poor in quality, or, if spectacular, are too inadequately timed to be of scientific use. Parallactic cinematography possibly by one permanent and one mobile team would have been most decisive in the determination of the orbit. This would require equipment appropriately studied both from the point of views of optics and timing, as well as personnel.

Only reliable independent observations of meridian crossings have been listed. These total 54 for 40 meridian crossings of 1957-Beta from date of launching, to January 7, 1958.

Meridian Crossings

a) Observed at the premises of the Geophysical Institute, College Alaska.

Latitude: 64°   51.3'   N.
Longitude: 147   49.8   W.

	Date	U.T. h	m	s	Year Day U.T. 1=1957 Jan 1.0	Elev.	No.Rev.*	Height km
1957-Alpha One	'57 Nov 25	01	57	42	329.08174	22°S		
	26	02	09	00	330.08958	15	16	
1957-Beta	'57 Nov 13	16	14	00	317.67638	35 S		280
	26	16	34	50	330.69083	66 S		
	28	16	27	30	332.68576	58 S		
	29	16	21	34	333.68159	52 S	14	
	30	16	16	17	334.67798	46 S	14	
	Dec 01	16	11	42	335.67479	43 S	14	
	02	16	04	30	336.66979		14	
	03	15	56	30	337.66423	38 S	14	
				32	.66429			
	04	15	48	25	338.65861	34 S	14	
	05	15	39	42	339.65256	32?S	14	
	06	15	29	38	340.64555	30 S	14	
	07	15	19	13	341.63833	28 S	14	
				20	.63847	29 S		
				23	.63854			
	'58 Jan 10	03	18	36	375.13792	88 S		
	12	01	57	30	377.08159	75 S		
		03	40	12	.15291	85 N	01	
	13	02	58	20	378.12376	80 N	14	
	14	02	15	29	379.09412	87 N	14	
				31	.09413	86 N		
				32	.09414	86 N		
				34	.09417	87 N		
	15	01	31	42	380.06368	84 N	14	480
		03	14	22	.13500	85 N	01	600
	16	02	29	42	381.10395	88 N	14	
		04	12	01	.17500	69 S	01	
				02	.17502	70 S		
				02	.17502	70 S		
	18	02	39	04	383.11048	90	28	
		04	21	46	.18181	50 S	01	
			21	55	.18187	51 S		
	20	02	44	16	385.11409	82 S	28	
				17	.11512	84 S		
		04	27	33	.18583	31 S	01	
	21	03	36	55	386.15062	47 S	14	
			37	00	.15069	45 S		

No. Rev.: Number of revolutions as from previously listed passage.

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Meridian Crossings (Cont'd.)

a) Observed at the premises of the Geophysical Institute, College, Alaska.

Latitude: 64° 51.3' N.
Longitude: 147 49.8 W.

Date	U.T. h	m	s	Year Day U.T. 1=1957 Jan 1.0	Elev.	No.Rev.*	Height km
'58 Jan 22	02	45	41	387.11506	61°S	14	
			42	.11508	64 S		
	04	29	27	.18715	23 S	01	
23	01	54	15	388.07937	81 S	13	
	03	36	20	.15021	36 S	01	
			27	.15028	38 S		
25	03	32	00	390.14722	25 S	30	
26	02	36	10	391.10847	45 S	14	
27	03	24	25	392.14194	16 S	15	
			30	.14201	15 S		

b) Observed at Nome, Alaska, -

Latitude: 64°   30' N
Longitude: 165°   19' W

	Date	h	m	Year Day U.T.	Elev.	No.Rev.*
1957 Beta	'57 Nov 11	17	45	315.73958		
	12	17	51	316.74375	48 S	14
	13	17	58	317.74861	68 S	14
	14	18	05	318.75347	90	14
	15	18	10	319.75694	76 N	14
	16	18	13	320.75908	69 N	14
	18	18	28	322.76944	60 N	28

III. GEOMETRY OF THE TRAJECTORY

Viewed from P (Fig. 1) the orbit would appear to an observer through a presumably transparent earth as an ellipse. The observer, located outside the plane of the orbit, and the orbit itself, would define an elliptical cone having its summit at P.

