

Секция 4 – Проблемы надежности машиностроения  
и машиностроительные технологии

## KINEMATICAL AND POWER FLOW ANALYSIS OF THE NUTATION SPEED REDUCER

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Nutation speed reducer with face teeth is a totally innovative gear transmission developed in recent years [1,2], which bases its functioning on the mathematical concept of nutation motion, that is the main feature that characterizes nutation gear from conventional gear. It has unique features which include high reduction ratio in one stage, very high profile contact ratio and an overall higher power density transmission. In addition, the contact is theoretically pure rolling in the gear, the heat generated should be lower and the efficiency higher.

The Nutation Speed Reducer is illustrated in Figure 1. The component names illustrated in Figure 1 are (1) an input shaft which drives the (3) nutator gear via bearings, and meshes with (2) the stator gear and drives (4) the rotor gear and (5) output shaft.

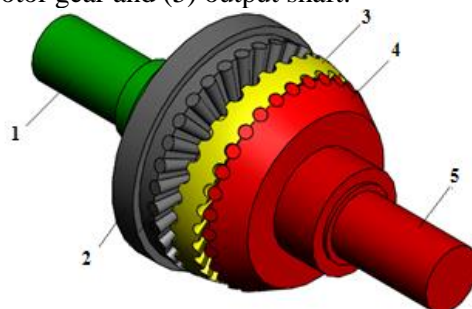


Figure 1 - The Nutation Speed Reducer

The nutator is mounted to the input shaft at a specified nutation angle (Typical range of the nutation angle  $\beta$  is 2-5° from vertical direction) [3]. The nutator design embodies back-to-back face-gears. The numbers of teeth on the stator gear 2 and rotor gear 4 is  $Z_1$ ,  $Z_4$  respectively. The numbers of teeth on the nutator gear 3 meshing with stator gear is  $Z_2$ . The numbers of teeth on the nutator gear 3 meshing with rotor gear is  $Z_3$ .

Due to the mounting arrangement, the nutator gear teeth on both sides are always in mesh with both the stator and the rotor gears. The meshing between teeth 2, 3 prevents the nutator gear 3 from rotating during nutation, so that the meshing between teeth 4, 3 is affected solely by means of the meshing between the nutator gear 3 and rotor gear 4. The nutator gears simultaneously share the tooth loads with high contact ratios as a result of its unique assembly [4]

As illustrated in Figure 2, input shaft, stator gear and rotor gear rotate about the Z-axis of the global fixed-coordinate frame  $S_f$ . The motion of the nutator gear consists of two motions. The first motion is with the rotation of the nutator gear with the input shaft about the Z axis. The second motion is a rotation about the  $z_p$  axis (nutational motion). This second rotation is caused by the meshing engagement with the stator gear. The  $z_p$  axis is fixed to the input shaft; thus, it rotates about the Z axis.

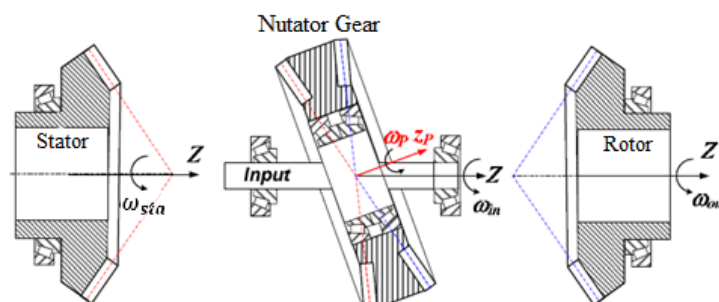


Figure 2 – Motion of the nutator gear

Coordinate relations of the  $S_f$  and  $S_p$  are illustrated in Figure 3. The  $S_f$  and  $S_p$  coordinate frames are coincident at the common origins  $O_f$  and  $O_p$ . The directions of the frames are set according to the right hand rule.

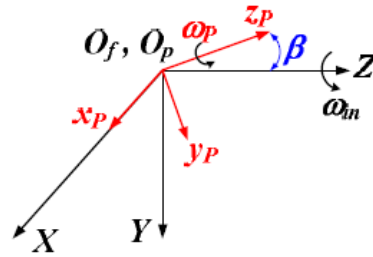


Figure 3 – Coordinate relations of the nutator gear and global frame  
The angular velocity vector of the input shaft is:

$$\vec{\omega}_{in} = \omega_{in} \cdot \vec{K} = \begin{bmatrix} 0 \\ 0 \\ \omega_{in} \end{bmatrix} \quad (1)$$

Where the symbol  $\vec{K}$  presents unit vector in the Z direction of the  $S_f$  coordinate.

The stator gear angular velocity is:

$$\vec{\omega}_{sta} = \omega_{sta} \cdot \vec{K} = \begin{bmatrix} 0 \\ 0 \\ \omega_{sta} \end{bmatrix} \quad (2)$$

where  $\omega_{sta}$  is the angular speed of the stator gear.

Lastly, the rotor gear angular velocity is:

$$\vec{\omega}_{out} = \omega_{out} \cdot \vec{K} = \begin{bmatrix} 0 \\ 0 \\ \omega_{out} \end{bmatrix} \quad (3)$$

The motion of the nutator gear member is in two components as explained before. It is:

$$\vec{\omega}_{PMC} = \omega_{in} \cdot \vec{K} + \omega_p \cdot \vec{k} \quad (4)$$

One of the significant characteristics of the Nutation Speed Reducer is the overall speed reduction ratio ( $i_v$ ). It is the ratio of the input speed to the output speed. The reduction ratio is presented as:

$$i_v = \frac{\omega_{in}}{\omega_{out}} = \frac{1}{1 - \frac{Z_1 \cdot Z_3}{Z_2 \cdot Z_4}} \quad (5)$$

#### References

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