A northern snowmelt model
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A NORTHERN SNOWMELT MODEL

by

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LIST OF SYMBOLS

Text	Definition	Computer
A_1	Initial active layer depth	A1
A_2	Final depth of the active layer	A2
A	Depth of the active layer	A
A_s	Albedo of the snowpack	AS
C_s	Solar constant	
d_m	Equivalent water depth of snowmelt	WEQ
d_{mA1}	Equivalent water depth of snowpack	WEQA2
D_s	Snowpack depth	D
D_{si}	Initial depth of snowpack prior to melting	XI, D1
d_w	Equivalent liquid water depth of cold content	
e_1	Vapor pressure at an elevation of 1 foot above the snow surface	
e_a	Vapor pressure at level Za (mb)	E1
e_s	Vapor pressure at the snow surface	ES
E_a	Emissivity of the air	EA
E_s	Emissivity of the snow surface	EP
H	Hourangles between sunrise and sunset (degrees)	
H_c	Convective heat transfer from the air	HCONV
H_{cc}	Cold content; heat required to raise snowpack to 0°C	HC
H_d	Total heat deficit, heat required to melt entire snowpack	HD
H_e	Latent heat of vaporization, released by condensation	HCOND
H_g	Conduction of heat from underlying soil	HG
H_I	Effective latent heat of fusion for active layer of a given temperature, density, and depth	HI
H_m	Quantity of heat involved in change of state from solid to a liquid	HMELT
H_p	Heat given off by rain	HR

LIST OF SYMBOLS (Continued)

Text	Definition	Computer
H_q	Increase in energy content of the snow	HQ
H_{rl}	Net long wave radiation exchange between the snowpack and its environment	HLWR
H_{rs}	Absorbed shortwave radiation	HSWR
H_T	Heat flux applied to snowpack to produce changes in its energy content	HT
I	Shortwave radiation reaching the snow	
I_o	Amount of solar radiation reaching a horizontal unit on the surface of the atmosphere	
K''	A coefficient	
K'_e	Coefficient used in the condensation equation	
L_f	Latent heat of fusion of ice (constant)	
L_{fs}	Latent heat of fusion of snow	
M	Snowmelt in inches	XMELT
N	Cloud cover	XN
P	Passive layer	P
P_1	Initial depth of passive layer	P1
P_r	Precipitation in inches	PR
r	Relative humidity	RH
r_e	Radius vector of the earth	
R	Water in the snowpack that can potentially be frozen	R
R_v	Radius vector of earth	RV
S	Solar constant	
t	Time variable hours, days	
Δt	time period (days)	
T_a	Surface air temperature	TA
T_A	Temperature of the active layer	TA
T_{A1}	Initial temperature of the active layer	TA1
T_{A2}	Final temperature of the active layer	TA2

LIST OF SYMBOLS (continued)

Text	Definition	Computer
T_d	Dewpoint temperature	TDP
T_p	Increase in temperature of the passive layer from heat of fusion	XTP
T_{p1}	Initial passive layer temperature	TPL
T_s	Snow surface temperature	TSK
U_b	Mean wind velocity at level Z_b	V
W	Water content of the snow pack	WC
W_{A1}	Initial water content of the active layer	WCA1
W_{A2}	Final water content of the active layer	WCA2
W_c	Potential water content of the passive layer	WC
W_d	Liquid water deficiency of the snowpack	WD
W_e	Excess water that percolates to the soil (inches)	WE
W_L	Water leaving the layer in inches	WE
W_{max}	Liquid water holding capacity of the snowpack	WHC
W_{maxA1}	Liquid water holding capacity of the active layer	WHCA1
W_p	Water percolated into the passive layer	WP
W_{p1}	Initial water content of the passive layer	WCP1
X_k	Variable used in the model equation for convective heat exchange	XKC
X_p	Percolated water that freezes in the passive layer	XWP
Y	Depth of condensate (inches/unit area)	Y
Y_T	Change in temperature due to refreezing	YTA
Z	Height to base of clouds	Z
ϕ	Geographic latitude	
ρ	Snow pack density	RO
ρ_o	Atmospheric pressure at sea level (mb)	
ρ_{A1}	Initial snow density of the active layer	ROA1
ρ_{A2}	Final density of the active layer	ROA2
ρ_{p1}	Initial density of the passive layer	ROP1

LIST OF SYMBOLS (continued)

Text	Definition	Computer
ρ_{p2}	Final density of the passive layer	ROP2
ρ_s	Atmospheric pressure at snowsurface	
ρ_{s1}	Initial average snowpack density	ROA1
ρ_w	Density of liquid water at 0°C (constant)	
σ	Boltzmanns constant	S
σ_d	Declination of the sun	DEC
Θ	Thermal quality of the snowpack	TQ
Θ_{A1}	Initial thermal quality of the active layer	TQA1
Θ_{A2}	Final thermal quality of the active layer	TQA2
Θ_s	Zenith angle of the sun	

INTRODUCTION

In early 1968, a large petroleum discovery was made in the Prudhoe Bay area of Alaska's Arctic Coastal Plain. This discovery has led Alaska into a period of development of unprecedented speed and magnitude. This development will require the construction of many engineering facilities which are affected by the water resources. The design of each of these requires an understanding of the hydrologic system, a system which is dominated in Alaska by low temperatures, high latitudes, large elevation differences and sparse data. The latter factor is unique to Alaska and makes application of common design techniques virtually impossible.

Of the 250,000 stream gaging stations in the United States in 1970, less than 1% or 235 of these stations were located in Alaska. These stations (see Figure 1) cover an area that is one-sixth the total area of the United States. This sparseness is also exhibited in the number of climatological stations and snow survey courses in the state (see Figures 2 and 3). As of 1970, there were about 200 basic climatological stations recording only daily temperature extremes and precipitation. Of these, only half make aviation-related observations which include cloud cover, cloud base height, visibility, wind and pressure. Most of the climatological stations are located along the coastline and at lower elevations along the major rivers and their tributaries. There were only 101 snow survey stations, the majority of which lie in the lower two-thirds of the state.

