Multi-Objective Optimization for Railway Maintenance Plans

Daniel Peralta, Ph.D.[1], Christoph Bergmeir, Ph.D.[2], Martin Krone[3], Marta Galende, Ph.D.[4], Manuel Menéndez[5], Gregorio I. Sainz-Palmero, Ph.D.[6], Carlos Martinez Bertrand[7], Frank Klawonn, Ph.D.[8], and Jose M. Benitez, Ph.D.[9]

[1](Corresponding Author) Post-doctoral researcher, Department of Applied Mathematics, Computer Science and Statistics, Ghent University, Ghent, Belgium; Data Mining and Modelling for Biomedicine group, VIB Center for Inflammation Research, Ghent, Belgium. e-mail: daniel.peralta@irc.vib-ugent.be

[2]Post-doctoral researcher, Faculty of Information Technology, Monash University, Melbourne, Australia

[3]Ph.D. Student, Department of Computer Science, Ostfalia University, Wolfenbüttel, Germany

[4]Post-doctoral researcher, CARTIF Centro Tecnológico, Parque Tecnológico de Boecillo, Boecillo, Valladolid, Spain

[5]Vías Y Construcciones, S.A. (Grupo ACS), Spain

[6]Professor, Dept. of Systems Engineering and Control, University of Valladolid, Valladolid, Spain; CARTIF Centro Tecnológico, Parque Tecnológico de Boecillo, Valladolid, Spain

[7]CEO, Ingeniería INSITU, Las Rozas, Madrid, Spain

[8]Full professor, Department of Computer Science, Ostfalia University, Wolfenbüttel, Germany

[9]Professor, Department of Computer Science and Artificial Intelligence, CITIC-UGR. University of Granada, Granada, Spain

ABSTRACT

Railway track maintenance is a critical problem for any railway administrator. More precisely, preventive maintenance scheduling is an NP-hard problem, which additionally involves multiple objectives such as economical cost, maximum capacity, serviceability, safety and passenger comfort. This paper proposes a multi-objective optimization approach to this problem, combined with a track deterioration model that takes into account the degradation caused by maintenance operations. The track behavior is simulated by an exponential deterioration model based on a two-level segmentation. The maintenance schedule is built using a Pareto-based algorithm with two objectives (cost and delay) and three constraints, on top of an initialization heuristic based on expert knowledge. The proposed approach has been tested with two different algorithms (NSGA-II and AMOSA), over a model of a real track, to create schedules for different horizons ranging between three and twenty years. The solutions obtained by AMOSA outperform those designed by human experts both in terms of time delay and economical cost, demonstrating the capability of the proposal to produce near-optimal long-term maintenance schedules.

INTRODUCTION

Railway track maintenance represents an important challenge for stakeholders in the railway sector, such as railway contractors and infrastructure administrators, both in terms of money, resources and safety (Ferreira and López-Pita 2015). The economical cost of railway infrastructure maintenance is up to \$150 000 per kilometer, two thirds of which are associated with the track maintenance. Additionally, the non-redundancy of railway tracks implies that maintenance has a direct impact on the level of service and safety that can be provided by the trains. Therefore, the elaboration of feasible maintenance plans is a critical issue for railway infrastructure administrators.

Traditionally, track maintenance can be corrective or preventive. Preventive maintenance is sought after by the maintenance policies in the industry world, and can lead to smaller costs and better quality of the track, while providing a higher flexibility and better management of the resources (Kong and Frangopol 2003). However, the preventive maintenance scheduling problem is NP-hard (non-deterministic polynomial-time hard) (Budai et al. 2006; Gustavsson 2015). A problem H belongs to the NP-hard family when every NP problem (this is, problems for which a solution can be verified in polynomial time) can be reduced in polynomial time to H, meaning that H is at least as complex as any NP problem (Garey and Johnson 1979). In practice, this implies that a globally optimal preventive maintenance schedule cannot be computed in a feasible time. Moreover, the difficulty of this task increases along with the time span of the schedule. Therefore, it is crucial to develop algorithms that can find near-optimal approximate solutions to this problem in an acceptable time.

An adequate preventive maintenance requires an accurate track deterioration model to anticipate future failures and demands. The specialized literature includes several proposals to model the track based on the workload of the rails and the ballast, either using linear (Esveld 2001; Ramos and Fonseca 2011b; Wen et al. 2016) or non-linear models (Jovanovic 2004; Zhao et al. 2006; Andrade and Teixeira 2016). Other works describe how maintenance operations affect the degradation rate of the track (Ramos and Fonseca 2013; Audley and Andrews 2013; Andrade and Teixeira 2016).

Traditional optimization algorithms aim at finding the solution that minimizes (or maximizes) the value of a function for a given problem. However, many real-world problems involve several objective functions. Multi-objective algorithms have been an important research topic for the last decades, as they attempt to optimize several objective functions altogether, allowing to handle a set of non-dominated solutions (Deb 2001; Deb et al. 2002; Bandyopadhyay et al. 2008).

Multi-objective algorithms have been successfully applied on the railway maintenance scheduling problem. In Caetano and Fonseca 2013, the authors optimize the track life-cycle cost and the track availability for scheduling the renewal strategy. In Ramos and Fonseca 2011a, a biobjective approach optimizes the economical cost and railway capacity after applying a maintenance plan, while complying with some constraints. Some authors propose a different way to tackle a similar problem, translating all the objectives into terms of economical cost (Higgins et al. 1996; Arasteh Khouy et al. 2014). Another multi-objective approach is presented in Podofillini et al. 2006, based on risks and on a Markov model to model the inspection operations. Finally, Caetano and Teixeira 2016 apply multi-objective algorithm to schedule tamping operations. However, the authors have been unable to find in the literature any attempt to combine a multi-objective strategy with a track deterioration model that involves the degradation caused by both tamping and renewal operations.

This paper describes a multi-objective optimization approach for preventive track maintenance scheduling. Two objective functions (cost and delay) and three sets of constraints (one for safety and two for resources) are defined to model the problem. Two multi-objective algorithms are considered, to obtain a non-dominated set of maintenance plans that satisfy all constraints while minimizing both cost and delay. Two possible initializations of the solution set, based on expert knowledge, are proposed. Each candidate solution to the problem is encoded into a binary vector that represents the maintenance plan of a track over an arbitrary number of trimesters. A non-linear deterioration model that simulates the behavior of a real track under the effects of time, tamping and renewal operations underlies the entire optimization process.

This manuscript is structured as follows. First, the background information about railway track maintenance and multi-objective algorithms is presented. Then, the proposal is described. The experiments performed and their results are then detailed. Finally, the conclusions that can be reached through the study carried out are explained.

BACKGROUND

Railway maintenance

In compliance with the European standard (European Committee for Standardization 2010), there are two possible reactions to insufficient track quality: lowering the maximum speed of service, and carrying out maintenance operations. Although the former is cheaper in the short-term, eventually the quality would decrease under the minimum allowed by the law and the safety constraints. The quality of the service could also be deteriorated. Additionally, lowering the speed lowers the maximum capacity of the track. Therefore, an adequate maintenance plan aims at finding a trade-off between maintenance costs and service capacity loss. This trade-off strongly depends on the particular perspective of the decision maker: maintenance subcontractors pursue a low cost, while in general train companies seek to maximize the capacity.

The specialized literature shows two groups of methods to optimize railway maintenance operations (Budai 2009). The first approach starts from a fixed set of necessary operations and aims at organizing them in an optimal schedule, taking into account resource restrictions (technological, production-related, human and organizational) (Budai et al. 2006; Macedo et al. 2017). The second approach is more complex, as it also involves modeling the deterioration process and computing the necessary operations before doing the scheduling (Vale and Ribeiro 2014; Wen et al. 2016). Therefore, both the maintenance operations and their scheduling have to be computed and optimized as a whole. The research carried out in this paper falls within the second category. Some recent proposals tackle the problem of scheduling the railway maintenance and traffic altogether (Lidén and Joborn 2017; Luan et al. 2017). However, in practice they fall very often under the responsibility of different agents (namely the maintenance contractor and the railway operator). This paper focuses on the maintenance scheduling, and takes into account an estimation of the total train delays that arise from this scheduling in combination with the track deterioration.

