

# The Symmetry Concept in the Perspective of Basic Education Teachers

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## Abstract

The curricular changes that have occurred with the Portuguese Mathematics programme of study of 2007 showed, among other situations, the introduction of new contents and the new concept of symmetry. For the teachers, these changes require scientific updates to be provided so that the outcomes of the mathematics teaching can be guaranteed. In the academic year 2011/2012 a questionnaire was applied to a 142 inservice basic education teachers of a particular region of Portugal, to analyze the content knowledge in this subject (symmetry) and the form of scientific update occurred. The processing of data allowed us to identify inadequate conceptions, beliefs, topics more or less successful in responses and also compare the teacher's responses of the different teaching groups.

*Key words: Isometry, symmetry, inservice teachers.*

## Introduction

Over the past 10 years in Portugal were adopted three Mathematics Programmes of Study for Basic Education with significant differences in the content knowledge and in the organization between them. In the 1990/1991 the Mathematic Programme was already noticeable the growing value of the teaching of geometry that is even more considerable in the programme approved in 2007 and the last one of 2013. In the 1990/1991 programme, more exactly in the 1st<sup>1</sup> and 2nd<sup>2</sup> cycles of Basic Education, the study of isometries had adopted a more intuitive character, less formal and restricted essentially to the study of reflection at the time designated by symmetry. With the 2007 mathematic programme, many changes have occurred, for example, the introduction of a new concept of symmetry, new teaching approaches, new contents have been included (rosettes, friezes, glide reflection) and also some issues were supposed to be taught in previous years comparing with the last programme. The current one, approved in 2013, has increased the mathematical formalism, and, with concern to the isometric transformations, the approach of some content also occurs in academic years before the ones that were mentioned in 2007 programme. Besides that, this last programme doesn't include the study of friezes and rosettes but it is supposed to study the rotational and translation symmetry. However, as an aim of the mathematic programme of basic education we have the analysis of the natural world and the interpretation of society. In other words a high-quality mathematics education should provide a foundation for understanding the world, an appreciation of the beauty and power of mathematics, and also gain sense of enjoyment and curiosity about the subject. Thereby, the study of

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<sup>1</sup> Four academic years for pupils with 6 to 9 years old.

<sup>2</sup> Two academic years for pupils with 10 and 11 years old.

rosettes and friezes would be the best way to study the isometries because using the nature it would be possible to study mathematics theory with natural evidence.

From the 1990/1991 mathematics programme to the 2007 were introduced "more or less profound changes in aspects that are considered more important" (Ponte & Sousa, 2010, p. 98). The 2007 programme was distinguished from the one before, also because of the extending of the content as a curriculum component, not limited to knowledge, but considering the values, attitudes, skills and abilities to develop (Rodrigues, 2009). Thus, in addition to changes of the concepts and the introduction of new isometric transformations, the innovative character is made also in the *new* appeal to a different mathematical experience of students, *new* in line with the current international curriculum guidelines for the teaching of mathematics, and *new* in the status of the cross capabilities that were already mentioned in the previous program (Canavarro, Tudella & Pires, 2009).

Although it is sometimes assumed that the new contents introduced in basic education mathematic programme are elementary and commonly understood, in the specific case of isometries, the theme is not all superficial, and teachers who will teach have the need to deepen their knowledge to understand it in a comprehensive way, otherwise it can generate confusing concepts that can cause difficulties and obstacles on later study of the concepts (Silva, 2010).

It should also be noted that, by the academic year of 2011/2012, investment in inservice teacher training was a concern of the Ministry of Education, and a strategy recognized by the positive impact it had on student learning. Currently, in Portugal we have a disinvestment in inservice teacher training which many educators and organizations see as a setback in the mathematics education with consequences for the learning of students (APM, 2013; Brunheira, 2013; Santos & Serrazina, 2013; Veloso, Brunheira & Rodrigues, 2013; Viana, 2013).

In the study presented here, we tried to identify difficulties in the content knowledge in respect to the isometric transformations, and the preferred form of scientific update of the 142 basic education teachers. These teachers, in the academic year of 2011/2012, were teaching children with 6 to 14 years old. In Portugal our Basic Education is organized in three cycles: 1<sup>st</sup> cycle (6 to 9 years old), 2<sup>nd</sup> cycle (10 to 11 years old) and 3<sup>rd</sup> cycle (12 to 14 years old). The investigation takes on characteristics of a survey study that has therefore the survey data collection as a main resource.

