## USER'S GUIDE FOR

ATMOSPHERIC CARBON MONOXIDE TRANSPORT MODEL


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User's guide for atmospheric carbon monoxide transport model
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Technical Report Documentation Page


## I. FOREWORD

## INTRODUCTION AND BACKGROUND

It's an often observed but disquieting fact that the concentration of man's activities in a limited area can result in conditions which tend toward envirommental degradation and, in some cases, a public health concern. It is not unusual for this process to be magnified in the fragile and extreme conditions found in Alaska, and only through a proper balance of man's activities and the natural environment can long-term, healthy and prosperous community development take place.

Notwithstanding a genuine desire to achieve the optimum balance with nature, a major problem arises when dealing with large-scale prototype systems such as an urban area. This condition is due to the complexity and scale of physical situation and the diverse and interacting character of the processes which cause problems. The need for more regions and comprehensive planning and analysis become essential if an understanding of, let alone control over, urban problems is to be achieved.

Modern technology is moving forward in a number of areas, however, and the techniques of mathematical modeling and simulation are seen as one of the most promising in being able to supply the information necessary to make intelligent and proper decisions for environmental control. The description of such a mathematical model, this one for simulation of carbon monoxide (CO) in the atmosphere, is the subject of this report.

The chief motivation for the development of the model described herein arises from carbon monoxide problems observed in and near Fairbanks, Alaska. In the winter months (generally November through March) a highly stable, near-surface temperature inversion often exists in the area. One adverse
result of this inversion is to induce high levels of carbon monoxide in response to the various local $C 0$ inputs (automobiles, stationary sources, etc.). If the public safety is to be assured there seems little question that control measures will have to be instituted at some time. The question is, of course, what is the best policy, and what impact will potential control alternatives have on the quality of the airshed? Such questions are not easily answered and it is hoped that with proper use and interpretation this model will aid in the decision process.

## MODEL OVERVIEW

The model described herein represents the efforts associated with the first phase of a really complete modeling building effort. What is meant by this is that the current version of the model has been formulated and structured within the state-of-the-art, and reflects the best available information on how $C O$ is generated and distributed in the atmosphere in an Alaskan airshed; the model is now at the stage where it can, and should, be tested and compared to actual CO behavior in the atmosphere. This exercise will have the dual purpose of improving our understanding of how $C O$ moves in the atmosphere, and also of identifying any weaknesses or deficiencies in the model which can be strengthened by some combination of research and/or model modification. The point is that one should not view this version of the model as the ultimate development, for only with use and experience will the real worth of the technique be understood.

Along a more technical line, the $C O$ model described in the following paragraphs, named ACOSP (Atmospheric CO Simulation Program) predicts expected CO concentrations within a specified geographical area for an arbitrarily defined set of $C O$ sources. At the present time the model is programmed to consider automobile emissions as the major source of $C O$, and estimates $C O$ inputs as either automobile emissions or specified sources.

The user must supply the basic physical description of the airshed (aerial extent, height of the inversion layer, wind velocities, etc.), along with expected movement and concentration of automobiles. The model will
then calculate the $C 0$ input to the airshed and route it spatially and temporally in accordance with the principle of conservation of mass. The output from the model consists of tabulated values of $C O$ concentrations throughout the airshed over the time period of simulation.

The model is coded for computer solution in the FORTRAN programming language, and uses the so-called Finite Element Method (FEM) of numerical solution. The program has a large number of user options (explained in following sections) and can be operated in either the steady state or dynamic mode. With its present formulation, the model holds the potential. for real-time analysis and control of $C 0$ problems, although this will not be achieved without significant additional work. Currently the model will be most valuable for investigation and understanding the physical processes which are responsible for the production and distribution of $C O$ in the airshed, and for testing the effectiveness of remedial control measures in the abstract at high speed and low cost. It is hoped that this capability will aid in the eventual solution to a serious and important problem.

## REPORT FORMAT AND OBJECTIVES

The objective of this report is to bring together, in a single document, a self contained description of the theory, assumptions, mathematics, and computer code which make up the mathematical model ACOSP. Additionally, several example problems are presented which, together with the information on program input and output, should allow one unfamiliar with the program to be able to set up and operate the computer program for simulation of $C 0$. In other words, this report is directed both at the theory of the mathematical model and to its practical application. The user is cautioned to fully appreciate both features of the presentation, for it is only with a clear understanding of what goes into the model that one can expect to get useful and meaningful results as output.

In format, this report is comprised of five chapters and two appendices. This chapter, Chapter $I$, is simply a brief introduction to the model and
describes its status and expected uses. A brief sumary of the mathematical background for the model and its solution technique (the FEM), is given in Chapter II. Chapter III outlines the procedure for implementing the model and presents several example problems to demonstrate the way problems are structured and solved; comparisons between analytical and numerical solutions to the same problem are presented for two problems. In Chapter IV the details of input data preparation are presented, and the formats for each card used as program input is explained in detail. Also, there is an explanation of what program output looks like, which user options control what program functions, and how to interpret the computer output. The actual documentation of the computer program ACOSP is the subject of Chapter V. Overall program organizations is indicated, and is followed by a fairly complete description of each program routine and its function in the scheme of solution. Finally, there are two appendices. The first; Appendix $A$, is used to develop the complete mathematical and numeric basis for the model and includes all details of the model's formulation and solution. This will be of concern chiefly to those who are involved in model building or who have an interest in applied mathematics. Appendix $B$ is included to complement the example problems of Chapter III, and reproduce the computer output generated by the example problems. This output should be fully studied by anyone attempting to reproduce the example problems or to perform original simulations.
II. THEORETICAL CONSIDERATIONS

INTRODUCTION

Every mathematical model must have as its basis some relationship or expression which interrelates system variables in a way such that they may be formulated into a consistent problem statement. The model may be very simple or quite complicated, but it is important to remember that a user can never get more out of a model than the builder put in. For this reason it is quite important that the details of the $C O$ model, including both its mathematical formulation and the technique used for its numerical solution, be presented for review. Only by understanding the foundation of the model will it be possible to use the technique to its fullest advantage.

Before moving to the technical discussion, it may be useful to clarify several definitions. First, the model will be formulated in terms of known inputs, such as prescribed CO loads, and unknown CO concentrations, which must be solved for. In the jargon of the modeler, known or specified quantities are usually called "inputs," or "constants" even if they change over time; in either case, the distinguishing feature is that the model does not predict their behavior, but rather reacts to their specification. The values for which the model solves are called the "system state variables," and represent the unknowns of the problem. In the ACOSP model the only system state variable, with one exception, is the carbon monoxide concentration.

With this brief introduction we shall now proceed to an explanation of the governing equations, the CO input generator, and the finite element method. Please recall that a more complete treatment of governing equations and the solution technique can be found in Appendix $A$ of this report.

The governing equation for the CO model is based on the principle of conservation of mass. Basically, this relationship states that all mass introduced into a system must be accounted for, and will be distributed according to the processes of wind movement (convection) and turbulence (diffusion). For this reason, governing equation is often referred to as the convection-diffusion equation. The model assumes the wind field is known and specified, and that the $C O$ inputs are prescribed according to the relationships presented in the next section of this chapter. A summary of the technical formulation of the mass transport equation is developed in the following paragraphs.

Consider a control volume as shown graphically in Figure II-1.


Figure II-1. CONTROL VOLUME FOR MASS TRANSPORT EQUATION

If carbon monoxide is a nonconservative substance which is transported by convection and turbulent mixing, then a mass balance for a vertically mixed control volume may be found by summing inflows and outflows. When this is done the resulting expression is of the form

$$
\begin{align*}
\frac{\partial}{\partial t}(V c) & +\frac{\partial}{\partial x}(\text { cuhdy }) d x+\frac{\partial}{\partial y}(\operatorname{cvhdx}) d y \\
& -\frac{\partial}{\partial x}\left(D_{x} h d y \frac{\partial c}{\partial x}\right) d x-\frac{\partial}{\partial y}\left(D_{y} h d x \frac{\partial c}{\partial y}\right) d y-\sigma d x d y+K V c=0 \tag{II-1}
\end{align*}
$$

where
$\mathrm{V}=\mathrm{the}$ volume of the control volume, $\mathrm{L}^{3}$,
$c \quad=$ the concentration of carbon monoxide, $\mathrm{ML}^{-3}$,
$u \quad=$ the average velocity in the $X$ direction, $L T^{-1}$,
$v=$ the average velocity in the $Y$ direction, $L T^{-1}$,
$h \quad=$ the height of the inversion layer, $L$,
$\mathrm{D}_{\mathrm{x}} \quad=$ the turbulent mixing (diffusion) coefficient associated with the X direction, $\mathrm{L}^{2} \mathrm{~T}^{-1}$,
$D_{y}=$ the turbulent mixing (diffusion) coefficient associated with the $Y$ direction, $\mathrm{L}^{2} \mathrm{~T}^{-1}$,
$\sigma \quad=$ a local term for sources or sinks of carbon monoxide, $\mathrm{MT}^{-1} \mathrm{~L}^{-2}$, and
$K \quad=$ the decay of $C O$ from the control value (first order rate reaction) $\mathrm{T}^{-1}$.

If we recognize the relationship $V=h d x d y$, Equation (II-1) can be simplified to
$\frac{\partial}{\partial t}(h c)+\frac{\partial}{\partial x}(c u h)+\frac{\partial}{\partial y}(c v h)-\frac{\partial}{\partial x}\left(D_{x} h \frac{\partial c}{\partial x}\right)-\frac{\partial}{\partial y}\left(D_{y} h \frac{\partial c}{\partial y}\right)-\sigma+K h c=0$ (II-2)

Further, if we consider the mixing coefficients, $D_{x}$ and $D_{y}$ to be approximately constant over time at a given location, and that $h$ may vary spatially, Equation (II-2) may be expanded to

$$
\begin{align*}
& c \frac{\partial h}{\partial t}+h \frac{\partial c}{\partial t}+u h \frac{\partial c}{\partial x}+h c \frac{\partial u}{\partial x}+c u \frac{\partial h}{\partial x}+v h \frac{\partial c}{\partial y}+h c \frac{\partial v}{\partial y}+c v \frac{\partial h}{\partial y} \\
&-D_{x} h \frac{\partial^{2} c}{\partial x^{2}}-D_{x} \frac{\partial c}{\partial x} \frac{\partial h}{\partial x}-D_{y} h \frac{\partial^{2} c}{\partial y^{2}}-D_{y} \frac{\partial c}{\partial y} \frac{\partial h}{\partial y}-\sigma+K h c=0 \tag{II-3}
\end{align*}
$$

From fluid mechanics the continuity equation for an incompressible fluid has the form $h \partial u / \partial x+h \partial v / \partial y+\partial h / \partial t=0$. Substituting in Equation (II-3) gives

$$
\begin{align*}
& h \frac{\partial c}{\partial t}+u h \frac{\partial c}{\partial x}+\operatorname{vh} \frac{\partial c}{\partial y}-D_{x} h \frac{\partial^{2} c}{\partial x^{2}}-D_{y} h \frac{\partial^{2} c}{\partial y^{2}} \\
& -D_{x} \frac{\partial h}{\partial x} \frac{\partial c}{\partial x}-D_{y} \frac{\partial h}{\partial y} \frac{\partial c}{\partial y}-\sigma+\text { Khc }=0 \tag{II-4}
\end{align*}
$$

Equation (II-4) is the basic routing equation for conservative substances in an incompressible fluid, and is the governing equation for the carbon monoxide model.

## CARBON MONOXIDE EMISSIONS

The above expression includes a term, $\sigma$, which represents the input of $C O$ to the airshed from local sources. For the purposes of this model it has been assumed that the major source of $C O$ is from automobile emissions, and an expression has been developed to quantify this condition.

The co emission from automobiles is broken down into three categories: traveling, idling and cold starts. Mathematically, the formulation of the daily input of carbon monoxide for a particular geographic area, corresponding to an element in the finite element grid, is calculated from the expression

$$
\begin{equation*}
\sigma=\sigma_{m}+\sigma_{i}+\sigma_{s}+\sigma_{c} \tag{II-5}
\end{equation*}
$$

where

$$
\left.\begin{array}{rl}
\sigma= & \text { the total mass rate of production of carbon } \\
& \text { monoxide in a particular area, gm/day, and }
\end{array}\right\}
$$

where

$$
\begin{align*}
& \sigma_{m}=M \alpha_{m} s^{\beta}  \tag{II-6}\\
& \sigma_{i}=\alpha_{i} T  \tag{II-7}\\
& \sigma_{s}=\alpha_{s} N \tag{II-8}
\end{align*}
$$

and

| M | $=$ the number of vehicle miles traveled per day, mi , |
| :---: | :---: |
| S | $=$ the average speed of travel, mi/hr, |
| $\alpha_{m}, \beta$ | $=$ empirically determined emission coefficients, |
| $\alpha_{i}$ | ```= the mass of carbon monoxide produced by idling automobiles, gm/hr ,``` |
| T | ```= the number of automobile hours spent idling in the local area, hr/day ,``` |
| $\alpha_{s}$ | ```= the mass of carbon monoxide produced by cold starting an automobile, gin/start, and``` |
| N | ```= the number of automobile coldstarts in the local area, starts/day.``` |

The total daily $C 0$ emission for each separate area (element) in the system is computed from Equation (II-5). As the model will generally be operated on a one-hour time interval, the rate of emission for each area at a particular hour of the day is found from the expression

$$
\begin{equation*}
\sigma_{k_{\ell}}=\gamma_{m_{\ell}} \sigma_{m_{k}}+\gamma_{i_{\ell}} \sigma_{i_{k}}+\gamma_{s_{\ell}} \sigma_{s_{k}}+\gamma_{c_{\ell}} \sigma_{c_{k}} \tag{II-9}
\end{equation*}
$$

where

| $\sigma_{k_{\ell}}$ | $=$the total rate of carbon monoxide production in area $k$ <br> at hour $\ell$, gm/day, |
| ---: | :--- |
| $\sigma_{m_{k}}, \sigma_{i_{k}}, \sigma_{s_{k}}, \sigma_{c_{k}}=$the rates of production of carbon monoxide in area $k$ <br> associated with traveling, idling, starting and other sources, <br> respectively, gm/day, and |  |
| $\gamma_{m_{l}}, \gamma_{i_{\ell}}, \gamma_{s_{l}}, \gamma_{c_{l}}=$the fractions of the daily carbon monoxide production |  |
| asociated with travel, idling, starting and other sources <br> which are released at hour $\ell$, unitless. |  |

As the model is now programmed, constant values are used for the factors $\alpha_{m}, \beta, \alpha_{i}$ and $\alpha_{s}$, while $M, S, T$, and $N$ are geographically variable over the study area. As more and better data become available it will be both possible and desirable to expand and refine the specification of the factors for estimating the input of carbon monoxide in urban areas.

SOLUTION TECHNIQUE--THE FINITE ELEMENT METHOD

The Finite Element Method is a numerical technique suitable for solving differential equations of the type derived for the mass balance relationship. It differs from the finite difference method in that it assumes a continuous description of state variables as opposed to values defined only at specific points. In a general sense, the finite element is said to provide an "exact" answer to an approximation of the problem, whereas finite differences provide an approximate answer to the "exact" problem. From the user's point of view, the FEM offers significant simplication in data input, provides a great deal of flexibility in problem setup, and seems well suited to the CO problem to which it has been applied.

The method was first applied by structural analysts seeking the solution to complex problems in stress analysis of a continum. The initial applications were in the aerospace industry where the resistance capability
of continuous aircraft skins had to be maximized in keeping with the overall objective of weight minimization. The solution of such continuum systems using traditional structural analysis methods would have required an infinite number of joints, each connecting infinitesimally small elements.

In the FEM, the continuum is represented by a series of discrete polygons each interconnected at a finite number of locations called nodal points. These nodal points are directly equivalent to joints in structural problems and the elements are equivalent to beams and columns. In the first applications, the elements were allowed to respond in only restricted and simple displacement patterns such that continuity was maintained along element faces, but force transfer was allowed only at the nodes. With the system thus approximated, the method of assemblage of elements and solutions of the governing equations was exactly that of conventional structural analysis.

The mid-1960's saw a rapid expansion of the finite element into many structural problems such as plate bending, axisymmetric structures, solid systems, and shell structures. The procedure was also generalized and solutions were developed for nonstructural problems such as electric fields and pore pressures. Simultaneously the method was applied to nonlinear and time dependent problems in elasticity and from these problems experience was built up allowing the systematic solution of nonlinear problems.

More recently, the method has been developed as a special form of the method of weighted residuals in which the governing equations are satisfied in an area integral sense. This process removes the need for a functional and allows solution of equations such as the convection-diffusion equations for which there are no convenient functionals.

The first step in the application of the FEM is the construction of acceptable single element approximation to the physical system. A suitable element will have an approximating function with the same number of independent variables as the number of node points. The approximation must be such that the variation of the approximating function along the boundary of an element exactly matches that of an adjacent element.

For example, a triangular element with nodal (corner) point degrees of freedom at the vertices must have a linear approximating function. The necessary condition is met because the function in linear along the sides of the element and the adjacent elements match exactly. If mid-side nodes are used, a quadratic approximating function may be used, as is the case in the model ACOSP.

A necessary condition for solution of the equations with decreasing element size is that the fundamental equations do not contain derivatives two orders higher than those that match across the element boundary. Also the function must be capable of representing a constant value of the function or its derivatives which may be present in the integral equation describing the element.

As an example, consider a region as shown in Figure II-2. Over each triangular element the state variables of the governing equations are allowed to vary in a prespecified form and elements are interconnected only at nodal points.


Figure II-2. FINITE ELEMENT REPRESENTATION OF A REGION

In the conceptual representation of the region, let the nodal points be the vertices of the triangles. Then for a typical triangle (i, j, k) a linear approximation for a state variable, $\phi$, may be written as

$$
\phi=\left\langle N_{i}, N_{j}, N_{k}\right\rangle \quad\left\{\begin{array}{c}
\phi_{i}  \tag{II-10}\\
\phi_{j} \\
\phi_{k}
\end{array}\right\}
$$

or

$$
\begin{equation*}
\phi=\langle N\rangle\left\{\phi_{e}\right\} \tag{II-11}
\end{equation*}
$$

in which $N_{i}$ is a linear function of the coordinate system such the $N_{i}=$ unity at node $i$ and zero at nodes $j$ and $k$. A similar relationship exists for $N_{j}$ and $N_{k}$. $N_{i}$ is the so-called "shape function." In all finite element applications the shape functions have the same definition, i.e.,

$$
\begin{aligned}
& N_{i} \text { is continuous over the element, and } \\
& N_{i}=\left\{\begin{array}{l}
1 @ \text { node } i \\
0 @ \text { all other nodes }
\end{array}\right.
\end{aligned}
$$

The coefficients of the linear expansions for $N_{i}$ etc. are all known functions of the element geometry. For example, in an element with vertices at locations $(0,0)(2,0)(0,2)$ as shown in Figure II-2. (A 45-degree triangle with base length equal 2). The shape functions are

$$
\begin{align*}
& N_{1}=1 / 2(2-x-y) \\
& N_{2}=1 / 2 x \\
& N_{3}=1 / 2 y \tag{II-12}
\end{align*}
$$



Figure II-3. SHAPE FUNCTIONS FOR A $45^{\circ}$ TRIANGLE

Similar functions may be written for the general linear triangle i, j, k, i.e.,

$$
\begin{equation*}
N_{i}=a_{i}+b_{i} x+c_{i} y \tag{II-13}
\end{equation*}
$$

where $a_{i}, b_{i}, c_{i}$ are purely geometric constants.

In developing the solution to the problem, it may be said that the continuous function has been replaced by a new, piecewise continuous function which is defined in terms of the values at the nodal points.

Now, to obtain a solution to the equations, we make use of the method of weighted residuals which restates the basic differential equation, $h=f(\phi, x, y)$ in terms of an error function and weighting function, $w$.

Ideally, the error function should be identically zero at all points. If this is the case, the integral for all weighting functions will be zero, or

$$
\begin{equation*}
\iint \mathrm{wh} d \mathrm{~A}=0 \tag{II-I4}
\end{equation*}
$$

Then for a finite element approximation with $n$ degrees of freedom, the single integral may be replaced by $n$ integrals with independent weighting functions

$$
\begin{equation*}
\iint w_{n} h d A=0 \tag{II-15}
\end{equation*}
$$

where $w_{n}$ represents the shape function associated with a degree of freedom. If the weighting function is defined to be equal to the shape function ( $W=N$ ), there results what is known as the Galerkin method of weighted residuals which is the most accurate overall form.

In the complete formulation the contribution from a single finite element will be to all degrees of freedom (nodes) for that element and the total system will be a set of equations representing the sum of these contributions. This contribution to the equations may be written $\iint \mathrm{N} h \mathrm{dA}$ where $N$ is the vector of shape functions for all degrees of freedom of the element.

The final step in the process is the assemblage and solution of the set of simultaneous equations representing the system. The equations must be integrated and summed for each nodal point.

Figure II-4(a) shows a typical system consisting of six elements. The element is connected to the nodes as listed below.

\left.| ELEMENT |  | NODES |  |
| :---: | :---: | :---: | :---: |
|  |  | 1 | 4 |$\right) 3$

If the three contributions are formed in terms of global coordinates, they can be added in the appropriate space of the global matrix. Figure II-4 (b) shows the form of the six elements before addition and Figure II-4 (c) the final form where non-zero elements are shaded.

After the equations are completely formed the boundary conditions must be inserted. These usually consist of specifying values of $\phi$ or its derivative at one or more nodes on the boundary of the region. Present values of $\phi$ may be inserted as an overriding equation replacing the previously formed equations or by eliminating the equation by direct substitution of the specified value into the remaining equations. Values of the derivative of $\phi$ must be inserted by evaluating their influence on the right-hand side vector. The final step is the solution of linear simultaneous equations for the unknown nodal values of $\phi$.

（a）element connections

| 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ロmロロRma | Qamammb | Mammmma | Mramman | H．urni．．．．．． |  |
|  | ODQDDMa | manadar | ［0］${ }^{\text {ana }}$ |  | － |
| QmロロMm\％ |  | 日R［1macy | M日ロamat | Mロ吅回 | 以П7T |
|  | adabman＋ | marabom＋ | 口以口口丁口冋＋ | 以下Tロロ口口＋ | Qn¢u¢ |
|  |  | manadun |  | ตпnTMrin | 7ron |
| Q0＠10MT | T日G60日 |  | Mएवロロロ\％ | Qumbuad |  |
| manganma | CEAECADE］ | CWETCMEIC | CRTERDa |  | กmLadab |

（b）element contributions to global matrix
（c）final global matrix


Figure II－4．TYPICAL SIX－ELEMENT SYSTEM

## III. PROGRAM OPERATION

## INTRODUCTION

An explanation of the mathematical basis for the carbon monoxide model has been given in the previous chapter and is more fully developed in Appendix A. The purpose of this chapter is to review the most significant items of data which need be considered when using the computer program, and to discuss ways of structuring problems for efficient and accurate simulations. The concepts and procedures presented are equally applicable to both dynamic and steady state simulations and should be considered in each application of the program. Subsequent to the discussion of data preparation, the results from several example problems are presented to demonstrate the method of solving problems with the model.

## DATA REQUIREMENTS

One of the first questions which arises in the application of the model is how much data are required to obtain useful results. Unfortunately this question does not have a straightforward answer, as the data requirements, and the adequacy of the data base, depend on what the model's results are to be used for. One overriding fact should always be kept in mind, how-ever-the model supplies no independent information and the results it produces are no better than the information it uses for input. This means that one cannot expect the model to produce answers that are better than the information from which they were derived. An example of this is found in the case where an average daily velocity field is input to the model together with hourly $C O$ inputs. The model will calculate $C O$ concentrations on an
hourly basis, but the effects of wind shifts of less than daily frequency will not be represented. If one were actually interested in hourly occurrences, it would be necessary to prepare all input data at an hourly frequency.

Regardless of their frequency of specification, the data for the model can be broken down into six categories:

```
- Geographical Data
- Carbon Monoxide Source Data
- Wind Field Data
- Boundary Condition Data
- Mixing Coefficients
- Initial Conditions
```

A brief discussion of the importance and specification of each data item is provided below.

GEOGRAPHICAL DATA

The carbon monoxide model operates within finite and specified geometric boundaries for a given problem. The user of the model is thus confronted with the problem of interfacing the region to be modeled with the rest of the world. The basic rule to use is to move the boundaries of the model far enough from the area of major interest that if something is happening at the boundary its effect will be insignificant in the area of interest. For a typical urban problem, this means moving the limits of the model into the suburban or rural regions where experience and/or observation has demonstrated little or no system response to CO emissions.

A second geometrical consideration relates to surface topography and CO source locations. If the area to be modeled has high surface relief, or widely separated $C O$ sources, the domain of the model should be extended to cover all such features. Again, the extent of the model should include a large enough area such that any features not included in the model will not influence its results in the area of interest.

The specification of the sources of carbon monoxide requires four types of data:

- The unit rate of CO emission from a typical automobile during starting, idling and traveling
- The geographical distribution and activity of automobiles on a daily basis
- The hourly distributions of starting, idling and traveling over a day
- The rate and distribution of CO emissions from sources other than automobiles

It is up to the user to determine the magnitude of these data, and typically they will come from a number of diverse sources. For purposes of model validation or short-term operational control, the data should be collected from field measurements and actual observations. For broad scale studies, such as planning activities, typical operating schedules and emission rates are appropriate. Especially for planning work, rates of automobile emissions can be estimated from observed traffic patterns and 0 and $D$ surveys.

WIND FIELD DATA

In most atmospheric situations the major factor in $C O$ transport is wind movement. Thus, the accuracy of the model's calculations is dependent on the proper temporal and spatial estimate of the wind field. It is the responsibility of the model's user to specify the magnitude of the wind field at all nodal points within the region to be modeled. The effects of the wind field are manifest in the terms of Equation (II-4) which contain the variables $u$ and $v$, and thus the magnitude of the wind velocity in the $X$ and $Y$ directions at all node points is required.

At the present time no technology is available to accurately forecast future wind behavior and just making consistent estimates from:observed data
can be a difficult task. From the derivation of the equations it is apparent that an error in the wind field will be carried through to the $C 0$ estimates, and care must be exercised if overall mass is to be maintained.

For the purposes of this model, the user has three options for specifying the wind field. First, he may simply estimate some representative wind behavior from his experience or by statistical analysis. This is usually a fairly straightforward task, and may be suitable for planning work. A second, and somewhat more sophisticated approach, can be taken with the help of the model. In this case the user supplies to the model an estimate of expected wind velocities along the boundaries of the model, and an internal calculation is made which estimates a smooth wind field over the entire region. Be aware that this is not a physical simulation of the wind field, but rather a simplified technique which satisfies only the specified boundary conditions and the equation of continuity.

The final, and most difficult, method of estimating the wind field is for the user to specify a mass consistent field at all points in time and space over the model area. No general method of doing this is suggested, but if any attempt is made, please recall that flow continuity must be satisfied and that changes in elevation of the ground surface must be accounted for.

BOUNDARY CONDITION DATA

Boundary conditions refer to the conditions which interface the model region with its surrounding area. The formulation of the model requires that the user specify the concentration of CO along all external model boundaries across which $C O$ comes into the system. If the user does not specify the value at these locations the model will assume the concentration is zero, and proceed normally. Such specifications can be most easily developed by use of observed data or by extending the model area to a point where $C O$ concentrations are either known or can be assumed.

## MIXING COEFFICIENTS

The mathematical model developed in the previous chapter indicates two "mixing coefficients" which reflect the influence of turbulent behavior in the convective field. These coefficients are sometimes referred to as dispersion coefficients or eddy diffusion coefficients. Due to their somewhat artificial physical basis (the analogy to Fickian diffusion) these coefficients should be estimated from observed data. For preliminary studies, or when moving into areas where no data exist, however, one is faced with estimating appropriate values a priori. If this is the case, values known to be satisfactory in similar physical locations, or those which are conservative in regard to the objectives of the study, are most appropriate.

Before leaving the topic of mixing coefficients, two things should be pointed out. First, the values chosen will be somewhat time dependent, with larger values required as the detail in the wind field is diminished. This comes directly from the definition of the coefficients. If the wind field were known exactly on a very short-term basis, the "mixing" would approximate zero. At the other extreme the specification of a net zero wind field (i.e., long term, steady state) would require all transport to take place via the diffusion process. It is important, therefore, to keep time scales in mind when estimating mixing coefficients.

The second point is that under short interval simulations of the type usually required for the study of automobile emissions, convection is the dominant transport process and that modeling results may be relatively insensitive to the mixing coefficients. This being the case, it is often beneficial to operate the model with the range of possible coefficients to determine how sensitive the predicted levels of $C O$ are to these values. If, within the objectives of the study, the sensitivity is such that nominal values can be used the modeling task is simplified. On the other hand if the sensitivity to mixing coefficients is important a field study and model calibration exercise will be necessary to evaluate the precise values.

In the time-dependent mode of operation the carbon monoxide model moves forward in time from some beginning or initial condition. The CO concentrations at the starting point are called "initial conditions." The CO concentrations computed by the model are highly influenced by the initial conditions at first, and to a lesser and lesser degree as time progresses. Two options are open to overcome the problems associated with initial conditions. First, if one has a good estimate of the system at a particular time, this can be input to the model and the simulation started. Such detailed information can usually be obtained from an extensive field program or from the output of the computer program based on an earlier starting point.

A second, and sometimes more practical approach, is to start the simulation from an arbitrary state and simply let it proceed with specified inputs until the state of the system is independent of the starting condition. With this approach the user is relieved of having to worry about the initial conditions, but encounters the problem of how long the simulation should run to achieve a condition independent of the initial condition. This can be determined by letting the simulation run for a period of approximately twice the time it takes the wind field to move across the area being simulated. After this amount of time the initial conditions will no longer influence the model's results.

## PROBLEM SETUP

Setup and implementation of the CO model follows a straightforward and logical sequence regardless of the type of problem to be solved. The procedure can be considered to consist of three steps as outlined below. Please be aware that the model is assumed to exist in the horizontal, $X-Y$, plane, with complete mixing in the vertical direction. Also, the model operates in metric units, with all length scales defined in meters; the $C O$ sources are specified in grams/day.

Proper network layout is the most fundamental aspect of FEM modeling and one which becomes considerably easier with experience. Examples of networks for two different types of problems are shown in the next section and the reader is referred to these for a graphical interpretation of the procedure outlined for network layout.

The first step in developing the network is to get a map or diagram of the prototype system and sketch out the problem's overall boundaries. This should be done with the prototype's major characteristics in mind so as not to be overly concerned with local details. As a general rule, the system should be described in a way such that boundary conditions can be reasonably specified.

Once the overall limits have been set the next step is to construct the interior detail. This is done using any combination of straight- or curved-sided elements (linear, triangular or quadralateral), and will reflect both internal physical features as well as CO sources. The network should have the highest number of nodes in locations where $C O$ gradients are expected to be large, and a fewer number as gradients are expected to diminish. In all cases, enough nodes and elements must be included to give the minimum desired spatial detail for the problem.

With the network complete the nodes and elements should be numbered and checked against the capacity of the program (note: the order of element numbering is unimportant but node numbering may be important). If program capacity has been exceeded one can either consider enlarging the program dimensions (assuming a large enough computer) or restructuring the network to a lesser number of nodes and elements. It goes without saying that the best possible network is the smallest one which will adequately and accurately describe the prototype system.

Each node point in the network must be assigned a unique number for interpretation of input and output, as well as for internal computation. The objective in node numbering is to make the maximum difference in node numbers across any element a minimum. Be sure to remember that numbers must be assigned to the mid-side nodes of elements as well as the corner nodes.

In numbering the network, it is most straightforward to simply specify the numbers in their optimum order. Often this is not done for at least two reasons; numbering errors and network modification. The program has an option of overcoming this problem, however.

If one errs in network numbering or discovers an order which will reduce the maximum node number difference across an element, or if one wishes to add nodes to an existing network which do not fit with the existing numbering system, there is no need to renumber and repunch all the data for the network. An option has been included in the computer programs which will allow a new internal renumbering of the system, while at the same time allowing the old, external numbering system to be used for input and output. Additionally, the program incorporates an "automatic" renumbering option which attempts to find a better order than that specified by the user. If the user wishes to renumber the nodal order he may invoke this routine, but there is no guarantee that a better order can or will be found.

## DATA PREPARATION

The final step is to make the input data machine ready and perform the desired simulation. To do this requires that all program inputs be punched into cards according to the formats specified in Chapter IV. It is suggested that the user begin with relatively simple problems, and that more complex and subtle situations be taken on only as experience is gained. In the next section, several example problems are presented which attempt to follow this format.

## EXAMPLE PROBLEM NUMBER 1

A simple, steady state problem has been chosen as for first example of the operation of program ACOSP. The problem is basically a one-dimensional situation in which the concentration at two ends of an infinitely long region are specified at known concentrations; the program computes the resulting internal concentration distribution. A schematic description of the problem is shown as Figure III-1.