Table 1 shows an overview of the different maintenance operations and how they are triggered (Patra et al. 2009). The operations that are performed on a time or failure basis do not need any special considerations to be scheduled; therefore, this paper focuses on operations that are triggered by a certain condition: this involves tamping, ballast cleaning and component renewal.

The effect of tamping has already been modeled in previous research (Jovanovic 2004; Zhao et al. 2006). This modeling is based on geometric data gathered from the tracks, which must be properly align prior its use (Xu et al. 2015). However, modeling the exact effect of renewal operations that only involve certain components of the infrastructure proves to be more difficult (Lévi 2001). Following the approach of other work on the topic, in this paper a single renewal operation is assumed for all the elements of the track, which leaves it in an as-good-as-new condition (Ramos and Fonseca 2011a). Consequently, the remainder of this paper considers two maintenance operations: tamping and renewal.

In this context, the aim of a maintenance schedule is to determine when and where to perform tamping and renewal operations in an optimal way. This optimality can depend on many criteria that may be contradictory of conflicting, and the exact criterion remains in hands of the final decision maker, which is usually the railway administrator. It is not desirable to automatically build a schedule that optimizes a single criterion, or even a fixed combination of them. The next section describes how multi-objective algorithms can overcome this problem.

Multi-objective algorithms

Let $\mathcal{S}$ be the set of all possible solutions to a given problem. Single-objective optimization consists of looking for a solution $S^* \in \mathcal{S}$ that yields the best value of a function f, which can be the minimum or the maximum, depending on the context (Deb 2001). Hence the problem is called minimization or maximization, respectively. For the sake of simplicity, this paper focuses on minimization problems (Equation 1).

$$f(S^*) \leq f(S) \quad \forall S \in \mathcal{S} \tag{1}$$

On the other hand, a multi-objective problem involves a set of n objective functions $\mathcal{F} = \{f_1, ..., f_n\}$. Thus, the optimization becomes much more difficult, especially when these functions have conflicting behaviors, as it happens in most cases. Considering a single objective at a time is not feasible: the remaining objectives would get extremely bad values. There are two main ways to achieve multi-objective optimization (Deb 2001):

- Aggregating the objectives into a single function, thus converting the problem to a single-objective one.
- Looking for non-dominated solutions. A solution S_a dominates S_b if $f_i(S_a) \leq f_i(S_b)$, $\forall f_i \in \mathcal{F}$. In that case, S_b can be safely discarded because S_a is undoubtedly better. However, if a solution S_c is better than S_a for some functions but not for all of them, S_c and S_a do not dominate each other and none of them can be said to be better than the other. A set of non-dominated solutions is called a Pareto front.

Some proposals use the first approach to model the railway maintenance problem. For example, in Arasteh Khouy et al. 2014 all the objective functions are translated into an overall cost C_T that is optimized. Although this simplifies the handling of the objectives, it forces to establish a balance factor between the objectives prior the execution of the algorithm, fixing their priority. However, the decision criteria for railway maintenance can change according to many factors, and such an approach could avoid reaching potentially interesting solutions (Das and Dennis 1997). Therefore, this paper focuses on approaches that use a Pareto front, which have been proven to yield good results in similar problems (Caetano and Teixeira 2016; Aminbakhsh and Sonmez 2017). The main advantage of this alternative is the flexibility of the result: a set of solutions is made available under different balances of the objectives, and the decision maker can select one of them according to their specific needs.

Many real-world problems include constraints that restrict the solution space. A solution that does not comply with the constraints is said to be *non-feasible*, and in general terms should not be taken into account as a valid solution for the problem. Algorithms based on Pareto front usually include the constraints into the dominance criterion, so that a feasible solution always dominates a non-feasible one, independently of the value of the objective functions (Deb et al. 2002; Bandyopadhyay et al. 2008).

The number of objectives is one of many categorizations that can be done of optimization algorithms. Another popular manner to group them is according to how many solutions they handle at a time (Blum and Roli 2003). On the one hand, trajectory-based algorithms start from a single solution and modify it looking for improvements in the objective function(s). One of the most well-known algorithms in this category for multi-objective optimization is AMOSA (Bandyopadhyay et al. 2008). On the other hand, population-based algorithms maintain a pool of solutions and generate new solutions from them, increasing the diversification of the search. One of the most used ones is NSGA-II (Deb et al. 2002).

AMOSA

Simulated Annealing (SA) (Kirkpatrick et al. 1983) is one of the most popular trajectory-based algorithms. It starts with a randomly generated solution S_c. Then, a new solution S'_c is generated by modifying slightly S_c. If S'_c is better than S_c it is selected as current solution; otherwise, it can still be picked according to a certain probability based on a temperature value, which is gradually reduced as the search goes on until it reaches a minimum value, signaling the end of the search.

AMOSA (Bandyopadhyay et al. 2008) is a multi-objective adaptation of SA. Instead of using a single current solution, it maintains a so-called “Archive” of non-dominated solutions. Therefore, the Archive is the Pareto front of the search. First, the Archive is randomly initialized, a hill-climbing algorithm is applied to its members, and only the non-dominated solutions are kept. Then, a random solution is picked and SA is applied, introducing the domination criterion. In addition to the basic domination definition described earlier, AMOSA defines an *amount of domination*, which takes into account the numeric difference between the values of the objective functions. When the Archive gets too large, a similar solutions are clustered to reduce its size.

The main advantage of AMOSA is its capability to intensify the search towards promising areas of the search space. This is achieved first by the hill-climbing algorithm, which quickly improves the fitness of the initial solutions. Then, SA is also based on a hill-climbing procedure, although allowing for more exploratory capabilities thanks to the probability generated by the temperature.

NSGA-II

Evolutionary algorithms use a population of solutions (called individuals) that evolve together. New individuals are obtained by combining (crossing) several individuals (generally two) and introducing random mutations. A number of muti-objective evolutionary algorithms have been suggested in the literature (Knowles and Corne 2000). One of the most well-known of them is NSGA-II (Deb et al. 2002).

NSGA-II is based on the concept of nondominated sorting: when a new population is generated, the individuals are grouped into fronts according their domination. The first front corresponds to the Pareto front; the second one includes the solutions that would form the Pareto front if the first front was removed, and so forth. The following steps summarize the NSGA-II algorithm (for the full description, please refer to the original publication (Deb et al. 2002)):

1. Population initialization: with N randomly generated individuals.
2. Binary tournament: select N random pairs of individuals and pick the best of each pair.
3. Crossover: the N selected individuals are grouped in pairs. Each pair is combined by a crossover operator that generates two new individuals, for a total of N new individuals.
4. Mutation: each new individual suffers a random mutation with a given probability.

5. Evaluation: of the new individuals.
6. Nondominated sorting: sort old and new individuals together.
7. Selection of the new population: the fronts are included into the new population in order, until size N is reached. If the last selected front does not fit entirely, select the individuals so that they are as spread as possible across the front.
8. Go to step 2, until a stop criterion (typically a fixed number of generations) is met.

The algorithm design is focused on reducing the computational complexity of the nondominated and crowding sorting. NSGA-II favors a wide exploration of the search space rather than a deep intensification towards already known areas. This makes NSGA-II especially powerful when dealing with problems of which little knowledge is possessed, or where the structure of the search space is unknown or highly complex (El-Abbasy et al. 2017).

PROPOSAL

This section describes our proposal for building maintenance plans using optimization based on multi-objective algorithms, including a modeling of track to simulate the whole maintenance process, the encoding of the generated maintenance plans, the evaluation of the cost and delay functions and the safety and resources constraints that are used to model the problem, the solution initialization process, the operators and other particular considerations for the design and implementation of the algorithms and the proof that the problem is NP-hard.

