Trading Costs and Informational Efficiency*

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Abstract

We study the effect of trading costs on information aggregation and acquisition in financial markets. For a given precision of investors' private information, an irrelevance result emerges when investors are ex-ante identical: price informativeness is independent of the level of trading costs. This result holds for quadratic, linear, and fixed trading costs in competitive and strategic environments. When investors are ex-ante heterogeneous, trading costs reduce (increase) price informativeness if and only if investors who disproportionately trade on information are more (less) elastic than investors who mostly trade on hedging. Through a reduction in information acquisition, trading costs reduce price informativeness.

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1 Introduction

Technological advances have dramatically reduced the cost of trading in financial markets. However, has this reduction in trading costs made financial markets better at aggregating information? Has the ability to trade more cheaply encouraged information acquisition in financial markets? More broadly, what are the implications of changes in trading costs for the aggregation and generation of information in financial markets? In this paper, we seek to provide an answer to these questions by systematically studying the implications of trading costs for information aggregation and information acquisition in financial markets.

In our model, investors trade for two reasons. They trade on private information, after receiving a private signal about asset payoffs, and due to a privately known hedging demand, which is stochastic and uncertain in the aggregate. The combination of trading based on private information and the aggregate uncertainty in hedging motives makes prices only partially informative. This forces investors – or any interested external observer – to solve a filtering problem to recover the information about asset payoffs aggregated by asset prices. Using this framework as the core building block, in the spirit of Modigliani and Miller (1958), we structure our paper around several irrelevance results that emerge in different canonical models of financial trading.

Our first main result is an irrelevance theorem that applies to competitive economies with ex-ante identical investors. We show that, for a given precision of investors' private signals, price informativeness is independent of the level of trading costs. The logic behind our main result is elementary and intuitive. The effect of trading costs on how prices aggregate information is a function of how the relevant signal-to-noise ratio contained in asset prices is affected. For example, an increase in trading costs necessarily reduces the amount of trading due to information motives, reducing the informational content of prices. However, this same increase in trading costs also reduces trading due to hedging needs, reducing the noise component of asset prices. When investors are ex-ante identical, the ratio of these trading motives – which becomes the relevant signal-to-noise ratio of the economy – remains constant as trading costs change. This is the logic that underlies our irrelevance results.

Our second set of results illustrates how specific forms of heterogeneity break our irrelevance result. We show that only when investors who disproportionally trade on information are more price sensitive than investors who disproportionally trade for hedging reasons, do we expect prices to become less informative when trading costs are higher and vice versa. Formally, we allow for heterogeneity in the precision of the private signals about the fundamental, in the variance of hedging needs, and in investors' risk aversion. First, we show that all three sources of heterogeneity, in isolation, are associated with a reduction in price informativeness when trading costs increase. This result arises because investors with more precise information, either about the fundamental or the aggregate hedging, or with relatively high risk tolerance, trade more aggressively in general and react more to trading costs, while putting more weight on their private signal about the fundamental and contributing relatively more information to the price. Next, we formally establish that for most combinations of two-dimensional heterogeneity across the three sources of heterogeneity that we consider, an increase in trading costs is associated with a reduction in price informativeness. Intuitively, only very specific forms of two-dimensional heterogeneity are able to overturn the one-dimensional result. Finally, we provide a characterization of the effect of trading costs on price informativeness in terms of demand sensitivities to private signals and hedging needs that applies to multidimensional forms of heterogeneity. This characterization illustrates the intuition behind the economic mechanisms that drive the results.

Since our model with heterogeneous investors nests the classic noise trading formulation, we are able to highlight the importance of how economic "noise" is modeled when studying information aggregation.[1] Classic noise trading, as in Grossman and Stiglitz (1980), is often modeled as an exogenous stochastic demand or supply shock and it is often justified as standing in for hedging needs of unmodeled traders. Although a classic noise trading formulation may be a useful shortcut at times, it is not satisfactory when we seek to understand the effects of trading costs on price informativeness: it is silent on how noise traders react to changes in the level of trading costs, a form of Lucas (1976) Critique.

Subsequently, we allow investors to choose the precision of their private signal about the fundamental. In our benchmark model with ex-ante identical investors, we show that an increase in trading costs endogenously reduces the precision of the signal about the fundamental chosen by investors. Intuitively, high trading costs make it harder for a given investor to profit from acquiring private information. Since the investors anticipate that they will be able to profit less from having better information, they choose less precise signals, which reduces equilibrium price informativeness.[2] We can draw two conclusions from this exercise. First, trading costs have sharply different implications for information aggregation and information acquisition. Second, trading costs tend to reduce the endogenous precision of signals about the fundamental, decreasing equilibrium price informativeness.

We return to the benchmark model without information acquisition and show that our irrelevance theorem extends to economies with a) alternative forms of trading costs, b) random heterogeneous priors as a source of aggregate uncertainty, and c) strategic investors. First, we show that our irrelevance result continues to hold when investors face linear trading costs or fixed trading costs, instead of quadratic, the sustained assumption in most of the paper. Second, we allow investors to have stochastic privately known heterogeneous priors, which are random in the aggregate. This shows that our irrelevance result is robust to having other sources of aggregate uncertainty, in addition to hedging. Third, we show that changes in trading costs in economies in which investors' strategic behavior matters (for instance, when there is a finite number of investors) do not affect the level of price informativeness. Strategic behavior changes the trading sensitivities of investors, but it does so symmetrically. Therefore, the logic underlying the results in the competitive model with a continuum of investors still applies.

[1] A consequence of modeling aggregate noise from first principles is that our model features multiple equilibria. Our formulation, similar but not identical to the one used in Ganguli and Yang (2009) and Manzano and Vives (2011), who also find multiple equilibria, guarantees equilibrium existence for any set of primitives. Note that our irrelevance result remains valid under their formulation.

[2] In the Online Appendix, we extend the model by allowing investors to choose the precision of a private signal about the aggregate hedging need (noise). We show that investors also choose less precise signals about the noise when they face higher trading costs. Less precise signals on the noise also reduce the equilibrium level of price informativeness.

In our final extension, we consider a model in which investors have general preferences and signals.[3] We show that the condition for the irrelevance result to hold when investors are ex-ante identical in a symmetric equilibrium is that the average *ex-post* demand sensitivities to information and noise (hedging needs) react identically to a change in trading costs. This result shows that the forces behind our irrelevance argument apply generally.

In addition to improving the understanding of whether the secular trend of reduction in trading costs has affected the role played by financial markets in aggregating information, our results have important practical implications for the broader discussion on the effect of transaction taxes as a policy instrument. It is somewhat surprising that our irrelevance results and our directional result in the model with endogenous information acquisition have been absent from policy discussions. Stiglitz (1989) and Summers and Summers (1989) are good examples of policy-oriented articles which would have benefited from using the results of this paper as a benchmark for policy analysis.

Like other irrelevance results in finance and economics, e.g., Modigliani and Miller (1958), Barro (1974), Wallace (1981), Krueger and Lustig (2010), our irrelevance results are pedagogical in nature. We do not argue that changes in trading costs do not affect price informativeness in practice.[4] Our main contribution is identifying the set of assumptions (investor homogeneity and exogenous information precision) that must be violated for trading costs to affect price informativeness, as well as those that are irrelevant (among others, the form of trading costs or the presence of market power). Our results allow us to shed light on how different forms of investor heterogeneity affect the relation between trading costs and price informativeness.

Related Literature

This paper lies at the intersection of two major strands of literature. On the one hand, we share the emphasis of the work that studies the role played by financial markets in aggregating and originating information, following Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981). From a modeling perspective, our benchmark formulation with a continuum of investors is closest to the large economy model in Admati (1985). Investors in our model have private information about both the fundamental and the noise contained in the price. The existence and multiplicity properties of the equilibria in related – but not identical – setups have been studied by Ganguli and Yang (2009) and by Manzano and Vives (2011). In contrast to these papers, the noise structure we assume in our model guarantees that an equilibrium always exists. In our model, aggregate hedging needs are stochastic and not observable, similar to Manzano and Vives (2011) and Hatchondo, Krusell and Schneider (2014). Goldstein, Li and Yang (2014) find that multiple equilibria may arise when market segmentation leads to heterogeneous hedging needs. Our result in the case of general

[3] We consider a CARA-Gaussian setup for most of the paper. Therefore, our results should be interpreted as a first-order approximation to more general environments (Ingersoll (1987), Huang and Litzenberger (1988)).

[4] For instance, our results associating a reduction in trading costs with an increase in information acquisition can be used to rationalize the rise in the share of trading-type financial activities in aggregate GDP since the mid-1970's, as documented by Philippon (2015) and Greenwood and Scharfstein (2013).

preferences and noise structure relates to the work of Barlevy and Veronesi (2000), Yuan (2005), Albagli, Tsyvinski and Hellwig (2012), Breon-Drish (2015), and Chabakauri, Yuan and Zachariadis (2015), which are relevant examples of the growing literature that explores information aggregation and acquisition in alternative environments to the canonical CARA-Gaussian model.

Our results on endogenous information acquisition are related to the large literature that follows Verrecchia (1982) and Kyle (1989). See Biais, Glosten and Spatt (2005), Vives (2008), Veldkamp (2009) for recent thorough reviews of this line of work. We first allow investors to acquire information about the fundamental as in Hellwig and Veldkamp (2009), Van Nieuwerburgh and Veldkamp (2010), and Manzano and Vives (2011). Ganguli and Yang (2009) and Farboodi and Veldkamp (2016) study the choice of whether to acquire information about fundamental or non-fundamental variables. These papers abstract from modeling trading costs, which is the focus of our paper.

On the other hand, our results also relate to the body of literature that studies the effects of transaction costs/taxes on financial markets, following Constantinides (1986) and Amihud and Mendelson (1986). More recent contributions are Vayanos (1998), Vayanos and Vila (1999), Gârleanu and Pedersen (2013), Abel, Eberly and Panageas (2013), and Gârleanu, Panageas and Yu (2014). These papers focus on the implications of trading costs for volume or prices, while we focus on the effects on information aggregation and information acquisition. We refer the reader to Vayanos and Wang (2012) for a recent survey of this vast literature.

Only a handful of papers feature both *technological* trading costs and learning, as ours. Vives (2016) shows in a linear-quadratic market game that introducing a quadratic trading cost can be welfare improving by reducing the degree of private information acquisition. Subrahmanyam (1998) and Dow and Rahi (2000) discuss the effect of quadratic trading costs in models of trading with strategic agents. The inherent asymmetry among investors embedded in these papers explains their findings regarding the effects of trading costs. Budish, Cramton and Shim (2015) show that a tax on trading is a coarse instrument to reduce high frequency trading in a model with learning. In the context of a model of bilateral trading with information acquisition but without information aggregation, Dang and Morath (2015) compare profit and transaction taxes.

Finally, our paper is related to the body of work that studies whether structural changes in the financial industry, as those motivated by the reduction in the cost of trading, have affected the role played by financial markets in modern economies. Greenwood and Shleifer (2013), Philippon (2015), Bai, Philippon and Savov (2015), and Turley (2012) document and interpret these trends, explaining the forces behind them.

Outline Section 2 describes the benchmark model and Section 3 characterizes the equilibrium of the model for the cases with ex-ante identical and ex-ante heterogeneous investors. Section 4 illustrates how to break the main irrelevance result by varying the form of ex-ante heterogeneity. Section 5 allows for endogenous information acquisition and Section 6 extends the irrelevance results to the cases with linear and fixed trading costs, random heterogeneous priors, strategic investors, and general utility and signal structure. Section 7 concludes. The Appendix contains derivations and proofs. The Online Appendix contains additional derivations and results.

2 Benchmark model: competitive investors with trading costs

As a benchmark, we initially study a competitive model of trading in financial markets with rational investors who receive private signals about asset payoffs and have stochastic hedging needs. Within this canonical framework, we characterize the conditions under which trading costs affect price informativeness. In Section 6, we extend our model in multiple dimensions.