It is obvious that any arc of the orbit viewed from P will be an arc of ellipse, and this is the trajectory which the satellite will follow in the sky provided the passage is not a zenith passage. This arc will be symmetrical (curve III of Fig. 1) for an azimuth $A = \dfrac{A_r + A_s}{2}$ where A_r and A_s are the azimuths of the rising point and the setting point of the satellite respectively. However if a part of the trajectory is in shadow the illuminated portion will tend to appear asymmetrical giving the impression as of an arc of a tilted ellipse (curves I and II of Fig. 1).

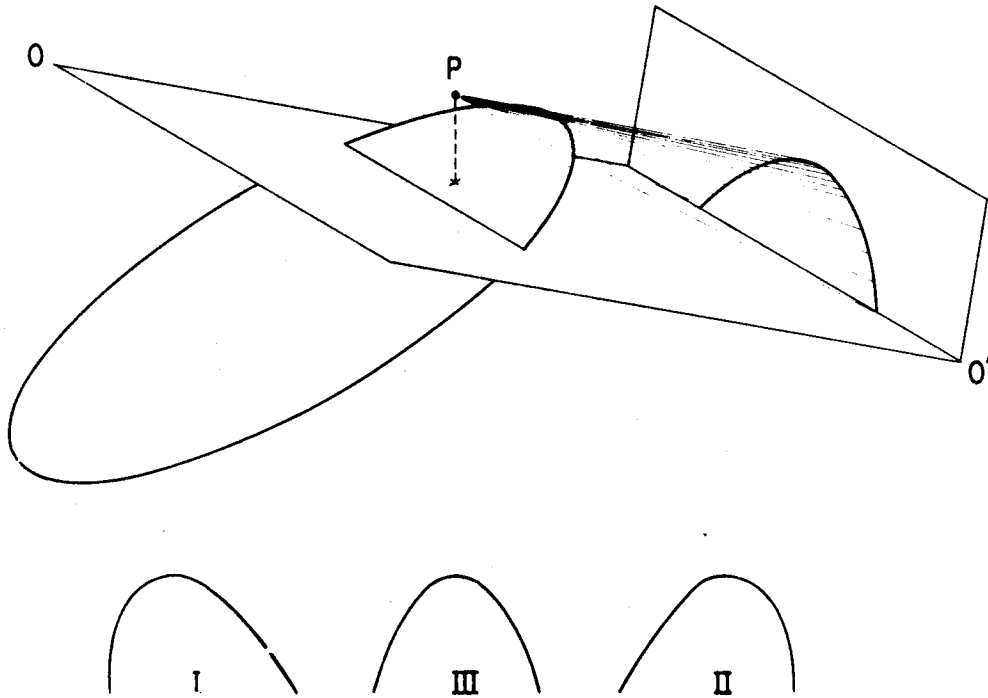For the sake of illustration the curves I, II, and III of Fig. 1 have been very much exaggerated.

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Fig. 1

Geometry of the Trajectory over the Horizon.

IV. SEASONAL ILLUMINATION OF A CIRCUMPOLAR EARTH SATELLITE AT ITS EXTREME LATITUDE ORBIT - POINT.

The illumination by the sun of an earth satellite revolving on a highly inclined orbit as is the case with the Russian earth-satellites is examined in this section.

As the orbit gradually swings around the earth with a velocity of $\frac{d\Omega}{dt}$ its summit points defines a locus L, (Fig. 2) which is a small circumpolar, Arctic or Antarctic circle; centered on the earth's axis. Verticals passing through the summit point and its sub-points define a conical surface which is a circular cone having its summit at the center of the earth. Locus L, is the basis of this cone (Figs. 1 and 2) and the intersection of this cone with the illumination cylinder 1. Cy. of the solar rays will define two arcs of this circle, the illuminated arc abc, and the arc ac plunged in the shadow. The angles of these are respectively, $2(\pi - K)$ and $2K$. We may derive K, through the trigonometrical relations rendered clear in Fig. 2. In what follows we shall consider an inclination of i = 65° and a height at the north summit point of 400 km. This is the case of 1957 Alpha 2.