Railway modeling

The core of a good optimization framework for any real-world problem is an adequate representation. In this case, it must simulate the response of the track over time and the different maintenance operations that are performed upon it. This section describes the railway segmentation process, the deterioration model and the modeling of maintenance operations used in this paper.

Railway segmentation

The behavior of the track depends on a wide variety of factors such as curvature, traffic, ballast type and previously applied maintenance. Thus, the track cannot be modeled as a whole: it must be segmented and each segment must be treated separately (Jovanovic 2004). There are two main types of segmentation strategies: static segmentation divides the track into segments of the same length, and dynamic segmentation takes into account the factors that affect its behavior.

This paper describes a two-level segmentation procedure that combines both approaches. First, the track is dynamically divided into *sections*, according to the curvature, age and type of the track, previously applied maintenance operations, and the presence of elements such as switches, bridges or tunnels. This design ensures that the characteristics of quality, deterioration and maximum allowed speed remain constant within each section. Then, each section is statically divided into *segments* of lengths between 25 and 100 meters. This approach allows to accurately model a real track where tamping and renewal operations have different ranges: tamping is carried out throughout a segment, whilst the renewal is performed on an entire section. Note that the number of segments within each section is variable because there is no constraint on the length of the sections.

Deterioration model

Deterioration models can be categorized into mechanistic and stochastic approaches (Cárdenas-Gallo et al. 2017). Mechanistic models are based on a simulation of the track geometry taking into account physical factors such as ballast and sleeper type, weather conditions, workload and wheel geometry. These models provide insight into the behavior of different components of the railway infrastructure from a physical point of view; however, their use for predictive modeling is hindered by large uncertainties (Nguyen et al. 2016). Stochastic approaches produce a model from data measured from the tracks themselves. These can be broadly classified into linear (Esveld 2001; Ramos and Fonseca 2011b; Wen et al. 2016) and non-linear models (Jovanovic 2004; Zhao et al. 2006; Andrade and Teixeira 2016). The latter assume the deterioration of the track to be inversely proportional to the current quality, which reflects the behavior measured from the tracks more accurately (Hummitzsch 2009). Furthermore, maintenance operations also affect this degradation rate (Ramos and Fonseca 2013; Audley and Andrews 2013; Andrade and Teixeira 2016).

In our approach, we consider an exponential fitting model (Hummitzsch 2009), combined with a mixed maintenance model where tamping operations restore the quality of the track while increasing the deterioration rate and renewals restore the track to its maximum quality, as suggested in Ramos and Fonseca 2011a. This is shown in Equation 2, where Q_0 is the initial quality, b is the deterioration rate, and t is the time expressed in days. Although all track segments are based on the same exponential model, the parameters Q_0 and b are different for each segment. These parameters can be estimated from geometric auscultation data.

$$\frac{dQ(t)}{dt} = b \cdot Q(t) \quad \Leftrightarrow \quad Q(t) = Q_0 \cdot e^{bt} \tag{2}$$

This work considers the standard deviation of longitudinal level D1 (σ) as the quality measure, following European regulations (European Committee for Standardization 2010). Therefore, Equation 3 gives the quality σ_{ijk} of segment j of section i in trimester k, considering that no maintenance operations have been performed in that time period. Figure 1 shows the exponential behavior of a segment between successive tamping operations.

$$\sigma_{ijk} = \sigma_{ij0} \cdot e^{b_{ijk}(90t)} \tag{3}$$

Maintenance operations modeling

When the quality level attains a certain threshold, maintenance operations are performed in order to take it to an appropriate value. This introduces a break in the model, as the quality is changed. Moreover, maintenance operations also change the deterioration rate (Ramos and Fonseca 2013; Audley and Andrews 2013), which makes the modeling problem much more difficult, in particular with respect to the estimation of Q_0 and b.

Previous works in the literature consider that tamping induces a constant change in the first derivative of the exponential deterioration model curve (Hummitzsch 2009). This model starts from the first derivative of σ in trimester $k = 0$ (Equation 4) and assumes a constant ratio c between this value before and after a tamping (Equation 5). Then, it estimates $\sigma_{ij(k+1)}$ with a linear fitting using the age of the track, so that the new deterioration rate is given by Equation 6.

$$\sigma'_{ij0} = \sigma_{ij0} \cdot b_{ij0} \tag{4}$$

$$\sigma'_{ij(k+1)} = c \cdot \sigma'_{ijk} \tag{5}$$

$$b_{ij(k+1)} = \frac{\sigma'_{ij(k+1)}}{\sigma_{ij(k+1)}} \tag{6}$$

Figure 1 depicts an example of the quality of a segment over the years after successive tamping operations, for two slightly different quality thresholds. It can be seen that the more tampings are performed, the faster the track deteriorates, and the smaller the quality gain is. Moreover, the small difference in the threshold causes serious disturbances of the degradation forecast for large time horizons. This highlights the difficulty of the tackled problem: decisions that are made for early stages of the scheduling might have important long-term effects on the track behavior.

The modeling of a renewal operation is simpler. It is considered to be applied to a whole section of the track, whose quality is restored to some level Q_{best}, with a certain deterioration rate b_{best}. This operation resets the deterioration model to the optimal state of a new track.

Solution modeling

Maintenance operations can be encoded as a vector of binary values that indicate if the operation is performed or not at a certain time and location. Focusing on tamping operations, the vector is of the form $\mathbf{x} = \{x_{ijk}\}$, where i denotes the track section, j is the segment within a section and k is the trimester. Likewise, complete renewal operations are represented as a vector $\mathbf{y} = \{y_{ik}\}$.

The length of these vectors is $N_g N_k$ and $N_s N_k$ respectively, where N_s, N_g and N_k are the number of sections, segments and trimesters. Each solution to the scheduling problem is represented by the concatenation of $\mathbf{x}$ and $\mathbf{y}$, as shown in Figure 2, where N_i is the number of segments in section i. Note that each section can be split into a different number of segments, according to the segmentation procedure previously described. This gives an overview of the difficulty of the problem, which involves a very high dimensionality. More precisely, the size of the search space is $2^{N_k(N_s+N_g)}$, making brute-force or even exact approaches not feasible.

Objective functions and constraints

The proposed approach to the optimization of maintenance plans uses two different objective functions: economical cost of the maintenance and time delay of the trains. This design complies with other approaches (Patra et al. 2009; Ramos and Fonseca 2011a). Other secondary objectives are considered to be included within these, such as the durability of the track (reflected as a higher cost), or level of service (which reflects the deterioration state in the same way as the delay). However, two more factors must be taken into account: safety and resources. These have been implemented as constraints, so that a solution that violates any constraint is said to be non-feasible.

Cost

The economical cost of railway maintenance includes costs of track inspection and maintenance operations. In the literature, various approaches to assess these costs can be found (Patra et al. 2009; Guler 2013), which involve duration and length of the operations and cost of the workforce and equipment. Based upon the cost functions defined in Patra et al. 2009; Guler 2013, the maintenance cost is defined as the sum of tamping cost (C_T) and renewal cost (C_R) for all sections, segments and trimesters, as shown in Equation 7, where C_t is the cost of a tamping operation per meter, L_{ij} is the segment length, C_r is the renewal cost per meter, L_i is the section length and r is the discount rate (which models the economic impact of the investment). Equation 7 is the first objective function for the modeling of the problem, and it is to be minimized.

$$f_1(\mathbf{x}, \mathbf{y}) = C_T + C_R = \sum_k \frac{\sum_{ij} \left(C_t \cdot L_{ij} \cdot x_{ijk}\right) + \sum_i \left(C_r \cdot L_i \cdot y_{ik}\right)}{(1+r)^k} \tag{7}$$

Delay

The other main objective for railway maintenance is the maximization of the track availability and capacity. Usually, maintenance operations are performed when no trains are scheduled, so that the availability is not affected. As for the capacity, it can be translated into terms of overall time delay of the trains (Ramos and Fonseca 2011a). Table 2 shows the maximum speed of the track depending on its measured quality, according to European standards (European Committee for Standardization 2010).