Because of Alaska's severe cold climate, the surface hydrology for most of the state is only active for four to six months. As a result, the spring snow and ice breakup dominate the cycle and the consequent snowmelt runoff is the major contributor to stream flow and an important source of floods. This fact, when coupled with the sparseness of hydrologic data, suggests some type of modeling activity be undertaken with the intent of increasing our knowledge of the spring breakup. This report describes that portion of the study which attempts to model the snowmelt process.

Lacking empirical knowledge relating air temperatures to snowmelt, we have described the snowmelt process by using a rather complete model with a strong physical basis. Our final model considers heat transfer between the air and the snowpack surface and heat and mass transfer within the snowpack itself. In the model we have used as few empirical parameters as possible in order to allow for the use of the model in other areas and to allow full magnitude for improvement of the model.

SNOWMELT PHYSICS

To understand the snowmelt process and the concepts upon which we have based the model, it is important to look at the physics of snowmelt. A report by the US Army Corps of Engineers (Rockwood, 1956) presents a detailed discussion of the snowmelt process. The discussion in this section closely follows Rockwood's account.

During the winter season, the surface of the snowpack is subjected to the effects of radiation, rain, turbulent heat exchange and ground heat exchange.

According to Wilson (1941), the inflow of heat to the snow (positive heat flux) and the losses from it (negative heat flux) can be classified as follows:

heat inflow:

1. incoming shortwave and longwave radiation
2. conduction and convection exchange from the air
3. condensation
4. conduction from soil
5. rainwater
6. air

heat losses:

1. outgoing longwave radiation
2. evaporation and/or sublimation
3. conduction to soil
4. conduction and convection exchange to the air
5. reflected shortwave radiation

From the above, the following equation which expresses conservation of energy can be written

$$H_{rs} + H_{rl} + H_c + H_e + H_p + H_g - H_q - H_m = 0 \tag{1}$$

where

H_{rs} = absorbed shortwave radiation

H_{r1} = net longwave radiation exchange between the snowpack and its environment

H_c = convective heat transfer from the air above

H_e = latent heat of vapor released by condensation

H_p = heat given off by rain

H_g = conduction of heat from underlying soil

H_q = increase in energy content of the snow

H_m = quantity of heat involved in change of state from solid to liquid

If the sum of H_q and H_m is designated by H_T, it follows that the heat flux applied to the snowpack to produce changes in its energy content as well as state, can be expressed by the relation

$$H_T = H_{rs} + H_{r1} = H_c + H_e + H_p = H_g \tag{2}$$

A detailed discussion of each of the above components is presented later in the report.

Snowmelt is defined as the liquid water which can leave the snowpack. Both the cold content and the thermal quality of the snowpack determine the quantity of heat energy available to produce snowmelt.

The latent heat of fusion is 80 cal/gram or 144 BTU/lb for pure ice at 0°C. However, a snowpack under natural conditions is a mixture of liquid water, ice, and air often at a temperature less than 0°C. The heat required to raise the snowpack to 0°C is called the "cold content" H_{cc} If the heat capacity of the entrapped air is neglected then

$$H_{cc} = \rho_s C_s D_s T_s \qquad (cal/cm^2) \tag{3}$$

where

ρ_s = average snowpack density (g/cm³)
C_s = snowpack specific heat (cal/g-°C)
D_s = snowpack depth (cm)
T_s = average snowpack temperature (°C)

It is convenient to express H_{cc} in terms of an equivalent depth of water, d_w, produced by either rain or melt which would raise the temperature to 0°C upon refreezing within the snowpack. This may be written as

$$H_{cc} = L_f \rho_w d_w \qquad (cal/cm^2) \tag{4}$$

where

L_f = latent heat of fusion of ice = 80 cal/g
ρ_w = density of liquid water at 0°C = 1 g/cm³
d_w = water depth equivalent of cold content (cm)

Combining (3) and (4) and setting $C_s = 0.5\ cal/g°C$ gives

$$d_w = \frac{\rho_s C_s D_s T_s}{\rho_w L_f} = \frac{\rho_s D_s T_s}{160} \tag{5}$$

Thermal quality, a measure of the thermal state of a snowpack, determines the amount of melt which results from an input of heat energy. It is defined by Eagleson (1970) as the ratio of the amount of heat required to produce a given amount of water from snow, to the amount of heat required to produce the same quantity of melt from pure ice at 0°C. The cold content and thermal quality of a snowpack are related through the total heat deficit, H_d. This is the heat required to melt the entire snowpack. It is given by

$$H_d = \rho_s D_s L_{fs} + H_{cc} \tag{6}$$

where L_{fs} is the latent heat of fusion of snow.

The heat necessary to produce d_m cm of melt from pure ice is $\rho_w d_m L_f$. Thus, since $d_m = (\rho_s/\rho_w) d_s$,

$$\Theta = \frac{H_d}{\rho_w d_m L_f} = \frac{L_{fs}}{L_f} + \frac{C_s T_s}{L_f} \tag{7}$$

For snow below 0°C, $L_{fs} = L_f$ and the thermal quality is simply related to the snow temperature by the expression

$$\Theta = 1 - \frac{T_s}{160} \tag{8}$$

When melting, the snowpack is usually isothermal at 0°C and contains free water. When the ice matrix is melted less water is produced than would be expected from melting pure ice particles, since free water is also released. Under this situation $L_{fs} < L_f$ and the relationship for the thermal quality is

$$\Theta = 1 - W \tag{9}$$

where W is the water content of the snowpack expressed as a decimal percentage by weight.

If H_T represents the total heat supplied to the snowpack per unit time and area (cal/cm^2/hr) and since 80 cal/cm^2 are required to melt a 1 cm depth of water from pure ice at 0°C, a general expression for the melt of any snowpack is

$$M = \frac{H_T}{80\Theta} \tag{10}$$

If the melt is expressed in inches per hour (80 x 2.54 = 203), then

$$M = \frac{H_T}{203\Theta} \tag{11}$$

To this point we have discussed the heat transfer between the snowpack and its environment and the amount of melt that can be expected from a given heat input. The heat transfer and the movement of melt water within the snowpack will now be examined.