We have the following relations:

$$\cos K = \frac{\cos \nu - \cos \Phi \cos \mu}{\sin \nu \sin \mu} \qquad (R + h) \sin \nu = R$$

$$\Phi = \pi - P$$
$$\mu = 90° - i = 25°$$

And the following quantities:

$\sin 25° = 0.42262 = \cos 65°$
$\cos 25° = 0.90631 = \sin 65°$

R:Earth's med. radius = 6,350 km
h = 400 km.
$\sin \nu = 0.94071$
$\cos \nu = 0.33901$
$\nu = 70° 11'$

We have:

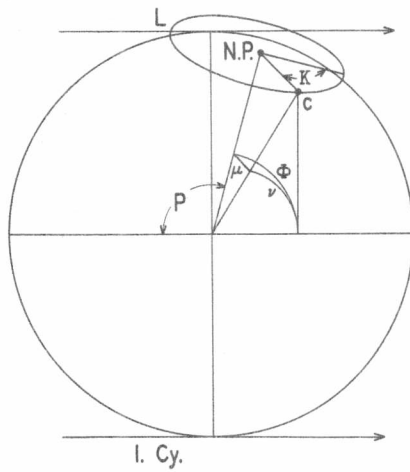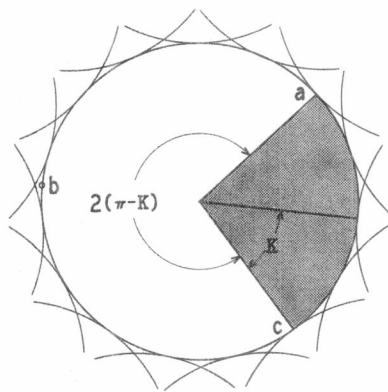$$\cos K = \frac{0.33901 + \cos P \times 0.90631}{\sin P \times 0.42262}$$

Fig. 2

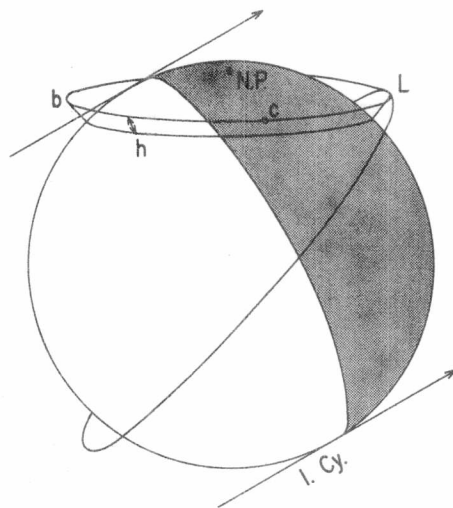Seasonal Variation of the Illumination of the Circumpolar Orbit-Summit Point.

Arcs <u>ab</u> = 2K, and <u>abc</u> = 2(π - K) are functions of the polar distance P, of the sun. Table I contains the values of these arcs corresponding to values of P.

Table I

P			K			2(π - K)	
°	'		°	'		°	'
084	50.0		000	00.0		360	00.0
090	00.0		036	39.5		286	40.0
100	00.0		064	07.5		231	45.0
110	00.0		085	48.7		188	22.6
113	26.5 winter solstice		093	10.0		173	40.0
120	00.0		108	10.7		143	38.6
125	00.0		121	31.2		116	57.6
134	49.0		360	00.0		000	00.0

these values are plotted in Fig. 3. The inner scale of the abscissae axis gives the arc 2(π -K) and the outside scale the hours, before and after solar noon, when the summit point would still be in sunlight. This signifies that when the satellite goes over that extreme northmost (for the Northern hemisphere ,southmost for the south hemisphere) point of the orbit it will catch the sun's rays even if the rest of the orbit is in shadow. Actually due to the refraction, the illuminated portion of the orbit around the top will be more than a point. For the Northern hemisphere the summit point of the orbit will be constantly sunlit as from April 5 to September 8. For the Southern hemisphere the dates will be October 8 to March 6. Refraction has not been taken into consideration.