In order to calculate the delay, the maximum permissible nominal speed s_i^{max} of section i is defined as the minimum speed across every segment j within the section, considering that the track is in perfect condition, and depends mainly on the curvature of the track. Accordingly, the maximum speed that a train t, whose average speed is s_t^{mean}, can attain within section i is denoted s_i^t. Equation 8 presents the maximum speed for a train t in section i and trimester k, where $s_{ik}(\sigma_{ik})$ is the maximum speed in the section taking into account the deterioration state of the track ($\sigma_{ik} = \max_j\{\sigma_{ijk}\}$), as defined in Table 2.

$$s_{ik}^t = \min\{s_i^t, s_{ik}(\sigma_{ik})\}, \quad s_i^t = \min\{s_i^{max}, s_t^{mean}\}, \quad s_i^{max} = \min_j\{s_{ij}^{max}\} \tag{8}$$

Based on these equations, the second objective function is defined by calculating the overall delay in hours, as detailed in Equation 9, where N_t is the number of trains and L_i is the length of the section. Note that $\mathbf{x}$ and $\mathbf{y}$ are not explicitly shown, but they are used to calculate σ_{ik}. For each train, each trimester and each section, the time difference is calculated with respect to the same track in perfect conditions. Therefore, the time delay would be zero in such a case where all σ_{ik} are low enough to allow $s_{ik}(\sigma_{ik}) \geq s_i^t \; \forall i, k, t$.

$$f_2(\mathbf{x}, \mathbf{y}) = \sum_{ik} \sum_{t=1}^{N_t} \frac{L_i}{1000} \left(\frac{1}{s_i^t} - \frac{1}{s_{ik}^t} \right) \tag{9}$$

Safety and resource constraints

Even though a low quality of the track can be palliated by reducing the speed, each segment has to be kept above the acceptable minimum determined by the legal and technical normative for safety reasons. Table 2 shows the quality limit values for each speed in the experiments, which were extracted from European Committee for Standardization 2010. Thus, the safety constraint can be represented as shown in Equation 10.

$$1 - \frac{\sigma_{ijk}}{\max\{L_{QN3}\}} \geq 0 \quad \forall i, j, k \tag{10}$$

The other constraint to be included into the model refers to the available resources. In particular, the limits of the resources for tamping and renewal operations (Equations 11 and 12, respectively) are modeled by establishing a maximum extent of operations per trimester (max_t and max_r, respectively), measured in meters.

$$1 - \frac{\sum_{ij} L_{ij} x_{ijk}}{max_t} \geq 0 \quad \forall k \tag{11}$$

$$1 - \frac{\sum_{i} L_{i} y_{ik}}{max_r} \geq 0 \quad \forall k \tag{12}$$

Proof that railway maintenance planning is NP-hard

The problem defined above can be proven to be NP-hard. Let us consider a simplification of the problem that involves only tamping operations ($\mathbf{y} = \mathbf{0}, C_r = 0$), the cost function f_1 with no discount rate ($r = 0$) as a single objective, and a deterioration model where tamping does not change the deterioration rate ($c = 1$). With these conditions, the safety constraint is held if and only if the period between two consecutive tampings on the same segment is kept under a threshold T_{ij}.

This simplification can be expressed as an integer linear programming problem with a binary decision variable $\mathbf{x}$ (Equation 13). As integer programming problems are known to be NP-hard (Garey and Johnson 1979), this simplified version of railway maintenance scheduling is also NP-hard, and so is the full non-linear multi-objective problem that is tackled in this paper.

$$\begin{aligned} &\text{Minimize}: C_t \sum_{ijk} L_{ij} x_{ijk} && && \text{(cost function)} \\ &\text{Subject to:} \sum_{k=l+1}^{N_k - T_{ij}} x_{ijk} \geq 1 && \forall l = 0, \ldots, N_k && \text{(safety constraint)} \\ &\sum_{ij} L_{ij} x_{ijk} \leq max_t && \forall k = 1, \ldots, N_k && \text{(resource constraint)} \end{aligned} \tag{13}$$

Solution initialization

The search space of the tackled optimization problem has two main difficulties: its very high dimensionality ($N_k(N_s + N_g)$ dimensions), and its complexity due to the constraints that restrict the feasibility of the solutions. Moreover, the objectives of a maintenance plan differ depending on the horizon of the schedule: a short-term scheduling usually prioritizes tamping operations, while a long-term approach must make an adequate use of renewal operations.

On the other hand, there are experts on railway maintenance scheduling that possess information about how to build good maintenance plans. Therefore, this proposal does not use a randomly generated initial set of solutions. Instead, those solutions are generated following certain heuristic rules given by experts, to conform an initial set of feasible and reasonably good solutions. Then, it falls upon the algorithm to improve those solutions and obtain maintenance plans that are better than those designed by the experts. This design ensures that the quality of the obtained solutions to the problem will be at least as high as that of the human-designed initial set. Furthermore, the improvement can be measured by simply evaluating the differences between the initial solution set and the final Pareto front.

When considering short-term scheduling, each solution is initialized as follows:

1. For the first trimester, tamping is programmed in the segments whose deterioration is above the threshold ($\max\{L_{QN3}\}$).

    - If the tamping capacity is insufficient, a renewal is performed in the section with the largest number of segments needing action.
    - Otherwise, and if there is some remaining tamping capacity, a random number of tampings are programmed in the segments with worst quality among those that do not have tamping scheduled.

    The same operation is performed for the remaining renewal capacity.

2. After the maintenance of the first trimester has been scheduled, the deterioration model simulates the quality for the second trimester, and the operations are scheduled following

step 1. This procedure is iteratively applied for the whole simulation time span.

3. If no renewal is planned, it is randomly determined if a single renewal should be introduced into the solution.

This procedure aims at ensuring the generation of feasible solutions. Note that there may be cases in which the track is in such a bad state that the available resources do not suffice to mend it within a single trimester. This situation can also arise when the first trimesters are assigned a low amount of tampings and renewals. In extremely bad quality tracks, feasible solutions might be entirely non-existent. However, this kind of solutions could also be interesting as a starting point for the algorithm, because they introduce diversity into the search. Eventually, as non-feasible solutions are dominated by feasible ones, these solutions will disappear from the population, but their information could have been used to generate new promising solutions.

Different rules apply for long-term horizons, as renewal must often be preferred over tamping in order to obtain feasible schedules. Therefore, a different initialization heuristic was used:

1. The total number of renewals is randomly fixed between the maximum and half of the maximum.
2. These operations are randomly distributed among all the trimesters in the schedule.
3. For each trimester:

    1. The deterioration model is applied.
    2. If this trimester had a renewal operation scheduled, it is performed over the most deteriorated section in terms of $dQ(t)/dt$ (see Equation 2).
    3. Tamping is applied over any section above the threshold ($\max\{L_{QN3}\}$).
    4. If there is any remaining tamping capacity, a random fraction of it is used to schedule tamping over the sections with worst quality.

Operators and implementation particularities

NSGA-II uses single-point crossover and bitwise mutation, as suggested in the original paper for binary problems (Deb et al. 2002). AMOSA uses only the bitwise mutation, as it does not involve any crossover operations.

The main difference in the implementation with respect to the originally published algorithms lies in the hill-climbing technique for AMOSA. Although the same algorithm was implemented, an additional criterion was added to allow for handling such a high dimension problem (note that the number of dimensions is $2N_k(N_s + N_g)$, see Table 5 for the dimensionality of the track evaluated in this paper). Instead of performing the hill-climbing procedure until no improvement is reached, the procedure is interrupted when the solution has been improved more than than a fixed number of times max_{HC}. Otherwise, the search space for the hill-climbing procedure would be too large to be used as initial greedy algorithm to improve the solutions.