Preferences There are two dates $t = 1, 2$ and a unit measure of investors, indexed by i. Investors choose their portfolio allocation at date 1 and consume at date 2. They maximize constant absolute risk aversion (CARA) expected utility. Therefore, expected utility of investor i is given by

$$\mathbb{E}\left[U_i\left(w_{2i}\right)\right] \quad \text{with} \quad U_i\left(w_{2i}\right) = -e^{-\gamma_i w_{2i}}, \tag{1}$$

where Eq. (1) imposes that investors consume all their terminal wealth w_{2i}. The parameter $\gamma_i > 0$ represents the coefficient of absolute risk aversion $\gamma_i \equiv -\frac{U_i''}{U_i'}$.

Investment opportunities There are two assets in the economy, a riskless asset and a risky asset. The riskless asset is in elastic supply and pays a gross interest rate R. Without loss of generality, we normalize R to 1. The risky asset is in exogenously fixed supply $Q \geq 0$. This asset is traded in a competitive market at date 1 at price p. This price is quoted in terms of an underlying consumption good (dollar), which acts as numeraire. Each investor i is endowed with q_{0i} units of the risky asset at date 1, where $\int q_{0i} di = Q$, since investors must hold as a whole the total supply of the asset Q. Similarly, market clearing at date 1 implies that $\int q_{1i} di = Q$, where q_{1i} denotes investor i's final holdings of the risky asset. Investors face no constraints when choosing portfolios: they can borrow and short sell freely.

The per unit asset payoff at date 2 is normally distributed and denoted by θ, where

$$\theta \sim N\left(\overline{\theta}, \tau_\theta^{-1}\right). \tag{2}$$

This formulation implies that there is aggregate uncertainty about the expected asset payoff. The unconditional expected asset payoff is given by the constant $\overline{\theta} \geq 0$, while its precision (the inverse of its variance) corresponds to τ_θ.

Hedging needs Every investor i has a stochastic endowment of the consumption good at date 2, denoted by n_{2i}. This random endowment is normally distributed and potentially correlated with the risky asset payoff θ.[5] This endowment captures the fundamental risks associated with each individual

[5]Formally, each investor i draws a random pair $(h_i, \mathbb{V}ar\left[n_{2i}\right])$ that characterizes the joint normal distribution of his endowment process n_{2i} and the risky asset payoff θ, where $\mathbb{V}ar\left[n_{2i}\right]$ is such that the variance-covariance matrix of the joint distribution is positive semi-definite. To be more specific, each investor i draws a random pair $(h_i, \mathbb{V}ar\left[n_{2i}\right])$ that characterizes the joint normal distribution of the endowment process n_{2i} and the risky asset payoff θ, where $\begin{pmatrix} n_{2i} \\ \theta \end{pmatrix} \sim N\left(\begin{pmatrix} \mathbb{E}\left[n_{2i}\right] \\ 0 \end{pmatrix}, \Sigma^i\right)$ with $\Sigma^i = \begin{bmatrix} \mathbb{V}ar\left[n_{2i}\right] & h_i \\ h_i & \tau_{hi}^{-1} + \tau_\delta^{-1} \end{bmatrix}$ and the restriction that, given h_i, $\mathbb{V}ar\left[n_{2i}\right]$ is such that Σ^i is positive semi-definite.

investor's normal economic activity. The covariance $h_i \equiv \mathbb{C}ov\left[n_{2i}, \theta\right]$ determines whether the risky asset is a good hedge for investor i (if $h_i < 0$) or not (if $h_i > 0$). More specifically, we assume that the conditional covariance between the consumption good endowment and the asset payoff is constant and equal to h_i.[6] At the beginning of date 1, before trading, every investor i learns the realization of his individual hedging needs h_i, given by

$$h_i = \delta + \varepsilon_{hi}, \tag{3}$$

where

$$\delta \sim N\left(0, \tau_\delta^{-1}\right) \qquad \text{and} \qquad \varepsilon_{hi} \sim N\left(0, \tau_{hi}^{-1}\right), \tag{4}$$

and the realizations of ε_{hi} are independent across investors. This formulation implies that there is uncertainty about the aggregate magnitude of hedging needs δ. The expected level of total hedging needs is zero. Without loss of generality, we normalize the initial endowment n_{1i} to zero for all investors and assume that $\mathbb{E}\left[n_{2i}\right] - \frac{\gamma_i}{2}\mathbb{V}ar\left[n_{2i}\right] = 0$.

Information structure Investors do not observe the actual realization of the risky asset payoff, θ. However, every investor observes a private signal s_i about the asset payoff θ, with the following structure

$$s_i = \theta + \varepsilon_{si},$$

where

$$\varepsilon_{si} \sim N\left(0, \tau_{si}^{-1}\right).$$

The realizations of ε_{si} are independent across investors. In principle, we allow for the precision of the private signal to be different for each investor. For now, we take the precisions of investors' private signals $\left\{\tau_{si}\right\}_i$ as a primitive of the economy.

Investors do not observe the aggregate hedging needs in the economy either. Investors only observe their own realization of the hedging need, that is, h_i is private information of investor i. Given the formulation of h_i in Eq. (3), h_i contains information about the aggregate hedging need δ.

Trading costs Investors face quadratic trading costs. In particular, a change in the asset holdings of the risky asset $|q_{1i} - q_{0i}|$ incurs a trading cost, in terms of the numeraire, due at the same time the transaction occurs, for both the buyer and the seller of

$$\frac{c}{2}\left(\Delta q_{1i}\right)^2,$$

where $\Delta q_{1i} \equiv q_{1i} - q_{0i}$. We model trading costs as quadratic in the size of the trade to preserve tractability.[7] Whether c corresponds to the use of economic resources (a trading cost) – our sustained

[6] This formulation is isomorphic to assuming that n_{2i} covaries only with an unlearnable component of the asset payoff. To avoid introducing additional notation, we relegate the specifics of this formulation to the Appendix. This assumption is only important to guarantee existence of equilibrium but not to attain our main results. We thank our discussant Liyan Yang for this suggestion.

[7] Our results easily extend to the case of trading costs that are proportional to the asset price level, as in $\frac{c}{2}p\left(\Delta q_{1i}\right)^2$. We show in Section 6.1 that our irrelevance results also extend to the cases of linear and fixed costs.

assumption – or whether it corresponds to a transfer (a transaction tax) is irrelevant for every positive result in this paper.

The consumption/wealth of a given investor i at $t = 2$ is given by his stochastic endowment n_{2i}, the stochastic payoff of his asset holdings $q_{1i}\theta$, and the return on the investment in the riskless asset. This includes the net purchase or sale of the risky asset $(q_{0i} - q_{1i})\, p$ and the total trading cost $-\frac{c}{2}(\Delta q_{1i})^2$. Formally, the final wealth of investor i is

$$w_{2i} = n_{2i} + q_{1i}\theta + q_{0i}p - q_{1i}p - \frac{c}{2}(\Delta q_{1i})^2 . \tag{5}$$

Remark. There are four relevant dimensions of ex-ante heterogeneity among investors.

Ex-ante, investors can have different risk aversion γ_i, different initial asset holdings q_{0i}, different precision of their hedging needs τ_{hi}, and different precision of their private signals τ_{si}. Ex-post, they also differ in the realizations of their hedging needs h_i and their signal s_i, which are stochastic.

Remark. Aggregate uncertainty on the level of stochastic hedging needs make the filtering problem non-trivial, given that there are no exogenous noise traders in the model.

The presence of aggregate stochastic hedging needs make the filtering problem non-trivial: if $\tau_\delta \to \infty$, the equilibrium of the economy becomes fully revealing. In order to have a meaningful filtering problem, many papers studying learning introduce an unmodeled stochastic demand shock or, equivalently, a shock to the number of shares available: this modeling approach is often referred to as having "noise traders". Allowing for noise traders in its standard form – as in Grossman and Stiglitz (1980) – is not appropriate to study the effects of trading costs. In particular, in those models it is hard to understand how the behavior of noise traders varies with the level of trading costs: this is a form of Lucas (1976) Critique. Our theoretical results allow us to elaborate on this remark, which we do at the end of Section 3.

3 Equilibrium

We restrict our attention to rational expectations equilibria in which net demands are linear in the investor's private signal, his private hedging needs, and the price.

Definition. (Equilibrium) A rational expectations equilibrium in linear strategies consists of a linear net portfolio demand Δq_{1i} for each every investor i and a price function p such that: a) each investor i chooses Δq_{1i} to maximize his expected utility subject to his budget constraint and given his information set and b) the price function p is such the market for the risky asset clears, that is $\int \Delta q_{1i} di = 0$.[8]

To characterize the equilibrium, we first study the portfolio problem of an individual investor i. Subsequently, we study the equilibrium of the model with ex-ante identical investors, which allows us to introduce our first irrelevance result. Finally, we characterize the equilibrium of the model in the general

[8] Because we adopt a formulation with a continuum of investors, as Admati (1985), our investors do not suffer from the schizophrenia critique of Hellwig (1980).

case with ex-ante heterogeneous investors and qualify the conditions under which trading costs affect price informativeness and the direction of these effects.

Investors' portfolio choice

Because of the CARA-Gaussian structure of preferences and returns, the demand for the risky asset of every investor i is given by the solution to a mean-variance problem in q_{1i}. Note that an investor i knows the actual realization of his hedging needs when trading, although that realization is not known to other investors. In particular, investor i chooses q_{1i} to solve

$$\max_{q_{1i}} \left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma_i h_i - p\right) q_{1i} - \frac{\gamma_i}{2}\mathbb{V}ar\left[\theta|s_i,h_i,p\right] q_{1i}^2 - \frac{c}{2}\left(\Delta q_{1i}\right)^2. \tag{6}$$

The first term in the objective function of investor i represents the expected payoff of holding q_{1i} units of the risky asset. This expected payoff increases with the investor's expected value of the fundamental, $\mathbb{E}\left[\theta|s_i,h_i,p\right]$, decreases with the level of his realized hedging needs, h_i, and decreases with the price he has to pay for the risky asset, p. The second term captures the utility loss suffered by a risk-averse investor who faces uncertainty about the asset payoff. The last term represents the trading cost the investor must pay to adjust his asset holdings from q_{0i} to q_{1i}.

The first order condition of the problem stated in (6) yields the following demand for the risky asset

$$q_{1i} = \frac{\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma_i h_i - p + c q_{0i}}{\gamma_i \mathbb{V}ar\left[\theta|s_i,h_i,p\right] + c}. \tag{7}$$

Intuitively, investor i demands more shares of the risky asset when the expected asset payoff $\mathbb{E}\left[\theta|s_i,h_i,p\right]$ is high, when the risky asset is a good hedge ($h_i < 0$), when the price of the risky asset is low, and when the variance of risky asset $\mathbb{V}ar\left[\theta|s_i,h_i,p\right]$ is low. More risk averse investors demand fewer shares of the risky asset.

To interpret the effect of trading costs on the investors' demands more easily, we rewrite the investors' optimal portfolio decisions in the form of net demands

$$\Delta q_{1i} = \omega_i\left(c\right)\Delta \hat{q}_{1i}, \tag{8}$$

where

$$\Delta \hat{q}_{1i} = \frac{\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma_i h_i - p}{\gamma_i \mathbb{V}ar\left[\theta|s_i,h_i,p\right]} - q_{0i} \qquad \text{and} \qquad \omega_i\left(c\right) = \frac{\gamma_i \mathbb{V}ar\left[\theta|s_i,h_i,p\right]}{\gamma_i \mathbb{V}ar\left[\theta|s_i,h_i,p\right] + c}. \tag{9}$$

Eq. (8) decomposes the investor's net demand in two components: $\Delta \hat{q}_{1i}$ and $\omega_i\left(c\right)$. $\Delta \hat{q}_{1i}$ represents the net demand of investor i if he did not face trading costs. $\omega_i\left(c\right)$ takes into account how trading costs affect the net demand for the risky asset. $\omega_i(c) \in [0,1]$, it is a decreasing function of the trading cost c and it satisfies $\lim_{c\to 0}\omega_i\left(c\right) = 1$ and $\lim_{c\to\infty}\omega_i\left(c\right) = 0$. This coefficient $\omega_i\left(c\right)$ can be interpreted as an attenuation weight that measures how the net demand of an investor changes relative to the case in which the investor faces no trading costs. Alternatively, we can write Eq. (7) in the form of a weighted average of investors' initial asset holdings q_{0i} and the hypothetical optimal portfolio demand in the absence of trading costs, that is, $q_{1i} = \omega_i\left(c\right)\hat{q}_{1i} + \left(1-\omega_i\left(c\right)\right)q_{0i}$.