### **Isometries in the recent Portuguese Mathematics Programmes of Study**

The differences between the Mathematics programmes of study of 1990/1991 and 2007 are very significant in respect to the mathematical content that should be taught in the 2<sup>nd</sup> cycle (see Table 1). The programme of study of 2007 for this stage referred the study of three isometries (rotation, translation and reflection) and the composition between them, as well as the more formal study of some representations: the friezes and rosettes. The exploitation of friezes and rosettes provided in the 2007 mathematic program of study is clearly distinct from the expected in the 1990/1991 programme.

Table 1.  
Comparative table of the 1991 and 2007 programmes of study of 2<sup>nd</sup> cycle

1991 Mathematic Programme of Study	2007 Mathematic Programme of Study (p. 38)
<b>Specification of content</b>	<b>Topics</b>
Reflection symmetry: <ul style="list-style-type: none"> <li>• Symmetry axes</li> <li>• Bisectrix</li> </ul>	Reflection, rotation and translation <ul style="list-style-type: none"> <li>• Concept and properties of reflection, rotation and translation</li> <li>• Axial and rotational symmetry</li> </ul>
<b>Goals</b>	<b>Specific goals</b>
<ul style="list-style-type: none"> <li>• Discover and draw symmetry axes of geometric figures.</li> <li>• Recognize that the bisectrix of an angle is a symmetry axis.</li> <li>• Draw, on graph paper, the symmetry image of a figure.</li> </ul>	<ul style="list-style-type: none"> <li>• Identify, predict and describe the isometry given the geometric figure and its image.</li> <li>• Draw the image of a figure, from an isometry or a composition of isometry.</li> <li>• Understand the concept of axial and rotational symmetry and identify the symmetries a figure.</li> <li>• Complete, draw and explore geometric pattern that involve symmetries.</li> <li>• Identify symmetries in friezes and rosettes.</li> <li>• Draw friezes and rosettes.</li> </ul>

Nowadays and since 2013 Portugal has a new Mathematic Programme which compared with the previous one, has, once again, significant differences in the study of isometric transformations, which include:

- The lack, throughout the all basic education, of the study of friezes and rosettes;
- In the 1<sup>st</sup> cycle programme of study the reflection symmetry is the only symmetry to be taught;
- In the 2<sup>nd</sup> cycle programme specified the study of reflection as an isometry and the study of reflection and the rotational symmetry. Here it is also mentioned the study of the central reflection as an isometry;
- In the 3<sup>rd</sup> cycle programme it begins the study of translation and the translation symmetry and once again, in an increasingly complex way, the study of the previous symmetries;
- The study of glide reflection as an isometry and as a symmetry, which was mentioned in the previous programme in the 1<sup>st</sup> and 2<sup>nd</sup> cycle, now appears only in the 3<sup>rd</sup> cycle.

In this new document although there is no reference to the study of rosettes and friezes in basic education, with the study of rotational symmetry rosettes are analyzed and the study of translational symmetry (in one direction) involves the study of the friezes. So it isn't understandable why this new program has taken the study of those representations. However, despite the studies of rosettes and friezes are directly related with the 2013 programme goals, by not present the specific content in this document, the teach will depend on the development that the teacher wants to give to this issue but hardly have the place achieved in the previous programme.

The change of the contents in mathematic programme study, the anticipation of the teaching of some issues, and the existence of a new concept of symmetry it requires teachers to update scientifically in order to reach the proposed objectives in the program.

### **Teacher's knowledge and the isometric transformations**

Over time, the educational currents tend to value certain teaching knowledge in relation to others. For example, in 1875, in United States of America, teachers were submit to assessment tests to testing their competence in subject matter and pedagogical skill in which 90% to 95% of the question is on the content, the subject matter to be taught, and the others about pedagogical practice (Shulman, 1986). The content and pedagogical knowledge are two fundamental skills for good teaching performance as advocated Shulman (1986, 2005) and several other educators. In this paper we will focus on the Shulman (1986, 2005) studies who contributed greatly to highlight the importance of content knowledge that have motivated and still motivate the worldwide researchers.

