OPTIMIZED NURBS CURVE BASED G-CODE PART PROGRAM FOR CNC SYSTEMS

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Dedicated to my family and friends.

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ABSTRACT

Kanna, Sai Ashish. M.S.M.E., Purdue University, December 2018. Optimized NURBS Curve Based G-Code Part Program for CNC Systems. Major Professor: Andres Tovar and Hazim El-Mounayri.

Computer Numerical Control (CNC) is widely used in many industries that needs high speed machining of the parts with high precision, accuracy and good surface finish. In order to avail this the generation of the CNC part program size will be immensely big and leads to an inefficient process, which increases the delivery time and cost of products. This work presents the automation of high-accuracy CNC tool trajectory planning from CAD to G-code generation through optimal NURBs surface approximation. The proposed optimization method finds the minimum number of NURBS control points for a given admissible theoretical cord error between the desired and manufactured surfaces. The result is a compact part program that is less sensitive to data starvation than circular and spline interpolations with potential better surface finish. The proposed approach is demonstrated with the tool path generation of an involute gear profile and a topologically optimized structure is developed using this approach and then finally it is 3D printed.

1. INTRODUCTION

1.1 Background of High Speed CNC Systems

Computer Numerical Control (CNC) is widely used in high speed machining of ultra-precision parts due to its accuracy, speed, and repeatability. The generation of CNC part programs requires a workflow that begins with the 3D model of the part designcomputer-aided design (CAD), and it is followed by the tool path representationcomputer-aided manufacturing (CAM), numerical control of the part program (NC-program) via CAM post-processing (PP), workpiece manufacturing through CNC commands, and computer-aided quality (CAQ) assurance. During the manufacturing optimization process of a new part, the CAD-CAM-PP-CNC-CAQ workflow needs to be repeated for each and every design iteration. This leads to an inefficient process, which increases the delivery time and cost of new products. To meet the demand to reduce the cost of processing of new parts, the CNC generation time must also be reduced.

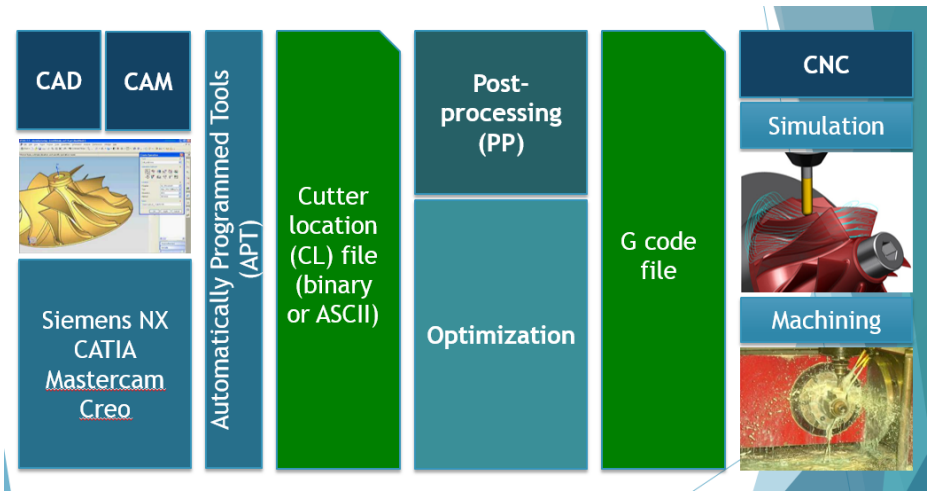

Fig. 1.1. Optimized Process workflow for CNC machining

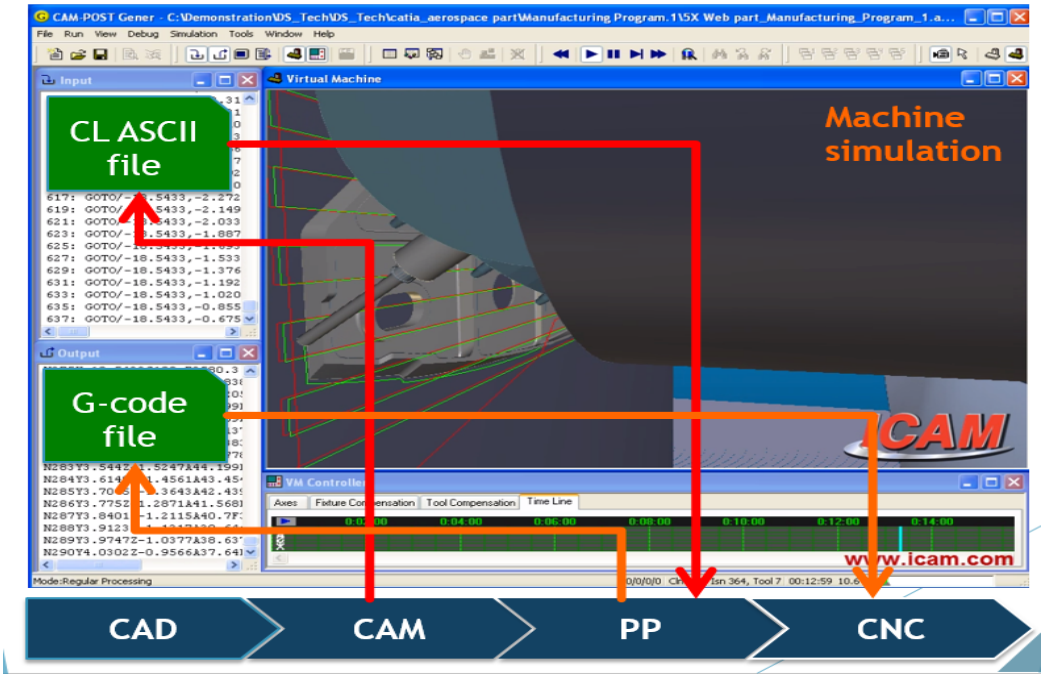

Fig. 1.2. CNC Part Programming workflow

One conventional approach to reduce CNC generation time is by post-processing the tool paths using piecewise linear (G01) and circular (G02/G03) interpolations [1]. However, this approach is of relatively low accuracy. For complex surfaces, the size of the part program may dramatically increase with tighter tolerances (lower cord error). Furthermore, a long part program also leads to increased storage requirements and longer data transmission times compromising the high speed machining effectiveness. If the part data cannot be processed fast enough, i.e., block execution time is shorter than the block processing time, then the machine starts and stops in a machine gun fashion causing path errors due to repeated acceleration and deceleration [2]. Even if the data can be processed fast enough, there is always rate fluctuations and velocity discontinuity between two consecutive line segments, which promote vibration and reduce machining quality [1] [2] [3].

An alternative to linear and circular interpolations is the use of quadratic B-splines (G05.1) and non-uniform rational B- splines (NURBS) (G05.2/G05.3/G06.2).

These approaches allow the representation of complex (free form) curves with the promise of compact part programs. However, B-spline and NURBS interpolations are traditionally generated from the CAM post-processing of piecewise linear segments. While this approach increases the accuracy of the curve, it does not have a major impact on the size of the part program.

The NURBS interpolations has been used in industries with many different approaches such as Ship Hull design, minimizing tool path fluctuation, smoothen the feedrate for high speed machining, Optimizing Shell Structures and many more.

1.2 Applications on NURBS

This research work reconstructs a concise and accurate approximated curve or surface of the pinion gear using the NURBS geometry. There are many optimization methods used to minimize the error generated between the targeted curve and the approximated curve. A new interpolation method called Adams Bashforth interpolation of NURBS curve in order to realize higher precision under the condition of satisfying speed demand [4]. A new adaptive acc-jerk-limited non-uniform rational B-spline (NURBS) interpolation method based on an optimized S-shaped C quintic feedrate planning scheme [5].The improved quantum-behaved particle swarm optimization (QPSO) algorithm based on NURBS is used on Ship design [6]. The isogeometric topology optimization on NURBS Surfaces [7]. A tool path optimization algorithm of spatial cam flank milling based on NURBS surface [8]. Reconstruction of free-form shapes in space from their arbitrary perspective images using NURBS [9, 10].The general bilinear transformations are used in optimizing NURBS surfaces [11]. Optimize the control points by a web-like search algorithm [12]. An evolutionary programming algorithm called Simulated Annealing is used to optimize the weight and the knot by minimizing the sum square error between the fitted and target curve and surface [13]. The optimal spatial positions and weights of a fixed number of NURBS control points are determined using a quasi Newton optimization

algorithm in order to approximate a general planar target curve [14].The arrangement of the knot vector and location of control points are optimized locally for every segment of the object [15].In this research work, to obtain the NURBS curve data an approximate curve is found based on the given data points and different optimization algorithms have been implemented that are capable to identify the optimal NURBS curve parameters. The Optimization algorithms such as SQP, Nelder-Mead, and SA are compared in the error minimization between the approximate curve and the target curve. One-dimensional parametric search is used to find the minimum number of points. Lastly, the optimized data of the location of control points and their respective weights have been used in generating the G-Codes using the NURBS Interpolation.

1.3 Motivation and Objectives of this research

Our work adopts a less traditional but intuitive method consisting on generating the NURBS curves directly from Cutter location(CL) data offset from a CAD model considering constant length segments in-order to achieve continuous smooth surface finish [16]. In this approach, the required control points, weights, and knots required by the NURBS curves are converted directly from CL data in two steps, first we identify the constant length segments from CL data and then we implement the NURBS interpolation to convert that into a G-code program [16–20] . While this strategy is currently used by some high-end CNC systems such as FANUC and SIMENS [3] [21] the technical challenge that remains unanswered is how to optimally generate the NURBS-based part program using the minimum amount of information for a given admissible cord error. Therefore, the objective of our work is to establish a design approach to find the optimal location of control points and their weights with minimum number of control points that lead to the most compact part program for the required product profile and ultimately, to the highest part quality.

1.4 Contributions of this research

In this research we developed a NURBS function that would create the optimized NURBS curve with their optimal parameters which are used in generating the NURBS interpolation. We evaluated this function by performing some case studies to described how this function would create the optimized NURBS curve using some well-known geometric entities such as Periodic signal wave, star shape and Trident curve, then we used this function to develop NURBS curve representation for pinion gear and its comparison with different optimization methods, comparison between the meta-models such as kriging function with NURBS representation and finally representation of topologically optimized structures using NURBS Curve and 3D printing their structures.

2. DERIVATION OF NURBS

In this chapter, we discuss briefly about how the curves are represented considering very common methods of representing the curves such as analytical and parametric representation of the curves. The significance of using the parametric approach over the analytical approach is expressed. The parametric form of equations solving using the interpolation methods such as LaGrange and Hermite interpolation are well discussed with their importance. Then the curve representation using Bezier curves, B-Spline Curve and the most important curve representation on which this research is based is the NURBS curve. The NURBS curve and its interpolation is well described in this chapter with its advantages over the other curve representations.

2.1 Parametric Representation

The representation of the curves and surfaces geometric modeling can be implemented by two common methods one being the classical representation of the curve and surfaces: 1. Analytical (explicit or implicit) representation and other being the most effective method in terms of its control over the intricate shapes: 2. Parametric representation.

The drawbacks of analytical representation:

- The analytical representation do not offer a natural procedure to evaluate the points on the curve or surfaces
- The representation depends on the co-ordinate system used.
- The analytical representation have no flexibility to represent intricate shapes and to manipulate it.

Thus, to overcome the limitations of the analytical representation and to allow the flexibility to control the geometric entities shapes the parametric representation have been used. This would provide a large number of points for the geometric entities through interpolation and they form the piecewise segments that represents entire geometric entity with relatively simple representation.

2.1.1 Parametric Cubic Polynomial Curves

The geometric representation for 3D- modeling requires to describe the non-planar curves, to avoid the computational difficulties and unwanted undulations that might be introduced by higher-order polynomial curves. These requirements are satisfied by using cubical polynomial which is the lower order polynomial that would represent the non-planar curves. The current technology uses the polynomial representation to represent the geometric curves and surfaces. This would establish relationship between the coordinates and parameters.

2.1.2 Boundary Conditions for Cubical Polynomial

The boundary conditions can be described based on the geometric entity such as a line can be define by at least two points, a parabola can be defined by three points minimum and a cubic polynomial need at least 4 points minimum. Thus, cubic polynomial requires four boundary conditions to describe its entity. These can be as follows:

1. Four points that lie on the curve
2. Two points and two tangents

The equation of the cubic polynomial in the parametric form is as follows

$$x = a_1 + b_1 u + c_1 u^2 + d_1 u^3 \tag{2.1}$$

$$y = a_2 + b_2 u + c_2 u^2 + d_2 u^3 \qquad 0 \leq u \leq 1 \tag{2.2}$$

$$z = a_3 + b_3 u + c_3 u^2 + d_3 u^3 \tag{2.3}$$

The equations above have 12 unknowns and to solve this we need to solve for 12 unknowns. This can be solved considering two methods: 1) Lagrange Interpolation and 2) Hermite Interpolation.

Lagrange Interpolation: The fitting of the curve through points.

Fig. 2.1. Lagrange interpolation of a curve

Hermite Interpolation: The fitting of the curves through points and tangents.

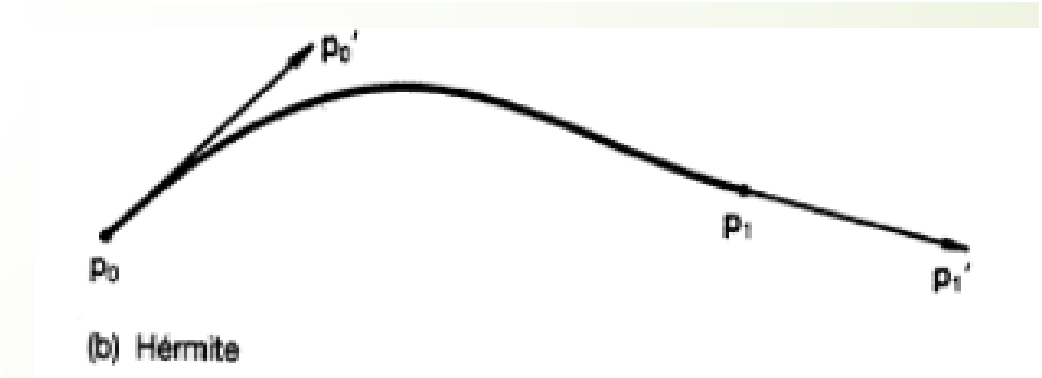

Fig. 2.2. Hermite interpolation of a curve

2.1.3 Blending Functions

Bending function is defined as sum of a number of functions in u parametric variable multiplied by their boundary conditions. The shape of the hermite cubic may be expressed as a function of the defining points and slopes multiplied by invariant interpolation functions called bending functions. These blending functions would represent the influence of the control points on the shape of the curve. .

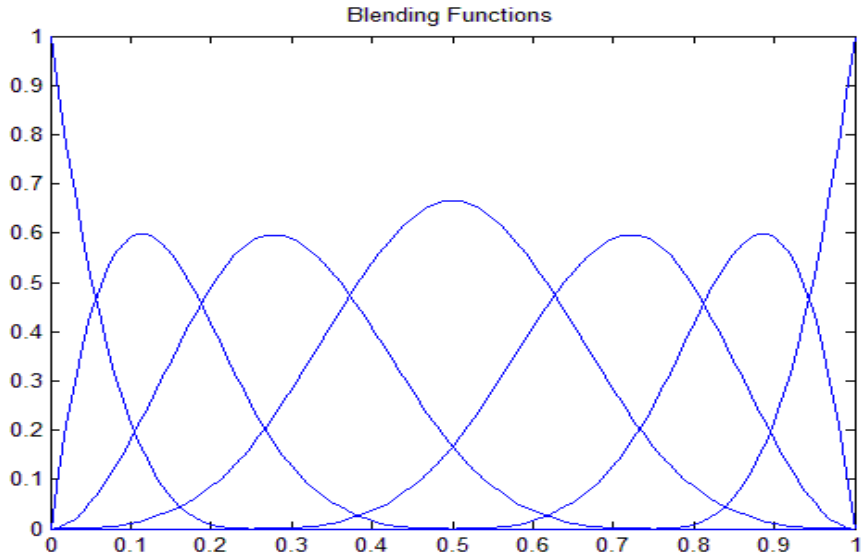

Fig. 2.3. Blending Functions

2.2 Curves and Surface Representation

2.2.1 Bezier Curves

The representation of the curve using points and tangent vectors have been obsolete since user would not have any feel to enter slope with numerical values. Thus, the use of Bezier curves would resolve this difficulty.

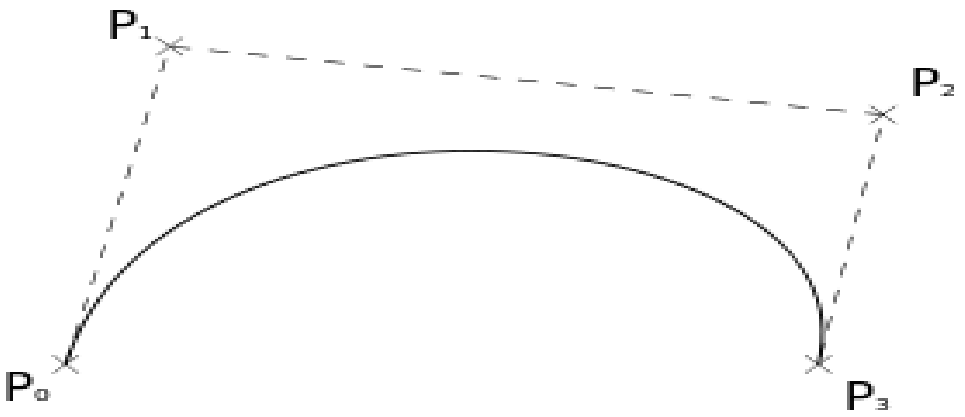

Fig. 2.4. Bezier Curve

Bezier Curve Formulation:

Given $n+1$ points $(p_0, p_1 ..., p_n)$, bezier curve that fits those points can be expressed as follow:

$$P(u) = \sum_{i=1}^{n} B_{i,n}(u) p_i \qquad 0 \leq u \leq 1 \tag{2.4}$$

Where blending function $B_{i,n}(u)$ are given by:.

$$B_{i,n}(u) = \frac{n!}{[(n-i)!i!]} u^i (1-u)^{n-i} p_i \tag{2.5}$$

where

- $p_0, p_1, ..., p_n$ are the control points,
- n order of the curve and
- $n+1$ number of control points

Example of Bezier curve is as follows:

Consider Control Points are $p_0(1,1), p_1(3,6), p_2(5,7) and p_3(7,2)$.

Therefore,

Number of controls, $n + 1 = 4$

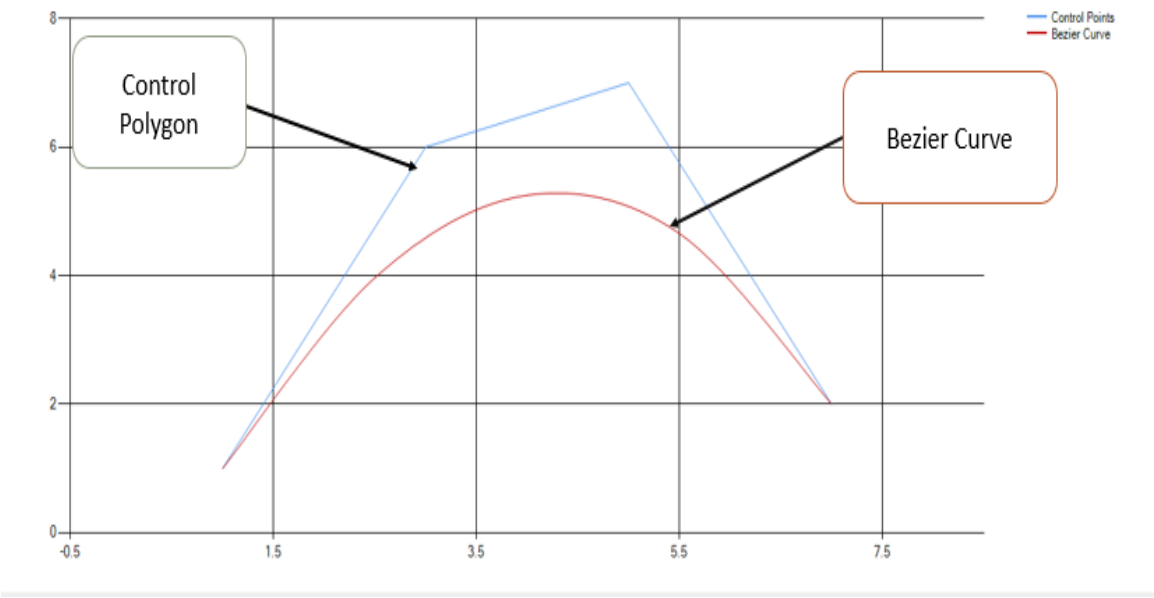

Fig. 2.5. Example of Bezier Curve

2.2.2 B-Spline Curve

The B-spline curves are the generalization of the bezier curves and they have unique characteristics to control or to manipulate the shape of the curve. They provide the local control of the curve with the use of specific set of blending or basis functions. They provide the ability to add control points without increasing its degree of the curve.

B-Spline curve formulation:

Given $n+1$ control points $(p_0, p_1, ., p_n)$, the B-spline curve can be expressed as follows:

$$P(u) = \sum_{i=1}^{n} N_{i,k}(u) p_i \qquad 0 \leq u \leq u_{max} \tag{2.6}$$

Where basis function $N_{i,k}(u)$ are given by:

$$N_{i,k}(u) = \frac{N_{i,k}(u)}{\sum_{i=1}^{n} N_{i,k}(u)} \tag{2.7}$$

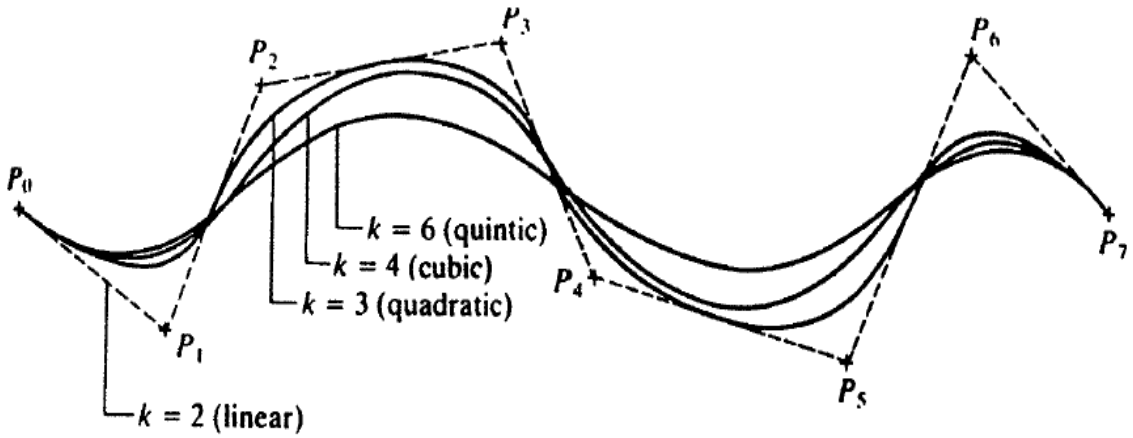

Fig. 2.6. B-Spline Curve with degree variation

Where,

$$N_{i,1}(u) = \begin{cases} 1, if \quad U_i \leq U_{i+1} \\ 0, otherwise \end{cases} \tag{2.8}$$

and where $p_0, p_1, ., p_n$ are the control points,

k order of the curve and

n+1 number of control points

Example for B-Spline Curves is as follows:

Consider control points are $p0(1,1)$, $p1(3,6)$, $p2(5,7)$ and $p3(7,2)$.

Therefore,

- Number of control points, $n+1 = 4$,
- Order of the curve, $k = 4$

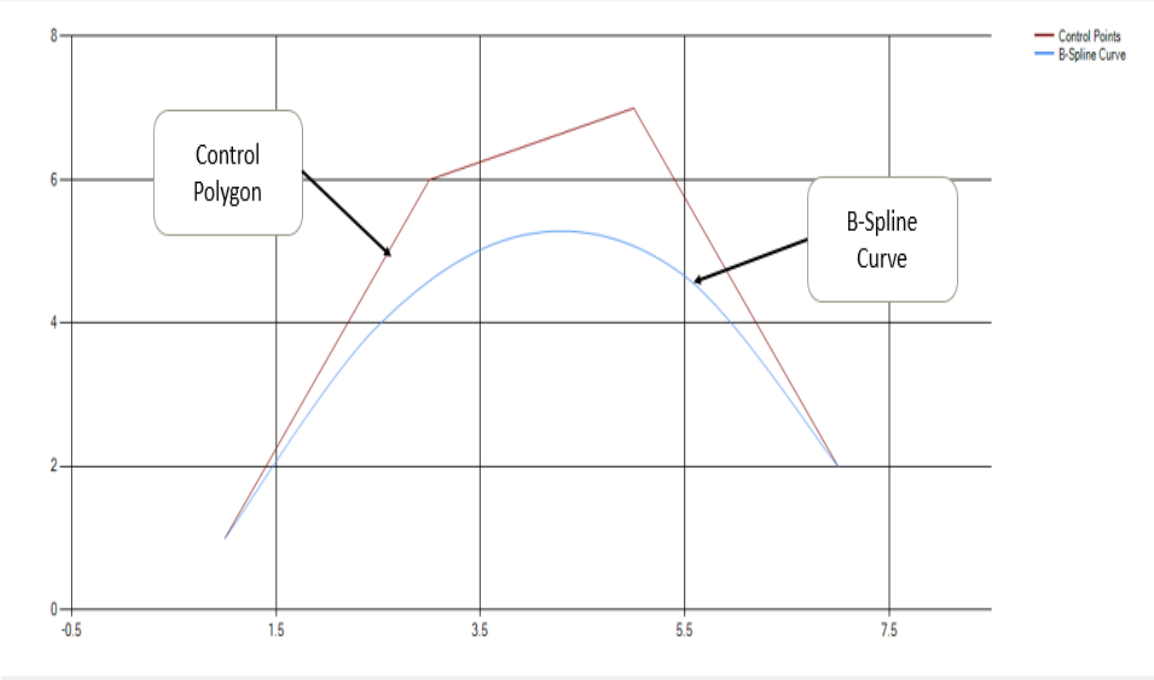

Fig. 2.7. Example of B-Spline Curve

2.2.3 NURBS Curve

NURBS (Non-Uniform rational B-spline) has been used in many CAD/CAM systems to represent all the internal geometric entities used in their systems. It formulates and represents the curve and surface with the unified approach and it is one of the convenient way of designing the geometric entities with the smooth curves and surfaces. NURBS are widely adopted in representing the curve and surface modeling since it offers a common mathematical form for representing both free form and analytical geometry [22, 23]. The NURBS development was started in the late 1970s when Boeing has developed the Tiger CAD system by integrating the B-spline with the rational B-splines.

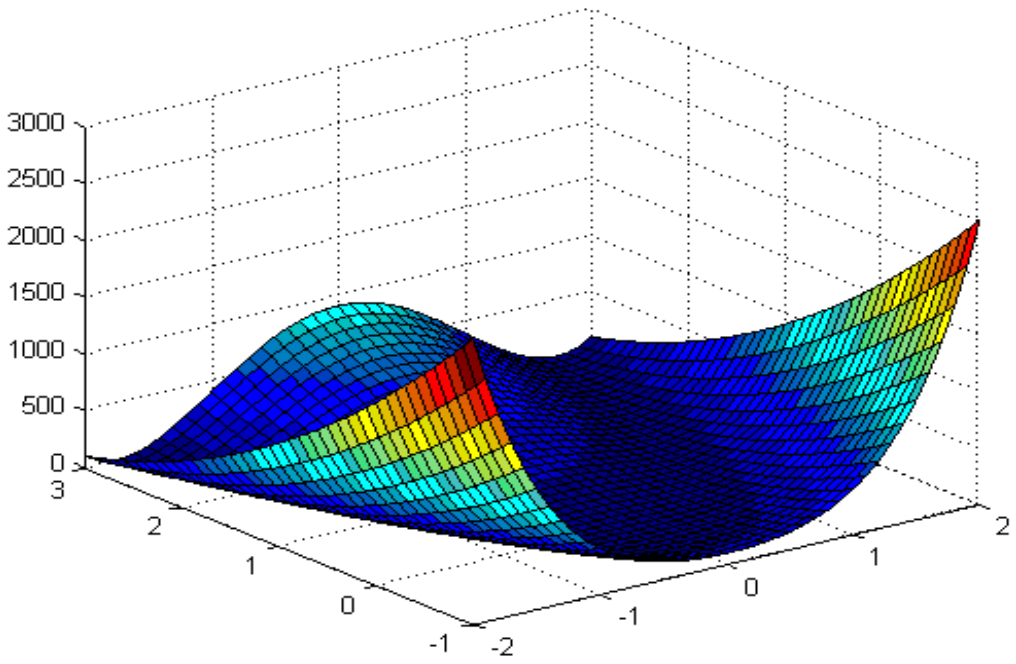

Fig. 2.8. NURBS Surface

Importance of NURBS

NURBS are widely used in CAD/CAM systems for the following reasons as below:

- The representation of both the standard analytical shapes and free-form shapes with the unified common mathematical form.
- NURBS provides the flexibility to design the most complex intricate shapes by manipulating the control points as well as weights.
- It is fast and computationally stable.
- They have lucid geometrical representation that are useful for the designer.
- NURBS are invariant under the scaling, rotation, translation, and perspective transformation of control points.