The equilibrium of the model is fully characterized by combining the portfolio decision of investors, characterized in Eq. (7), with the market clearing condition for the risky asset, accounting for the filtering problem solved by the investors. When forming their expectations about the fundamental, investors use all the information available to them. Each investor i observes two signals about the fundamental θ: the private signal s_i and the public signal revealed by the price p. Moreover, the realization of the individual hedging h_i need reveals information about the aggregate hedging need in the economy, δ, and, thus, about the noise contained in the asset price.[9]

Equilibrium with ex-ante identical investors

As a benchmark, we consider the case in which all investors are ex-ante identical. That is, we assume that all investors have identical risk aversion, initial asset holdings, variance of their hedging motives, and precision of the private signal. Formally, $\gamma_i = \gamma$, $q_{0i} = q_0$, $\tau_{hi} = \tau_h$, and $\tau_{si} = \tau_s$, $\forall i$.

In the class of symmetric equilibria in linear strategies that we study, we guess (and subsequently verify) that investor i's optimal net portfolio demand takes the form

$$\Delta q_{1i} = \alpha_s s_i - \alpha_h h_i - \alpha_p p + \psi, \tag{10}$$

where α_s, α_h, and α_p are positive scalars, while ψ can take positive or negative values. α_s, α_h, and α_p respectively represent the demand sensitivities of investor i to his private signal, his realized hedging needs and the price. All these sensitivities take into account the informational content of the relevant variable. In particular, the price sensitivity α_p accounts for the pecuniary cost of acquiring the asset and for the informational content of prices, while the sensitivity to the hedging needs, α_h, captures the level of risk aversion as well as the informativeness of the individual hedging need about the aggregate level of hedging needs in the economy, δ.

Market clearing implies that the equilibrium price takes the form

$$p = \frac{\alpha_s}{\alpha_p}\theta - \frac{\alpha_h}{\alpha_p}\delta + \frac{\psi}{\alpha_p}. \tag{11}$$

A higher fundamental value of the asset θ and higher aggregate hedging needs, low δ, increase the asset price. The last term in Eq. (11) embeds both the unconditional expected payoff of the risky asset and a risk premium.

The price p contains information about the fundamental value of the asset and about the aggregate hedging needs in the economy, as can be seen from Eq. (11). While investors intrinsically care about the value of θ, they care about the aggregate hedging need δ only insofar it allows them to predict θ more accurately from prices. Therefore, an investor i uses his information about the aggregate hedging need when extracting information from prices. Let $\hat{p} = \frac{\alpha_p}{\alpha_s}p - \frac{\psi}{\alpha_s}$ be the unbiased signal of θ contained in the price p for an external observer. Then, the (augmented) unbiased signal of the fundamental contained

[9] For reference, we characterize the equilibrium of our model when investors do not learn from prices in the Online Appendix.

in the price for an investor with hedging needs h_i is

$$\hat{p} + \frac{\alpha_h}{\alpha_s}\mathbb{E}\left[\delta|\, h_i\right]\Big|\, \theta \sim N\left(\theta, \tau_{\hat{p}}^{-1}\right),$$

where

$$\mathbb{E}\left[\delta|\, h_i\right] = \frac{\tau_h}{\tau_\delta + \tau_h} h_i \quad \text{and} \quad \tau_{\hat{p}} = \left(\frac{\alpha_s}{\alpha_h}\right)^2 (\tau_\delta + \tau_h)\,. \tag{12}$$

This signal corresponds to the unbiased signal in prices $\hat{p}$, augmented by the information contained in the private hedging needs. When the realization of h_i is high, investor i assigns a high probability to the aggregate level of hedging needs δ also being high, which, for a given price p, increases the perceived expected payoff θ.

After solving the filtering problem, investor i's conditional expectation of the fundamental value of the asset $\mathbb{E}\left[\theta|s_i, h_i, p\right]$ takes the form

$$\mathbb{E}\left[\theta|\, s_i, h_i, p\right] = \frac{\tau_\theta \overline{\theta} + \tau_s s_i + \tau_{\hat{p}}\left(\hat{p} + \frac{\alpha_h}{\alpha_s}\mathbb{E}\left[\delta|h_i\right]\right)}{\tau_\theta + \tau_s + \tau_{\hat{p}}}. \tag{13}$$

The expectation in Eq. (13) is a weighted average of the prior on the fundamental $\overline{\theta}$, the private signal s_i, and the augmented signal contained in prices, $\hat{p} + \frac{\alpha_h}{\alpha_s}\mathbb{E}\left[\delta|h_i\right]$.

An external observer only gathers information from the asset price. Therefore, from an external observer's perspective, the unbiased signal contained in the price is distributed as follows

$$\hat{p}|\, \theta \sim N\left(\theta, \left(\tau_{\hat{p}}^e\right)^{-1}\right), \quad \text{where} \quad \tau_{\hat{p}}^e = \left(\frac{\alpha_s}{\alpha_h}\right)^2 \tau_\delta. \tag{14}$$

Not surprisingly, an investor i extracts more precise information from the price than an external observer, i.e., $\tau_{\hat{p}} \geq \tau_{\hat{p}}^e$, because the investor can filter out part of the aggregate noise. When $\tau_{\hat{p}}^e \to 0$, observing the asset price does not reveal any information about the asset payoff θ. Alternatively, when $\tau_{\hat{p}}^e \to \infty$, asset prices are arbitrarily precise and observing the asset price perfectly reveals the realization of θ. Without aggregate risk on hedging needs, that is, $\tau_\delta \to \infty$, it is evident from Eq. (12) that the equilibrium price is fully revealing and that Grossman (1976) paradox applies.

Definition. (Price Informativeness) We define price informativeness as the precision of the unbiased signal of the payoff θ contained in the asset price, from the perspective of an external observer. Formally, we use $\tau_{\hat{p}}^e$, as defined in Eq. (14), as the relevant measure of price informativeness.

This measure of price informativeness, which captures the precision of the information about fundamentals contained in the price, is the relevant welfare measure for an outside observer whose utility depends on knowing the value of θ.[10] See the Online Appendix for a derivation. This result justifies why we use price informativeness as our variable of interest, as opposed to focusing on the welfare of the investors within the model, which is only driven by risk-sharing considerations. In our model, it's

[10]For clarity, we abstract from production in our model. It is easy to append a production side to this model which exclusively uses asset prices as a source of information to guide production decisions, as we show in the Online Appendix. It is somewhat more involved to introduce feedback effects between real and financial markets, as discussed in Bond, Edmans and Goldstein (2012). There is no a priori reason for why that would affect our results.

straightforward to show that whenever the irrelevance results apply, investors are worse (better) off when trading costs are higher (lower), regardless of whether trading costs are rebated or not.

Lemma 1. (Existence and multiplicity) *An equilibrium always exists. There are at most three equilibria.*

The existence and uniqueness properties of the equilibrium are determined by studying the solutions of the following cubic equation in $\frac{\alpha_s}{\alpha_h}$

$$\gamma\left(\tau_\delta+\tau_h\right)\left(\frac{\alpha_s}{\alpha_h}\right)^3-\tau_h\left(\frac{\alpha_s}{\alpha_h}\right)^2+\gamma\left(\tau_s+\tau_\theta\right)\left(\frac{\alpha_s}{\alpha_h}\right)-\tau_s=0. \tag{15}$$

In the Appendix, we show that Eq. (15) has at least one positive real solution, establishing equilibrium existence. We also show that Eq. (15) generically has one or three positive real solutions, depending on primitives. Moreover, we also show in the Appendix that, if there are multiple equilibria, the middle equilibrium is not stable. This allows us to direct our analysis to the higher and lower equilibria, which can be made stable under plausible assumptions on equilibrium convergence.

Multiple equilibria in this environment arise when strategic complementarities in learning are sufficiently strong. The equilibrium price contains information about the fundamental asset payoff θ and about the aggregate hedging need in the economy δ, which acts as noise and makes the price only partially revealing. Therefore, the price is a public signal of the fundamental and the noise. When price informativeness increases, the precision of the price as a signal of the fundamental increases while its precision as a signal of the noise decreases. Intuitively, an increase in price informativeness $\frac{\alpha_s}{\alpha_h}$ has two effects on an individual investor's behavior. First, given that the price becomes a better public signal about θ, the investor optimally assigns less weight to his private signal, which reduces α_s and the informativeness of the individual investor's demand, $\frac{\alpha_s}{\alpha_h}$. This channel makes investors' decisions strategic substitutes and pushes towards a unique equilibrium. Second, since an increase in price informativeness also makes the price a worse public signal about δ, the investor optimally assigns more weight to his hedging need h_i as a private signal about the aggregate noise and reduces his α_h, which increases the price informativeness of the investor's demand $\frac{\alpha_s}{\alpha_h}$.[11] This channel makes investors' decisions strategic complements and, when strong enough, can generate multiple equilibria.

The two stable equilibria share the following properties:

$$a)\frac{\partial\left(\frac{\alpha_s}{\alpha_h}\right)}{\partial\tau_h}>0,\quad b)\frac{\partial\left(\frac{\alpha_s}{\alpha_h}\right)}{\partial\tau_s}>0,\quad c)\frac{\partial\left(\frac{\alpha_s}{\alpha_h}\right)}{\partial\gamma}<0,\quad d)\frac{\partial\left(\frac{\alpha_s}{\alpha_h}\right)}{\partial\tau_\theta}<0,\text{ and } e)\frac{\partial\left(\frac{\alpha_s}{\alpha_h}\right)}{\partial\tau_\delta}<0. \tag{16}$$

Figure 1 illustrates how the equilibrium values of $\frac{\alpha_s}{\alpha_h}$ vary with γ, τ_s, and τ_h, for the reference parameters in Table 1. As described above, the ratio $\frac{\alpha_s}{\alpha_h}$ measures the demand's relative sensitivity to information versus hedging needs. On the one hand, as shown by *a*) and *b*) above, very precise private signals and very small dispersion of hedging needs make investors relatively more willing to trade

[11] The more an investor relies on h_i as a signal about δ, the lower α_h. Intuitively, a high h_i for an investor that learns about δ from the equilibrium price suggests that other investors are selling the risky asset for reasons not related to its payoffs, which dampens the desire to sell purely for hedging reasons, reducing the sensitivity of investors' demand to h_i.

on information, as opposed to trading based on their hedging needs. On the other hand, high levels of risk aversion and low degrees of prior uncertainty (high precision) either about the fundamental or aggregate hedging needs make investors relatively more willing to trade on hedging needs as opposed to information, as can be seen in *c*), *d*), and *e*) above.