To Shulman (1986) the content knowledge “refers to the amount and organization of knowledge per se in the mind of the teacher” (p. 9). In this type on knowledge the teacher needs to understand not only that something is so but why is so, under what circumstances can be asserted, weakened or even denied (Shulman, 1986). The pedagogical content knowledge goes beyond knowledge of subject matter and include, in a specific area, “the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations” in a way that could be comprehensible to others, in this case, to the pupils. Here we could also add the way the teachers managed the classroom, organized time and activities, structured the sequence tasks, planned classes and assessed (Shulman, 1986, p. 9).

Other skills are also distinguished by this author and many others, but the domain of the content knowledge assumes, for many researchers, a major role in how this scientific area, in particular, is taught (e.g., Amaral, Ralha & Gomes, 2011; Gomes, 2012; Ma, 2009; Price & Ball, 1997; Shulman, 1986).

After about a century, in 1986, the content knowledge was conspicuously absent and the emphasis was given to aspects related to pedagogical issues. The assessment by teachers, says Shulman (1986), gave special emphasis to the ability to teach and little or no attention to the content knowledge. Today, taking into account the educational policy of the last Portuguese government, it seems to be clear that the domain of content knowledge return to gain relevance because of the introduction of exams to evaluate the teacher's content knowledge. These tests prevent teachers from teaching if they fail.

Regarding the changes occurred in the mathematic programme of study (2007) about isometric transformations several Portuguese educators noted the existence of old conceptions, in particular in concern to the symmetry concept, which continued to be understood by teachers as being the axial reflection (Canavarro, 2010; GTG, n.d.; Loureiro, 2009; Veloso, 2012). GTG2 (2010) and Canavarro (2010) highlighted the need for a theoretical study of these new themes. Besides the lack of studies involving inservice math teachers of basic education in this issue, some of them reveal difficulties

in this area as a result of many years of a secondary role that geometry have in previous mathematical curriculum in Portugal. In the international sphere there are few studies about teacher performance in teaching this issue. Ilaslan (2013) studied six teachers and concluded that difficulties existed in various aspects: visualization, lack of confidence in teaching these topics, lack of connection with other areas, difficulty to define each one of the transformations. Mabuchi (2000) presented a study that reported the existence of similar shortcomings to those found in studies with elementary school students mainly by lack of mastery of mathematical content knowledge.

In October 2005, the Portuguese Ministry of Education has developed continuous training programs for mathematics teachers of the 1<sup>st</sup> cycle and extended to 2<sup>nd</sup> cycle teachers in the following year (Pinheiro & Cabrita, 2012). After this, in the following years, there were two other inservice programs that were finished in 2012. With these programs, the improvement of the results of Portuguese students not only increased in international studies, but also, in Portuguese schools as notes Viana (2013). Since 2012 the inservice programs organized by the government finished although the enormous content changes of mathematical programme of study and, many educators, expecting the negative consequences for student learning and also for the teachers.

## **The study**

### ***Research objectives***

The study developed focuses the attention in the teacher content knowledge regarding to the concept of symmetry and its representations, considered essential to the teaching of this subject. Thus, the main objectives of this study were:

- identify how teachers update their knowledge about this content;
- to know the teacher's concept image of symmetry;
- to detect difficulties in recognizing the properties of rosettes and friezes.

### ***Participants***

The study was conducted during the academic year of 2011/2012 and was attended by 142 teachers who were teaching in basic education levels in Portuguese schools: 12 were teaching in the 1<sup>st</sup> cycle, 60 in the 2<sup>nd</sup> cycle and 70 in the 3<sup>rd</sup> cycle of a given region of the country.

The participants in this study attended in an inservice teacher training programme about isometries conducted by the researchers in various schools in the country. This programme was free and the number of teachers interested in this sessions exceeded expectations and, in one of the schools was necessary to divide the group in two classes.

This situation could be justified by two important factors:

- at the moment of collecting data the 2007 math program was already being implemented in all the years of schooling and in all schools;
- inservice teacher training programs were almost inexistent because of the financial situation of Portugal.

All de teachers present in these sessions agreed in participating in this study.

In concern to the participants there is some important information that we will try to describe.

As we can see in Table 2, 3% of the teachers had less than 3 years as a teacher and 66% taught for over 10 years.