Figure III-1. EXAMPLE PROBLEM NUMBER 1

In addition to the boundary condition specifications ( $c=$ constant $@ x=0$, $c=0 @ x=\infty)$, a convective velocity is assumed to be moving in the negative $X$ direction. Under steady state conditions, the analytic solution to the problem is found from the expression

$$
\begin{equation*}
c=c_{o} e^{-\frac{U x}{D}} \tag{III-1}
\end{equation*}
$$

where
c $\quad=$ the concentration at $x>0$,

```
co = the concentration at x = 0,
U = the convective velocity,
D = the diffusion coefficient, and
x = distance.
```

In order to solve this problem with the computer program, we shall proceed as follows. First, a network is constructed which is representative of the problem. Since we are interested in a one-dimensional approximation, a network which is long relative to its width is suggested; Figure III-2 shows the network developed for this problem. Each major horizontal and vertical line is separated by a 0.25 meter spacing, and the overall dimensions of the network are 1.0 meters wide by 2.5 meters long.

Since there are no internal sources or sinks in the problems, all internal emissions have been set to zero. The wind field is specified at a value of $-0.05 \mathrm{~m} / \mathrm{sec}$, which means it moves from right to left. The boundary conditions reflect the original assumptions, with values of 1.0 specified at the left end (nodes 1-7) and 0.0 specified at the right end (nodes 131-137).

The program was executed with these specifications and the results are presented graphically in Figure III-3. The figure shows the computed steady state concentration vs. distance from the left end of the network. The analytical solution, Equation (III-1) is plotted as the solid line, with the results produced by the computer model designated by the small stars.

As can be seen in the plot, good agrement is obtained between the analytical and model solutions, and for practical purposes they are the same. This plot provides confidence that the numerical technique gives a good solution to the equations, and that the model will respond to a properly formulated problem. A complete reproduction of the computer output for this problem can be found in Appendix $B$.


Figure III-2. ONE-DIMENSIONAL EXAMPLE PROBLEM NETWORK


Figure III-3. EXAMPLE PROBLEM 1--STEADY STATE CONCENTRATIONS

The second example problem illustrates the dynamic operation of ACOSP. As in the previous problem, the physical problem is a one-dimentional situation in which the concentration at each end is of an "infinitely" long region are specified, and the program computes the resulting concentration distribution at internal points as a function of time. A diagrammatic representation of the problem is shown in Figure III-4. The main conceptual difference between this problem and the previous example is that we shall


Figure III-4. TIME DEPENDENT EXAMPLE PROBLEM
begin the solution at time zero with zero concentration at all points except at $\mathrm{x}=0$, where a concentration of 1.0 has been defined. Also, the convective velocity is now directed in the positive or $X$ direction rather than in the negative direction as was the earlier case. For purposes of reference, the analytic solution for the problem at tine $t$ is given by

$$
\begin{equation*}
\frac{c}{c_{o}}=\frac{1}{2} e^{\frac{U x}{D}} \operatorname{erfc}\left(\frac{x+U t}{2 \sqrt{D t}}\right)+\frac{1}{2} \operatorname{erfc}\left(\frac{x-U t}{2 \sqrt{D t}}\right) \tag{III-2}
\end{equation*}
$$

and all the symbols are as previously defined.

To solve this problem with ACOSP we use the procedure outlined earlier. The same criteria for network configuration are valid for this problem as in the steady state case, and the network shown in Figure III-2 can be adopted. Once again there are no internal sources to worry about and all loads are set to zero. The wind field is input at a constant value of $0.05 \mathrm{~m} / \mathrm{sec}$, and is constant over time. The boundary conditions reflect the original assumptions, with values of 1.0 specified at the left end (nodes $1-7$ ). In this case there is no need to specify the right end boundary conditions, as flow can only leave the system at the point and thus they have no influence on the problem solution. This is a general feature of ACOSP in that boundary conditions need be specified only at points where Co enters the system, and the program calculates the concentrations at exit points.

The results of the computer solution and the analytic solution, Equation (III-3), are presented in Figure III-5. The analytic solution appears as the solid line, with the model output plotted as stars. The output is shown for three times, 1,3 and 6 seconds, and was computed on the basis of one second time steps.

The agreement between the analytic solution and the computer model is quite reasonable at all times, and is almost identical as time progresses. The small inconsistencies after the first step (1 sec.) are a result of the fact that it is never possible to start a finite approximation to the analytic solution in a precisely consistent fashion. As the influence of the starting condition is diminished by stepping through time, however, the two solutions become more and more coincident, and continue to converge as long as the simulation continued. In a reassuring fashion, this problem demonstrates that the program gives a reasonable solution to the governing equations, and suggests the way in which problems should be set up and operated from the point of view of network construction and boundary conditions. A complete listing of the computer output from this problem is also reproduced in Appendix B.


Figure III-5. EXAMPLE PROBLEM 2--TIME DEPENDENT CONCENTRATIONS

The third and final example problem is intended to demonstrate the type of network which might be developed for a real problem, and to indicate the various element shapes available to the user. The example network, Figure III-6, is asimplification of an actual network used in the Fairbanks area (the CBD is in the area of dense element construction), and is representative of prototype network construction.

The first feature to note is the combination of element shapes, and the relative size of elements. In the area of high emissions, and expected high concentration gradients, there are a large number of small elements. At the outer edge of the network, particularly along the top (northern) boundary, where $C 0$ emissions are low, the region is represented by a few large quadralaterals. This construction is in line with the idea of moving the network outward to a point where boundary conditions can be reasonably specified.

For illustration purposes, two curved-sided elements have been included in the network. Element 66 is a triangle with one curved side, while element 69 is a quadralateral with a curved side. Such elements are automatically generated in the computer program by specification of the coordinates of the mid-side node point. More information on the use of curved sided elements is given in Chapter IV.

A very simple prototype condition was used for this problem. The wind field was assumed to be out of the north at $0.25 \mathrm{~m} / \mathrm{sec}$, and there was no east-west wind movement ( $u=0, v=-0.25$ ). The boundary conditions were prescribed as zero $C O$ concentration along the western and northern limits of the network (13 values), with program computed, natural, boundaries at all other points.

A schedule of six elements were assumed to produce carbon monoxide due to automobile emissions, and all other sources were set to zero. The elements and their relative $C O$ inputs were specified as follows (grams/day/ element):


Figure III-6. NETWORK FOR EXANPLE PROBLEM 3

| Element | Travel | Idling | Starting |
| :---: | ---: | ---: | ---: | ---: |
|  |  |  |  |
| 18 | 222,100 | 75,000 | 225,000 |
| 19 | 5,114 | 37,200 | 127,500 |
| 34 | 39,984 | 139,800 | 475,660 |
| 51 | 3,142 | 9,000 | 31,280 |
| 52 | 3,142 | 9,000 | 31,280 |
| 53 | $1,486,714$ | 0 | 0 |

The effective inversion layer was set at an elevation of five meters, and the diffusion coefficients at $0.1 \mathrm{~m}^{2} / \mathrm{sec}$. The program was operated in the steady state mode, with an hourly CO distribution factor of 0.08 , a value representative of afternoon, rush hour-conditions.

The results of the program's operation are pretty much as expected, with virtually no CO predicted in the area north of the sources, the highest concentrations near the sources and lesser values in the lateral directions. The maximum steady state concentration predicted under these conditions was about $3.0 \mathrm{mg} / \mathrm{m}^{3}$, and fell away to roughly one-sixth this level at the southern boundary. The complete computer output for this problem is reproduced in Appendix $B$.

Before leaving this problem, two things should be mentioned. First, in regard to the network construction, there are a couple of lessons to be learned. While, in general, the network is well formed it gets quite "busy" and irregular in the downtown area. As a result of this, the numbering gets confused, and the problem demands a relatively large amount of computer storage. A better construction might be found by adding more elements, with an eye to making the elements more regular in size and to diminishing the number of of elements which connect to a single node.

The second point has to do with the generation of negative concentrations as are seen in the computer output. Such values, if small, should not concern
the user and are a reflection of the solution technique. What they indicate is that the user's specifications of the problem (network diffusion coefficients and CO loads) represents a problem that is too "stiff" for the numerical technique, and small numerical waves result. If relatively large negative concentrations appear, the user is cautioned to examine the definition of the problem, identify its source, and affect its cure. This can usually be most easily accomplished by adding more detail in the troublesome region or increasing the mixing coefficients in area.

## INTRODUCTION

This chapter explains the details of the input to, and output from, the computer program ACOSP. It presumes the reader is familiar with the governing equation and the finite element method for its solution as presented in Chapter II. The reader is referred also to the example problems of Chapter III and the computer output of Appendix B for examples on how the program operates.

## PROGRAM OPTIONS

The computer program ACOSP has a number of options which the user may invoke as he sees fit. The options are selected and controlled via card input and are designated by entering certain codes in the input data. Certain of the options are controlled by what is called an option switch, which is the FORTRAN program variable which internally controls the particular program option. In the same order as they are specified, the program options are:

Option 1: OUTPUT PRINT OPTION (Option switch is IPRT)

This option controls the amount of information output on the line printer during program execution. If the option switch is set to the value one (1), the program will exercise all its printing capabilities, and produce the maximum amount of output. If a value of zero ( 0 ) is specified the volume of printing will be reduced somewhat. In the second case neither the input geometric data nor the input CO load specifications will be printed. The zero option is most useful when making repeated solutions on a problem which has been fully checked out geometrically and one wishes to concentrate on the quality
routing aspects of the model. The value one (1) should always be used when a new problem is attempted.

Option 2: LINES FOR CONTINUTTY (Option switch is NCL)

This option instructs the program to calculate the total mass flow rate of $C O$ across a specified line of nodes by convective transport. The option is used as follows. The user sets the option switch to the number of individual lines across which the flow rate is to be calculated. Later in the input data stream, just after input of the diffusion coefficients, the nodal points which specify each line will be read from the card reader. One input card is used for each line, and up to 16 node points may be specified along a line. The line may take any type of continuous irregular shape (not across elements), but the sign of calculated flow will reflect the direction in which the line was defined. For example, a positive $X$ direction flow across a line directed upward to the right will be positive, while the same flow across the same line specified in the opposite direction will be negative. A little experience with specification of these lines will indicate how to make the definitions obtain the best results. If this option is selected, the program will print the mass flow rates for each line just after output of the results for the quality simulation. If the option is not selected, no printing will occur. At the time the output is printed, the flow across each line is compared to the flow across the first line on a percentage basis.

Option 3: TYPE OF VELOCITY FIELD (Option switch is NVEL)

In its present form, ACOSP has three options for specification of a velocity field. If NVEL is made equal to zero, the program will expect to input exactly one card which contains an $X$ and $Y$ velocity specification. The values so specified will be assigned to all nodal points in the system, and the solution will proceed in its normal fashion. If the option switch is made greater than zero, the program will expect to read as many cards as the numerical value of NVEL. Two things can happen at this point. If the VELOCITY FIELD OPTION, explained below, is zero, the program will make a
linear interpolation for any velocities not specified between corner nodal points and proceed normally. If the VELOCITY FIELD OPTION is greater than zero, the velocity values input at this point will be used as boundary conditions for internally computing a mass consistent estimate of the flow field at all other points in the network. If the VELOCITY FIELD OPTION is greater than zero, velocities should be input for all points along the external boundary of the system (both inflow and outflow) such that flow continuity is maintained in an overall sense. Please note that any velocity not specified either, directly or indriectly, will be set to zero.

Option 4: TYPE OF INITIAL CONDITION (Option switch is IS1)

In all cases the $C O$ model will expect its initial conditions to be defined, although they have meaning only in time-dependent simulations. If the option switch is set to zero, the program will expect to input one card, and to assign the value input to all node points in the system. If the option switch is greater than zero, the program will expect to read as many initial condition cards as the numerical value of ISl. Please note that in the second case, cards are needed only for those node numbers which are active in the solution, and no data are ever required for missing node numbers.

Option 5: REORDERING OPTION (Option switch is IRO)

In Chapter II, explanation was given of how the FEM collects and stores the data which interconnects node points. The total amount of computer storage necessary to accomplish this can be seen to be a function of the numbering order assigned to the node points. In simple systems the best numbering system is easy to recognize, but in complicated or modified systems it may be difficult to recognize the best order, or difficult to repunch all the input data to get a better numbering system. For this reason, a REORDERING OPTION has been included in the program, which works in either of two modes. If the option switch, IRO, is set to a value of one, the program will expect to input a new internal order for the network node numbers from cards. The new sequence will tell the program that for storage purposes the network nodes are to be considered in a different order than that indicated on the element
cards. If, for example, it was desirable to consider nodes 1,2 and 3 as 46, 13 and 97, the program input cards would contain 46,13 and 97 as the first three entries in the list. Note that if $\operatorname{IRO}=1$, a value must be input for all node numbers up to and including the maximum number in the system; if the order of node is not to be changed, its location can be left blank or set to zero.

In the second operational mode, the option switch is set to two and the program will attempt to automatically develop a new ordering sequence which is better than the original. The technique cannot guarantee a better order, but has proven useful in a number of cases and should always be tried in difficult and/or complicated networks. One can tell if any improvement has been made from the storage allocation message printed by the program, with the smallest total storage being the most desirable.

A final word on renumbering. At first it may appear that using this program option, especially with $\operatorname{IRO}=1$, is confusing and difficult. With experience, however, the user will find that it is much easier to prepare a new ordering list for input than to restructure, renumber and repunch all his program output. Also, if one has a problem where the same network is used for a number of different cases, and the automatic renumbering option (IRO $=2$ ) gives a better order, the new order can be punched into cards and input with an attendant savings in computer time. The nodal order, regardless of how specified, appears in the output with the coordinate points, and will print as zero if no reordering option of any type has been used.

Option 6: VELOCITY FIELD OPTION (Option switch is IS3)

Under the description of the TYPE OF VELOCITY FTELD option; it was mentioned that a velocity field for internal network points could be calculated if suitable boundary velocities were specified. To exercise this feature of the program, the option switch IS3 is set to the value one. This directs the program through an internal procedure which solves the so-called "potential flow problem," and gives a mass consistent flow field at all network grid points for the specified boundary conditions.

For those unfamiliar with this solution technique it is formulated as follows: assume that there exists a potential function $\phi$ such that the flow at any point is defined by the gradient at that point, or

$$
\begin{align*}
& q_{x}=\frac{\partial \phi}{\partial x}  \tag{IV-1}\\
& q_{y}=\frac{\partial \phi}{\partial y} \tag{IV-2}
\end{align*}
$$

where

$$
\begin{aligned}
& \mathrm{q}_{\mathrm{x}}=\text { the unit flow in the } \mathrm{x} \text { direction, } \\
& \mathrm{q}_{\mathrm{y}}=\text { the unit flow in the } \mathrm{y} \text { direction, and } \\
& \phi \quad=\text { the flow potential function. }
\end{aligned}
$$

From the equation of continuity $\partial q_{x} / \partial x+\partial q_{y} / \partial y=0$, Equation (IV-1) and Equation (IV-2) may be combined as

$$
\begin{align*}
& \frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial y}\right)=0  \tag{IV-3}\\
& \frac{\partial^{2} \phi}{\partial x^{2}}+\frac{\partial^{2} \phi}{\partial y^{2}}=0 \tag{IV-4}
\end{align*}
$$

The problem of defining a flow field which satisfies continuity is thus reduced to finding the solution to Equation (IV-4), and then calculating the velocities according to Equation (IV-1) and Equation (IV-2). This is the procedure utilized by ACOSP.

In using this option, the user should keep a couple of things in mind. First, this expression is at best approximate in that it is a steady state approximation to a very dynamic and variable problem, and the solution does
not include the effects of momentum or other physically complicating factors. Secondly, the user must be careful to define his boundary conditions around the grid in a manner such that continuity is satisfied in an overall sense. If the continuity is not satisfied on the outside boundary, the internal field will also fail to achieve mass continuity, but in an unpredictable manner.

Option 7: DYNAMIC INPUT OPTION (Option switch is IS5)

The CO model may be run with either steady (not variable in time) or dynamic (variable in time) boundary conditions and/or wind fields. If either of these input is to be dynamic, the option switch must be set to the value one for proper program operation. This option keys the reading of the card which defines the dynamic files, but has no other function.

Option 8: VERTICAL DIST OPTION (Option switch is IS8)

ACOSP assumes vertical mixing between the ground surface and the upper (inversion) layer. Subject to the condition that the wind velocities are uniform in the vertical, the concentration of $C O$ may be assumed to have a vertical distribution which is defined in terms of the mean vertical concentration computed by the model. The option of computing such $C O$ distributions, which can be used to estimate concentrations near the ground or at "nose level," has been included in the model.

To use the distribution option, the option switch is set equal to the type of distribution desired (see below). If distribution parameters are required, they are entered later on the appropriate input card. On output, the model first prints the mean concentration at each node point, and then prints the $C O$ concentration at the specified height, subject to the assumed vertical distribution.

Any of six separate distributions may be specified. The mathematical definition of each is given below, with the general shape indicated graphically in Figure IV-1. Note that in all cases the user must select the



$\stackrel{H}{4}$




Figure IV-1. VERTICAL DISTRIBUTION FUNGTIONS
elevation at which the local concentration is to be computed and, with certain of the functions, an estimate must be supplied of the ratio of the ground level $C O$ concentration to the mean concentration.
(1) Linear Distribution

$$
\begin{equation*}
c(h)=2 \overline{\mathrm{c}}(1-h / H) \tag{IV-5}
\end{equation*}
$$

where
$c(h)=$ the concentration at height $h$,
$h=$ the user specified height for local CO concentration,
$\bar{c} \quad=$ the local average vertical $C O$ concentration, and
$\mathrm{H} \quad=$ the vertical distance to the inversion layer.
(2) Parabolic Distribution

$$
\begin{equation*}
c(h)=1.5 \bar{c}\left[1-(2 h / H-1)^{2}\right] \tag{IV-6}
\end{equation*}
$$

where
$c(h)=$ the concentration at height $h$,
$h \quad=$ the user specified height for local $C 0$ concentration,
$\overline{\mathrm{c}} \quad=$ the local average vertical $C O$ concentration, and
$\mathrm{H}=$ the vertical distance to the inversion layer.

$$
\begin{equation*}
c(h)=2 \bar{c} h / h^{\prime} \quad \text { for } 0 \leq h \leq h^{\prime} \tag{IV-7}
\end{equation*}
$$

and

$$
\begin{equation*}
c(h)=2 \bar{c}\left(1-\left(h-h^{\prime}\right) /\left(H-h^{\prime}\right) \quad \text { for } h^{\prime} \leq h \leq H\right. \tag{IV-8}
\end{equation*}
$$

where
$c(h)=$ the concentration at height $h$,
$h \quad=$ the user specified height for local CO concentration,
$h^{\prime}=$ the user specified height of maximum vertical $C 0$ concentration (referred to as the "vertical distribubution parameter" in the program input instructions),
$\bar{c} \quad=$ the local average vertical $C O$ concentration, and
$\mathrm{H}=$ the vertical distance to the inversion layer.
(4) Trapezoidal Distribution

$$
\begin{equation*}
c(h)=\vec{c}(R-2 h(R-1) / H) \tag{IV-9}
\end{equation*}
$$

where
$c(h)=$ the $C O$ concentration at height $h$,
$h \quad=$ the user specified height for local $C O$ concentration,
$\mathrm{R}=$ the ratio of ground level $C O$ concentration to average vertical $C O$ concentration. This value is referred to as the "vertical distribution parameter" in the program input instructions and must have a value less than 2.0 ,
$\overrightarrow{\mathrm{c}} \quad=$ the local average vertical $C O$ concentration, and
$\mathrm{H}=\mathrm{the}$ vertical distance to the inversion layer.

## (5) Exponential Distribution

$$
\begin{equation*}
c(h)=\frac{\vec{c} R}{1-e^{-a}}\left(e^{-a h / H}-e^{-a}\right) \tag{IV-10}
\end{equation*}
$$

where
$c(h)=$ the $C O$ concentration at height $h$,
$h \quad=$ the user specified height for local $C O$ concentration,
$R=$ the ratio of ground level $C O$ concentration to average vertical $C O$ concentration. This value is referred to as the "vertical distribution parameter" in the program input instructions and must have a value greater than 2.0,

H $\quad=$ the vertical distance to the inversion layer, and
a $\quad=$ an internally calculated distribution parameter.
(6) Gaussian Distribution

$$
\begin{equation*}
c(h)=R \bar{c} e^{-\frac{h^{2}}{2 a^{2} H^{2}}} \tag{IV-1I}
\end{equation*}
$$

where


In many cases the data for either geometric and/or wind data are most readily available in units other than the metric units used in the model. For this reason, the user is allowed to specify a scale factor for any or all input of this type. The factors are used to convert the card input values to consistent values for internal use.

A second type of conversion factor is available for CO concentration units. Internally, the program has concentration units of $\mathrm{gm} / \mathrm{m}^{3}$. If the user wishes to have his input and output in other units, he may input an appropriate conversion factor and a five character title for column headings. The program uses a default value of 0.001 , which means it expects to have user units of $\mathrm{mg} / \mathrm{m}^{3}$. The conversion factor is found from the relationship that user mass units $x$ conversion factor $=$ grams.

Option 10: TIME CONTROL

The user has control over the time step used by the model, and the total length of time to be simulated. The basic time step used by the model is one hour, and this is the shortest period for which wind fields, boundary conditions and $C O$ sources may be specified. If, however, one hour is long compared to velocity of the wind and the size of the grid, the user may specify shorter time steps for internal calculation. If, for instance, a time step of 0.20 hr . is input, the program will take five internal steps in each hour, although the output will be printed only once per hour. Under normal circumstances, a onehour time step should be used, and shorter values input only under extraordinary conditions.

To have the program compute a steady state solution, only one simple action is required; set the length of the time step to zero. All other inputs will remain the same, and the program will compute, in a single pass, the steady state solution. When this option is used the CO source loads are defined to be those input for hour one.

Program ACOSP will accept any combination of linear, triangular or quadrilateral element shapes. Each side of each element is defined in terms of the spatial coordinates at the three nodal points which lie along the element side. In all cases there exists a nodal point at each end of an element side with an internal, or mid-side node at an arbitrary location between the end points. For straight element sides the position of the midside node is internally calculated from the position of the end nodes, and lies exactly midway between the end nodes.

If the user wishes to define an element side as being other than straight, he simply inputs to the program the coordinates of the mid-side node in addition to the usual specification of the end nodes. The program will then pass a smooth curve through the three points which define the element side. Ths shape of the curve is approximately quadratic and will pass exactly through the end and mid-side nodes. The network shown in Figure III-6 contains two curved sided elements, and the shape of these curves is typical of those generated by the program.

The only practical way for a user to examine the precise shapre of a curved sided element is to plot the internally generated function. A separate computer routine, coded to accept data in exactly the same format as ACOSP, has been prepared for network plotting and is available to users upon request.

## CARD INPUT FORMATS

The following description indicates the exact FORTRAN card formats to be used for computer program. All input data are transmitted to the program via the card reader on logical unit 5. The data should be submitted in the same order as it is described, and the input stream is the same regardless of whether the simulation is dynamic or steady state. Only one difference exists in the logical sequence in which data are provided to the
program. If the DYNAMIC INPUT OPTION (number 7) has been taken, the input of the boundary conditions and/or wind field data is different from that used for steady input data. The complete job stream for each case is covered in this section, and the user is encouraged to make sure he is using the proper instructions for his particular problem.

In the preparation of the card input all values should be right justified in the card columns indicated. All real data are read with "E" type FORTRAN formats, and thus any type of real number representation is allowed in the input fields.


| Card Sequence <br> No./No. <br> of cards <br> Column | FORTRAN <br> Name | Format |
| :--- | :--- | :--- |


| Card Seque No. / No. of cards | Card Columns | FORTRAN <br> Name | Format | Type* | Description |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{2 / 1}{\left(\operatorname{con}^{\prime} d\right)}$ | 66-70 | IS4 | I5 | I | The number of sets of hourly distribution schedules used to distribute CO emissions throughout a 24 -hour period |
| 3/1 | 1-10 | XSCALE | E10.0 | R | Scale factor for X coordinate card inputs to convert to meters |
|  | 11-20 | YSCALE | E10.0 | R | Scale factor for $Y$ coordinate card inputs to convert to meters |
|  | 21-30 | WSCALE | E10.0 | R | Scale factor for depth inputs to convert to meters |
|  | $31-40$ | USCALE | E10.0 | R | ```Scale factor for X direction velocity card inputs to convert to m/sec.``` |
|  | 41-50 | VSCALE | E10.0 | R | ```Scale factor for Y direction velocity card inputs to convert to m/sec.``` |
|  | 51-60 | CVF | E10.0 | R | Conversion factor between the user's concentration units and the programs internal units of $\mathrm{gm} / \mathrm{m}^{3}$. If left blank the value 0.001 is assumed and the user's units will be $\mathrm{mg} / \mathrm{m}^{3}$ |
|  | 6I-65 | -- | -- | -- | Blank |
|  | 66-70 | NSU | A4, A1 | A | A 5-character alphabetic description of the user's concentration units. This description appears on all program output and will be assumed to be mg/m if left blank |
| 4/1 | 1-5 | NTIME | I5 | I | Length of simulation, hours |
|  | 6-10 | NTSEG | I5 | I | Frequency of printout, hours |
|  | 11-20 | DELT | E10.0 | R | Length of dynamic time step in hours, usually one (1.0); for steady state set equal to zero |


| Card Sequ No. /No. of cards | Card Columns | FORTRAN Name | Format | Type* | Description |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 4 / 1 \\ \left(\text { cont }^{\prime} \mathrm{d}\right) \end{gathered}$ | 21-30 | HTN | E10.0 | R | The height above ground (meters) at which vertical CO concentrations are to be computed. This value is required only if the option switch IS8 is greater than zero |
|  | 31-40 | ADIST | E10.0 | R | The "vertical distribution parameter," necessary for triangular, trapezoidal exponential and Gaussian distributions; see above for the precise definition. This value is required only if the variable IS8 is greater than zero. |
| 5/NMAT | 1-10 | J | I10 | I | Fluid property type number |
|  | 11-20 | $\operatorname{ORT}(J, 1)$ | E10.0 | R | X-Diffusion coefficient (sq.m./sec.) |
|  | 21-30 | ORT (J, 2) | E10.0 | R | Y-Diffusion coefficient (sq.m./sec.) |
|  | $31-40$ | $\operatorname{ORT}(J, 3)$ | E10.0 | R | The decay rate of CO from an element (hr. ${ }^{-1}$ ). This is the $K$ value in Equation (II-4) |
| .6/NCL | 1-80 | NBTLIN( $\mathrm{N}, \mathrm{J}$ ) | 16 I 5 | I | An array used to define the nodes in a line across which the convective mass flow rate is to be computed and printed. Values are input only if NCL $>0$; a line may have up to 16 nodes and a maximum of 10 lines may be specified |
| 7/NPX | 1-10 | J | I10 | I | Node number at which coordinate is to be specified |
|  | 11-20 | $\operatorname{CORD}(\mathrm{J}, 1)$ | E10.0 | R | X -coordinate of node J |
|  | 21-30 | $\operatorname{CORD}(\mathrm{J}, 2)$ | E10.0 | R | $Y$-coordinate of node $J$ |
|  | $31-40$ | WD (J) | E10.0 | R | Vertical distance from the ground to the upper bounding (inversion) layer |

Card Sequence

| No. /No. <br> of cards | Card <br> Columns | FORTRAN <br> Name | Format | Type* | Description |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 8/K | 1-80 | NFIXH (J) | 16 I 5 | I | An array of node numbers in the order which their associated equations are to be ordered. Number of cards is $\mathrm{K}=\operatorname{INTEGER}(\mathrm{NP} / 16)+1$ |
|  |  |  |  | Note: | IRO $=0$, no cards are read <br> IRO $=1, K$ cards are read <br> $N P=$ maximum network node number |
| 9/NCD | 1-5 | NAME (N) | A4 | I | Any 4 character designation for an element |
|  | 6-45 | NOP (J, K) | 815 | I | The node numbers associated with element J listed counter-clockwise around the element. For a linear element 3 nodes will be listed, with 6 and 8 for triangular and quadralateral elements respectively. Fill all unused locations with zeros (the maximum allowable node number is 1000) |
|  | 46-50 | $\operatorname{IMAT}(\mathrm{J})$ | I5 | I | A four digit number which designates the emission schedule and the diffusion/decay coefficients to be associated with the element. The first two digits are the emission schedule, and the second two the diffusion/decay coefficients. (Example: to specify emission schedule 3 and coefficient set 5, enter 0305) |
| 10/IS1 | 1-5 | N | I5 | I | Node of initial condition |
|  | 6-15 | $\operatorname{TOLD}(\mathrm{N}, 1)$ | E10.0 | R | Value of initial condition for carbon monoxide, user units |
|  |  |  |  | Note: | If ISl $=0$ only one card is read; if IS1 > 0, ISI cards are required |

NOTE: To this point all input specifications are uniform regardless of the type of simulation desired. At this point, however, if velocities and/or boundary conditions are to be time dependent, skip the following instructions and go to the section entitled "Time-Dependent Input Formats." Please recall that the variable IS5 on input card 2 must be set equal to 1 if time dependent inputs are to be used.

Input Formats when Neither Velocities Nor Boundary Conditions are Time Dependent

| Card Sequ No. / No. of cards | Card Columns | FORTRAN Name | Format | Type* | Description |
| :---: | :---: | :---: | :---: | :---: | :---: |
| . 11/NVEL | 1-10 | N | I10 | I | Node number for specified velocity |
|  | 11-20 | UL (N) | E10.0 | R | X-velocity of node N |
|  | 21-30 | VL (N) | E10.0 | R | Y-velocity of node N |
|  |  |  |  | Note: | NVEL $=0$ uniform velocity field and only one velocity card is read <br> NVEL > 0. A velocity distribution is required and the program will expect NVEL input cards. If the value IS 3 on input card 2 is greater than zero the input velocities will be used as boundary conditions for a solution to the velocity distribution based on potential flow theory |
| 12/NBX | 1-10 | NBLOC (L) | I10 | I | Node number of specified boundary condition |
|  | 11-20 | $\operatorname{SPEC}(\mathrm{L}, \mathrm{M})$ | E10.0 | R | Fixed value of carbon monoxide at node $N$, user units |
| 13/1 | 11-20 | FIDLE | E10.0 | R | The amount of carbon monoxide produced by a car at idle, $\mathrm{gm} / \mathrm{hr}$ |
|  | 21-30 | COLDSF | E10.0 | R | The amount of carbon monoxide produced by a cold start, gm/start |
|  | 31-40 | ALF | E10.0 | R | The coefficient $\alpha_{m}$ used to calculate emissions from traveling automobiles (Equation II-6) |
|  | 41-50 | BET | E10.0 | R | The coefficient $\beta$ used to calculate emissions from traveling automobiles (Equation II-6) |

Card Sequence
No. /No. Card FOR'TRAN
of cards Columns Name


Input Formats when Either Velocities or Boundary Conditions are Time Dependent (the Variable IS $5 \neq 0$ on Input Card 2)

| Card Sequence <br> No. /No. <br> of cards | Card Columns | FORTRAN Name | Format | Type* | Description |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 11/1 | 1-5 | NTV | I5 | I | The logical unit number (scratch) upon which the time dependent velocities will be written. Enter zero if velocities are not time dependent |
|  | 6-10 | NVV | I5 | I | The number of sets of velocity data (only used if NTV > 0) |
|  | 11-15 | NTBC | I5 | I | The logical unit number (scratch) upon which the time dependent boundary conditions will be written. Enter zero if boundary conditions are not time dependent |
|  | 16-20 | NBCV | 15 | I | The number of sets of boundary condition data (only used if NTBC >0) |
| 12/1NVV sets | 1-5 | ITM | I5 | I | The hour (referenced to the set of velocity values) at which the following velocities apply. The initial hour must be zero in all cases |
|  | 6-10 | NPTS | I5 | I | The number of velocity values to be read. Enter zero if one card is to be read and all system values set to those values |
| Read | 1-10 | N | 110 | I | Node number for specified velocity |
| NPTS | 11-20 | UL (N) | E10.0 | R | X -velocity of node N |
| Cards | 21-30 | $\mathrm{VL}(\mathrm{N})$ | E10.0 | R | Y-velocity of node N |
| 13/NBVC sets |  |  |  |  |  |
|  | 1-5 | ITM | I5 | I | The hour (references to the set of boundary condition values) at which the following boundary conditions apply. <br> The initial hour must be zero in all cases |


| Card Sequence No. / No. of cards | Card Columns | FORTRAN Name | Format | Type* | Description |
| :---: | :---: | :---: | :---: | :---: | :---: |
| .13/NBCV sets (cont'd) | 6-10 | NPTS | I5 | I | The number of boundary condition values to be read |
| Read | 1-10 | NBLOC (L) | 110 | I | Node number of specified boundary condition |
| Cards | 11-20 | $\operatorname{SPEC}(\mathrm{L}, \mathrm{M})$ | E10.0 | $\mathrm{R}$ | Fixed value of carbon monoxide at node $N$, user units |
|  | END OF AND/OR | UT SPECIFT NDARY COND | FOR TIME-DEPENDENT VELOCITIES |  |  |
| 14/1 | 11-20 | FIDLE | E10.0 | R | The amount of carbon monoxide produced by a car at idle, gm/hr |
|  | 21-30 | COLDSF | E10.0 | R | The amount of carbon monoxide produced by a cold start, gm/start |
|  | 31-40 | ALF | E10.0 | R | The coefficient $\alpha_{m}$ used to calculate emissions from traveling automobiles (Equation II-6) |
|  | 41-50 | BET | E10.0 | R | The coefficient $\beta$ used to calculate emissions from traveling automobiles (Equation II-6) |

Card Sequence
No./No. Card FORTRAN
of cards Columns Format Type
Description


All output from the program ACOSP is via the line printer, FORTRAN logical unit 6, and closely parallels the format and logic sequence of the input data. For purposes of illustration, the output produced by a simulation quite similar to the second example problem is discussed in detail. The reader is referred to Plates IV-1 to IV-8 which are reproductions of the computer output, and which serve as the illustration for the following discussion.