Drawbacks of NURBS

However, there are some draw backs of representing the curves and surface entities with NURBS or any other free-form schemes.

- To represent the traditional curves and surfaces extra storage is required. For example, to represent a full circle requires a seven control points and ten knots. But, the traditional representation requires only the center, radius and the normal vector to the plane of the circle.
- The curves and surfaces representation can be destroyed with the improper implementation of weights and also can result in very bad parameterization.

2.3 NURBS Formulation

The NURBS interpolation has been widely adopted in curve and surface modeling since it not only offers a common mathematical form for representing free form and analytical geometry [22, 24]. Besides its use as a mathematical description tool, NURBS are used for CNC part programs due to its flexibility to control shape. In this section, lets us summarize the main features of the NURBS formulation.

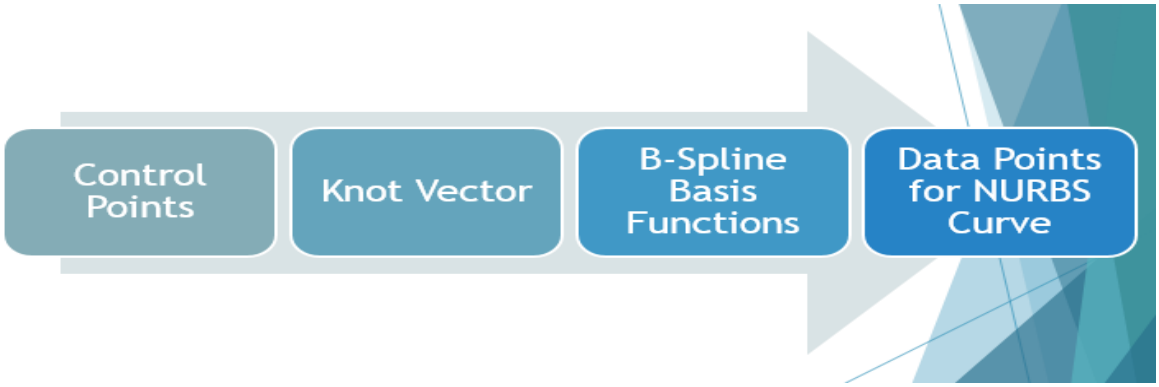

Fig. 2.9. Block diagram of NURBS formulation

Parameters of NURBS

Control Points: Control points are defined as a set of points that determine the shape of the given profile. Figure below shows the control polygon that connect all the control points which are represented by circles in red and the smooth blue color curve is the required NURBS curve. The second curve below in figure is same curve, but one of the control point is moved a bit. We can notice that entire curve has not altered but, only at the small area near the changed control point. Thus this allows to perform some localized changes by moving the individual control points and the influence of the control points is rendered to the part of the curve nearest to it but, has little or no effect to the part of the curve farther away.

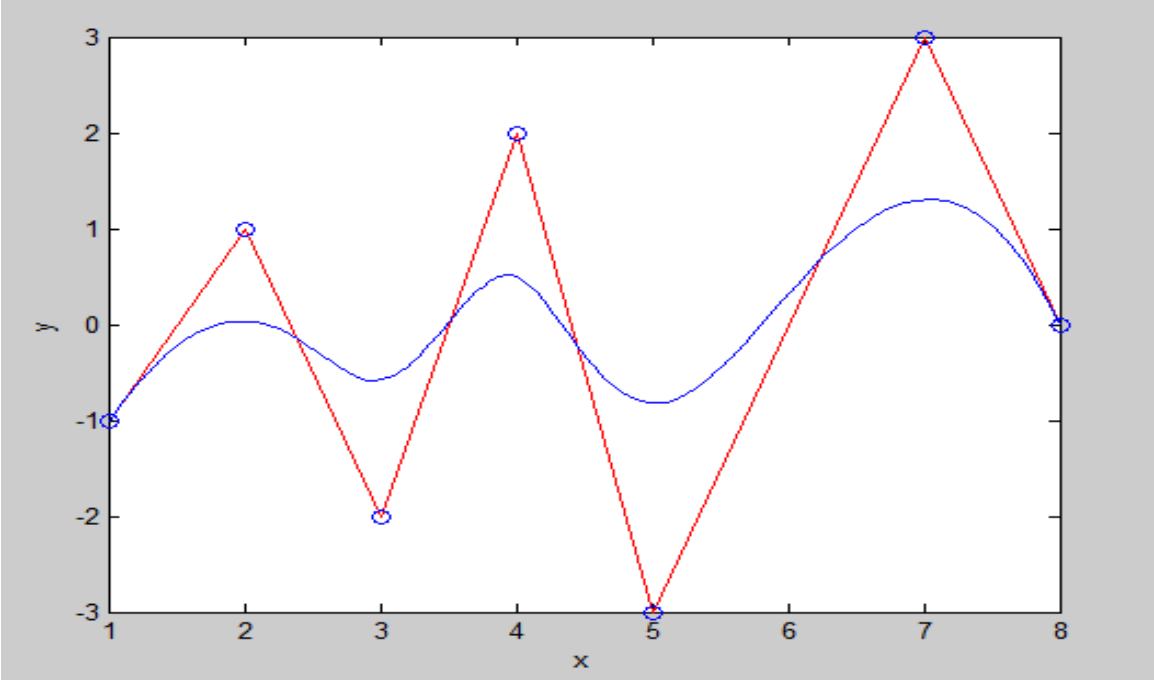

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Fig. 2.10. Control Points before variation in their position

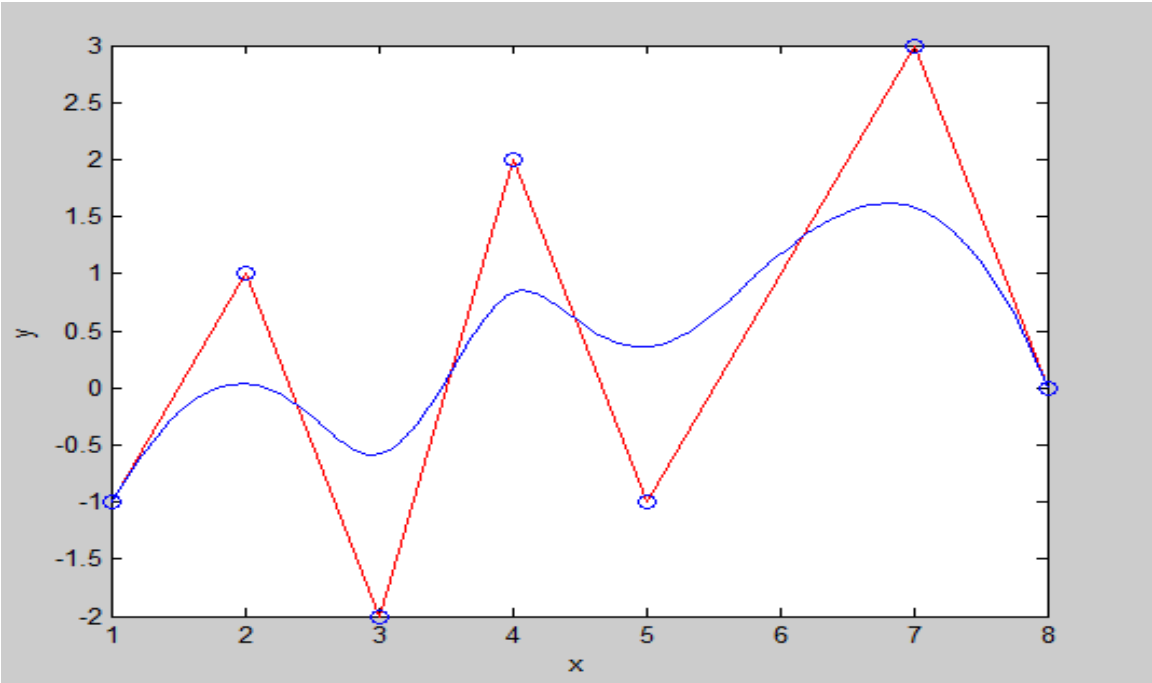

Fig. 2.11. Control Points after variation in their position

Knot Vector: The knot vector in B-Spline curves are evenly spaced over the range of u with unit variation between non-coincident knots, but in non uniform rational b-spline curves the knot vector is non uniform. There can be multiple knots with same values in the knot vector. The length of the knot vector, $m = n + k + 1$ and m knots are required to describe the curve of order k with $(n + 1)$ control points.

The knot vector $U = [U_1, ..., U_{n+k+1}]^T$ is given by

$$Knot \quad Vector, U = [U_0, U_1, ..., U_m]^T, \tag{2.9}$$

There is a limit on order of the curve k which is determined with the number of control points $(n + 1)$ used and is given by,

$$n - k + 2 > 0 \tag{2.10}$$

The above equation results in the requirement of two, three, and four control points to define a line, parabola and cubic curves. The knot values depends on whether the curve is open(nonperiodic) and closed(periodic) curve.

$$Open\ Loop\ Curves, u_j = \begin{cases} 0, & j < k \\ j - k + 1, & k \leq j \leq n \\ n - k + 2, & j > n \end{cases} \tag{2.11}$$

$$Closed\ Loop\ Curves, u_j = j, \quad 0 \leq j \leq (n+1) \tag{2.12}$$

The knot vector of the non uniform rational b-splines are non uniform, thus the name NURBS. The knot vector of NURBS have their first k knots with value 0 and the lask k knots have a value 1 as shown in the equation below,

$$U = [U_0, U_1 ... U_m]^T = [U_0 ... U_0, U_k ... U_{n_{cp}}, U_{m-k} ... U_m]^T \tag{2.13}$$

$$U = [0...0, U_k ... U_{n_{cp}}, 1...1]^T \tag{2.14}$$

Examples of Knot vectors:

1. Uniform Knot Vector (B-Spline Curves),

$$U = [000123444]^{TT} \tag{2.15}$$

2. Non-Uniform Knot Vector (NURBS Curve),

$$U = [000\frac{1}{5}\frac{2}{5}\frac{4}{5}111]^{TT} \tag{2.16}$$

Note: the multiple use of knots of same value in the middle of the knot vector lead to sharp kinks in the curve or less smooth of the curve, but the multiple use of the end knots are desirable to make sure that the curve passes through the first and last control points.

Basis Functions: The function $N_{i,k}(u)$, that strongly determines the influence of the control point P_i on the curve at the parametric value u is called basis function for that control point [**?**]. The value of this function is a real number such as 0.5, particular $C(u)$ can be defined as, say 15% of one control point position, plus 40% of another plus, so on. Thus to complete the NURBS equation we need to specify the basis functions for each control point. It is desired that the each region of the spline would be the average of some control points close to that region. We know that when the moving point is far away from the control points it has influence and when it gets closer to the control point its effect increases. Later the effect would be reduced as the moving point passes away from control point.

Basis Function for open loop curves,

$$N_{i,k}(u) = \frac{N_{i,k}(u)}{\sum_{j=1}^{n_{cp}} N_{j,k}(u)} \tag{2.17}$$

Basis Function for Closed loop curves,

$$N_{i,k}(u) = N_{0,k}((u - i + n + 1) mod(n + 1)) \tag{2.18}$$

Weights: The weight vector of the NURBS curve is an effective design tool to control the NURBS curve locally and it has been implemented in many graphic software which are being used in many industries. The values of weight vector are considered to be positive and each control point p has a corresponding weight w. the functionality of the weight in the NURBS curve is such that when the value of certain weight is increased the curve starts moving towards the control point and similarly, when the weight value for the corresponding control point is decreased the curve moves away from control point. The movement of the curve with variation of the weight w at the control point p is linear considering that the parametric variable u is constant. The values of the weight vector are chosen by the NURBS curve designer.

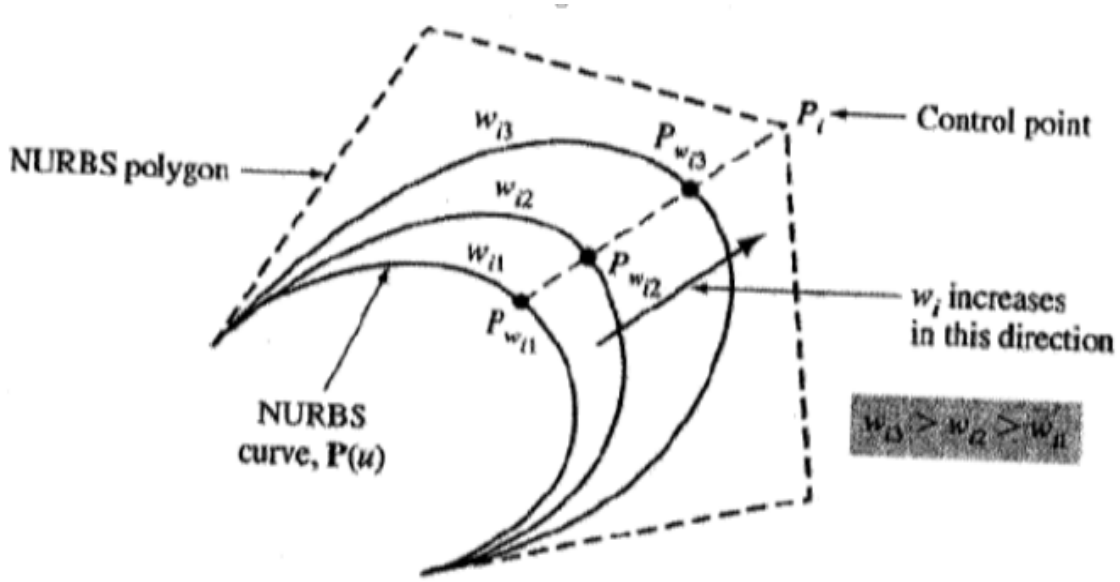

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Fig. 2.12. NUBRS curve with weight variation

The general form of a NURBS interpolation C(u) for a given point u is defined as

$$C(u) = \sum_{i=1}^{n_{cp}} R_{i,k}(u) P_i \tag{2.19}$$

Where, $P_i = [P_{Xi}, P_{Yi}]^T$, $i = 1, , n_{cp}$ contains the coordinates of the i-th control point, n_{cp} is the number of control points and

$R_{i,k}(u)$ is the rational B-spline basis function defined as

$$R_{i,k}(u) = \frac{N_{i,k}(u) w_i}{\sum_{j=1}^{n_{cp}} N_{j,k}(u) w_j}, \tag{2.20}$$

Where,

$N_(i,k)(u)$ is the $k-th$-degree B-spline basis function and

w_i is the corresponding weight.

The B-spline basis function is expressed as

$$N_{i,1}(u) = \begin{cases} 1, & U_i \leq u < U_{i+1} \\ 0, & otherwise \end{cases} \tag{2.21}$$

$$N_{i,k}(u) = \frac{u - U_i}{U_{i+k-1} - U_i} N_{i,k-1}(u) + \frac{U_{i+k} - u}{U_{i+k} - U_{i+1}} N_{i+1,k-1}(u), \quad (\text{for } k > 1) \tag{2.22}$$

Where,

$U_i, i = 1, ..., k + n_{cp}$ are the non-uniform knot values that define the knot vector U. The knot vector $U = [U_1, ..., U_{n+k+1}]^T$ is given by

$$U = [0...0, U_k, ..., U_{n_{cp}}, 1...1]^T, \tag{2.23}$$

Where,

the $n_{cp} - k$ interior knot values are calculated as follows:

$$U_k = U_{k-1} + \Delta U = 0 + \Delta U = \Delta U \tag{2.24}$$

$$U_{k+1} = U_k + \Delta U \tag{2.25}$$

$$\vdots \tag{2.26}$$

$$U_{n+1} = U_n + \Delta U \tag{2.27}$$

where,

$$\Delta U = \frac{1}{(n - k + 2)}. \tag{2.28}$$

Examples of NURBS curve is as follows:

Case 1: NURBS curve is as follows:

Control Points are $p_0(1, -1)$, $p_1(2, 1)$, $p_2(3, -2)$, $p_3(4, 2)$, $p_4(5, -3)$, $p_5(7, 3)$ and $p_6(8, 0)$.

Therefore,

Number of control points, $n + 1 = 7$,

Order of the curve $k = 4$

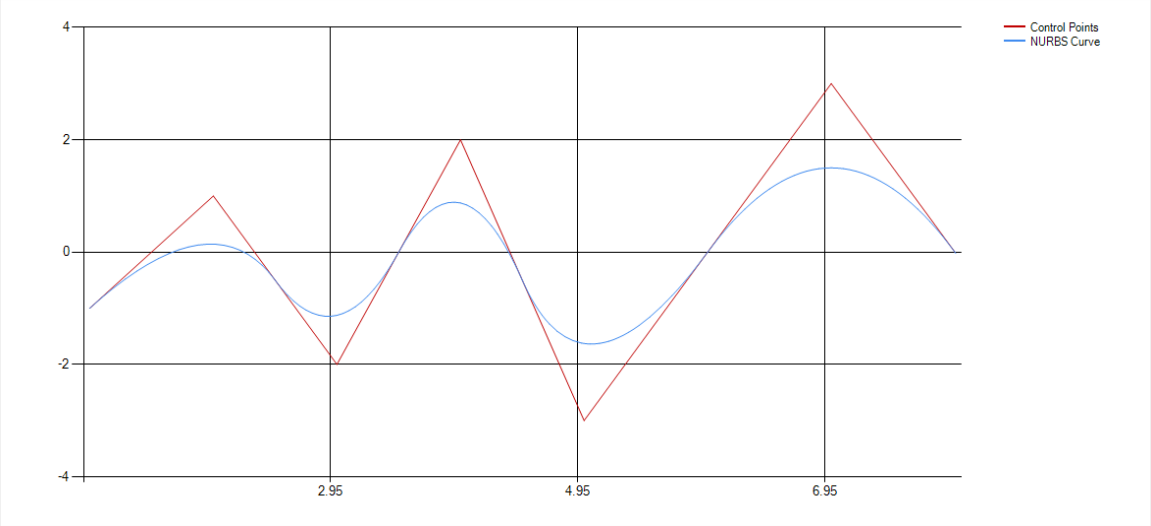

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Fig. 2.13. NURBS Curve with Weight Vector = 1,1,1,1,1,1,1

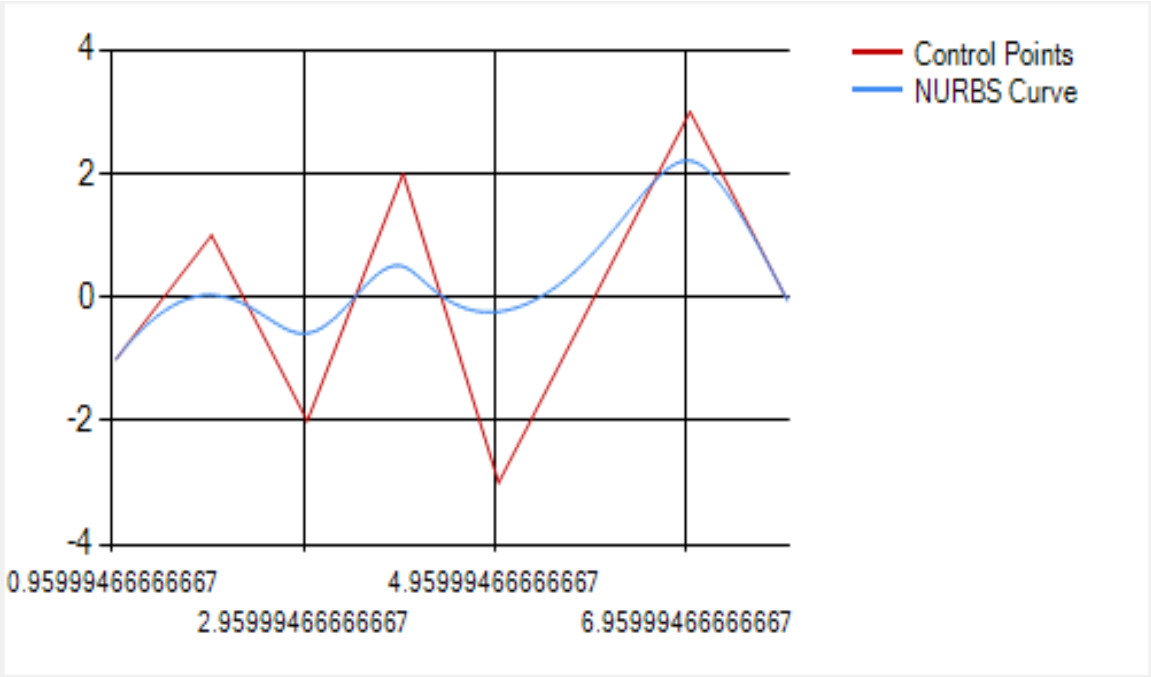

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Fig. 2.14. NURBS Curve with Weight Vector = 1,1,1,1,1,3,1

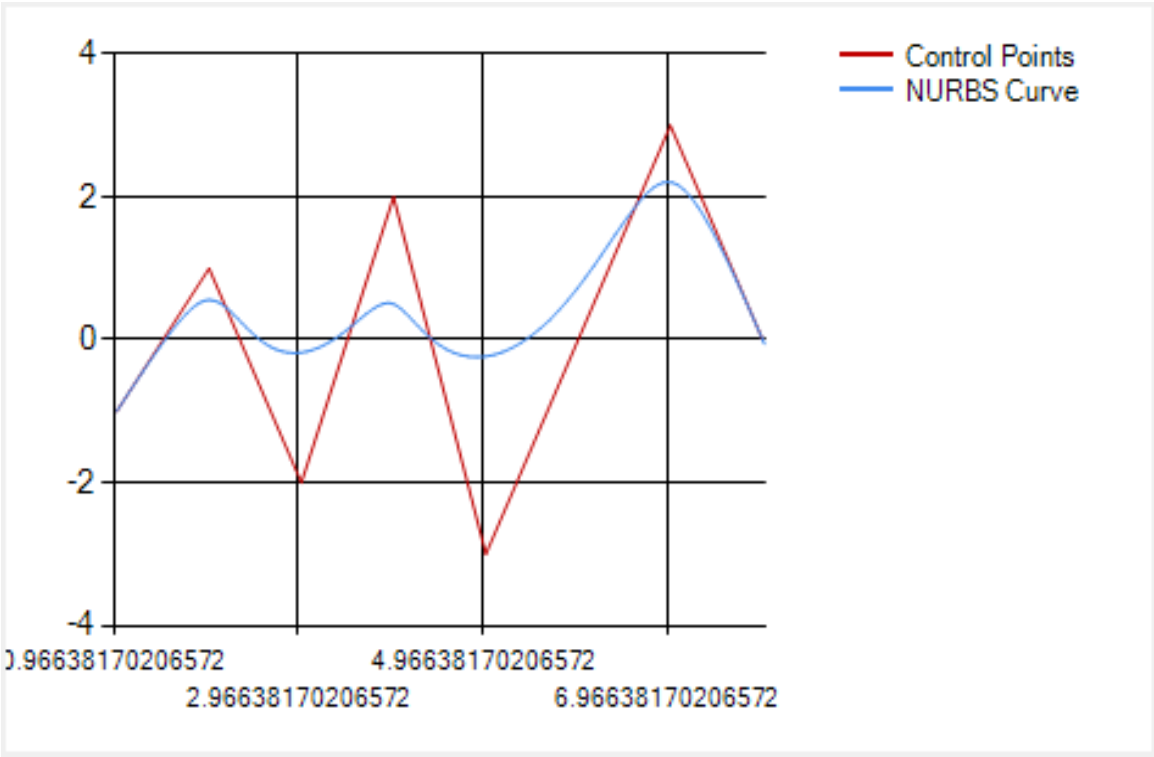

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Fig. 2.15. NURBS Curve with Weight Vector = 1,3,1,1,1,3,1

Case 2: NURBS Surface is as follows

Control Mesh of Points:

	p0	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	p12	p13	p14	p15
x	0	0	0	0	2	2	2	2	3	3	3	3	4	4	4	4
y	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
z	-1	-1	-1	-1	0	0	0	0	1	1	1	1	-2	-2	-2	-2
w	1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

.

Here,

Number of Rows of Control Points, $m = 4$

Number if Columns of Control Points, $n = 4$

Order of the Curve in u direction, $k = 4$

Order of the Curve in v direction, $l = 4$

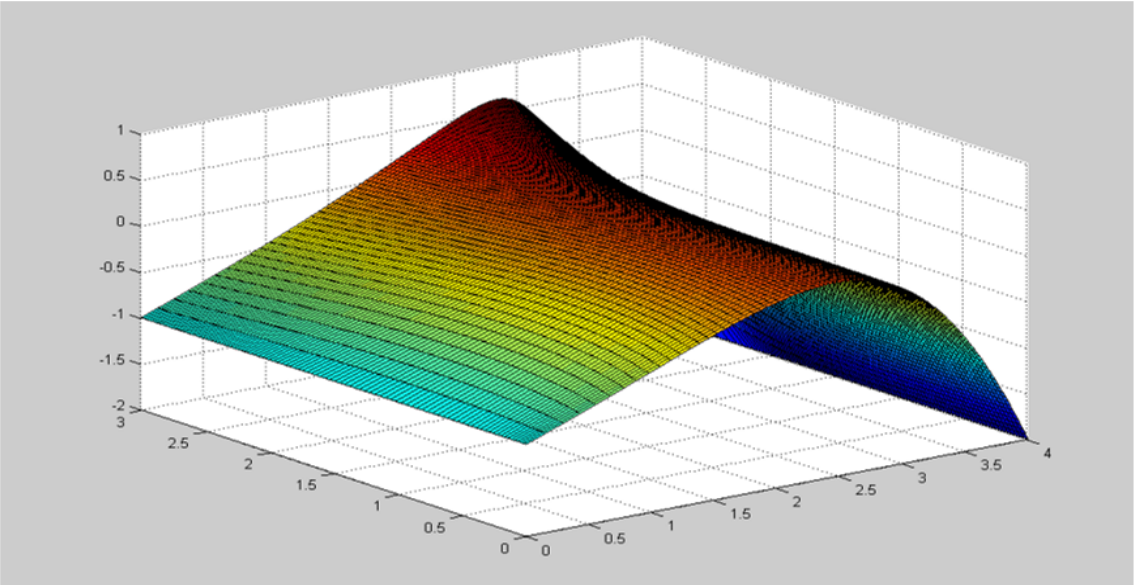

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Fig. 2.16. NURBS Surface

2.4 NURBS Interpolation

The development of the high speed and high accuracy machining has increased gradually in the present industries and the necessity to support this machining advance functions are required such as conventional CNC systems in which the sequential line segments or arcs are used to machine the surfaces [25–34]. The tool moves in a discontinuous manner leading the poor surface finish of the product and also it requires a huge number of program blocks leading to increase in the part program size

and data storage. Therefore, to overcome these drawbacks NURBS interpolation has been implemented based on its ability to define a curves and surface with precision and flexibility.

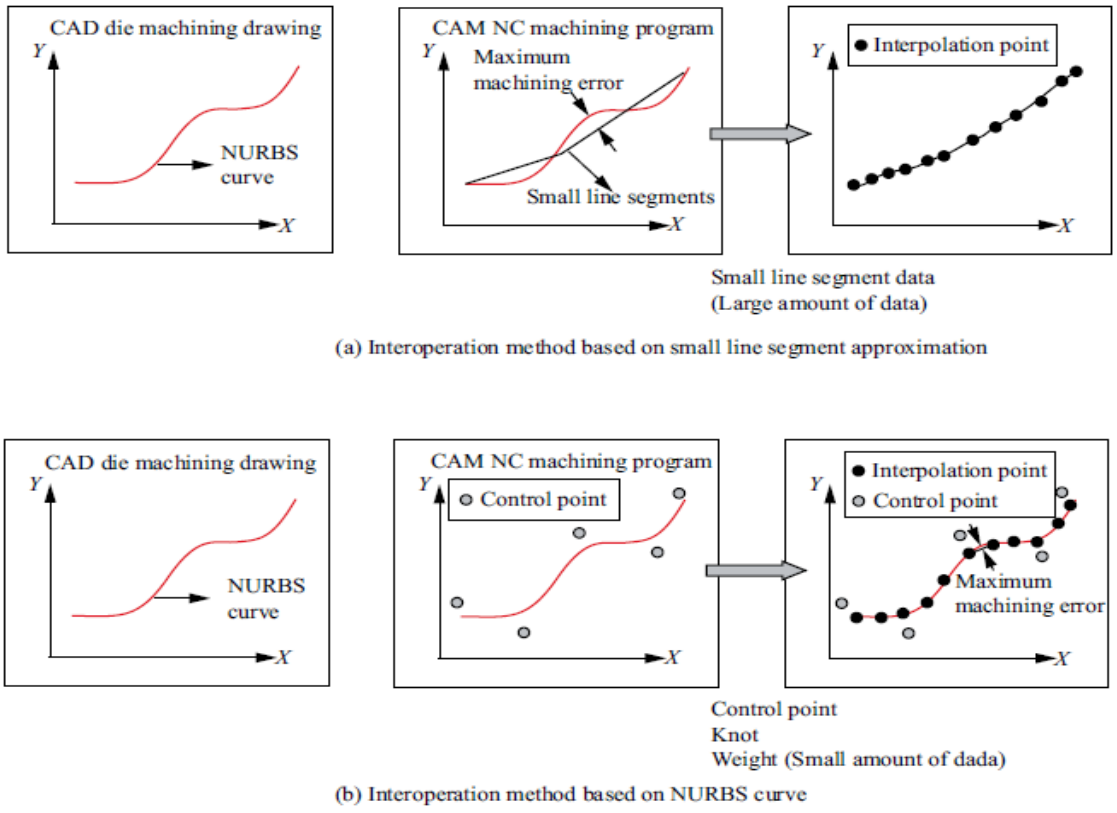

Fig. 2.17. Comparison between line segment and NURBS interpolation

In NURBS interpolation, NURBS curve data (e.g. control points, weights, and knot vector) are considered as inputs parameters and they are directly sent to the CNC system where generates the interpolation points based on the NURBS curve data, instead of the small line segment data that are defined by the G01 command. The figure above shows the implementation of interpolation based on line segment and NURBS.