Table 2.  
*Numbers of years as a teacher*

	Frequency	%
less than 3 years	4	3
between 3 years and 10 years (including)	44	31
more than 10 years	94	66
Total	142	100

The initial training of these math teachers were varied given the lack of math teachers for decades ago. Thus, in the 2<sup>nd</sup> cycle, some teachers had training in teaching, and others in Engineering, Pharmacy, Nutritionist, Economics or Management, Biology and Ecology. In the 3<sup>rd</sup> cycle, the most teachers were graduated in mathematics and the others teachers in Engineering. The teachers training in other areas different from mathematics represented 17% of the participants. The diversity of teachers with initial graduate in different areas besides math is higher in the 2<sup>nd</sup> cycle (25 teachers) than in the 3<sup>rd</sup> cycle (7 teachers).

### ***Data collection and analysis***

The data collection method was the application of a questionnaire during an inservice programme and six sessions specially dedicated to symmetries and isometric transformations. Therefore, all questions were answered in the presence of the researchers and in individual form.

The questionnaire consisted of three main parts: the first dedicated to the professional situation of the teacher; the second related to the pedagogical content knowledge and the curricular content knowledge of the teaching of symmetries and isometries, and the third and final part, related to the content knowledge. In this last part, that we will give the main emphasis, we focus in four themes (isometries, symmetries, rosettes and friezes) from the point of view of content knowledge and in the way how teachers are updated scientifically.

This study assumed characteristics of a descriptive and quantitative study.

## **Results**

The programmatic changes that occurred in the math basic education program (2007), with respect to isometric transformations were seen as positive by 52% of teachers surveyed who considered the theme appropriate and important for the development of geometric thinking. However, we cannot fail to highlight that 34% of our participants believes that the approach of this content in the 6th grade was somewhat complex for students. These different views present percentages that should not be ignored. There are two possible causes for that: on one hand, it may still have some resistance that characterize the period of adjustment to curricular changes (e.g., Fisher, 2007; Martins, 2011; Price & Ball, 1997; Shulman & Sherin, 2004), on the other, also is notorious the decrease of these behaviors result with over time and with increased scientific guidance (Priestly, 2003 cited by Fisher, 2007). We refer to inservice programs promoted by the last Minister of Education since the start of implementation of the 2007 program.

However, in these times of curricular changes that require scientific updates by teachers, and with this study we were able to observe that the principal source for teachers updating was the textbook (75%). This fact was also mentioned in the Ilaslan study (2013). However, the approaches of the themes in portuguese school textbooks do not allow a sufficient depth level that provides a deep knowledge of the subject. For example, the definition of rosette, found in several textbooks, did not exist or was incomplete.

A significant proportion of our participants (47%) mentioned the initial training as a source for their knowledge. Only a few teachers mentioned this source in an exclusive way. However, as Veloso (2012) has mentioned, the knowledge obtained in the initial training wasn't sufficient to teach the content set out in programme (2007), because there were different concepts of this themes, or the content was not taught or was in a superficial way, or could have been taught with so high complexity that didn't promote a significant comprehension. The Internet resource and the small group sessions were selected by 40% of these teachers.

With regard to scientific knowledge of the matter, as noted by many authors as synonymous of content knowledge, we will focus on some aspects that were introduced in the programs under the thematic highlighted: the symmetry.

The new concept of symmetry present in the last two mathematic programme (2007 and 2013) did not include only the reflection symmetry, but all the others - rotational symmetry, translation and gliding reflection. Now, the study of this concept allows the study of other structures and new representations.

In our questionnaire the questions about the content knowledge included issues or presented goals in the math basic education programme of 2007, and special features that enable assessment of the level of knowledge of teachers.

We started with a question to understand if teachers identified the equilateral triangle as a rosette. In the results obtained (Figure 1), 50% of the teachers in the 1<sup>st</sup> or 2<sup>nd</sup> cycle and 74% of 3<sup>rd</sup> cycle teachers answered correctly.