EXPERIMENTS AND RESULTS

Case study and parameters

Two multi-objective algorithms have been used for the experimental framework of this paper: NSGA-II and AMOSA. Both algorithms have been executed up to a total of 500 000 evaluations of the objective functions, and the corresponding parameters have been set up accordingly (Table 3). The horizon of the prediction was 3 years, which corresponds to an average contract period for maintenance contractors. Both algorithms started from the same set of initial solutions. The value for max_{HC} was chosen so as to invest approximately 2 000 evaluations for the hill-climbing procedure, and the remaining evaluations for the simulated annealing optimization.

The experiments have been performed upon a model of a real railway track from the Swedish Iron Ore Line, which is 152 km long and runs in the northern part of Sweden, subject to temperatures between $-40°$C and $25°$C and heavy snowfalls during winter. A total of 19 geometrical auscultations with a resolution of 25cm performed between 2007 and 2012 are available. These data were spatially aligned to match the measurements taken at different points in time, using correlation-based alignment on the curvature. This information was used to estimate the initial Q_0 and b for every segment of the track by an exponential fitting. Tables 4 and 5 contain the parameters that define the track modeling and the solutions to the problem for this case study, respectively.

To complete the study and give an overview of the potential of the proposed multi-objective approach, a complementary study is presented in a subsequent section, with horizons longer than 3 years for the maintenance plans, namely 5, 10 and 20 years. Due to the computational constraints, the number of evaluations was reduced to 20 000 for these tests.

Scheduling for 3 Years

Tables 6 and 7 present a summary of the solutions in the final Pareto fronts obtained by NSGA-II and AMOSA, respectively. These show clearly that renewal and tamping operations increase the maintenance cost and decrease the time delay. They also reflect that renewal improves the track quality more than tamping, allowing for a higher nominal speed.

Table 7 shows the flexibility provided by the Pareto front. The difference between the two extremes of the Pareto (first and last rows of the table) states that the delay can be reduced by 55% by increasing the cost by around 23%. However, railway maintenance companies may be more interested in the intermediate results, seeking a trade-off between cost and delay. The approach proposed in this paper allows to consider a wide set of non-dominated solutions that provides a rich decision support for railway maintenance companies.

Figure 3 depicts the initial population and the final Pareto fronts of NSGA-II and AMOSA. At first sight, it is observed that the AMOSA Pareto front outperforms that of NSGA-II. This behavior arises because the initial local search performed by AMOSA proves to be crucial for the algorithm convergence. The initial population of solutions is not random; quite oppositely, it has been generated according to directions and constraints given by experts, so they all have a reasonable quality. AMOSA's local search focuses on further improving these solutions, rather than exploring entirely new areas of the search space for unknown solutions to the problem, which is the strategy followed by NSGA-II. Thus, AMOSA starts its exploratory search from a set of already optimized solutions, which yields far better results, as demonstrated by the distance between the initial population and the Pareto front in Figure 3: the solution of minimal cost is reduced from about 6.2M€to 5.7M€, and that with minimal delay is improved from 100 hours to 62. Moreover, it is able to explore solutions with different amounts of renewals than initially provided in the expert-based solutions, demonstrating a considerable diversification of the search as well. Note that the solutions with the lowest delays, which involve 10 renewal operations, also involve a high number of tampings; this highlights the heavy maintenance that would be required to keep the track at an optimal quality at all times. On the other hand, it can be seen that the combinations of existing solutions favored by NSGA-II do not suffice to reach the performance of AMOSA.

To further illustrate this behavior, Figure 4 gives an overall view of all the 500 000 solutions explored by AMOSA. It shows that even though AMOSA focuses on improving the good solutions, a good deal of exploration effort is made. This plot also shows the structure of the problems: each of the vertical stripes represents a certain number of renewals (the three stripes with solutions in the Pareto correspond respectively and from left to right 8, 9 and 10), and each additional renewal increases the cost of the maintenance plan, but reduces the delay. It can be seen that the search explored feasible maintenance plans with 11 renewals, but they did not yield better delays than solutions with 10 renewals. Some plans with 7 or 6 renewals and a very low cost were also generated, but they did not comply with the constraints and therefore were not included into the final set of solutions.

To summarize, the proposed approach has been shown to greatly improve the quality of solutions in both objectives. In addition, by design the obtained solutions will never be worse than those obtained by human experts. While metaheuristics have no guarantee for quality assurance, they are usually better than other simpler methods. In addition, due to the large budgets of the maintenance contracts, the improvement in solutions easily leads to large economical savings.

Long-term scheduling

It is well-known that models and solutions for long-term horizons are subject to important uncertainties and therefore cannot be considered as an exact forecast (Ramos and Fonseca 2011b). However, the results presented in this complementary study are useful to illustrate the behavior of the multi-objective approach, and they represent the long-term point of view of the railway owner. A similar study is presented in Ramos and Fonseca 2011a, in which a small custom track is simulated over 30 years; the authors are able to generate nine non-dominated feasible solutions. Nevertheless, the results cannot be compared to those obtained in this paper because they do not take into account the deterioration caused by tamping operations, which simplifies the problem and the search space they consider.

This section presents the results obtained after additional executions of the algorithms for a simulation of the track over 5, 10 and 20 years. This is reflected in a linear increase in the size of the solutions and therefore an exponential growth of the search space. The initialization rules for the population are also different, as human experts follow different scheduling patterns for such long-term situations. Due to the higher computational cost of the objective and constraint functions, only 20 000 evaluations of the objective functions were performed for each horizon and algorithm. Note that the difficulty of the problem is such that NSGA-II did not obtain any improvement with respect to the initial population; therefore, only the results from AMOSA are presented hereby.

Figure 5 represents the initial populations and the Pareto fronts obtained by AMOSA, in terms of average cost and delay per year. For the sake of simplicity, only feasible solutions are shown in the initial populations. The plot shows great improvements on both objectives for all three horizons. The initial solutions are in general worse for distant horizons because the complexity of the scheduling (which is an NP-hard problem) increases greatly as the horizon grows.

However, the Pareto fronts surprisingly follow the opposite behavior: the larger the horizon, the better the final Pareto front of solutions. This means that the proposed scheduling procedure works best with more distant horizons than with small ones, despite the exponential growth of the search space. This behavior arises because for long-term simulations, the cost of the renewal operations can be amortized over the years, yielding better quality railways at lower costs per year, which in turn leads to lower average delays. In this manner, our approach has been able to improve altogether two objectives that are a-priori opposed to each other. Furthermore, it implies an improvement of the average track quality after applying the computed maintenance schedules with respect to the current state of the tracks, which is the result of a maintenance plan carefully designed by experts.

The case with the largest horizon is especially illustrative: both the average cost per year and the average delay are reduced by a factor of at least 20. This reflects the advantages of the proposed metaheuristics over human-designed approaches and assesses the quality of the obtained solutions.

The limiting factors for most optimization algorithms are the size of the solution space and the number of evaluations. The results in this paper demonstrate that the proposal is able to explore very large solution spaces and reach good solutions in very few iterations. As an example, the number of possible solutions for the considered railway along 20 years is more than 10^{35000}, and the proposal is able to provide high-quality solutions after evaluating only 20 000 of them.

CONCLUSIONS

In this paper, a multi-objective approach has been described to tackle the railway track maintenance scheduling problem. Two objective functions have been considered (maintenance costs and train delays), as well as three sets of constraints that model safety limits and resources. The proposal includes a deterioration model based on exponential fitting and a two-level segmentation, that takes into account the variations in the deterioration curve caused by tamping and renewal operations. Two multi-objective algorithms (AMOSA and NSGA-II) have been applied to the problem, starting from an initial population of solutions generated heuristically according to expert knowledge.

The described approach has been tested over a model of a real railway from northern Sweden to generate a maintenance schedule for 3 years. Both algorithms have been run with equivalent parameters and started from the same initial population. Then, an additional set of experiments for longer horizons (namely 5, 10 and 20 years) has been performed.