	FANUC	SIEMENS	OKUMA	Mitsubishi	Toshiba Machine
CNC Model	15 Series, 16 Series, 18 Series, 30 Series	840D	OSP700M (spar H1 CNC)	M700	TOSNUC 888
CPU	64bit RISC	RISC	RISC		
G-code	G06.2	Language Type: BSPLINE	G132	G70.0 G70.1	

.

Fig. 2.18. NUBRS interpolation formats by different CNC makers

The above table shows the implementation of NURBS interpolation function by different CNC companies in different formats. A sample format of the NURBS interpolation format by FANUC CNC maker has been given below,

$G06.2[P]KXYZ[R][F]$;

$KXYZ[R]$

$KXYZ[R]$

$KXYZ[R]$

Where,

G06.2 : NURBS interpolation mode ON

P : the order of NURBS curve(default of 4)

XYZ : the control points

R : the weight

K : the Knot

F : Feed rate

Example of NURBS interpolation:

N1 G06.2 P4 K0 X19.0407 Z4.9824

N2 K0 X18.7908 Z5.1331

N3 K0 X18.3069 Z5.404

N4 K0 X17.6201 Z5.7361

N5 K0.083333333 X16.9658 Z6.0063

N6 K0.166666667 X16.3398 Z6.2228

N7 K0.25 X15.739 Z6.3915

N8 K0.333333333 X15.161 Z6.5166

N9 K0.416666667 X14.6046 Z6.6007

N10 K0.5 X14.0687 Z6.6459

N11 K0.583333333 X13.5534 Z6.6519

N12 K0.666666667 X13.0587 Z6.6187

N13 K0.75 X12.5876 Z6.5406

N14 K0.833333333 X12.2916 Z6.4542

15 K0.916666667 X12.1538 Z6.3915

N16 K1

N17 K1

N18 K1

N19 K1

3. METHODOLOGY TO GENERATE CNC PART PROGRAM FROM OPTIMIZED NURBS

In this chapter a method is described to represent a given set of data points (surface points) using the NURBS geometric formulation curve. The approximation curve is defined for the given data points by representing them with the NURBS curve, in this the data points are converted into set of NURBS parameters such as control points, knots, weights etc., which represent all the set of data points with a smooth NURBS curve. Then an error evaluation curve is defined to evaluate the error between the approximate curve and the target curve. This error between the curves should be as minimum as possible, in order to that some of the NURBS parameters must be altered to minimize the error and for this reason optimization methods are used. These optimization methods used would provide the optimal NURBS parameters that would better represent the target curve with minimum error and with minimum number of control points and optimized weight. Thus these optimum NURBS parameters are used in generating the g-codes using the NURBS interpolation.

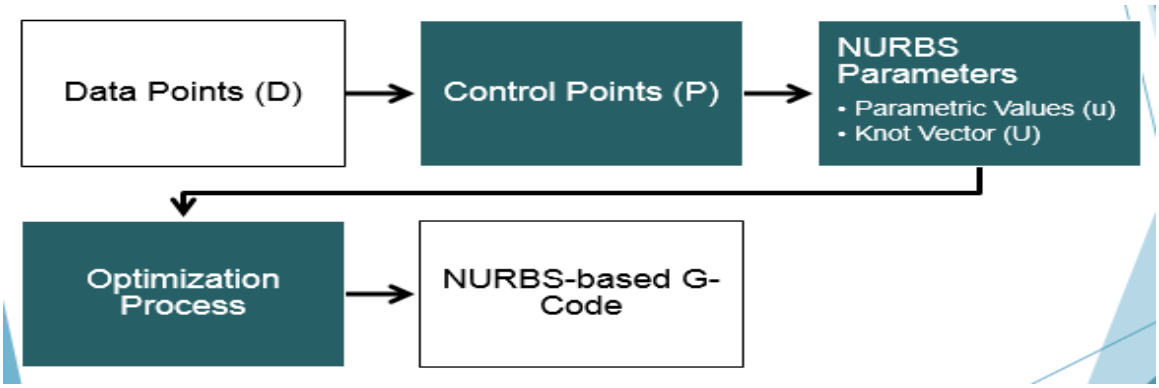

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Fig. 3.1. Block diagram for NURBS function methodology

3.1 Curve Approximation

Curve approximation is the inverse process to point interpolation: given a set of n_{dp} data points $D_j = [D_{Xj}, D_{Yj}]^T, j = 1, ..., n_{dp}$ from the target curve, the objective is to find the n_{cp} control points $P_i = [P_{Xi}, P_{Yi}]^T, i = 1, , n_{cp}$ of the NURBS approximation that better approximates the target curve. The approximation of a given curve is performed in four steps:

Step 1: Parameter extraction - the parameter u_j for each data point D_j is calculated using chord length parameterization. This parameter u_j for each data point is a measure of the distance of the data point along the curve. Specifically, for the n_{dp} data points, the parameter value at the $l - th$ data point is given as

$$u_1 = 0, \tag{3.1}$$

$$u_l = u_{max} \frac{\sum_{s=2}^{l} |D_{Xs} - D_{X,s-1}|}{\sum_{s=2}^{n_{dp}} |D_{Xs} - D_{X,s-1}|}, \quad \text{(for } l = 2,...,n_{dp}) \tag{3.2}$$

where D_{Xs} represent the X-coordinate of the data point D_s.

Step 2: Non-uniform knot valuesFind the knot vector $U = [U_1, ..., U_{n+k+1}]^T$from Eqs. (4) to (6).

Step 3: Control points - The point interpolation in Eq. (1) can be expressed in matrix from as using data points as

$$\begin{bmatrix} D_1(u_1) \\ D_2(u_2) \\ D_3(u_3) \\ \vdots \\ D_{dp}(u_{dp}) \end{bmatrix} = \begin{bmatrix} R_{1,k}(u_1) & R_{2,k}(u_1) & R_{3,k}(u_1) & \dots & R_{cp,k}(u_1) \\ R_{1,k}(u_2) & R_{2,k}(u_2) & R_{3,k}(u_2) & \dots & R_{cp,k}(u_2) \\ R_{1,k}(u_3) & R_{2,k}(u_3) & R_{3,k}(u_3) & \dots & R_{cp,k}(u_3) \\ \vdots & \vdots & \vdots & \vdots & \dots \\ R_{1,k}(u_{dp}) & R_{2,k}(u_{dp}) & R_{3,k}(u_{dp}) & & R_{cp,k}(u_{dp}) \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_{cp} \end{bmatrix}$$

$$D = R(u)P \tag{3.3}$$

where D is a vector of data points, $P = [P_X, P_Y]$ is a matrix containing the control points, and $R(u)$ is a matrix containing the rational B-spline basis functions where u are the parameters from Step 1. The control points P for the approximation curve are calculated as

$$P = (R^T R)^{-1} R^T D \tag{3.4}$$

Thus the NURBS curve is obtained with sample number of control points and the weights corresponding to each control point is considered to be ones, which is same as the B-spline curve.

Step 4: Error calculation - The error between the given $D(u)$ and approximated curves $C(u)$ is defined as the sum of all the differences between the data points of target curve and the approximation curve. This is given by equation as below:

$$f = \sqrt{\int_u (C(u) - D(u))^2 du} \approx \sqrt{\sum_i (C(u_i) - D(u_i))^2} \tag{3.5}$$

In this work, a gradient-free optimization algorithm is implemented to find the NURBS parameters that minimize this error.

3.2 Parameters for Extraction of Control Points

Parametric Variable:

The above procedure describes the extraction of the control points of the NURBS curve for the given data points (surface points) of the target curve. The parametric value for each data point is extracted and it is assumed that their design variable coordinates are at increasing order other than their responses. Thus the algorithm has been developed to extract the parametric values for each individual data point for the given objective curve. The start value of the parametric variable shall be 0 and the end shall be 1. The intermediate values are computed based on the above mentioned formula and an example has been given below.

Example:

a= -3:7

a = [-3 -2 -1 0 1 2 3 4 5 6 7]

A = ParametricVariable1(a)

A = [0 0.1000 0.2000 0.3000 0.4000 0.5000 0.6000 0.7000 0.8000 0.9000 1.0000]

Arrangement of the given data points: For 2D coordinates, we use an algorithm named arrangement that will convert the extracted parametric values of the design variables into a grid like structure and finally arranging them in single column arrays.

Example:

x = -3:1

x =

-3 -2 -1 0 1

y = -2:2

y =

-2 -1 0 1 2

A = arrangement([x(:),y(:)])

ans =

-3 -2
-3 -1
-3 0
-3 1
-3 2
-2 -2
-2 -1
-2 0
-2 1
-2 2
-1 -2
-1 -1
-1 0
-1 1
-1 2
0 -2
0 -1
0 0
0 1
0 2
1 -2
1 -1
1 0
1 1
1 2

Knot Vector:

Compute the Knot vector assuming that there are sample number of control points and order of the curve to be default value (say $k = 4$).In this algorithm we use the non-uniform knot vector in which the first set of values till the count reached the order of the curve value, k shall be zero and the last set of value for the count equivalent to order of the curve shall be equal to one as shown in below example.

Example: Consider number of control points to be, $n = 8$ and the order of the curve to be, $k = 4$.

Knot Vector = [0 0 0 0 0.2000 0.4000 0.6000 0.8000 1.0000 1.0000 1.0000 1.0000]

Computation of Basis Function:

The normalized basis functions are defined recursively as below,

$$N_{i,1}(u) = \begin{cases} 1 & u_i \leq u < u_{i+1} \\ 0 & (otherwise) \end{cases} \tag{3.6}$$

$$N_{i,k}(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_{i,k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1,k-1}(u) \tag{3.7}$$

Here,

u - parametric value

u_i - Elements from knot vector, where ranges from $i = 1$ to $n + k$.

Note: The size of the basis function would be of the form $mxnp$

Where,

m - Number of Rows in parametric variable u

np - Product of elements in Number of Control Points vector, n

For the computation of the basis functions for assumed sample number of control points n_{cp} we use Cox-de Boor recursion formula as mentioned above. Some of the basis functions are calculated using the developed algorithm are shown below and if we

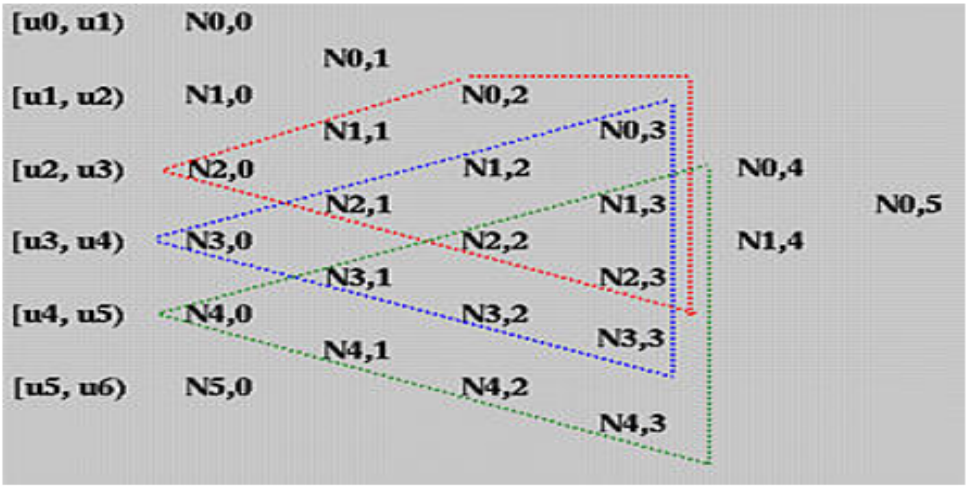

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Fig. 3.2. Triangular Evaluation scheme for Basis Functions

observe the figure showing the basis functions with degree 1 (order, $k = 2$) it consists of straight lines for all the functions in the plot. But, if we observe the basis functions of degree 2 (order, $k = 3$) it consists of parabolic paths. In this recursion formula two consecutive zero degree basis functions $N_{i,0}$ and $N_{i+1,0}$ are used to evaluate the higher degree basis function $N_{i,2}$ and thus following the triangular evaluation scheme. This triangular evaluation scheme is being implemented in the algorithm that has been developed for the evaluation of the basis functions. The basis functions are validated such that the computed basis functions shall be non-zero only in their knot span elsewhere it should be zero as mentioned in the Cox-De door recursion formula.

Example:

N = Nfun(u(i),j,k(i),U(i,:));

Here,

$u(i) = 0.5$

$j = 6$

$k(i) = 4$

$U' = [0\ 0\ 0\ 0\ 0.2000\ 0.4000\ 0.6000\ 0.8000\ 1.0000\ 1.0000\ 1.0000\ 1.0000]$

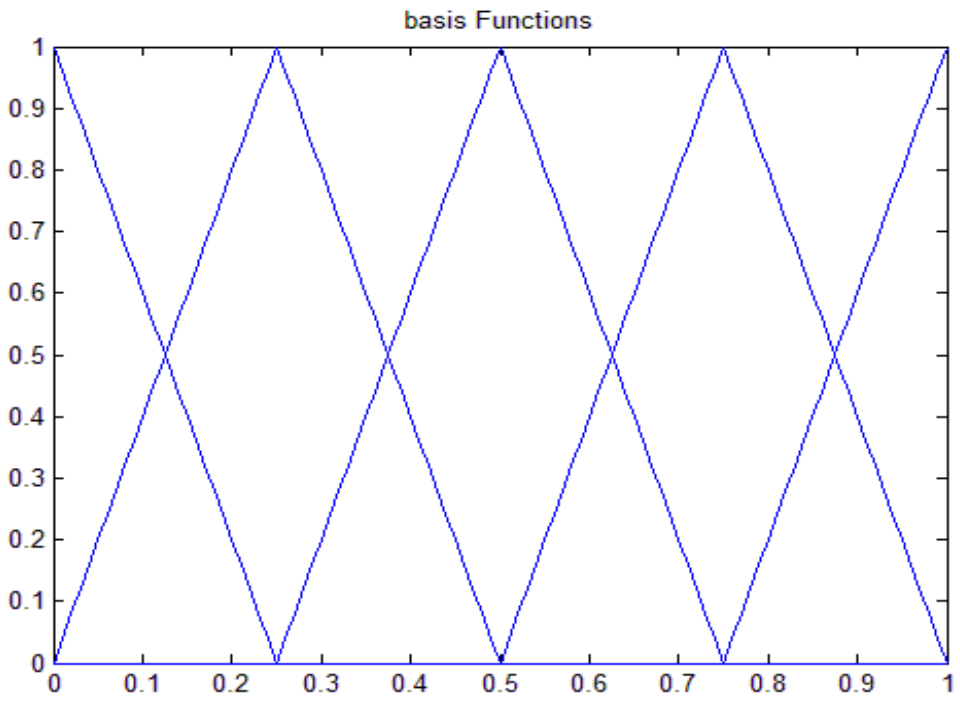

Fig. 3.3. Basis Function for order, k=2 which are straight lines

N = Nfun(0.5,6,4,A)

N = 0.0208

Computation of Rational B-Spline Functions:

The rational b-spline functions are computed by considering the weights and initially all the weights are considered as one. The rational b-spline bases functions will be same as b-spline basis functions if all the weights of the control points are equal. Thus, in this case NURBS curve will be same as the B-Spline curve. The approximation curve is constructed based on the assumption that all the weights of the control points will be equal and then later the error is evaluated between the target curve and approximate curve.

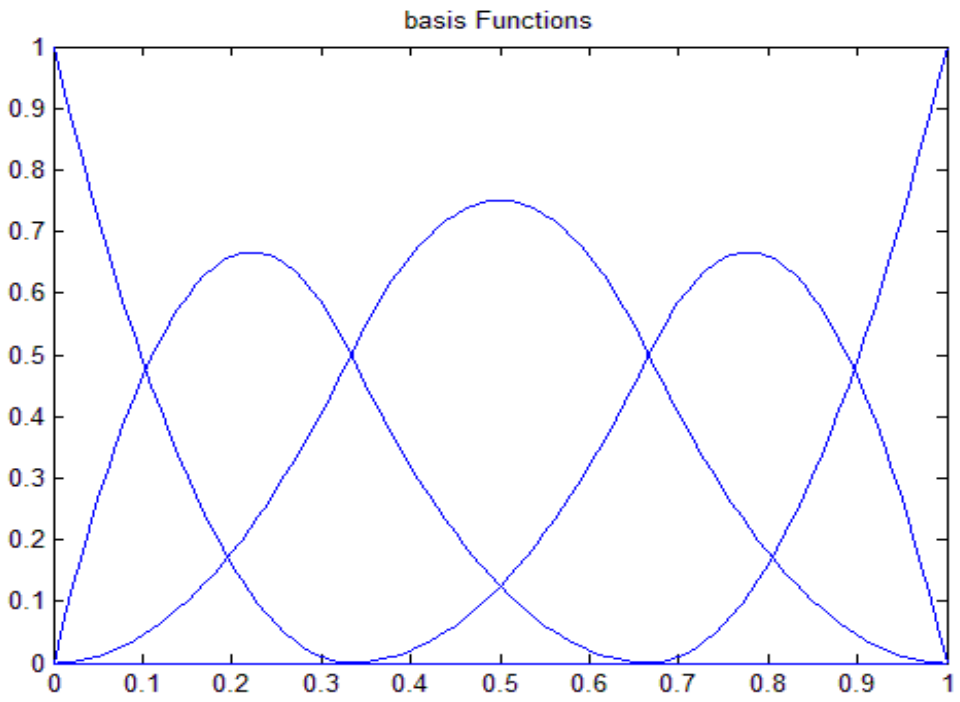

Fig. 3.4. Basis Function for order, k=3 which are parabolic paths

Example:

R = (w(j)*N)/bsum;

Here,

N = 0.0208

j = 6

bsum = 1

w(j) = 1

R = 0.0208

Computing the Control Points for the given Data Points or Surface Points:

In this the control points are calculated based on the above computed rational b-spline basis function matrix and other NURBS parameters. Here an assumption is made by considering the sample number of control points. The control points are

verified by checking the first and last control point of the curve and the first and last data point of the curve are same or not. If they are similar we consider moving to the next evaluation of error between the curves or else we revisit the above computation.

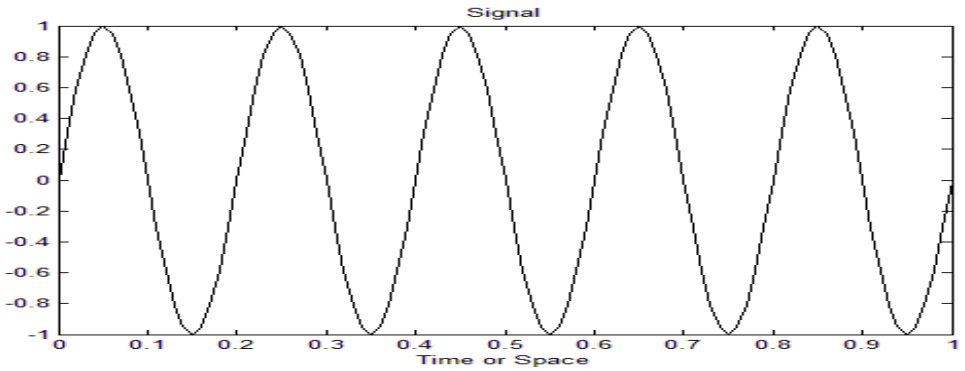

Fig. 3.5. Data point plot from the periodic signal function

The Figure shown above the representation the periodic Signal Wave function. Here we have considered a periodic signal function from which we got our data points or surface points.

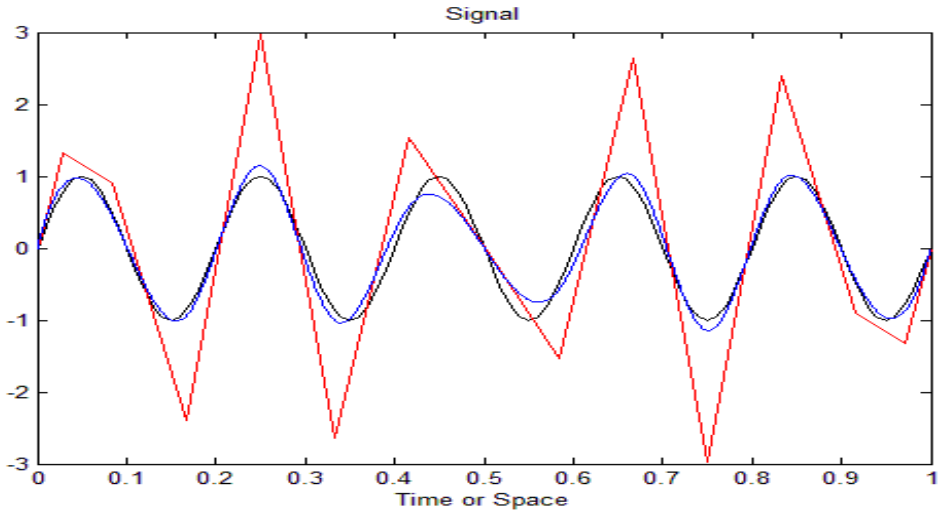

Fig. 3.6. Plots of Data points curve (black), control points polygon (red) and approximate NURBS curve (blue)

Above figure shows an example how to extract the control points for the given set of data points. The black color curve in the above figure represents the target curve. The developed algorithm is being used to extract the given sample number of control points and it is being represented with red in color. This curve which represents the control points is called as the control polygon which connect all the control points. The curve which is in blue in color is our approximate NURBS curve which nearly represents the target curve. The shape of the curve can be altered by moving the control points and varying the weight associated with each control point of the curve.

Error Evaluation between the curves:

Based on the above computed control points and assumed weight vector an approximate NURBS curve is constructed with the same parametric values that were assigned to each data point of the target curve. Thus, a one to one mapping between the target curve and approximate curve for each parametric value is obtained. The difference between these points is calculated and their sum is evaluated as the error between the target curve and approximate curve. This error between the curves is minimized by varying some of the NURBS parameter and to accomplish this we consider some of the optimization methods that will reduce the error between the curves. Thus, it provide the required NURBS curve for the given target curve and then NURBS interpolation is applied onto it based on the optimal parameters obtained after the optimization methods were applied. This optimal parameters of the NURBS curve will be considered in generating the NURBS interpolation G Codes.

The figure below shows the error between the target curve and the approximate curve and it is being highlighted with the red dashed lines. The summation of all the errors between the data points curve and the approximate curve is considered to be error that needs to be minimized in order to better represent the given target curve using NURBS curve with minimum number of control points and with appropriate weight associated with each control point. A truncated view of the above figure is shown below which shows the error between the curves with better resolution.

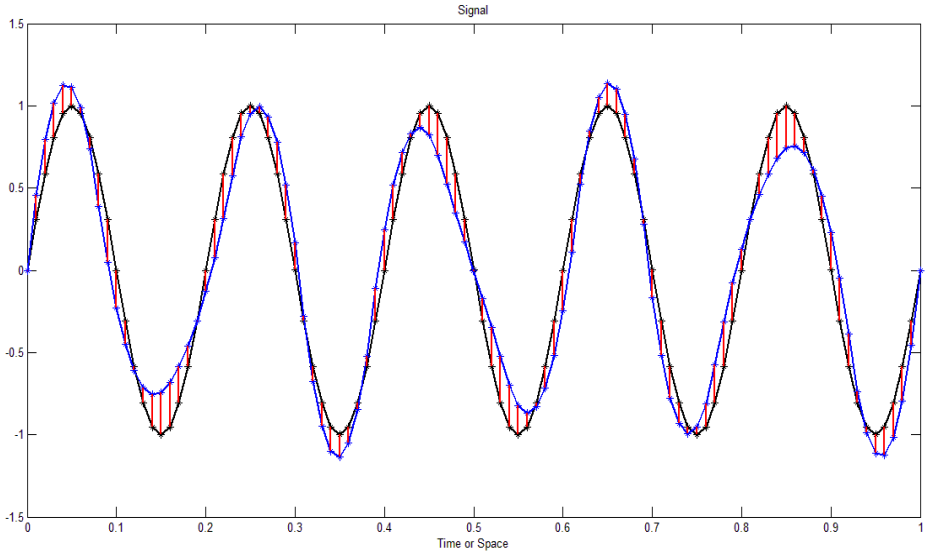

Fig. 3.7. Error between the data points curve and the approximate curve

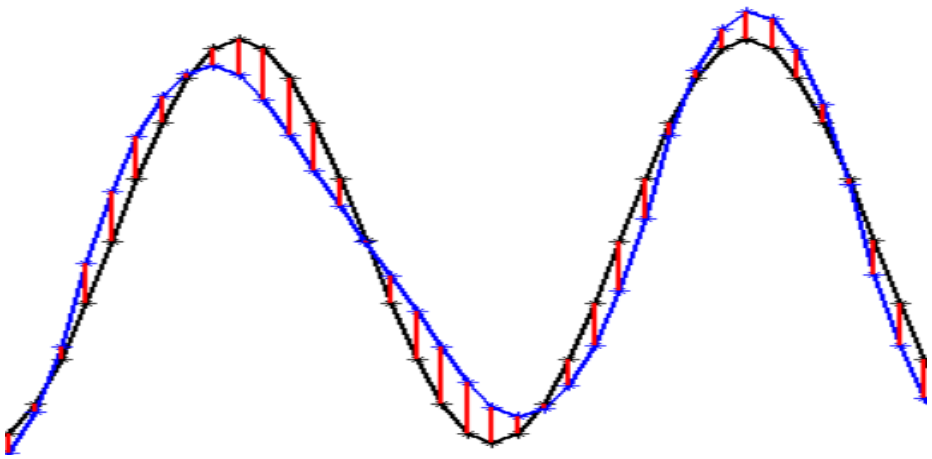

Fig. 3.8. Truncated View of error between the data points curve and the approximate curve

Thus, this error is minimized using the optimization methods which uses the NURBS parameters such as control points, weight vector, etc. as the constraint variables and optimal values of these variables will better or closely represent the target curve with minumum error.

3.3 NURBS Parameter Optimization

NURBS representation is very flexible and can be deformed to any intricate desired shape required by the user. It provides enough set of parameters through which the curve can be deformed to the desired shape. Some of the parameters that help in achieving the desired curve shape or deformation are given below.

- Moving of the control points p_i
- Increasing or decreasing the number of control points.
- Multiple locating of the control points.
- Changing the order k of the basis functions.
- Modifying the weights w_i for each control point.
- Replacing the basis functions
- Rearranging the knot vector U.
- Using multiple knot values in knot vector.
- Changing the relative spacing of the knots.

In this algorithm we have considered the optimization of the NURBS curve using the NURBS parameters such as Control points and their respective weight that would minimize the evaluated error function to near zero and better represent the target curve with the NURBS curve with minimum number of control points and their optimal weight vector associated with it. Once the optimization process is completed as desired then the NURBS optimal parameters are used in generating the NURBS interpolation structure as explained in the previous chapter and then the NURBS G-Codes are implemented as required.