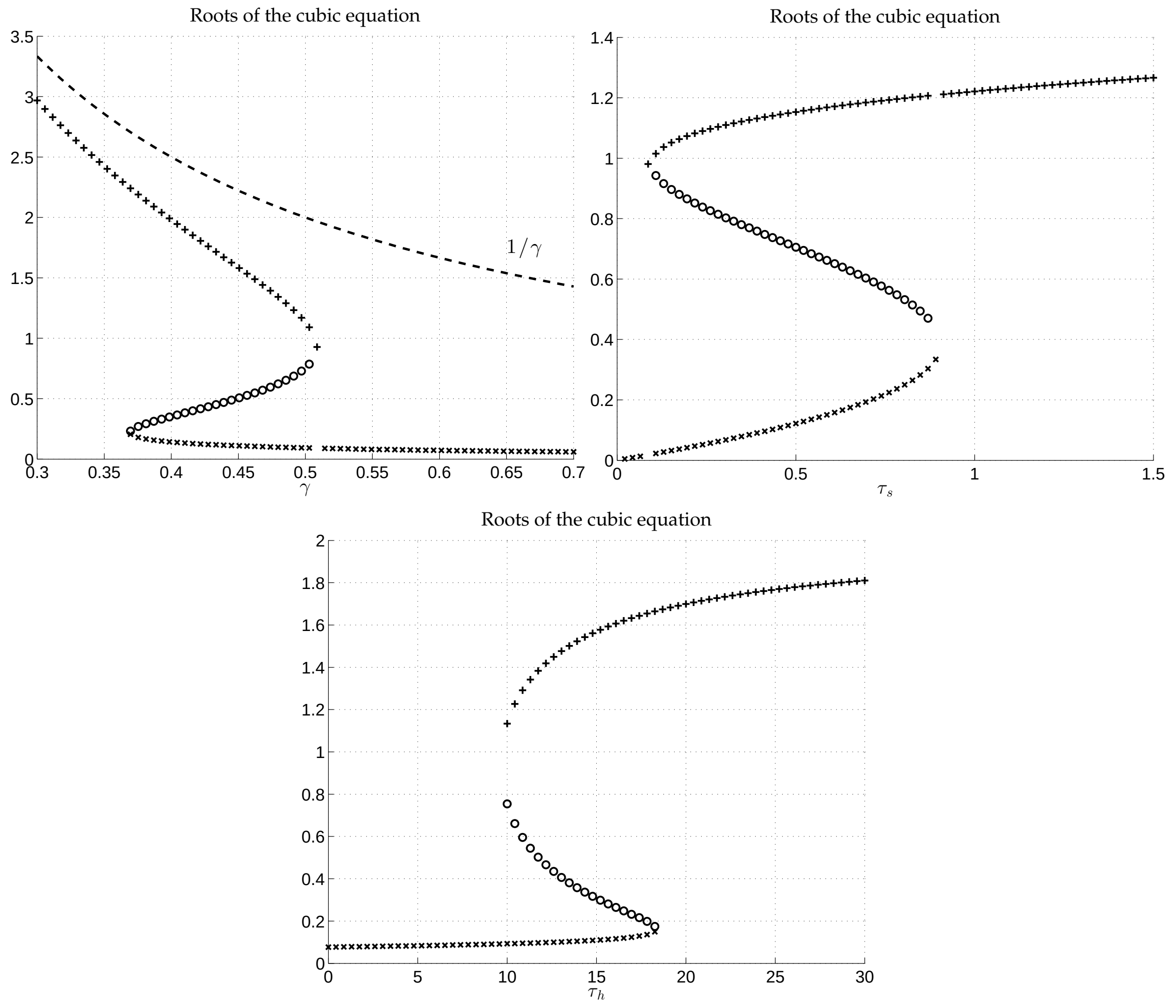

Figure 1: Equilibrium values of $\frac{\alpha_s}{\alpha_h}$ for different γ, τ_s, and τ_h

Table 1: Reference parameters for Figure 1

$\gamma = 0.5$	$\tau_s = 0.4$	$\tau_h = 10$	$\tau_\theta = 10$	$\tau_\delta = 0.1$

Figure 2 provides heat maps of the multiplicity regions for different combinations of γ, τ_s, and τ_h. When γ is sufficiently high, only the unique equilibrium with low price informativeness survives. On the contrary, when γ is sufficiently low, only the unique equilibrium with high price informativeness survives. For intermediate values of risk aversion, increased precision of private information and hedging needs make more likely the unique equilibrium with high price informativeness and vice versa.

All other equilibrium objects are uniquely pinned down given an equilibrium value of $\frac{\alpha_s}{\alpha_h}$. The

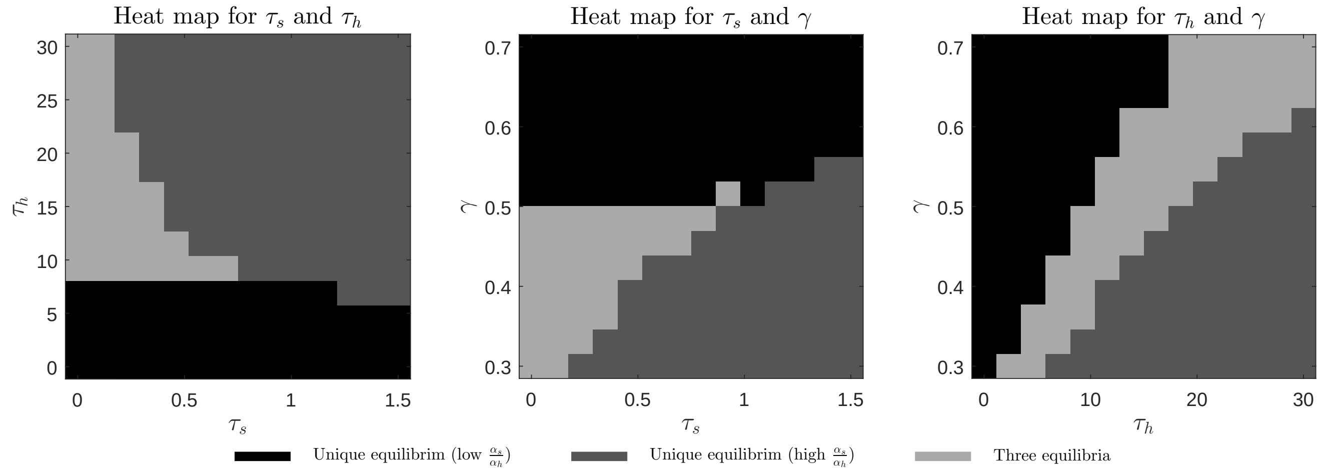

Figure 2: Uniqueness/multiplicity regions for different combinations of γ, τ_s, and τ_h

conjectured coefficients of investors' net demands are given in equilibrium by

$$\alpha_s = \frac{1}{\kappa}\frac{\tau_s}{\tau_\theta + \tau_s + \tau_{\hat{p}}}, \qquad \alpha_p = \frac{1}{\kappa}\frac{\tau_s}{\tau_s + \tau_{\hat{p}}},$$

$$\alpha_h = \frac{1}{\kappa}\left(\gamma - \frac{\alpha_s}{\alpha_h}\frac{\tau_h}{\tau_\theta + \tau_s + \tau_{\hat{p}}}\right), \quad \text{and} \qquad \psi = \alpha_p\left(\frac{\tau_\theta}{\tau_\theta + \tau_s + \tau_{\hat{p}}}\overline{\theta} - \gamma\mathbb{V}ar\left[\theta|s_i, p\right]q_0\right),$$

where we define $\kappa \equiv \gamma\mathbb{V}ar\left[\theta|s_i, p\right] + c$.

The coefficient α_s, which determines the sensitivity of the demand for the risky asset with respect to investors' private signals, is increasing in the precision of investors private signals τ_s. When the signal is more informative, investors put more weight on their signals since a higher realization of the signal increases the expected payoff of the asset.

The coefficient α_h determines the sensitivity of the demand for the risky asset with respect to hedging needs. Naturally, more risk averse investors react more to their hedging needs, as captured by γ. Because investors' partially infer the aggregate component of hedging from their individual realization, they dampen their demand response to h_i. Intuitively, a high realization of h_i, which induces investors to sell for hedging reasons, implies that other investors also desire to sell for hedging reasons, which, for a given price, makes investing in the risky asset more desirable.

The coefficient α_p, which determines the sensitivity of the demand for the risky asset with respect to the asset price, features a substitution effect and an information effect. When $\tau_{\hat{p}} \to 0$, there is no information effect and $\alpha_p \to \frac{1}{\kappa}$. In this case, the elasticity of investor i portfolio demand the prices is given by $\frac{1}{\kappa}$, as in the model without learning: this is the standard substitution effect caused by price changes. When prices are somewhat informative, i.e., when $\tau_{\hat{p}} > 0$, an information effect arises. Investors are less sensitive to price changes since high prices induce investors to infer that the expected asset payoff is high and vice versa. The value of information contained in asset prices $\tau_{\hat{p}}$ relative to the information in private signals τ_s determines the relative sensitivity of the investor's demand to the asset price α_p.

The coefficient ψ determines the autonomous demand for the risky asset, which does not depend on private signals, prices or hedging needs. This autonomous demand is proportional to the price coefficient

α_p and it has two components. Its first component captures the (weighted) unconditional expected value of the asset. Its second component captures the risk premium associated with holding the risky asset.

Importantly, the equilibrium values of α_s, α_h, and α_p are directly modulated by κ, which is a measure of investors risk tolerance and trading costs. The fact that κ enters multiplicatively in all three variables makes the ratios $\frac{\alpha_s}{\alpha_h}$, $\frac{\alpha_s}{\alpha_p}$, and $\frac{\alpha_h}{\alpha_p}$ independent of the level of trading costs, which is crucial to establish our main result.

Theorem 1. (Irrelevance theorem with ex-ante identical investors) *When investors are ex-ante identical, price informativeness in any equilibrium is independent of the level of trading costs. Formally, the precision of the unbiased signal about the fundamental revealed by the asset price* τ_p^e *does not depend on* c*.*

Theorem 1 establishes the first main irrelevance result of the paper. Theorem 1 shows that price informativeness is independent of the level of trading costs. Two identical economies with different levels of trading costs c have equally informative prices. Intuitively, high trading costs make investors less willing to trade on both their private information and their hedging needs, leaving unchanged the total relative demand sensitivities to hedging and information and, consequently, the signal-to-noise ratio in asset prices. Therefore, price informativeness is not affected by changes in the level of trading costs. Moreover, changes in the level of trading costs do not affect the structure of the set of equilibria. That is, in the context of Theorem 1, the set of equilibrium levels of price informativeness is invariant to the level of trading costs. Theorem 1 provides a natural benchmark to understand the role of trading costs on the informational efficiency of the economy: only departures from ex-ante homogeneity across investors can generate an effect of trading costs on information aggregation.

Although this paper focuses on the effects of trading costs on learning and price informativeness, Theorem 1 (and all other irrelevance results in this paper) apply to the unconditional volatility of asset prices, as we show in the Appendix. Intuitively, given that the reduction on buying and selling pressures is symmetric across all investors, asset prices remain unaffected by variations in the level of trading costs.[12]

Even though price informativeness and volatility are independent of c, other equilibrium outcomes, like portfolio holdings and trading volume do depend on the level of trading costs. The net trading in equilibrium by investor i can be written as a function of the realizations of ε_{si} and ε_{hi} as follows

$$\Delta q_{1i} = \alpha_s \varepsilon_{si} - \alpha_h \varepsilon_{hi}.$$

Because α_s and α_h are decreasing in the level of trading costs c, the level of net trading by an individual investor is decreasing in c.

The effects on aggregate trading volume are similar. Using a Law of Large Numbers, we can exactly

[12] In Davila and Parlatore (2017), we systematically study the relation between price informativeness and price volatility in a general class of models of financial market trading.

express trading volume in this economy, defined as the number of shares traded and denoted by $\mathcal{V}$, as

$$\mathcal{V} = \frac{1}{2}\int |\Delta q_{1i}|\, di = \frac{1}{\sqrt{2\pi}}\left(\frac{\alpha_s^2}{\tau_s} + \frac{\alpha_h^2}{\tau_h}\right)^{\frac{1}{2}}.$$

Because α_s and α_h are decreasing in the level of trading costs c, the level of aggregate trading volume is decreasing in c. Formally, we show that

$$\frac{d\mathcal{V}}{dc} < 0.$$

Therefore, even when price informativeness remains unchanged, trading volume will decrease when trading costs are higher.

Equilibrium with ex-ante heterogeneous investors

Theorem 1 is an important benchmark to understand how trading costs affect informational efficiency. However, investors may be ex-ante heterogeneous along different dimensions. In this section, we study how ex-ante asymmetries among investors break our irrelevance result. Formally, we let γ_i, τ_{si}, τ_{hi}, and q_{0i} take arbitrary values across the distribution of investors.

Given a price p, Eq. (7) continues to determine investor i's demand for the risky asset. In the equilibrium in linear strategies that we study, we guess and subsequently verify that the optimal portfolio of investor i takes the form

$$\Delta q_{1i} = \alpha_{si} s_i - \alpha_{hi} h_i - \alpha_{pi} p + \psi_i, \tag{17}$$

where α_{si}, α_{hi}, and α_{pi} are positive scalars for every investor i and ψ_i can be positive or negative.