The first page of output, Plate IV-1, reproduces an echo check of all mandatory input data. At the head of each page, after the constant heading block, comes the information input on the comment card. The content of the comment is completely at the user's option, and is intended to provide the mechanism for distinguishing between computer runs. The next group of output prints the values of the various option switches, plus the scale and unit conversion factors. The time and vertical distribution control parameters come next, and are followed immediately by the various diffusion coefficients. Please note that the number of different values printed under the heading "ELEMENT CHARACTERISTICS" is equal to the number specified as "DIFFUSION COEFFS" in the first group of output. If a larger number of diffusion coefficients had been specified, all would have been printed at this point. Following the mixing and decay coefficients, the lists of node numbers defining the lines across which convective mass transport is to be computed are printed. In this problem the same line has been defined in two directions to show the influence of order.

In every computer run the output contained on Plate IV-1 will be printed. If the "OUTPUT PRINT OPTION" is set equal to zero, the information contained in Plate IV-2 to IV-5 will not be printed. In this case the option is set equal to one and the data were output. Plate IV-2 indicates the location of the node points, the distance to the upper bounding layer, and the nodal order. The $X$ and $Y$ coordinates are printed for all points up to the maximum node number in the network. Note that the coordinates and vertical depths are printed after scale factor conversion, and are the final values used by

MATHFMATICAL SIMULATIUN OF CAHBON MUNOXIDE DISTHIBUIION IN THE ATHOSPHEHE TKII DIMENSIONAL FIHITE ELEMENT FOH HORI ZONTAL PRUBLEMS UF VARIABLE THICKNESS INIIIAL APPLICATION TG THE FAIHHANKS, ALASKA AIRSHED...
***** EXAMPLE PHOBLEM FUR OLSCUSSION UF PROGRAM UUTPUT *****

| NJMAFR OF FLEMENTS | 50 |
| :---: | :---: |
| NIJMAER OF $\mathrm{F}=\mathrm{C}$ |  |
| OIFFUSIJN COEFFS |  |
| CURNER NDDFS | 4 |
| OUTPUT PRINT OPTION |  |
| LINES FOH CONTIVIJNITY |  |
| TYut OF VEL field |  |
| TYPE OF INITIAL COND |  |
| HETHEDERING OPTION | 0 |
| LUGICAL UNIT(SCRATCH) | 1 |
| Velucity field option |  |
| DYMAMIC INPIT OPTION |  |
| VEHTICAL DIST OPTION |  |
| NUM EMISSIUN SCHEDLS |  |


| $x$ SCALE | $=$ | .250 |
| :--- | ---: | ---: |
| $Y$ SCALE | $=$ | .250 |
| $W$ SCALE $=$ | 10.000 |  |
| $U S C A L E ~$ | 1.000 |  |
| $V$ SCLAE $=$ | 1.000 |  |


|  | $V S C A L E$ | 1.000 |
| :--- | :--- | :--- |
| $H$ | 1.000 |  |

UNITS CTNVERSION IS $1,000-03$
EXTERNAL INTTS ARE MG/M3

TImE CUATROL AND VERTICAL DISTRIGUTION SPECS

| TOTAL HUN TIME(HOURS) | -3 |
| :--- | ---: |
| PHINT INTERVAL(HOURS) | 1 |
| TIME STEP(HOURS) | .00 |
| VERT OIST HFIGHY(H) | 1.80 |
| VERT DIST PARAMETEA | 3.00 |

## ELEMENT CHARACTERISTICS

| number | X DJFFUSION <br> (MZ/SEC) | Y DIFFUSICN <br> (MZ/SEC) | RATE OF LOSS (1/HOUR) |
| :---: | :---: | :---: | :---: |
| 1 | $1.000-02$ | 0,000 | 0,000 |

## CINTINUNITY CHECK LINES

| LINE | NODES |  |  |  |  |  |  |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 0 | 0 | 0 | 0 | 0 |
| 2 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 |  |  |  |  |  |  |  |

Plate IV-1. EXAMPLE OF PROGRAM OUTPUT
mathfiatical simulation gf cahetin monoxide distrigutiun in the atmosphebe THO DIMENSIOHAL FINITE ELEMENT FDH HORILUNTAL HRGBLEMS OF VARIABLE THICKNESS INITIAL APPLICATION TO THE FAIHBANKS, ALASKA AIHSHED...
***** EXAHPLE PROBLEM FOR DISCUSSION UF PROGRAM OUTPUT *****


the program. If nodal reordering has been input or internally calculated, the new order will appear under the column "NODAL ORDER." The user need concern himself with this order only if he wishes to check input card specifications or wants to punch an internally generated order for subsequent program input.

The element identification, connections, types and areas comprise the information shown on Plate IV-3. The specifications for each element, up to the maximum element number in the system, is printed together with its nodal points taken in a counter-clockwise direction. The element type designation is also shown, and this value corresponds to the particular set of emission schedules and coefficients assigned to the element. The element area is computed internally and is output primarily for reference and data-checking purposes. The comment "VELOCITIES AND BOUNDARY CONDITIONS ARE CONSTANT FOR RUN" is the next line of output. This comment appears as a result of the fact that the "DYNAMIC INPUT OPTION" switch has been set to zero. If this were not the case, the values associated with the input files and number of dynamic files would be printed at this point.

The data used to compute the CO loads is the next item in the output stream. Plate IV-4 shows the four constant emission factors used to estimate the CO loads for a given level of automobile activity within an element, and is followed by the hourly schedules of distribution factors for each mode of emission behavior; if more than one emission schedule is input (IS4 $>\mathrm{J}$ ), each will be printed on a separate page.

Recall that in steady state simulations the model will use only the first of these distribution factors. The next illustration, Plate IV-5, shows the input and computed $C O$ data for each system element from each source. These daily values, in grams/day/element, are distributed according to the factors shown on Plate IV-4 and their sum is the total input used by the model. At this point it should again be mentioned that the output shown on Plates IV-2 through IV-5 will not be printed if the print option switch is turned off.

```
MATHFMATICAL SIYILLATIIN OF CARBIIN MUNGXIDE OISTHIHUTIJN IN THE ATMOSPHERE
TWO DIYENSIIJNAL FINITE ELEMENT FIJR HORIZONTAL PROHLEMS IF VARIAGLE THICKNESS
INITIAL AHPLICATTINN TO THE FAIRGANKS, ALASKA AIKSHED.:.
```

***** EXAMPLE PROULEM FUR DISCUSSION DF PROGRAM DUTPUT *****

NUDAL CONNECTIONS ANO MATERIAL NUMBERS....
ELEMENT NOOES(COUNTERCLOCKWISE) TYPE AREA(MZ) ELEMENT NUDES(COUNTEHCLUCKWISE) TYPE AREA(HE)

| $\times \times 1$ | 5 | 12 | 20 | 13 | 7 | 6 | 0 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 5 | 11 | 18 | 19 | 20 | 12 | 0 | 0 |
| 3 | 18 | 25 | 33 | 20 | 20 | 19 | 0 |  |
| 4 | 18 | 24 | 31 | 32 | 33 | 25 | 0 |  |
| 5 | 31 | 3 H | 46 | 34 | 33 | 32 | 0 |  |
| 0 | 31 | 37 | 44 | 45 | 46 | 38 | 0 |  |
| 7 | 44 | 51 | 59 | 52 | 46 | 45 | 0 |  |
| 8 | ald | 50 | 57 | 58 | 59 | 51 | 0 |  |
| 0 | 57 | 64 | 72 | 65 | 59 | 59 | 0 |  |
| 10 | 57 | 63 | 70 | 71 | 72 | 64 | 0 |  |
| 11 | 70 | 77 | 85 | 78 | 72 | 71 | 0 |  |
| 12 | 70 | 76 | 83 | 84 | 45 | 77 | 0 |  |
| 13 | ${ }^{4} 3$ | 90 | 98 | 91 | 85 | 84 |  |  |
| 14 | B3 | 89 | 96 | 97 | 9月 | 90 | 0 |  |
| 15 | 95 | 105 | 111 | 104 | 98 | 97 | 0 |  |
| 10 | 96 | 102 | 109 | 110 | 111 | 103 | 0 |  |
| 17 | 109 | 116 | 124 | 117 | 111 | 110 | 0 |  |
| 18 | 109 | 115 | 132 | 123 | 124 | 116 | 0 |  |
| 19 | $1 ? 2$ | 129 | 137 | 130 | 17.4 | 123 | 0 |  |
| 20 | 12d | 128 | 134 | 136 | 137 | 129 | 0 |  |
| 21 | 3 | 10 | 10 | 17 | 18 | 11 | 5 |  |
| 22 | 10 | 23 | 27 | 31. | 31 | 24 | 18 | 17 |
| 23 | 29 | 36 | 4 ? | 43 | 44 | 31 | 31. | 30 |
| 24 | 42 | 49 | 55 | 56 | 57 | 50 | 14 | 5 |
| 23 | 55 | 62 | 68 | 69 | 70 | 65 | S7 | 56 |



| 26 | 68 | 75 | 81 | 82 | 83 | 76 | 70 | 69 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 27 | 81 | 84 | 94 | 93 | 90 | 89 | 83 | 82 |
| 28 | 94 | 101 | 107 | 108 | 109 | 102 | 96 | 95 |
| 24 | 107 | 114 | 120 | 121 | 122 | 115 | 109 | 108 |
| 30 | 120 | 127 | 133 | 134 | 135 | 128 | 122 | 121 |
| 31 | 1 | 8 | 14 | 9 | 3 | 2 | 0 | 0 |
| 32 | 14 | 15 | 16 | 10 | 3 | 9 | 0 | 0 |
| 33 | 14 | 21 | 27 | 22 | 16 | 15 | 0 | 0 |
| 34 | 27 | 28 | 29 | 23 | 16 | 22 | 0 | 0 |
| 35 | 27 | 34 | 40 | 35 | 29 | 24 | 0 | 0 |
| 36 | 40 | 41 | 42 | 36 | 29 | 35 | 0 | 0 |
| 37 | 40 | 47 | 53 | 48 | 42 | 41 | 0 | 0 |
| 34 | 53 | 54 | 55 | 44 | 42 | 48 | 0 | 0 |
| 39 | 53 | 60 | 66 | 61 | 55 | 54 | 0 | 0 |
| 40 | 66 | 67 | 68 | 62 | 55 | 61 | 0 | 0 |
| 41 | 66 | 73 | 79 | 74 | 60 | 67 | 0 | 0 |
| 42 | 79 | 80 | 81 | 75 | 68 | 74 | 0 | 0 |
| 43 | 79 | 86 | 92 | 87 | 81 | 80 | 0 | 0 |
| 44 | 92 | 93 | 94 | 88 | 81 | 81 | 0 | 0 |
| 45 | 92 | 99 | 105 | 100 | 94 | 43 | 0 | 0 |
| 46 | 105 | 106 | 107 | 101 | 94 | 100 | 0 | 0 |
| 47 | 105 | 112 | 118 | 113 | 107 | 106 | 0 | 0 |
| 48 | 118 | 119 | 120 | 114 | 107 | 113 | 0 | 0 |
| 49 | 116 | 125 | 131 | 126 | 120 | 119 | 0 | 0 |
| $\times 50$ | 131 | 132 | 133 | 127 | 120 | 126 | 0 | 0 |

.... VELOCITIES AND GUUNDARY CUNDITIONS ARE CONSTANT FOR THIS RUN .....

Plate IV-3. EXAMPLE OF PROGRAM OUTPUT

BUGMAGY OF IGPUT DATA JSED TO CALCILLATE CAPHiJN MONOXIDE LOADS
FUF INPIJT TH THE FAIRHATJS AIR IJUALITY MDMODEL...
***** EXAHPLF PRDBLEM FUR DISCUSSIUN UF PRIJGRAM UUTPUT *****


FRACTION IF DAILY C O LOAU FOH TRAVELING, IULING, STARTING AND DTHER SOUREES... A:

| hour | thavel | IDLING | STARTING | OTHER |
| :---: | :---: | :---: | :---: | :---: |
| 1 | - 000 | .000 | .000 | .000 |
| 2 | .000 | . 000 | .000 | .000 |
| 3 | . 000 | .000 | .000 | .000 |
| 4 | .000 | .000 | .000 | .000 |
| 5 | . 000 | .000 | .000 | .000 |
| 6 | . 000 | . 000 | .000 | .000 |
| 7 | .000 | .000 | .000 | .000 |
| 8 | . 000 | .000 | .000 | .000 |
| 9 | -100 | .000 | .000 | .000 |
| 10 | . 000 | .000 | .000 | .000 |
| 11 | . 00010 | .000 | .000 | . 000 |
| 12 | . 000 | .000 | .0100 | .000 |
| 13 | .000 | .000 | . 000 | . 000 |
| 14 | - 000 | . 000 | . 000 | .000 |
| 15 | .000 | .000 | .000 | .000 |
| 16 | .000 | .000 | .000 | . 000 |
| 17 | - 000 | .000 | .000 | .000 |
| 18 | -100 | .000 | . 000 | . 000 |
| 19 | . 000 | . 000 | .000 | .000 |
| 20 | .000 | . 000 | .000 | . 000 |
| 21 | -000 | .000 | .000 | .000 |
| ? ? | .000 | .000 | . 000 | .000 |
| 23 | .000 | .000 | .000 | . 000 |
| 24 | .000 | .000 | .000 | .000 |

Plate IV-4. EXAMPLE OF PROGRAM OUTPUT
$\qquad$

FRACTIUN OF DAILY C D LOAD FOR TRAVELING, IOLING, STARTING AND OTHER SOURCES.: **

| HOUR | travel | 10LING | STARTING | OTHER |
| :---: | :---: | :---: | :---: | :---: |
| 1 | . 000 | .000 | . 000 | . 000 |
| 2 | .000 | . 000 | .000 | .000 |
| 3 | .000 | .000 | .000 | .000 |
| 4 | .000 | .000 | .000 | . 000 |
| 3 | . 000 | .000 | .000 | . 000 |
| 6 | .1000 | .000 | .000 | .000 |
| 7 | .000 | .000 | .000 | . 000 |
| 8 | - 000 | .000 | .000 | . 000 |
| 9 | .000 | .000 | . 000 | . 000 |
| 10 | .000 | .000 | .000 | . 000 |
| 11 | 1000 | .000 | .000 | . 000 |
| 12 | .000 | .000 | .000 | . 000 |
| 13 | .000 | .000 | .000 | .000 |
| 14 | .000 | .000 | .000 | .000 |
| 15 | .000 | .000 | .000 | . 000 |
| 16 | . 000 | .000 | .000 | .000 |
| 17 | .000 | .000 | .000 | .000 |
| 18 | . 000 | .000 | . 000 | . 000 |
| 19 | .000 | , 000 | .000 | , 000 |
| 20 | . 000 | . 000 | .000 | .000 |
| 21 | . 000 | . 000 | .000 | .000 |
| 22 | , 000 | .000 | .000 | .000 |
| 23 | .000 | .000 | .000 | . 000 |
| ? 4 | - 000 | .000 | .000 | .000 |

## *A** EXAMPLE PROBLEM FOH DISCUSSION UF PROGRAM UUTPUT *****

AILY MASS INPUTS OF $C$ O FOR EACH FLEHENT FKOM EACH SORCE THE LOAD INPUT UNITS ARE (GM/DAY/ELEMENT)...

|  | NUMHER ELEMENT | M゙̈ロ <br> (DAY) | $\begin{aligned} & \text { BP: .DATA } \\ & \text { SPEFD } \\ & \text { (MPH) } \end{aligned}$ | IMPUTS <br> IDLING <br> (H/DY) | CLO STS <br> (DAY) | OTHER <br> (DAY) | $\begin{aligned} & \text { IRÄVEL } \\ & \text { CGMAV } \end{aligned}$ | ULATED C 1DLING (GM/DY) | EMISSION <br> STARTS <br> (GM/DY) | OTHEH <br> (GM/OY) | (GM/DY) | $\begin{aligned} & \text { CO LUAD } \\ & \text { (GM/H2) } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\times \times 1^{-}$ | (0ars | 0 | - 0 | - 0 | - 0 | . 0 | \% 0 | - 0 | (6M/O 0 | 0. | (GM.00 |
|  | X 2 | . 0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | 0. | .00 |
|  | 3 | .0 | .0 | .0 | . 0 | .0 | .0 | .0 | . 0 | .0 | 0 . | . 00 |
|  | 4 | 0 | - 0 | 0 | . 0 | .0 | . 0 | , 0 | .0 | . 0 | 0. | . 00 |
|  | 5 | .0 | .0 | .0 | . 0 | .0 | . 0 | .0 | .0 | .0 | 0. | , 00 |
|  | - 6 | . 0 | . 0 | 10 | .0 | .0 | , 0 | . 0 | .0 | . 0 | 0 , | . 00 |
|  | 7 | . 0 | .0 | .0 | . 0 | $\cdots$ | .0 | .0 | .0 | . 0 | 0. | .00 |
|  | - 8 | .0 | .0 | .0 | $\bigcirc$ | . 0 | . 0 | .0 | . 0 | . 0 | 0. | .00 |
|  | 9 | .0 | .0 | .0 | .0 | .0 | .0 | -. 0 | .0 | .0 | 0. | .00 |
|  | 10 | .0 | .0 | .0 | . 0 | .0 | . 0 | .0 | .0 | . 0 | 0. | .00 |
|  | 11 | . 0 | . 0 | .0 | . 0 | .0 | .0 | . 0 | .0 | , 0 | 0 | .00 |
|  | 12 | .0 | .0 | . 0 | . 0 | . 0 | .0 | .0 | . 0 | .0 | 0, | , 00 |
|  | 15 | .0 | .0 | 0 | . 0 | .0 | - 0 | . 0 | . 0 | .0 | 0. | .00 |
|  | 14 | . 0 | .0 | 15 | $\bigcirc$ | .0 | .0 | .0 | .0 | . 0 | 0. | .00 |
|  | 15 | .0 | . 0 | 0 | .0 | .0 | .0 | .0 | .0 | .0 | 0. | .08 |
|  | 16 | . 0 | .0 | .0 | . 0 | . 0 | . 0 | . 0 | .0 | .0 | 0. | . .00 |
|  | 17 | . 0 | .0 | . 0 | 10 | .0 | . 0 | . 0 | .0 | .0 | 0. | .00 |
|  | 18 | .0 | . 0 | .0 | . 0 | .0 | .0 | .0 | .0 | .0 | 0. | .00 |
| $\stackrel{1}{1}$ | 19 | . 0 | .0 | .0 | , 0 | .0 | .0 | .0 | . 0 | . 0 | 0. | . 00 |
| N | 20 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | . 0 | .0 | 0, | .00 |
|  | 21 | - 0 | .0 | .0 | . 0 | , 0 | .0 | - 0 | . 0 | .0 | 0. | . 00 |
|  | 22 | .0 | . 0 | . 0 | . 0 | .0 | .0 | . 0 | .0 | . 0 | 0 , | .00 |
|  | 23 | .0 | .0 | $\because 0$ | . 0 | .0 | .0 | .0 | .0 | .0 | 0. | .00 |
|  | 24 | .0 | .0 | .0 | . 0 | .0 | .0 | . 0 | . 0 | .0 | 0. | .00 |
|  | 25 | .0 | .0 | .0 | . 0 | .0 | - 0 | . 0 | .0 | .0 | 0. | .00 |
|  | 26 | .0 | .0 | .0 | .0 | .0 | .0 | 10 | .0 | .0 | 0. | .00 |
|  | 27 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | $: 0$ | 0. | .00 |
|  | 2 H | . 0 | .0 | .0 | . 0 | .0 | .0 | . 0 | . 0 | .0 | 0. | .00 |
|  | 29 | .10 | .0 | .0 | . 0 | $\bigcirc$ | . 0 | .0 | , 0 | . 0 | 0. | .00 |
|  | 50 | .0 | . 0 | .0 | . 0 | .0 | .0 | . 0 | . 0 | . 0 | 0. | .00 |
|  | 31 | .0 | .0 | .0 | .0 | .0 | - 0 | .0 | . 0 | .0 | 0. | . 00 |
|  | 32 | .0 | .0 | .0 | . 0 | .0 | .0 | .0 | $\bigcirc$ | .0 | 0. | .00 |
|  | 33 | .0 | .0 | : 0 | .0 | . 0 | .0 | .0 | .0 | . 0 | 0. | . 00 |
|  | 34 | .0 | .0 | .0 | .0 | 10 | . 0 | .0 | .0 | .0 | 0. | . 00 |
|  | 35 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | , 0 | .0 | 0. | .00 |
|  | 36 | . 0 | . 0 | . 0 | . 0 | .0 | . 0 | . 0 | . 0 | . 0 | 0. | .00 |
|  | 31 | .0 | . 0 | .0 | .0 | .0 | . 0 | . 0 | . 0 | .0 | 0. | . 00 |
|  | 3 H | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | .0 | 0. | .00 |
|  | 59 | . 0 | .0 | .0 | 10 | .0 | .0 | . 0 | . 0 | .0 | 0. | .00 |
|  | 40 | .0 | .0 | .0 | . 0 | .0 | .0 | .0 | .0 | .0 | 0. | .00 |

Plate IV-5. EXAMPLE OF PROGRAM OUTPUT

SUHMARY OF INDUT DATA HSEO TO CALCULATE CARHON MUN(IXIDE LOAOS FGR INPUT ITI THF FAIRGANKS AIR JUALITY MIIMOUEL...
***** EXAMPLE PRGBLEM FUK OISCUSSION UF PRUGRAM OUTPUT *****
daILy mass inputs of c o for each element from each sorce THE LOAD INPUT UNITS ARE (GM/DAY/ELEMENT)."


```
..SETUP CALL FOR C 0 ROUTING...
..NUMBER OF EQUATIONS IS 130 .,TOTAL STORAGE IS 2898 ...."
```

The next line of output (on Plate IV-5 In this case) is very important and should be understood fully. Either of several messages will now be printed which indicate a call to the subroutine SETUP. These calls will be for velocity generation and/or CO routing. The message for $C O$ routing will always appear, and will be preceded by the velocity generation message if the VELOCITY FIELD OPTION has been taken. The message will appear two extra times if the REORDERING OPTION has been used.

Regardless of the type of call the user should interpret the output as follows. The number of equations should be exactly equal to the total number of node points in the system minus the number of specified boundary conditions. This is a straightforward calculation for the CO routing; for the velocity field calculation it is the total number of active node points minus one. Use this value to check the proper specification of the number of boundary conditions. The second piece of information is the total storage requirement needed by the problem. If this value is 25,000 or less it will so state and the program will continue. If the requirement is more than 25,000 , the program will print a message and stop. In order to continue, the problem must be recast such that its storage requirement is less than 25,000. This will mean a decrease in the number of nodes in the system or renumbering for greater efficiency. The first thing to do is to try the REORDERING OPTION. If this does not correct the problem the user will either have to find a better order manually, reduce and renumber the network, or contact the author of the program to expand the program limits (assuming a computer with larger core storage is available).

Assuming storage requirements have been met, the output now turns to actual simulation results, and is simply repeated over and over until the simulation is complete. For dynamic problems the first output, Plate IV-6, shows the initial conditions for the mean concentrations. Each node number, its $C O$ concentration (in user units) and the wind velocities for each direction are printed in tabular form under the appropriate heading. Wherever a boundary condition has been specified, a star will appear to the right of the number. This pattern of output, with the hour to which it applies, is printed at the designated frequency until the total run time has elapsed.
***** EXAMPLE. PROBLEM FOH DISCUSSION OF PROGRAM DUTPUT *****
INITIAL CONDITIUNS


If the VERTICAL DISTRIBUTION OPTION has been used, the concentrations at the given height under the assumed distribution are printed in the same format directly after the mean concentrations. Plates IV-7 and IV-8 are examples of mean and vertically distributed output.

The output for steady state simulations is the same as shown with two minor exceptions. Since the initial conditions have no influence on the steady state results, they are not printed. Secondly, since time is not defined for steady state, the hour designation is replaced by a message indicating that the results are "STEADY STATE."

After each print cycle, the convective mass flow will be computed and printed for each "CONTINUITY LINE." The percentage calculations reference the first line designated, and the sign of the result depends on the order in which the nodes were input. This conditions is seen on Plate IV-8, where the calculation for the same line has a different sign depending on order in which its nodal points were input. The units of the output are mass flow in concentration (user units) $x$ velocity $x$ area $=$ mass/time. Please note that the mass flows follow the last page of output and will follow the mean concentrations if vertical concentrations are not computed.

MATHEMAIICAL SJMULATION (IF CAHRON MUNIXIDE DISTRIRUTIUN IN THE ATMOSHHERE IWiJ OIHENSIITAAL FINITE EI.EMENT FOK HOHIZUMTAL PHOHLEMS OF VARIABLE THICKNESS INITIAL APPLICAIIINN TU THE FAIRBANKS, ALASKA AIKSHEU...
***** EXAMPLF PRJHLEM FOR DISCUSSION OF PROGRAM OUTPUT*****


Plate IV-7. EXAMPLE OF PROGRAM OUTPUT

MATHFMATICAL SIMUIATIUN OF CAHAON HUNOXIOE IIISTRIBUTIUN IN THE ATMOSPHERF
TMU DIMENSTUHAL FINITE ELENENT FIH HIJHI LONTAL PRIAHIEMS UF VARIABLE THICKNESS
INITIAL APPLICAIIUN TO THE FAIRBANKS, QLASKA AIRSHED...
$\omega_{*}^{*} * *$ EXAMPLE PRUHLEM FUR DISCLSSION OF PROGRAM DUTPUT **** -.,RESULTS AFTER 1 HOURS UF DYNAMIC SIMULATION...

CONCENTHATIUNS ARE 1.80 METENS ABOVE GRUUIVD. "ASSUMED VERTICAL DISTRIEUTIUN IS GAUSSIAN -• OUTPUT FOR NOOES 1 TO 137 ... IHE SYMHOL DENOTES A SPECIFIED VALUE ...

|  | NODE: | CIINC | $x=V E L$ | Y-VFL | NOUE | CONC | X-VEL | Y-VEL | NUDE | CONC MG/HS | $X-V F L$ $M / Y E C$ | Y-VEL | NTOE | CONC MG/HS | $X-V E L$ $M / S E C$ | $Y=V E L$ <br> M/St |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | NUM | M(\%) M 3 | M/SEC | M/SEC | NUM | MG/MS | M/SEC | M/SEE | NUM | MG/HS | M/SEC | M/SEC | NUM | MG/HS | M/SEC | M/SEC |
|  | 1 | 2,39a | . 05 | . 00 | 36 | . 02 | . 05 | . 00 | 71 | .00 | . 05 | . 00 | 106 | . 00 | . 05 | . 00 |
|  | 2 | 2. 59. | .05 | . 00 | 37 | .02 | . 05 | .00 | 72 | .00 | . 05 | . 00 | 107 | .00 | . 05 | .00 |
|  | 3 | 2.394 | .05 | - 00 | 38 | . 02 | .05 | .00 | 73 | .00 | .05 | .00 | 108 | .00 | . 05 | .00 |
|  | 4 | 2. $30 \%$ | .05 | .00 | 39 | .02 | .05 | .00 | 74 | .00 | .05 | .00 | 109 | .00 | .05 | .00 |
|  | 5 | 2.39* | .05 | .00 | 40 | .01 | .05 | .00 | 75 | .00 | .05 | .00 | 110 | .00 | . 05 | .00 |
|  | 6 | 2,39* | .05 | .00 | 41 | . 01 | .05 | .00 | 76 | .00 | .05 | .00 | 111 | . 00 | . 05 | .00 |
|  | 7 | 2.59* | .105 | .00 | 42 | .01 | . 05 | .00 | 77 | .00 | .05 | .00 | 112 | .00 | . 05 | .60 |
|  | 8 | 1:05 | .05 | .00 | 43 | .01 | . 05 | .00 | 78 | .00 | .05 | .00 | 113 | . 00 | . 05 | . 00 |
|  | 9 | 1.02 | . 05 | .00 | 44 | .01 | . 05 | , 00 | 79 | .00 | .05 | .00 | 114 | .00 | .15 | .00 |
|  | 10 | 1.00 | .05 | .00 | 45 | .01 | .05 | . 00 | 80 | .00 | .05 | .00 | 115 | .00 | 105 | .00 |
|  | 11 | 1.00 | .05 | .00 | 46 | , 01 | .05 | .00 | 81 | .00 | .05 | .00 | 116 | . 00 | .05 | .00 |
|  | 12 | 1.0 ? | .05 | - 00 | 47 | .00 | . 05 | .00 | 82 | . 00 | . 05 | . 00 | 117 | .00 | .45 | .00 |
|  | 15 | 1.05 | .05 | .00 | $4{ }^{4}$ | .00 | .05 | .00 | 83 | .00 | .05 | .00 | . 118 | .00 | . 05 | .00 |
|  | 14 | - 29 | .05 | .00 | 49 | . 00 | . 05 | .00 | 84 | .00 | .05 | .00 | 119 | -. 00 | . 05 | .00 |
| < | 15 | .33 | .05 | .00 | 50 | .00 | .05 | . 00 | 85 | .00 | .05 | .00 | 120 | .00 | .05 | .00 |
| W | 10 | .31 | .05 | .00 | 51 | . 00 | .05 | .00 | 86 | .00 | .05 | . 00 | 121 | .00 | . 05 | .00 |
| $\infty$ | 17 | .32 | .05 | . 00 | 52 | .00 | .05 | .00 | * 7 | .00 | .05 | .00 | 122 | .00 | .05 | .00 |
|  | 18 | , 51 | .05 | .00 | 53 | .00 | .05 | .00 | 88 | .00 | .05 | .00 | 123 | -. 00 | . 05 | .00 |
|  | 19 | .33 | - 05 | . 00 | 54 | .00 | . 05 | .00 | 89 | . 00 | .05 | . 00 | 124 | .00 | . 05 | . 00 |
|  | 20 | .24 | .05 | . 00 | 55 | .00 | .US | .00 | 90 | .00 | . 05 | .00 | 125 | . 00 | . 05 | .00 |
|  | 21 | . 10 | .05 | .00 | 56 | .00 | . 05 | .00 | 91 | .00 | . 05 | .00 | 126 | -. 00 | . 05 | . 00 |
|  | 22 | .13 | . 05 | .00 | 57 | .00 | .05 | .00 | 92 | .00 | .05 | .00 | 127 | .00 | . 05 | .00 |
|  | 23 | .1 ? | .15 | .00 | SH | .00 | .05 | , 00 | 93 | .00 | . 05 | . 00 | 128 | . 00 | .05 | .00 |
|  | 2.4 | .12 | .05 | .00 | 59 | . 00 | .05 | .00 | 94 | .00 | .05 | .00 | 129 | -. 00 | . 05 | .00 |
|  | 2ら | .13 | .05 | .00 | 60 | .00 | .105 | .150 | 95 | . 00 | .05 | .00 | 150 | . 00 | . 05 | .00 |
|  | 20 | .10 | .05 | .00 | 01 | .00 | . 05 | , 00 | 96 | . 00 | . 05 | - 00 | 131 | .00 | . 05 | .60 |
|  | 27 | . 04 | .05 | . 00 | 62 | . 00 | . 05 | -00 | 97 | .00 | . 05 | .00 | 132 | -. 00 | . 05 | . 00 |
|  | 28 | .05 | .05 | .00 | 63 | .00 | .05 | .00 | 98 | .00 | , 05 | .00 | 153 | . 00 | .05 | .00 |
|  | 29 | .05 | .05 | , 00 | 64 | .00 | .05 | .00 | 99 | .00 | . 05 | .00 | 134 | .00 | . 05 | .00 |
|  | 30 | . 05 | .05 | .00 | 65 | .00 | .05 | . 00 | 100 | .00 | .05 | .00 | 135 | . 00 | . 05 | .00 |
|  | 31 | . 05 | .05 | .00 | ab | .00 | . 05 | .00 | 101 | .00 | . 05 | . 00 | 136 | .00 | . 05 | .00 |
|  | 32 | .05 | . 05 | .00 | 67 | .00 | .05 | - 00 | 102 | .00 | . 05 | .00 | 117 | .00 | .05 | 00 |
|  | 33. | .04 | . 05 | . 00 | 68 | .00 | .05 | .00 | 103 | .00 | .05 | .00 |  | - | .... |  |
|  | 34 | .02 | . 05 | .00 | 69 | .00 | . 05 | . 00 | 104 | .00 | . 05 | .00 |  |  |  |  |
|  | 35 | . 02 | .03 | .00 | 70 | .00 | .05 | .00 | 105 | .00 | .05 | .00 |  | * |  |  |

CONTINUNITY CHECKS

| LINE | TIYAL | X-FLIJN | Y-FLUW | PERCENT |
| ---: | ---: | ---: | ---: | ---: |
| 1 | $6.6 A 1-02$ | $6.681-02$ | 0.000 | 100.0 |
| 2 | $-0.661-02$ | $-6.681-02$ | 0.000 | -100.0 |

P1ate IV-8. EXAMPLE OF PROGRAM OUTPUT

## INTRODUCTION

The computer program which embodies the features of the $C O$ model is written in the FORTRAN IV computer language, and is composed of somewhat over 2000 source statement cards. The program features one main or executive routine, ACOSP, and twelve operational subroutines; a BLOCK DATA subroutine is also included. The program has been run on Univac, IBM and Honeywell computer systems, and is written to be as system independent as possible.