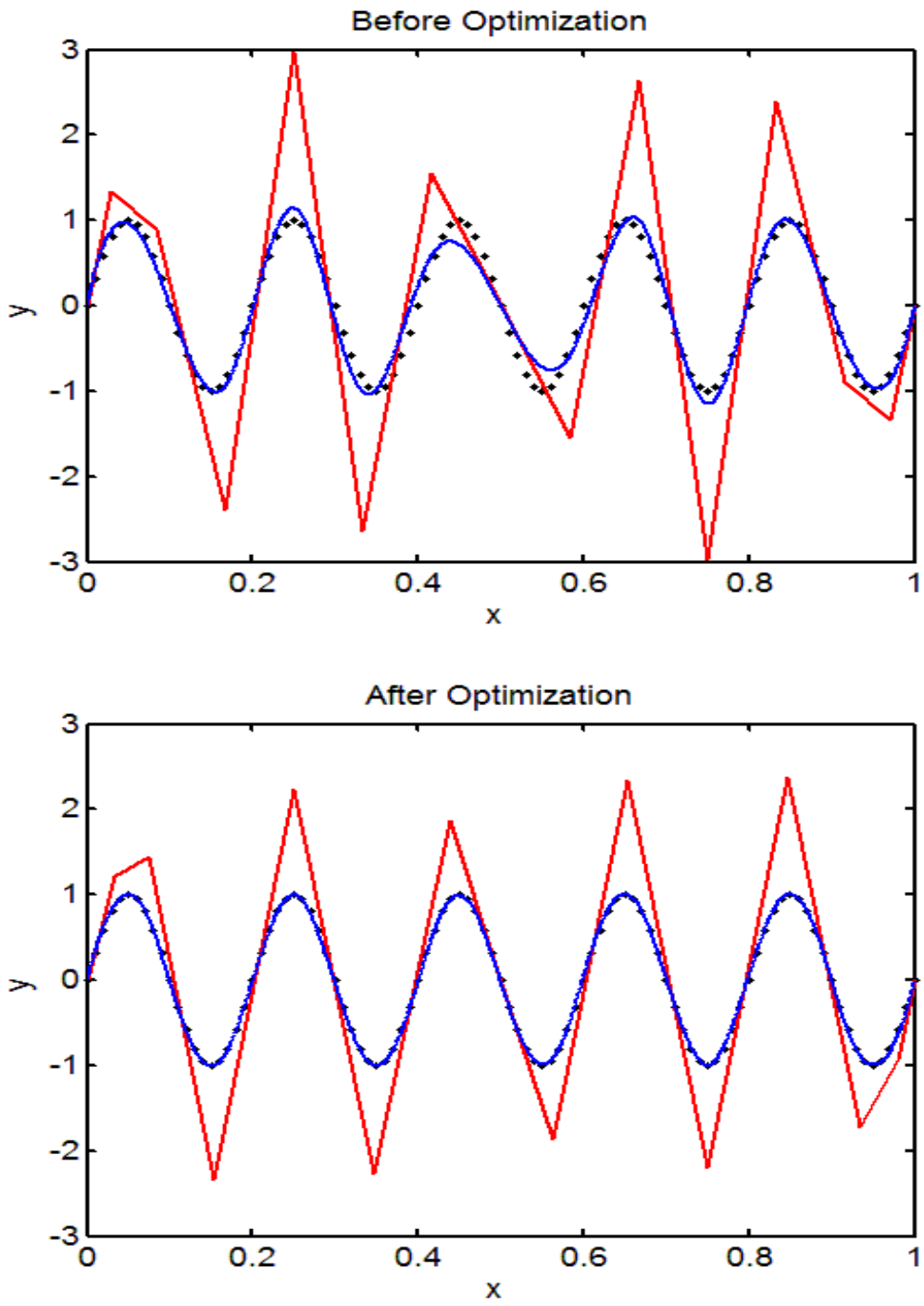

Fig. 3.9. Approximate NURBS curve before optimization and after optimization

The figures above show the approximate NURBS curve before optimization and after optimization of the periodic signal function. We can observe the variation of the control points position and their weights associated with it. Thus providing us with the optimal NURBS parameters required in generating the NURBS interpolation of the given product using NURBS G-Codes.

4. NURBS OPTIMIZATION METHODS

In this chapter optimization of the NURBS curve is done by adjusting the NURBS parameters to optimal values so as to minimize the evaluated error function that was calculated in the previous chapter subjected to certain constraints. The optimization of the NURBS curve is performed using various optimization methods, such as Fmincon, FminSearch, Simulated Annealing and their results are shown in this chapter. Also the NURBS Curve representation is compared with commercially available Meta models such as Krigging function. Thus, this optimization results will be used in generating the NURBS Interpolation G-Codes with their optimal NURBS parameters.

4.1 Introduction

In the present industries the Computer Numerical Control (CNC) machining is required with promising accuracy, speed and repeatability. Also, they are machining the most complex products that has the complex intricate designs. This involves the workflow CAD-CAM-PP-CNC-CAQ to manufacture a product and it has to be repeated. This will increase the cost of the product and delivery time. There are many other methods that meets the requirements but they have relatively lower accuracy, size of the part program increases which leads to increased storage requirements. Thus, in order to reduce the time and the cost associated with its manufacturing we have implemented a new methodology that will reduce both the time and cost using the NURBS interpolation. This will compact the size of the part program associated with complex product shapes and reduces delivery time. This NURBS interpolation requires the use of optimal NURBS parameters in their G-Codes to better represent the given product shape with NURBS Curves with the promising accuracy.

From literature it is found that many efforts have been made to optimize NURBS

interpolation parameters. In Chapter 1 their review and summary are clearly explained. There are many optimization methods used to minimize the error generated between the targeted curve and the approximated curve. Some of them are Adams Bashforth interpolation, web-like search algorithm, Simulated Annealing and quasi Newton optimization algorithm are used to optimize the NURBS interpolation Parameters. In this research work, to obtain the NURBS curve data an approximate curve is found based on the given data points and different optimization algorithms have been implemented that are capable to identify the optimal NURBS curve parameters. The Optimization algorithms such as SQP, Nelder-Mead, and SA are compared in the error minimization between the approximate curve and the target curve. One-dimensional parametric search is used to find the minimum number of points. Lastly, the optimized data of the location of control points and their respective weights have been used in generating the G-Codes using the NURBS Interpolation.

4.2 Objective Function

The objective function for this research will be the evaluated error bound function that calculates the error between the target curve and the approximate curve. This objective needs to be minimized by varying the NURBS parameters to its optimal level. The weight vector will be initialized to all ones before the optimization.

The optimization problem is described to as

$$
\begin{aligned}
& & \text{given:} \quad & f^{target} \\
& O1 & \text{minimize:} \quad & n_{cp} \in \{1,2...\} \\
& & \text{subject to:} \quad & f(n_{cp}) \leq f^{target}
\end{aligned}
\tag{4.1}
$$

coupled with

$$O2 \qquad \begin{array}{l} \text{given:} \quad n_{cp} \\ \text{find:} \quad \theta \in R^{n_{dv}} \\ \text{minimize:} \quad f(\theta) : R^{n_{dv}} \rightarrow R) \end{array} \tag{4.2}$$

where $f(\theta)$ is the error function defined by $Eq.(3.5)$, where f^* is a the target error, $f^* = f(^*)$, n_{cp} is the number of control points, and is the set of n_{dv} design variables. In this work, three design variables are considered: the control point coordinates P_{Xi} and P_{Yi} and their corresponding weights w_i for $i = 1, ..., n_{cp}$. In other words $= [P_X, P_Y, W]$ or

$$\theta \begin{bmatrix} P_{X1} & P_{Y1} & w_1 \\ P_{X2} & P_{Y2} & w_2 \\ \vdots & \vdots & \vdots \\ P_{Xn_{cp}} & P_{Yn_{cp}} & w_{n_{cp}} \end{bmatrix}$$

Therefore, $n_{dv} = 3n_{cp}$. The coupling between the two optimization algorithms can be expressed as O1[O2], where the solution of O2 is required to solve O1.

4.3 Parametric Optimization

For an initial number of control points n_{cp}, the optimization algorithm solves O2 to find θ^* that drives the error to its minimum values $f^* = f(\theta^*)$.

If $f^* > f^{target}$ (unfeasible condition) then the algorithm increases the number of control points, e.g., $n_{cp} \leftarrow n_{cp} + 1$.

If $f^* f^{target}$, then the algorithm reduces the number of control points, e.g., $n_{cp} \leftarrow n_{cp} - 1$. The algorithm terminates when n_cp cannot be further reduced without violating the feasibility condition.

The calculation of f^* requires the solution of O2. In this paper NelderMead algorithm, Sequential Quadratic Programming Algorithm and Simulated Annealing Optimization Methods are used to solve the optimization problem O2.

4.4 Optimization Methods

4.4.1 Nelder Mead Simplex Algorithm

The Nelder-Mead (NM) simplex algorithm is one of the most used gradient-free search method for solving the unconstrained optimization problems [35,36]. A simplex is a geometric gure in n_{dv} dimensions that is the convex hull of n_{dv}+1 vertices identified as $_1, _2, ..., _(n_{dv}+1)$. The vertices are ordered according to the objective function values

$$f(\theta_1) \leq f(\theta_2)... \leq f(\theta_{n_{nd+1}}), \tag{4.3}$$

where θ_1 is referred to as the best vertex and to θ_{n_dv+1} to as the worst vertex. Lagarias et al. [13] provide tie-breaking rules such as those given in are required for the method when two vertices have the same objective value.

The NM algorithm uses four operations: reection, expansion, contraction, and shrink, each being associated with a scalar parameter: α (reflection), β (expansion), γ (contraction), and δ (shrink). The values of these parameters satisfy $\alpha > 0$, $\beta > 1$, $0 < \gamma < 1$, and $0 < \delta < 1$. In the standard implementation of the NM method, the parameters are chosen to be $\{\alpha, \beta, \gamma, \delta\} = \{1, 2, 1/2, 1/2\}$. We now outline the NM algorithm as follows [13]:

Step 1: *Sort* - Evaluate f at the $n_{dv} + 1$ vertices of and sort the vertices.

Step 2. *Reflection* - Let $\overline{\theta}$ be the centroid of the n_{dv} best vertices. Compute the reection point $_r$ from

$$\theta_r = \overline{\theta} + \alpha(\overline{\theta} - \theta_{n_d v+1}). \tag{4.4}$$

The evaluate

$$f_r = f(\theta_r) \tag{4.5}$$

If $f_1 \leq f_r < f_{n_{dv}}$ then replace $\theta_{n_d v+1}$ with θ_r.

Step 3: *Expansion* - If $f_r < f_1$ then compute the expansion pointθ_e from

$$\theta_e = \overline{\theta} + \beta(\theta_r - \overline{\theta}). \tag{4.6}$$

and evaluate $f_e = f(\theta_e)$. If $f_e < f_r$ then replace $\theta_{n_d v+1}$ with θ_e, otherwise replace $\theta_{n_d v+1}$ with θ_r.

Step 4: *Outside contraction* - If $f_n \leq f_r < f_{n+1}$, compute the outside contraction point

$$\theta_{oc} = \overline{\theta} + \gamma(\theta_r - \overline{\theta}) \tag{4.7}$$

and evaluate $f_{oc} = f(\theta_{oc})$. If $f_{oc} \leq f_r$ then replace $\theta_{n_{dv}+1}$ with θ_{oc}, otherwise go to Step 6.

Step 5: *Inside contraction* - If $f_r > f_{n_{dv}+1}$ then compute the inside contraction point θ_{ic} from

$$\theta_{ic} = \overline{\theta} - \gamma(\theta_r - \overline{\theta}) \tag{4.8}$$

and evaluate $f_{ic} = f(_{ic})$. If $f_{ic} < f_{n_{dv}+1}$ then replace $\theta_{n_{dv}+1}$ with θ_{ic}, otherwise go to Step 6.

Step 6. *Shrink* - Calculate the n_{dv} vertices

$$\theta_i = \theta_1 + \delta(\theta_i - \theta_1) \tag{4.9}$$

and calculate $f_i = f(\theta_i)$ for $2 \leq n_{dv} + 1$.

Step 7: *Termination criteria* - The vertices of the simplex at the next iteration are $\theta_1, ..., \theta_{n_{dv}+1}$. Finally, verify termination criteria,

$$|f_1 - f_{n_{dv}+1}| \leq \epsilon_f \tag{4.10}$$

and

$$|\theta_1 - \theta_{n_{dv}+1}| \leq \epsilon_\theta \tag{4.11}$$

where ϵ_f and ϵ_θ are small numbers, $\epsilon_f = \epsilon_\theta = 10^{-6}$. If the algorithm is terminated then the optimizer is θ_1 with the minimum value f_1. To demonstrate the proposed optimization method, let us consider the manufacturing of a precise rack and pinion gear.

4.4.2 Fmincon Optimization Method

Fmincon optimization method is a nonlinear programming solver which finds the minimum of the constrained nonlinear multivariable functions. An example is shown below,

Finds the minimum of a problem specified by,

$$min f(x) \quad such \quad that \begin{cases} c(x) \leq 0 \\ ceq(x) = 0 \\ A.x \leq b \\ Aeq.x = beq \\ lb \leq x \leq ub \end{cases} \tag{4.12}$$

Here,

- b and beq are vectors,
- A and Aeq are matrices,
- c(x) and ceq(x) are functions that returns vectors
- f(x) is a function that returns scalar
- f(x), c(x) and ceq(x) can be nonlinear functions.

The algorithms used in solving these constrained nonlinear functions are:

1. Interior-point optimization
2. SQP Optimization
3. Active-Set Optimization
4. Trust-Region-Reflective Optimization

Interior Point Algorithm:

The interior point algorithm uses a sequence of approximate problems to solve the constrained nonlinear minimization function. The original problem is defined as below,

$$minf(x), \quad subject \quad to \quad h(x) = 0 \quad and \quad g(x) \leq 0, \tag{4.13}$$

For each $\mu < 0$, the approximate problem is,

$$minf_{\mu}(x, s) = minf(x) - \mu \sum_{i} ln(s_i), subject \quad to \quad h(x) = 0 \quad and \quad g(x) + s = 0, \tag{4.14}$$

There are as many slack variables s_i as there are inequality constraints g. The si are restricted to be positive to keep $ln(s_i)$ bounded. As μ decreases to zero, the minimum of f_{μ} should approach the minimum of f. The added logarithmic term is called a *barrier function*.

To solve the approximate problem, the algorithm uses one of two main types of steps at each iteration:

- A direct step in (x, s). This step attempts to solve the KKT equations
- A CG (conjugate gradient) step, using a trust region.

By default, the algorithm first attempts to take a direct step. If it cannot, it attempts a CG step. One case where it does not take a direct step is when the approximate problem is not locally convex near the current iterate

At each iteration the algorithm decreases a merit function, such as

$$minf(x,s) + v||(h(x), g(x) + s)||. \tag{4.15}$$

The parameter may increase with iteration number in order to force the solution towards feasibility. If an attempted step does not decrease the merit function, the algorithm rejects the attempted step, and attempts a new step. If either the objective function or a nonlinear constraint function returns a complex value, NaN, Inf, or an error at an iterate x_j, the algorithm rejects $x + j$. The rejection has the same effect as if the merit function did not decrease sufficiently: the algorithm then attempts a different, shorter step.

Active Set Optimization Algorithm:

The active set optimization is a constrained optimization method that transforms the given problem into an easier sub problem that can be solved based on the iterative process. In the early methods large class constrained problems are transformed into unconstrained problems using a penalty function for the given constraints considering their constraint boundaries. Now these methods are considered to be inefficient and these methods are replaced with the use of Karush Kuhn Tucker (KKT) equations. These KKT equations are necessary to find the optimality of the given constrained optimization problem and if the given problem is convex programming problem and the constraints are convex function then the KKT equation are sufficient enough to get the global solution point for the given constrained optimization problem.

$$\nabla f(x^*) + \sum_{i=1}^{m} \lambda_i . \nabla G_i(x^*) = 0 \tag{4.16}$$

$$\lambda_i . G_i(x^*) = 0, i = 1, .., m_e \tag{4.17}$$

$$\lambda_i \geq 0, i = m_e + 1, \ldots, m, \tag{4.18}$$

The above equation shows that the gradients between the objective function and the active constraints are removed at their solution point. The gradients are replaced with the LaGrange multipliers $(_i, i = 1, 2m)$ that would balance the deviation of the magnitude of the objective function and constraint gradients. In this operation only the active constraints are considered and all the other non-active constraints are multiplied by Lagrange multiplier$_i = 0$. Thus the solution of the KKT equations is considered to be the bases for many nonlinear constrained optimization problems. The Lagrange multipliers are computed directly with the use of algorithms such as quasi-newton methods.

Sequential Quadratic Programming Algorithm:

The sequential quadratic programming algorithms is similar to the act set optimization algorithm but the only different between them is as below:

1. Strict Feasibility with respect to Bounds
2. Robustness to Non-Double Results
3. Refactored Linear Algebra Routines
4. Reformulated Feasibility Routines

Trust-Region Methods for Nonlinear Minimization:

To understand the trust-region approach to optimization, consider the unconstrained minimization problem, minimize $f(x)$, where the function takes vector arguments and returns scalars. Suppose you are at a point x in n-space and you want

to improve, i.e., move to a point with a lower function value. The basic idea is to approximate f with a simpler function q, which reasonably reflects the behavior of function f in a neighborhood N around the point x. This neighborhood is the trust region. A trial step s is computed by minimizing (or approximately minimizing) over N. This is the trust-region subproblem,

$$min\{q(s), s \in N\}, \tag{4.19}$$

The current point is updated to be $x + s$ if $f(x + s) < f(x)$; otherwise, the current point remains unchanged and N, the region of trust, is shrunk and the trial step computation is repeated.

4.4.3 Simulated Annealing Optimization Method

The Simulated annealing optimization methods is used where the number of objects increase for certain optimization problem which cannot be solved by other methods. It is a very practical algorithm as the name indicate the process of heating the metal and then cooling it slowly. This method would provide with a very good solution even in the presence of noisy data unlikely with the optimal solution. An example for this methods is a traveling salesman who visits many cities with minimizing the total travel mileage. Suppose if the salesman started his travel with a random ticket and has planned to reduce his total travel mileage in the upcoming travel at each exchange. This approach will provide the local minimum but not the Global minimum as required. The simulated annealing has improved this with the inclusion of two algorithms. The first being the metropolis algorithm that includes some trades that do not reduce the total mileage to allow the solver to have extra space for its solution. These bad trades are allowed with the formula indicated below,

$$e^{-\Delta D/T} > R(0, 1), \tag{4.20}$$

Where,

- is the change of distance implied by the trade (negative for a "good" trade; positive for a "bad" trade),
- is a "synthetic temperature," and
- is a random number in the interval .
- is called a "cost function," and corresponds to the free energy in the case of annealing a metal

The space for solution can be increased with the inclusion of bad trades and this can be possible if the T is larger.

The second methods uses the process of annealing to lower the temperature and it would restrict the size of the allowed bad trades. This is done because after making many trades we observe that the cost function is reduced very slowly and that is the reason we lower the temperature in order to restrict the size of the allowed bad trades. This is done periodically many times and then it starts to accept only the good trades that provides us with the local minimum of the cost function. There is another methods that is faster than the rest of the two and it is based on how the temperature is reduced in annealing process. In this method they use the threshold acceptance which will allow the bad trades to be included in the space of solution up to the level of threshold mark and this threshold is decreased periodically. Thus, this will eliminate the use of exponentiation and random number computation in the Boltzmann criteria.

Some of the observations before optimization and after optimization of the generated approximate NURBS curve are shown below. For this observation we have considered some of the well know geometric entities such as periodic signal wave, star shape and trident curve. We have shown an example below using the periodic signal wave, the nature how the number of control points are very important in generating the approximate NURBS curve which is close enough to represent the target curve. After the selection of the sample number of control points for the approximate curve

which is close enough to target curve, we then optimize the weight vector of the NURBS approximate curve using the above optimization methods and some of the examples are shown below.

Case 1: Selection of Sample number of Control Points for the approximate Trident Curve using NURBS Curve.

The trident curve is a fourth order NURBS curve ($k = 4$) generated using the following control points P, knot vector U, and weights W:

$P = [(10, 0), (20, 20), (12, 8), (10, 20), (8, 8), (0, 20), (10, 0)]^T$

$U = [00000.250.500.751111]^T$

$W = [1111111]^T$

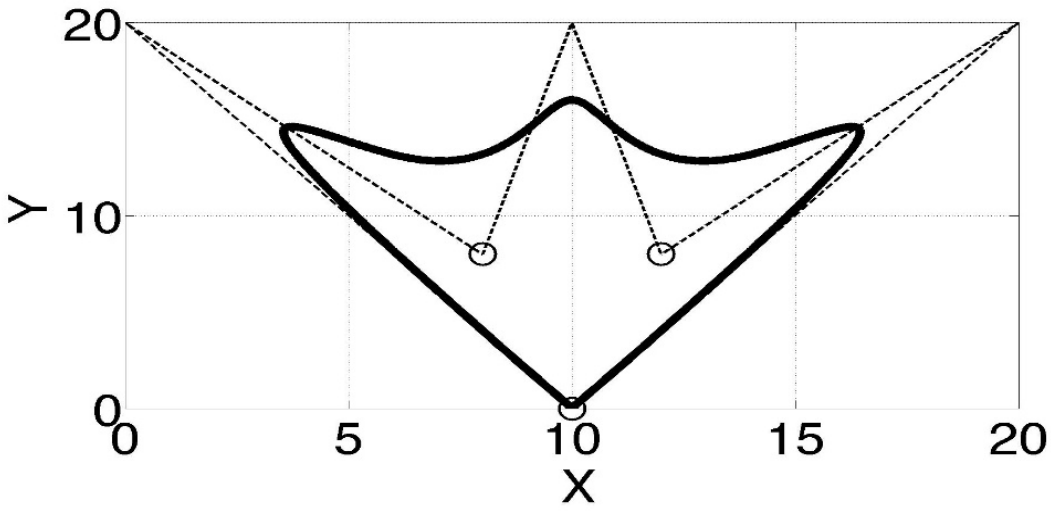

Fig. 4.1. Target Trident Curve

The objective of this test problem is to use the proposed optimization approach to find the NURBS parameters of the trident curve, which are unknown for the algorithm, based on collected data $D(u_i)$. To illustrate this, let us present the results for $n_{cp} = 5, 6, 7, 8$.

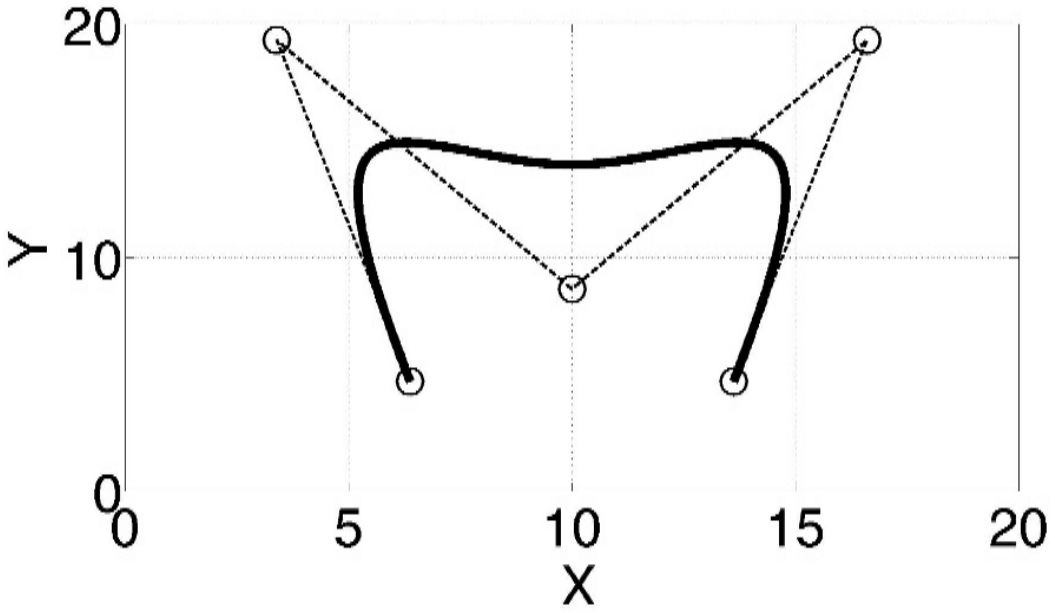

Fig. 4.2. n_{cp}=5,$f^* = 1.8074$

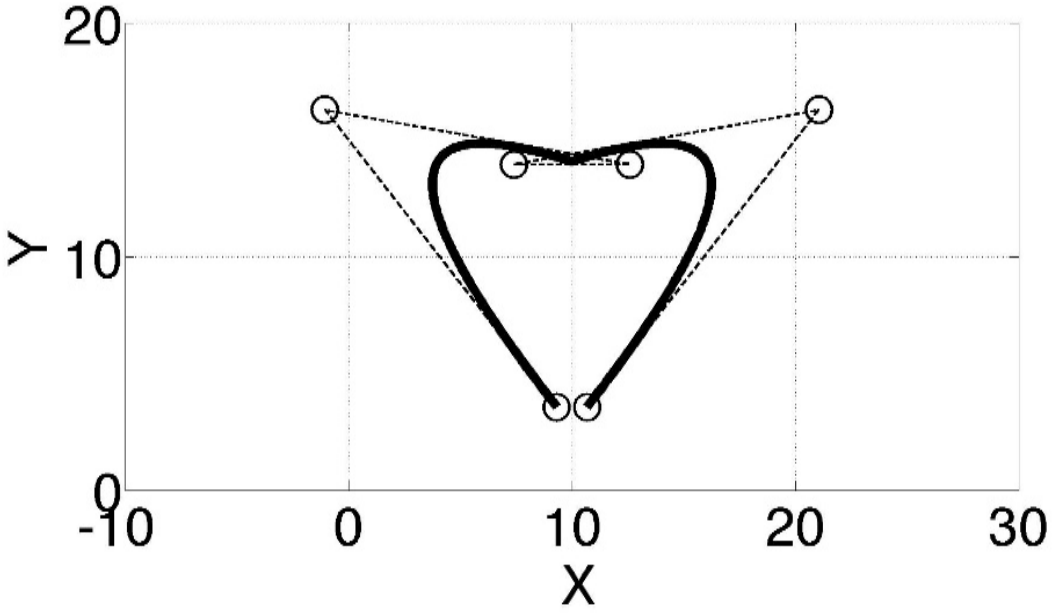

Fig. 4.3. n_{cp}=6,$f^* = 1.8074$

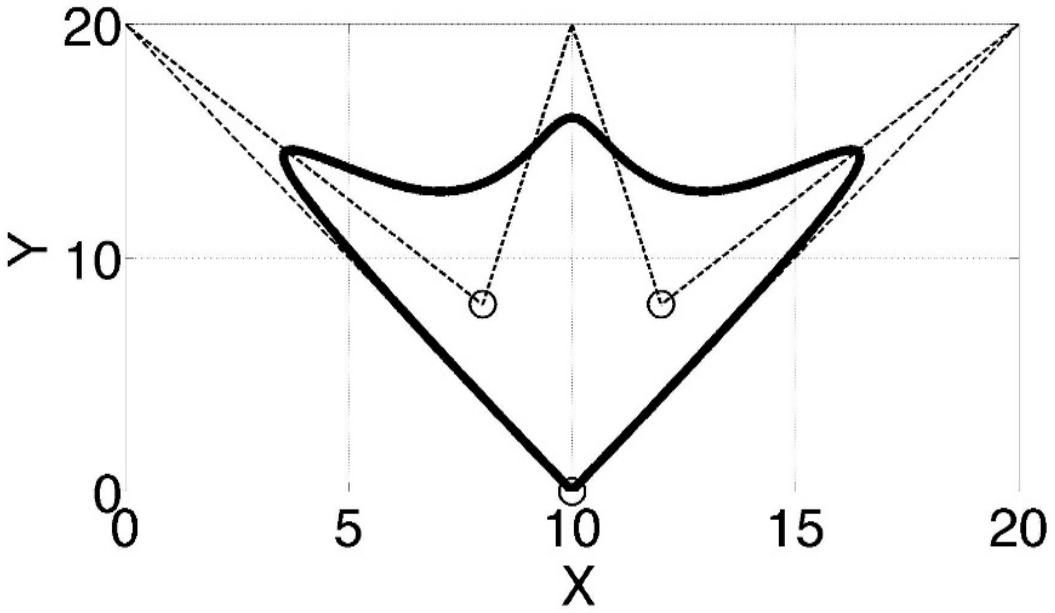

Fig. 4.4. n_{cp}=7,$f^* = 1.2510^{-14}$

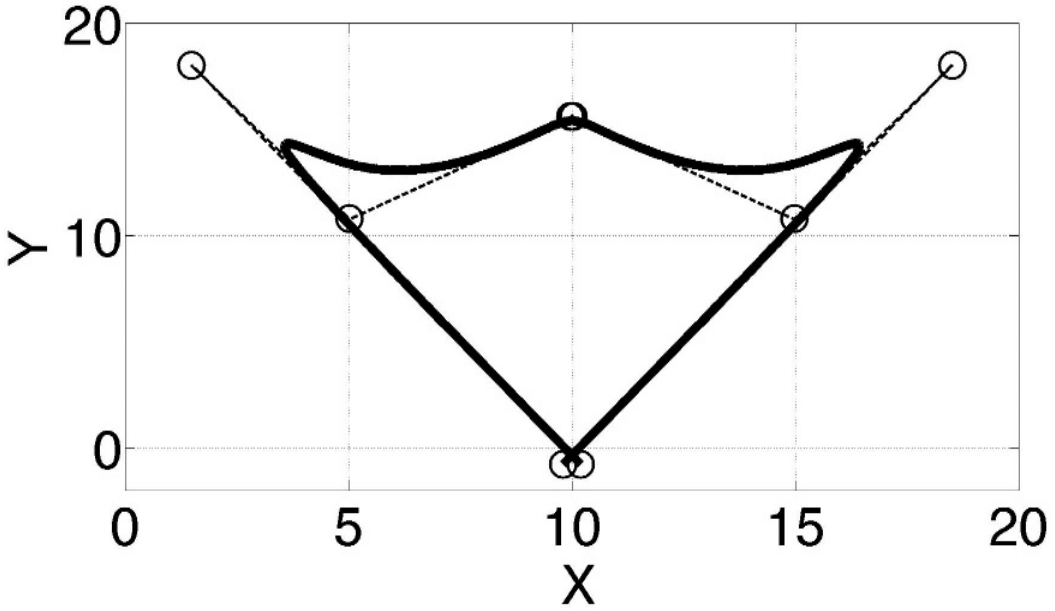

Fig. 4.5. n_{cp}=8,$f^* = 0.36$

Case 2: Selection of Sample number of Control Points for the approximate Periodic Signal Wave NURBS Curve