During the winter and sometimes during clear nights in the melting period, the snowpack temperature is below 0°C. In this state no liquid water can be present. Any heat supplied to the snowpack will first raise the snow surface temperature to 0°C. The surplus heat will then produce melt, and liquid water will begin to accumulate within the snow matrix. Liquid water may exist in one of several forms (Eagleson, 1970)

1. *Hygroscopic* water, which consists of a thin film of absorbed water on the snow crystals and cannot appear as runoff until the crystal melts;

2. *Capillary* water, which is held and moved by surface forces in the pores of the snowpack and cannot become runoff until the capillaries increase in size and/or the snow melts;

3. *Gravitational* water, which is draining through the the pack under the action of gravity.

It is now necessary to consider both the actual liquid water content, W, of the snowpack and the liquid water-holding capacity, W_{max}, (maximum hygroscopic and capillary water) of the snowpack.

The liquid water content is defined as the ratio, by weight, of liquid water within the snowpack. For a snowpack at 0°C, $H_{cc} = 0$ and from (7)

$$W = 1 - \Theta \qquad T_s = 0°C \tag{12}$$

The water-holding capacity of a snowpack depends upon the snow density and depth, the distribution of ice layers, the degree of channelization of the snow matrix, and the size, shape and spacing of the snow crystals. This dependence is complex and may vary throughout the season; however, a large part of this variable may be accounted for by an empirical correlation with snowpack density (ρ). For spring snow with densities varying from 0.35 to 0.46, Gerdel (1954) obtained values of W_{max} ranging from -02 to -05. Amorocho and Espildora (1966) have adopted the following relationships:

$$W_{max} = 0.025\rho + .03 \qquad \rho \leq .4 \tag{13}$$

$$W_{max} = 0.20\rho - .04 \qquad .4 < \rho \leq .55 \tag{14}$$

$$W_{max} = 0.111\rho = 0.131 \qquad .55 < \rho \leq 0.9 \tag{15}$$

The difference between the water-holding capacity and the water content represents a certain liquid water storage capacity called liquid water deficiency, W_d, expressed per unit water equivalent. When the liquid water content exceeds the liquid water-holding capacity, gravitational water is present. This water will percolate downward according to the drainage condition of the pack. If the inner layers of the pack are below freezing, some or all of this water will refreeze, liberating its heat of fusion which will in turn raise the temperature of the inner layers.

The cumulative supply of heat to the snow will increase the temperatures of all layers to the melting point. At this point the snowpack is considered "ripe". A ripe snowpack is defined by Rockwood (1951) as "one which is isothermal at 0°C and has all of its liquid water-holding capacity satisfied." The density of a snowpack will increase during the ripening and then remain fairly constant during the melting period.

Additional energy to the ripe snowpack causes melt water to appear as runoff. If the snowpack loses energy, however, it will refreeze, causing a heat deficiency that must be satisfied before further melting can occur.

THE SNOWMELT MODEL

The model described in this section is based on the actual physical processes described in the previous section. The model, which is based on present knowledge, practical hydrology considerations and the availability of data, is closely patterned after that of Amorocho and Espildora (1966) with minor modifications. In their model, they consider both snow accumulation and melting. As our interest was only in the melting processes, that portion which considers snow accumulation has been deleted. For further simplification, we have not considered the effects of precipitation during the melting period.

MODEL GEOMETRY

In order to account for the heterogeneity and layering of the snowpack, two different snow layers are considered in the model. First, it is proper to single out a surface or exchange layer, which is assumed to be directly affected by the heat transfer between the atmosphere and the snowpack. This exchange layer is called the "active layer." Its depth is designated by A.

Whenever the snowpack depth, D, is equal to or greater than a certain depth, the model assumes a constant depth of the active layer. On the basis of studies on the penetration of solar radiation into the snowpack by the U.S. Army Corps of Engineers, and on data regarding the usual range of depths of nocturnal crusts, the maximum active layer depth is taken as 10 inches. Whenever the snowpack depth is smaller than 10 inches, the entire snowpack is considered the active layer.

Any snowdepth in excess of the depth of the active layer is called the passive layer, P. This passive layer is assumed to be influenced only by water that percolates from the active layer. The heat exchange between snowpack and ground is assumed to be negligible.

A general block diagram of the model is shown in Figure 4. Boxes representing a decision, calculation, or results of a process are numbered for reference in describing the general operation of the model.

The model is designed to use meteorological data and initial snowpack conditions as input information (Box 1). The snowmelt process is considered for specific time intervals at the end of which the final conditions of the snowpack are calculated and introduced as initial conditions for the continuation of the computational process. Time intervals of one or two hours are used.

In the time interval under consideration, the model computes the heat flux between the atmosphere and the active layer (Box 2). Once the heat flux received (positive) or emitted (negative) by the active layer has been computed, the melting processes of the active layer are considered. The computation of potential parameters is the performed (Box 3). The parameters computed for each layer are density, depth, water equivalent, water content, temperature, thermal quality, and water-holding capacity.

The model then compares the potential water content of the active layer with the liquid water-holding capacity (Box 4). If the water-holding capacity is surpassed, the excess water percolates downward into the passive layer and is considered as a heat input to that layer (Box 5).

If the passive layer is at 0°C, water that has percolated downward from the active layer is compared with the water-holding capacity of the passive layer (Box 6). If the water-holding capacity is exceeded, (Box 7) water percolates through the passive layer and appears as a net input to the watershed (Box 8).

If the passive layer temperature is below 0°C, some or all of the percolated water from the active layer refreezes, liberating its latent heat of fusion and thus increasing the passive layer temperature (Box 9).

The ablation process is also simulated in the model (Box 10). It is assumed that ablation (decrease in snowdepth) occurs only when there is melting in the active layer. If ablation does occur, it is calculated (Box 11) and a redistribution of the layers is made in accordance with the computed decrease in snowpack depth (Box 12).

These new parameters represent the final conditions for the time interval. If there is no ablation, the parameters computed in the melting process (after considering the water-holding capacity of the active layer in Box 4) represent the final conditions of the interval.

The cycle is then repeated with the final parameters taken as initial conditions for the next time interval.

INITIAL CONDITIONS AND METEOROLOGICAL INPUTS

The measured meteorological data considered as input to the model are: surface air temperature, T_a; dewpoint temperature, T_d; relative humidity, r; cloud cover, N; height to the base of clouds, Z; and precipitation, P_r. These data are read into the model on a daily basis. The declination of the sun, σ_d, and the radius vector of the earth, R_v; are also read in each day.