Market clearing implies that the equilibrium price takes the form

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\delta + \frac{\overline{\psi}}{\overline{\alpha_p}}, \tag{18}$$

where we denote the cross sectional averages of the individual coefficients by $\overline{\alpha_s} = \int \alpha_{si} di$, $\overline{\alpha_h} = \int \alpha_{hi} di$, $\overline{\alpha_p} = \int \alpha_{pi} di$, and $\overline{\psi} = \int \psi_i di$. The interpretation of Eq. (17) and Eq. (18) is the analogous to the interpretation of Eq. (10) and Eq. (11) in the model with ex-ante identical investors. We denote by $\hat{p}^e = \frac{\overline{\alpha_p}}{\overline{\alpha_s}} p - \frac{\overline{\psi}}{\overline{\alpha_s}}$ the unbiased signal of θ from the perspective of an external observer, which is distributed as follows

$$\hat{p}^e|\theta \sim N\left(\theta, \left(\tau_{\hat{p}}^e\right)^{-1}\right), \quad \text{where} \quad \tau_{\hat{p}}^e = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 \tau_\delta.$$

As before, we adopt $\tau_{\hat{p}}^e$ as the relevant measure of price informativeness. We relegate the exact characterization of the equilibrium to the Appendix, and exclusively focus on the implications of trading costs for price informativeness.

Theorem 2 focuses on the case in which two groups of investors are heterogeneous across a single dimension. In that case, we show that an increase in trading costs is associated with a reduction in price informativeness for any set of primitives.

Theorem 2. (One-dimensional heterogeneity) *When two groups of investors differ along one dimension of heterogeneity in i) risk aversion, ii) precision of private information, or iii) precision* *of hedging needs across, an increase in trading costs is always associated with a decrease in price informativeness.*

We show that one dimensional heterogeneity in primitives across two groups of investors in risk aversion, the precision of private information, or hedging needs implies that trading costs reduce price informativeness. This result arises because investors with more precise information, either about the fundamental or the aggregate hedging, or with relatively high risk tolerance, trade more aggressively in general and react more to trading costs, while putting more weight on their private signal about the fundamental and contributing relatively more information to the price. Intuitively, all three forms of heterogeneity endogenously generate a positive cross-sectional correlation between aggressive trading behavior and relative sensitivities to information versus hedging. We describe in detail how this pattern emerges endogenously in our numerical illustration in Section 4.

Theorem 3 extends the result in Theorem 2 to multi-dimensional heterogeneity. When investors differ in two dimensions, we show that an increase in trading costs is associated with a reduction in price informativeness for most combinations of primitives.[13] We also provide a general characterization of the directional change in price informativeness that accommodates multi-dimensional heterogeneity. This directional change is expressed as a function of asset demand sensitivities, which in our model are equilibrium objects, and clearly illustrates the intuition behind the economic mechanisms that drive the results.

Theorem 3. (Two-dimensional heterogeneity and general directional effects of trading costs with ex-ante heterogeneous investors) *a) When two groups of investors differ along two out of the three following dimensions: i) risk aversion, ii) precision of private information, or iii) precision of hedging needs across, an increase in trading costs is associated with a decrease in price informativeness for most parameter combinations.*

b) When the difference in relative-to-the-average sensitivities between information and hedging motives for trading, $\frac{\alpha_{si}}{\overline{\alpha}_s} - \frac{\alpha_{hi}}{\overline{\alpha}_h}$*, is positively (negatively) correlated in the cross-section of investors with the demand sensitivity* $\frac{1}{\kappa_i}$*, an increase in trading costs* c *decreases (increases) price informativeness in a given equilibrium. Formally, the sign of* $\frac{d\tau^e_{\hat{p}}}{dc}$ *is determined by*

$$\operatorname{sgn}\left(\frac{d\tau^e_{\hat{p}}}{dc}\right) = -\operatorname{sgn}\left(\mathbb{C}ov_i\left[\frac{\alpha_{si}}{\overline{\alpha}_s} - \frac{\alpha_{hi}}{\overline{\alpha}_h}, \frac{1}{\kappa_i}\right]\right). \tag{19}$$

When investors are heterogeneous along multiple dimensions of heterogeneity, price informativeness can either increase or decrease with trading costs. For example, when a group of investors with high risk aversion also have a high precision of private information, an increase in trading costs disproportionally reduces the amount of information incorporated into the price and price informativeness increases when trading costs increase. In Section 4, we provide specific examples of this phenomenon and illustrate the specific combinations that are associated with a positive value for $\frac{d\tau^e_{\hat{p}}}{dc}$.

[13] By most combinations, we formally mean over 50% of the parameter space.

In Theorem 3b, independently of the primitives of the economy, the equilibrium objects α_{si}, α_{hi}, and κ_i are sufficient statistics to determine how changes in the level of trading costs affect price informativeness. This characterization illustrates the economic mechanisms at play. In general, when investors are heterogeneous, an increase in trading costs can increase or decrease price informativeness, depending on the sign of $-\mathbb{C}ov_i\left[\frac{\alpha_{si}}{\alpha_s}-\frac{\alpha_{hi}}{\alpha_h},\frac{1}{\kappa_i}\right]$. This is the negative of the cross-sectional covariance of two terms. The first term corresponds to the difference between relative sensitivities to private signals on the fundamental and relative sensitivities to hedging. The second term corresponds to the demand sensitivity of investors to trading costs: when $\frac{1}{\kappa_i}$ is high, investors trade aggressively and their overall demand is highly sensitive to price changes and trading costs. Intuitively, when the investors who are relatively more sensitive to information than to hedging needs, that is, those with a high $\frac{\alpha_{si}}{\alpha_s}-\frac{\alpha_{hi}}{\alpha_h}$, are also the more responsive to changes in trading costs, that is, those for which $\frac{1}{\kappa_i}$ is high, we show that high trading costs reduce price informativeness and vice versa.

Not every form of heterogeneity breaks down the irrelevance result. In particular, heterogeneity about initial positions leaves price informativeness unaffected by changes in the level of trading costs, since α_{si}, α_{hi}, and κ_i remain unaffected. Trading costs affect price informativeness as long as investors differentially trade on information and hedging needs. Therefore, whenever γ_i, τ_{si}, and τ_{hi} are constant, demand sensitivities are identical across all investors, which leaves the signal-to-noise raise unchanged.

Remark. (Comparison with standard noise trading formulations) Our irrelevance results crucially depend on the fact that all investors are symmetrically affected by the change in trading costs. At times, for tractability, models of learning in financial markets assume an ad-hoc supply/demand shock, often referred to as "noise trading". Taken at face value, this assumption leads us to believe that high trading costs are associated with low price informativeness. In these models, an increase in trading costs reduces the amount of information in asset prices because only informed investors react to this change, while noise traders' demand is fully inelastic. The classic noise trading formulation can be viewed as a special case of our model in which a group of investors inelastically trades on hedging motives. Theorem 2 shows that increasing trading costs in an economy with a set of perfectly inelastic investors who do not trade on information makes prices less informative.

4 Numerical illustration

To provide a deeper understanding of Theorems 1 through 3, we conduct three different numerical exercises. First, we illustrate how price informativeness and trading volume vary with the level of trading costs for different combinations of risk aversion and precision about the fundamental for a subset of investors. Second, we illustrate how most combinations of heterogeneity in risk aversion and the precision of the private signal about the fundamental are associated with a decrease in price informativeness when trading costs increase. Third, we show that most combinations of risk aversion and the precision of hedging needs are also associated with a decrease in price informativeness when trading costs increase.

4.1 Effects of trading costs on price informativeness and volume

Figure 3 illustrates the effect of trading costs in the equilibrium price informativeness and trading volume. To build on the insights from Theorems 2 and 3, we assume that there are two groups of investors, denoted by $i = A, B$, each of them accounting for one half of the total population. Investors' initially own a single share of the risky asset, so $q_{0i} = Q = 1$, $\forall i$. We assume that all investors have identically distributed hedging needs, i.e., $\tau_{hi} = 1$, $\forall i$. We also assume that $\tau_{\delta} = \tau_{\theta} = 1$. This choice of parameters guarantees that we are in a region with a unique equilibrium.

We compare five different parameter configurations. First, we consider the benchmark with ex-ante identical investors assume that $\tau_{sA} = \tau_{sB} = 1$ and $\gamma_A = \gamma_B = 1$. In that case, the irrelevance result of Theorem 1 applies and $\tau_{\hat{p}}$ is independent of the level of trading costs. Trading volume, as expected, decreases with the level of trading costs.

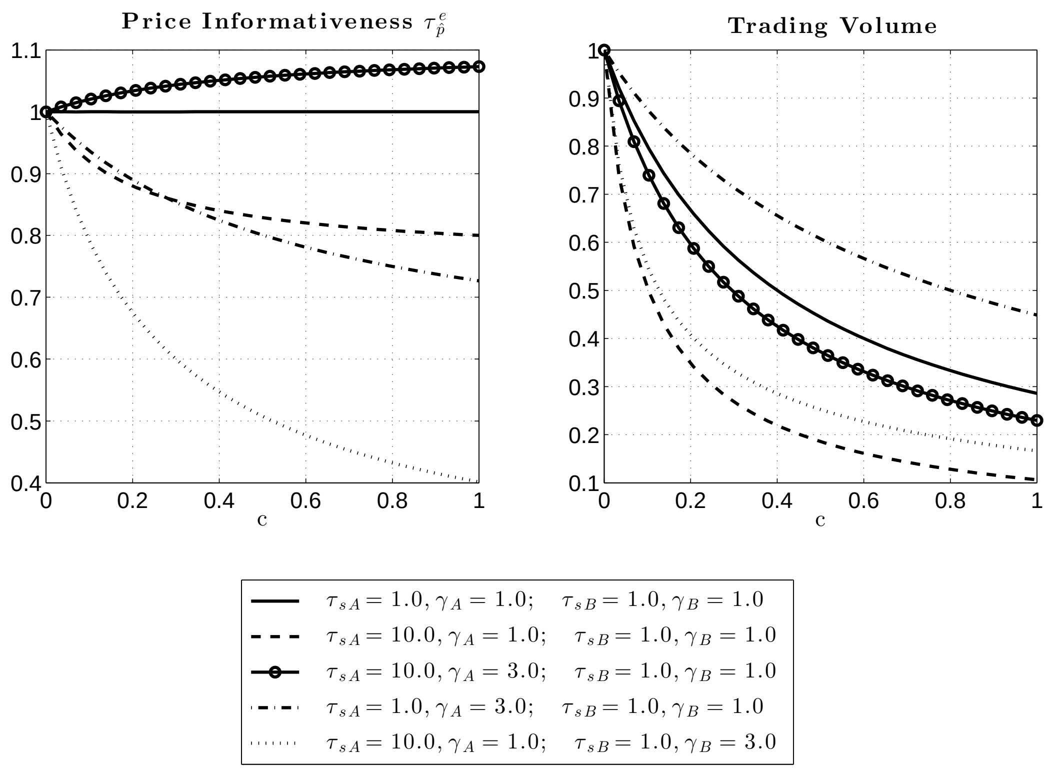

Figure 3: Comparative statics on trading costs c (values are relative to $c = 0$)

Second, we assume that A-investors are better informed than B-investors by increasing the precision of their private signal about the fundamental. Specifically, we set $\tau_{sA} = 10$ and $\gamma_A = 1$. In this case, $\tau_{\hat{p}}$ decreases with the level of trading costs. With this parametrization, A-investors are more informed and more price sensitive than B-investors. Therefore, as shown in Theorem 2, price informativeness must decrease when trading costs increase: the reduction in trading by the more informed and more sensitive A-investors makes prices less informative.

Third, we preserve the asymmetry on information precision while also making A-investors more risk averse. In particular, we set $\tau_{sA} = 10$ and $\gamma_A = 3$. In this case, A-investors are more informed and less price sensitive than B-investors at the margin. Exploiting Theorem 3b), we expect an increase in trading costs to increase price informativeness. Less informed but more sensitive B-investors disproportionally trade less, while the smaller reduction in trading by the less sensitive and better informed A-investors makes prices more informative.