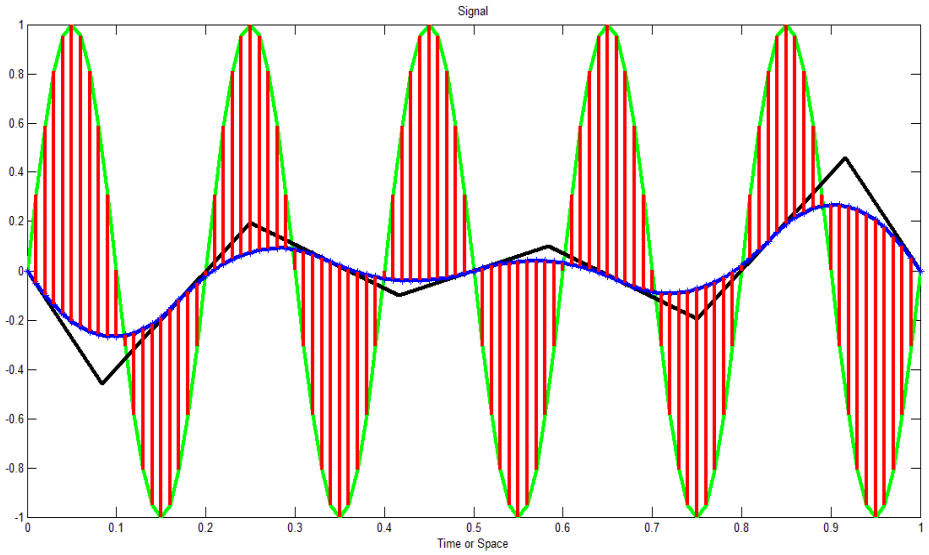

Fig. 4.6. (a) Number of Control Points = 8

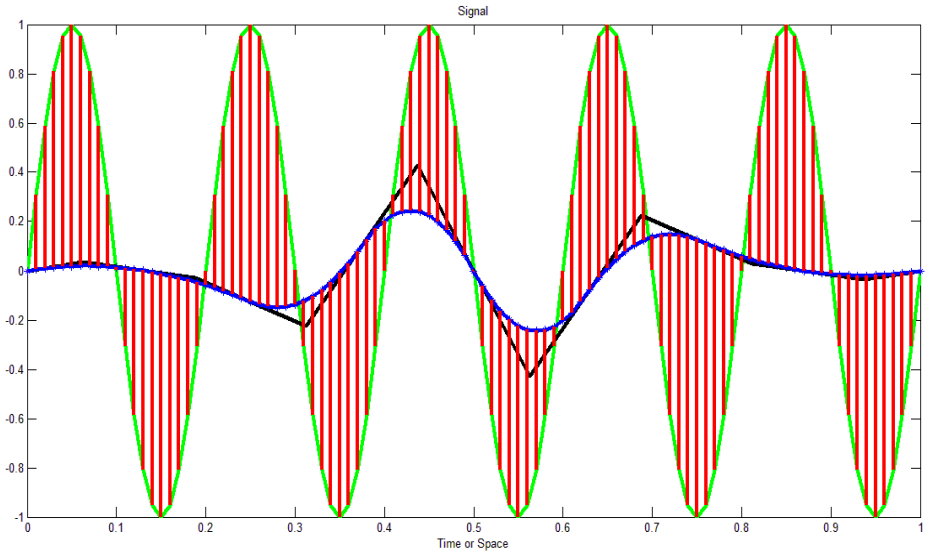

Fig. 4.7. (b) Number of Control Points = 9

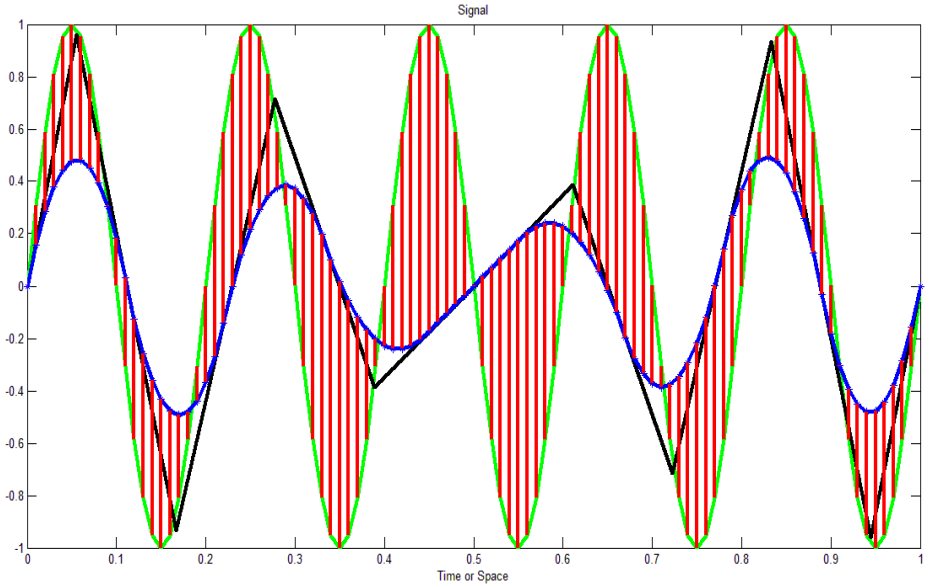

Fig. 4.8. (c) Number of Control Points = 10

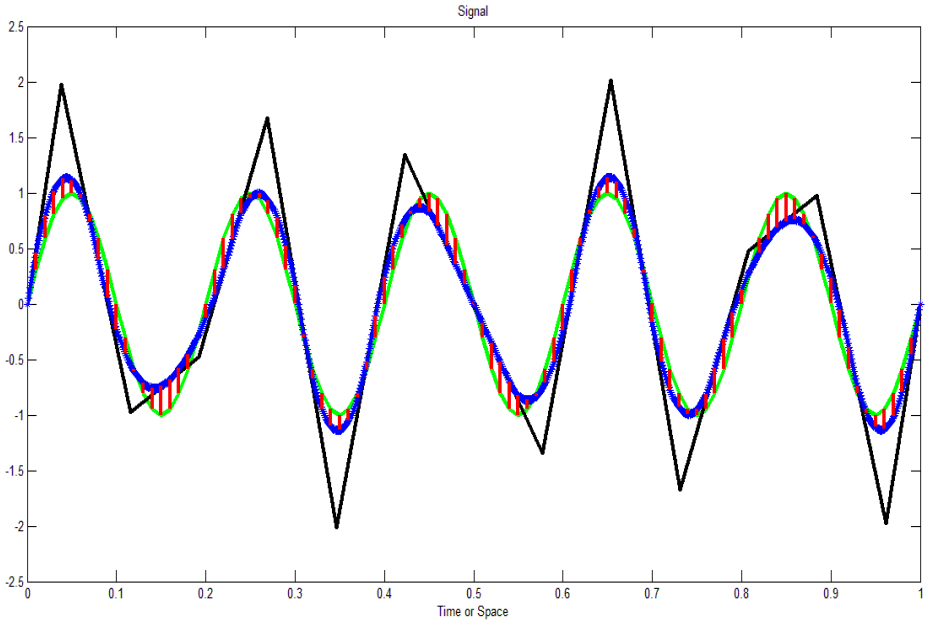

Fig. 4.9. (d) Number of Control Points = 15

Case 3: Comparison of the approximate NURBS curve before optimization and after optimization.

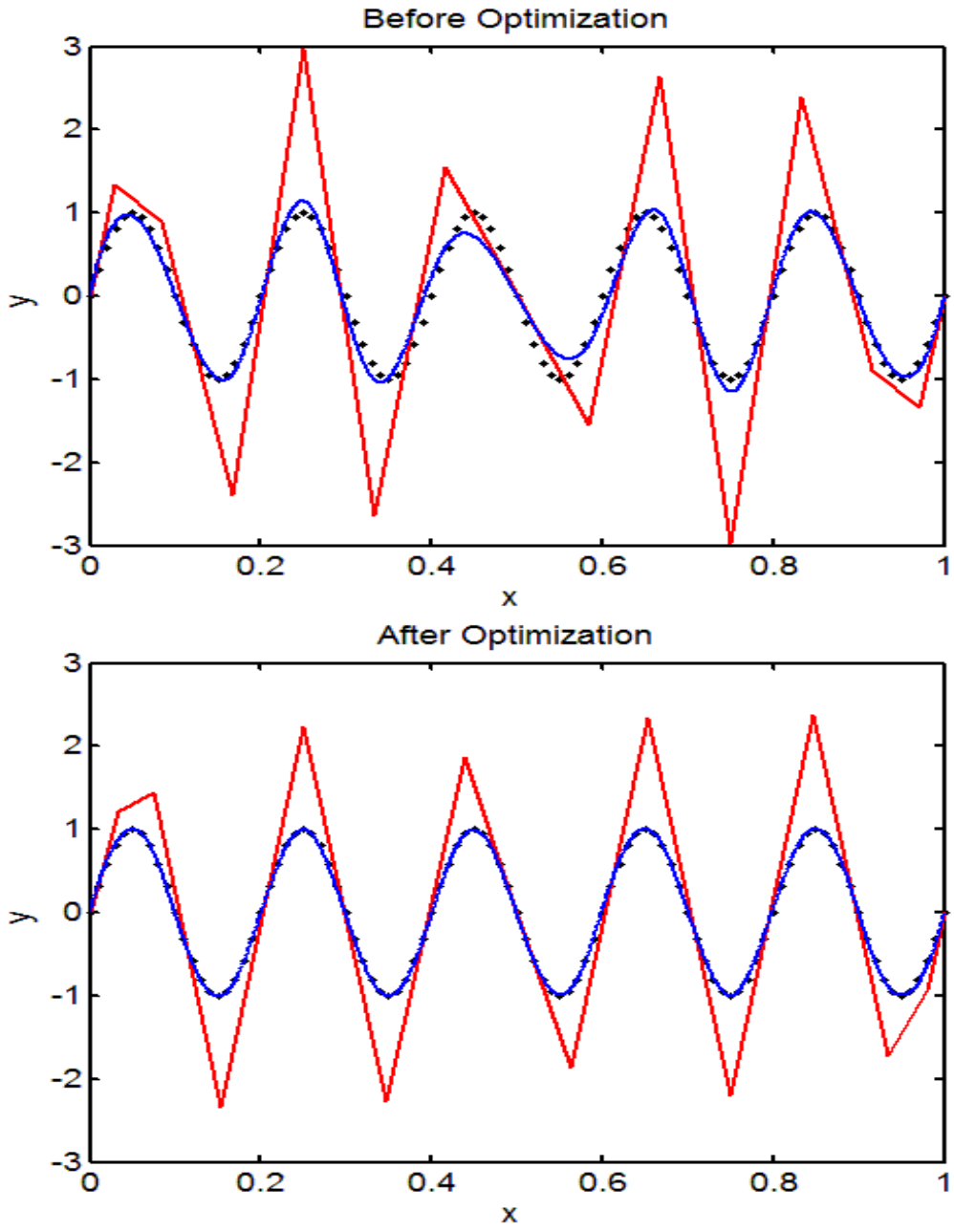

Fig. 4.10. Comparison of the approximate NURBS curve before optimization and after optimization for Periodic Signal Wave

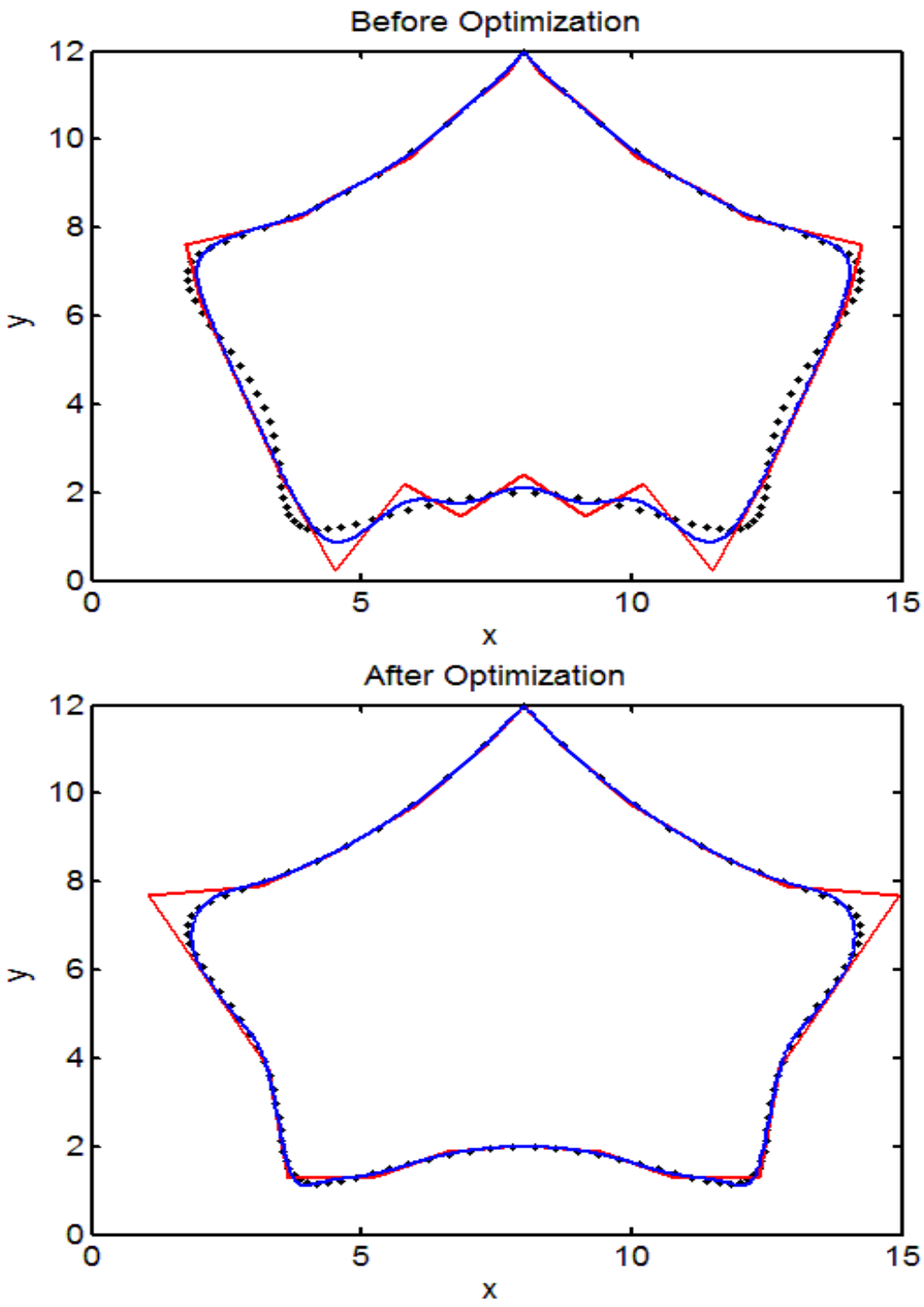

Fig. 4.11. Comparison of the approximate NURBS curve before optimization and after optimization for Star Shape.

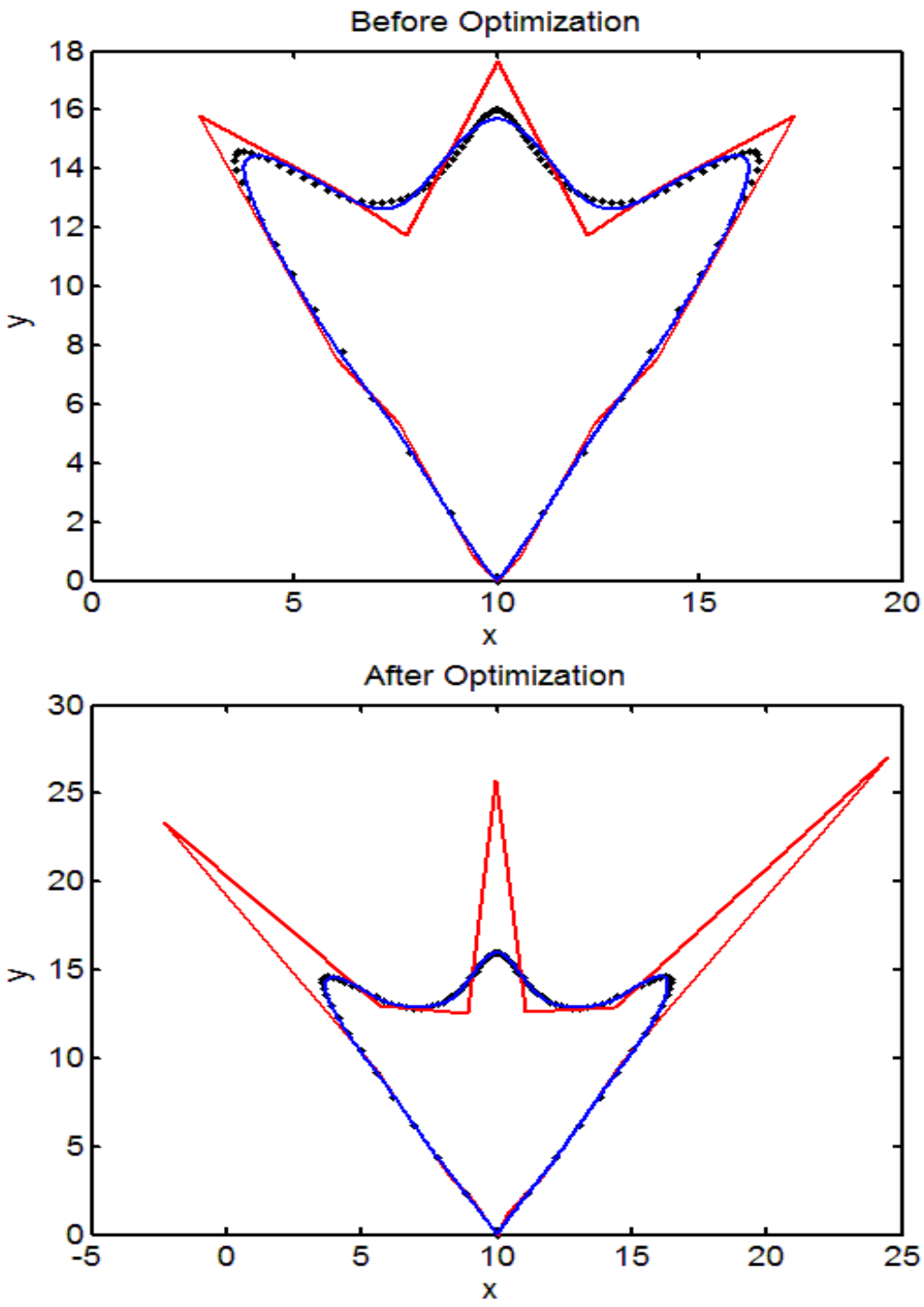

Fig. 4.12. Comparison of the approximate NURBS curve before optimization and after optimization for Trident Curve.

5. OPTIMAL NURBS FUNCTION

In this chapter the developed NURBS function is described with some applications. These applications would follow the methodology described in the previous chapters and the optimized NURBS curve with their optimal parameters are used in generating the NURBS interpolation. Some of the case studies described in this chapter are the optimized NURBS curve for the well-known geometric entities, NURBS curve representation for pinion gear and its comparison with different optimization methods, comparison between the meta-models such as kriging function with NURBS representation and finally representation of topologically optimized structures using NURBS Curve and 3D printing their structures.

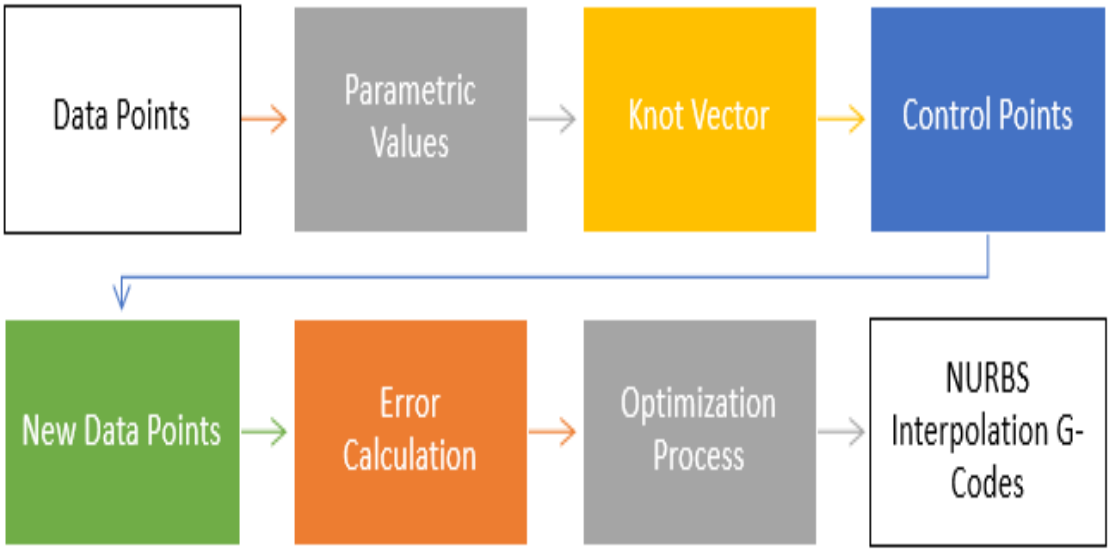

Fig. 5.1. NURBS function block diagram

5.1 Optimized NURBS Curves

The NURBS curve for the given target curve is found based on the given data points or surface points. An approximate NURBS curve is found based on these data points considering the sample number of control points. Then the approximate NURBS curve is optimized by varying the NURBS parameters such as control points and weights. This optimization process is carried out using different optimization methods such as SQP, Nelder-Mead and Simulated Annealing. The error between the target curve and the approximate NURBS curve is considered as an objective function for the above mentioned optimization methods and their comparison is shown in this chapter. Thus, the optimized location of control points and their respective weights are used in generating the G-Codes with the NURBS interpolation. Some of the examples with their comparison with different optimization methods are shown below.

Case 1: Optimized Periodic Signal Wave using NURBS function

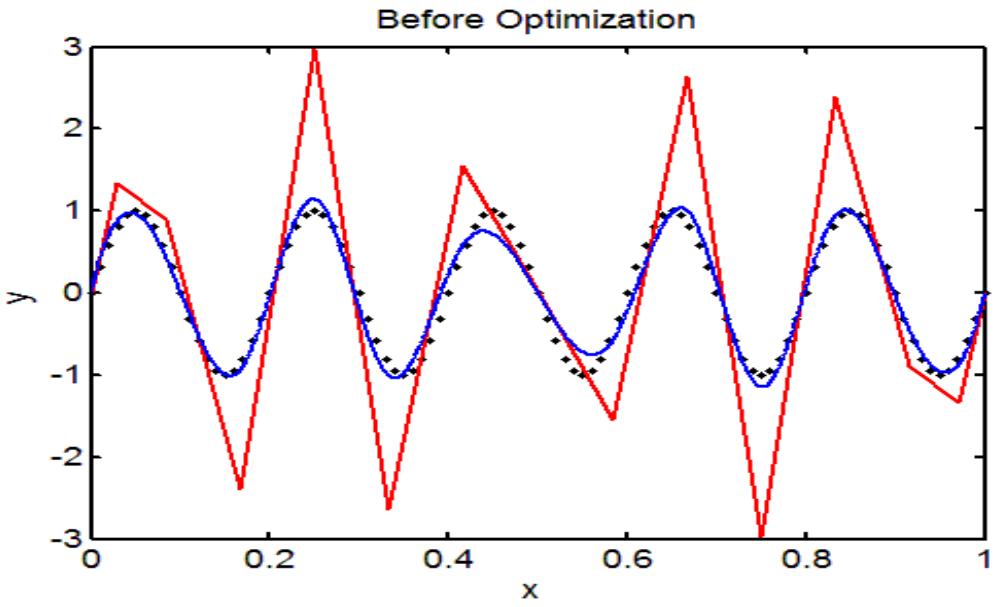

Fig. 5.2. Periodic Signal Wave using NURBS before optimization

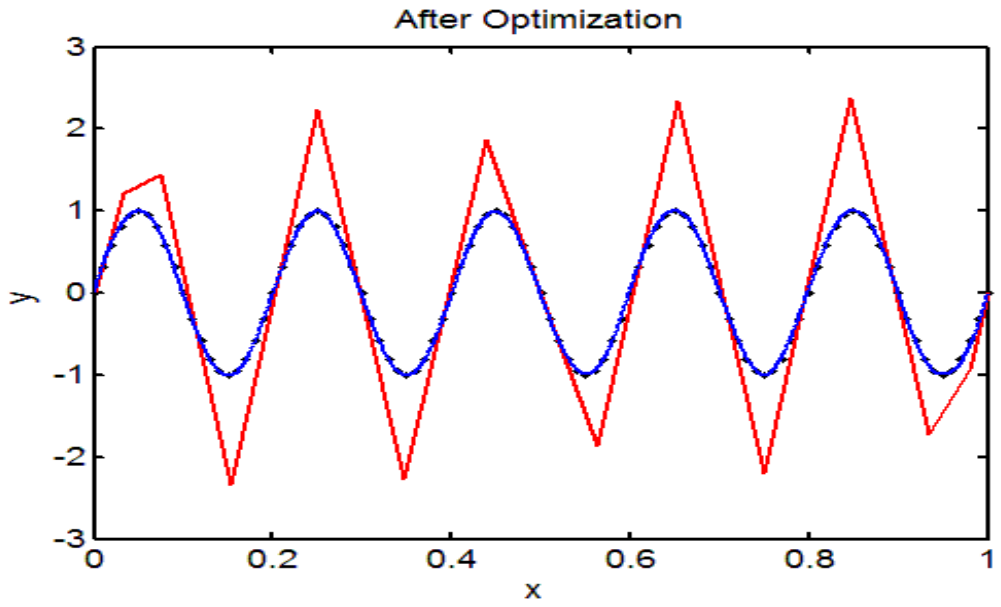

Fig. 5.3. Periodic Signal Wave using NURBS after optimization

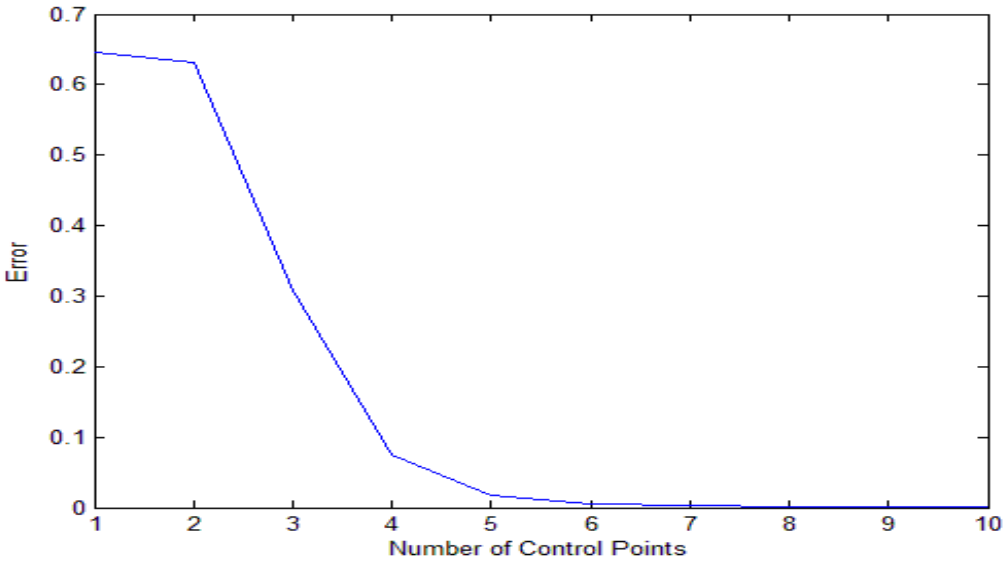

Fig. 5.4. Error Vs Number of Control Points for Periodic Signal Wave

Periodic Signal			
Methods	iter	func calls	error
fmincon	29	497	0.004258
fminsearch	2260	3000	0.004326
simulated Annealing	33500	37191	0.008071