This chapter develops the information necessary to explore the logic of the computer program in detail. A narrative description of the function and operation of each active program routine is included, together with a flow chart for the logic of the program's sequential operations. It should be noted that the documentation provided below is not intended as a line-byline explanation of each FORTRAN statement, but rather as a mechanism of understanding, at a fine level, the sequence and logic of the computer solution. For the sake of completeness, a definition of all program variables allocated in FORTRAN labeled 'common' has also been prepared.

## PROGRAM LIMITATIONS AND OPERATIONAL REQUIREMENTS

The program ACOSP requires a large-scale digital computer system with approximately 60,000 words of core storage and a FORTRAN IV compiler. Additionally the program needs one "scratch tape" capability for all simulations, and three if dynamic boundary conditions and wind fields are used. All card input is through the system card reader, logical unit 5, and printed output on the system line printer, logical unit 6 .

The program's present dimension statements have the size limitations listed below. Problems whose size is in excess of these limits will necessitate recoding certain parts of the program.

```
Maximum number of elements = 300
Maximum node number = 1000
Maximum number of boundary conditions = 100
Maximun number of sets of diffusion coefficients = 20
Maximum number of continuity check lines = 10
Maximum equation storage = 25,000
```

PROGRAM DOCUMENTATION

OVERALL PROGRAM LOGIC

Program ACOSP is comprised of one main routine, $A C O S P$, and thirteen subroutines (including one BLOCK DATA subroutine). Program execution is initiated in $A C O S P$ and continues in a sequential manner as suggested in Figure V-1. Note that the operation of subroutines CHECK, RENUM, VDIST and VELGEN are under control of user options, and may or may not be executed in a given computer run.

Main Program ACOSP: (Flowchart is Figure V-2)

Program ACOSP is the executive routine for the carbon monoxide model and as such directs the main flow of operation during program execution. It contains no unique mathematical functions and is comprised mainly of logical operations and subroutine calls. ACOSP is the entry point for program execution and provides the normal program termination. The only other point of normal program termination is found in subroutine SETUP.

ACOSP begins by initializing a large number of program arrays and variables. The intent is to zero out all arrays prior to program execution such that the model will be computer system independent. Next, calls are made to subroutines INPUT, VELGEN, COEMIS and SETUP to accomplish data input and bookkeeping functions prior to beginning simulation.


Figure v-1. OVERALL PROGRAM LOGIC--PROGRAM ACOSP:


Figure $\mathrm{V}-2$. FLOWCHART-MAIN PROGRAM ACOSP

Next, the routine sets the time parameters in accordance with the user specifications. If a steady state solution is requested, the program will process one time loop step, and the various parameters are set accordingly. Dynamic operation requires a number of solution steps and the loop limits and time step are set by input. Note that for dynamic solutions with time steps shorter than one hour, the program will loop through the iteration cycle until an even hour has been reached. In dynamic operation, the time factor is set to 1.00 for the first step and thereafter to 1.50 . Subroutine FANDSL is next called to form and solve the equations, with the solution returned in the array $R 1$; the time derivative is computed from Equation (A-23). The program then checks to see if an output cycle has been reached and, if so, calls subroutines PRINT and CHECK. After finishing the time loop, the program makes sure the results of the last time step have been printed, after which it terminates execution.

Subroutine CHECK: (Flowchart is Figure V-3)

Subroutine CHECK has the purpose of computing and printing the convective mass flux which crosses a user specified line which is defined in terms of a list of node numbers. The output from this routine is helpful in that a user can define lines along both internal and external sections of the network and the program will automatically determine the net convective mass flux and thus provide information on overall patterns of $C O$ movement, allowing comparisons to be made for different types of wind fields and emission patterns. CHECK is called from ACOSP for each print čycle, and makes no further subroutine calls itself.

To compute the fluxes, the program integrates the components of flow in each orthogonal direction, and sums the components for the total. The program does this by entering a loop on the number of input lines and a nested loop on the user specified list of nodal connections. Each set of three nodes in a line is assumed to define an element side, and is integrated by computing the flux at each node point and weighting the end points by the factor $1 / 6$ and the mid-point by $2 / 3$. This process is repeated until either a zero node number, or the 16 th node number has been reached. CHECK prints


Figure V-3. FLOWCHART--SUBROUTINE CHECK
the flux rate across each line and makes a percentage comparison for each line compared to the first line computed. When the flux crossing all lines (maximum $=10$ ) has been computed and printed control is returned to the calling routine.

Subroutine COEMIS: (Flowchart is Figure V-4)

Subroutine COEMIS has the purpose of reading the data for, and computing the hourly rates of, carbon monoxide emissions from automobiles. The routine uses as its basis the mathematical functions indicated in Chapter II, Equation (II-5) through Equation (II-9), and makes an estimate of the CO load (in grams) for each element in the system for each hour of simulation. COEMIS is called once from ACOSP during each program execution, and makes no further subroutine calls itself.

In an operational sense, COEMIS begins by reading from the card reader the four constant emission factors. Next, the specified number of hourly distribution functions associated with traveling, idling, starting, and other sources are read. This input is followed by a loop which inputs the emission activity associated with each system element. From these data the component and the total emission rates are computed for each element by hour and written to scratch file IS2. This tape is simply rewound and reread over and over in dynamic operation. Finally, the program checks to see if the user desires printed output. If so, a summary of the CO loads is printed; otherwise, control is sent directly back to ACOSP.

Before leaving COEMIS, two things should be pointed out. First, if a steady state problem is to be run, the appropriate emission factors must be read in as the first hour's values for proper program operation. Secondly, even though COEMIS expects data for each element in the system, if there are elements with zero loads their cards may be omitted from the input stream and the program will still operate properly.


Figure V-4. FLOWCHART--SUBROUTINE COEMIS

Subroutine FANDSL has the purpose of accumulating the contribution of each nodal point to each system equation, and then of solving the resulting set of simultaneous equations for the $C O$ concentrations. The routine accumulates the nodal contributions on an element by element basis; the R.H.S. vector is calculated and modified at the same time. FANDSL is fairly general in nature and is suitable for both solution to the CO problem and the potential flow problem.

During program operation this routine is called once for each time step. If an internally generated flow field is requested, FANDSL will be called from VELGEN. In normal operation FANDSL will be called from ACOSP once for a steady state solution and once for each time step in a dynamic problem. In return, FANDSL calls STIFR once for each element active in the problem, and returns control directly to the calling routine.

Operationally, FANDSL begins by zeroing out the temporary part of the R.H.S. vector, RI. It then loops through all elements with a call to STIFR. The nodal contribution from each element is returned in ESTIFM, and stored and accumulated in SK. If the velocity field has not been updated there is no change made to $S K$; in either case, the R.H.S: vector is updated. Subsequent to completion of the formation of $S K$, the resulting set of equations, Equation ( $\mathrm{A}-26$ ), is solved. This is done by Gaussian elimination, with the forward elimination omitted if there has been no change in the velocity field. The back substitution is completed in all cases, and the solution vector resides in the array $R 1$. Control is then passed back to the calling routine where the $R I$ vector is used to supply the desired solution.

Subroutine INPUT: (Flowchart is Figure V-6)

The purpose of subroutine INPUT is to read from the card reader (logical unit 5), and to output on the line printer (logical unit 6) the majority of non-time-dependent data needed by ACOSP. The routine works in a sequential fashion, reading and printing data, making certain calculations


Figure V-5. FLOWCHART--SUBROUTINE FANDSL


Figure V-6. FLOWCHART-SUBROUTINE INPUT
and generally getting the program going. The only program data not read in INPUT are those associated with the boundary conditions, the wind field. and the $C O$ emissions, which are read in subroutines INPUT2 and COEMIS.

Subroutine INPUT is called from ACOSP once in each program execution, and calls subroutines INPUT2 and SIZE. Upon completion, control is given back to ACOSP.

INPUT always begins by reading and printing the first five program control cards. This input consists of the title, program options, scale factors, time parameters and diffusion/decay coefficients. Next is the optional input of the lines for mass flow checking but no cards are read unless requested by the user. The network coordinate and element data come next, but are not printed at this time. If the user has specified either of the non-zero renumbering options the cards are read or subroutine RENUM called as required. INPUT then computes the coordinates for any mid-side nodes which have not been input, applies the various scale factors, and zeros out the coordinates of nodes which are not included in the network (up to the maximum node number in the network). Subroutine SIZE is then called to compute all element areas.

If the print option switch, IPRT, is turned on, the coordinate and element data are now printed; otherwise output of these values is omitted. The initial conditions are the next input, with the option of either applying a single value to all system nodes or reading a specified number of cards. Subroutine INPUT2 is called to input, then called to read the wind and boundary condition data, after which progran control is returned to ACOSP.

Subroutine INPUT2: (Flowchart is Figure V-7)

Subroutine INPUT2 has the purpose of performing the input/output functions for all wind field and boundary condition data. In reality, the routine is composed of two logically separate parts, each of which has a separate function. The first part is programmed to input data from the card reader and, for dynamic problems, to write it to scratch files. The second part of the routine is used to rewind the scratch files and to input the wind, boundary condition and $C O$ emission data as the solution progresses in time.


Figure V-7. FLOWCHART--SUBROUTINE INPUT2

The choice of which section of the routine is used is made through a variable named KEY, which is in the calling sequence. If $K E Y=1$, the first part of the routine is used; if $K E Y=2$, the second.

Control reaches INPUT2 from two points. INPUT calls INPUT2 once during program execution to execute the first part of the routine. This happens only once; thereafter all calls come from ACOSP and are directed to the second half of the routine. INPUT2 is called once from ACOSP to rewind all the files, and then once for each time step. INPUT2 makes no subroutine calls of its own.

The routine starts its operations by checking the DYNAMIC INPUT OPITON. If this option has been selected a card is read, setting up the time-dependent files; otherwise the wind and $B-C$ data will be constant for the run. The wind field card data are then input, with a scratch file built if the data are dynamic. Unless a wind field is to be internally calculated, the program will interpolate any missing velocities at mid-side nodes. The input of $B-C$ data is next, and follows the same pattern. The initial data are read, and a scratch file built if the data are time varient. This operation completes the first section of the routine, and control is given back to ACOSP.

The second section of the program simply reads the data files as appropriate, and rewinds them when necessary. The first input is the wind data, followed by the boundary condition data, and the operation is completed by input of the CO emissions. The emission file is always read, but the wind and $B-C$ files are used only if they have been set up by user specification.

Subroutine PRINT: (Flowchart is Figure V-8)

Subroutine PRINT has the purpose of outputting on the line printer (logical unit 6) the CO concentrations at all node points in the system. The output is presented in terms of user units and provides the $X$ and $Y$ wind velocities in addition to the quality values. PRINT outputs both the mean vertical concentrations and the concentrations at a particular height under an assumed vertical distribution. Subroutine PRINT is called only from ACOSP, and calls subroutine VDIST if so specified. Control is always returned to ACOSP.


Figure V-8. FLOWCHART--SUBROUTINE PRINT

Operationally, PRINT begins by copying the internal concentration units ( $\mathrm{gm} / \mathrm{m}^{3}$ ) into the array TA and converting them to user units; the stars that indicate boundary conditions are entered into array TB for output. Appropriate page headings are next determined and output, followed by the concentrations and velocities at each node point up to the maximum nodal number in the network. The routine checks to see if VDIST is to be called, or if it already has been and the results printed. In either case, if the output is found to be complete, control is returned to ACOSP. If output is not complete, VDIST is called and the printing operation repeated. Subroutine PRINT is the only routine in the program which prints nodal concentrations and wind velocities.

Subroutine RENUM: (Flowchart is Figure V-9)

Subroutine RENUM has the purpose of developing an internal order for the system of network equations which will result in a more efficient storage allocation than was specified by the user. The routine uses a systematic trial and error procedure, and essentially tries a very large number of combinations to find the best sequence. Because of its trial and error nature the routine cannot be guaranteed to improve, let along optimize, the numbering sequence although it has done a good job in many difficult cases.

Subroutine RENUM is called from INPUT based on the value of the REORDERING OPTION. At most it will be called once in program execution and may not be executed at all. RENUM calls setup three times to find the storage requirements of its sequences, and returns control to INPUT.

The routine tries two separate passes through the data, once forward through the elements and once backwards. It first develops a list of all nodes connected to all other nodes from the element connections. It then works through this list in a systematic fashion, trying numbering sequences starting at different points. It will keep a sequence going until it finds it worse than the best previous attempt, or until complete. When complete, three ordering sequences are potentially available: the original, the one from the forward pass and the one from the backward pass. SETUP is called for each, and the best moved into the array NFIXH. Control is then passed back to INPUT.


Figure V-9. FLOWCHART--SUBROUTINE RENUM

Subroutine SETUP: (Flowchart is Figure V-10)

Subroutine SETUP has the purpose of developing bookkeeping arrays which define the location of each equation in the storage and solution scheme and also of assessing the total storage requirement of a problem. The routine contains several novel features which make the program flexible and allow the user or the program to internally renumber the system equations with no outward restructuring of the problem. The major intent of the routine is to allow the global coefficient matrix, $S K$, to be stored as a one-dimensional array, resulting in a considerable reduction in computer storage.

SETUP is called whenever it is necessary to structure a problem for solution or to find the storage requirements of a problem. SETUP may be called from VELGEN in a solution to the wind field problem, from RENUM as the program tries to improve its equation order, or from ACOSP in normal operation. In no case does SETUP make any further calls, and control is returned directly to the calling routine.

Operationally, the routine begins by finding those node points for which equations are to be formed. All nodes not mentioned in the definition of an element, or nodes for which a boundary conditions have been specified are deleted from consideration. Next, the order of the equations is developed, either as a straight sequence or in a manner indicated by an override in array NFIXH. A loop for each element develops the connectivity of each equation, and the maximum number of columns to the left, NLEFT, to the right, NRIGHT, is calculated. The final information saved is the sequential location of the diagonal term which is put in the array NFIX, and the maximum column to the right, NRIGHT. These two pieces of information allow the storage allocation to be handled as a one-dimensional array. The final operation is to compare the problem to the program's limits. If excess storage is required, the program halts; otherwise, it returns control to the calling routine. In either case, a message is printed indicating the results of the storage allocation.


Figure V-10. FLOWCHART--SUBROUTINE SETUP

Subroutine SIZE has the single purpose of calculating the area for each element in the network. In the case of linear elements, SIZE computes the length of the element. This subroutine is needed as the calculation of the area (or length) of an element with curved sides is not straightforward in the general sense.

To compute element areas, SIZE in effect integrates (numerically) a constant function over the element with the result that the value of the integration is the element area. To do this, the routine uses the expressions given as Equation ( $\mathrm{A}-36$ ), Equation $(A-37)$ and Equation ( $A-38$ ). The technique is quite similar to the numerical integration used in the general solution to the convection-diffusion equation, and relies on many of the same factors. SIZE is called once, from subroutine INPUT, in each program execution, and makes no further subroutine calls.

Logically, the routine consists of one large DO loop for each network element. Within this loop the program finds the type of element (linear, triangular or quadralateral) and copies the proper numerical integration factors into working storage locations. A loop is then executed for each numerical integration point (3 for linear, 7 for triangular and 9 for quadralateral) for the particular element, and the result summed into XAREA. At the conclusion of these two loops, control returns to INPUT.

Subroutine STIFR: (Flowchart is Figure V-12)

Subroutine STIFR has the purpose of developing the known part of the left and right hand side of Equation (A-17) for a single element. The routine uses numerical integration techniques to evaluate and sum the contribution of each node point in an element to the overall equation for that node. Additionally, the routine modifies the element R.H.S. vector and eliminates the boundary condition values on an element-by-element basis.


Figure V-11. FLOWCHART--SUBROUTINE SIZE


Figure V-12. FLOWCHART--SUBROUTINE STIFR

This routine is called from FANDSL for each element in the network for each solution step. It makes no further subroutine calls and returns control to FANDSL.

The routine begins by assigning the proper diffusion coefficients (for $C O$ or velocity generation) and then copies the proper numerical integration factors for the type element into working storage; the element's nodal coordinates are all localized to minimize numerical problems. A loop on the number of numerical integration points is now entered. First the Jacobian is formed, and the shape function derivatives transformed into local coordinates. Next, the value of the various system variables is evaluated at the integration point. If a new wind field has been specified, the contribution to each node point is summed into ESTIFM and copied to ELN; otherwise only the R.H.S. vector FDK is modified for time dependence. If no new boundary conditions are input, the formation is complete, and control given back to FANDSL. With new boundary conditions, the effects are eliminated from the element array ELN and stored in the permanent R.H.S. vector, RSAVE. Control is now returned to FANDSL.

Subroutine VDIST: (Flowchart is Figure V-13)

Subroutine VDIST has the purpose of computing the expected $C O$ concentration at a known distance above the ground according to an assumed vertical distribution, provided the mean vertical concentration is known. This routine is used to provide estimates of the $C O$ concentrations which might be experienced at a local "nose height" for a given vertically mixed concentration computed by the mathematical model.

To make vertical estimates, VDIST may use any of six mathematical functions which are formulated to relate the vertical distribution of $C O$ to its mean concentration. The distribution functions available are defined as Equation (IV-5) to Equation (IV-11) in Chapter IV, and require the user to provide one or two parameters for proper operation. Subroutine VDIST is called only if the user has elected the VERTICAL DISTRIBUTION OPTION in the program input. It is called from subroutine PRINT, and makes no further subroutine calls.


Figure V-13. FLOWCHART--SUBROUTINE VDIST

Operationally, VDIST simply moves to the proper calculation loop based only on the value of the option switch, IS8, and computes the vertical concentration from the mean concentration (array TOLD), and the distribution parameters $H T N$ and ADIST. The results, in user units, are returned to PRINT in the array TA.

Subroutine VELGEN: (Flowchart is Figure V-14)

Subroutine VELGEN has the purpose of developing a solution to the "potential flow problem" as formulated in Equation (IV-4). In doing so, VELGEN employs the same procedures and subroutines which are used in the CO simulation, but solves a different problem and interprets the answers in a different way. As an operational device, VELGEN copies the variables used for CO simulation into temporary storage, uses regular storage for its solution, and then restores the main arrays to their original values.

VELGEN is called once from ACOSP, and if operational will call SETUP and FANDSL; control is returned to ACOSP.

The routine begins by copying all necessary arrays to temporary storage, and calling SETUP. Next, the R.H.S. vector is modified to reflect the fact that the specified velocities are gradients of the function rather than function itself. This requires special programming to differentiate the function at the specified points. FANDSL is then called to solve the problem, and returns the value of the function at all node points. Since these values are in the transformed coordinates, they have to be modified back to the global coordinate system. The final velocities are stored in the UL and VL arrays, the original arrays restored, and control given back to ACOSP.


Figure V-14. FLOWCHART--SUBROUTINE VELGEN

In the program ACOSP all FORTRAN variables are defined in the routine in which they are used except for those values passed through LABELED COMMON, It may be helpful therefore to define these values with the knowledge that the reader can work out any other local definition in which he may be interested.

| Variable Name | $\begin{aligned} & \text { In COMMON } \\ & \text { BLOCK } \end{aligned}$ | Variable Definition |
| :---: | :---: | :---: |
| ADIST | BLK 1 | User-supplied parameter for vertical concentration distributions |
| ALPHA | BLK 1 | Weighting factor in time-integration scheme |
| $\operatorname{ATEMP}(\mathrm{I}, \mathrm{J})$ | BLK 2 | Not used |
| BVAL (I) | BLK 2 | Not used |
| $\operatorname{CORD}(\mathrm{I}, \mathrm{J})$ | BLK 3 | $X$ and $Y$ coordinates of a node |
| CVF | BLK 1 | Conversion factor to convert user mass units to grams |
| DECAY ( $I, J$ ) | BLK 4 | Not used |
| DELT | BLK 1 | The time step, seconds |
| DNAL (I, J) | BLK L | ```Derivatives of linear element shape functions evaluated at numerical integration points (defined in BLOCK DATA)``` |
| DNAR ( $\mathrm{I}, \mathrm{J}$ ) | BLK R | X derivatives of quadrilateral element shape functions evaluated at numerical integration points (defined in BLOCK DATA) |
| DNAT ( $\mathrm{I}, \mathrm{J}$ ) | BLK T | $X$ derivatives of triangular element shape function evaluated at numerical integration points (defined in BLOCK DATA) |
| $\operatorname{DNBR}(\mathrm{I}, \mathrm{J})$ | BLK R | Y derivatives of quadrilateral element shape functions evaluated at numerical integration points (defined in BLOCK DATA) |
| $\operatorname{DNBT}(\mathrm{I}, \mathrm{J})$ | BLK T | Y derivatives of triangular element shape function evaluated at numerical integration points (defined in BLOCK DATA) |
| $\operatorname{ELN}(\mathrm{I}, \mathrm{J}, \mathrm{K})$ | BLK 4 | A permanent array of the element equations which is preserved until the wind field is updated |
| $\operatorname{ESTIFM}(\mathrm{I}, \mathrm{J})$ | BLK 2 | Element contribution for node point equations |
| EXTLD (I) | BLK 4 | The element $C O$ source inputs, $\mathrm{gm} / \mathrm{sec} / \mathrm{m}^{2}$ |
| $F(I)$ | BLK 2 | Temporary element R.H.S. vector |
| FDK (I) | BLK 2 | Temporary element R.H.S. vector |


| Variable <br> Name | In COMON <br> BLOCK |
| :--- | :--- |
| HTN | BLK 1 |


| Variable Name | $\begin{gathered} \text { In COMMON } \\ \text { BLOCK } \end{gathered}$ | Variable Definition |
| :---: | :---: | :---: |
| NBLOC (I) | BLK 2 | Node numbers at which boundary conditions are to be applied |
| NBTLIN(I, J) | BLK 2 | Array of node numbers defining flux calculation Iines |
| NBX | BLK 1 | Number of specified CO boundary conditions |
| NCL | BLK 1 | Maximum number of lines across which convective transfer is to be calculated |
| NCN | BLK 1 | Number of corner nodes for a particular element (3, 6 or 8 ) |
| NCYC | BLK 1 | Not used |
| NDF | BLK 1 | Number of degrees of freedom $=1$ |
| NE | BLK 1 | Maximum network element number |
| NFIX (I) | BLK 3 | A temporary bookkeeping array used in the formation of the equation count |
| NFIXH (I) | BLK 3 | An array containing the order in which the nodes are to be considered in equation formation |
| NMAT | BLK I | Number of different set of element diffusion coefficients |
| $\operatorname{NOP}(1, J)$ | BLK 4 | The node numbers (counter clockwise) around an element |
| NP | BLK 1 | Maximum network node number |
| NPR | BLK 1 | Not used |
| NPX | BLK 1 | Number of user input coordinate points |
| NQAL | BLK 1 | Number of different quality constituents $=1$ |
| NRIGHT ( I ) | BLK 3 | Number of columns to the right of the diagonal for a particular equation |
| NSN | BLK 1 | Not used |
| NSTEP | BLK 1 | Current simulation step, hours |
| NSU (I) | BLK 2 | User concentration units |
| NSZF | BLK 1 | Maximum equation (row) number : |


| Variable Name | In corpron BLOCK | Variable Definition |
| :---: | :---: | :---: |
| NTINE | BLK 1 | Total simulation time, hours |
| NTSEG | BLK 1 | Output print interval, hours |
| NVEL | BLK 1 | Option switch for type or velocity field card input |
| ORT ( $\mathrm{I}, \mathrm{J}$ ) | BLK 2 | Array of element diffusion coefficients |
| R1(I) | BLK 3 | R.H.S. vector before solution, solution vector after |
| RSAVE (I, J) | BLK 3 | The permanent part of the R.H.S. vector which reflects elimination of boundary conditions |
| $\operatorname{SPEC}(\mathrm{I}, \mathrm{J})$ | BLK 2 | Array of specified CO boundary conditions |
| SSF | BLK 1 | One for dynamic simulations and zero for steady state problems |
| TA(I) | BLK 3 | Temporary array used in VDIST, PRINT and VELGEN |
| TB (I) | BLK 3 | Temporary array used in PRINT and VELGEN |
| TITLE ( I ) | BLK 2 | User input comment |
| TOLD (I, J) | BLK 3 | First column contains internal value of co concentrations, and second column its time derivative |
| UL (I) | BLK 3 | The X direction velocities, $\mathrm{m} / \mathrm{sec}$ |
| USCALE | BLK 1 | Scale factor for $X$ velocities to convert inputs to $\mathrm{m} / \mathrm{sec}$ |
| VL (I) | BLK 3 | The Y direction velocities, $\mathrm{m} / \mathrm{sec}$ |
| vSCALE | BLK 1 | Scale factor for $Y$ velocities to convert inputs to $\mathrm{m} / \mathrm{sec}$ |
| WAIT (I) | BLK 2 | Temporary storage of weighting factors for numerical integration |
| WaItl ( I ) | BLK L | Weighting factors for numerical integration of linear elements (defined in BLOCK DATA) |
| WAITR(I) | BLK R | Weighting factors for numerical integration of triangles (defined in BLOCK DATA) |
| WAITT(I) | BLK T | Weighting factors for numerical integration of triangles (defined in BLOCK DATA) |
|  |  | V-31 |


| Variable Name | $\begin{gathered} \text { In CONDION } \\ \text { BLOCK } \end{gathered}$ | Variable <br> Definition |
| :---: | :---: | :---: |
|  |  |  |
| WD (I) | BLK 3 | The distance to the inversion layer |
| WSCALE | BLK 1 | Scale factor for height of inversion layer to convert inputs to meters |
| XAREA ( I ) | BLK 4 | The element area, $\mathrm{m}^{2}$ |
| XKCOEF (I) | BLK 3 | Not used |
| XNL ( $\mathrm{I}, \mathrm{J}$ ) | BLK L | Linear element shape functions evaluated at numerical integration points (defined in BLOCK DATA) |
| $\mathrm{XNR}(\mathrm{I}, \mathrm{J})$ | BLK R | ```Quadrilateral element shape functions evaluated at the numerical integration points (defined in BLOCK DATA)``` |
| XNT ( $I, ~ J)$ | BLK T | ```Triangular element shape functions evaluated at the numerical integration points (defined in BLOCK DATA)``` |
| XSCALE | BLK 1 | Scale factor for $X$ coordinates to convert inputs to meters |
| YKCOEF (I) | BLK 3 | Not used |
| YSCALE | BLK 1 | Scale factor for $Y$ coordinates to convert inputs to meters |

# Appendix A <br> DERIVATION OF THE MATHEMATICAL MODEL 

## INTRODUCTION

As the basis for prediction of carbon monoxide in the atmosphere, it is necessary to develop a mathematical expression for the temporal and spatial behavior of carbon monoxide concentrations which can be formulated in terms of known physical quantities. Such an expression is available through the use of the so-called convection-diffusion equation for turbulent, incompressible flow. This relationship, which is developed in the following paragraphs, expresses a mass balance for material movement in a fixed, or Euclidian, frame of reference. The derivation given below first indicates the basic differential equation to be solved and then outlines the application of the Finite Element Method (FEM) to the solution of the equation. Further explanation and information concerning the solution to the convection-diffusion by the FEM technique can be found in several references [1,2], and the reader should investigate these sources for more detailed information if so desired.

CONVECTION-DIFFUSION EQUATION FOR TURBULENT FLOW

Consider a small fluid control volume with dimensions dx, dy and dz as suggested graphically in Figure A-1.
[1] Smith, I.M., R.V.Farraday, B.A. $0^{\prime}$ Connor. "Rayleigh-Ritz and Galerkin Finite Elements for Diffusion-Convection Problems." Water Resources Research, Vol. 9, No. 3, June 1973. pp 593-606
[2] Zienkiewicz, O.C. The Finite Element Method in Engineering Science. McGraw-Hill. London. 1971. 521 p.


Figure A-1. Flux of Substance c Through a Fluid Element

If it can be assumed that carbon monoxide is a substance which is transported by convection and turbulent mixing, and that molecular diffusion can be neglected relative to macroscopic turbulent mixing, then the mass flux of substance into the element in the $X$ direction is ( $\rho c u$ ) $d x d y$, where

$$
\begin{aligned}
u= & \text { instantaneous fluid velocity component in the } \mathrm{X} \\
& \text { direction, } \\
\mathrm{c}= & \text { density }=\frac{\text { mass of solution }}{\text { volume of solution }}, \\
& =\frac{\text { mass of substance }}{\text { mass of solution }}
\end{aligned}
$$

The net change (inflow-outflow) in mass of the volume due to flow in the $X$ direction is given by
$(\rho c u) d x d y-\left[(\rho c u) d x d y+\frac{\partial}{\partial x}(\rho c u) d x d y d z\right]=-\frac{\partial}{\partial x}(\rho c u) d x d y d z$

Performing a similar operation for the other coordinate directions, and setting the time rate of change of mass of substance within the volume to the net inflow-outflow, and accounting for local sources and sinks of material, yields the equation (after dividing by the volume, dxdydz)

$$
\begin{equation*}
\frac{\partial}{\partial t}(\rho c)+\frac{\partial}{\partial x}(\rho c u)+\frac{\partial}{\partial y}(\rho c v)+\frac{\partial}{\partial z}(\rho c w)-\rho \sigma+\rho k c=0 \tag{A-2}
\end{equation*}
$$

where

$$
\begin{aligned}
& v=\text { instantaneous fluid velocity in the } Y \text { direction, } \\
& w=\text { instantaneous fluid velocity in the } Z \text { direction, } \\
& \sigma=\text { any local sources of mass of substance, and } \\
& k=\text { the rate of decay of material from the control volume. }
\end{aligned}
$$

In most applications the density change due to changes in the diffusing substance is in the order of a few percent and may be safely neglected. If this is the case, as can be reasonably assumed to be for CO, Equation (A-2) can be simplified to the form

$$
\begin{equation*}
\frac{\partial c}{\partial t}+\frac{\partial}{\partial x}(c u)+\frac{\partial}{\partial y}(c v)+\frac{\partial}{\partial z}(c w)-\sigma+k c=0 \tag{A-3}
\end{equation*}
$$

Now, while Equation $(A-3)$ is a good approximation at a particular instant in time, in a turbulent shear flow, velocities can be expected to fluctuate around some average or mean value as suggested in Figure (A-2).

## Velocity



Figure A-2. Turbulent Unsteady Flow

The velocity field shown in this figure suggests that while a mean velocity exists for the flow, there may be large deviations about this value at any particular instant of time. To handle this mathematically, we shall define the instantaneous velocities and concentrations in Equation ( $\mathrm{A}-3$ ) in terms of a time average value (denoted by an overbar) and a fluctuating term (denoted by a prime). When this is done we can write the expressions

$$
\begin{aligned}
& \mathrm{u}=\overline{\mathrm{u}}+\mathrm{u}^{\prime} \\
& \mathrm{v}=\overline{\mathrm{v}}+\mathrm{v}^{\prime} \\
& \mathrm{w}=\overline{\mathrm{w}}+\mathrm{w}^{\prime} \\
& \mathrm{c}=\overline{\mathrm{c}}+\mathrm{c}^{\prime}
\end{aligned}
$$

By substitution of the instantaneous velocities and concentrations into Equation (A-3) and after averaging each term with respect to time,

$$
\begin{align*}
& \frac{\partial \overline{\left(\bar{c}+c^{\prime}\right)}}{\partial t}+\frac{\partial}{\partial x} \overline{\left[\left(\vec{c}+c^{\prime}\right)\left(\bar{u}+u^{\prime}\right)\right]}+\frac{\partial}{\partial y} \overline{\left[\left(\bar{c}+c^{\prime}\right)\left(\vec{v}+v^{\prime}\right)\right]} \\
&+\frac{\partial}{\partial z} \overline{\left[\left(\bar{c}+c^{\prime}\right)\left(\bar{w}+w^{\prime}\right)\right]}-\sigma+k \overline{(\bar{c}+\bar{c})}=0 \tag{A-4}
\end{align*}
$$

From the definition of our terms, the time average of fluctuating component is zero (i.e., $\int^{\Delta t} u^{\prime} d t \equiv 0$ ), and the product of a fluctuating and a mean term such as $\bar{u} \mathbf{c}^{\text { }}$ is also zero. Recognizing this condition, Equation (A-4) may be simplified to

$$
\begin{gather*}
\frac{\partial \vec{c}}{\partial t}+\frac{\partial}{\partial x}(\vec{u} \bar{c})+\frac{\partial}{\partial x} \overline{\left(u^{\prime} c^{\prime}\right)}+\frac{\partial}{\partial y}(\vec{v} \bar{c})+\frac{\partial}{\partial z} \overline{\left(v^{\prime} c^{\prime}\right)} \\
+\frac{\partial}{\partial z}(\overline{w c})+\frac{\partial}{\partial z} \overline{\left(w^{-} c^{\prime}\right)}-\sigma+k \bar{c}=0 \tag{A-5}
\end{gather*}
$$

Further, from basic fluid mechanics we know that the time averaged continuity equation for an incompressive fluid of constant density has the form

$$
\begin{equation*}
\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}+\frac{\partial \vec{w}}{\partial z}=0 \tag{A-6}
\end{equation*}
$$

which when substituted into Equation (A-5) results in

$$
\begin{align*}
& \frac{\partial \vec{c}}{\partial t}+\bar{u} \frac{\partial \bar{c}}{\partial x}+\vec{v} \frac{\partial \vec{c}}{\partial y}+\bar{w} \frac{\partial \vec{c}}{\partial z}+\frac{\partial}{\partial x} \overline{\left(u^{\prime} c^{\prime}\right)}+\frac{\partial}{\partial y} \overline{\left(v^{\prime} c^{\prime}\right)} \\
&  \tag{A-7}\\
& +\frac{\partial}{\partial z} \overline{\left(w^{\prime} c^{\prime}\right)}-\sigma+k \vec{c}=0
\end{align*}
$$

In Equation ( $\mathrm{A}-7$ ) the fluctuation product terms such as $\overline{u^{\prime} c^{\prime}}$ represent the flux of mass due to turbulent mixing, while the terms such as $\bar{u} \frac{\partial \bar{c}}{\partial \bar{x}}$ represent the changes due to average field velocities.