As for the results obtained, AMOSA outperformed NSGA-II due to its stronger intensification strategy. Furthermore, both the Pareto front and the solution space explored by AMOSA showed that a wide range of solutions were analyzed, providing the decision maker with a fair variety of possible maintenance schedules. All the solutions provided in the Pareto front for the three years horizon were non-constrained, which stresses the adequacy of the proposed scheme. Moreover, the results obtained for long-term horizons show a very important decrease of the cost and delay, and this decrease is higher for more distant horizons, assessing the capabilities of the proposed scheme to schedule railway maintenance plans.

The main limitation of the proposal is the computational complexity of simulating of the degradation model for each generated schedule, which limits the number of evaluations that can be carried out during the optimization algorithm. Therefore, even though the obtained solutions were of very high quality, it would be of interest to develop new approaches that can make use of parallel computing infrastructures to solve this problem, which would allow us to deal with longer railways (which would have an impact on the dimensionality of the search space and the complexity of the problem). Another possibility of extending the work consists of considering more complex maintenance schedules, including availability of human and material resources and time slots.

ACKNOWLEDGMENTS

This work was supported by the projects OPTIRAIL (FP7-GA314031), P12-TIC-2958, TIN2013-47210-P, TIN2016-81113-R.

REFERENCES

Aminbakhsh, S. and Sonmez, R. (2017). "Pareto Front Particle Swarm Optimizer for Discrete Time-Cost Trade-Off Problem." *Journal of Computing in Civil Engineering*, 31(1), 04016040.

Andrade, A. R. and Teixeira, P. F. (2016). "Exploring Different Alert Limit Strategies in the Maintenance of Railway Track Geometry." *Journal of Transportation Engineering*, 142(9), 04016037.

Arasteh Khouy, I., Larsson-Kråik, P.-O., Nissen, A., Juntti, U., and Schunnesson, H. (2014). "Optimisation of track geometry inspection interval." *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 228(5), 546–556.

Audley, M. and Andrews, J. D. (2013). "The effects of tamping on railway track geometry degradation." *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 376–391.

Bandyopadhyay, S., Saha, S., Maulik, U., and Deb, K. (2008). "A simulated annealing-based multi-objective optimization algorithm: AMOSA." *IEEE Transactions on Evolutionary Computation*, 12(3), 269–283.

Blum, C. and Roli, A. (2003). "Metaheuristics in Combinatorial Optimization: Overview and Conceptual Comparison Metaheuristics in Combinatorial Optimization." *ACM Computing Surveys*, 35(3), 268–308.

Budai, G. (2009). "Operations research models for scheduling railway infrastructure maintenance." Ph.D. thesis, Erasmus University Rotterdam, Rotterdam, Nederlands.

Budai, G., Huisman, D., and Dekker, R. (2006). "Scheduling preventive railway maintenance activities." *Journal of the Operational Research Society*, 57(9), 1035–1044.

Caetano, L. F. and Fonseca, P. (2013). "Availability approach to optimizing railway track renewal operations." *Journal of Transportation Engineering*, 139(9), 941–948.

Caetano, L. F. and Teixeira, P. F. (2016). "Predictive Maintenance Model for Ballast Tamping." *Journal of Transportation Engineering*, 142(4), 04016006.

Cárdenas-Gallo, I., Sarmiento, C. A., Morales, G. A., Bolivar, M. A., and Akhavan-Tabatabaei, R. (2017). "An ensemble classifier to predict track geometry degradation." *Reliability Engineering & System Safety*, 161, 53–60.

Das, I. and Dennis, J. (1997). "A closer look at drawbacks of minimizing weighted sums of objectives for Pareto set generation in multicriteria optimization problems." *Structural Optimization*, 14(1), 63–69.

Deb, K. (2001). *Multi-objective optimization using evolutionary algorithms*. John Wiley & Sons.

Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T. (2002). "A fast and elitist multiobjective genetic algorithm: NSGA-II." *IEEE Transactions on Evolutionary Computation*, 6(2), 181–197.

El-Abbasy, M. S., Elazouni, A., and Zayed, T. (2017). "Generic Scheduling Optimization Model for Multiple Construction Projects." *Journal of Computing in Civil Engineering*, 31(4), 04017003.

Esveld, C. (2001). *Modern railway track*. MRT-productions Zaltbommel.

European Committee for Standardization (2010). "European Standard EN 13848-5: Railway applications: Track - Track geometry quality - Part 5: Geometric quality levels.

Ferreira, P. and López-Pita, A. (2015). "Numerical modelling of high speed train/track system for the reduction of vibration levels and maintenance needs of railway tracks." *Construction and*

Building Materials, 79, 14–21.

Garey, M. R. and Johnson, D. S. (1979). *Computers and intractability : a guide to the theory of NP-completeness*. W.H. Freeman.

Guler, H. (2013). “Decision support system for railway track maintenance and renewal management.” *Journal of Computing in Civil Engineering*, 27(3), 292–306.

Gustavsson, E. (2015). “Scheduling tamping operations on railway tracks using mixed integer linear programming.” *EURO Journal on Transportation and Logistics*, 4, 97–112.

Higgins, A., Kozan, E., and Ferreira, L. (1996). “Optimal scheduling of trains on a single line track.” *Transportation Research Part B: Methodological*, 30(2), 147–161.

Hummitzsch, R. (2009). “For the Predictability of Track Quality Behaviour - Statistical Analysis of Track Behaviour for the Creation of a Prediction Model.” Ph.D. thesis, Technische Universität Graz, Graz, Austria.

Jovanovic, S. (2004). “Railway track quality assessment and related decision making.” *IEEE International Conference on Systems, Man and Cybernetics*, Vol. 6, IEEE, 5038–5043.

Kirkpatrick, S., Gelatt Jr., C. D., and Vecchi, M. P. (1983). “Optimization by simulated annealing.” *Science*, 220(4598), 671–680.

Knowles, J. D. and Corne, D. W. (2000). “Approximating the nondominated front using the Pareto archived evolution strategy.” *Evolutionary computation*, 8(2), 149–172.

Kong, J. S. and Frangopol, D. M. (2003). “Evaluation of expected life-cycle maintenance cost of deteriorating structures.” *Journal of Structural Engineering*, 129(5), 682–691.

Lévi, D. (2001). “Optimization of track renewal policy.” *World Congress on Railway Research, Cologne*.

Lidén, T. and Joborn, M. (2017). “An optimization model for integrated planning of railway traffic and network maintenance.” *Transportation Research Part C: Emerging Technologies*, 74, 327–347.

Luan, X., Miao, J., Meng, L., Corman, F., and Lodewijks, G. (2017). “Integrated optimization on train scheduling and preventive maintenance time slots planning.” *Transportation Research Part*

C: Emerging Technologies, 80, 329–359.

Macedo, R., Benmansour, R., Artiba, A., Mladenović, N., and Urošević, D. (2017). "Scheduling preventive railway maintenance activities with resource constraints." *Electronic Notes in Discrete Mathematics*, 58, 215–222.

Nguyen, K., Villalmanzo, D. I., Goicolea, J. M., and Gabaldon, F. (2016). "A computational procedure for prediction of ballasted track profile degradation under railway traffic loading." *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 230(8), 1812–1827.

Patra, A. P., Söderholm, P., and Kumar, U. (2009). "Uncertainty estimation in railway track life-cycle cost: A case study from Swedish National Rail Administration." *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 223(3), 285–293.

Podofillini, L., Zio, E., and Vatn, J. (2006). "Risk-informed optimisation of railway tracks inspection and maintenance procedures." *Reliability Engineering and System Safety*, 91(1), 20–35.

Ramos, A. and Fonseca, P. (2011a). "Biobjective optimization model for maintenance and renewal decisions related to rail track geometry." *Transportation Research Record*, 2261, 163–170.

Ramos, A. and Fonseca, P. (2011b). "Uncertainty in rail-track geometry degradation: Lisbon-Oporto line case study." *Journal of Transportation Engineering*, 137(3), 193–200.

Ramos, A. and Fonseca, P. (2013). "Hierarchical Bayesian modeling of rail track geometry degradation." *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 364–375.