Fig. 5.5. Table with optimization results for Periodic Signal Wave using NURBS

Case 2: Optimized Star Shape using the NURBS Function

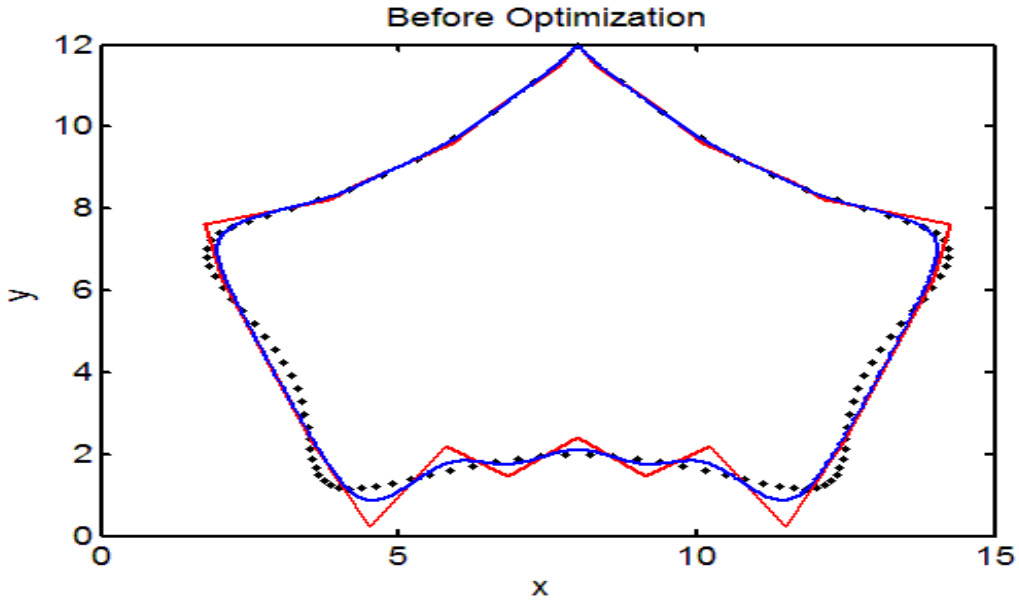

Fig. 5.6. Star Shape curve using NURBS before optimization

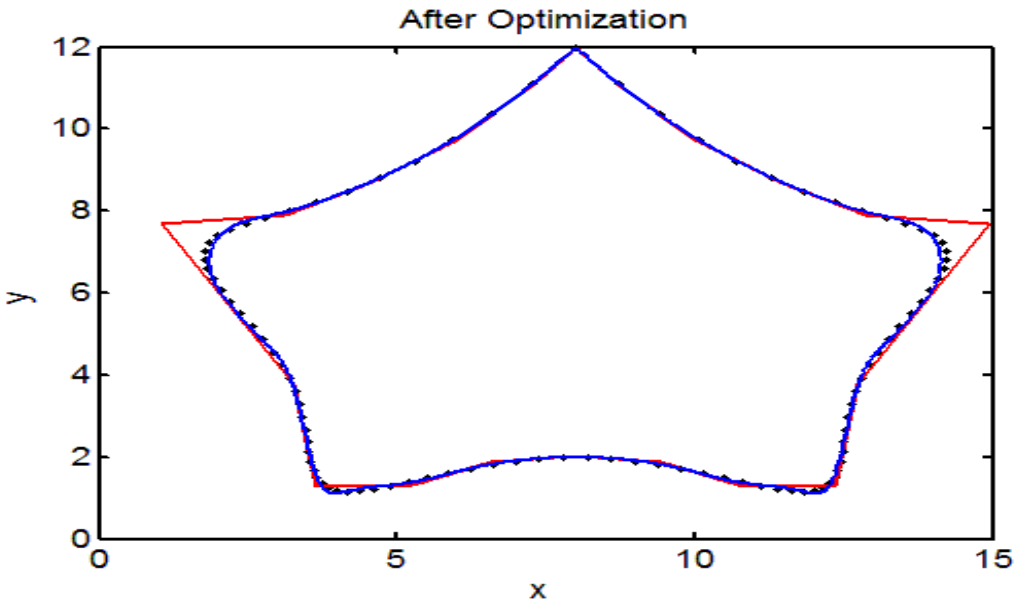

Fig. 5.7. Star Shape curve using NURBS after optimization

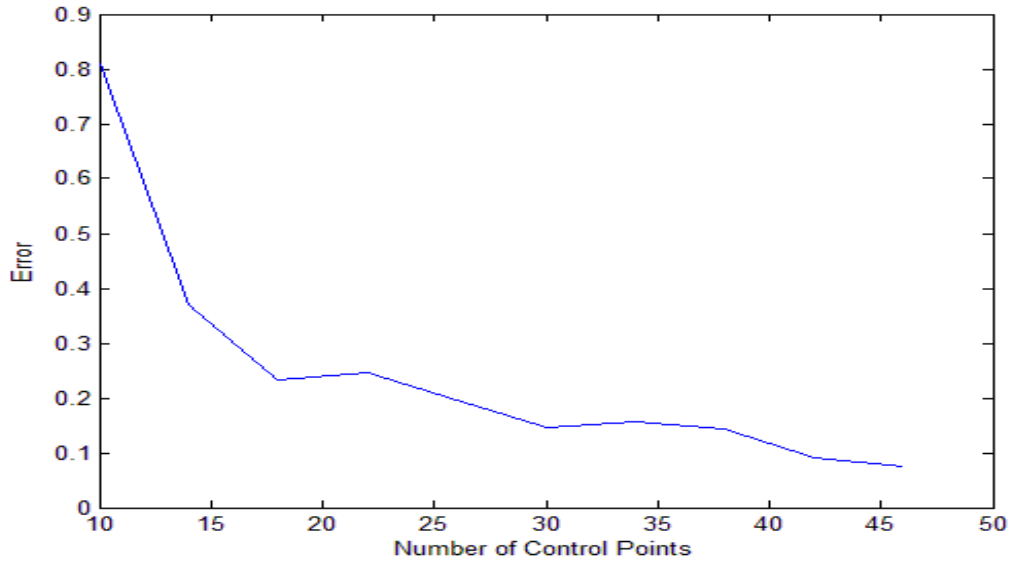

Fig. 5.8. Error Vs Number of Control Points for Star Shape curve

Star Shape			
Methods	iter	func calls	error
fmincon	69	1828	0.069233
fminsearch	4050	5001	0.079847
simulated Annealing	21750	25726	0.11756

Fig. 5.9. Table with optimization results for Star Shape curve using NURBS

Case 3: Optimized Trident Curve using NURBS Function

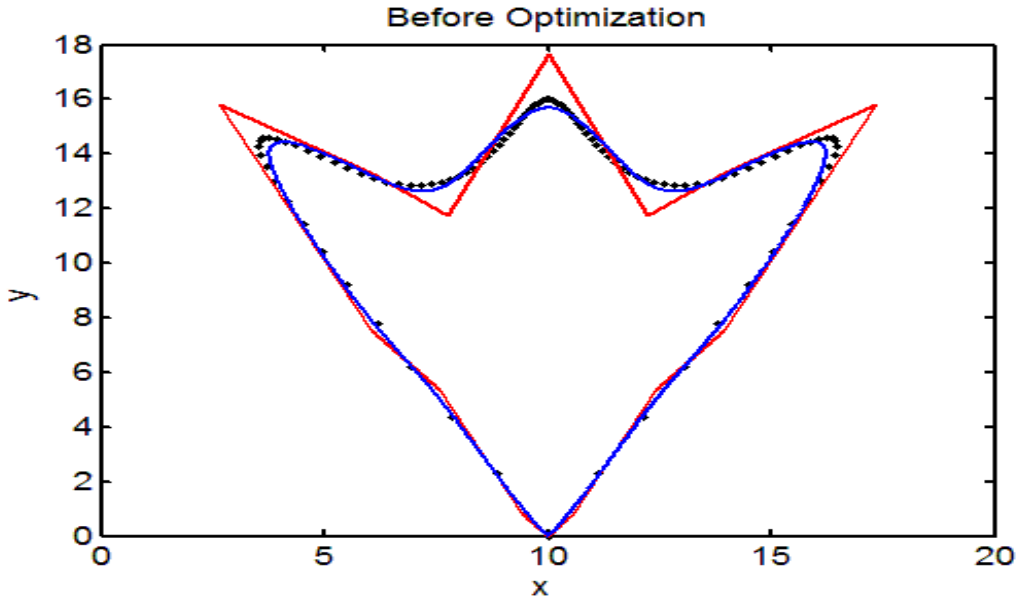

Fig. 5.10. Trident Curve using NURBS before optimization

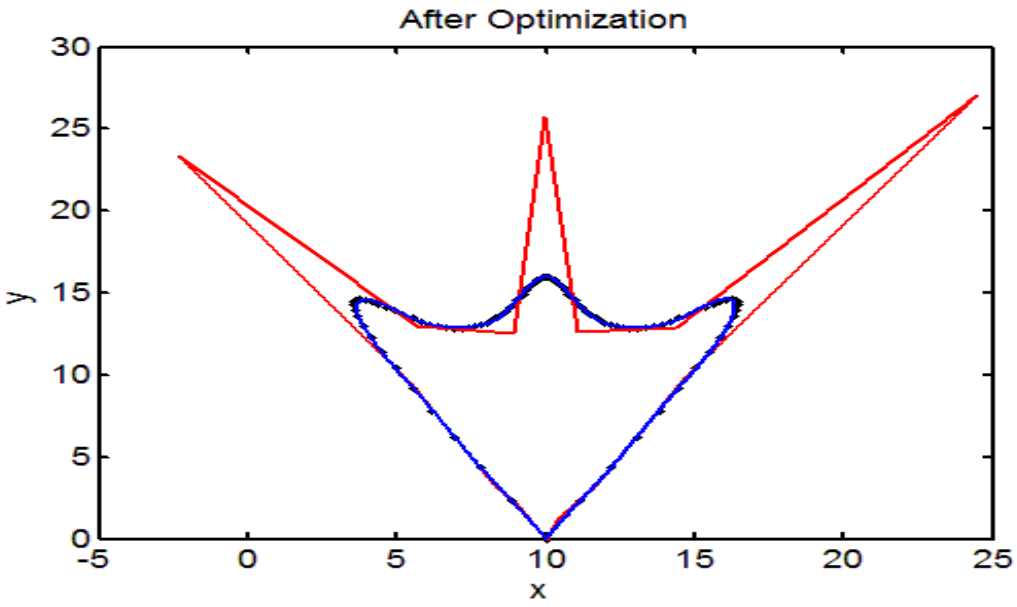

Fig. 5.11. Trident Curve using NURBS after optimization

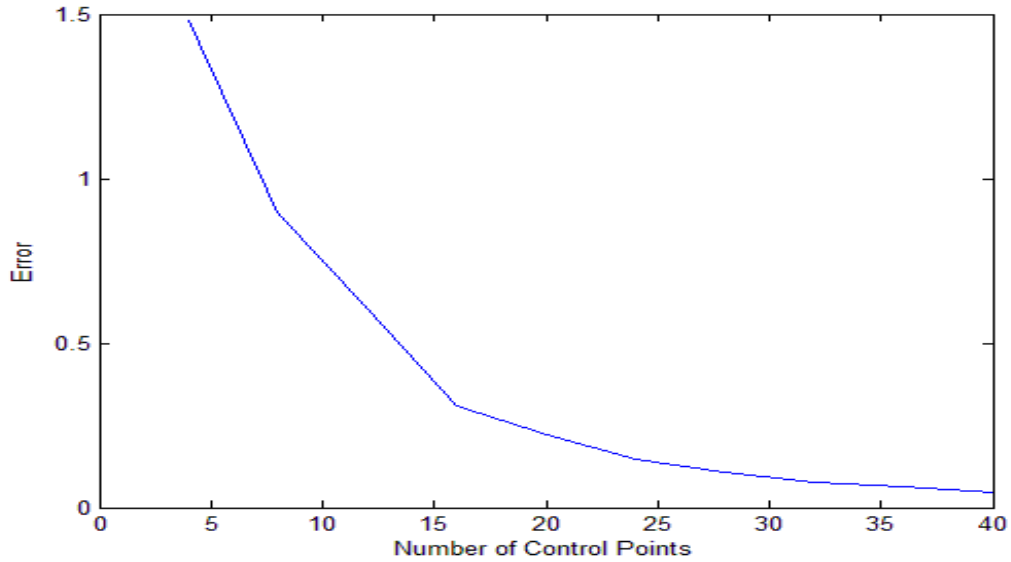

Fig. 5.12. Error Vs Number of Control Points for Trident Curve

Trident Curve			
Methods	iter	func calls	error
fmincon	51	792	0.142841
fminsearch	2091	2800	0.145614
simulated Annealing	12300	13575	0.149159

Fig. 5.13. Table with optimization results for Trident Curve using NURBS

The case studies shown above for different geometric entities significantly show how the NURBS function can be used to represent the given target curve with the best curve fit. It provides the flexibility to modify the shape of the entity as desired by the developer with the change of NURBS parameters to appropriate values. In this approach we have optimized the NURBS parameter as required to meet the requirements of representing the target curve with the NURBS curve function with minimum error between them. The optimization methods used in achieving this are tabulated for each case study as shown above. If we observe that in the above case studies the SQP method of optimization has better results than the other optimization methods with less function calls and iterations.

5.2 NURBS Curve Representation for Pinion Gear

RACK AND PINION GEARS

A rack and pinion is a pair of gears that convert rotational motion to linear motion. The circular pinion engages teeth on a flat bar the rack. Rotational motion applied to the pinion by the steering wheel will cause the rack to move to one side

or the other, right up to the limit of its travel. The rack and pinion arrangement is commonly found in the steering mechanism of cars or other wheeled, steered vehicles. This arrangement provides a lesser mechanical advantage than other mechanisms such as recirculating balls, but much less backlash and greater feedback, or steering feel. The advantages of rack and pinion steering are that it gives easier and more accurate control of the vehicle, improving handling and steering response. In this paper we use the NURBS geometry to develop the design of the prototype pinion gear profile and then these geometrical input parameters are then used to generate the machining G-Codes for the mass production of pinion gears.

Fig. 5.14. Rack and Pinion Gears

In this approach we have considered a involute pinion gear profile with specification as stated below and this profile is been developed using spur gear matlab code which serves as the data points for the NURBS curve function.

Specification of Involute pinion gear profile:

- Module : 3.0
- Pressure angle : 25 degrees
- Addendum : 1.0
- Dedendum : 1.0

- Number of teeth : 10
- Pitch Circle Diameter : 30mm (Module x Number of teeth)
- Base Circle Diameter : 27.1892mm (PCD x cos(Pressure Angle))
- Tip Circle Diameter : 36mm

Based on the above specification of the involute pinion gear profile the data points required for the NURBS curve function are extracted using the spur gear matlab code and the figure(a) below shows the profile of complete involute pinion gear. In this case we have considered one side of a single involute gear teeth as the target curve because the other side of the teeth is a mirror image of the former at defined datum plane and this is being shown in the figure (b). Now, this target curve serves as the data points for the NURBS curve function then the approximate NURBS curve is generated with minimum number of control points and then it is optimized using different optimization methods to reduce the error between the target curve and the approximate curve to as minimum as possible. Thus, these optimal NURBS parameters with minimum error are used in generating the G-Codes using the NURBS interpolation.

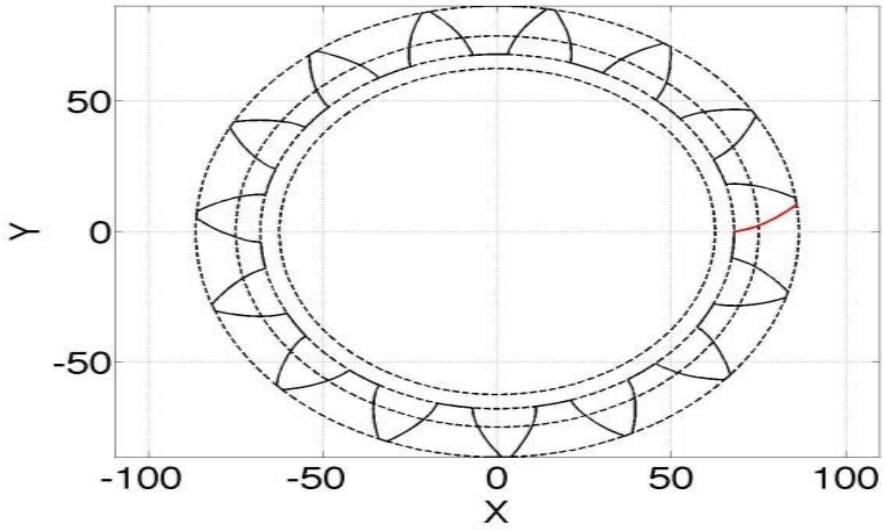

Fig. 5.15. (a) Complete Involute Pinion Gear Profile

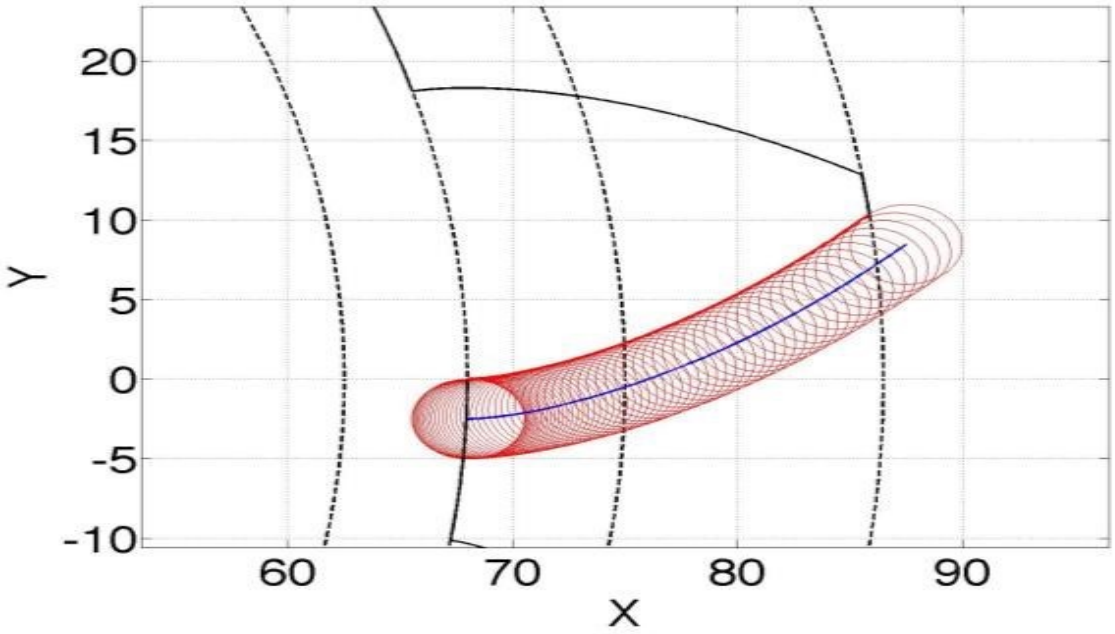

Fig. 5.16. (b) Involute Gear Profile of a single tooth

Case 4: Optimized Involute Gear Profile using NURBS Function

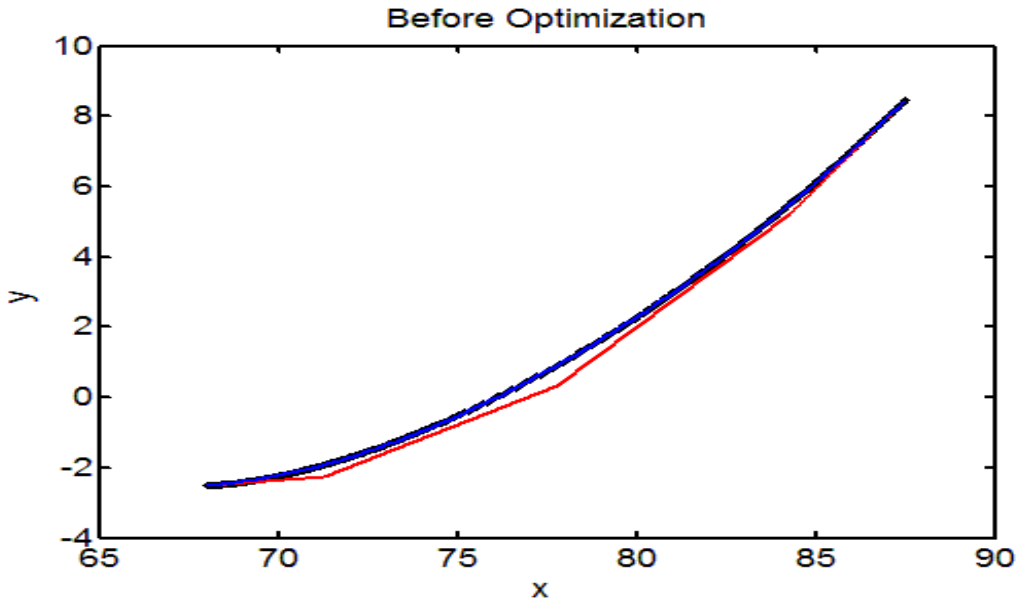

Fig. 5.17. (a) Involute gear profile before optimization using NURBS function

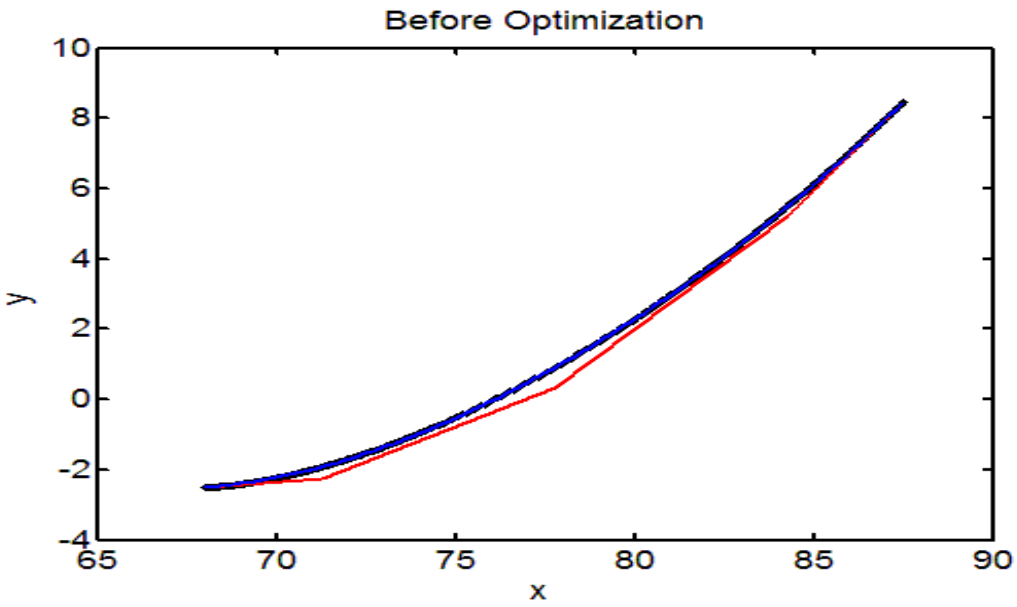

Fig. 5.18. (b) Involute gear profile after optimization using NURBS function

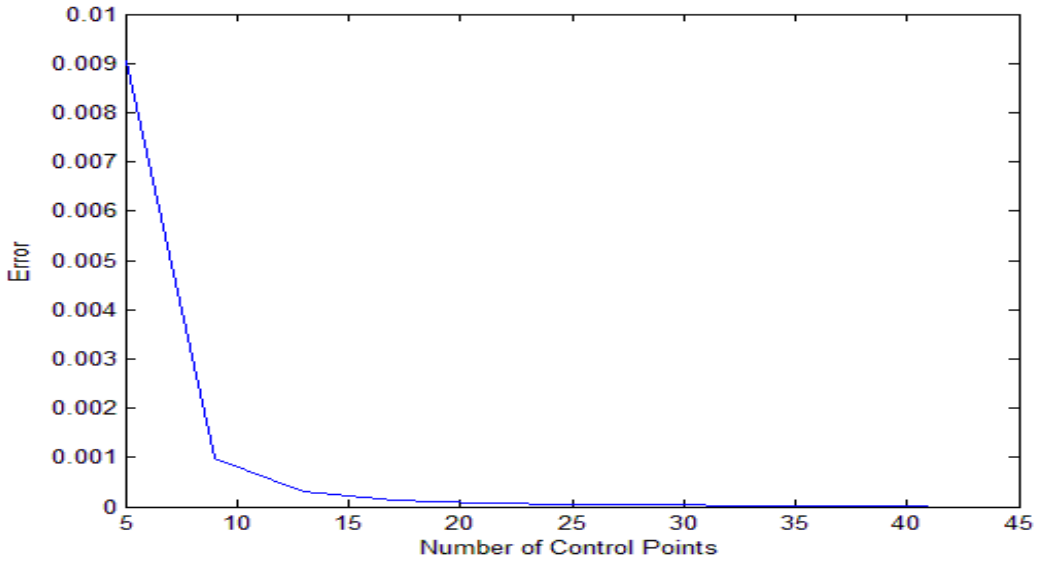

Fig. 5.19. (c) Error Vs Number of Control Points for Involute gear profile

Involute Curve			
Methods	iter	func calls	error
fmincon	29	193	0.001186
fminsearch	310	605	0.001186
simulated Annealing	4212	4368	0.00588

Fig. 5.20. (d) Table with optimization results for Involute Gear Profile using NURBS

Above case study shows how the involute gear profile is being represented by NURBS curve using NURBS function. The figure (c) shows the nature of the control points with relation to error between the target curve and the approximate NURBS curve. An approximate curve is described that better represents the target curve with the minimum number of control points and it has been shown in the figure (a). This approximate curve is then optimized by varying the NURBS parameters to the optimal values at which the error between the target curve and the approximate NURBS curve is minimum. The figure (b) shows the optimized curve with optimal NURBS parameters such as optimal location of control points and their respective weighs. This optimization process is performed using different optimization methods and their results are tabulated in figure (d). The detailed list of NURBS parameters for the required involute gear profile before optimization and after optimization is tabulated in figure 5.21.

Details of first side of the involute teeth profile					
Unit weights (B-spine)			Optimized weights (NURBS)		
Control Points		W	Control Points		W
P_X	P_Y		P_X	P_Y	
67.9780	-2.521	1	67.97	-2.5051	0.3904
71.2309	-2.240	1	70.12	-2.426	0.5923
77.7367	0.3265	1	76.21	-0.6090	0.9409
84.2424	5.2098	1	83.99	4.974	1.1782
87.4953	8.4673	1	87.49	8.460	1.2683

Fig. 5.21. Table with detailed list of NURBS parameters before and after optimization for the involute gear profile.

Based on above optimal NURBS parameters we can generate the G-codes using the NURBS interpolation for the involute gear profile.A sample NURBS G-Code part program with the above NURBS parameters is shown below:

FANUC's Example of NURBS interpolation:

N1 G06.2 P4 K0 X19.0407 Z4.9824

N2 K0 X18.7908 Z5.1331

N3 K0 X18.3069 Z5.404

N4 K0 X17.6201 Z5.7361

N5 K0.083333333 X16.9658 Z6.0063

N6 K0.166666667 X16.3398 Z6.2228

N7 K0.25 X15.739 Z6.3915

N8 K0.333333333 X15.161 Z6.5166

N9 K0.416666667 X14.6046 Z6.6007

N10 K0.5 X14.0687 Z6.6459

N11 K0.583333333 X13.5534 Z6.6519

N12 K0.666666667 X13.0587 Z6.6187

N13 K0.75 X12.5876 Z6.5406

N14 K0.833333333 X12.2916 Z6.4542

N15 K0.916666667 X12.1538 Z6.3915

N16 K1

N17 K1

N18 K1

N19 K1

NURBS G-Code part program for Involute Gear Profile:

N1 G06.2 P4 K0 X67.9774 Z-2.50513 R0.3904

N2 K0 X70.1219 Z-2.4266 R0.5926

N3 K0 X76.2164 Z-0.6090 R0.9409

N4 K0 X83.9926 Z4.9740 R1.1782

N5 K0.5 X87.4954 Z8.4609 R1.2683

N6 K1

N7 K1

N8 K1

N9 K1

5.3 Comparison between the Kriging Curve and NURBS Curve

In this research project a new technique is implemented known as metamodel optimization using the NURBS function and this technique is used to find the maximum or minimum of the function as desired. The response surface methodology is used to develop an approximate low degree polynomial model which consists of group of mathematical and statistical techniques used in development of an adequate functional relationship between the response of interest y, and a number of associated control input variables. Thus the model developed using the response surface methodology will be used as a metamodel for this new technique and the optimization is performed. Furthermore, the optimum results obtained through this new technique

are compared with the kriging (DACE function) which is commercially available technique. Some of the examples are shown below with their comparison.