The snow parameters representinginitial conditions of the snowpack are: depth, D_s; density, ρ; water equivalent, d_m; water content, W; temperature, T_s; thermal quality, Θ; and water-holding capacity, W_{max}. If the initial depth of the snowpack is sufficient to permit a subdivision into layers, the above parameters should be given for each of the layers present.

The initial conditions of the snowpack can be assumed or may be obtained from actual field measurements.

THE HEAT BUDGET PROCESSES

As previously discussed in the section on snowmelt physics, the total energy exchange between the snowpack and its environment may be expressed as

$$H_T = H_{rs} + H_{rl} + H_c + H_e + H_\rho + H_g \tag{16}$$

Neglecting the heat input from precipitation and the energy exchange between the snowpack and the ground surface, the above equation simplifies to

$$H_t = H_{rs} + H_{rl} + H_c + H_e \tag{17}$$

A discussion of each component in (17) follows. A more complete presentation may be found in Eagleson (1970) pp 252 to 259.

H_{rs}, *Shortwave Radiation Exchange*

Shortwave radiation is that radiation from the sun which reaches the surface of the snowpack. The empirical equation for computing the amount of solar radiation (I_o) reaching a unit horizontal area at the top of the atmosphere over a period of time, t, is given by

$$\frac{d\,I_o}{dt} = \frac{s}{r_e^{\,2}} \cos \Theta_s \tag{18}$$

where

Θ_s = the zenith angle of the sun (degrees)

s = the colar constant (1.94 cal/cm^2 min);

r_e = the radius vector of the earth (distance from the center of the earth to the center of the sun expressed in terms of the length of the semimajor axis of the earth's orbit)

t = time (hr)

To determine the extraterrestrial solar radiation, (18) is integrated during the period of sunrise to sunset. The zenith distance to the sun may be defined by

$$\cos\Theta_s = \sin\phi \,\sin\delta + \cos\phi \,\cos\delta_d \,\cos H \tag{19}$$

where

ϕ = geographic latitude (degrees)

δ_d = declination of the sun (degrees)

H = hour angles between sunrise and sunset (degrees)

Substitution of the above equation in (19) gives

$$\int dI_o = \frac{s}{r_e^{\,2}} \int (\sin\phi \,\sin\delta + \cos\phi \,\cos\delta \,\cos H)dt \tag{20}$$

The portion of incident solar radiation actually reaching the snow surface can be expressed by

$$I = (1 - K'\,N)I_o \tag{21}$$

where

I = shortwave radiation reaching the snowpack

N = portion of sky covered by clouds (tenths)

K' = empirically derived coefficient given by equation (22)

$$K' = 0.82 - .024Z \qquad (22)$$

where

Z = height to the base of clouds in thousands of feet

A portion of the shortwave radiation that reaches the snow surface is reflected. The amount that is absorbed is

$$H_{rs} = I(1 - A_s) \qquad (23)$$

where A_s is the albedo of the snowpack. From studies by Cubley and others (1971) the albedo of the snowpack may be approximated by

$$A_s = .38 + \frac{.46\, D_s}{D_{si}} \qquad (24)$$

where

D_s = depth of the snowpack (in)

D_{si} = initial depth of the snowpack prior to melting (in)

Combining (20) and (21) into the form used in the model, the energy input into the snowpack by shortwave radiation is

$$H_{rs} = I_o \left(1 - \frac{K'N}{10}\right)\left(1 - \frac{A_s}{100}\right) \qquad (25)$$

where

I_o = possible shortwave radiation at the top of the atmosphere given by (20) cal/cm^2/day

K' = coefficient derived in (22)

N = portion of sky covered by clouds

A_s = albedo of snow surface as defined by (24)

H_{rl}, *Longwave Radiation Exchange*

Longwave radiation is the radiation emitted from objects. In our model, longwave emitters of importance are the snowpack and the clouds.

Eagleson (1970) gives the empirical relationship for the net longwave radiation exchange between snowpack and clouds as

$$H_{rl} = (1 - K'N)(E_a \sigma T_a^4 - E_s \sigma T_s^4)(cal/cm^2/min) \qquad (26)$$

where

N = portion of sky covered by clouds (tenths)

σ = Boltzmans constant = 0.826 x 10^{-10} cal/cm^2/min/°K

T_a = air temperature (°K)

T_s = snow surface temperature (°K)

E_a = emmissivity of air

E_s = emissivity of the snow surface

K = empirically derived coefficient given by (27)

$$K = 1 - 0.024Z \qquad (27)$$

where

Z = height to the base of clouds in thousands of feet

The emissivity of the snow surface, E_s, is usually assumed to be one and the emissivity of the air, E_a, may be estimated by

$$E_a = .74 + .0125\ N + .0049\ e_1 \qquad (28)$$

where

N = portion of skies covered by clouds (tenths)

e_1 = is the vapor pressure (mb) at an elevation of one foot above the snow surface

H_e, Heat of Condensation of Sublimation

The depth of condensate that will result with increased vapor pressure above a snowpack can be estimated by the diffusion method presented in Eagleson (1970). The moisture transfer per unit area is given by the equation.

$$Y = K_e (Z_a\ Z_b)^{-1/6} (e_a - e_s) U_b\ \Delta t \qquad (29)$$

where

Y = depth of condensate (inches/unit area)

e_a = vapor pressure at level z_a (mb)

e_s = vapor pressure at snow surface (mb)

U_b = mean wind velocity at level z_b (mph)

Δt = time period (days)