Vale, C. and Ribeiro, I. M. (2014). "Railway condition-based maintenance model with stochastic deterioration." *Journal of Civil Engineering and Management*, 20(5), 686–692.

Wen, M., Li, R., and Salling, K. (2016). "Optimization of preventive condition-based tamping for railway tracks." *European Journal of Operational Research*, 252(2), 455–465.

Xu, P., Sun, Q., Liu, R., Souleyrette, R. R., and Wang, F. (2015). "Optimizing the Alignment of Inspection Data from Track Geometry Cars." *Computer-Aided Civil and Infrastructure Engineering*, 30(1), 19–35.

Zhao, J., Chan, A. H. C., Stirling, A. B., and Madelin, K. B. (2006). "Optimizing policies of railway ballast tamping and renewal." *Transportation Research Record*, 1943, 50–56.

List of Tables

TABLE 1. Maintenance operations and their triggers

Maintenance operation	Trigger
Rail grinding Rail lubrication Track inspection	Time
Tamping Ballast cleaning Rail renewal Ballast renewal Sleeper renewal Fasteners renewal	Condition
Rail replacement	Failure

TABLE 2. Maximum speed and minimum quality values according to EN13848-5

Standard deviation in longitudinal level D1 (mm)	Speed (km/h)	L_{QN3}
2.3 to 3.0	$s \leq 80$	3.1
1.8 to 2.7	$80 < s \leq 120$	2.7
1.4 to 2.4	$120 < s \leq 160$	2.2
1.2 to 1.9	$160 < s \leq 230$	2.0
1.0 to 1.5	$230 < s \leq 300$	1.7

TABLE 3. Parameters for the optimization algorithms

Algorithm	Parameter	Value
NSGA-II	Size of the population	104
	Number of generations	4810
	Crossover probability	0.6
	Mutation probability	0.3
AMOSA	HL	104
	SL	104
	γ	1
	α	0.9183544
	max_{HC}	20
	Initial temperature	500
	Minimum temperature	0.1
	Iterations per temperature	5000

TABLE 4. Parameters concerning the considered track

Parameter	Description	Value
C_t	Tamping cost	10
C_r	Renewal cost	150
max_t	Maximum tamping (meters)	5100
max_r	Maximum renewal (meters)	12 000
s_t^{mean}	Average speed of train	Between 60 and 135
r	Discount rate	0.03
N_s	Number of sections	24
N_g	Number of segments	1435

TABLE 5. Parameters concerning the solutions

Parameter	Description	Horizon			
		3 years	5 years	10 years	20 years
N_k	Number of trimesters	12	20	40	80
$N_g N_k$	Length of **x**	17 220	28 700	57 400	114 800
$N_s N_k$	Length of **y**	288	480	960	1920
$N_k(N_s + N_g)$	Length of the solution	17 508	29 180	58 360	116 720

TABLE 6. Summary of the Pareto front obtained by NSGA-II

Cost (€)	Delay (hours)	Tampings	Renewals
6 215 034	334.01	501	9
6 224 578	263.46	517	9
6 235 872	239.43	536	9
6 251 445	212.07	551	9
6 259 503	207.73	568	9
6 259 854	178.66	556	9
6 287 670	172.05	608	9
6 293 251	164.96	601	9
6 299 343	159.31	618	9
6 305 647	158.21	629	9
6 305 650	148.29	631	9
6 318 460	148.12	648	9
6 326 540	138.54	667	9
6 329 879	127.06	672	9
6 345 281	121.64	695	9
6 362 485	100.86	725	9

TABLE 7. Summary of the Pareto front obtained by AMOSA

Cost (€)	Delay (hours)	Tampings	Renewals
5 689 494	137.98	665	8
5 690 557	126.63	667	8
6 286 985	126.63	623	9
6 289 973	126.14	625	9
6 293 171	119.84	630	9
6 293 924	118.34	630	9
6 297 452	116.68	636	9
6 299 088	113.50	640	9
6 321 768	110.84	663	9
6 323 540	110.83	666	9
6 325 447	109.16	670	9
6 325 871	103.03	669	9
6 326 664	100.72	668	9
6 335 103	99.06	680	9
6 339 205	99.03	689	9
6 343 664	96.25	691	9
6 345 849	93.09	694	9
6 347 311	91.43	696	9
6 350 716	89.45	699	9
6 351 128	86.29	702	9
6 364 458	86.26	723	9
6 364 458	86.26	724	9
6 369 357	84.75	729	9
6 382 535	84.68	758	9
6 393 818	81.53	774	9
6 397 428	78.81	780	9
6 996 854	72.01	839	10
7 006 133	68.85	852	10
7 007 671	62.06	855	10
7 007 671	62.06	856	10

List of Figures

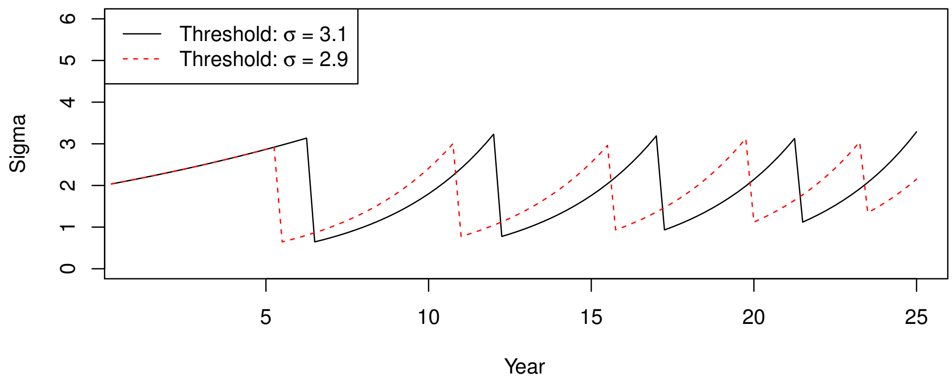

Fig. 1. Example of quality simulation with the deterioration model. Both lines simulate the same segment, with a slightly different quality threshold for tamping.

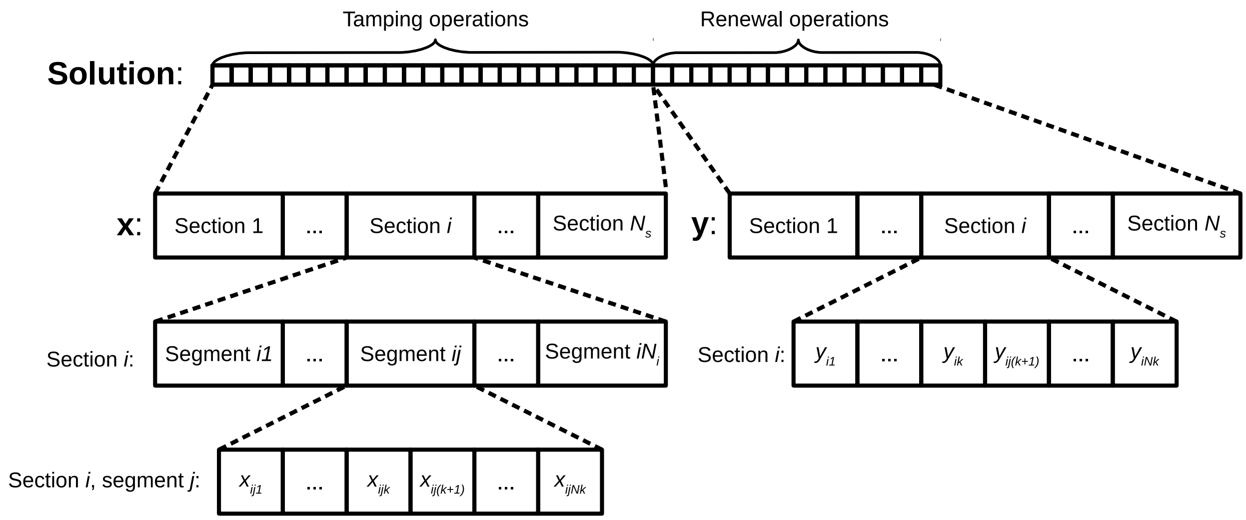

Fig. 2. Representation of a single solution to the maintenance scheduling problem.