Case 1: 1-Dimensional Comparison between Kriging and NURBS for a open box function

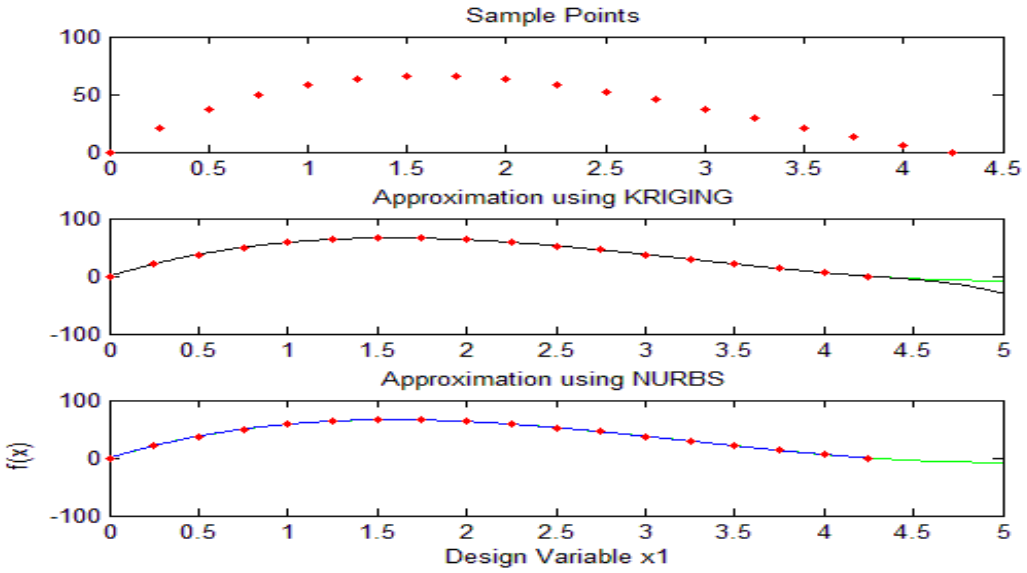

Fig. 5.22. Comparison between Kriging and NURBS

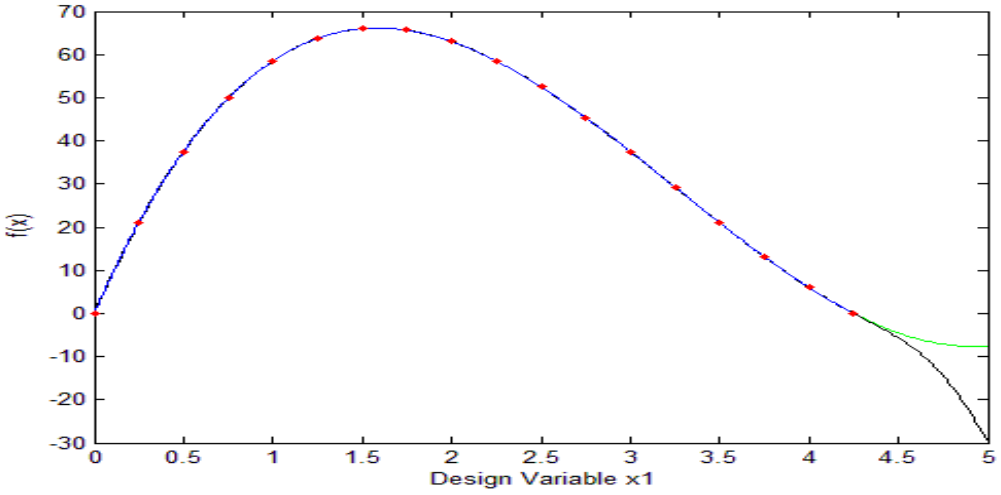

Fig. 5.23. Comparison between Kriging and NURBS

Case 2: 1-Dimensional Comparison between Kriging and NURBS for sine function

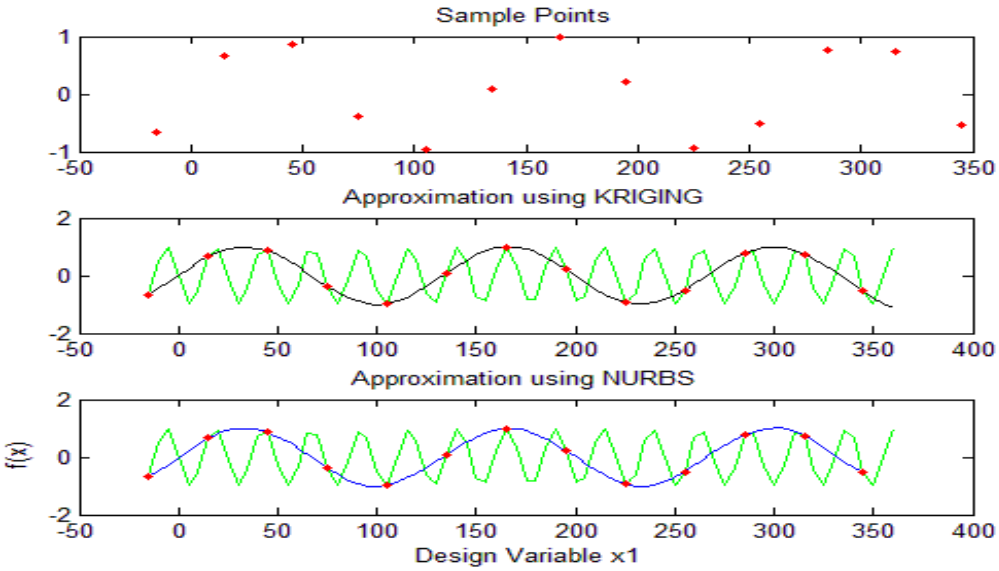

Fig. 5.24. Comparison between Kriging and NURBS using subplots

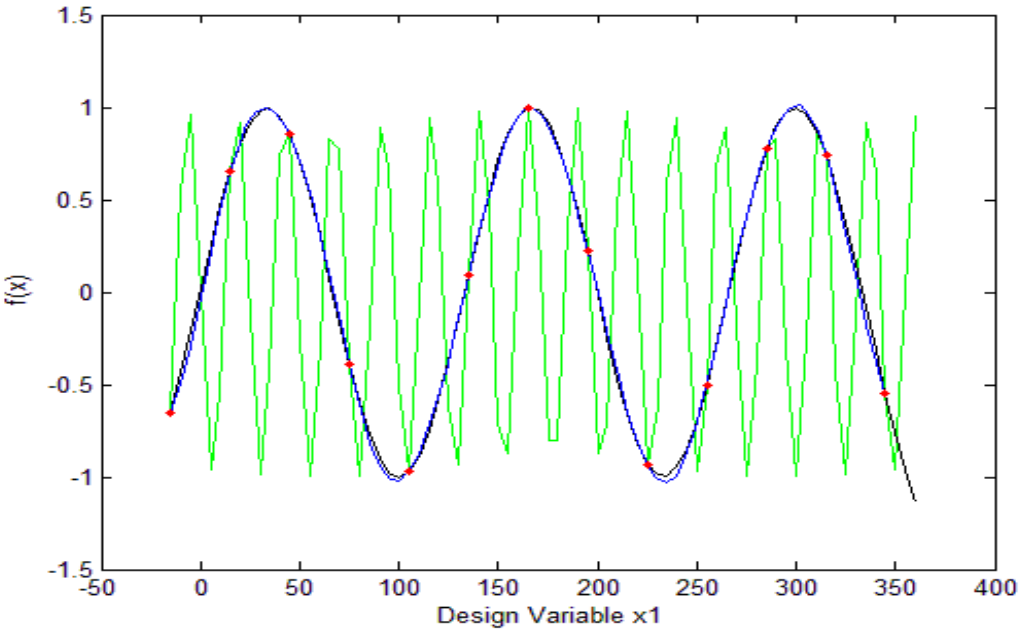

Fig. 5.25. Comparison between Kriging and NURBS

Case 3: 2-Dimensional approach using NURBS for the Banana function

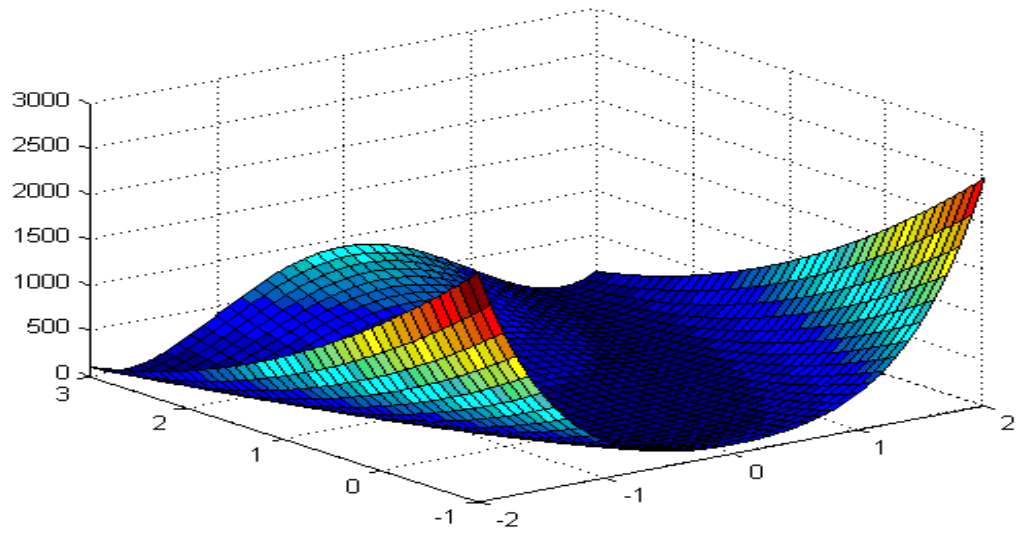

Fig. 5.26. Surface plot of banana function

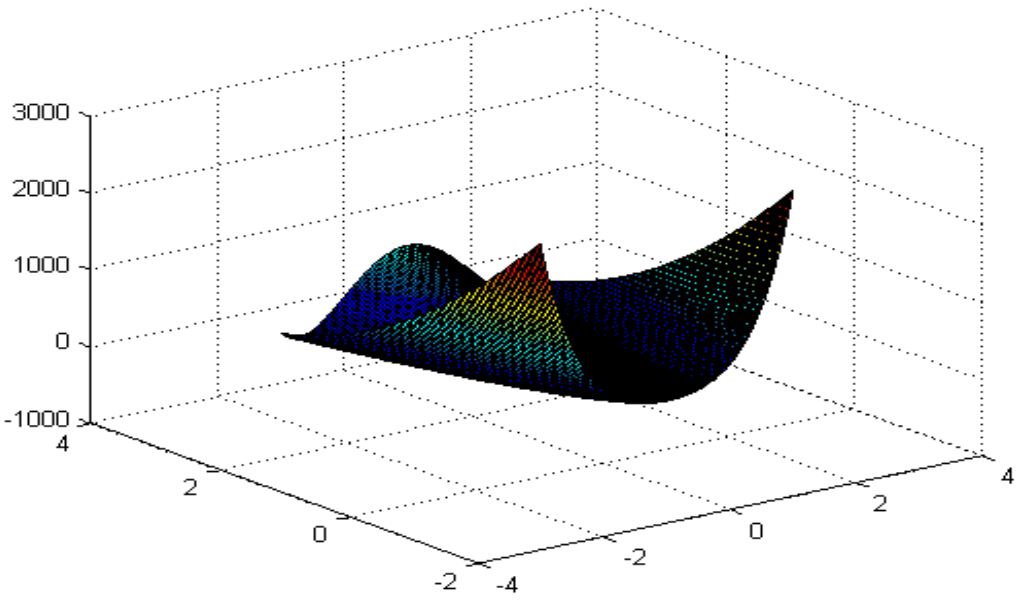

Fig. 5.27. Surface plot of banana function using NURBS

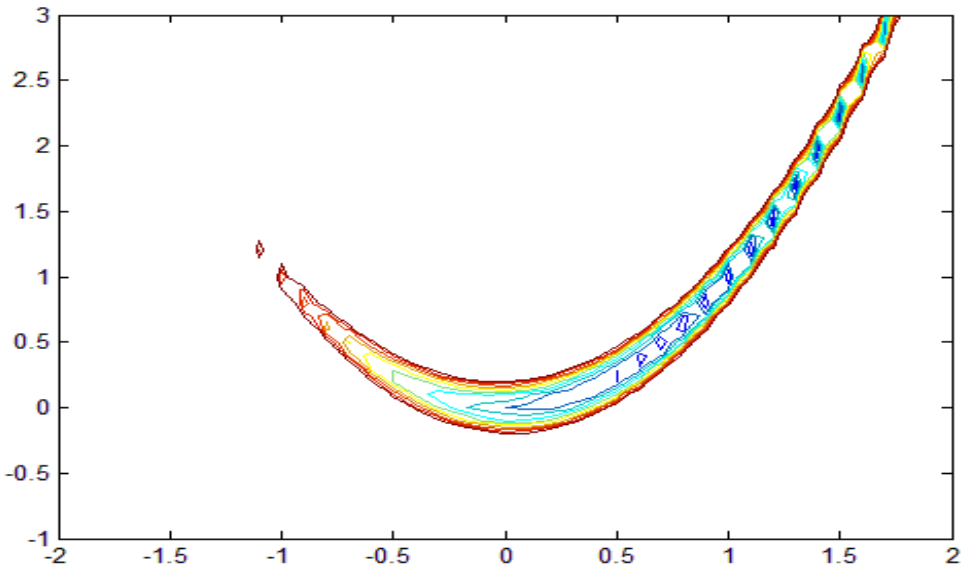

Fig. 5.28. Contour plot of banana function

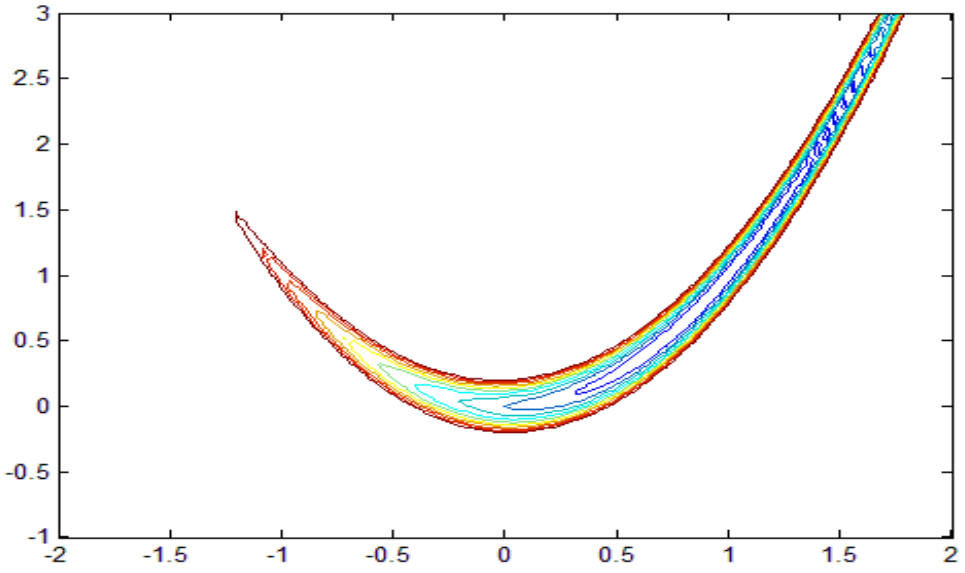

Fig. 5.29. Contour plot of banana function using NURBS

5.4 Topologically Optimized Structures using NURBS Curve

In this we have considered topologically optimized structures for some engineering design problems. The design problem is solved using the top3d a 3D topology optimization tool [37, 38]. The optimized structure image is then represented with pixel values ranging in between 0 and 1. These pixel values represent with different colors starting from white to dark (black) color based on the value of the pixel at a point on the structure. In our procedure we have considered to represent the topologically optimized structures with 0s and 1s pixel values based on some tolerance threshold value. The pixel values which fall below this threshold will be represented as 0s and pixel values which fall above the threshold are considered to be 1s. Now, this structure is used to be represented by NURBS function. The developed NURBS structure for the considered topologically optimized structure is then converted into a stereo lithography file (.STL) using the optimal NURBS parameters, which is then used in 3D printer as an input file. Finally, the 3d printer develops the 3D NURBS structure for the considered topologically optimized structure.

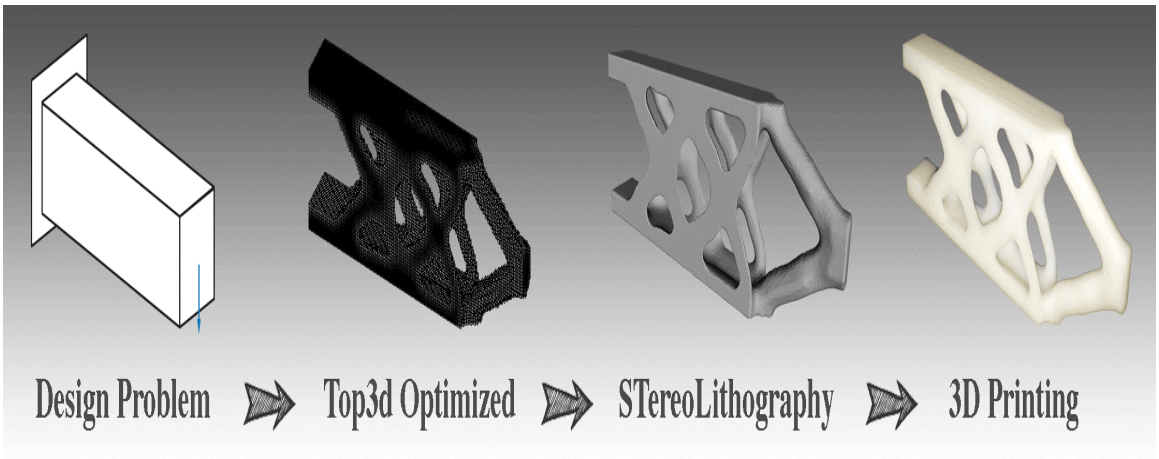

Fig. 5.30. Topology Optimized Structure

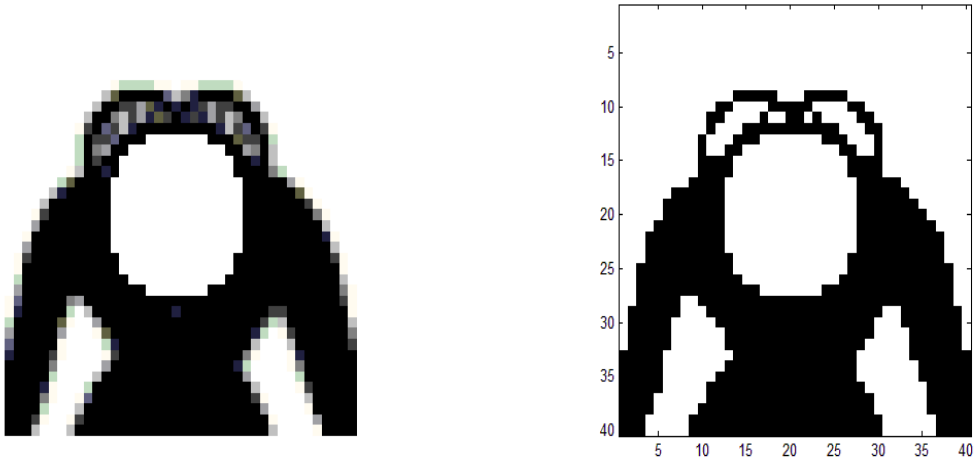

Fig. 5.31. (a) Topology optimized structure and (b) Topology optimized structure with pixel values 0s and 1s

The figures above show the pixel representation of the considered topologically optimized structure. The figure (a) represents structure before conversion and figure (b) represents structure after conversion of the pixel values based on the threshold value to represent the above structure using the NURBS function.

Boundary Representation and Generation of NURBS curves:

Once the conversion of the pixel values based on the threshold is done of the considered structure, then we find all the boundaries of the structure based on the pixel locations on the structure for both closed and open loop. These boundary representation points serves as a data or surface points for the NURBS function. Then these data points are used in representation of the structure using NURBS for each boundary of the structure. An example is shown below for both boundary representation and NURBS Structure generation using NURBS function.

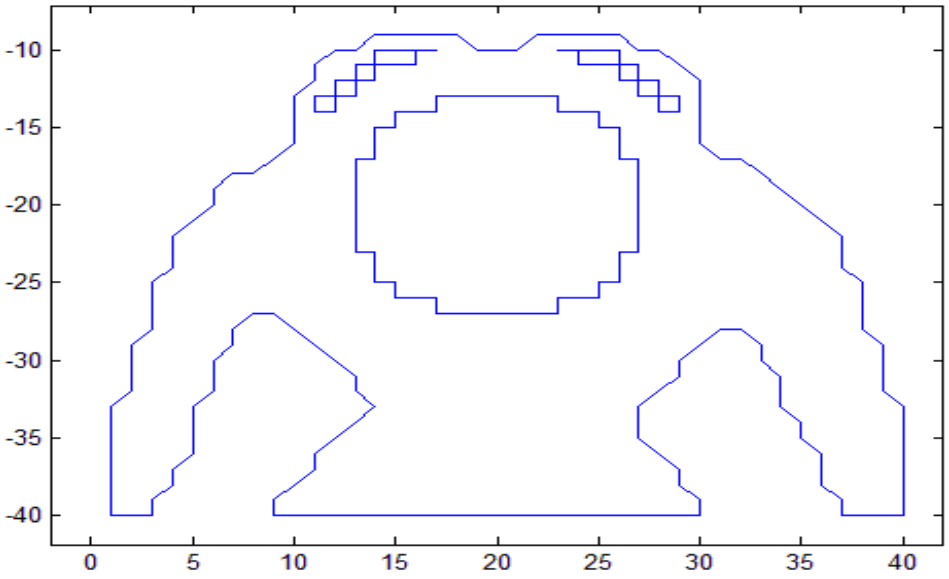

Fig. 5.32. Boundary Representation

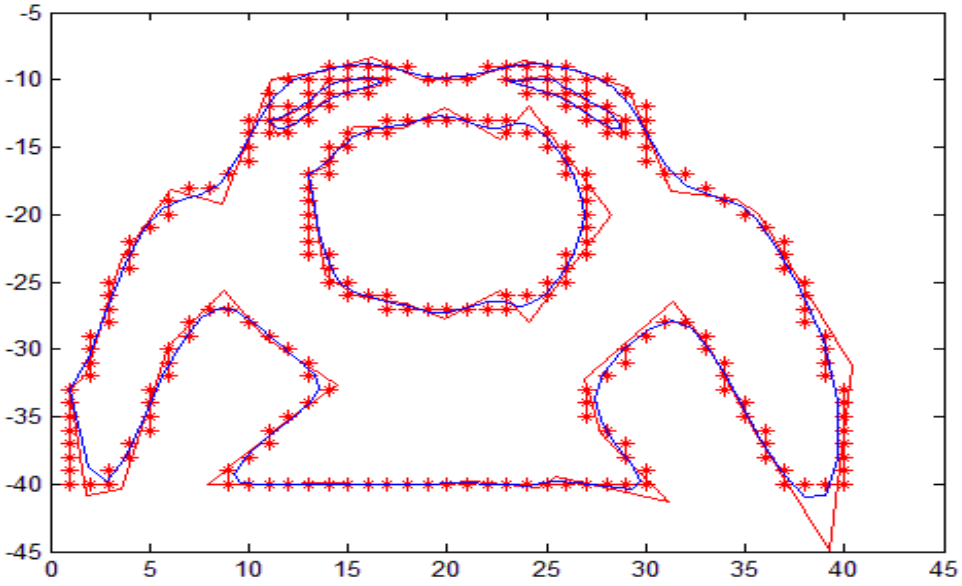

Fig. 5.33. NURBS Curve Representation

3D structure development using 3ds max software with NURBS optimal parameter:

The 3ds max software is used in developing the 3D structure using the NURBS optimal parameters. In this software we consider the NURBS optimal control points and their respective weights in developing the topologically optimized structure. After development of the structure using the NURBS parameter it is then imported into Pro-Engineering Software which then converts the 3D structure developed in 3ds max into stereo lithography file (.stl) which is our required file format which is using in the 3D Printer. An example is shown below with the developed structure using 3ds software and then importing in Pro-Engineering software for correct file format. Thus, the file created in Pro-E is used as an input file in 3d printer and a sample structure is shown in the figure below.

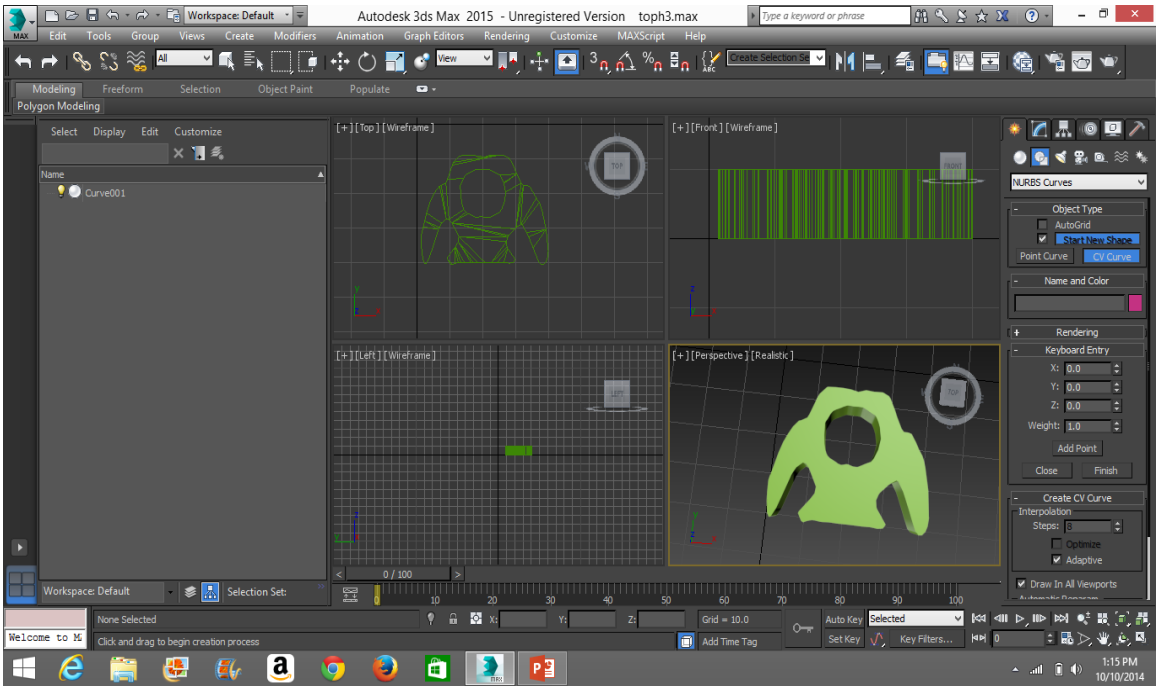

Fig. 5.34. Topological optimized structure developed using 3ds software with NURBS optimal parameters.

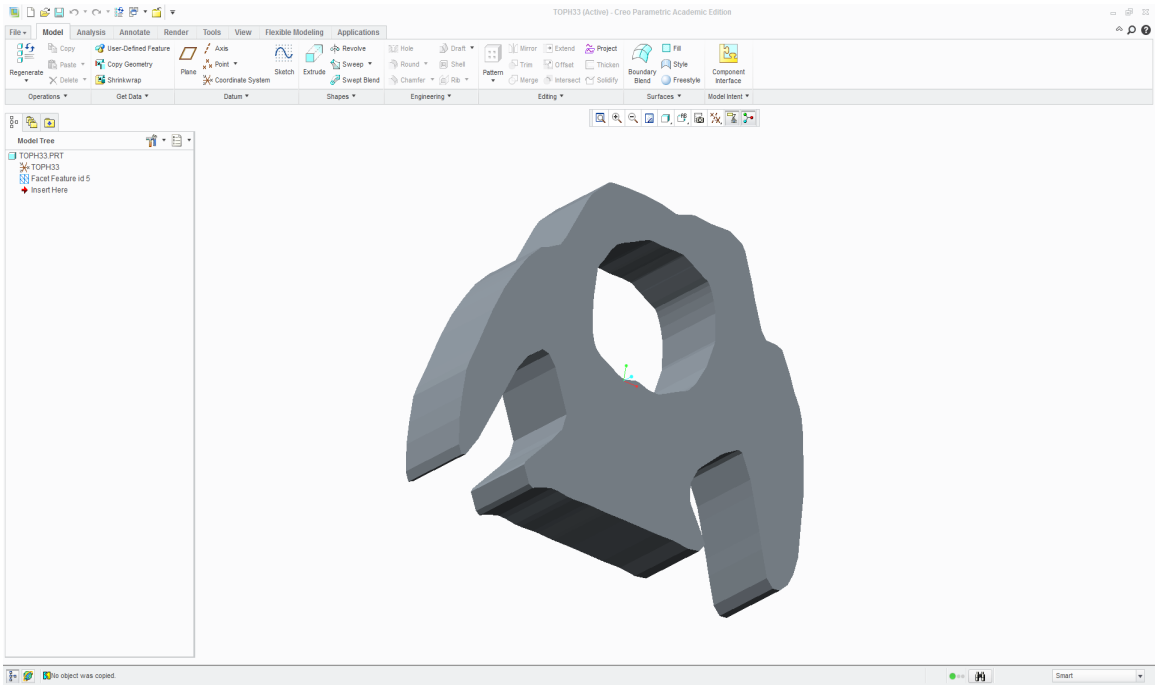

Fig. 5.35. Topological optimized structure developed using Pro-E software with NURBS optimal parameters

Fig. 5.36. Topological optimized structure developed using 3D printer with NURBS optimal parameters

6. CONCLUSIONS

This chapter presents an overview and general conclusions related to the work developed in this dissertation and recommendations for future work.

6.1 Summary

6.1.1 Optimized NURBS Curves

1. Successful implementation of developed NURBS function on different geometric entities such as Periodic Signal wave, Trident Curve and Star shape, Pinion Gear Profile was demonstrated.

2. The approximate NURBS curve is optimized using different optimization methods and their comparison are shown. It was observed that the SQP optimization methods is the most efficient way to find the best location of control points and their corresponding weights to represent the target curve with the results and observations shown in chapter 5.

3. Comparison between the developed NURBS function and the kriging function (DACE function) is demonstrated with several examples.

4. Examples: Representation of Periodic signal wave, Trident Curve and star shape, Pinion Gear Profile using the NURBS function was demonstrated with all the optimization results and their comparison. Also, shows the graph between the number of control points and the error function which results in effective selection of number of control points.

5. Finally, these optimized data of location of control points and their corresponding weights have been used in generating the G-Codes using the NURBS Interpolation for the pinion gear.

6.1.2 Topologically Optimized Structures using NURBS Curve

1. In this research approach we have considered a topologically optimized structure and then we have reconstructed the structure with desired pixel values. This incorporates in defining all the boundaries of the given structure. These boundaries are then used as the data points for the NURBS function and then a structure is developed using the NURBS function with optimal parameters.

2. These optimal parameters are used in 3ds max software which incorporates the use of NURBS functionality which is used in developing a 3D structure and this is been successfully demonstrated in chapter 5.

3. This 3D structure from 3ds max software is then imported into Pro-Engineer software to convert into required stereo lithography file and it is also been shown in chapter 5.

4. Thus, this stereo lithography file is been used as an input file for 3D Printer machine and finally, 3D product of the given topologically optimized structure with NURBS function is developed and well demonstrated.

6.2 Original Contributions

The requirements for advanced functions to support high-speed machining and high-accuracy machining which are generally used is growing. In particular, when conventional CNC systems where free curve is defined by sequential small line segments or arcs are used for machining free-form surfaces, the tool moves in a discontinuous manner and this makes the quality of the machined surface poor. The size

of the program will be large as it involves a lot of block programs as required. To overcome this problem NURBS interpolation was developed.