With Δt in days, the coefficient K_e has been estimated by the Central Sierra Snow Laboratory to be

$$K_e = 0.00635\ (in\ ft^{1/3}\ hr/(day\ mb\ mile) \qquad (30)$$

Since the latent heat of condensation is 600 cal/gm or 600 cal/cm^3, 1 inch of condensate will release 1542 cal/cm^2. We then have

$$H_e = 1524Y \qquad (cal/cm^2) \tag{31}$$

H_c, Convective Heat Exchange

Convective (or sensible) heat is that heat which is transferred directly from the air to the snowpack or vice versa depending on the temperature gradient. The empirical equation for the convective heat exchange as presented in Eagleson (1970) is

$$H_c = K_c \frac{\rho s}{\rho o} (Z_a \; Z_b)^{-1/6} (T_a - T_s) U_b \; \Delta t \qquad (cal/cm^2) \tag{32}$$

where

ρ_s = atmospheric pressure at snow surface (mb)
ρ_o = atmospheric pressure at sea level (mb)
T_a = air temperature at level Z_a (°F)
T_s = snow surface temperature (°F)
U_b = mean wind velocity at level Z_b (mph)
Δt = time period (days)

With Δt days, the coefficient K_c has been estimated by the Central Sierra Snow Laboratory to be

$$K_c = 1.28 \;\; (cal/cm^2 \; ft^{1/3} hr/(day \;\; °F \; mile) \tag{33}$$

Rewriting (32) into the form used in the model, the convective heat transfer is

$$H_c = X_k \ (T_a - T_s) \tag{34}$$

where

T_a = air temperature (°F)
T_s = snow surface temperature (°F)
V = wind velocity (mph)

$$X_k = K_c \ \rho_s/\rho_o (Z_a \ Z_b)^{-1/6} \tag{35}$$

The value of X_k varies with the height at which vapor pressures and wind velocities are measured, and the ratio of ground level pressure to sea level pressure. As an initial estimate, ρ_s/ρ_o was assumed equal to one since the model was applied to areas relatively close to sea level. The range of values for X_k are from .64 to 1.28. The value chosen for the model was 0.808 cal/cm^2 ft$^{1/3}$ hr/(day °F mile).

MELTING AND REFREEZING PROCESSES

The name "melting processes" is given to the process of melting of the snow layers, change in heat content, refreezing of liquid water, and the downward percolation of water when the liquid water-holding capacity of the snow is surpassed. Some of these processes can occur independently or in combination, depending on the relative characteristics of each one of the snow layers.

The melting processes in the active layer will be analyzed separately from the melting processes in passive layers. In each layer, several cases can occur.

MELTING PROCESSES IN THE ACTIVE LAYER

In describing these processes, reference to Figure 5 will be useful. The symbols used in Figure 5 as well as those appearing in the foregoing equations are explained in the List of Symbols.

If the initial thermal quality of the active layer is less than, or equal to 100%, the water content is positive or equal to zero, respectively, and the active layer temperature is initially at 32°F.

Under these initial conditions the heat supplied to the snowpack, H_T, will produce immediately an amount of melt, M, (Box 2) given by the equation

$$M = \frac{H_T}{2.03\ \Theta_{A1}} \qquad \textit{(inches)} \tag{36}$$

where Θ_{A1} is the initial thermal quality in percent.

After the amount of melt is calculated, it is necessary to test for the presence of a passive layer (Box 3). This is done by comparing the initial snowpack depth D_{si} with 10 inches which is the selected maximum depth of the active layer.

Assuming that initially there is only an active layer ($D_{s1} \leq 10\ in$), the amount of melt must be compared with the water equivalent in order to test whether the water melted is sufficient to produce complete ablation of the snowpack (Box 4).

If the snow melted is less than the water equivalent, there is not complete ablation of the active layer and a potential water content can be computed by the following relation

$$W = W_{A1} + \frac{M}{\rho_{A1}\ A_1} \qquad \% \tag{37}$$

where M is given in inches of water and water contents are given in inches of water per unit water equivalent.

If this potential water content is greater than the initial water-holding capacity, W_{max1}, the actual and final water content of the active layer will not exceed the water-holding capacity (Box 8), which is given also per unit water equivalent. Hence

$$W_2 = W_{maxA2} \tag{38}$$

The water in excess of W_{max1} percolates downward and can be computed by the expression

$$W_e = (W - W_{maxA1})\, \rho_{A1}\, A_1 \tag{39}$$

where W_e is the excess water in inches that percolates to the soil (no passive layers in this case).

If the potential water content is less than, or equal to, the water-holding capacity of the active layer, the final water content equals the potential water content and no water percolates (Box 7). Therefore

$$W_2 = W \tag{40}$$

$$W_e = 0 \tag{41}$$

In both cases, for any value of W, the final temperature and thermal quality are given by the following equations (Box 9)

$$T_{a2} = 32 \tag{42}$$

$$\Theta_2 = (1 - W_{A2})\, 100 \tag{43}$$

The final density is given by the following expression (Box 10)

$$\rho_2 = \frac{\rho_1 \cdot A_1 - W_e}{A_1 - \frac{M}{\rho_1}} \tag{44}$$

The final water-holding capacity is given as a function of the final density (Box 10).

The active layer depth changes according to the amount of ablation, and the following equations can be written (Box 11, Box 12)

$$A_2 = A_1 - \frac{M}{\rho_{A1}} \tag{45}$$

$$D_2 = D_1 - \frac{M}{\rho_{A1}} \tag{46}$$

$$d_{ma2} = \rho_{A2} + A_2 \tag{47}$$

Since the analysis refers to the case in which there is only one active layer, the calculation of the above parameters completes the melting process for the period under consideration. The next period can now be considered (Box 13).

If the melt that H_T can produce is greater than or equal to the initial water equivalent, complete ablation of the active layer occurs. Because there is no passive layer, the snowpack disappears during the interval considered. The water reaching the soil is given by the expression (Box 14, 15)

$$W_e = d_{ma1} \tag{48}$$

If the snowpack contains a passive layer (D_1 is greater than 10), the amount of melt computed (Box 2) must also be compared with the water equivalent of the active layer (Box 16).

When the total possible melt is greater than or equal to the water equivalent of the active layer, the actual melt is assumed to be equal to d_{m1} (Box 17) and a complete ablation of the active layer occurs. This possibility is very remote under normal conditions. In any event, if complete ablation of the active layer occurs, the snowpack depth decreases by 10 inches (Box 18) and the amount of water that percolates to the passive layer is equal to the water equivalent of the initial active layer (Box 18). Under this situation a special redistribution of the snowpack layers takes place. This process will be discussed later.