In order to simplify the turbulent mixing terms, a turbulent analogy is made with Fick's law of diffusion. This analogy assumes that the mass flux is in the direction of decreasing concentration. If this is the case, then

$$
\begin{aligned}
& \overline{u^{\prime} c^{\prime}}=-D_{t x} \frac{\partial \vec{c}}{\partial x} \\
& \overline{v^{\prime} c^{\prime}}=-D_{t y} \frac{\partial \bar{c}}{\partial y} \\
& \overline{w^{\prime} c^{\prime}}=-D_{t z} \frac{\partial \bar{c}}{\partial z}
\end{aligned}
$$

With these substitutions, Equation (A-7) becomes the three-dimensional convective-diffusion equation for turbulent incompressible flow:

$$
\begin{align*}
\frac{\partial \bar{c}}{\partial t}+\bar{u} \frac{\partial \bar{c}}{\partial x} & +\bar{v} \frac{\partial \bar{c}}{\partial y}+\bar{w} \frac{\partial \bar{c}}{\partial z}-\frac{\partial}{\partial x}\left(D_{t x} \frac{\partial \bar{c}}{\partial x}\right)-\frac{\partial}{\partial y}\left(D_{t y} \frac{\partial \bar{c}}{\partial y}\right) \\
& -\frac{\partial}{\partial z}\left(D_{t z} \frac{\partial \bar{c}}{\partial z}\right)-\sigma+k \bar{c}=0 \tag{A-8}
\end{align*}
$$

The turbulent diffusion coefficients $D_{t x}, D_{t y}, D_{t z}$ are not necessarily the same in the three coordinate directions. The subscript $t$ is used to designate diffusion due to the macroscopic turbulence of the fluid. The first four terms of Equation (A-8), with the exception of $\partial \bar{c} / \partial t$, represent the convective mass transfer associated with the fluid velocity. The terms on the right represent the nonadvective turbulent mass transfer. In the following development the bars denoting the time averages and the $t$ subscripts. on the diffusion coefficients are omitted.

Equation (A-8) is the complete statement of the convection-diffusion equation in three dimensions. For the purposes of $C 0$ modeling, we shall reduce this expression to a two-dimensional equation in the $x-y$ plane by assuming complete mixing in the third, or $z$, direction and integrating each term along the $z$ axis. If we assume the limit of the integration is some known value, $h$, a typical term has the form

$$
\begin{equation*}
\int_{0}^{h} \frac{\partial c}{\partial x} d z=u \frac{\partial}{\partial x} \int_{0}^{h} c d z=u \frac{\partial}{\partial x}(c h) \tag{A-9}
\end{equation*}
$$

Using this approach on each term of Equation $A-8$, and differentiating and expanding the terms, we find the relationship

$$
\begin{gather*}
h \frac{\partial c}{\partial t}+u h \frac{\partial c}{\partial x}+c u \frac{\partial h}{\partial x}+v h \frac{\partial c}{\partial y}+c v \frac{\partial h}{\partial y}-D_{x} h \frac{\partial^{2} c}{\partial x^{2}}-D_{y} h \frac{\partial^{2} c}{\partial y^{2}} \\
\therefore-D_{x} \frac{\partial h}{\partial x} \frac{\partial c}{\partial x}-D_{y} \frac{\partial h}{\partial y} \frac{\partial c}{\partial y}-h \sigma+k h c=0 \tag{A-10}
\end{gather*}
$$

Equation (A-10) is the final form of the two-dimensional, vertically integrated, convection-diffusion equation for a conservative substance in an incompressible, turbulent shear flow. Please note that this expression applies to conditions over a particular, infinitesimal control volume, and that this equation must be integrated over the problem domain to achieve a solution. The method of doing this is presented next.

The concept of the FEM has been outlined in Chapter 2. The details of how it is applied to our particular problem is now presented.

To develop a proper mathematical form for the application of the finite element method to Equation ( $\mathrm{A}-10$ ), we shall take advantage of Galerkin's technique which is a particular form of the method of weighted residuals. The idea behind this approach is that if a trial function satisfying the boundary conditions is written for a differential equation, there may be some error. Mathematically, if a function is of the form $\delta(x, y, \phi)=0$, and we assume a solution $g(x, y, \phi)$, we will have the condition

$$
\begin{equation*}
\mathrm{g}(\mathrm{x}, \mathrm{y}, \phi)=\mathrm{R} \neq 0 \tag{A-11}
\end{equation*}
$$

where
$\mathrm{R}=$ the solution residual or error.

The best trial solution will be one that, in some sense, reduces the residual (error) to a minimum at all points. One obvious way to achieve this is to formulate the problem such that

$$
\begin{equation*}
\iint W R d x d y=0 \tag{A-12}
\end{equation*}
$$

where $W$ is any function of the coordinates $x$ and $y$. This general procedure is known as the method of weighted residuals, and if the weighting function, $W$, is made equal to the shape functions defining the trial functions (see Chapter 2), the technique is called the Galerkin's method. For example, employing the normal finite element definition for variables, we can write

$$
\begin{equation*}
\phi(x, y, t)=\langle N\rangle_{i}\left\{\phi_{e}\right\}_{i} \tag{A-13}
\end{equation*}
$$

where
$\phi(x, y, t)=$ the value of $\phi$ at an arbitrary point in time and space in element i ,
$\langle\mathbb{N}\rangle_{i} \quad=$ the finite element shape (a row vector), and
$\begin{aligned}\left\{\phi_{e}\right\}_{i}= & \text { the values of } \phi \text { at the nodal points of } \\ & \text { element } i \text { (a column vector). }\end{aligned}$

If Galerkin's method is applied to Equation (A-13) we obtain

$$
\begin{equation*}
\iint\langle N\rangle \underset{i}{T} \phi d x d y=0 \tag{A-14}
\end{equation*}
$$

Equation (A-14) is the generalized statement for the application of Galerkin's method to the state variables of finite element $i$. This definition results in one linear, independent equation for each nodal point for each element. In the final solution the contribution from all elements are combined for each nodal point and solved as one set of equations.

The final statement of the finite element model is developed as follows. First, for each element in the system the Galerkin method is applied to Equation ( $\mathrm{A}-10$ ). This leads to the general form

$$
\begin{array}{r}
\iint<N>\left\{h \frac{\partial c}{\partial t}+u h \frac{\partial c}{\partial x}+c u \frac{\partial h}{\partial x}+v h \frac{\partial c}{\partial y}+c v \frac{\partial h}{\partial y}-D_{x} h \frac{\partial^{2} c}{\partial x^{2}}\right. \\
\left.\quad-D_{y} h \frac{\partial^{2} c}{\partial y^{2}}-D_{x} \frac{\partial h}{\partial x} \frac{\partial c}{\partial x}-D_{y} \frac{\partial h}{\partial y} \frac{\partial c}{\partial y}-\sigma+k c\right\} d x d y=0 \tag{A-15}
\end{array}
$$

where
$\mathrm{c}=$ the concentration of $\mathrm{CO}, \mathrm{ML}^{-3}$,
$u=$ the average velocity in the $X$ direction, $\mathrm{LT}^{-1}$,
$v \quad=$ the average velocity in the $Y$ direction, $\mathrm{LT}^{-1}$,
$h \quad=$ the height of the bounding layer, L ,

```
D
    the x direction, L' }\mp@subsup{}{}{2}\mp@subsup{\textrm{T}}{}{-1}\mathrm{ ,
D D = = the turbulent mixing coefficient associated with 
\sigma = a local term for sources or sinks of CO. MT -1 L
    (note change in units from ho to \sigma)
k = the decay rate of CO, T T , and
<N> = the finite element shape functions, unitless.
```

Next, to achieve the necessary inter-element consistency in the order of the approximation, we shall reduce the second order derivatives of Equation (A-15) to first order terms by an application of Green's transformation. This is accomplished by writing the identity

$$
\begin{equation*}
\iint \rho \frac{\partial^{2} \phi}{\partial x^{2}} d x d y=\oint \rho \frac{\partial \phi}{\partial x} d y-\iint \frac{\partial \rho}{\partial x} \frac{\partial \phi}{\partial x} d x d y \tag{A-16}
\end{equation*}
$$

Recognizing that $\rho=h D<N\rangle$, and that $\left.\frac{\partial \rho}{\partial x}=h D<N,{ }_{x}\right\rangle+D N \frac{\partial h}{\partial x}$, Equation (A-16) can be substituted into Equation (A-15), yielding

$$
\begin{align*}
& \iint\left[<N>\left\{h \frac{\partial c}{\partial t}+u h \frac{\partial c}{\partial x}+c u \frac{\partial h}{\partial x}+v h \frac{\partial c}{\partial y}+c v \frac{\partial h}{\partial y}-\sigma+k c\right\}\right. \\
&\left.\left.+D_{x} h<N,>_{x}\right\rangle^{T} \frac{\partial c}{\partial x}+D_{y} h<N,_{y}>\frac{\partial c}{\partial y}\right] d x d y \\
&+\oint h D_{x}<N>\frac{\partial c}{\partial x} d y+\oint h D_{y}<N>\frac{\partial c}{\partial y} d x=0 \tag{A-17}
\end{align*}
$$

In Equation ( $A-17$ ) it should be recognized that for all interior element boundaries the line integrals exactly cancel and their net influence at any node is zero. For external element boundaries, however, these terms do not cancel and appropriate boundary conditions must be specified or assumed. Equation (A-17) is the form of the transport equation for carbon monoxide and is the expression coded for solution in the computer code. The equation
is numerically integrated for each element in the system and a system of linear, simultaneous equations developed. These are then integrated in time to provide the desired temporal and spatial description for carbon monoxide.

## INTEGRATION IN TIME

The expression developed above is valid at all points in space at a particular instant in time. If a steady state solution is desired ( $\frac{\partial c}{\partial t} \equiv 0$ ), the appropriate terms are set to zero and a single solution is made of the equations. If a dynamic problem is specified, however, it is necessary to integrate Equation (A-17) in time as well as space. This is accomplished as follows. First, it is assumed that the value of $c$ at a particular point varies over a discrete time interval according to the relationship

$$
\begin{equation*}
c=c_{1}+a t+b t^{\alpha} \tag{A-18}
\end{equation*}
$$

where

$$
\begin{aligned}
& c_{1}=\text { the value of } c \text { at the beginning of the time step, } \\
& c \quad=\text { the value of } c \text { at the time } t, \\
& t \quad=\text { elapsed time from the beginning of the interval, and } \\
& a, b=\text { coefficients. }
\end{aligned}
$$

If Equation (A-18) is differentiated w.r.t. t, we find

$$
\begin{equation*}
\frac{\partial c}{\partial t}=a+\alpha b t^{\alpha-1} \tag{A-19}
\end{equation*}
$$

which can be substituted into Equation (A-18) to eliminate b and gives

$$
\begin{equation*}
\frac{\partial c}{\partial t}=a+\frac{\alpha}{t}\left(c-c_{1}\right)-\alpha a \tag{A-20}
\end{equation*}
$$

We also know that at $t=0, \frac{\partial c}{\partial t}=a$, or $\left(\frac{\partial c}{\partial t}\right)_{1}=a$. If this relationship is applied, and the subscript 2 is used to denote the value at the end of time interval ( $t=\Delta t$ ), we find

$$
\begin{equation*}
\left(\frac{\partial c}{\partial t}\right)_{2}=\frac{\alpha}{\Delta t}\left(c_{2}-c_{1}\right)+(1-\alpha)\left(\frac{\partial c}{\partial t}\right)_{1} \tag{A-21}
\end{equation*}
$$

Note that if $\alpha=1$, this reduces to the conventional linear integration scheme and if $\alpha=2$, the scheme is identical to the quadratic integration method. A value of $\alpha=1.5$ has been found stable for large time steps and also to give good accuracy.

To incorporate Equation ( $\mathrm{A}-21$ ) into the CO model, we make use of the idea that our solution is to be valid at a particular instant in time (the end of the time step), and that complete information is available for $c$ and its time derivative at the beginning of the time step. This being the case, Equation (A-21) can be rearranged as
or

$$
\begin{align*}
& \left(\frac{\partial c}{\partial t}\right)_{2}=\frac{\alpha}{\Delta t} c_{2}+(1-\alpha)\left(\frac{\partial c}{\partial t}\right)_{1}-\frac{\alpha}{\Delta t} c_{1}  \tag{A-22}\\
& \left(\frac{\partial c}{\partial t}\right)_{2}=\frac{\alpha}{\Delta t} c_{2}+p \tag{A-23}
\end{align*}
$$

where

$$
\begin{aligned}
& \mathrm{p}=(1-\alpha)\left(\frac{\partial c}{\partial t}\right)_{1}-\frac{\alpha}{\Delta t} c_{1} \text {, and } \\
& p=\text { known values at time } 2
\end{aligned}
$$

It can be deduced from Equation (A-17) that once the spatial and temporal terms have been collected for all elements in the system, the resulting set of equations will have the form
$[H]\left\{\frac{\partial c}{\partial t}\right\}+[K]\{C\}-\{R\}=0$
where

$$
\left.\left.\begin{array}{rl}
{[\mathrm{H}]=} & \text { a matrix of the weighting functions, } \\
{[\mathrm{K}]=} & \text { a matrix of diffusion, convection and weighting } \\
& \text { function terms, }
\end{array}\right\} \begin{array}{rl}
\left\{\frac{\partial c}{\partial t}\right\}= & \text { a vector of the time derivatives of carbon monoxide } \\
& \text { at the end of the time step, }
\end{array}\right\}
$$

If Equation (A-23) is put in matrix form, it can be substituted into Equation (A-24) with the result

$$
\begin{equation*}
\frac{\alpha}{\Delta \mathrm{t}}[\mathrm{H}]\{\mathrm{C}\}+[\mathrm{K}]\{\mathrm{C}\}-\{\mathrm{R}\}+\{\mathrm{P}\}=0 \tag{A-25}
\end{equation*}
$$

or

$$
\begin{equation*}
\left[K^{*}\right]\{C\}-\left\{R^{*}\right\}=0 \tag{A-26}
\end{equation*}
$$

where

$$
\begin{aligned}
& {\left[K^{*}\right]=\frac{\alpha}{\Delta t}[H]+[K]} \\
& \left\{R^{*}\right\}=\{R\}-\{P\}
\end{aligned}
$$

Equation (A-26) represents the final formulation of the $C O$ model; when integrated and solved these equations are said to simulate the behavior of CO within the airshed.

It will be noted that Equation $(A-26)$ is a system of integral equations which must be evaluated in the solution process. For reasons of speed and efficiency, these equations are integrated numerically rather than analytically in the computer model. A brief explanation of numerical integration is as follows.

The process of numerical integration is the evaluation of the function at specified locations within the element and then multiplying the value of the function by appropriate weighting factors and summing the result, i.e.,

$$
\begin{equation*}
\iint_{A} g(x, y) d x d y \cong A \sum_{i=1}^{q} w_{i} g\left(x_{i}, y_{i}\right) \tag{A-27}
\end{equation*}
$$

where

```
    A \(\quad=\) the area of the element,
    \(\mathrm{w}_{\mathrm{i}} \quad=\) a specified weighting factor,
    \(x_{i}, y_{i}=\) the specified coordinate locations for evaluation of \(g\), and
    \(q=\) the number of integration points.
```

In this process, $x_{i}$ and $y_{i}$ are invariant with respect to actual shape of the element. Appropriate numbers of points and their locations for given orders of error are available in the literature. In the CO model a threepoint scheme is used on linear elements, a seven-point scheme on triangles and a nine-point procedure for quadralaterals. The location and weighting factors for each of the two dimensional elements in the $C O$ model is indicated in Figure ( $\mathrm{A}-3$ ).


$$
\begin{array}{lc}
\left(x_{1}, y_{1}\right) & w_{1} \\
(0,0) & 16 / 81 \\
\left( \pm \sqrt{\frac{3}{5}} h, \pm \sqrt{\frac{3}{5}} h\right) & 25 / 324 \\
\left(0, \pm \sqrt{\frac{3}{5}} h\right) & 10 / 81 \\
\left( \pm \sqrt{\frac{3}{5}} h, 0\right) & 10 / 81
\end{array}
$$



$$
\begin{array}{ll}
\left(x_{i}, y_{i}\right) & w_{i} \\
(0,0) & 270 / 1200 \\
\left(\left(\frac{\sqrt{15}+1}{7}\right) h, 0\right) & \frac{155-\sqrt{15}}{1200} \\
\left(\left(\frac{-\sqrt{15}+1}{14}\right) h,\right. \\
\left. \pm\left(\frac{\sqrt{15}+1}{14}\right) \sqrt{3} h\right) \\
\left(\left(-\frac{\sqrt{15}-1}{7}\right) h, 0\right) & \\
\left(\left(\frac{\sqrt{15}-1}{14}\right) h, \pm\left(\frac{\sqrt{15}-1}{14}\right) \sqrt{3} h\right)
\end{array}
$$

Numerical Integration For Quadralaterals

Numerical Integration
For Triangles

Figure A-3. Numerical Integration Relationships

The CO model has the added feature that the sides of any of its elements (linear, triangular, or quadrilateral) may take on a curved shape. This is accomplished by means of a coordinate transformation as outlined below.

Recalling that basic finite element definition takes the form $\phi=\langle N\rangle\left\{\phi^{e}\right\}$, it seems a logical process to also consider the idea that the coordinates of an element could also be expressed in terms of a shape function, say $\left\langle N^{\prime}\right\rangle$, and its specified coordinate points. If this is the case we can write

$$
\begin{align*}
& x=\left\langle N^{\prime}\right\rangle\left\{x^{e}\right\}  \tag{A-28}\\
& y=\left\langle N^{\prime}\right\rangle\left\{y^{e}\right\} \tag{A-29}
\end{align*}
$$

When the shape functions for the coordinates are equal to the shape functions for the state variables ( $\left\langle N^{\prime}\right\rangle=\langle N\rangle$ ) the elements are said to be isoparametric; this is the case for the CO model.

Next, we wish to develop the expressions necessary to transform the arbitrarily specified, curved sided elements into a standard form in order that we may carry out an efficient and proper formulation, integration and solution to the problem. To do this, we must recognize first that two transformations are necessary. Since $\langle N\rangle$ is defined in terms of the local (curvilinear) coordinates, the first transformation is necessary to devise a means expressing the global derivatives in terms of the local derivatives. The second transformation is used to define the surface over which the numerical integration is to be carried out in terms of the local coordinates after an appropriate change in limits of integration.

For example, consider a set of local coordinates (associated with the numerical integration elements) $\xi, \eta$, and a corresponding set of global coordinates $x$ and $y$. By usual rules of partial differentiation we can write, for example, the $\xi$ derivative of the shape function as

$$
\begin{equation*}
\frac{\partial N_{i}}{\partial \xi}=\frac{\partial N_{i}}{\partial x} \frac{\partial x}{\partial \xi}+\frac{\partial N_{i}}{\partial y} \frac{\partial y}{\partial \xi} \tag{A-30}
\end{equation*}
$$

If this process is carried out for all terms, in matrix form we have the expression

$$
\left\{\begin{array}{c}
\frac{\partial N_{i}}{\partial \xi}  \tag{A-31}\\
\frac{\partial N_{i}}{\partial \eta}
\end{array}\right\}=\left[\begin{array}{cc}
\frac{\partial x}{\partial \xi}, & \frac{\partial y}{\partial \xi} \\
\frac{\partial x}{\partial \eta}, & \frac{\partial y}{\partial \eta},
\end{array}\right]\left\{\begin{array}{c}
\frac{\partial N_{i}}{\partial x} \\
\frac{\partial N_{i}}{\partial y}
\end{array}\right\}=[J]\left\{\begin{array}{c}
\frac{\partial N_{i}}{\partial x} \\
\frac{\partial N_{i}}{\partial y}
\end{array}\right\}
$$

In the above, the left-hand side can be evaluated as the functions of $N_{i}$ and are specified in local coordinates. Further, as $x$ and $y$ are explicitly given by the relation defining the curvilinear coordinates, the matrix [J] can be found explicitly in terms of the local coordinates. This matrix is known as the Jacobian matrix.

To find the global derivatives we invert [J] and write

$$
\left\{\begin{array}{c}
\frac{\partial N_{i}}{\partial x}  \tag{A-32}\\
\frac{\partial N_{i}}{\partial y}
\end{array}\right\}=[\mathrm{J}]^{-1}\left\{\begin{array}{c}
\frac{\partial N_{i}}{\partial \xi} \\
\frac{\partial N_{i}}{\partial \eta}
\end{array}\right\}
$$

In terms of the shape function defining the coordinate transformation we have

$$
[J]=\left[\begin{array}{cc}
\sum \frac{\partial N_{i}^{\prime}}{\partial \xi} x_{i}, & \sum \frac{\partial N_{i}^{\prime}}{\partial \xi_{i}} y_{i},  \tag{A-33}\\
& \\
\sum \frac{\partial N_{i}^{\prime}}{\partial n} x_{i}, & \sum \frac{\partial N_{i}^{\prime}}{\partial \eta} y_{i},
\end{array}\right]
$$


or, in the case of a two dimensional element

$$
[J]=\left[\begin{array}{ll}
\mathrm{J}_{11} & \mathrm{~J}_{12}  \tag{A-35}\\
\mathrm{~J}_{21} & \mathrm{~J}_{22}
\end{array}\right]
$$

For which we can find the expression

$$
[\mathrm{J}]^{-1}=\frac{1}{\operatorname{det}[\mathrm{~J}]} \quad\left[\begin{array}{cc}
\mathrm{J}_{22} & -\mathrm{J}_{12}  \tag{A-36}\\
-\mathrm{J} & \\
21 & \mathrm{~J}_{11}
\end{array}\right]
$$

and det $[\mathrm{J}]=\mathrm{J}_{11} \mathrm{~J}_{22}-\mathrm{J}_{12} \mathrm{~J}_{21}$

To transform the variables and the region with respect to the integration is a process which involves the determinant of [J]. For instance,

$$
\begin{equation*}
\operatorname{dxdy}=\operatorname{det}[J] d \xi \mathrm{~d} \eta \tag{A-37}
\end{equation*}
$$

This type of transformation is valid irrespective of the number of coordinates used. For its justification the reader is referred to standard mathematical texts.

We have now reduced the evaluation of the element properties to that of finding integrals of the form,

$$
\begin{equation*}
\int_{-1}^{1} \int_{-1}^{1}[G(\xi, \eta, \zeta,)] d \xi d \eta \tag{A-38}
\end{equation*}
$$

if the curvilinear coordinates are of the normalized type. Indeed, the integration is carried out within the normalized area and not in the complicated distorted shape, thus accounting for the simple integration limits.
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| 3 | - Uv | . 25 | 10.0 | 0 | 72 | 1.25 | 1.00 | 10.0 | 0 |
| 4 | . 00 | . 50 | 10.0 | 0 | 73 | 1.37 | .00 | 10.0 | U |
| 5 | .00 | . 75 | 10.0 | 0 | 74 | 1.37 | .15 | 10.0 | 0 |
| 6 | .00 | . 88 | 10.0 | 0 | 75 | 1.37 | . 25 | 10.0 | 0 |
| 1 | .04 | 1.00 | 10.0 | 0 | 76 | 1.37 | .75 | 10.0 | 0 |
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| 11 | .15 | . 25 | 10.0 | 0 | 79 | 1.50 | .00 | 10.0 | 0 |
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| 19 | - 2 | . H | 10.0 | 0 | P88 | 1.02 | .cb | 10.0 | 0 |
| 20 | . 25 | 1.00 | 10.0 | 0 | 49 | 1.62 | . 75 | 10.0 | 0 |
| c1 | . 3 H | .00 | 10.0 | 0 | 40 | 1.62 | . 86 | 10.0 | 0 |
| 22 | . st | .13 | 10.0 | 0 | 41 | 1.62 | 1.00 | 10.0 | 0 |
| 23 | .30 | -23 | 10.0 | 0 | 92 | 1.75 | . 60 | 10.0 | 0 |
| 24 | . 50 | . 75 | 10.0 | 0 | 93 | 1.75 | . 13 | 10.0 | 0 |
| 25. | . 38 | . 48 | 10.0 | 0 | 94 | 1.15 | - 25 | 10.0 | 0 |
| 20 | .10 | 1.00 | 10.0 | 0 | 95 | 1.75 | .50 | 10.0 | 0 |
| 21. | . 5" | . 00 | 10.0 | 0 | 96 | 1.75 | . 75 | 10.0 | 0 |
| 20 | . 50 | .13 | 10.0 | 0 | 97 | 1.75 | . 6 | 10.0 | 0 |
| 24 | - 50 | -23 | 10.0 | 0 | 94 | 1.75 | 1.00 | 10.0 | U |
| 3 u | . $\mathrm{S}^{0}$ | . 50 | 10.0 | 0 | 99 | 1.88 | . 00 | 10.0 | 0 |
| 31. | - ${ }^{4}$ | . 75 | 10.0 | 0 | 100 | 1.80 | .13 | 10.0 | 0 |
| 32 | - 56 | . 88 | 10.0 | 0 | 101 | 1.88 | . 25 | 10.0 | 0 |
| -33 | - 24 | 1.00 | 10.0 | 0 | 102 | 1.80 | . 75 | 10.0 | 0 |
| 34. | .63 | . U0 | 10.0 | $\checkmark$ | 103 | 1.88 | . 88 | 10.0 | 0 |
| 35 | .03 | .13 | 10.0 | 0 | 104 | 1.88 | 1.00 | 10.0 | 0 |
| 50 | .63 | .25 | 10.0 | 0 | 105 | 2.00 | .00 | 10.0 | 0 |
| 37 | .63 | . 15 | 10.0 | 0 | 106 | C.00 | .13 | 10.0 | 0 |
| 30 | .6. 3 | .80 | 10.0 | 0 | 107 | 2.00 | . 25 | 10.0 | 0 |
| 14 | .63 | 1.00 | 10.0 | 0 | 108 | 2.00 | . 50 | 10.0 | 0 |
| 40 | . 75 | . 04 | 10.0 | 0 | 109 | 2.00 | .75 | 10.0 | 0 |
| 41 | . 75 | .15 | 10.0 | 0 | 110 | 2.00 | . 88 | 10.0 | 0 |
| $4{ }^{4}$ | .75 | . 25 | 10.0 | 0 | 111 | 2.00 | 1.00 | 10.0 | 0 |
| 43 | . 75 | . 50 | 10.0 | 0 | 112 | C. 12 | .60 | 10.0 | 0 |
| 44 | . 76 | . 15 | 10.0 | 0 | 113 | 2.12 | . 15 | 10.0 | 0 |


|  | 45 | .15 | .88 | 10.0 | 0 | 114 | 2.12 | . 23 |  | 10.0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 40 | .75 | 1.00 | 10.0 | 0 | 115 | 2.12 | .75 |  | 10.0 | $\cup$ |
|  | 41 | . 68 | . 00 | 10.0 | 0 | 110 | 2.12 | .86 |  | 10.0 | 0 |
|  | 48 | . 68 | .15 | 10.0 | 0 | 117 | 2.12 | 1.00 |  | 10.0 | 0 |
|  | 49 | -tc | . 23 | 10.0 | 0 | 118 | 2.25 | .00 |  | 10.0 | 0 |
|  | 50 | . 68 | .15 | 10.0 | 0 | 119 | 2.25 | .13 |  | 10.0 | 0 |
|  | 51 | . 6 H | . 88 | 10.0 | 0 | 120 | 2.25 | . 25 |  | 10.0 | 0 |
|  | 56 | .86 | 1.00 | 10.0 | 0 | 121 | 2.25 | . 50 |  | 10.0 | 0 |
|  | 53 | 1.00 | .00 | 10.0 | 0 | 122 | 2.25 | . 75 |  | 10.0 | 0 |
|  | 34 | 1.00 | . 13 | 10.0 | 0 | 123 | 2.25 | . 88 |  | 10.0 | 0 |
|  | 5 | 1.00 | . 25 | 10.0 | 0 | 124 | 2.25 | 1.00 |  | 10.0 | 0 |
|  | 30 | 1.00 | . 50 | 10.0 | 0 | 125 | 2.37 | . 00 |  | 10.0 | 0 |
|  | 31 | 1.00 | . 75 | 10.0 | 0 | 126 | 2.37 | .13 |  | 10.0 | 0 |
|  | 54 | 1.60 | . 80 | 10.0 | 0 | 127 | 2.37 | . 25 |  | 10.0 | 0 |
|  | 59 | 1.00 | 1.00 | 10.0 | 0 | 128 | 2.37 | . 75 |  | 10.0 | 0 |
|  | 60 | 1.12 | .00 | 10.0 | 0 | 129 | 2.37 | . $\mathrm{H8}$ |  | 10.0 | 0 |
|  | 01 | 1.12 | .13 | 10.0 | 0 | 130 | 2. 37 | 1.00 |  | 10.0 | 0 |
|  | 02 | 1.12 | . 25 | 10.0 | 0 | 131 | 2.50 | . 60 |  | 10.0 | 0 |
|  | 03 | 1.12 | . 13 | 10.0 | 0 | 132 | 2.50 | .13 |  | 10.0 | 0 |
|  | 04 | 1.12 | -60 | 10.0 | 0 | 133 | 2.50 | . 25 |  | 10.0 | 0 |
|  | 65 | 1.12 | 1.00 | 10.0 | 0 | 134 | 2.50 | .50 |  | 10.0 | 0 |
|  | 00 | 1.25 | . 010 | 10.0 | 0 | 135 | 2.50 | . 75 | - | 10.0 | 0 |
| bod | 67 | 1.25 | .13 | 10.0 | 0 | 136 | 2.50 | .88 |  | 10.0 | 0 |
| $\omega$ | 06 | 1.25 | . 25 | 10.0 | 0 | 137 | 2.50 | 1.00 |  | 10.0 | 0 |
|  | 04 | 1.25 | . 50 | 10.0 | 0 |  |  |  |  |  |  |

MATMENATICAL SIMULATIUN UF CAKBUN MUNUXIUE UISIKIBUIIUN IF THE ATMOSPHERE
 INITIAL APPLICAIILN IU THE FAIHBANKS, ALASKA AIKSHEU..
.....t天AMPLE PHUbLEM fuNGEK 1 .... STEAOY STATE PKUOLEM WITH CONSTANI GUUNUARY