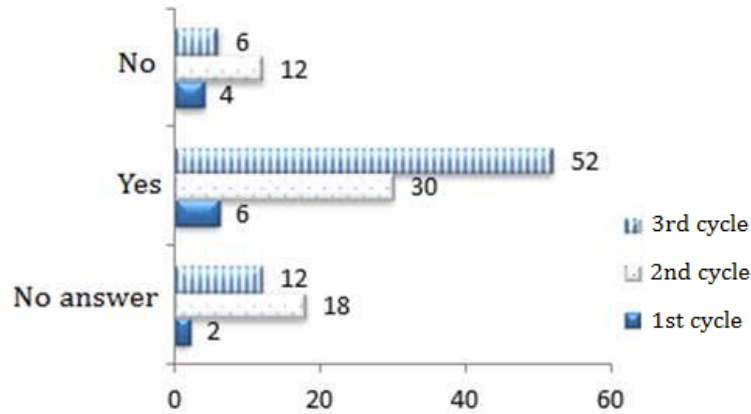


Figure 1. Is the equilateral triangle a rosette?

However, when asked about the existence of a relationship between the number of symmetries of a given regular polygon and the number of sides of the same polygon, as envisaged in the mathematic programme, the teachers revealed some inconsistencies with what they had responded in the aforementioned question (Figure 2).

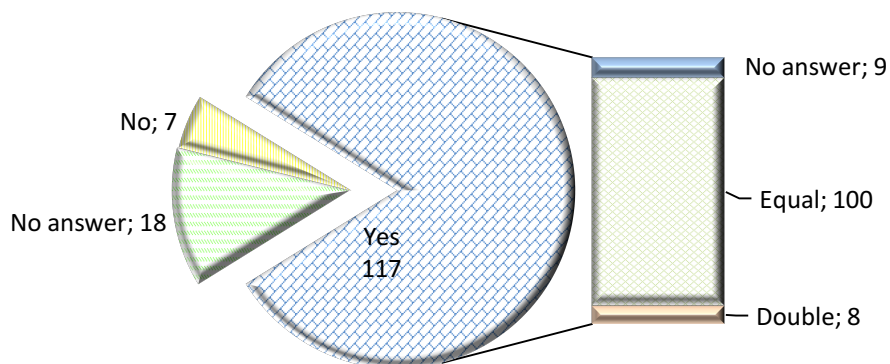


Figure 2. The relationship between the number of symmetries and the number of sides of a regular polygon.

From the 117 teachers who recognized the existence of such a relationship have mentioned, a very large majority (100 teachers), answered that the number of symmetries in regular polygons was equal to the number of sides of the polygon. These teachers have identified, only the reflection symmetries and haven't remembered the rotational symmetry, whereby, this situation indicates the prevalence of the old conceptualization of symmetry. This situation is also evident in the identification, by



63% of the teachers, of the  $r$ -axis shown in figure 3 as a symmetry axis when given two triangles.

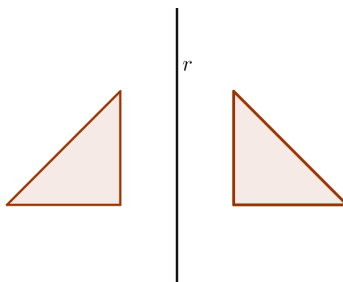


Figure 3. Example of a reflection.

The identification of identity symmetries was also a problem for teachers. In the answers obtained, about 35% of the teachers were grouped by their diversity into four broad groups:

- Group 1 (6 answers): mentioned some isometries like rotation, translation and reflection without other details.
- Group 2 (5 answers): mentioned different symmetries, like a teacher that wrote "symmetry of symmetry".
- Group 3 (12 answer): mentioned the term overlap - "when figures overlap".
- Group 4 (3 answer): mentioned the orientation, the similarly form or the congruence of figures.

Only three teachers correctly identified the identity symmetries.

The in-depth knowledge of the symmetry concept include, of course, the recognition of the two types of rosettes, cyclic and dihedral, but, however, only 36% of the teachers revealed to know these designations.

The properties of the representations studied, rosettes and friezes, are central to its approach. For example, the particularity of the existence of a limited number of rotational symmetries to be a rosette excludes from this group the circumference.

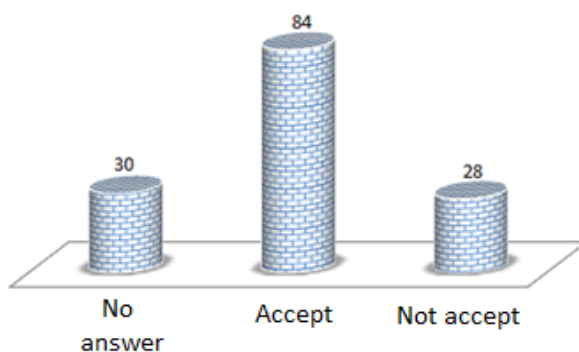


Figure 4. Is the circumference a rosette?