Fourth, we assume that A-investors are more risk averse than B-investors, although both groups are equally informed. In this case, A-investors give a higher weight to trading due to hedging needs at the same time that they have a less sensitive demand. An increase in trading costs has a bigger impact on the trades of B-investors, who are relatively more demand sensitive, reducing price informativeness. This is again consistent with Theorem 2.

Finally, we assume that B-investors are more risk averse than A-investors, who are better informed. This configuration is similar to the second one. An increase in trading costs in that case disproportionally reduces the demand by the relatively better informed A-investors, reducing price informativeness.

Figure 3 illustrates how price informativeness and trading volume vary with the level of trading costs c for the different parameter combinations. We express all variables as a ratio relative to the $c = 0$ reference point. For all parameter configurations, trading volume goes down, as expected.

4.2 Heterogeneity on the precision of signals on fundamental and risk aversion

We systematically study how two-dimensional heterogeneity in risk aversion and in the precision of private information about the fundamental determines the effect of trading costs on price informativeness. We adopt as reference the case in which $\gamma_B = 1$ and $\tau_{sB} = 1$. In Figure 4, we plot equilibrium price informativeness relative to the case when $c = 0$ for different combinations of γ_A, in the horizontal axis, and τ_{sA}, in the vertical axis. This analysis generalizes Figure 3. By design, when $\gamma_A = 1$ and $\tau_{sA} = 1$, the heat map takes a unit value, because price informativeness is invariant to the level of trading costs.

Figure 4 shows that most combinations of risk aversion and precision of the signal on the fundamental heat map values are less than unity.[14] Intuitively, investors with relatively high risk aversion become less aggressive traders and, at the same time, more prone to trade on their hedging. This implies that an increase in trading costs disproportionally reduces the trading of the less risk averse investors, who are those adding more information to the price. Similarly, investors with precise information on the fundamental are, in general, more aggressive and at the same time more willing to trade on their private information. This implies that an increase in trading costs disproportionally reduces the trades of the investors with more precise signals, who are those adding more information to the price.

Only parameter combinations in which a group of investors has high risk aversion, making them less aggressive traders, and high precision of their signal on the fundamental, implying that they react strongly to their private information on the fundamental, are associated with increases in price informativeness when trading costs increase. As expected, the higher the level of trading costs, the stronger the effects on equilibrium price informativeness.

[14] Theorem 1 implies that the point $(\gamma_A, \tau_{sA}) = (1, 1)$ takes a unit value. Theorem 2 implies that points of the form $(x, 1)$ or $(1, y)$, for any $x \neq 1$ or $y \neq 1$, take values strictly less than unity.

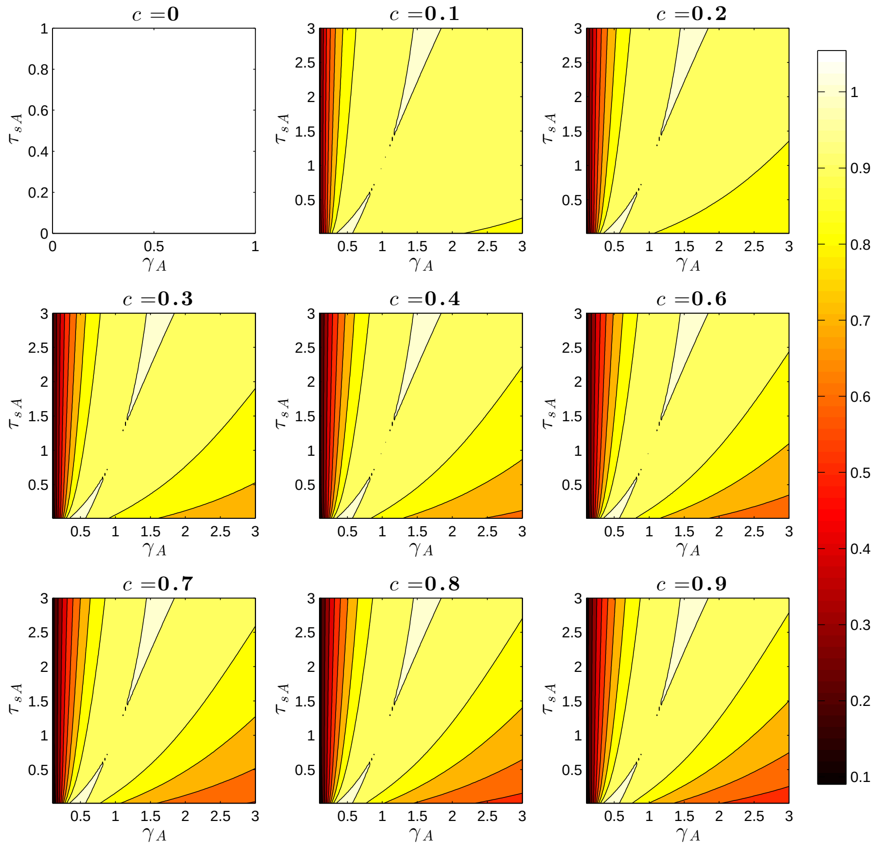

Figure 4: Heat map of price informativeness (values are relative to $c = 0$)

4.3 Heterogeneity on the precision of signals on hedging needs and risk aversion

Finally, we study how two-dimensional heterogeneity in the precision (volatility) of investors' hedging needs determines the effect of trading costs on price informativeness. In this case, we assume that $\tau_{sA} = \tau_{sB} = 1$, and take as reference the case in which $\gamma_B = 1$ and $\tau_{hB} = 1$. In Figure 4, we plot equilibrium price informativeness relative to the case when $c = 0$ for different combinations of γ_A, in the horizontal axis, and τ_{hA}, in the vertical axis. Again, by design, when $\gamma_A = 1$ and $\tau_{hA} = 1$, the heat map takes a unit value, because price informativeness is invariant to the level of trading costs.

Figure 5 also shows that most combinations of risk aversion and precision of hedging needs for A-investors are associated with reductions in price informativeness. Intuitively, investors with very volatile hedging needs (low precision τ_h) in general trade less aggressively, because their perceived variance of the fundamental $\mathbb{V}ar\left[\theta \middle| s_i, h_i, p\right]$ is higher. At the same time, because they are relatively less informed about the noise in asset prices, they react more strongly to the realization of h_i. This implies that an increase in trading costs disproportionally reduces trading by investors with less volatile hedging needs, which are those who react more strongly in relative terms to their private signals about the fundamental, reducing price informativeness. As before, the higher the level of trading costs, the stronger the effects on equilibrium price informativeness.

We summarize the new insights that emerge from Figures 4 and 5 in the following remark.

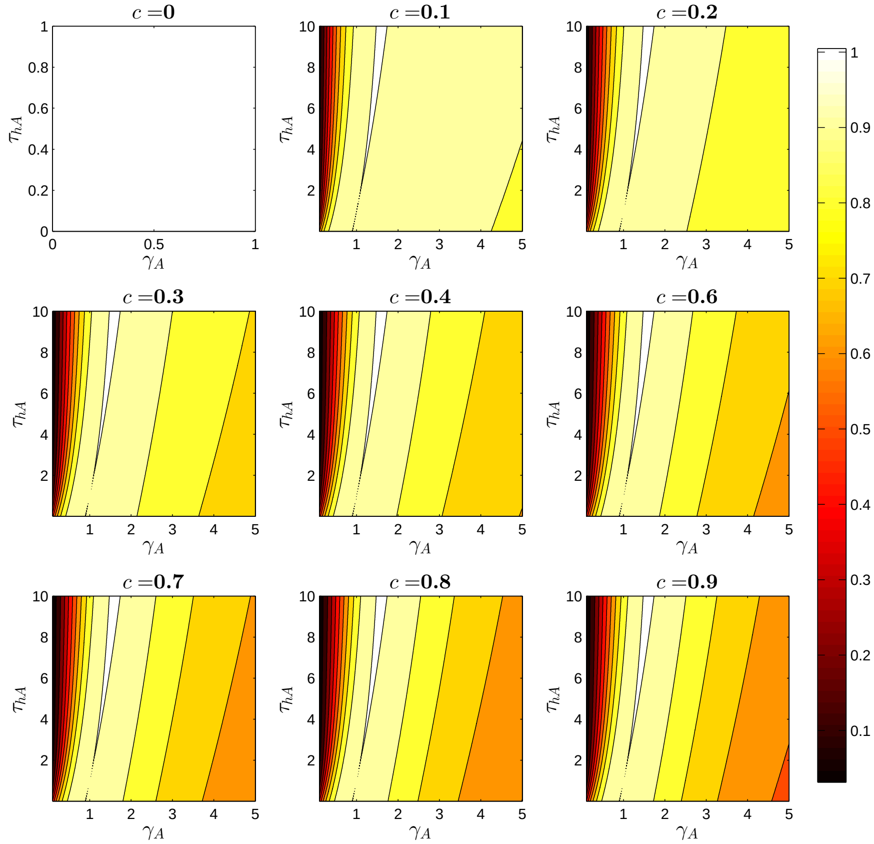

Figure 5: Heat map of price informativeness (values are relative to $c = 0$)

Remark. (Heterogeneity and price informativeness) Consistent with Theorem 3, Figures 4 and 5, show that most combinations of parameters involving risk aversion, the precision of the private signal about the fundamental, or the precision of hedging needs, that generate heterogeneity across investors are associated with a negative response of price informativeness to trading costs. Consistent with Theorem 2, one-dimensional heterogeneity is also associated with negative responses of price informativeness to trading costs.

5 Endogenous information acquisition

So far, our analysis has treated the precision of investors' private information as a primitive of the model. In this section, we allow investors to optimally choose the precision of their private signals.[15] In the paper, we consider the case in which investors choose the precision of their private signals about the fundamental θ. In the Online Appendix, we extend the benchmark model by allowing investors to receive a private signal about the aggregate hedging need δ and allow them to choose the precision of that signal. Both scenarios yield identical insights. To isolate the effects coming from information acquisition, we

[15]The model with exogenous precision can be interpreted as modeling the short-run response to trading costs changes, when investors have not adjusted their information gathering technology. The model with endogenous information acquisition can be interpreted as modeling long-run responses, after investors adjust how they gather information.

focus our attention on the case with ex-ante identical investors. To simplify our calculations, we assume that the risky asset is in zero net supply throughout this section.

5.1 Endogenous precision of the signal about the fundamental

The exact timing of the investors' choices is represented in Figure 6. As in the benchmark model, investors choose their portfolio allocation q_{1i} at date 1, after observing the realizations of s_i and h_i, while filtering the information contained in the asset price. Now, at date 0, every investor chooses the precision of his private signal τ_{si} at a cost $\lambda(\tau_{si})$, where $\lambda(\cdot)$ is continuous and twice differentiable and it satisfies, $\lambda'(\cdot) > 0$, $\lambda''(\cdot) \geq 0$ and the Inada condition $\lim_{\tau_{si}\to\infty} \lambda'(\tau_{si}) = \infty$. In the simulations, we assume that $\lambda(\cdot)$ is quadratic.

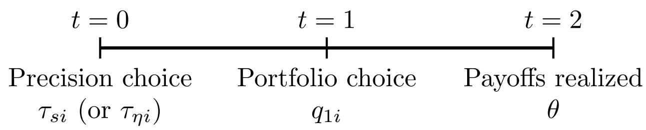

Figure 6: Timeline endogenous information acquisition

The equilibrium of this augmented game is subgame perfect, i.e., it takes into account the equilibrium played in the trading stage. However, since there may be multiple equilibria in the trading stage of the game, the probability with which each equilibrium is played at date 1 is also an equilibrium outcome. We continue to restrict our attention to equilibria in linear strategies in the trading stage.

Definition. (Equilibrium) An equilibrium in the information acquisition game is a set of precision choices for each investor i, $\{\tau_{si}\}_i$, and a probability π with which the high equilibrium is played if there are multiple equilibria in the trading game such that a) each investor chooses the precision of his private signal τ_{si} to maximize his expected utility $V\left(\tau_{si}; \{\tau_{sj}\}_{j\neq i}\right)$, as defined in Eq. (20), given π and the other investors' precision choices $\{\tau_{sj}\}_{j\neq i}$, and b) the probability π is a sunspot equilibrium of the trading game at date 1 given the precision choices $\{\tau_{sj}\}_i$.