These limit values for P change with h, the height of the summit-point. Curves h, of Fig. 3 furnish the values of 2(π - K) and the hours of insolation of the summit-point as a function of P. No explanation is required, the interpretation of the diagram being quite straightforward. Table II contains the values of P which

Relation between Hours of Insolation of Summit-Point of
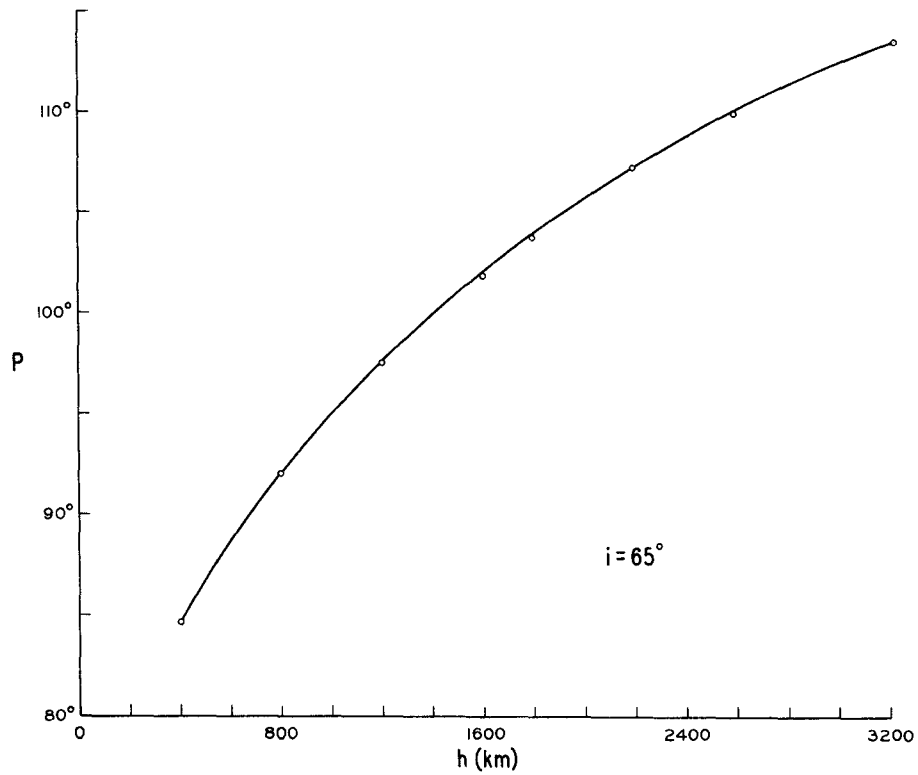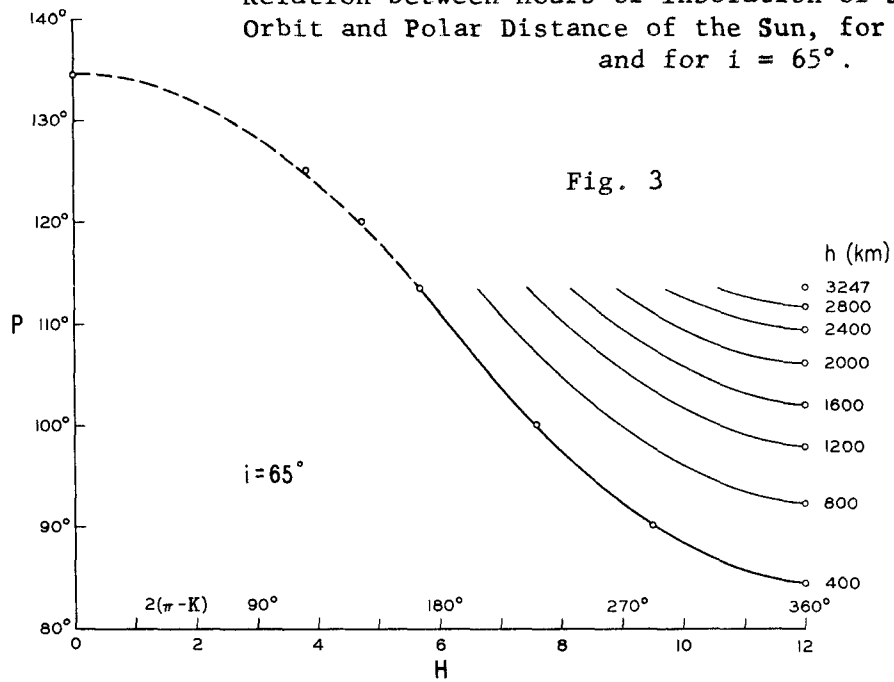Orbit and Polar Distance of the Sun, for Various Heights
and for i = 65°.