As we can see in figure 4, there were only 28 teachers who said that the circumference was not a rosette, however, in the justification for their answer, only 9 teachers (6%) justified properly.

Were also asked if there was a possibility of having a rosette as a minimum rotation angle of 62 degrees. These responses are shown in figure 5.

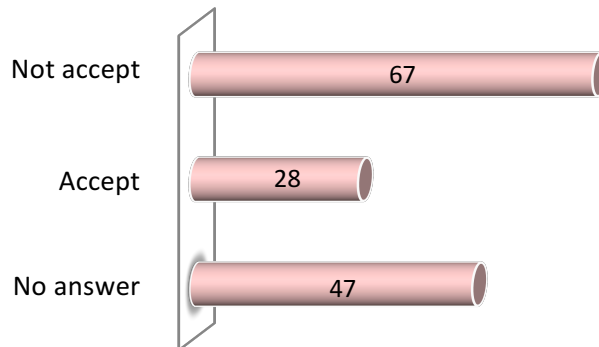


Figure 5. Can in a rosette the minimum angle of rotation be 62 degrees?

In the analysis of these answers, we highlight the large number of teachers that have not responded, and 28 of those that said they have to see the rosette to answer that question. Also 67 teachers said it was not possible, 22 did not present any justification, and only 38 properly justified.

Referring now to the representations characterized by the presence of translational symmetry in one direction (friezes), which present themselves as rich representations because they allow the study of other symmetries, the response rate of the teachers surveyed decreased dramatically. Although 59 teachers (42%) had recognized the existence of 7 types of friezes, we realize that this knowledge had no theoretical foundation sustained. Only 11 teachers had stated that the existence of gliding reflection symmetry did not imply the existence of an axis reflection parallel to the translation vector, and vice versa.

In the analysis of some friezes there were only 14 teachers who correctly identified all existing symmetries in them. This exercise, once again, highlighted the high number of absent responses, representing 32% of the teachers surveyed.

In these specific contents, the deep understanding would require, for example, understand why there are only seven types of friezes, or why the gliding reflection wasn't considered if it was trivial or even why include the identity of rotational symmetry in the set of the symmetries of a given figure it is always present.

As has stated Shulman (1986), the deep content knowledge is assumed that the teacher knows, not just because something is so, but why that is so. The knowledge, patent in the answers given by the participants, have several weaknesses like a superficial knowledge of the subject and it only allow a limited exploration of this concept.

### Closing remarks

Programmatic changes that occur in Mathematics Programmes of Study, regardless of the grade, but with particular emphasis in the elementary years, and which included

conceptual changes or the introduction of new content, must be accompanied by a continuous training to support teachers in its scientific update and to the perception of the importance of the modifications that took place. The new concept of symmetry, clearly distinct from the one stated before the 2007 programme has an important role in the approach of all symmetries.

In a time that inservice teaching training programmes provided by the government are almost nonexistent and considering the content knowledge as fundamental to the process of teaching and learning mathematics we will now revisit the goals to achieve with this study:

- identify how teachers update their knowledge about this content;
- to know the teacher's concept image of symmetry;
- to detect difficulties in recognizing the properties of rosettes and friezes.

In related to the first one we could see that the teachers tried to study the new contents by themselves because their prime source for the scientific development was the textbooks, does not allow in-depth knowledge of these matters, requiring additional training. Besides this we cannot forget that the initial training of some teachers not always allows this update autonomously (Veloso, 2012).

In concern to the concept of symmetry we observed the strong presence of the old conceptualization of symmetry and all the ensuing consequences in the study of rosettes and friezes. For this reason we can question how teachers are teaching the rotation, translation and glide reflection symmetries if in their minds the symmetry continues to be seen as a reflection. This concern is shared by some Portuguese educators.

Therefore, we advocate that teachers should investigate other ways and learn more about the subject matter hereof, together with the concern by the Ministry of Education to ensure access to continuous training for the teachers in order to validate the known Bernard Shaw metaphor that was corrected by Shulman (1986, p. 14) and reproduced here: "those who can, do, those who understand, teach".

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