NUDAL CONIVECTIOIVS ATUO MATEHIAL NUNBEFS....
ELEMENT NUOES(CGUNIEHCLUCKWISE) TYPE AKEA(MZ) ELEMENT NUDES(CUUNTEHCLUCKHISE) TYHE AREA(M\&)
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Summany uf INPUT dala ustu iU calculafe carbuig moinuxjot luadS
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```
*...EXAMPLE PMOHLEM NUMBER 1 .... SIEADY STATE PKOBLEN WITH CUNSTANI GUUNDARY
```

| GM/HH AI IULE |  | . 00 |
| :---: | :---: | :---: |
| GM/LILUSIAMT |  | 00 |
| Srttu tmissium | faclum atpha | . 00 |
| SPEEU EMISSIUN | FaClum btla | .00 |

FHACIIUN UF UAILY C U LUAD FUR IKAVELING, IULEING, ANU STARTING......

| JU | TKavel | 10LFING | STAFIING |
| :---: | :---: | :---: | :---: |
| 1 | - U00 | . 000 | . 000 |
| 2 | - buo | . 0 U0 | 000 |
| 3 | . 000 | -006 | . 000 |
| 4 | . 000 | . 000 | . 000 |
| 3 | . 040 | -100 | . 000 |
| 0 | - 000 | . 000 | . 000 |
| 7 | . 000 | . 000 | . 000 |
| $\checkmark$ | -0uJ | . 600 | . 000 |
| 9 | . 060 | ciuo | . 000 |
| 10 | - 00.0 | . 600 | . 000 |
| 11 | -000 | . Gu0 | .000 |
| 12 | . OUO | . unu | . 000 |
| 13 | - U00 | . 000 | . 000 |
| 14 | . 000 | .000 | . 000 |
| 15 | . 000 | - Live | . 000 |
| 16 | - uou | - uou | . 0001 |
| 11 | - UuO | . 000 | . 00 |
| 18 | . Uuv | -0tu | .000 |
| 14 | - wou | . 0 U6 | . 000 |
| 20 | - uou | . 000 | . 600 |
| 21 | - 000 | . 000 | . 000 |
| 22 | . 000 | . 000 | .000 |
| 23 | . 000 | .000 | . 000 |
| 24 | - JUO | .000 | .000 |

Summary uf dnpul lafa useu iu calculait carour munuxide luads
Fuk lnful fu fht radrbafok alh gualify munulut ...

DAILY IAASS INPUIS UF $C$ O FOH EACH ELEMENT FHUM EACH SURCE
IHE LUAU INPUT UPIITS AHE (GA/OAY/ELEMENT)...

|  | elemefit | travel | IOLEIING | SIAKIING | ELEMENI | Jhavel | 1DLEING | STAH11NG |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | . 00 | . 00 | .60 | $\longleftarrow$ | . 00 | . 00 | . 00 |
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|  | 5 | . 00 | .00 | .10 | 0 | .00 | . 00 | .00 |
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|  | 9 | . U0 | .00 | . 00 | 10 | .00 | .60 | . 00 |
|  | 11 | . 00 | .00 | .00 | 12 | .00 | .00 | . 00 |
|  | 15 | .00 | .00 | .00 | 14 | .00 | . 00 | .00 |
|  | 15 | . 00 | . 00 | .00 | 16 | .00 | .00 | .00 |
|  | 11 | . 00 | .00 | .00 | 18 | .00 | .60 | .00 |
|  | 19 | . 00 | .00 | .00 | 20 | .00 | .00 | .06 |
|  | 21 | -00 | .00 | .00 | 22 | .00 | .00 | .00 |
|  | 23 | . 00 | .00 | .00 | 24 | .00 | . 00. | .00 |
|  | 25 | - 010 | .00 | .00 | 20 | .00 | .00 | . 00 |
|  | 27 | .uv | .00 | .00 | 20 | .00 | . 00 | .10 |
|  | 24 | . 00 | .00 | .00 | 30 | . 00 | . 00 | .00 |
| 1 | 31 | . 00 | .60 | .00 | 32 | .00 | . 00 | .00 |
| 0 | 31 | .00 | .00 | . 00 | 34 | .00 | . 00 | .00 |
|  | 35 | . 00 | . 00 | .00 | 30 | . 00 | - (1) | .00 |
|  | 37 | . 00 | . 00 | .00 | 90 | .00 | . 00 | .00 |
|  | 39 | . 00 | .00 | .00 | 40 | .00 | . 00 | .00 |
|  | 41 | . 60 | .00 | .00 | 42 | . 00 | .00 | . 00 |
|  | 45 | . 06 | .00 | .00 | 44 | .00 | - U̇0 | . 00 |
|  | 45 | . 00 | . 00 | . 00 | - 46 | .00 | .00 | . 00. |
|  | 47 | -10 | . 00 | . 00 | 40 | .00 | . 00 | -60. |
|  | 49 | .40 | . 00 | .00 | S0 | .00 | .00 | . 00 |

...StiUP CALL Fuh © U kuUtlig...
...Numoth uf enualiuits is 1 és ...Iutal siuhage is
2687 ....

## Malmtmalilal 5anulaijun uf caktun munuxive disiribuliun in jht atmuspmefe

## 

 INIIIAL ARPLILATILN IU IHE FAIKUANKS，ALASKA AlHSHED．．．．．．．．exariple phuolen inurbeh 1 ．．．．steauy stale phublem mith constant buundahy ．．．．Sifagr siait Sulutilin．．．．．

## ．．．Conceivihalluns afe mealr vehilcal values．．．

|  | nuve | counc | $x-v \in L$ | YOVEL | nulit | conc | $x=v \in L$ | Y－VEL | NUDE | CUNC | $x=V E L$ | Y－VEL | NUUE | ciunc | $x-v E_{L}$ | Y－VEL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | NUM | MG／P．S | M／SEC | N／SEL | Num | MG／M3 | M／SEL | M／SEC | NuM | MG／Ni3 | M／SEC | M／SEC | NUM | MG／M3 | M／DEC | M／StC |
|  | 1 | 1．01\％ | －．05 | ．$V 0$ | 36 | ． 15 | －．05 | .00 | 11 | ． 00 | －． 05 | ． 00 | 106 | －． 00 | －． 05 | ． 00 |
|  | 2 | 1．60＊ | －． 05 | ．vo | 31 | ． 6 | －． 05 | .00 | 72 | .00 | －．06 | .00 | 107 | ． 00 | －．05 | .00 |
|  | 3 | 1．00＊ | －．us | ． 00 | 36 | ． 14 | －． 05 | .00 | 73 | ． 00 | －． 05 | .00 | 148 | ． 00 | －． 0.05 | .00 |
|  | 4 | 1．0u＊ | －． 05 | －Uu | 39 | ． 05 | ． .05 | .00 | 74 | －10 | －． 05 | .00 | 109 | .00 | －． 05 | ． 00 |
|  | 5 | 1．00＊ | －．05 | ． 00 | 40 | .03 | －． 05 | .00 | 75 | .00 | －． 0.05 | .40 | 110 | －． 00 | －． 05 | .00 |
|  | 6 | 1.00 | －．us | －U0 | 41 | ． 02 | －． 0.05 | .00 | 76 | .00 | －． 05 | ． 0 | 111 | .00 | －． 0.05 | ． 00 |
|  | 7 | 1．UU＊ | －．ub | .00 | $4{ }^{4}$ | .03 | －． 05 | ． 00 | 77 | .00 | －． 05 | .00 | 112 | ．64 | －． 05 | ． 00 |
|  | 8 | ． 51 | －．ub | ．00 | 43 | ． 02 | －． 05 | － 00 | 78 | .00 | －． 05 | .00 | 113 | －． 00 | －． 03 | .00 |
|  | 9 | －ל | －．03 | ． 00 | 44 | ． 03 | －．us | .00 | 79 | ． 00 | －． 05 | ． 00 | 114 | .00 | －． 0.05 | .00 |
|  | 10 | .34 | －．us | ． 610 | 45 | .02 | －．us | .00 | 80 | －． 00 | －． 0.05 | .00 | 115 | ． 00 | －． 05 | ． 010 |
|  | 11 | －bu | －．us | － 00 | 40 | .03 | $=.05$ | .00 | E！ | ． 00 | －．0b | ． 00 | 110 | －． 00 | －． 05 | ． 00 |
| 0 | 12 | ． 35 | －．us | ． 00 | 47 | ． 02 | －．05 | ． 00 | $8 \bar{c}$ | .60 | －．05 | .00 | 117 | .00 | －． 05 | ． 00 |
| $\stackrel{1}{*}$ | 13 | － 3 | －． 05 | ．OU | 48 | .01 | －． 05 | .00 | 85 | ． 00 | －． 05 | ． 00 | 115 | .00 | －． 05 | .00 |
|  | 14 | ． 31 | －． 05 | .00 | 49 | .02 | －． 0.05 | .00 | H4 | $=.00$ | －． 0.05 | ． 00 | 119 | －． 00 | －． 05 | .00 |
|  | 15 | ． $2 ⿰ 口 口$ | －． 05 | .60 | Su | .02 | －．13 | .00 | 85 | ． 00 | －．05 | .00 | 120 | ． 00 | －． 05 | ． 00 |
|  | 16 | .29 | －．05 | －60 | 51 | .01 | －．03 | .00 | H6 | ． 00 | －． 05 | ． 00 | 121 | .00 | －．us | .00 |
|  | 11 | ． 20 | －． 05 | .00 | Si | .02 | －．05 | .00 | 41 | －． 00 | －． 05 | .00 | 122 | ． 00 | －． 05 | ． 00 |
|  | 18 | .24 | －．03 | ．U0 | 53 | ． 01 | －．05 | .00 | 88 | ． 00 | －． 05 | .00 | 123 | －． 00 | －．05 | .00 |
|  | 19 | ． 68 | －．0s | ． 00 | 54 | ． 00 | －．us | .00 | 89 | .00 | －． 05 | .00 | 124 | ． 00 | －． 05 | .60 |
|  | 20 | .31 | －． 05 | .00 | bs | ．01 | －． 0.05 | .00 | 90 | －． 00 | －． 05 | .00 | $1<5$ | ． 00 | －． 05 | .00 |
|  | 21 | .11 | －． 05 | ． 00 | 20 | ．01 | －．0b | ． 00 | 91 | ． 00 | －．ub | .60 | 120 | －． 60 | －． 05 | .00 |
|  | 22 | ． 15 | －US | － 00 | S 1 | ． 01 | －．05 | .00 | 92 | .00 | －． 05 | .00 | 127 | .00 | －． 05 | .60 |
|  | 23 | .10 | －－US | .60 | 58 | .00 | －． 05 | .00 | 43 | －． 00 | －． 05 | ． 00 | 126 | .00 | －． 05 | ． 00 |
|  | 24 | .10 | －．us | ． 00 | 54 | ． 01 | －． 05 | ． 00 | 44 | ． 00 | －． 05 | ． 00 | 129 | －． 00 | －． 05 | .00 |
|  | 23 | .15 | －．us | ． 00 | 00 | .01 | －． 05 | .00 | 95 | .00 | ． .05 | .00 | 130 | .00 | －． 05 | － 00 |
|  | 20 | .11 | －． 05 | ． 010 | 01 | .00 | －．03 | ． 00 | 90 | ． 00 | －． 05 | ． 00 | 131 | ． 00 ＊ | －． 0.05 | ． 00 |
|  | 27 | .14 | －． 05 | ．vo | O2 | ．U1 | －． 05 | .00 | 97 | －． 00 | －． 05 | .00 | 132 | .00 | ． 0.05 | ． 00 |
|  | 20 | .00 | －．us | ． 60 | 03 | .01 | －． 05 | .00 | 98 | ． 00 | －． 05 | .00 | 133 | ． 00 ＊ | －． 0.05 | ． 00 |
|  | 29． | ． 49 | －． 05 | .60 | 04 | .00 | －． 05 | .00 | 99 | ． 00 | －． 05 | ． 00 | 154 | $.00 *$ | －．05 | ． 00 |
|  | 30 | ．08 | －． 05 | ． 60 | 0 | ． 01 | －．06 | ． 00 | 100 | －． .00 | －． 05 | ． 00 | 135 | ． 00 ＊ | －．03 | ． 00 |
|  | 31 | ． 04 | －．03 | －U U | 00 | .00 | －． 65 | ． 00 | 101 | ． 00 | －． 05 | .00 | 130 | ． 60 ＊ | －．03 | ． 00 |
|  | 32 | ． 63 | －．0b | －しu | 07 | .00 | －．05 | ．00 | 102 | .00 | －． 05 | .00 | 137 | ． $00 \%$ | －． 05 | .00 |
|  | 33 | .10 | －．03 | －0u | ct | .00 | －．05 | .00 | 103 | $=.00$ | ＝． 05 | .00 |  |  |  |  |
|  | 34 | ． 0 | －． 05 | ． 60 | 09 | .00 | －． 65 | .00 | 104 | ． 00 | －． .05 | .60 |  |  |  |  |
|  | 35 | .04 | －． 05 | ．00 | 70 | .00 | －． 05 | .00 | 105 | .00 | －． 05 | .00 |  |  |  |  |

MATHEMATICAL SIMULAIIUN UF CGHUGA MGPUXIUE DISIGIGUTIUN IN THE ATMOSR HEHE
 INIIIAL AYPLICAIICIV IU IHE FALHGANKS, ALASKA AIMSHED...
.....EXAMPLE PhGELEM NUMBEH 2.... DYNAMIC HKUBLEM wITH CUNSTAMI GUUNDAKY.......

| - vonitith uf Elentivis | 50 |
| :---: | :---: |
|  | 1 |
| CItFUSIU'S CUEFFS | 1 |
| Cuniatr audes | 4 a |
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| Lltits far Luividmurily | 2 |
| frpt uf vel flelv | 0 |
| TYpt UF d'IIINL LCl* | 0 |
| MEUHUEKING UFITLA | 0 |
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| velucily fielo urijun | 0 |
| DYNAMIL INPIT UHILCN | 0 |
| VERIICAL OISI UF゙IIUN | 0 |


fIME CUNIMUL ADA VERIICAL UISAFIGUIJUN SFECS


CUNIINULYIII CHECK LIMES

| LINE | iruuts | 15 | 10 | 17 | 18 | 19 | 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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| 2 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

.... VELUCIIIES AND GUUNDAHY CUNUITIUNS ARE CONSTANI FOH THIS HUN.....

 INIIIAL APPLICATIUA IU THE FAIKGANKS, ALASKA AIHSHEO. .


FIXEU NUUAL SrECIFLCAIIUKS.....

|  | NuUt NUNBEH | $x=L(1 C$ <br> (MEIEFS) | $\begin{array}{r} Y-L U C \\ \text { (ME IEHS } \end{array}$ | $\begin{aligned} & \text { MEIGHI } \\ & \text { (METEKS) } \end{aligned}$ | nulal UKDER | NuIE number | $\begin{array}{r} x-L G C \\ \text { (METERS) } \end{array}$ | $\begin{array}{r} Y=L U C \\ (M E T E K S) \end{array}$ | HELGMI (METEKS) | nOUAL OROEN |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | . 00 | - 0 | 10.0 | 0 | 70 | 1.25 | . 7 | 10.0 | 0 |
|  | 2 | . 00 | .13 | 10.0 | 0 | 71 | 1.25 | . 88 | 10.0 | 0 |
|  | 3 | .00 | . 25 | 10.0 | 0 | 72 | 1.25 | 1.00 | 10.0 | 0 |
|  | 4 | . 60 | . 50 | 10.0 | 0 | 13 | 1.37 | .00 | 10.0 | 0 |
|  | $\rangle$ | .00 | .15 | 10.0 | 0 | 74 | 1.37 | .13 | 10.0 | 0 |
|  | 0 | .00 | . 88 | 10.0 | 0 | 75 | 1.37 | . 25 | 10.0 | 0 |
|  | 1 | . 06 | 1.00 | 10.0 | 0 | 70 | 1.37 | .75 | 10.0 | 0 |
|  | . 8 | .13 | .00 | 10.0 | 0 | 77 | 1.37 | . 68 | 10.0 | 0 |
|  | 4 | .13 | .15 | 10.0 | 0 | 78 | 1.37 | 1.00 | 10.0 | 0 |
|  | 10 | .13 | . 25 | 10.0 | 0 | 79 | 1.50 | . 00 | 10.0 | 0 |
|  | 11. | .13 | . 75 | 10.0 | 0 | 80 | 1.50 | .13 | 10.0 | 0 |
|  | 12 | .13 | . 88 | 10.0 | 0 | 41 | 1.50 | . 25 | 10.0 | 0 |
|  | 13 | 113 | 1.00 | 10.0 | 0 | 82 | 1.50 | .50 | 10.0 | 0 |
|  | 14. | - 25 | . 00 | 10.0 | 0 | H3 | 1.50 | .75 | 10.0 | 0 |
| 1 | 15 | - < | .13 | 10.0 | 0 | 64 | 1.50 | .08 | 10.0 | 0 |
| 10 | 16. | .25 | . 25 | 10.0 | 0 | 85 | 1.50 | 1.00 | 10.0 | 0 |
|  | 17. | - ${ }^{\text {c }}$ | . 30 | 10.0 | 0 | 86 | 1.62 | .00 | 10.0 | 0 |
|  | 16 | . 25 | .7b | 10.0 | 0 | 87 | 1.02 | . 13 | 10.0 | 0 |
|  | 14 | . 25 | . 8 H | 10.0 | 0 | H8 | 1.02 | - ל | 10.0 | $u$ |
|  | 20 | . 25 | 1.60 | 10.0 | 0 | 84 | 1.62 | . 75 | 10.0 | 0 |
|  | 21 | . 38 | .00 | 10.0 | 0 | 90 | 1.02 | . 06 | 10.0 | 0 |
|  | ca | . 36 | .15 | 10.0 | 0 | 91 | 1.02 | 1.00 | 10.0 | 0 |
|  | 23 | .3 n | ל | 10.0 | 0 | 92 | 1.15 | .00 | 10.0 | 0 |
|  | 24 | . 3 H | . 15 | 10.0 | 0 | 93 | 1.75 | .13 | 10.0 | 0 |
|  | d | . 34 | -ts | 10.0 | 0 | 94 | 1.75 | . 25 | 10.0 | 0 |
|  | 20 | , 3\% | 1.00 | 10.0 | 0 | 95 | 1.75 | .50 | 10.0 | 0 |
|  | c) | . 50 | .00 | 10.0 | 0 | 40 | 1.75 | . 75 | 10.0 | 0 |
|  | 20 | . 50 | .13 | 10.0 | 0 | 97 | 1.75 | . 8 H | 10.0 | U |
|  | 24 | . 50 | . 25 | 10.11 | 0 | 98 | 1.75 | 1.00 | 10.0 | 0 |
|  | 30 | . 50 | . 50 | 10.0 | 0 | 99 | 1.48 | . 00 | 10.0 | 0 |
|  | 31 | . 50 | . 75 | 10.0 | 0 | 100 | 1.48 | .13 | 10.0 | 0 |
|  | Sc | . 50 | cts | 10.0 | 0 | 101 | 1.88 | . 25 | 10.0 | 0 |
|  | 33 | . 50 | 1.00 | 10.0 | 0 | 102 | 1. BH | . 75 | 10.0 | 0 |
|  | 54 | . 03 | .0u | 10.0 | 0 | 103 | 1.80 | . 84 | 10.0 | 0 |
|  | 35 | .63 | . 15 | 10.0 | 0 | 104 | 1.88 | 1.00 | 10.0 | 0 |
|  | 30 | .63 | . 25 | 10.0 | 0 | 105 | 2.00 | .00 | 10.0 | 0 |
|  | 31 | .03 | . 75 | 10.0 | 0 | 106 | 2.00 | .13 | 10.0 | 0 |
|  | 3 H | .0s | - 4n | 10.0 | 0 | 107 | 2.00 | - 25 | 10.0 | 0 |
|  | 34 | .03 | 1.00 | 10.0 | 0 | 108 | 2.00 | . 50 | 10.0 | 0 |
|  | 40 | .75 | . 00 | 10.0 | 0 | 109 | 2.00 | .75 | 10.0 | 0 |
|  | 41 | . 15 | . 13 | 16.0 | 0 | 110 | 2.00 | . 8 | 10.0 | 0 |
|  | 4 | . 13 | -25 | 10.0 | $u$ | 111 | 2.00 | 1.00 | 10.0 | 0 |
|  | 43 | . 75 | . 50 | 10.0 | 0 | 112 | 2.12 | .00 | 10.0 | 0 |
|  | 44 | . 75 | .75 | 10.0 | 0 | 113 | 2.12 | .13 | 10.0 | 0 |



MATHEMAIICAL SINULAIIUN UF CAKBUN MUNUXJUE UISTHIBUIIUN IN IME AIMUSHHEHE
 INIIIAL APHLICAIIGN IG IHE FAIKGANKS, ALASKA AlKSHED...

nUOAL CUNIVECTIUNS ANO MAIERIAL NUMBENS....
ELEmENJ NUDES(CUUNIEHLLUCAVISE) TYHE AKEA(m己)

| 1 | 5 | 12 | 20 | 13 | 7 | - | 0 | 0 | 1 | .03 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 5 | 11 | $1 \%$ | 14 | 20 | 12 | 0 | 0 | 1 | .03 |
| 3 | 18 | 25 | 33 | 26 | 20 | 14 | 0 | 0 | 1 | . 05 |
| 4 | 18 | Cu | 51 | Si | is | c ${ }^{\text {d }}$ | 0 | U | 1 | . 03 |
| 5 | 31 | 50 | 40 | 39 | 55 | 32 | 0 | 0 | 1 | .03 |
| 6 | 31 | 57 | 44 | 45 | 40 | 5 | 0 | 0 | 1 | .03 |
| 7 | 44 | 31 | 59 | $5{ }^{5}$ | at | 45 | 0 | 0 | 1 | .03 |
| $\theta$ | 44 | 50 | 51 | be | 59 | 31 | 0 | 4 | 1 | . 03 |
| 9 | 57 | 04 | 12 | 65 | 59 | 50 | 0 | 0 | 1 | .03 |
| 10 | 57 | 03 | 10 | 71 | 72 | 04 | 0 | 0 | 1 | .03 |
| 11 | 70 | 77 | 85 | 7 H | 16 | 11 | 0 | 0 | 1 | . 03 |
| 12 | 70 | 10 | 03 | 84 | 45 | 77 | $\checkmark$ | 0 | 1 | .03 |
| 13 | 03 | 911 | 94 | 41 | H | $\mathrm{H}_{4}$ | 0 | 0 | 1 | . 03 |
| 1 is | 83 | 04 | 90 | 91 | 9 M | 90 | 0 | 0 | 1 | .03 |
| 15 | 40 | 10s | 111 | 104 | 48 | 47 | 0 | 0 | 1 | . 03 |
| 16 | 40 | 102 | 104 | 110 | 111 | 103 | 0 | 0 | 1 | -0 0 |
| 17 | 104 | 110 | 124 | 117 | 111 | 114 | 0 | 0 | 1 | . 03 |
| 18 | 104 | 113 | 122 | 123 | $1<4$ | 110 | 0 | 0 | 1 | .03 |
| 14 | 1 く2 | 124 | 157 | 150 | 124 | $1{ }^{\text {c }}$ | 0 | 0 | 1 | . 03 |
| 20 | 122 | 120 | 15 | 156 | 131 | 124 | 0 | 0 | 1 | .43 |
| 21 | 3 | 10 | 10 | 11 | $1{ }^{1}$ | 11 | 5 | 4 | 1 | .12 |
| 22 | 10 | 23 | 24 | 30 | 31 | 24 | 18 | 17 | 1 | .12 |
| 23 | c9 | 36 | 42 | 4 | 44 | 31 | 11 | 30 | 1 | .12 |
| 24 | W2 | 44 | 5 | 50 | לl | 50 | 44 | 43 | 1 | .16 |
| 25 | 5 | 0.6 | 0 O | 64 | 10 | 03 | b7 | 50 | 1 | . 12 |


| 20 | 06 | 15 | 81 | H2 | 83 | 70 | 70 | 09 | 1 | .12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\prec 7$ | 81 | 8 H | 94 | 45 | 40 | 69 | 83 | 82 | 1 | . 12 |
| 28 | 94 | 101 | 107 | 108 | 104 | 102 | 90 | 95 | 1 | .12 |
| 29 | 107 | 110 | 120 | 121 | 122 | 115 | 104 | 106 | 1 | .12 |
| 30 | 120 | 127 | 133 | 134 | 135 | 126 | 12 L | 121 | 1 | .12 |
| 31 | 1 | 8 | 14 | 4 | 3 | 2 | 0 | 0 | 1 | .03 |
| 32 | 14 | 15 | 16 | 10 | 3 | 9 | 0 | 0 | 1 | .03 |
| 33 | 14 | 21 | 27 | 22 | 10 | 13 | 0 | 0 | 1 | . 05 |
| 34 | 21 | 28 | 29 | 23 | 16 | 22 | 0 | 0 | 1 | .03 |
| 35 | 27 | 54 | 40 | 35 | 29 | C | 0 | 0 | 1 | .05 |
| 30 | 40 | 41 | 42 | 36 | 29 | 55 | 0 | 0 | 1 | .13 |
| 37 | 40 | 41 | bs | 48 | 42 | 41 | $\checkmark$ | 0 | 1 | .03 |
| 38 | 51 | 54 | 56 | 49 | 42 | 40 | 0 | 0 | 1 | . 05 |
| 34 | 33 | 06 | 66 | 61 | bs | 54 | 0 | 0 | 1 | .03 |
| 40 | 06 | 67 | 68 | $6{ }^{6}$ | 55 | 61 | 0 | 0 | 1 | .05 |
| 41 | 60 | 13 | 79 | 74 | 68 | 67 | 0 | 0 | 1 | . 03 |
| 42 | 19 | ¢0 | H1 | 75 | 68 | 14 | 0 | 0 | 1 | .03 |
| 43 | 19 | 66 | 92 | $\triangle 7$ | 61 | 80 | 0 | 0 | 1 | .03 |
| 44 | 92 | 93 | 94 | 88 | 81 | 87 | $v$ | 0 | 1 | .03 |
| 45 | 42 | 99 | 103 | 100 | 94 | 43 | 0 | 0 | 1 | .03 |
| 40 | 105 | 106 | 107 | 101 | 94 | 100 | 0 | 0 | 1 | .03 |
| 47 | 105 | 112 | 118 | 113 | 107 | 100 | 0 | 0 | 1 | .43 |
| 4t | 118 | 114 | 120 | 114 | 101 | 113 | 0 | 0 | 1 | .03 |
| 49 | 118 | 125 | 131 | 126 | 120 | 119 | 0 | 0 | 1 | .03 |
| bo | 131 | 132 | 133 | 127 | 120 | 120 | 0 | 0 | , | 03 |

sumimart uf infut uata usté iu calculate cahbun munuxide luaus FUR INHUI IU IHE FAIGBARAS AIH ULALIIY MGNUULL..
.....EXAMPLE PHLGLEM NUMBEE 2 .... UYNAMIC FKUGLEM WIIH CUNSTANT BUUNOARY.......

| GM/HH AJ IULE |  | . 00 |
| :---: | :---: | :---: |
| LM/CULUSIAKT |  |  |
| Sretu tmissiuv | factin altra | 00 |
| spttu tMLSSIUIV | faciun otia | .00 |

FHACIIUN UF DAILY C U LUAD FUH IHAVELING, IDLEING, AND STAKIING.....

| HUUR | thavel | IDLEJUG | STAFIING |
| :---: | :---: | :---: | :---: |
| 1 | . UUO | . 000 | . 000 |
| 2 | -uvo | .000 | .000 |
| 3 | .003 | . Guo | .1100 |
| 4 | - U00 | . 000 | .000 |
| 5 | - UuO | .600 | .000 |
| 6 | . 000 | . 000 | .000 |
| 7 | . 000 | - und | . 000 |
| 8 | . 000 | .000 | . 000 |
| 4 | - uvo | . 000 | .000 |
| 10 | . 000 | . 000 | .000 |
| 11 | .000 | .000 | .000 |
| 12 | . 000 | . 000 | .000 |
| 13 | - uou | . 000 | . 000 |
| 14 | . 000 | . 610 | . 000 |
| 15 | . 000 | -600 | .000 |
| 10 | - UリU | . 600 | .000 |
| 17 | -000 | . 000 | .000 |
| 18 | . 00 ¢ | .000 | .600 |
| 19 | . 000 | .001) | - 000 |
| 20 | - uuu | . 0 00 | . 000 |
| 21 | . UVO | .000 | . 010 |
| 22 | . 000 | .000 | - 000 |
| 23 | - uou | . 000 | . 000 |
| 24 | . 000 | - 000 | .000 |

SUMMAFIY GF IYPUT UATA USEL IU CALLULATE CAKUUIY MUNUXIDE LUAUS FUK IIVPUI TO IHE FAImbAMAS AIH GUALIIY MGMUUEL...

DAILY MASS INPUTS UF L U FUH EACH ELEMENT FRUM EACH SURLE IHE LUAD INPUT UNIIS AHt (GM/UAY/ELEMENI)...

|  | ELEMENI | fhavel | IOLEIMG | STARTING | ELEMENI | ikAvel | IDLELIVG | STAKTING |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | - wo | . 10 | .00 | 2 | . 00 | . 00 | . 00 |
|  | 5 | .00 | . Uu | .00 | 4 | .00 | . 00 | .00 |
|  | 5 | .00 | . 00 | .00 | 0 | .00 | .00 | -ub |
|  | 7 | . 00 | . 04 | .00 | 8 | . 00 | . 00 | . 00 |
|  | 9 | .00 | .00 | .00 | 10 | .00 | .00 | .00 |
|  | 11 | .00 | . 00 | .00 | 12 | . 00 | . 00 | .00 |
|  | 13 | . 00 | . U6 | .00 | 14 | .00 | .00 | .00 |
|  | 15 | .00 | . 00 | . 00 | 10 | . 00 | .00 | . 00 |
|  | 17 | . 00 | .60 | .60 | 18 | .00 | .un | .00 |
|  | 14 | -00 | - UO | .00 | 20 | . 00 | .60 | .60 |
|  | 21 | - U0 | . 00 | . 00 | 22 | .00 | .00. | .00 |
|  | CS | .60 | .00 | .00 | 24 | .00 | . 00 | .00 |
|  | 25 | . 010 | .00 | .00 | co | .00 | . Uu | .00 |
|  | 27 | . 16 | . 00 | .00 | 20 | .00 | .00 | .00 |
|  | 24 | .60 | . U0 | .00 | 30 | . 00 | . 06 | .00 |
|  | 31 | . 60 | .00 | .00 | 32 | .00 | .60 | .60 |
| $\stackrel{1}{\sim}$ | 33 | . 06 | .00 | . 00 | 34 | . 00 | .00 | . 00 |
| $\omega$ | 35 | .60 | .00 | . 00 | 36 | . 00 | .00 | . 00 |
|  | 37 | .00 | . 00 | . 00 | 58 | .00 | .00 | .00 |
|  | 34 | .00 | .00 | . 00 | 40 | - 00 | . 00 | . 00 |
|  | 41 | .60 | .40 | .00 | 42 | . 00 | .00 | . 00 |
|  | 43 | .00 | . 00 | .00 | 44 | .00 | . 00 | . 00 |
|  | 45 | . 00 | . 00 | - 00 | 46 | .00 | . 00 | . 00 |
|  | 41 | . 00 | . 00 | .00 | 48 | .00 | .00 | . 00 |
|  | 44 | .00 | .00 | .00 | 50 | .00 | .00 | .00 |

*..Stiup call fuh l u huuijag...

2098 .....
malhematical simulailuiv gif catibur munuxiue disikiguilun in the atmosphehe Tind dIMENSIUNAL FINIIE ELENENI FQH HUKIZLIVTAL PRUBLENS UF VAHIABLE TMICKNESS LNIJIAL APPLICATIUN TU JHE FAIHGANKS, ALASKA AlHSHEU...
.....EXAMPLE PHUBLEN ivUMHEH $2 \ldots$... DYNAMIC PHU日LEM WIIH CUNSTANT BOUNDAKY.......

INITIAL CUNUITIUNS

|  | .... output fuh inuuts |  |  | 110 | $157 \ldots$ | Tht SyMbul * U |  | IES A SPECIFItD |  | value. |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | NUUE | C 6114 | $x \sim V E L$ | r-vtL | Nưt | CUNL | $x$-vtL | Y-VEL | nude | cunc | $x$-VEL | $Y$-VEL | nube | colve | $x-v t L$ | Y-VEL |
|  | NUM | MG/m3 | M/SEC | M/SEC | Ivum. | MG/MS | M/SEC | M/SEC | NUM | MG/M3 | M/SEC | M/SEC | MuM | MG/MS | M/StC | M/SEC |
|  | 1 | 1.00* | . 05 | . 00 | 36 | - U 0 | . 05 | . 00 | 71 | . 00 | .05 | . 00 | 106 | .00 | . 05 | . 00 |
|  | 2 | 1.00 * | . | - 40 | $3 \%$ | .00 | . 05 | .00 | 72 | .00 | . 05 | .00 | 107 | . 10 | . 05 | . 00 |
|  | . 3 | $1.00 *$ | . 05 | .00 | 38 | . 00 | . 05 | .00 | 73 | . 00 | . 05 | .00 | 108 | . 00 | . 05 | . 00 |
|  | . 4 | 1.00* | .05 | . 10 | 34 | . 00 | . 05 | .00 | 74 | .00 | . 05 | .00 | 164 | .00 | . 05 | - 00 |
|  | $b$ | 1.00* | -US | .00 | 40 | . 00 | . 05 | .00 | 75 | .00 | .05 | .00 | 110 | . 00 | . 05 | - 00 |
|  | 6 | 1.00 * | . 05 | . 00 | 41 | . 00 | . 05 | .00 | 70 | .00 | . 05 | . 00 | 111 | . 00 | . 05 | - 04 |
|  | 1 | 1.00 . | .05 | .60 | 42 | .00 | . US | .00 | 71 | .00 | .05 | .00 | 112 | . 00 | . 05 | . 10 |
|  | 8 | - UU) | .05 | . 00 | 43 | . 60 | . 05 | .00 | 78 | .00 | . 05 | . 00 | 113 | . 00 | . 05 | . 00 |
|  | 9 | .00 | .us | - (10) | 44 | .00 | . 0 | . 00 | 79 | .00 | . 05 | .00 | 114 | .00 | .05 | .00 |
|  | 10 | .00 | .05 | .00 | 45 | . 00 | . 03 | .00 | 80 | . 00 | . 05 | . 00 | 115 | .00 | . 05 | . 00 |
| 1 | 11 | . 00 | . 05 | . 00 | 00 | - U0 | . 05 | .00 | 81 | . 00 | . 05 | . 00 | 116 | .00 | .05 | . 00 |
| $\xrightarrow{+}$ | 12 | . 00 | . 05 | . 00 | 47 | . 00 | .05 | .00 | 42 | . 00 | .05 | .00 | 117 | . 00 | . 05 | . 00 |
| - | 13 | . 00 | .us | - vio | ut | .00 | . 05 | .00 | 83 | . 00 | . 05 | . 00 | 118 | . 00 | . 05 | . 00 |
|  | 14 | .00 | . 05 | . 00 | 49 | . 00 | . 03 | .00 | 84 | . 00 | . 05 | .60 | 114 | .00 | .05 | . 00 |
|  | 13 | -vo | . 03 | . 04 | bu | .00 | .us | . 00 | 85 | .00 | . 05 | . 00 | 120 | . 04 | .05 | .00 |
|  | 10 | - 00 | . 05 | - Uu | 51 | .00 | . 05 | .00 | 86 | .00 | . 05 | . 00 | 121 | . 00 | .05 | . 00 |
|  | 17 | .00 | -us | -0u. | bid | . 00 | . 0 | . U 0 | 87 | . 00 | . 05 | . 00 | 122 | .00 | .05 | . 00 |
|  | 18 | .00 | - 4 | . 00 | 53 | . U0 | . 05 | . 00 | 88 | . 00 | - is | . 00 | 123 | .00 | . 05 | . 00 |
|  | 14 | . Uu | .us | .vo | b4 | . 40 | . 05 | .00 | 89 | . 00 | . 05 | . 00 | 124 | . 00 | . 05 | . 00 |
|  | 20 | . 00 | . is | . 00 | Ss: | .00 | .us | .00 | 40 | .00 | . 05 | . 00 | 125 | . 00 | . 05 | .00 |
|  | 21 | . 00 | .us | . UV | 50 | .00 | .0) | .00 | 91 | .00 | . 05 | .00 | 120 | .00 | . 05 | . 00 |
|  | 22 | .uv | . 05 | -110 | b7 | . 00 | . 05 | . 00 | 92 | .00 | . 05 | . 00 | 127 | - 00 | .05 | . 00 |
|  | 23 | . 00 | .05 | . 00 | 58 | .00 | . 05 | .00 | 43 | .00 | . 05 | -00 | 128 | . 00 | . 05 | . 00 |
|  | 24 | -00 | . 05 | .00 | 59 | . 00 | . 05 | .100 | 44 | .00 | . 05 | .00 | 124 | . 00 | .05 | .00 |
|  | 25 | -uv | .05 | . 00 | no | . 00 | .05 | .00 | 95 | . 00 | . 15 | . 00 | 130 | . 00 | .05 | . 00 |