We prove Theorem 4 allowing for $\pi \in [0,1]$. To simplify the analysis and highlight the economic mechanisms, we focus on equilibria with a degenerate distribution $\pi \in \{0,1\}$ in what follows.

Investors' information choice Each investor i takes the equilibrium of the model starting at date 1 and the other investors' precision choices as given when he chooses his own precision. Specifically, an investor i optimally chooses τ_{si} by solving

$$\max_{\tau_{si}} V\left(\tau_{si}; \{\tau_{sj}\}_{j\neq i}\right), \quad \text{where} \quad V\left(\tau_{si}; \{\tau_{sj}\}_{j\neq i}\right) = \mathbb{E}[v_i] - \lambda(\tau_{si}), \tag{20}$$

and $\mathbb{E}[v_i]$ is given by[16]

$$\mathbb{E}[v_i] = \mathbb{C}ov\left[\left(\mathbb{E}[\theta|s_i,h_i,p] - \gamma h_i - p\right), q_{1i}^*\right] - \frac{1}{2}\left(\gamma \mathbb{V}ar[\theta|s_i,h_i,p] + c\right)\mathbb{E}\left[(q_{1i}^*)^2\right],$$

[16]Our choice of objective function is standard in these environments. It is studied and justified in Veldkamp (2009) and Van Nieuwerburgh and Veldkamp (2010). They show that the expected utility case delivers analogous qualitative insights.

where q_{1i}^* and p correspond to the date 1 equilibrium outcomes, which are a function of the precision choices of all investors.

Best responses and equilibrium determination The first order condition of the investor's problem in Eq. (20) fully characterizes the best response of investor i – we show in the Appendix that the second order condition for the investors' problem always holds. Formally, the best response $\tau_{si}\left(\{\tau_{sj}\}_{j\neq i}\right)$ is given by the solution to

$$\underbrace{\frac{\partial \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]}{\partial \tau_{si}}}_{\Delta \text{ in accuracy}}-\underbrace{\frac{\gamma}{2}\frac{\partial \mathbb{V}ar\left[\theta|s_i,h_i,p\right]}{\partial \tau_{si}}\mathbb{E}\left[\left(q_{1i}^*\right)^2\right]}_{\Delta \text{ in perceived risk}} \tag{21}$$
$$=\underbrace{\frac{\gamma}{2}\mathbb{V}ar\left[\theta|s_i,h_i,p\right]\frac{\partial \mathbb{V}ar\left[q_{1i}^*\right]}{\partial \tau_{si}}}_{\Delta \text{ in risk taking}}+\underbrace{\frac{c}{2}\frac{\partial \mathbb{V}ar\left[q_{1i}^*\right]}{\partial \tau_{si}}}_{\Delta \text{ in trading costs}}+\underbrace{\lambda'\left(\tau_{si}\right)}_{\Delta \text{ in information cost}}.$$

The left hand side of Eq. (21) represents the marginal benefit of increasing the precision of the private signal. It has two terms. First, increasing the precision of the signal about the fundamental changes the accuracy of the demand function submitted by an investor. An investor wants to have a high demand for the risky asset when it offers a good return, and vice versa. Second, increasing the precision of the signal about the fundamental reduces the level of risk perceived by the investor. The right hand side of Eq. (21) represents the marginal cost of increasing the precision of the private signal. It has three terms. The first term captures the change in risk born by the investor when the expected final asset holdings change. The second term corresponds to the marginal change in trading costs. The last term is the marginal cost of increasing the precision of the signal.

In the following lemma, we establish that there is an equilibrium in the information acquisition stage and that all equilibria are symmetric.

Lemma 2. (Existence and symmetry of equilibrium) *There always exists an equilibrium in the information acquisition stage. Any equilibrium is symmetric.*

A higher precision of the private signal received by investors, increases the accuracy of their demand and reduces their perceived variance of the fundamental. Then, by inspecting Eq. (21), we can see that, since investors can benefit less from acquiring information when trading costs are higher, information acquisition decreases with trading costs. This is the main result of this section, formally stated in Theorem 4.

Theorem 4. (Effect of trading costs with endogenous precision of the fundamental signal) *When investors are ex-ante identical, an increase in trading costs decreases the information acquired about the fundamental in equilibrium, i.e.,*

$$\frac{d\tau_{si}^*}{dc}<0.$$

In the two well-behaved equilibria, this reduction in information acquisition also generates a reduction in price informativeness, hence $\frac{d\tau_{\hat{p}}^e}{dc}<0$.

As Theorem 4 shows, higher trading costs induce investors to choose less precise signals in equilibrium, which makes prices less informative whenever investors coordinate on the stable equilibria of the trading stage, as described in Eq. (16). Figure 7 further illustrates the effect of trading costs on the equilibrium on information acquisition choices and price informativeness.

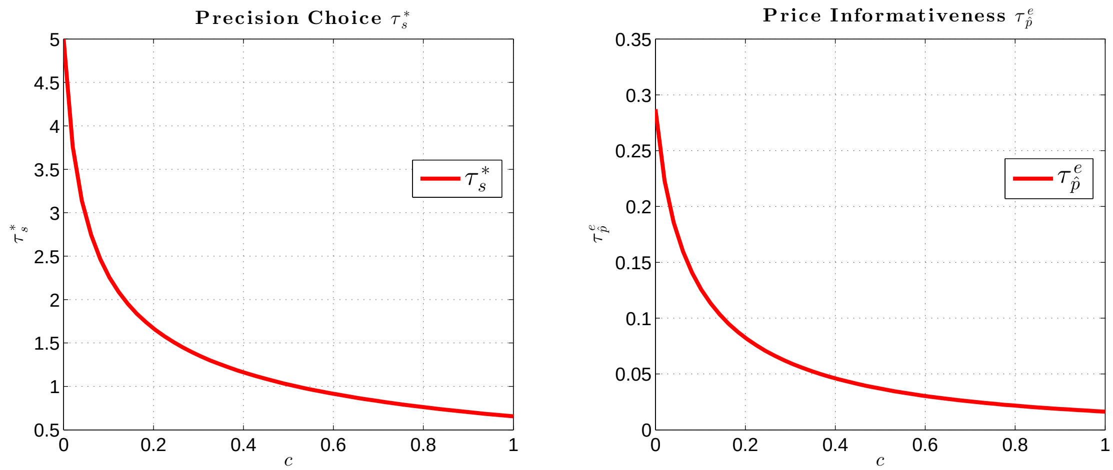

Figure 7: Equilibrium comparative statics

Our irrelevance result derived in the case of exogenously given information precisions does not extend to situations in which investors acquire information. Intuitively, the presence of trading costs makes acquiring information less profitable for every individual investor. In equilibrium, even though the reduction on the precision of information acquired by every other investor in the economy due to the trading costs increases the incentives for an individual investor to acquire information, this effect is not large enough to overcome the original reduction of information precision choice caused by the higher trading cost.

6 Extensions: generalizing the irrelevance result

Finally, we show that our irrelevance argument is valid in more general economies. In particular, we extend our benchmark model with ex-ante identical investor to show that it remains valid in environments with linear trading costs, fixed trading costs, random heterogeneous priors, strategic investors, and general utility and signal structure.

When trading costs are linear on the number of shares traded or fixed, as opposed to quadratic, some investors decide not to trade all, changing the nature of the equilibrium. However, price informativeness remains unaffected. Allowing for random heterogeneous priors shows that the irrelevance argument does not rely on aggregate hedging noise, but that aggregate uncertainty regarding the level of other trading motives preserves the irrelevance. We also show that strategic behavior considerations do not affect our irrelevance result when investors are ex-ante identical.[17] Departures from homogeneity would break our irrelevance results in a similar way to Theorem 3 and Section 4.

[17]In the Online Appendix, we extend our irrelevance result to the case of multiple rounds of trading. Although investors'

6.1 Linear trading costs

In this extension, we modify the form of the trading costs faced by investors.[18] We now assume that investors face a linear trading cost $\phi \geq 0$ per share traded of the risky asset. In particular, a change in the asset holdings of the risky asset $|\Delta q_{1i}|$ incurs a trading cost of

$$\phi\, |\Delta q_{1i}|$$

There are two benefits modeling trading costs as linear. First, they overcome the problem of order slicing associated with any nonlinear trading cost. Second, they can be derived as the compensation to a group of outside agents that operate a constant returns to scale trading technology that facilitates trading.

The demand for the risky asset of every investor i is given by the solution to

$$\max_{q_{1i}} \left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i - p\right) q_{1i} - \frac{\gamma}{2}\mathbb{V}ar\left[\theta|s_i,h_i,p\right] q_{1i}^2 - \phi\,|\Delta q_{1i}|\,, \tag{22}$$

where their optimal portfolio choice, which features an inaction region, is given by

$$\Delta q_{1i} = \begin{cases} \Delta q_{1i}^{+} = \frac{\mathbb{E}[\theta|s_i,h_i,p]-\gamma h_i - p - \phi}{\gamma \mathbb{V}ar[\theta|s_i,h_i,p]} - q_0, & \text{if} \quad \Delta q_{1i}^{+} > 0 \\ 0, & \text{if} \quad \Delta q_{1i}^{+} \leq 0,\ \Delta q_{1i}^{-} \geq 0 \\ \Delta q_{1i}^{-} = \frac{\mathbb{E}[\theta|s_i,h_i,p]-\gamma h_i - p + \phi}{\gamma \mathbb{V}ar[\theta|s_i,h_i,p]} - q_0, & \text{if} \quad \Delta q_{1i}^{-} < 0. \end{cases} \tag{23}$$

In a symmetric equilibrium in linear strategies, we postulate net demand functions for buyers (Δq_{1i}^{+}) and sellers (Δq_{1i}^{-}) respectively given by

$$\begin{aligned} \Delta q_{1i}^{+} &= \alpha_s s_i - \alpha_h h_i - \alpha_p p + \psi^{+} \\ \Delta q_{1i}^{-} &= \alpha_s s_i - \alpha_h h_i - \alpha_p p + \psi^{-}, \end{aligned} \tag{24}$$

where α_s, α_h, and α_p are positive scalars, while ψ^{+} and ψ^{-} can take positive or negative values.

Market clearing in the asset market implies that the equilibrium price takes the form

$$p = \frac{\alpha_s}{\alpha_p}\theta - \frac{\alpha_h}{\alpha_p}\delta + \frac{\psi}{\alpha_p}, \tag{25}$$

where we define $\psi = \frac{\psi^{+}+\psi^{-}}{2}$. The derivation of Eq. (25) exploits equilibrium symmetry and a Law of Large Numbers.

The precision of the unbiased signal of θ from the perspective of an external observer, which we denote by $\tau_{\hat{p}}^{e}$, is the relevant measure of price informativeness. As in the benchmark model, price informativeness is given by

$$\tau_{\hat{p}}^{e} = \left(\frac{\alpha_s}{\alpha_h}\right)^2 \tau_\delta.$$

We can then establish a new irrelevance result.

portfolio sensitivities vary in that case, they do so symmetrically, allowing us to find another irrelevance result. There is scope for further research on the interaction of how trading costs affect price informativeness when investors are heterogeneous in their dynamic trading considerations.

[18] Our extensions with linear and fixed trading costs are a standalone contribution by itself. To our knowledge, this is the first paper to solve for a Rational Expectations Equilibrium (REE) with many investors and linear trading costs, which endogenously generate inaction regions. Formally, the closest results are those of Yuan (2005, 2006), who solves a REE with kinked asset demands.

Theorem 5. (Irrelevance theorem with linear trading costs) *In an economy with linear trading costs, when investors are ex-ante identical, price informativeness in any equilibrium is independent of the level of trading costs. Formally, the precision of the unbiased signal about the fundamental revealed by the asset price $\tau_{\hat{p}}^{e}$ does not depend on c.*

Theorem 5 shows that our irrelevance argument is not specific to assuming quadratic trading costs, applying also when trading costs are linear. Interestingly, when trading costs are linear, an increase in trading costs is associated with a reduction in trading on both intensive and extensive margins – some investors cease to trade altogether.[19] However, because the decrease in trading at the extensive margin reduces both fundamental and hedging trades in equal proportions, price informativeness remains unchanged. It is trivial to prove the more general irrelevance result with both linear and quadratic trading costs, given by $\phi\left|\Delta q_{1i}\right| + \frac{c}{2}\left|\Delta q_{1i}\right|^2$.