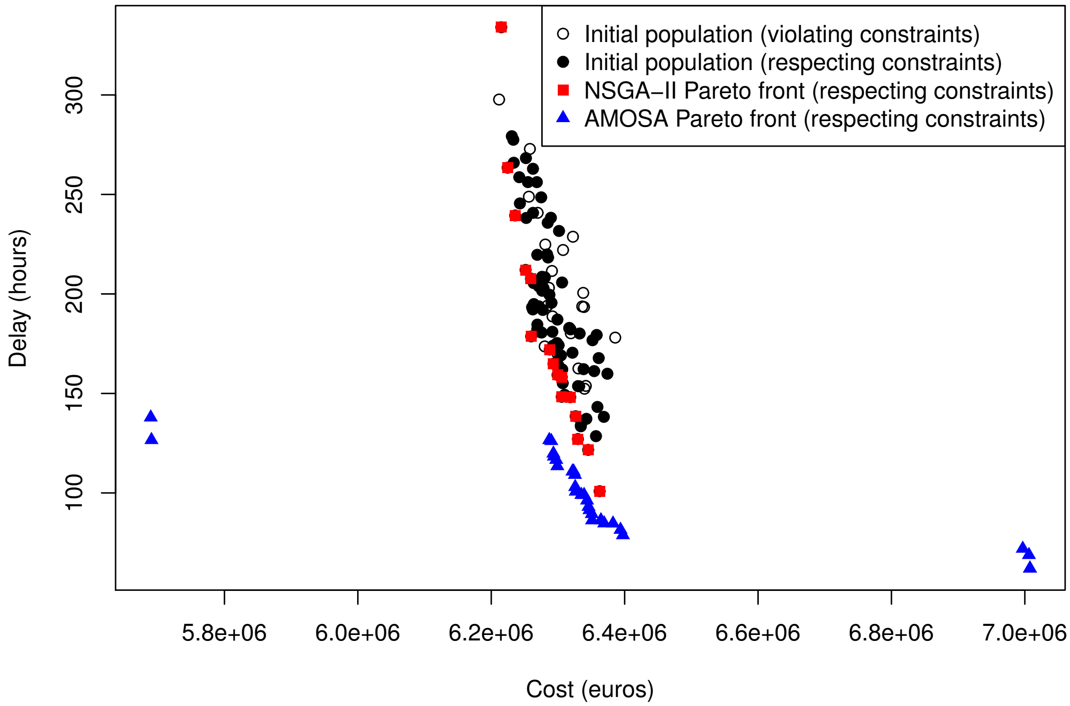

Fig. 3. Initial population and Pareto fronts of NSGA-II and AMOSA

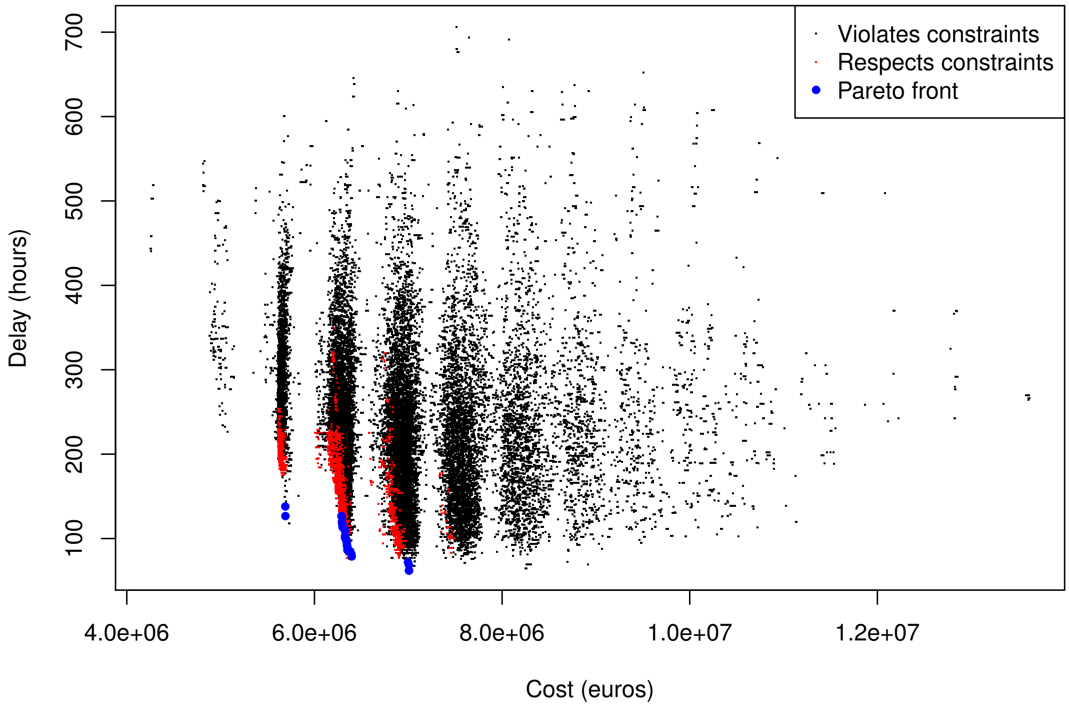

Fig. 4. All the solutions generated by AMOSA

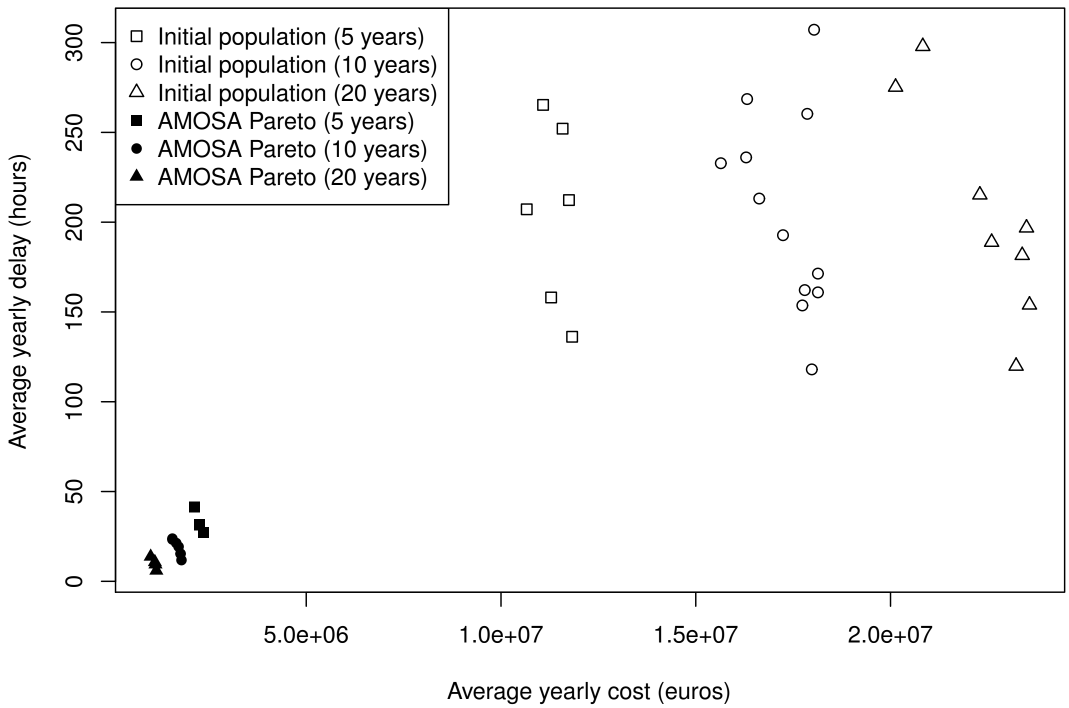

Fig. 5. Feasible solutions of the initial populations and AMOSA Pareto fronts for the three long-term horizons tested.