If the snowpack previously contained a passive layer and the potential melt (Box 2) is less than the active layer water equivalent (Box 16), a potential water content can be computed (Box 20) as before. The rest of the computations (Box 21 to 28) of the active layer parameters are similar to those described (Boxes 6 through 13). The only difference is, since there is a lower passive layer, the water in excess of the water-holding capacity of the active layer (W_{maxA1}) is called percolated water (W_p) and its volume is expressed in inches per unit water equivalent, rather than excess water (W_e). After these computations, the next step is the computation of the melting process in the passive layer (Box 28).

When the initial active layer thermal quality (Θ_{A2}) is greater than 100 per cent, the initial active layer has no liquid water and its temperature is below 32°F. The amount of heat necessary to raise the active layer temperature to 32°F in one hour can be expressed as an effective latent heat of fusion (Box 29)

$$H_I = 2.54 \cdot \rho_{A1} \cdot A_1 \cdot C_s (32 - T_A) 5/9 \quad (cal/cm^2) \tag{49}$$

where c represents the specific heat of the snow (taken to be 0.5).

If the actual heat supplied to the active layer is greater than H_I (Box 30), the active layer temperature will raise to the melting point. The surplus heat (H_T - H_I), will then produce a certain amount of melt water as expressed by the equation (Box 31)

$$M = \frac{H_T - H_I}{203} \qquad (in) \tag{50}$$

The rest of the processes and computations are the same as if the thermal quality of the active layer were 100 per cent or less. Therefore, as shown in Figure 2, control is transferred to (Box 3).

If H_T is less than or equal to H_I, the heat supplied will not be capable of producing melt in the active layer, and the only change will be in its heat content. The active layer temperature will increase to a value given by (Box 32)

$$T_{A2} = T_{A1} + \frac{1 \cdot 8 \cdot H_T}{2.54 \cdot \rho_{A1} \cdot A_1 \cdot C_s} \tag{51}$$

Its thermal quality can be computed as follows (Box 33)

$$\Theta_{A2} = 100 \quad 1 - \frac{(T_{A2} - 32)(5/9)}{160} \tag{52}$$

The other final parameters of the active layer do not change with respect to the initial conditions. The rest of the computations are the same as those following (Box 24).

Null heat budget: In this case, it is assumed that no changes occur in the snowpack parameters. Therefore, the initial parameters of the active layer are transferred unchanged to the next time interval.

Negative heat budget: Reference is made to flow diagram, Figure 6. If initial thermal quality of the active layer is less than 100 per cent, there is liquid water in the active layer matrix. Since there is an outgoing flux of heat, some or all of the water can be frozen. The water that potentially can be frozen is estimated by the following equation (Box 2)

$$R = -\frac{H_T}{203} \qquad \textit{inches} \tag{53}$$

If R (when expressed per unit water equivalent) is less than or equal to the actual water content of the active layer, the final water content will be given by the equation (Box 4)

$$W_{A2}\ W_{A1} - \frac{R}{\rho_{A1} \cdot A_1} \tag{54}$$

The final temperature will remain at 32°F (Box 5) but the thermal quality will decrease according to the value of the new water content (Box 6)

$$\Theta_{A2} = (1 - W_{A2})100 \tag{55}$$

The final depth, density, water equivalent and water-holding capacity of the active layer are computed by equations that appear in (Boxes 7 and 8).

If R (when expressed per unit water equivalent) is greater than the initial water content, the final water content will be zero (Box 9) and a change in heat content occurs. In refreezing, water releases its heat of fusion H_I (Box 10) and the final change in temperature (Box 11) can be expressed by the following equation:

$$Y_T = \frac{-H_T\ -H_I}{2.54\ C_s\ \rho_{A1}\ A_{A1}} \qquad (°C) \tag{56}$$

Hence, the final temperature and thermal quality of the active layer are given by (Box 12)

$$T_{A2} = T_{A1} - 1.8Y \tag{57}$$

$$\Theta_{A2} = 100 \quad 1 - \frac{(T_{A2} - 32)(5/9)}{160} \tag{58}$$

Other final parameters of the active layer are computed by the equations in (Boxes 7 and 8).

If, on the other hand, the initial thermal quality of the active layer is equal to or greater than 100 per cent, there is no liquid water present in the active layer and the temperature is equal to or less than 32°F.

Therefore, the "melting process" consists in this case in a change in the heat content of the active layer. The negative (outward) heat flux produces a decrease in temperature. This decrement is expressed by the following equation (Box 13)

$$Y = \frac{H_T}{2.54\ C_s\ \rho_{A1}\ A_{A1}} \qquad (^\circ C) \tag{59}$$

The final temperature and thermal quality are given by equations in (Boxes 14 and 15).

$$T_{A2} = T_{A1} - 1.8Y \qquad (^\circ C) \tag{60}$$

$$\Theta_{A2} = 100 \quad 1 - \frac{(T_{A2} - 32)(5/9)}{160} \tag{61}$$

The other parameters are calculated as shown in (Boxes 7 and 8).

In all cases the model proceeds to the next time interval.

The melting process in the passive layer: Two possibilities can occur, depending on the value of the thermal quality of the passive layer. Reference is made to Figure 7.

If the thermal quality of the passive layer is greater than 100 per cent, all or some of the water that may percolate from the active layer may refreeze when entering the passive layer. Consequently, its temperature is raised due to the heat of fusion. Therefore, the temperature of the passive layer may increase to a value T_P given by (Box 2, 3)

$$T_P = T_{P1} + 1.8 \quad \frac{203 \cdot W_P}{1.27} \tag{62}$$

The snow temperature cannot be higher than 32°F. Hence, in order to estimate the percolated water that is actually frozen, T_P has to be compared with 32 (Box 4), giving rise to the following alternatives:

(1) If T_P is less than or equal to 32°F, all the water percolated from the active layer can freeze and the final temperature is equal to T_P (Box 5). The final thermal quality is computed as a function of this temperature (Box 6). For this case, the final water content will remain zero and therefore no water can percolate downward (Box 7). Computation of the other final parameters will be explained later.