In NURBS interpolation, NURBS curve parameters such as the control points, weights and knot vector are directly fed as an input to the CNC system instead of the small line segment data that are defined by G01 command. As the CNC system generates interpolation points based on NURBS curve parameters, the programmed federate and the tolerance, it makes it possible to perform high-speed machining and high-accuracy machining.

The conclusion from this research is that the pinion gear profile was developed using the NURBS curves and the resultant approximate curves are optimized using NelderMead, SQP and Simulated annealing methods. We found that the SQP method for optimization is the most efficient way to find the best location of control points and their corresponding weights to represent the target curve with the results and observations shown. The comparison between the NURBS function and kriging function (DACE function) is served as a validation of the developed methodology approach. Finally, these optimized data of location of control points and their corresponding weights have been used in generating the G-Codes using the NURBS Interpolation for the pinion gear. Also, the use of NURBS function in representing the topologically optimized structures and 3D printing it.

6.3 Future Recommendations

1. The NURBS interpolation G-Codes can be used in 3D printer instead of using the linear interpolation and circular interpolation. This would enhance the performance and the surface finish of the product desired.

2. This approach of metalmodel using the NURBS function can used in Simulink Model especially in library blocks such as lookup tables where cubic interpolation is desired. This will help in selection of correct response for the given input parameters.

3. There is a high possibility that we can use this developed NURBS function for optimizing higher derivatives such as tool path fluctuation, feed rate and etc.

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APPENDICES

A. KRIGING FUNCTION

The kriging approximate model is constructed based on the experimental data and this is used as a surrogate model for the original experiment model [39–41]. This surrogate model has a collection of inputs and their responses based on the original experiment model. These inputs and their responses are usually considered to be high dimensional. The purpose of using this kriging model is to find the response of the original model at unknown or untried input parameter values. The result of the kriging is the expected value and variance computed for each and every point within the region. This is practically done on a fine enough grid.

Suppose we observe some variable Z along 1-dim space (X). There are 5 measurements made. We might ask ourselves, knowing the probabilistic behavior of the random field being observed, what are possible trajectories (realizations) of the random field, that agree with the data? This is answered by conditional simulation

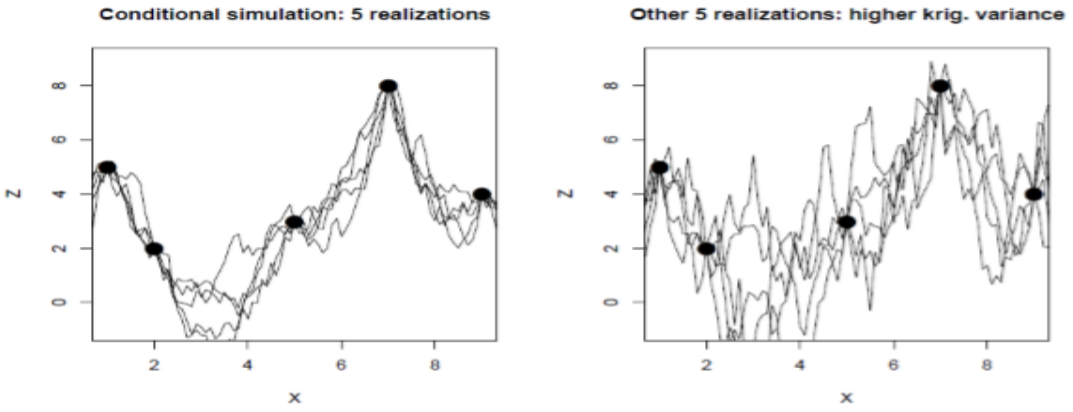

Fig. A.1. Representation of curves using Kriging

Some examples are shown below that describes the comparison between the kriging function and generated approximate NURBS curve.

Case 1: 1-Dimensional Comparison between Kriging and NURBS for sine function.

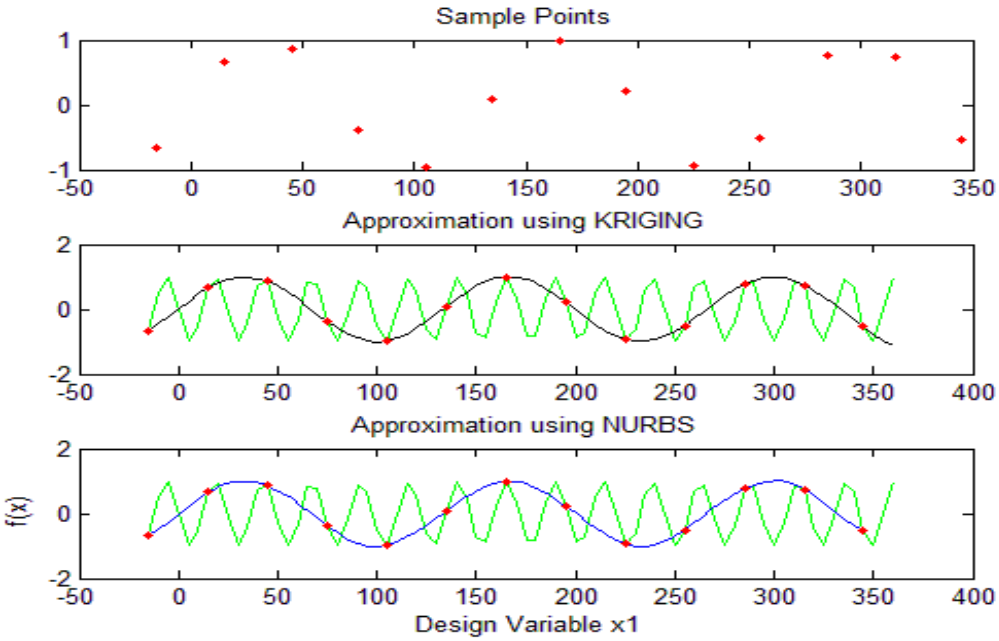

Fig. A.2. Comparison between Kriging and NURBS using subplots

Case 2: 1-Dimensional Comparison between Kriging and NURBS for a open box

Case 3: 2-Dimensional approach using NURBS for the Banana function

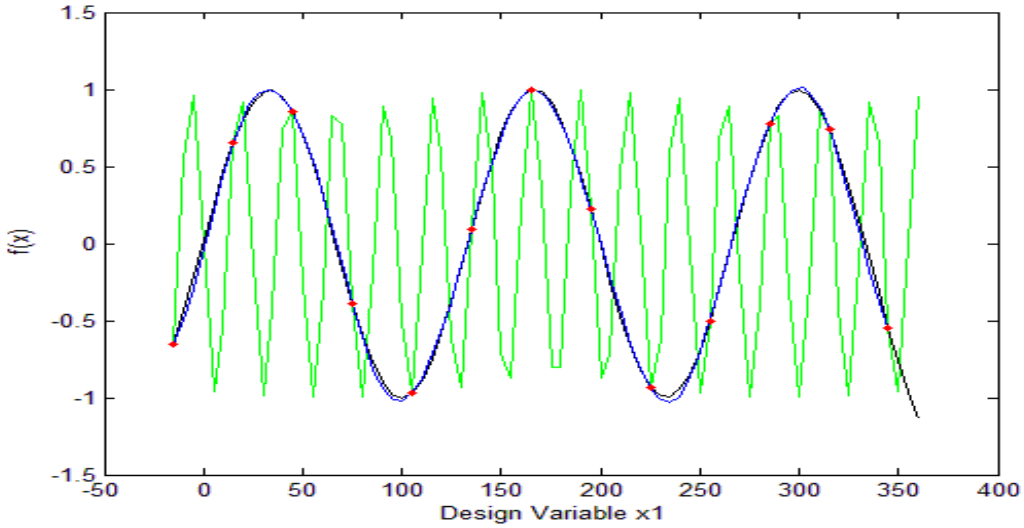

Fig. A.3. Comparison between Kriging and NURBS

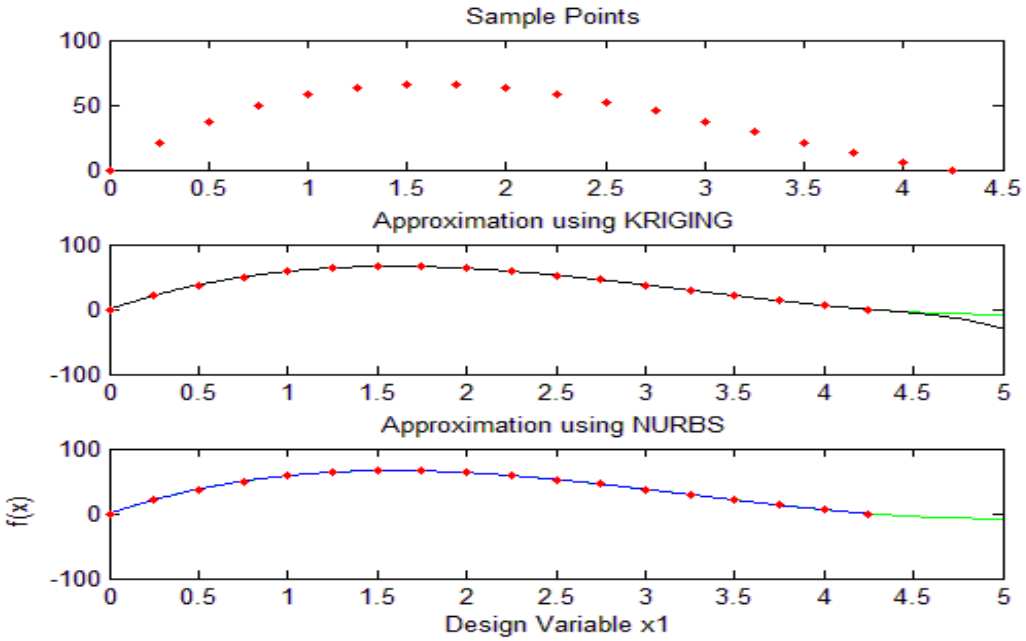

Fig. A.4. Comparison between Kriging and NURBS

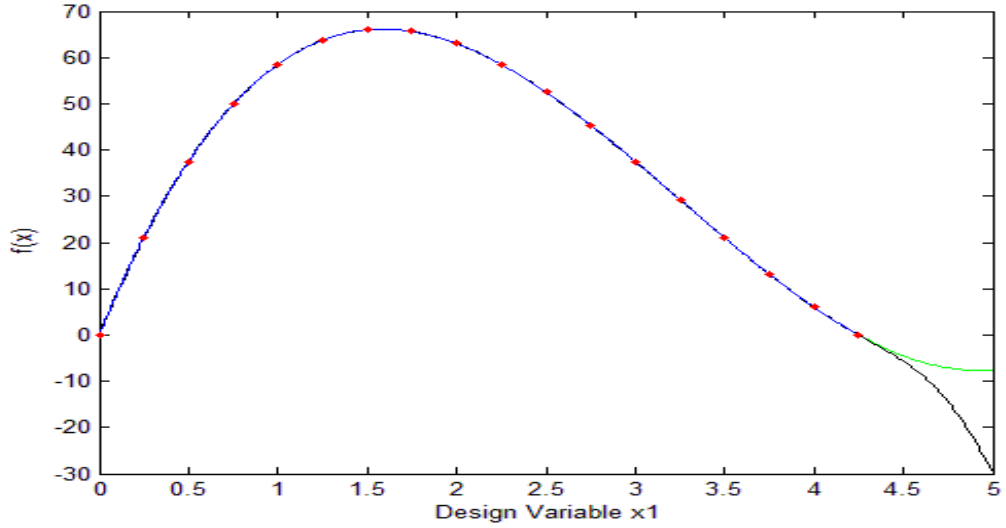

Fig. A.5. Comparison between Kriging and NURBS

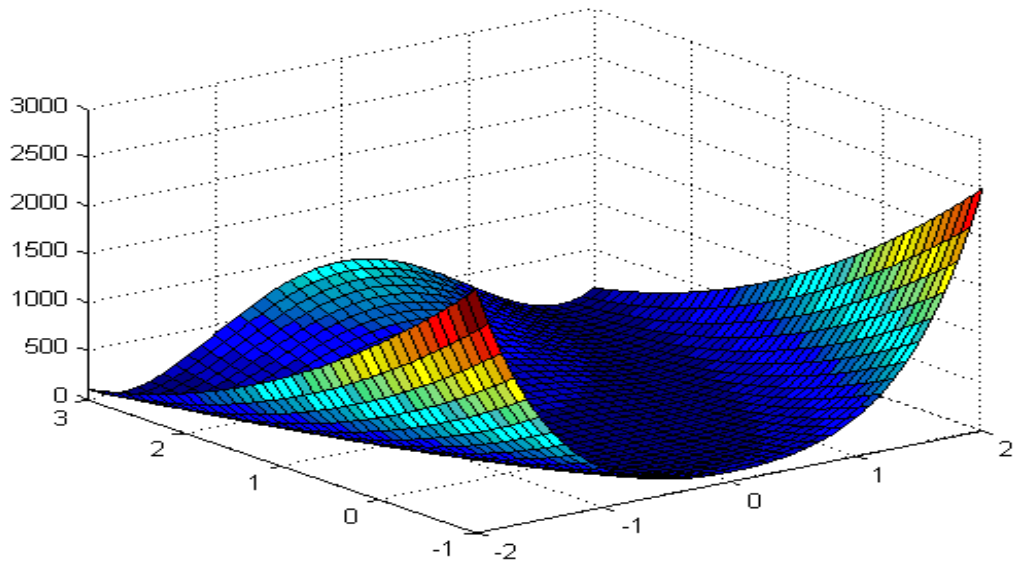

Fig. A.6. Surface plot of banana function

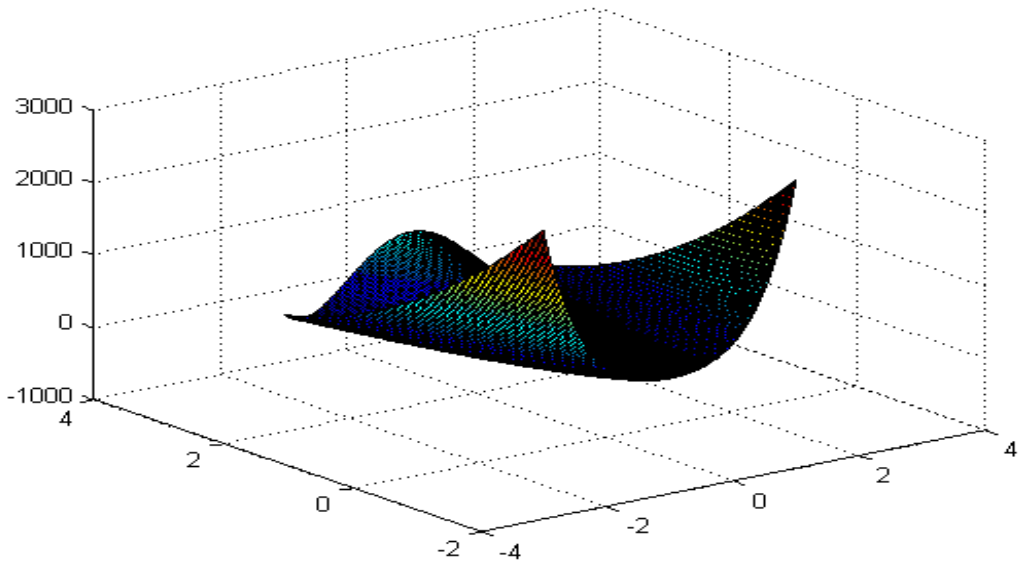

Fig. A.7. Surface plot of banana function using NURBS

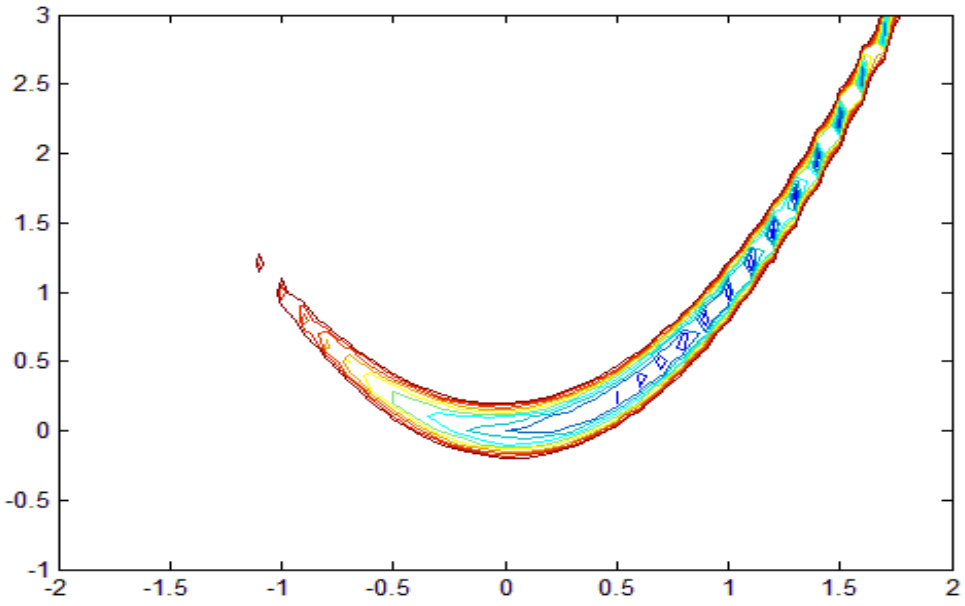

Fig. A.8. Contour plot of banana function

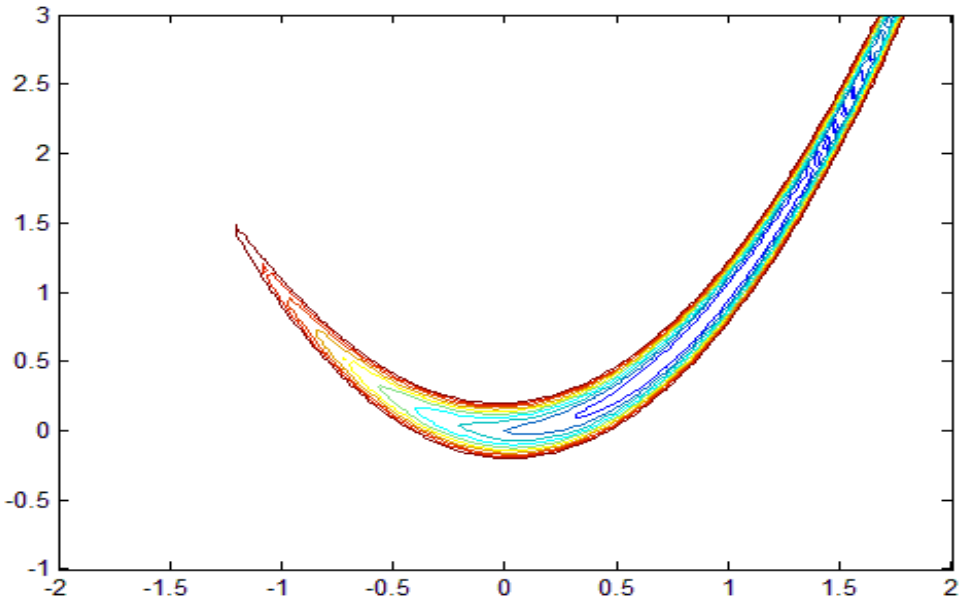

Fig. A.9. Contour plot of banana function using NURBS

B. MATLAB CODE

```
Implementing NURBS function on simple periodic signal using fmincon

parameters for simple periodic signal

Fs = 100;                   %  sample size
t0  = 0:1/Fs:1;             % time
Frq = 5;                    % frequency
x = @(t)  sin(2*pi*t*Frq); % function to calculate responses
y = x(t0);                  % responses

plotting
plot(t0,y,'k'); hold on       % data points plot
title('Signal');
xlabel('Time or Space')

% NURBS Parameters
P = [t0(:),y(:)];            % data points
n = 15;                      % number of control points
k = 4;                       % order of the curve (degree plus one)
w0 = ones(1,n);              % the weight vector

u_required = 0:1000;          % abscisa of the interpolation points

% Before optimization
[p1,P_Test1,Error1] = nurbs_function(P,n,k,w0,P(:,1),u_required(:));
```

```
figure(1)
plot(p1(:,1),p1(:,2),'-r');hold on  % control point polygon
plot(P_Test1(:,1),P_Test1(:,2));    % NURBS CURVE

%% Optimization using Fmincon
lb =1e-4* ones(1,n);
ub = [];
options =optimset('disp','iter');
[wopt, eopt] = fmincon(@(w)nurbs_error(P,n,k,w,P(:,1)),w0,
[],[],[],[],lb,ub,[],options);

%% After optimization
[p,P_Test,Error] = nurbs_function(P,n,k,wopt,P(:,1),u_required(:));
figure(2)
plot(t0,y,'k'), hold on   % data points
plot(p(:,1),p(:,2),'-r');hold on    % control point polygon
plot(P_Test(:,1),P_Test(:,2));      % NURBS CURVE
```

NURBS_FUNCTION.m:

```
function [p,P_Test1,Error] = nurbs_function(P,n,k,w,u_ini,u_required)
%%
% This function is used for generating the NURBS curve for the given
% data points that provide the deliverables as control points,
% resultant data points and error between the target curve and nurbs
% curve.

% P  - Data points of the given model
% n  - Number of Control Points
% k  - Order of the Curve
% w  - Weight vector for the selected Number of Control Points
% (initailly taken as all ones)
% u_ini - Design Variable Set
% u_required  - Interpolation Points
%%

a     = numel(u_ini(:,1));
b     = numel(u_ini(1,:));
u_mid = zeros(a,b);
% //////*************** Parametric Variable ***************//////
for i=1:numel(u_ini(1,:))
    u_mid(:,i) = ParametricVariable(u_ini(:,i));
end
u = arrangement(u_mid);
% /////************* Computing the Knot Vector *************/////
```

```
U = zeros(numel(n),n+k);
for i = 1:numel(n)
     U(i,:) = knotvector(n(i),k(i));
end

%/**************** Computing the Basis Functions *****************/
N = zeros(numel(u(:,1)),prod(n));
for i=1:numel(u(:,1))
    N(i,:)=BasisSum(u(i,:),n,k,U);
end

%/*********** Rational B-Spline Basis Function ******************/
R = zeros(numel(u(:,1)),prod(n));
for i=1:numel(u(:,1))
 bsum=0;
     for j=1:numel(N(1,:))
         bsum= bsum +(w(j)*N(i,j));
     end
     for j=1:numel(N(1,:))
         R(i,j) = (w(j)*N(i,j))/bsum;
     end
end

% Computing the Control Points for the given Data Points or Surface
% Points.

p = ((R')*R)\R'*P;            % p - Resultant Control Points Vector
if numel(n) == 1
    p(1,:) = P(1,:);
```

```
    p(numel(p(:,1)),:) = P(numel(P(:,1)),:);
end
% for error
P_Test = nurbstest(p,u,n,k,w);              % 'nurbstest' function
% is used to find the data points for the given control points

% for plotting
P_Test1 = nurbstest(p,u_required,n,k,w);
% Error Calculation
 di = 0;
 dif = zeros(1, numel(P(:,1)));
 for j=1:(numel(P(:,1)))
 dif(j)=((P(j,1)-P_Test(j,1))^2+(P(j,2)-P_Test(j,2))^2)^(1/2);
 di= di + dif(j);
 end
Error = di/(numel(P)/2);
end
```

PARAMETRIC VARIABLE:

```
function u=ParametricVariable(P)
% Note
% P - Position Vector
t=0;
temp=0;
t_max=1;

u=zeros(size(P));
```

```
for r=2:numel(u)
    z=P(r)- P(r-1);
    if z<0
        z=(-1)*z;
    end
    temp=temp+z;
end
for s=2:numel(u)

    for r=2:s
        x= P(r)- P(r-1);
        if x<0
            x=(-1)*x;
        end
        t=t+x;
    end
    u(s)=(t*(t_max/temp));
    t=0;
end
end
```

ARRANGEMENT CODE:

```
function N = arrangement(P)

%  P - Matrix consisting of all design variable vectors in
```

```
% column wise.

n = numel(P(1,:));                % n - Number of Columns
N = zeros(numel(P(:,1))^n,n);     % N - All the combinations
for the given desugn Variables



for i = 1:n

    c = 1;      % c - Count for number of rows
    l = 0;      % l - Number of intervals of each design variable

    for j=1:numel(P(:,i))^i

        for k=1:numel(P(:,i))^(n-i)
            N(c,i)= P(j-l*numel(P(:,i)),i);
            c=c+1;
        end
        if j == (l+1)*(numel(P(:,i)))^i/(numel(P(:,i))^(i-1))
        l= l+1;
        end

    end
end
end
```

KNOT VECTOR CODE:

```
function U=knotvector(n,k)
% n - Number of Control Points vector
% k - Order of the curve vector
d = n+k; % d - total number of elements in knot vector

x = 1/(d-2*k+1);
U=zeros((n+k),1); % U - Knot Vector for the given control points
% and order of the curve
for j=1:(n+k)
    if j<=k
        U(j) = 0;
    elseif j>k && j<=(d-k)
        U(j) = U(j-1)+x;
    else
        U(j) = 1;
    end
end
```

BASIS SUMMATION CODE:

```
function N = BasisSum(u,n,k,U)
 x=1;
% 'u' - Parametric Value
% 'n' - Control Point Vector
% 'k' - Order Vector of the Curve
% 'U' - Knot Vector

for i= numel(n):-1:1
```

```
    c=1;

    for j=1:n(i)
        for l=1:numel(x)
            if u(i)== 1
                if j == n(i)
                    N(c) = x(l);
                else
                    N(c)=0;
                end
            else
                N(c) = Nfun(u(i),j,k(i),U(i,:)) * x(l);
            end
                c=c+1;
        end
    end
    x=N;
end
```

NURBS_TEST CODE:

```
function P = nurbstest(p,u_ini,n,k,w)
% This function is used to generate the data points of the NURBS
% curves using the input parameter as given below:

% 'p' - Control Points
% 'n' - Number of Control Points Vector
% 'u_ini' - Parametric Values
```

```
% 'k' - Order of the curve
% 'w' - Weight vector

%************* Parametric Values **********************%
u_mid = zeros(numel(u_ini(:,1)), numel(u_ini(1,:)));
for i=1:numel(u_ini(1,:))
    u_mid(:,i) = ParametricVariable(u_ini(:,i));
end
u = arrangement(u_mid);
%************* Computing the Knot Vector***************
U = zeros(numel(n),n+k);
for i = 1:numel(n)
    U(i,:) = knotvector(n(i),k(i));
end

% Basis functions N and the parametric equation P
N = zeros(numel(u(:,1)),prod(n));
for i=1:numel(u(:,1))
        N(i,:)=BasisSum(u(i,:),n,k,U);
end

%/*********** Rational B-Spline Basis Function *****************/

R = zeros(numel(u(:,1)),prod(n));
for i=1:numel(u(:,1))
 bsum=0;
     for j=1:numel(N(1,:))
         bsum= bsum +(w(j)*N(i,j));
     end
```

```
     for j=1:numel(N(1,:))
         R(i,j) = (w(j)*N(i,j))/bsum;
     end
end

%Computing Position Vector for the corresponding values of parametric
% variable 'u'.

P=R*p;

end
```

NURBS FUNCTION ERROR CODE:

```
function [Error] = nurbs_error(P,n,k,w,u_ini)
%%
% P  - Data points of the given model
% n  - Number of Control Points
% k  - Order of the Curve
% w  - Weight vector for the selected Number of Control Points
% (initailly taken as all ones)
% u_ini - Design Variable Set
%%

a     = numel(u_ini(:,1));
b     = numel(u_ini(1,:));
u_mid = zeros(a,b);
```

```
% //////*************** Parametric Variable ****************//////
for i=1:numel(u_ini(1,:))
    u_mid(:,i) = ParametricVariable(u_ini(:,i));
end
u = arrangement(u_mid);
% /////************* Computing the Knot Vector *************/////
U = zeros(numel(n),n+k);
for i = 1:numel(n)
     U(i,:) = knotvector(n(i),k(i));
end

%/**************** Computing the Basis Functions *****************/
N = zeros(numel(u(:,1)),prod(n));
for i=1:numel(u(:,1))
    N(i,:)=BasisSum(u(i,:),n,k,U);
end

%/*********** Rational B-Spline Basis Function *****************/
R = zeros(numel(u(:,1)),prod(n));
for i=1:numel(u(:,1))
 bsum=0;
     for j=1:numel(N(1,:))
         bsum= bsum +(w(j)*N(i,j));
     end
     for j=1:numel(N(1,:))
         R(i,j) = (w(j)*N(i,j))/bsum;
     end
end
```

```
% Computing the Control Points for the given Data Points or Surface
% Points.

p = ((R')*R)\R'*P; % p - Resultant Control Points Vector
if numel(n) == 1
    p(1,:) = P(1,:);
    p(numel(p(:,1)),:) = P(numel(P(:,1)),:);
end
% for error
P_Test = nurbstest(p,u,n,k,w); % 'nurbstest2' function is used to
% find the data points for the given control points

%Error
 di = 0;
 dif = zeros(1, numel(P(:,1)));
 for j=1:(numel(P(:,1)))
 dif(j)=((P(j,1)-P_Test(j,1))^2+(P(j,2)-P_Test(j,2))^2)^(1/2);
 di= di + dif(j);
 end

Error = di/(numel(P)/2);



end
```