Fig. 3

Fig. 4

Relation between Height and Polar Distance of the Sun for Constant Insolation
of Summit-Point and for Orbit, i = 65°.

would be required to have continuous insolation of the summit-point, for various values of h.

Table II

h km	cos (90 - v)	90 - v		P	
		°	'	°	'
400	0.94074	019	49.0	034	49.0
800	0.88811	027	21.5	092	21.5
1,200	0.84106	032	45.0	097	45.0
1,600	0.79784	036	59.5	101	59.5
1,800	0.77914	038	49.0	103	49.0
2,200	0.74269	042	02.3	107	02.3
2,600	0.70950	044	48.5	109	48.5
3,247	0.66327	048	27.5	113	27.0

These values are plotted on Fig. 4 and we perceive that we shall have all year round insolation of the summit-point, i.e. of the entire locus L if the height of the earth satellite revolving around the earth on an orbit having an inclination of i = 65° is 3,247 km since P = 113° 27' is the polar distance of the sun at the winter solstice of the respective hemisphere.

V. TENTATIVE EVALUATION OF BROUWER'S MOMENT FACTORS.

For deriving Brouwer's moments the orbit of earth satellite 1957-Alpha Two, although perhapd too highly inclined, presents certain advantages over that of 1957-Beta. The orbit is less excentric, and the satellite itself, being spherical, was at least at the initial (immediately following the launching) stage, less susceptible to secular perturbations by drag. Furthermore, radio observations were more continuous and complete at this initial comparatively dragless stage and the positional data issued on October 18 and October 25, 1957 by Vanguard (N.R.L.) were probably the best ever made available on that particular earth satellite. The writer, although fully aware that the degree of precision in these earth satellite positions is far from approaching the orthodox standards warranting oblatenes measurements, has nevertheless tentatively tried to use these positions to derive Brouwer's moments through the mode exposed in Van Allen's Symposium "Scientific Uses of Earth Satellites."

$$\left[\frac{d\Omega}{dt} \right]_{i=0} = -5.5172 \times 10^{-8} \quad \text{rad/sec}$$

$$\left[\frac{d\bar{\omega}}{dt} \right]_{e^2=0} = +3.1997 \times 10^{-8} \quad \text{rad/sec}$$

Meaning that $d\Omega/dt$ has been reduced for an equatorial orbit, and $d\bar{\omega}/dt$ for a circular orbit. And we have:

$$\frac{d\bar{\omega}}{dt} + \frac{d\Omega}{dt} = 2.682 \times 10^{-8} \quad \text{rad/sec}$$

$$n' = 1.1015 \times 10^{-3} \quad \text{rad/sec}$$

$$n = 1.1018 \times 10^{-3} \quad \text{rad/sec}$$

$$f_o = 1.00052$$

And we obtain for Brouwer's moments:

$$\gamma_2 = 1.6 \times 10^{-3}$$

$$\gamma_4 = 8.6 \times 10^{-4}$$

If C is the moment of inertia of the earth about the axis of rotation and if A its moment of inertia about an equatorial axis, we would have for C-A/2M , where M:mass of the earth $= 5.983 \times 10^{27}$ gms.

$$\frac{C-A}{2M} = 6.509 \times 10^{14}$$

higher by a factor of 2.93 to the value 2.221×10^{14} calculated theoretically by Allen.