(2) If the computed potential temperature (T_P) is greater than 32°F, then the actual final temperature is limited to 32°F, and only the necessary percolated water will have frozen to produce the melting point in the passive layer. Therefore the actual percolated water that freezes, X_P, can be computed by (Box 9)

$$X_P = \frac{2.54 \; \rho_{P1} \; C_s P_1 \; (32 - T_{P1}) 5/9}{203} \tag{63}$$

A potential water content, W_C, is computed taking into account water percolated from above, W_P, and the amount of water frozen, X_P, by the expression

$$W_e = W_P + \frac{X_P}{P_1 \rho_{P1}} \tag{64}$$

Comparing W_C with the actual water-holding capacity of the layer (Box 10), the final water content, thermal quality and water that may be percolated can be computed as shown in (Boxes 11, 12, 13 and 14).

The depth of the passive layer is maintained (Box 15) and its final density can be computed considering the variation in water content, if any. The following expression gives this density (Box 16)

$$\rho_{P2} = \rho_{P1} - \frac{W_L}{P_1} + W_P \rho_{P1} \tag{65}$$

where W_e is the water leaving the layer in inches and W_P is the water entering the layer from above, expressed per unit water equivalent. The final water equivalent and water-holding capacity are computed as shown in (Box 17).

If the thermal quality of the passive layer is less than or equal to 100 per cent, the layer is already at 32°F, and the water that may percolate from above does not freeze. Hence, this water increases the water content of the passive layer. The potential water content, due to this process is (Box 18)

$$W_C = W_{P1} + W_P \tag{66}$$

As explained in the previous case, this water content, W_C, must be tested against the water-holding capacity of the layer before computing the other final parameters. The next sequence of computations is identical to those depicted following (Box 10).

The computed percolated water, if any, that has been calculated in the above processes enters the soil as a net watershed input.

The model then tests for the occurrence of ablation during the melting process of the active layer. This part is explained in detail in the next section.

REDISTRIBUTION PROCESS

Two possibilities for redistribution can occur, depending on the depth of the snowpack:

(1) The depth of the snowpack is greater than 10 inches, *i.e.* a passive layer exists.

In this case, the active layer depth is set to 10 inches and the passive layer depth is the depth of the snowpack less 10 inches.

The water equivalent of the active layer becomes the water equivalent of old active layer + the water equivalent of that portion of the passive layer used to increase the active layer depth to 10 inches.

The density of the active layer after redistribution is simply its new water equivalent divided by 10.

The water-holding capacity and water content are the initial values increased by the water-holding capacity and water content of that portion of the passive layer that is used to increase the depth of the active layer to 10 inches.

The temperature and thermal quality of the active layer are unchanged.

The passive layer depth is decreased by the amount necessary to increase the active layer dept to 10 inches.

The passive layer water equivalent, water content, and water-holding capacity decrease by the amount given up to the active layer.

The temperature and thermal quality of the active layer are unchanged.

(2) The depth of the snow pack is less than or equal to 10 inches.

In this case, no passive layer exists and the active layer properties remain the same as those computed in the melting process.

COMPUTER APPLICATION OF THE MODEL

Application of the model would be impossible without a digital computer. The model has been programmed in the Fortran language and is currently operational on the University of Alaska IBM 360/40F computer running under a DOS 25 operating system. A listing of the program and detailed instructions for the assembly of the data required by the program are included in an earlier report (Carlson, *et al.*, 1974). The following section is a brief discussion of the type and source of data required by the program and the output from the program.

The first step towards producing a snowmelt hydrograph is to define the initial parameters of the snowpack. These include depth, density, water equivalent, water content, temperature, water-holding capacity and thermal quality of the active and passive layers. The water content, water-holding capacity and thermal quality do not necessarily have to be measured in the field, but may be estimated knowing the temperature, density, and water equivalent of the layers, by equations 12, 14 and 8 respectively. Depth, water equivalent, and thus, density of the entire snowpack may be obtained from field measurements or from snow survey data published by the United States Department of Soil Conservation. When the snow survey data is used it is customary to equate the densities of both the active and passive layers. Temperature of the snowpack may be obtained from field measurements or, if not available, by averaging air temperatures of the three previous days. The initial conditions of the snowpack are printed out for reference on the first page of output as shown in Figure 8.

The driving function of the program is daily climatological data which, if not measured at the site, may usually be obtained from a nearby weather station. The data required are average daily values of height to the base of clouds, cloud cover, air temperature, dewpoint temperature, wind speed, and relative humidity. Also required are average daily values of the declination of the sun and the radius vector of the earth which may be obtained from a solar ephemeris. The above data must begin from the time that the initial conditions of the snowpack are defined and run through to the time when it is estimated that the snowpack will have melted.

The program computes from these parameters the hourly heat flow into or out of the snowpack and then considers the melting processes within the snowpack. At the end of the day, the total heat flux, the final snowpack parameters, and the amount of percolated water to the ground surface are printed. A typical page of output is shown in Figure 9. The final snowpack parameters for a given day are the initial parameters for the following day. When the snowpack has melted, the computation process ceases.

An option exists for calculating the snowmelt on an hourly or bihourly basis. Accuracy is increased by using one hour time increments rather than two hour increments. However, computation time is slightly longer. The option exists for printing the various parameters each hour or two hour period. A typical page of output is shown in Figure 10.

VERIFICATION OF THE MODEL

To test its accuracy, the model has been applied to the Chena River Basin located in Central Alaska. The basin, shown in Figure 11, is roughly 100 miles long, 40 miles wide, and covers an area of 1,980 square miles. Elevation with the basin range from 425 feet at Fairbanks located curve for the basin is shown in Figure 12. The basin was selected because of availability of data necessary for comparison and operation of the model. Snow survey data was obtained from the U.S. Soil Conservation Service for the five stations located within the basin. Climatological data necessary for the program was obtained from monthly summaries of the National Weather Service Office located at the Fairbanks International Airport.