|  | 20 | . 06 | .1) | . 40 | 01 | .00 | -ub | .00 | 90 | . 00 | . 05 | .00 | 131 | . 00 | . 05 | . 00 |
|  | 21 | . UV | . 05 | . 00 | 02 | .00 | . 05 | .00 | 97 | .00 | . 05 | . 00 | 152 | .00 | . 05 | .00 |
|  | 28. | . 00 | .05 | . 00 | cs | .00 | .05 | .00 | 98 | . 00 | . 05 | . 00 | 153 | .00 | .05 | .00 |
|  | 24. | .00 | .us | . 06 | 64 | .00 | . 05 | . 00 | 94 | . 00 | . 05 | . 00 | 154 | . 00 | . 05 | . 00 |
|  | 30 | . 00 | . 05 | . 00 | 05 | .00 | . 03 | . 00 | 100 | . 00 | . 05 | . 00 | 136 | . 00 | .05 | .00 |
|  | 31 | .00 | . 05 | . 10 | 06 | .60 | . 05 | .00 | 101 | .00 | . 65 | . 00 | 136 | . 00 | .05 | . 00 |
|  | 12 | . 60 | . 05 | . 00 | 07 | .00 | .05 | .00 | 102 | .00 | . 05 | . 00 | 137 | .00 | :05 | . 00 |
|  | 33 | . U0 | .us | .00 | 48 | .00 | . 05 | .00 | 103 | .00 | .05 | .00 |  |  |  |  |
|  | 34 | . 00 | -us | .00 | 09 | .60 | . 05 | .00 | 104 | . 00 | . 05 | . 00 |  |  |  |  |
|  | 35 | . 00 | . 05 | .00 | 10 | .00 | . 05 | .00 | 105 | . 00 | .05 | .00 |  |  |  |  |

$$
\begin{aligned}
& \text { HNIIIAL APPLICATIUN JU THE FAIKEANKS, ALASKA AIKSHEU.. } \\
& \text {.....tXAMPLE PhUGLEM IvUMBEH } 2 \ldots \text {.... UYNANIC FKGGLEM wIIH CGNSIANI GUUNUAKY...... }
\end{aligned}
$$

## ...RESULIS AFTEH 1 hUUKS UF DYNaMIC SIMULATIUA...

...Cuncerihatluns ahe meaf vehilcal valuts....


|  | NuUE NUM | $\begin{gathered} \text { CUIVC } \\ \text { MG/M3 } \end{gathered}$ | $\begin{aligned} & x-v E L \\ & \text { M/StC } \end{aligned}$ | $\begin{aligned} & Y-v E L \\ & M / s t C \end{aligned}$ | nult foum | $\begin{aligned} & \text { Culic } \\ & \text { MG/MS } \end{aligned}$ | $\begin{aligned} & x-V E L \\ & M / S E L \end{aligned}$ | $\begin{aligned} & Y=V E L \\ & M / S E C \end{aligned}$ | fulut NuM | $\begin{aligned} & \text { CUNE } \\ & M G / N, 3 \end{aligned}$ | $\begin{aligned} & x=V E L \\ & M / S t C \end{aligned}$ | $\begin{aligned} & Y-V E L \\ & M / S E C \end{aligned}$ | nUUE NUM | $\begin{array}{r} \text { culve } \\ \text { m.G/N. } 3 \end{array}$ | $\begin{aligned} & x-V E L \\ & M / \Delta E C \end{aligned}$ | $\begin{aligned} & Y-v E L \\ & M / S \in C \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 1.00* | . 0 | .00 | 56 | . 0101 | . U 5 | . 00 | 71 | . 00 | . 05 | . 00 | 100 | . 00 | . 05 | . 00 |
|  | 2 | 1.00 * | . 05 | -uv | 57 | .01 | . 05 | .00 | 72 | .00 | . 05 | .00 | 107 | .00 | . 05 | . 00 |
|  | 3 | 1.00 m | . 05 | . 00 | 36 | . 01 | .05 | . 00 | 73 | .00 | . 05 | .00 | 108 | . 00 | .05 | . 00 |
|  | 4 | 1.0U* | .us | .00 | 34 | .01 | -us | .00 | 74 | .00 | . 05 | . 00 | 109 | . 00 | . 05 | .00 |
|  | 5 | $1.00 \%$ | -15 | . 00 | 40 | .00 | . 05 | .00 | 75 | .00 | .05 | . 00 | 110 | . 00 | .05 | - 00 |
|  | 6 | 1.0ua | - | . 00 | 41 | . 00 | . 05 | - 0 | 76 | .00 | .05 | .00 | 111 | .00 | .05 | . 00 |
|  | 7 | 1.004 | - | . 00 | as | .00 | . 05 | . 10 | 11 | - 60 | .05 | .00 | 112 | -60 | .05 | - u |
|  | 8 | . 44 | .05 | . 00 | 45 | .00 | . 45 | . 00 | 78 | .00 | . 05 | .00 | 113 | . 00 | . 05 | . 00 |
|  | 9 | .45 | . 05 | .00 | 44 | . 00 | .05 | .00 | 19 | .00 | . 05 | . 00 | 114 | .00 | . 03 | . 00 |
|  | 10 | . 42 | .05 | . 010 | 45 | .00 | . 05 | . 00 | 60 | .00 | . 05 | . 00 | 115 | . GO | . 45 | .00 |
|  | 11 | -4ic | . 05 | . 04 | 40 | .00 | . 06 | . 00 | 81 | . 00 | . 05 | . 00 | 110 | . 40 | . 15 | . 00 |
| 0 | 12 | .43 | .05 | . 00 | 47 | .00 | . 03 | .00 | ¢ ${ }^{\text {c }}$ | .00 | . 63 | . 00 | 117 | .00 | . 05 | . 00 |
| $\xrightarrow{1}$ | 13 | .44 | . 05 | . 00 | 48 | .00 | . 05 | .00 | 83 | .00 | . 05 | . 00 | 116 | . 00 | . 05 | .00 |
| Un | 14 | .12 | .us | . 010 | 44 | .00 | . 05 | . 00 | 84 | .00 | .ub | . 00 | 114 | -. 00 | .05 | .00 |
|  | - 15 | .14 | . 05 | . 00 | 50 | . 00 | . 05 | . 00 | 65 | - Uu | . 05 | .00 | 120 | .00 | .05 | - 0 |
|  | 10 | .13 | . 03 | .00 | 51 | . 00 | . 05 | .60 | 60 | . 00 | . 05 | . 00 | 121 | . 00 | . 05 | . 40 |
|  | 17 | .13 | .us | .00 | 52 | . u | .05 | .00 | 87 | . 00 | .05 | .00 | 122 | . 00 | .05 | . 00 |
|  | 18 | .13 | . 0 | . 00 | 53 | . 00 | .45 | . 00 | 48 | . 00 | . 05 | .00 | 123 | -. 00 | . 05 | . 00 |
|  | 19 | . 14 | . 05 | .00 | 34 | - 10 | . 05 | .00 | 49 | . 00 | . 05 | .00 | 124 | .00 | . 05 | - 00 |
|  | 20 | .12 | . 05 | . 0 | 55 | .00 | - us | . 60 | 90 | .00 | . 05 | . 00 | 12 | . 00 | . 05 | .00 |
|  | 21 | . U4 | .ub | - (1) | 56 | . 00 | . 05 | . 00 | 91 | .00 | .05 | . 00 | 120 | -. 60 | . 05 | .60 |
|  | 2. | -4b | .03 | . U1) | 57 | . 10 | . 05 | . 00 | 92 | . 00 | .05 | .00 | 127 | .00 | . 15 | . 00 |
|  | 23 | .us | . 05 | . 010 | 54 | . 00 | .05 | .00 | 95 | .00 | . 05 | . 00 | 128 | . 00 | . 05 | .00 |
|  | 24 | .us | . 03 | .00) | 59 | .00 | . 05 | . 00 | 94 | .00 | .05 | . 00 | 124 | -. 00 | .us | .00 |
|  | $<5$ | . US | .us | . 00 | 00 | .00 | .05 | -00 | 95 | .00 | .05 | . 00 | 150 | . 00 | .0b | . 00 |
|  | 20 | .04 | . 05 | . Uu | 01 | .00 | . 05 | . 00 | 46 | . 00 | . 05 | - 00 | 131 | . 00 | . 05 | . 00 |
|  | 27 | . 02 | . 05 | . 00 | 02 | .00 | .05 | - 00 | 97 | . 00 | . 05 | .00 | 132 | -. 000 | . 05 | . 00 |
|  | 28 | . 02 | . 0 | . 00 | 63 | . 00 | .05 | . U0 | 98 | . 00 | . 03 | . 00 | 133 | . 00 | .05 | .00 |
|  | $29:$ | - 0 | - is | . 00 | 64 | . 00 | . 15 | .40 | 99 | .00 | . 05 | . 00 | 134 | . 00 | . 05 | .00 |
|  | 30 | . 02 | . 05 | .00 | 05 | .00 | . 05 | . 00 | 100 | .00 | . 05 | .00 | 135 | . 00 | .06 | . 00 |
|  | 31 | .0c | . 05 | .00 | 06 | .00 | .05 | .00 | 101 | . 00 | . 05 | . 00 | 136 | -. 00 | . 05 | .00 |
|  | 32 | . 02 | . 05 | .00 | $0 \%$ | . 00 | .05 | . 00 | 102 | .00 | . 05 | . 00 | 131 | .00 | . 05 | .00 |
|  | 35 | . U2 | .03 | .00 | -4 | .00 | .05 | .00 | 103 | .00 | . 05 | .00 |  |  |  |  |
|  | 34 | . 01 | .us | . 00 | 09 | .00 | .0b | . 00 | 104 | .00 | . 05 | .00 |  |  |  |  |
|  | 35 | .01 | . 15 | .00 | 70 | .60 | . 05 | .00 | 105 | .00 | . 05 | .00 |  |  |  |  |

CUNILNUINIIY CHtCKS

| LIHE | IUIAL | A-FLUN | Y-FLGN | PEHCENT |
| ---: | ---: | ---: | ---: | ---: |
| 1 | $0.0 N 1-02$ | $0.001-02$ | 0.000 | 100.0 |
| 2 | $-6.081-02$ | $-6.001-02$ | 0.000 | -100.0 |

## mathemailical simulatiuiv ur cakbuiv midnuxiot disirigutiun in the atmusphere

INO UIMENSIUNAL fiNIIE ELEMEINT +UK HUKJZLINIAL PKUGLEMS OF VAHIAGLE THICKNESS INIIIAL APPLICAIILH JU IHE FAIHBANKS. ALASMA AIKSMEU...
.....txample froblem mumger 2 .... dyivamic PkUblem with CONSTANI boundary
...HESULIS AFIEH $\quad 2$ hUUKS UF UYNAMIC SINULAlIUN...
...COnceflatiluns ahe nean vertical values...


| NODE | CUIVC MGIMS | $x=V E L$ $M / S \vdash C$ | $y=v t L$ $N / S t C$ | NULTE NUM | COHL MG/MS | $x=V E L$ | $Y=V E L$ <br> M/SEC | NUDE <br> NUM | Cunc <br> MG/M3 | $x=V E L$ <br> M/SEC | $Y=V E L$ <br> M/SEC | NuDE NuM | CONC $M G / N S$ | $x=V E L$ <br> M/SEC | $\begin{aligned} & Y-V E L \\ & M / S E C \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NUM | MG/ms | M/SEC | N/StC | fvum | MG/MS | M/SEL | M/SEC | NUM | MG/M3 | M/SEC | M/SEC | NuM | MG/NS | M/St.C | M/SEC |
| 1 | 1.00 m | .05 | . 00 | $56 *$ | . 02 | . 05 | . 00 | 71 | . 00 | . 05 | .00 | 106 | .00 | . 05 | . $V 0$ |
| 2 | 1.60* | . 05 | . 00 | 31 | . 02 | .05 | .00 | 72 | .00 | .05 | .00 | 107 | .00 | . 05 | . 00 |
| 3 | 1.00* | . 45 | .00 | 34 | .02 | .05 | .00 | 73 | .00 | . 05 | .00 | 108 | . 00 | . 05 | - 00 |
| 4 | 1.00* | . 05 | . 00 | 54 | .02 | . 05 | . 00 | 74 | .00 | . 05 | .00 | 104 | . 00 | .05 | . 00 |
| 5 | 1.0u* | .05 | . 00 | 40 | .01 | .05 | .00 | 15 | .00 | . 05 | .00 | 110 | .00 | . 05 | .00 |
| 0 | 1.0U* | .15 | .00 | 41 | .01 | . 05 | .00 | 70 | .00 | . 05 | .00 | .111 | .00 | . 05 | . 00 |
| 7 | 1.UU* | -6 | .00 | 42 | .01 | . 05 | .00 | 77 | .00 | .05 | . 00 | 112 | .00 | . 05 | .00 |
| H | . 00 | . Ub | . Uu | 43 | . 01 | .05 | .00 | 76 | . 00 | .05 | .00 | 113 | . 00 | .05 | .00 |
| 9 | .61 | .05 | . 00 | 44 | . 01 | . 05 | . 00 | 79 | .00 | . 05 | . 00 | 114 | . 00 | .05 | .00 |
| 10 | .06 | . 05 | . 00 | 45 | .01 | .05 | .00 | 60 | . 00 | .05 | .00 | 115 | .00 | . 05 | . 00 |
| 11 | . 00 | .05 | . 00 | 40 | .01 | .05 | .00 | 61 | . 00 | . 05 | . 00 | 116 | .00 | . 05 | . 00 |
| 12 | .07 | . 0.05 | .00 | 47 | .00 | . 05 | .00 | ${ }_{8}$ | .00 | .05 | .00 | 117 | .00 | . 65 | . 00 |
| 13 | .68 | . 05 | . 10 | 48 | .00 | .05 | .00 | 83 | . 00 | .05 | .00 | 116 | . 00 | . 05 | .00 |
| 14 | .32 | . 05 | .00 | 49 | .00 | . 05 | . 00 | 84 | .00 | . 05 | . 00 | 119 | $=.00$ | .05 | .00 |
| 15 | .34 | . 05 | .00 | 30 | .00 | . 05 | .00 | 85 | .00 | . 05 | .00 | 120 | . 00 | . 05 | .00 |
| 16 | . 36 | .05 | .60 | 51 | . 00 | . 05 | .00 | $\mathrm{HO}_{6}$ | .00 | .05 | . 00 | 121 | .00 | .05 | .00 |
| 17 | . 35 | . 05 | . 00 | 52 | .00 | . 05 | .00 | 87 | .00 | . 05 | .00 | 122 | . 00 | . 05 | . 00 |
| 18 | . 32 | -us | .0U | 53 | .00 | .05 | . 00 | 48 | .00 | . 05 | .00 | 123 | -. 00 | . 05 | .00 |
| 19 | . 34 | . 05 | - Ut | 54 | .00 | .05 | . 90 | 89 | .00 | . 05 | .00 | 124 | . 00 | . 05 | . 00 |
| 20 | .32 | .05 | .00 | 55 | .00 | . 05 | .00 | 90 | . 00 | .05 | . 00 | 1くら | . 00 | . 05 | . 60 |
| 21 | .15 | .03 | , u0 | 5o | .00 | .05 | .00 | 91 | .00 | . 05 | . 00 | 120 | -. 00 | . 05 | .00 |
| 22 | . 15 | . 05 | .00 | 57 | .00 | . 05 | .00 | 92 | . 00 | . 05 | .00 | 127 | .00 | . 05 | .00 |
| 23 | .14 | .05 | . 00 | $5{ }^{5}$ | .00 | .05 | . 00 | 93 | . 00 | . 05 | .00 | 128 | . 00 | -us | . 00 |
| 24 | .14 | . 05 | -60 | 54 | . 00 | .05 | . 00 | 44 | .00 | . 05 | . 00 | 129 | -. 00 | . 05 | . 00 |
| 23 | .15 | . 05 | . 40 | bo | . 00 | . 05 | .00 | 95 | . 00 | . 05 | .00 | 130 | .00 | . 05 | . 00 |
| 20 | . 13 | .05 | . 110 | 61 | .00 | .05 | .00 | 96 | .00 | . 05 | .00 | 131 | . 00 | . 45 | . 00 |
| 21 | .06 | . US | . 10 | $6 \dot{4}$ | . 00 | .05 | .00 | 97 | . 00 | . 05 | .00 | 132 | -. 00 | . 05 | . 00 |
| 28 | . 06 | . 05 | .60 | 03 | .00 | . 05 | .00 | 98 | .00 | .05 | .00 | 135 | . 00 | .05 | . 00 |
| 24 | - vo | .0b | . 00 | 04 | . 00 | . 05 | . 00 | 99 | . 00 | . 05 | . 00 | 134 | . 00 | .05 | . 00 |
| 30 | - 0 | .05 | . 00 | 05 | .00 | . 05 | .00 | 100 | .00 | . 05 | .00 | 135 | .00 | . 05 | .00 |
| 31 | . 00 | .05 | .00 | 06 | . 00 | . 05 | . 00 | 101 | .00 | . 05 | .00 | 130 | -. 00 | . 05 | . 00 |
| 32 | . 06 | . US | .00 | 07 | .00 | . 05 | .00 | 102 | . 00 | .05 | .00 | 137 | . 00 | . 05 | .00 |
| 33 | . 06 | . 05 | . 00 | 64 | .00 | .05 | .00 | 103 | .00 | . 05 | . 00 |  |  |  |  |
| 34 | . 02 | . 05 | .00 | 09 | .00 | .05 | .00 | 104 | . 00 | . 05 | . 00 |  |  |  |  |
| 35 | . 02 | . 05 | .00 | 70 | .00 | .05 | .00 | 105 | .00 | . 05 | .00 |  |  |  |  |

Cundinuinily checks

| LINE | IUTAL | $x=16 \mathrm{~L}$ | Y-rLuw | PEHCENT |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1.064-01 | 1.064-01 | 0.000 | 100.0 |
| 2 | -1.664-01 | -1.664-01 | 0.000 | $-100.0$ |

majmeniailical simblatiun uf canbun munuxjut ulsihinuidun lif imt atmuspreft
InU OIMENSIUNAL FIGIIt tLEMENI FUH HUHILUNIAL PKU甘LEMS OF VAHIABLE THICKNESS
INIIIAL APPLIGAJIUlv lu the falhuainks, alaska alkShed....
 *. .RESULIS AFItR 3 hULHS UF DYNAMIC SIMULAIIUN...
.. Congeminajiuns ahe meaf vefilical valuts...
... UUTPUI FUK IVIJOES $110137 \ldots$ THE SYMBUL ... UENUTES A SPECIFIED VALUE ...

| NODE <br> NUM | $\begin{aligned} & \text { CUMC } \\ & \mathrm{ML} / \mathrm{Mj} \end{aligned}$ | $x-V E L$ <br> M/SEL | r-vEL <br> N/bEL | nuve <br> NUM | $\begin{gathered} \text { CUTNL } \\ M G / M S \end{gathered}$ | $x=v \in L$ <br> M/SとC | Y-VEL M/SEC | NUDE INUN | CONC MG/M3 | $\begin{aligned} & X-V E L \\ & M / S E C \end{aligned}$ | $\begin{aligned} & Y-V E L \\ & M / S E C \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.00* | -ub | . 00 | 30 | . 06 | . 05 | .00 | 71 | . 00 | . 05 | . 00 |
| 2 | 1.00* | . 05 | . 40 | 51 | .06 | . 05 | .00 | 72 | .00 | . 0 | .00 |
| 3 | 1.00* | . 05 | .00 | 38 | . 06 | . 05 | .00 | 73 | .00 | .05 | .00 |
| 4 | 1.00\% | . 05 | . 00 | 39 | .05 | . 05 | .00 | 74 | . 00 | . 05 | .00 |
| 5 | 1.0U* | . 05 | . 00 | 40 | .03 | . 05 | .00 | 75 | .00 | .05 | .00 |
| 0 | $1.00 *$ | -13 | . UV | 41 | . 03 | . 05 | . 00 | 76 | .00 | . 05 | .00 |
| 7 | 1.00* | . 05 | . 00 | 42 | .03 | .05 | .00 | 77 | .00 | . 05 | . 00 |
| 8 | .17 | . 05 | . 00 | 43 | .03 | . 05 | .00 | 78 | . 00 | . 05 | . 00 |
| 9 | .77 | . 05 | .00 | 44 | .03 | . 03 | .00 | 79 | .00 | .05 | .00 |
| 10 | .70 | .05 | . 00 | 45 | .05 | . 05 | . 00 | 80 | . 00 | . 05 | . 00 |
| 11 | .70 | . 05 | . 00 | 46 | . 03 | . 03 | .00 | 81 | .00 | . 05 | .00 |
| 12 | .77 | . US | . 00 | 47 | .01 | . 05 | .00 | 82 | . 00 | . 05 | . 00 |
| 13 | .17 | .us | . 00 | 4) | .01 | .05 | . 00 | 83 | . 00 | . 05 | .00 |
| 14 | .49 | .05 | .00 | 49 | .01 | . 05 | .00 | 84 | .00 | . 05 | .00 |
| 15 | . 50 | . 05 | . 00 | 50 | .01 | . 05 | .00 | 85 | .00 | . 05 | . 00 |
| 10 | . 44 | .05 | .00 | 51 | .01 | . 05 | .00 | 86 | . 00 | . 05 | .00 |
| 17 | . 50 | . 05 | . 00 | 52 | .01 | . 05 | .00 | 47 | .00 | .05 | . 00 |
| 18 | .44 | . 05 | . 00 | 53 | .00 | . 05 | .00 | 88 | .00 | . 05 | .00 |
| 14 | . 50 | .0s | .100 | 54 | . 00 | . 05 | . 00 | 84 | . 00 | .05 | .00 |
| 20 | .44 | .05 | .00 | 55 | .00 | . 05 | . 00 | 90 | . 00 | . 05 | . 00 |
| 21 | - 20 | .ub | . 10 | 56 | . 00 | . 05 | .00 | 91 | .00 | . 05 | .00 |
| 22 | . 20 | . 05 | . Uu | 57 | .00 | . 05 | .00 | 92 | .00 | .05 | .00 |
| $\checkmark 3$ | - ${ }^{\text {c }}$ | .03 | .06 | 58 | .00 | . 13 | . 00 | 93 | .00 | .05 | . 00 |
| 24 | . 27 | . 03 | .00 | 59 | .00 | . 05 | . 00 | 94 | .00 | .05 | .00 |
| 25 | - 28 | .ub | . U6 | 60 | -10 | . 05 | .00 | 45 | . U0 | .05 | .00 |
| 20 | - 2 | . 05 | . UV | 01 | . 00 | .05 | .00 | 96 | . 00 | . 05 | .00 |
| 27 | .12 | . 05 | . 00 | 02 | . 00 | . 05 | .00 | 97 | . 00 | .0b | .00 |
| 28 | .13 | .ub | . 00 | 03 | .00 | . 05 | . 00 | 98 | .00 | .us | - 0 |
| 29 | . 13 | -us | . 60 | 64 | . 00 | .05 | .00 | 99 | .00 | . 05 | . 00 |
| 30 | .15 | . US | . 00 | 05 | .00 | -0 | .00 | 100 | .00 | . 05 | .00 |
| 31 | .15 | . 05 | . 00 | 00 | . 00 | . 05 | .04 | 101 | .00 | . 05 | .00 |
| 32 | .13 | .05 | . 10 | 61 | .00 | .us | .40 | 102 | .00 | . 05 | .00 |
| 33 | . 12 | .03 | . U0 | 08 | .00 | . 05 | .00 | 103 | .00 | .05 | . 00 |
| 34 | . 06 | .05 | .00 | 69 | . 00 | -05 | .00 | 104 | .00 | .05 | .00 |
| 35 | . 00 | .03 | .00 | 70 | .00 | . 05 | .00 | 105 | .00 | .05 | .00 |


| NUDE | CUNC | X-VEL | $Y=V E L$ |
| :---: | :---: | :---: | ---: |
| NUN | MG/M3 | M/SLC | M/SEC |
| 100 | .00 | .05 | .00 |
| 107 | .00 | .05 | .00 |
| 108 | .00 | .05 | .00 |
| 109 | .00 | .05 | .00 |
| 110 | .00 | .05 | .00 |
| 111 | .00 | .05 | .00 |
| 112 | .00 | .05 | .00 |
| 113 | .00 | .05 | .00 |
| 114 | .00 | .05 | .00 |
| 115 | .00 | .05 | .00 |
| 116 | .00 | .05 | .00 |
| 117 | .00 | .05 | .00 |
| 115 | .00 | .05 | .00 |
| 119 | .00 | .05 | .00 |
| 120 | .00 | .05 | .00 |
| 121 | .00 | .05 | .00 |
| 122 | .00 | .05 | .00 |
| 123 | .00 | .05 | .00 |
| 124 | .00 | .05 | .00 |
| 125 | .00 | .05 | .00 |
| 126 | .00 | .05 | .00 |
| 127 | .00 | .05 | .00 |
| 128 | .00 | .05 | .00 |
| 129 | .00 | .05 | .00 |
| 130 | .00 | .05 | .00 |
| 131 | .00 | .05 | .00 |
| 132 | .00 | .05 | .00 |
| 133 | .00 | .05 | .00 |
| 134 | .00 | .05 | .00 |
| 135 | .00 | .05 | .00 |
| 130 | .00 | .05 | .00 |
| 137 | .00 | .05 | .00 |

CuNIINUNIIY Cht CKS

| LINE | TGIAL | x-FLUF | Y-FLUN HERCEIYT |  |
| ---: | ---: | ---: | ---: | ---: |
| 1 | $2.442-01$ | $2.442-U 1$ | 0.000 | 100.0 |
| 2 | $-2.492-01$ | $-2.492-01$ | 0.000 | -100.0 |

MAIHENGIICAL SIFULAIIUH UF CAHBLN MUMUXIDE UISTHIHUTIUN IN IHE ATMCISPHEKE
 INIIIAL APMLICAIIUR IU THE FAIRGANKS, ALASNA AIHSHED...
 *..RESULIS AFIEK 4 hUUHS UF UYFAMIC SIMULATION....



MATHKMAIICAL SIMULATIUF OF LAhUUN MUNUXIJE UISTKIHUIIUN IH IHE AIMUSPHEHE
IVU OIMENSIONAL FIGIIE LLEMENT FUK HOKIZUNIAL PHUBLEMS OF VAHIABLE THICKNESS
INIIJAL APrLICATIUM IU IhE FAIHGAIKSS, aLASKA AIKShEU...
.....EXABPLE HKUBLEM NUMBEH $2 . .$. UYNAMIC PHUHLEM WIIH CUNSTANT BUUNDAHY.......
....RESULIS AFTER $\quad$ B MUUKS UF [YMAMIC SIMULATIUN...
... Cunceivinatluivs ahe nealy vehtical values...

|  | NUUE | cunce | $x$-VEL | Y-vt | nuot | Cunc | $x-V E L$ $M / S t$ | $Y-V E L$ | nuUE | CUNC | $x-v E L$ | Y-VEL | NOUE | Cunc | $x-v E L$ | $Y \sim V E L$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | NUM | MG/ms | M/SEC | N/StC | NUM | MG/N3 | M/StC | M/SEC | NUM | MG/M3 | M/SEL | M/SEC | NUN | $\mathrm{NG/H} 3$ | M/StC | M/SEC |
|  | 1 | $1.00 *$ | .05 | . 10 | So | .18 | . 05 | . 00 | 71 | . 01 | . 05 | . 00 | 100 | . 00 | . 05 | . 00 |
|  | 2 | 1.004 | ל0. | .00 | 37 | .14 | . 05 | .00 | 72 | . 01 | -is | .00 | 107 | .00 | .65 | .00 |
|  | 3 | 1.00* | . 05 | .00 | 36 | .14 | .05 | . 00 | 73 | .00 | -ub | .00 | 108 | .00 | . 05 | . 60 |
|  | 4 | 1.00* | .03 | .0u | 39 | . 16 | . 05 | .00 | 74 | . 00 | . 05 | . 00 | 109 | .00 | . 05 | .00 |
|  | 5 | 1.U0* | . 05 | . 00 | 40 | .10 | . 03 | .00 | 75 | .00 | . 05 | .00 . | 110 | .00 | . 05 | . 00 |
|  | 0 | 1.0U* | . 05 | . 00 | 41 | .10 | . 15 | . 00 | 76 | .00 | . 05 | .00 | 111 | .00 | .65 | .00 |
|  | 7 | 1.UU* | .05 | -US | 42 | .10 | - 0 | - 00 | 77 | . 00 | . 05 | . 00 | - 112 | .00 | . 05 | . 60 |
|  | 8 | .t1 | - | - U0 | 43 | .10 | -us | . 00 | 78 | .00 | . 03 | . 00 | 113 | .00 | .us | .00 |
|  | 9 | . 87 | . 05 | .100 | 44 | . 1 U | . US | . 00 | 79 | .00 | . 05 | .00 . | 114 | .00 | .05 | .00 |
|  | 10 | .00 | .0S | -uv | 45 | .10 | . 05 | . 00 | -80 | . 00 | .05 | .00 | 115 | . 00 | . 05 | . 00 |
|  | 11 | . Ho | . 05 | . vu | 46 | .10 | . 05 | . 00 | 81 | . 00 | . 05 | .00 | 116 | .00 | .05 | .00 |
|  | 12 | .07 | . 05 | .00 | 41 | . 05 | .05 | . 00 | d2 | .00 | . 05 | .00 | $11 \%$ | .00 | . 05 | .00 |
|  | 13 | .67 | . 05 | . 00 | 48 | .us | . 05 | . 00 | HS | .00 | . 05 | .00 | 118 | .00 | . 05 | . 00 |
| 0 | 14 | .00 | .US | .00 | 44 | . 05 | . 05 | .00 | 84 | .00 | . 05 | .00 | 119 | .00 | . 05 | . 00 |
| H | 15 | .04 | .0, | .0U | bo | . 05 | .0b | .00 | 85 | .00 | -is | . 00 | $1<0$ | .00 | . 05 | . 00 |
|  | 16 | . 08 | . 05 | .00 | bl | - | . 05 | . 00 | 46 | .00 | . 05 | .00 | 121 | .00 | . 05 | .00 |
|  | 17 | .08 | .05 | . 10 | be | . 05 | . 15 | .00 | 67 | .00 | . 05 | . 00 | 162 | .00 | . US | . 00 |
|  | 14 | . 0 | .05 | .00 | 53 | . 02 | -ub | . 00 | 80 | .00 | .03 | . 00 | 123 | - 00 | .05 | .40 |
|  | 14 | .64 | .05 | . uv | 54 | . 03 | . 05 | . 00 | 89 | .00 | . 05 | .00 | 124 | .60 | . 05 | .00 |
|  | $\bigcirc 0$ | .05 | -0 | .00 | 55 | . 62 | . 05 | .00 | 90 | .00 | . 05 | . 00 | 125 | .00 | .05 | .00 |
|  | 21 | .49 | .05 | . U0 | So | .02 | .us | . U0 | 91 | .00 | . 05 | . 00 | 120 | . 00 | . 05 | . 00 |
|  | $<2$ | .44 | . 05 | . 00 | ¢7 | . 02 | . 05 | . UU | 92 | .00 | .ub | . 60 | 127 | .00 | . 05 | -u |
|  | 23 | . 4 H | . 4 | .0u | SH | .03 | .05 | . 00 | 93 | . 00 | .0b | . 00 | 128 | .10 | . 05 | . 00 |
|  | 24 | .40 | . 05 | -10 | So | .02 | . 05 | . 00 | 94 | .00 | .05 | .00 | 129 | .00 | . 05 | . U0 |
|  | 25 | .49 | -us | . 00 | 00 | .01 | . 05 | . 00 | 95 | .00 | .45 | .00 | 120 | . 00 | .05 | . 00 |
|  | 20 | -4y | -us | - 113 | 01 | . 01 | . 45 | .00 | 46 | .00 | . 05 | .00 | 131 | .00 | . 05 | . 00 |
|  | 27 | .31 | - 15 | . 00 | 62 | .01 | . 05 | . 00 | 47 | - 0 | . 05 | . 00 | 132 | -. 40 | . 05 | . 00 |
|  | 28 | . 32 | . 05 | . Uu | 03 | . 01 | . 05 | .00 | 98 | .00 | . 05 | . 00 | 133 | .00 | .05 | - U 0 |
|  | 24 | . 31 | .05 | . 00 | 04 | .01 | . 05 | . 00 | 99 | .00 | . 05. | .00 | 134 | . 00 | . 05 | . 00 |
|  | 30 \% | - Sc | . 05 | . 00 | 05 | .01 | . 05 | . 00 | 100 | .00 | . 05 | .00 | 135 | . 00 | . 05 | . 00 |
|  | 31. | .31 | . 03 | . 00 | 00 | . 01 | .03 | .00 | 101 | . 00 | . 05 | . 00 | 130 | -. 00 | .05 | .00 |
|  | 32 | .32 | . 05 | -14 | 67 | .01 | . 05 | . 00 | 102 | .00 | . 05 | .00 | 157 | . 00 | .05 | . 00 |
|  | 33 | .31 | .05 | - 00 | no | .01 | . 05 | . 00 | 103 | .60 | . 05 | .00 |  |  |  |  |
|  | 34 | . 18 | . 05 | . 00 | 69 | .01 | . 05 | . 00 | 104 | . 00 | . 05 | .00 |  |  |  |  |
|  | 35 | .14 | . 0 | . 10 | 70 | .01 | . 05 | .00 | 105 | .00 | . 05 | .00 |  |  |  |  |

CUNTINUNITY CHt LKS

| LINt | tulal | $x=r \mathrm{~L}$ (10) |  | PhKCENt |
| :---: | :---: | :---: | :---: | :---: |
| 1 | S.42c-u1 | 3.4<2-01 | 0.000 | 100.0 |
| 2 | -3.422-01 | $-3.4 \leq$-01 | 0.000 | $-100.0$ |

MATHEMAIICAL SIMULAIIUH UF CARGUN MUNUXIDE DISTKIGUTIUN IN IHE ATMOSPMEKE
Iru UIMENSIUNAL + INITE ELEMEIH FOK HOHICUNTAL PKUBLEMS UF VAHIABLE THICKNESS INIILAL APPLILATLUN IU JHE FAIRUANKS, ALABKA AIHSHEU....
.....Example phutlem number 2 .... dynamic phoblem mith CoivStant buUndahy
...RESULIS 4 fith 6 mUUHS dF DYMAMIC SIMULAIIUN...
.. CUNGENTKATIUNS ake mear vertical values...


CUNIINUNIIY CHELKS

| LINE | TUTAL | X-FLUA | $Y-F L U N$ | PEHCENI |
| ---: | ---: | ---: | ---: | ---: |
| 1 | $3.709-01$ | $3.709-01$ | 0.000 | 100.0 |
| 2 | $-3.709-01$ | $-3.709-01$ | 0.000 | -100.0 |

A．AIHEMAIICAL SIMULAIJUN UF CAH甘UIV MUNUXIUE DISTRIGUTIUN IN IHE ATMUSPHERE IAU DIMENSIUNAL FINIIt ELEMENI FUK HUHIZONIAL PHOBLEMS OF VARIABLE THICKNESS INIIIAL AHPLICATIUN TU THE FAIHBANKS，ALASKA AJHSHED．．．

Example prublem number 3 ．．．FAIHBANKS NETWORK

| NUMOEN UF ELEMENIS | 60 |
| :---: | :---: |
| NUP「ロER UF $\quad$－C | 13 |
| DIFFUSIUN COEFFS | 1 |
| CURTEEA MUUES | 159 |
| UUIHUT PHINI UPTIUN | 1 |
| LIMES Fur Cuntinumity | 0 |
| Irfe uF vel fitud | 0 |
| IrPe ur INITIAL COND | 0 |
| reviuchiog thtion | 0 |
| LUGILAL UNIT（SCKAICH） | 11 |
| velucitr fielo upidun | $\checkmark$ |
| Urivaialc LNPUI（IPIION | 0 |
| VEHIICAL UIST GPTIUN | 0 |



Ifint CUMIKUL AIVU VEHIILAL DISIRIBUTIUN SPECS

．．．．．VELUCIIIES AND BUUNUARY CONDITIONS AKE CONSIANT FUR THIS KUN ．．．．．

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MAIHEMAIICAL SIMULATIUN UF CAHBON MUNUXIDE OISIRIBUTION IN IHE ATMOSPGERE
TAO UIMENSIONAL FINIIE ELEMENT FOH HUHIZONTAL PROBLEMS UF VARIABLE IHICKNESS
INIIIAL APPLICAILON TO THE FAIHBANKS, ALASKA AIRSHED...
```