As pointed out earlier, the snowmelt model being a lumped parameter system, is strictly only applicable to the location within the catchment where the climatological data is measured. To overcome this limitation, the basin was divided into four discrete elevation bands, as shown in Figure 13. The initial snow depth and water equivalent existing on March 1 was determined for the middle of each band by linearly interpolating between snowpack depth and water equivalents of the stations located within the band. The climatological conditions existing at Fairbanks International Airport were applied to each elevation band with the exception that air temperature and dewpoint temperature for each band were modified by the adiabatic lapse rate of -3°F per 1000 feet. The model then simulated the snowmelt in each of the elevation bands. The daily climatological inputs for the lowest elevation band are shown in Figure 14. Daily melt and snowpack depth for each elevation is shown in Figure 15. Comparison of the snowpack depth predicted by the model to actual snow course measurements for the dates March 15 and April 1 is shown in Figure 16. It is seen that the agreement between the two curves is very good, especially if one considers the variability in point measurements of the snowpack.

APPLICATIONS OF THE MODEL

Applications of the snowmelt model have included:

(1) The evaluation of satellite data as a tool for operational prediction of snowmelt runoff (Carlson and Kane, 1974).

(2) The coupling of the model with the runoff model to examine the spring breakup in Alaska's Arctic Coastal Plain (Carlson, Norton and McDougall, 1974).

(3) The creation of historical snowmelt hydrographs which are then statistically analyzed and converted via a model into flood frequency curves (Carlson and McDougall, 1974).

In the first application, the area percentage of snow cover vs. time, predicted by the model for the 1973 spring breakup of the Chena River basin, was compared with ERTS-1 satellite imagery (Figure 12). The model was further applied by using the weighted sum of the daily snowmelt for each elevation band (Figure 18) as input to a linear reservoir runoff model which simulates the effect of basin storage and channel delay. The runoff curve predicted by the runoff model was compared to actual runoff as measured by the United States Geological Survey (Figure 19).

In the second application, the model was again coupled with a similar linear reservoir model to examine spring runoff for the years 1970, 1971 in the Putuligauk, Kuparuk, and Sagavanirtok River basins located on the Arctic Coastal Plain of Alaska (Figure 20).

The Putuligayuk River basin, smallest of the three, is relatively flat. The basin is 56 km long, 22 km wide and has an area of 575 km^2. Elevations range from 0 to 67 m above sea level. The Kuparuk River basin is 218 km long, 81 km wide and has an area of 9220 km^2. Elevations range from 0 to 1433 m above sea level. The Sagavanirktok River basin, largest of the three, is 257 km long, extending far into the Alaska Range. The basin is 113 km wide and has an area of 13880 km^2. Elevations range from 0 to 2446 km above sea level.

Because little hydrologic and climatological data were available from the region, the basins were not divided into elevation bands. Initial snowpack depth and density interpolated from data taken at Pt. Barrow and Barter Island and climatological data from Prudhoe Bay were used as input to the snowmelt model. The resulting snowmelt hydrographs shown in Figures 21 and 22 were then used as input to the linear reservoir model shown in Figure 23. The resulting runoff hydrographs which are compared with runoff data obtained from the U.S. Geological Survey are shown in Figures 24 through 28.

In the third application, the model was used to create a simulated historical record (1950 through 1973) of snowmelt hydrographs for the Chena River basin. The method of obtaining these hydrographs was identical to that use in the previous section. From the hydrographs, probability density functions for snowmelt duration and intensity were calculated. The probability density functions were then fitted by least square methods to exponential functions, as illustrated in Figure 29. These functions, which are summarized by the exponents γ and β for duration and intensity are the principal input variables to a flood frequency model developed by Eagleson (1972) which has been adopted by the Institute of Water Resources to arctic and subarctic regions.

CONCLUSIONS

The results of this study have shown that the snowmelt model can be used as an effective tool in the prediction of snowmelt in watersheds. Comparison of the output from the model to field measurements showed good agreement. By using the same techniques, the model may be used to study snowmelt behavior in watersheds where little hydrological and climatological data has been collected. Such information will make it possible to draw preliminary conclusions about spring breakup which is the most dynamic portion of the hydrologic cycle in arctic and subarctic watersheds.

We expect that the snowmelt model when coupled with a runoff model such as shown in this study, will be a valuable tool in assessing the nature of spring runoff in ungaged watersheds. This will be particularly applicable in Alaska with the development of new transportation facilities such as roads, gas and oil pipelines. These routes will, for the most part, be built through what are now completely ungaged watersheds and for which the snowmelt process is known in only an approximate manner.

FIGURE 18

INITIAL SNOWPACK PARAMETERS AND SNOWMELT HYDROGRAPH
FOR ALASKA'S ARCTIC COASTAL PLAIN, 1970

Initial Snowpack Parameters

Parameter	Active Layer	Passive Layer
Depth (in)	10.000	4.285
Density (gm/cm^3)	0.350	0.350
Water Equivalent (in)	3.500	1.500
Water Content (%)	0.0	0.0
Temperature (°F)	26.000	26.000
Water Holding Capacity (%)	0.038	0.038
Thermal Quality (%)	102.000	102.000

Snowmelt Hydrograph

Date	Snowpack Depth (in)	Percolated Water (in)
710520	14.285	0.0
710521	14.285	0.0
710522	14.082	0.0
710523	14.082	0.0
710524	14.070	0.0
710525	14.070	0.0
710526	14.070	0.0
710527	13.857	0.0
710528	13.771	0.0
710529	13.771	0.0
710530	12.801	0.232
710531	9.600	1.194
710601	6.825	1.033
710602	0.264	2.442
710603	0.000	0.098

FIGURE 19

INITIAL SNOWPACK PARAMETERS AND SNOWMELT HYDROGRAPH
FOR ALASKA"S ARCTIC COASTAL PLAIN, 1970

Initial Snowpack Parameters

Parameter	Active Layer	Passive Layer
Depth (in)	9.04	0.0
Density (gm/cm^3)	0.36	0.0
Water Equivalent (in)	3.25	0.0
Water Content	0.0	0.0
Temperature (°F)	17.5	0.0
Water Holding Capacity (%)	0.039	0.0
Thermal Quality (%)	102.00	0.0

Snowmelt Hydrograph

Date	Snowpack Depth (in)	Percolated Water (in)
700529	9.04	0.0
700530	6.996	0.634
700531	5.753	0.465
700601	5.160	0.222
700602	5.110	0.019
700603	3.188	0.720
700604	2.143	0.391
700605	2.143	0.0
700606	0.981	0.420
700607	0.000	0.383

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