EXAMPLE PRUGLEM NUMGER 3 ... FAIRBANKS METWURK

トIXED NOUAL SPECIFICAIJUNS ......

|  | NUUE NUMBEH | $\begin{array}{r} x-L O C \\ (M E \mid E H S) \end{array}$ | $\begin{array}{r} Y-L U C \\ \text { (MEIEHS) } \end{array}$ | HEIGHI <br> (METEHS) | NODAL. <br> UHDEH | NODE NUMBER | $x-L O C$ <br> (METEHS) | $\begin{array}{r} Y-L U C \\ (M E I E N S) \end{array}$ | $\begin{gathered} \text { HEIGHT } \\ \text { (METEHS) } \end{gathered}$ | NUDAL OKUEN |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 17421.46 | 300603.53 | 5.0 | ט | 81 | 20289.39 | 36902\%.57 | 5.0 | 0 |
|  | 2 | 17453.99 | 307672.62 | 5.0 | 0 | 82 | 20620.39 | 569547.80 | 5.0 | 0 |
|  | 3 | 17406.53 | 308741.70 | 5.0 | 0 | 83 | 20672.87 | 369536.68 | 5.0 | 0 |
|  | 4 | 17535.33 | 370310.47 | 5.0 | 0 | 84 | 21734.98 | 369183.28 | 5.0 | 0 |
|  | 5 | 17584.14 | . 371879.24 | 5.0 | 0 | 85 | 21823.30 | 369041.52 | 5.0 | 0 |
|  | 0 | 18076.85 | 360887.07 | 5.0 | 0 | 86 | 22076.63 | 308472.11 | 5.0 | 0 |
|  | 1 | 1842S.40 | 367474.75 | 5.0 | 0 | 87 | 21576.94 | 368407.04 | 5.0 | 0 |
|  | 8 | 19429.40 | 369345.91 | 5.0 | 0 | 88 | 21181.85 | 308462.81 | 5.0 | 0 |
|  | 9 | 18458.00 | 309043.84 | 5.0 | 0 | 89 | 21181.85 | 368416.53 | 5.0 | 0 |
|  | 10 | 18500.81 | 370612.60 | 5.0 | 0 | 90. | 21284.11 | 368186.51. | 5.0 | 0 |
|  | 11 | 18897.26 | 370900.79 | 5.0 | 0 | 91. | 21291.08 | 367.903 .13 | 5.0 | 0 |
|  | 12 | 20210.37 | 369922.34 | 5.0 | 0 | 92 | 20735.02 | 367212.45 | 5.0 | 0 |
|  | 15 | 19987.26 | 371163.41 | 5.0 | 0 | 93 | 21753.58 | 367138.08 | 5.0 | 0 |
|  | 14 | 14704.15 | 372404.48 | 5.0 | 0 | 94 | 20545.04 | 368802.13 | 5.0 | 0 |
| $\square$ | 16 | 18732.25 | 367170.61 | 5.0 | 0 | 95 | 20193.12 | 308600.36 | 5.0 | 0 |
| N | 10 | 1925s.17 | 367324.00 | 5.0 | 0 | 96 | 20335.87 | 364201.87 | 5.0 | 0 |
| N | 17 | 19115.40 | 307030.79 | 5.0 | 0 | 97 | 22178.89 | 371284.27. | 5.0 | 0 |
|  | 18 | 19000.86 | 308258.29 | 5.0 | 0 | 98 | 21330.59 | 370628.67 | 5.0 | 0 |
|  | 19 | 19444.55 | 368090.95 | 5.0 | 0 | 99. | 20545.04 | 308214.13 | 5.0 | 0 |
|  | 20 | 19402.01 | 568/18.46 | 5.0 | 0 | 100 | 20289.39 | 308453.51 | 5.0 | 0 |
|  | 21 | 14638.04 | 364983.41 | 5.0 | 0 | 101 | 22976.05 | 370229.13 | 5.0 | 0 |
|  | 22 | 19854.43 | 309294.49 | 5.0 | 0 | 102 | 20533.42 | 367980.37 | 5.0 | 0 |
|  | C 5 | 19814.92 | 364634.16 | 5.0 | 0 | 103 | 20277.77 | 368225.75 | 5.0 | 0 |
|  | 24 | 20249.88 | 369547.68 | 5.0 | 0 | 104. | 21184.17 | 368513.94. | 5.0 | 0 |
|  | 2 | 20586.88 | 369722.44 | 5.0 | 0 | 105 | 20786.75 | $30+516.54$ | 5.0 | 0 |
|  | 20 | 201110.48 | 370275.61 | 5.0 | 0 | 106 | 20BU0.69 | 360802.15 | 5.0 | 0 |
|  | 21 | 20547.37 | 371516.68 | 5.0 | 0 | 107 | 21125.64 | 308481.41 | 5.0 | 0 |
|  | 28 | 20512.93 | 372371.95 | 5.0 | 0 | 108. | 22104.52 | 304207.16 | 5.0 | 0 |
|  | 29 | 19778.09 | 307477.39 | 5.0 | 0 | 109 | 22411.30 | 360137.94 | 5.0 | 0 |
|  | 30 | 19880.35 | 30H044.48 | 5.0 | 0 | 110 | 23010.91 | 308011.93 | 5.0 | 0 |
|  | 31 | 19891.91 | 368272.23 | 5.0 | 0 | 111 | 22994.65 | 367086.56 | 5.0 | 0 |
|  | 36 | 19891.97 | 36BSU4.77 | 5.0 | 0 | 112 | 21767.52 | 367544.79 | - 5.0 | 0 |
|  | 33 | 14671.18 | 306355.90 | 5.0 | 0 | 113 | 24293.25 | 308322.88 | 5.0 | 0 |
|  | 34 | 20289.39 | 364 ch3.00 | 5.0 | 0 | 114 | 24812.09 | 370238.43 | 5.0 | 0 |
|  | 35 | 20903.36 | 364522.60 | 5.0 | 0 | 115 | 23894.07 | 370235.77. | 5.0 | 0 |
|  | 30 | 21146.99 | 370075.74 | 5.0 | 0 | 116 | 23129.44 | 369910.73 | 5.0 | 0 |
|  | 31 | 20022.12 | 367737.69 | 5.0 | 0 | 117 | 22855.20 | 369027.57 | 5.0 | 0 |
|  | 38 | 21381.72 | 372339.41 | 5.0 | 0 | 118 | 22701.81 | 369345.97 | 5.0 | 0 |
|  | 59 | 18674.14 | 372141.80 | 5.0 | 0 | 119 | 22090.57 | 509020.59 | 5.0 | 0 |
|  | 40 | 21818.65 | 369308.79 | 5.0 | 0 | 120 | 22081.27 | 364087.99 | 5.0 | 0 |
|  | 41 | 2006H.60 | 368569.72 | 5.0 | 0 | . 121 | 22334.60 | 364401.75 | 5.0 | 0 |
|  | 4 C | 20064.60 | 364711.49 | 5.0 | 0 | 122 | 20289.39 | 304518.59 | 5.0 | 0 |
|  | 43 | 19636.32 | 367784.18 | 5.0 | 0 | 123 | 20538.07 | 364518.59 | 5.0 | 0 |
|  | 44 | 20289.39 | 368802.13 | 5.0 | 0 | 124 | 20538.07 | 368472.11 | 5.0 | 0 |


|  | 43 | 211US. 15 | 308902.49 | 5.0 | 0 |  | 125 | 21581.59 | 308509.29 | 5.0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 40 | 21058.67 | 369167.01 | 5.0 | 0 |  | 126 | 21651.32 | 368444.22 | 5.0 | 0 |
|  | 41 | 21191.14 | 308655.71. | 5.0 | 0 | - | 127 | 21753.58 | $368<16.46$ | 5.0 | 0 |
|  | 48 | 20786.75 | 368425.62 | 5.0 | 0 |  | 128 | 21679.20 | 360179.27 | 5.0 | 0 |
|  | 44 | 20800.69 | 301974.75 | 5.0 | 0 |  | 129 | 20538.07 | 368459.57 | 5.0 | 0 |
|  | 50 | 20250.80 | 307344.92 | 5.0 | 0 |  | 130 | 21579.27 | 308458.16 | 5.0 | 0 |
|  | 51 | 22140.35 | 372339.41 | 5.0 | 0 |  | 131 | 21653.64 | 368495.35 | 5.0 | 0 |
|  | 51 | 21350.15 | 371484.14 | 5.0 | 0 |  | 132 | 21658.29 | 368625.50 | 5.0 | 0 |
|  | 51 | 20266.15 | 307991.94 | 5.0 | 0 |  | 133 | 21586.24 | 368659.45 | 5.0 | 0 |
|  | 54 | 22153.32 | 370429.00 | 5.0 | 0 |  | 154 | 21709.42 | 368771.92 | 5.0 | 0 |
|  | 55 | 21032.60 | 370015.31 | 5.0 | 0 |  | 135 | 21195.79 | 368785.86 | 5.0 | 0 |
|  | So | 21514.62 | 369964.83 | 5.0 | 0 |  | 136 | 21776.82 | 306627.82 | 5.0 | 0 |
|  | 51 | 21391.02 | 369415.69 | 5.0 | 0 |  | 137 | 22074.30 | 367475.07 | 5.0 | 0 |
|  | 58 | 21349.10 | 369352.95 | 5.0 | 0 |  | 138 | 22084.25 | 367881.79 | 5.0 | 0 |
|  | 54 | 21827.95 | 306774.24 | 5.0 | 0 |  | 139 | 23887.10 | 312318.49 | 5.0 | 0 |
|  | 60 | 21590. 69 | 306769.59 | 5.0 | 0 |  | 140 | 24837.66 | 371208.00 | 5.0 | 0 |
|  | 61 | 21781.40 | 367951.51 | 5.0 | 0 |  | 141 | 23438.55 | 368576.70 | 5.0 | 0 |
|  | be | 20004.00 | 304930.93 | 5.0 | 0 |  | 142 | 23594.26 | 367501.00 | 5.0 | 0. |
|  | 03 | 20209.39 | 367120.01 | 5.0 | 0 |  | 143 | 20249.39 | 366060.56 | 5.0 | 0 |
|  | 04 | 20768.16 | 367593.60 | 5.0 | 0 | - | 144 | 23282.83 | 369592.32 | 5.0 | 0 |
|  | 06 | 21244.00 | 367175.26 | 5.0 | 0 |  | 145 | 21781.46 | 564478.76 | 5.0 | 0 |
|  | 60 | C1く71.14 | S67556.41 | 5.0 | 0 |  | 146 | 21776.82 | 369246.03 | 5.0 | 0 |
|  | 07 | 22910.90 | 372539.41 | 5.0 | 0 |  | 147 | 22076.63 | 56435b. 27 | 5.0 | 0 |
|  | 68 | 22943.51 | 371<84.27 | 5.0 | 0 |  | 148 | 24047.46 | 369915.37 | 5.0 | 0 |
| 1 | 69 | 19841.81 | 308620.85 | 5.0 | 0 |  | 149 | 23919.64 | 311263.50 | 5.0 | 0 |
| 心 | 10 | 22055.31 | 309015.44 | 5.0 | 0 |  | 150 | 20786.75 | 304472.11 | 5.0 | 0 |
|  | 71 | 22004.62 | 304748.04 | 5.0 | 0 |  | 151 | 20289.39 | 308480.05 | 5.0 | 0 |
|  | 1 1 | <2353.19 | 309500.95 | 5.0 | 0 |  | 152 | 20793.72 | 508200.14 | 5.0 | 0 |
|  | 75 | 22421.50 | S00462.81 | 5.0 | 0 |  | 153 | 20986.62 | 364460.17 | 5.0 | 0 |
|  | 74 | <2127.76 | 308018.55 | 5.0 | 0 |  | 154 | 22343.90 | 369334.35 | 5.0 | 0 |
|  | 13 | 22395.63 | 361812.07 | 5.0 | 0 |  | 155 | 22058.03 | 36705 CO | 5.0 | 0 |
|  | 76 | 22302.49 | 366966.09 | 5.0 | 0 |  | 156 | 22378.76 | 367389.08 | 5.0 | 0 |
|  | 11 | 20538.01 | 304600.36 | 5.0 | 0 |  | 157 | 24863.22 | 312297.57 | 5.0 | 0 |
|  | 70 | 20591.53 | 368470.43 | 5.0 | 0 |  | 158 | 23353.85 | 367174.55 | 5.0 | 0 |
|  | 79 | 20St2.36 | 369150.74 | 5.0 | 0 |  | 159 | 22390.38 | 368864.88 | 5.0 | 0 |
|  | 80 | 20535.67 | 308976.43 | 5.0 | 0 |  |  |  |  |  |  |

MAIHEMAIICAL SIMULATIUN UF CAKBGN MUNUXIDE DISTHIBUTION IN THE ATMOSPMERE INU UIMENSIUNAL FIVITE ELEMENT FUK HORILUNTAL PRU甘LEMS UF VARIABLE THICKNESS INIIIAL AHPLICAIIUN TU THE FAIHBANKS，ALASKA AIKSHED．．．

EXAMPLE PRUNLEM NUMBER 3 ．．．FAIKGANKS NETWURK

NUDAL COINIECTIOHS ANU MAIEHIAL NUNGERS．．．．
ELEMEII TUOLS（CUUNTEHCLOCKAISE）TYPE AHEA（MZ）ELEMENT NOOES（COUNTEHCLUCKWISE）TYPE AREA（MZ）


| 37 | 59 | 145 | 84 | 46 | 79 | 45 | 0 | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 38 | 106 | 135 | 60 | 153 | 79 | 78 | 0 | 0 |
| 39 | 125 | 133 | 60 | 135 | 106 | 47 | 0 | 0 |
| 40 | 105 | 104 | 125 | 47 | 106 | 95 | 0 | 0 |
| 41 | 87 | 130 | 125 | 104 | 105 | 68 | 0 | 0 |
| 42 | 48 | 89 | 87 | 88 | 105 | 150 | 0 | 0 |
| 43 | 61 | 128 | 87 | 49 | 48 | 40 | 0 | 0 |
| 44 | 49 | 91 | 61 | 90 | 48 | 152 | 0 | 0 |
| 45 | 93 | 112 | 61 | 91 | 49 | 60 | 0 | 0 |
| 40 | 92 | 65 | 93 | 66 | 49 | 64 | 0 | 0 |
| 47 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 48 | 101 | 115 | 114 | 140 | 157 | 139 | 61 | 68 |
| 49 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 50 | 144 | 148 | 114 | 115 | 101 | 116 | 0 | 0 |
| 51 | 73 | 117 | 144 | 116 | 101 | 118 | 0 | 0 |
| 52 | 73 | 118 | 101 | 71 | 72 | 159 | 0 | 0 |
| 53 | 72 | 71 | 101 | 70 | 121 | 154 | 0 | 0 |
| 54 | 59 | 119 | 72 | 154 | 121 | 120 | 0 | 0 |
| 55 | 59 | 120 | 121 | 147 | 40 | 85 | 0 | 0 |
| 56 | 73 | 154 | 72 | 119 | 59 | 74 | 0 | 0 |
| 57 | 107 | 86 | 73 | 74 | 59 | 136 | 0 | 0 |
| 54 | 107 | 136 | 59 | 134 | 60 | 132 | 0 | 0 |
| 59 | 125 | 131 | 107 | 132 | 60 | 133 | 0 | 0 |
| 60 | 61 | 126 | 107 | 131 | 125 | 130 | 0 | 0 |
| 61 | 61 | 121 | 107 | 126 | 87 | 128 | 0 | 0 |
| 62 | 61 | 108 | 73 | 66 | 107 | 121 | 0 | 0 |
| 63 | 61 | 138 | 75 | 109 | 73 | 108 | 0 | 0 |
| 64 | 93 | 137 | 75 | 138 | 61 | 112 | 0 | 0 |
| 65 | 93 | 155 | 76 | 156 | 75 | 137 | 0 | 0 |
| 66 | 76 | 158 | 142 | 111 | 75 | 156 | 0 | 0 |
| 67 | 75 | 111 | 142 | 110 | 73 | 109 | 0 | 0 |
| 68 | 44 | 80 | 79 | 96 | 34 | 81 | 0 | 0 |
| 69 | 75 | 110 | 142 | 113 | 114 | 148 | 144 | 117 |
| 70 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 71 | 59 | 85 | 40 | 146 | 84 | 145 | 0 | 0 |
| 72 | 60 | 134 | 59 | 45 | 79 | 153 | 0 | 0 |

278151.78
130433.36
102994. c1
112750.80
40661.03
36729.61
178076.78
220940.36
399218.50

390418．55
.00
3948630.59
.00
546020.27

445ち77．96
286210.11
50424.91
34981.57
138531.85
229506.21
103717.54
34472.07
18843.40
7452.57
41483.26
185443.10
201906.01
251491.40
260559.94
733380.50
394280.99
20957.80
3327411.19
.00
22945.32

47982．12

## GUMmAKY UF INPUT DAIA USED TU CALCULAIE CARHLJN MUNUXIDE LUADS

FUH I.VPU IU fHE FAIKBAfKS AIR UUALITY MUMUUEL..
EXAMPLE PHUGLEM NUMBER $3 \ldots$ FAIFGANKS NETWORK
GH/HK AI IDLE
GM/CULUSIAKT
SHEEU EMISSIUN FACIUK ALPHA
SPEEU EMISSIUN FACTUH BEIA
000.00
340.00
200.00
$-.55$

FRACIIUI UF DAILYC OLOAD FOK THAVELING, IDLEING, AND SIARTING.....

|  | HOUR | Itavel | 10LEIIVG | STARTING |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 | . UBO | . 0 H0 | . 080 |
|  | 2 | - U00 | .000 | . 000 |
|  | 3 | . 000 | . 000 | .000 |
|  | 4 | . 000 | .000 | .000 |
|  | 5 | .000 | .000 | . 000 |
|  | 6 | .000 | . 000 | .000 |
|  | 7 | .600 | .000 | .000 |
|  | 8 | . 000 | . 000 | .000 |
| 0 | 9 | . 000 | . 000 | . 000 |
| N | 10 | .000 | . 000 | . 000 |
| $\square$ | 11 | .000 | . 000 | .000 |
|  | 12 | .0U0 | . 000 | . 000 |
|  | 13 | . 000 | .000 | .000 |
|  | 14 | . 060 | .000 | . 000 |
|  | 13 | .000 | .000 | .000 |
|  | 10 | .000 | .000 | .000 |
|  | 17 | -000 | .000 | . 000 |
|  | 14 | . 000 | .000 | . 000 |
|  | 14 | .100 | .000 | .000 |
|  | 20 | . 000 | .000 | .000 |
|  | 21 | -000 | .004 | .000 |
|  | 22 | . 000 | . 0000 | .000 |
|  | 23 | .000 | .000 | .000 |
|  | 24 | .000 | .000 | .000 |

SUMmAKY UF linfut bata used iu calculate carbun munoxide loads FUR INPUT TO TME FAIHGANKS AIK. GUALITY MUMUDEL...

EXAMPLE PRUBLEM NUMBER 3 ... FAlRGANKS NETWURK
OAILY MASS INPUTS UF $C$ O FOR EACH LLEMENT FRUM EACH SORCE THE LUAD INPUT UNITS AKE (GM/OAY/ELEMENT)...

. . StiJup call fuh l o huuting...

MAIHEIAAIILAL SIMULAIIGN UF CARBUN MUNUXIDE DISIHIGUTIUN IN THE AIMUSPHEHE
IMU UIMEHSIUIVAL FINITE LLEMEINT．FUR MURILUNTAL PHUHLEMS OF VARIAGLE THICKNESS LNITIAL APPLICATIGN IU IHE FAIHGANKS，ALASKA AINSHED．．

ExAmple HRUHLEM nUMBER 3 ．．．FAIHBANKS NETMOHK
.....SIEAUY STATE SULUTIUN.....
．．．Cuiflemifatluns ahe nean vertical．values．．．
．．．UUIPUT FUH frUDES $110159 \ldots$ ．．．THE SYMBOL A ULNUTES A SPECIFIED VALUE ．．．

| muot | C0．di | $x-v t L$ | $y$ vevt | NUDE | conc | $x-V E L$ | $Y$－VEL | NUUE | CUNC | $X$－VEL | Y－VEL | MODE | cutuc | $x$－VEL | $Y$－VtL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NUM | HG／N， 3 | M／SEC | M／StC | NUM | MG／Pi3 | M／SEC | M／SEC | NUM | MG／M3 | M／SEC | M／SEC | NUM | MG／M3 | M／SEC | M／StL |
| 1 | ．00＊ | ． 00 | －． 25 | 41 | ．0S | ． 00 | $=.25$ | 81 | .25 | ． 00 | －． 25 | 121 | .35 | ． 00 | 25 |
| 2 | ．Uư | ． 00 | －． 25 | 42 | .11 | ． 00 | －． 25 | $\therefore 82$ | ． 23 | .00 | －． 25 | 122 | ． 19 | ． 00 | 2 |
| 3 | －U0＊ | － 00 | －． 25 | 43 | ． 01 | .00 | －． 25 | $\therefore 83$ | ． 23 | .00 | －． 25 | 123 | ． 44 | .00 | 2 |
| 4 | Uu＊ | ． 00 | －． 25 | 44 | .24 | .00 | －． 25 | 44 | .19 | .00 | －． 25 | 124 | ． 38 | ． 00 | － |
| $b$ | －Uu＊ | ． 00 | －． 25 | 45 | .85 | .00 | －． 25 | 45 | ． 04 | ． 00 | －． 25 | 125 | ． 49 | ． 00 | 2 |
| 0 | ． 04 | ． 00 | －． 25 | 46 | ． 92 | .00 | －． 25 | 86 | －． 23 | ． 00 | －． 25 | 126 | ． 29 | .00 | ． 2 |
| 7 | ！uu | .00 | －． 25 | 47 | ． 84 | .00 | －．23 | － 87 | .48 | .00 | －． 25 | 127 | .34 | .00 | －． 25 |
| 4 | －． 02 | ． .00 | －． 25 | 48 | .75 | ． 00 | －． 25 | ． 48 | ． 82 | .00 | －． 25 | 128 | ． 27 | .00 | 2 |
| 9 | ． 01 | .00 | －． 25 | 49 | ． 74 | .00 | －．cb | $\therefore \quad 69$ | ．88 | .00 | －． 25 | 129 | .38 | ． 00 | －． 25 |
| 10 | ．01 | .00 | －． 26 | Su | .18 | .00 | －． 25 | $\because 90$ | ． 76 | ． 00 | －． 25 | 150 | ． 50 | ． 00 | －． 25 |
| 11 | ．U0 | .00 | －． 25 | 51 | ． 00 ＊ | .00 | －． 25 | － 91 | .77 | .00 | －． 25 | 131 | .20 | .00 | ． 25 |
| 12 | －． 00 | .00 | －． 25 | 52 | －． 17 | .60 | －． 25 | ． 42 | .67 | ． 00 | －． 25 | 132 | ． 29 | ． 00 | －． 25 |
| 11 | .10 | .00 | －． 25 | 53 | .19 | .00 | －． 25 | $\therefore 93$ | ． 32 | .00 | －． 25 | 133 | ． 45 | ． 00 | $2{ }^{2}$ |
| 14 | ．00＊ | ． 00 | －． 25 | 54 | －． 10 | ． 00 | －． 25 | 94 | ． 48 | ． 00 | －． 25 | 134 | ． 25 | .00 | ． 25 |
| 15 | .10 | .00 | －． 25 | 55 | －． 19 | .00 | －． 25 | 45 | .72 | .00 | －． 25 | 135 | ． 84 | ． 00 | －． 25 |
| 10 | －． 01 | .00 | －． 25 | 56 | －． 19 | .00 | －． 25 | ． 96 | .76 | .00 | －． 25 | 156 | .24 | ． 00 | －．25 |
| 11 | －12 | ． 00 | －． 25 | 57 | ． 88 | .06 | －． 25 | 47 | .00 | .00 | －． 25 | 137 | ＝． 28 | ． 00 | ． 25 |
| 18 | ．us | .00 | －． 25 | 58 | 1.00 | .00 | －． 25 | 98 | .17 | .00 | －． 25 | 138 | －． 26 | ． 00 | －． 25 |
| 14 | ．U2 | .00 | －． 25 | 59 | ． 02 | ． 00 | －． 25 | 99 | .42 | .00 | －． 25 | 139 | ． 00. | .00 | －． 25 |
| 20 | －． 10 | ． 00 | －． 25 | 60 | ． 45 | .00 | －． 25 | 100 | .19 | .00 | －． 23 | 140 | .13 | ． 00 | 5 |
| 21 | ． 10 | .00 | －． 25 | 61 | ． 22 | .00 | －． 25 | 101 | .16 | .00 | －． 25 | 141 | ． 00 | ． 00 | －． 25 |
| 22 | .01 | .00 | －． 25 | 62 | ． 05 | ． 00 | －． 25 | 102 | .37 | .00 | －． 25 | 142 | .16 | ． 00 | － 2 |
| 23 | －． 05 | ． 00 | －． 25 | 63 | ． 20 | .00 | －． 25 | 103 | ． 20 | .00 | －． 25 | 143 | ． 26 | ． 00 | －． 25 |
| 24 | .34 | .00 | －． 25 | 04 | ． 70 | ． 00 | －． 25 | 104 | .82 | .00 | －． 25 | 144 | $=.09$ | .00 | － 23 |
| d | .10 | －uv | －． 25 | 05 | .77 | ． 00 | －． 25 | 105 | .77 | ． 00 | －． 25 | 145 | －． 05 | .00 | －．cs |
| 20 | －．01 | .00 | －． 25 | 66 | .76 | .00 | －． 25 | 106 | ． 68 | ． 00 | －． 25 | 146 | －． 45 | ． 00 | －．db |
| 27 | －リU | ． 00 | －． 25 | 07 | $.00 \%$ | .00 | －． 25 | 107 | ． $47^{\circ}$ | .00 | －． 25 | 147 | －． 05 | ． 00 | －． 65 |
| 20 | －りり＊ | ． 00 | －．2b | 68 | .13 | ． 00 | －． 25 | 108 | －． 15 | .00 | －． 25 | 148 | .29 | .60 | －．${ }^{\text {c }}$ |
| 24． | －Un | .00 | －． 25 | 64 | .05 | .00 | －． 25 | 109 | 2.90 | ． 00 | －． 25 | 149 | .00 | .00 | －． 25 |
| 30 | .01 | ． 00 | －． 25 | 70 | ． 32 | .00 | －． 25 | 110 | ． 54 | ． 00 | －． 25 | 150 | ． .67 | ． 00 | －． 25 |
| 31 | .61 | ． 10 | －． 25 | 71 | 1.62 | ． 00 | －． 25 | 111 | .64 | －U0 | －． 25 | 151 | ． 21 | .00 | －． 25 |
| $3{ }^{3}$ | .01 | .00 | －． 25 | 72 | 2.09 | ． 00 | －． 25 | 112 | ． 25 | .00 | －． 25 | 152 | .73 | .00 | －．ch |
| 33 | .12 | .00 | －． 25 | 73 | 2.91 | ． 00 | －． 25 | 113 | .07 | .00 | －． 25 | 153 | ． 82 | ． 00 | － |
| 34 | .10 | .00 | －．2 | 74 | －． 01 | .00 | $=.25$ | 114 | .10 | ． 00 | －． 25 | 154 | 1.29 | .00 | －．25 |
| 35 | .47 | .00 | －． 25 | 15 | 2.83 | ． 00 | －． 25 | 115 | －． 06 | .00 | －． 25 | 155 | －． 37 | ． 00 | 25 |
| 50 | .45 | .00 | －． 25 | 76 | 2． 51 | .00 | －． 25 | 116 | ． 04 | .00 | －． 25 | 156 | 2.65 | ．$V 0$ | －． 25 |
| 37 | .13 | .00 | －． 25 | 77 | .31 | .00 | －． 25 | 117 | ． 66 | .00 | －． 25 | 157 | ． $000 \%$ | ． 00 | －． 25 |
| 34 | － $00 *$ | ． 00 | －． 25 | 78 | ． 46 | ． 00 | $=.25$ | 118 | 2.07 | ． 00 | －． 25 | 158 | ． 00 | ． 00 | －． 25 |
| 39 | －Uu＊ | ． 00 | －． 25 | 79 | .13 | .00 | －． 25 | 119 | －． 17 | .00 | －． 25 | 159 | 2.52 | .00 | 25 |
| 40 | －． 0.04 | .00 | －． 25 | 80 | .08 | .00 | －． 25 | 120 | －． 18 | .00 | －． 25 |  |  |  |  |