6.2 Fixed trading costs

In this extension, we consider an alternative form of the trading cost. We now assume that investors face a fixed cost of trading $\Phi \geq 0$ and normalize $q_0 = 0$. Formally, investors' budget constraint now satisfies

$$w_{2i} = n_{2i} + q_{1i}\theta - q_{1i}p - \Phi \cdot \mathbf{1}\left[\Delta q_{1i} \neq 0\right]. \tag{26}$$

The problem solved by investors ceases to be convex in q_{1i}. Nonetheless, it's possible to conjecture and verify that the investors' optimal portfolio choice satisfies

$$\Delta q_{1i} = \begin{cases} \frac{\mathbb{E}[\theta|s_i,h_i,p]-\gamma h_i - p}{\gamma \mathbb{V}ar[\theta|s_i,h_i,p]}, & \text{if} \quad W\left(\Delta q_{1i} \neq 0\right) > W\left(\Delta q_{1i} = 0\right) \\ 0, & \text{otherwise,} \end{cases}$$

where $W = \frac{\gamma}{2}\mathbb{V}ar\left[\theta|s_i,h_i,p\right]\left(\Delta q_{1i}\right)^2 - \Phi \cdot \mathbf{1}\left[\Delta q_{1i} \neq 0\right]$. In a symmetric equilibrium in linear strategies, the portfolio demands of active investors and the equilibrium price satisfy familiar conditions

$$\Delta q_{1i} = \alpha_s s_i - \alpha_h h_i - \alpha_p p + \psi$$
$$p = \frac{\alpha_s}{\alpha_p}\theta - \frac{\alpha_h}{\alpha_p}\delta + \frac{\psi}{\alpha_p},$$

where α_s, α_h, and α_p are positive scalars. As in the case with linear costs, our equilibrium characterization exploits symmetry and a Law of Large Numbers.

We can then establish a new irrelevance result.

Theorem 6. (Irrelevance theorem with linear trading costs) *In an economy with linear trading costs, when investors are ex-ante identical, price informativeness in any equilibrium is independent of the level of trading costs. Formally, the precision of the unbiased signal about the fundamental revealed by the asset price $\tau_{\hat{p}}^{e}$ does not depend on c.*

[19] No-trade regions emerge because investors whose initial asset holdings are close to their optimal level of asset holdings experience a second-order welfare gain from adjusting their portfolios, but face a first-order welfare loss caused by the linear cost. When trading costs are quadratic, the welfare loss is second-order, so it is optimal for (almost) every investor to have a non-zero net trading position.

Intuitively, with fixed trading costs and homogeneous investors, the set of investors whose gains from trading are smaller ceases to trade altogether and become inactive. Those investors whose gains from trading are larger find optimal to pay the fixed cost of trading and submit the same demands as if the faced no trading costs. In this case, an increase in trading costs only has effects on the extensive margin, but not on investors' intensive margin. Since the extensive margin reduction on trading affects fundamental and hedging trades in equal proportions, price informativeness remains unchanged.

6.3 Random heterogeneous priors

In this extension, we add an alternative form of noise: privately known random heterogeneous priors that co-move in the aggregate. There are different ways to justify heterogeneity in priors: they may capture intrinsic differences in beliefs (optimistic versus pessimistic investors), they may be the result of having observed different private signals in the past, or, in some situations, they can also reflect heterogeneous private valuations for the risky asset. We preserve the structure of the symmetric benchmark model, but introduce stochastic heterogeneous priors as follows.[20]

From the point of view of investor i, the payoff of the risky asset is distributed according to

$$\theta \sim N\left(\overline{\theta}_i, \tau_\theta^{-1}\right),$$

where $\overline{\theta}_i$ denotes the prior expected value for investor i, which is also stochastic and distributed according to

$$\overline{\theta}_i = \overline{\theta} + \varepsilon_{ui},$$

where

$$\varepsilon_{ui} \sim N\left(0, \tau_u^{-1}\right) \qquad \text{and} \qquad \overline{\theta} \sim N\left(\mu_{\overline{\theta}}, \tau_{\overline{\theta}}^{-1}\right).$$

This formulation implies that the realized average prior mean is unknown, introducing a new source of aggregate uncertainty in addition to the aggregate hedging need and the payoff of the risky asset. Importantly, we assume that investors take their priors as given and do not use them to learn about the priors of others investors. For this reason, we could allow for heterogeneity in the precision of stochastic heterogeneous priors τ_{ui} without affecting the irrelevance result.

The demand for the risky asset of every investor i is given by the solution to

$$\max_{q_{1i}} \left(\mathbb{E}_i\left[\theta\right] - \gamma h_i - p\right) q_{1i} - \frac{\gamma}{2} \mathbb{V}ar_i\left[\theta\right] q_{1i}^2 - \frac{c}{2}\left(\Delta q_{1i}\right)^2,$$

where we denote the asset payoff posterior expected mean and variance for investor i by $\mathbb{E}_i\left[\theta\right] \equiv \mathbb{E}\left[\theta | \overline{\theta}_i, s_i, h_i, p\right]$ and $\mathbb{V}ar_i\left[\theta\right] = \mathbb{V}ar_i\left[\theta | \overline{\theta}_i, s_i, h_i, p\right]$.

In a symmetric equilibrium in linear strategies, we postulate net demand functions given by

$$\Delta q_{1i} = \alpha_\theta \overline{\theta_i} + \alpha_s s_i - \alpha_h h_i - \alpha_p p + \psi,$$

where α_θ, α_s, α_h, and α_p are positive scalars, while ψ can take positive or negative values.

[20] See Scheinkman and Xiong (2003) or Davila (2014) for models with trading costs and heterogeneous priors.

In contrast to the benchmark model, Δq_{1i} also depends on the individual realization of the heterogeneous prior $\overline{\theta}_i$. Market clearing in the asset market implies that the equilibrium price takes the form

$$p = \frac{\alpha_\theta}{\alpha_p}\overline{\theta} + \frac{\alpha_s}{\alpha_p}\theta - \frac{\alpha_h}{\alpha_p}\delta - \frac{\psi}{\alpha_p}.$$

In this case, the asset price depends on both the aggregate level of prior heterogeneity $\overline{\theta}$, and the actual payoff realization θ.

The variance of the unbiased signal of θ from the perspective of an external observer, which we denote by $\left(\tau_{\hat{p}}^e\right)^{-1}$ and whose inverse we adopt as the relevant measure of price informativeness, is given by

$$\left(\tau_{\hat{p}}^e\right)^{-1} = \left(\frac{\alpha_s}{\alpha_\theta}\right)^2 \tau_{\overline{\theta}}^{-1} + \left(\frac{\alpha_s}{\alpha_h}\right)^2 \tau_\delta^{-1}.$$

Unlike in the benchmark model, even if there are no trading motives due to differences in hedging needs, i.e., $\tau_\delta = 0$, the price of the risky asset is not fully revealing. This occurs because there is a new source of aggregate uncertainty coming from the average level of prior heterogeneity in the economy. Therefore, as long as either $\tau_{\overline{\theta}}$ or τ_δ are non-zero, the equilibrium price is not be fully revealing. In fact, when $\tau_\delta = 0$, the equilibrium of this model always exists and is unique, which makes the model with heterogeneous beliefs a tractable benchmark.

Theorem 7. (Irrelevance theorem with random heterogeneous priors) *In an economy in which investors have random heterogeneous priors, when investors are ex-ante identical, price informativeness in any equilibrium is independent of the level of trading costs. Formally, the precision of the unbiased signal about the fundamental revealed by the asset price $\tau_{\hat{p}}^e$ does not depend on c.*

Theorem 7 shows that our irrelevance argument is not specific to assuming hedging needs as the source of aggregate uncertainty. We can show that when hedging needs are not random, the model with random heterogeneous priors always has a unique equilibrium. This occurs because investors do not learn about the aggregate noise component of prices, which eliminates the strategic complementarities in investors' choices, since priors are fixed after they are realized. In this dimension, the model is even more tractable than our benchmark model. The logic behind Theorem 7 is similar to one behind Theorem 1. An increase in the level of trading costs equally reduces trading due to informational reasons and trading due to heterogeneity in priors, leaving price informativeness unchanged.

6.4 Strategic investors

In this extension, we assume an alternative market structure in which there are a finite number of investors who behave strategically.[21] In particular, we modify our symmetric benchmark model by assuming there are a finite number of investors N who internalize the effect of their demand on prices. We focus on equilibria in linear strategies in which strategic investors submit demand functions, conditional

[21] Both competitive and strategic models are used as frameworks to study trading in financial markets. See Vives (2008) for a recent overview of models of strategic behavior in financial markets.

on the price p. Modeling strategic behavior allows us to study the role of liquidity provision in more detail.

The demand for the risky asset of every investor i is given by the solution to

$$\max_{q_{1i}} \left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p_{-i}\right)q_{1i}-\frac{\gamma}{2}\mathbb{V}ar\left[\theta|s_i,h_i,p\right]\left(q_{1i}\right)^2+p_{-i}q_0-\frac{c}{2}\left(\Delta q_{1i}\right)^2, \tag{27}$$

where p_{-i}, a function of q_{1i}, corresponds to the residual demand faced by investor i given the portfolio choices of all other investors.

The first order condition of this problem yields the following net demand for the risky asset

$$\Delta q_{1i}=\frac{\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p-\gamma\mathbb{V}ar\left[\theta|s_i,h_i,p\right]q_0}{\gamma\mathbb{V}ar\left[\theta|s_i,h_i,p\right]+c+\frac{\partial p_{-i}}{\partial q_{1i}}}. \tag{28}$$

This expression is identical to the one in the benchmark model, with the exception of the price impact term $\frac{\partial p_{-i}}{\partial q_{1i}}$, which we show is positive in equilibrium. The term corresponding to the price impact of investors is similar to the one corresponding to the trading cost c. In fact, the term $c+\frac{\partial p_{-i}}{\partial q_{1i}}$ enters symmetrically into investors' portfolio decisions, with the caveat that $\frac{\partial p_{-i}}{\partial q_{1i}}$ is an equilibrium object while c is a primitive of the model.

In a symmetric equilibrium in linear strategies, we postulate net demand functions given by

$$\Delta q_{1i}=\alpha_s s_i-\alpha_h h_i-\alpha_p p+\psi,$$

where α_s, α_h, and α_p are positive scalars, while ψ can take positive or negative values.

Market clearing in the asset market, $\sum_{i=1}^{N}\Delta q_{1i}=0$, implies that the equilibrium price takes the form

$$p=\frac{\alpha_s}{\alpha_p}\left(\theta+\frac{\sum_{j=1}^{N}\varepsilon_{sj}^{M}}{N}\right)-\frac{\alpha_h}{\alpha_p}\left(\delta+\frac{\sum_{j=1}^{N}\varepsilon_{hj}^{M}}{N}\right)+\frac{\overline{\psi}}{\alpha_p},$$

An important input for the investors' portfolio demands is the residual price elasticity, given by $\frac{\partial p_{-i}}{\partial q_{1i}}$, which takes the value

$$\frac{\partial p_{-i}}{\partial q_{1i}}=\frac{1}{\sum_{j\neq i}\alpha_p}=\frac{1}{(N-1)\,\overline{\alpha_p}}>0.$$

In the strategic case, the inference problem solved by investors must account for the non-negligible effect that the signal of investor i has on the asset price. The variance of the unbiased signal of θ from the perspective of an external observer, which we denote by $\left(\tau_{\hat{p}}^{e}\right)^{-1}$ and whose inverse we adopt as the relevant measure of price informativeness, is given by

$$\left(\tau_{\hat{p}}^{e}\right)^{-1}=\left(\frac{\alpha_h}{\alpha_s}\right)^2\left(\tau_\delta^{-1}+\frac{\tau_h^{-1}}{N}\right)+\frac{\tau_s^{-1}}{N}. \tag{29}$$

In the strategic case, the equilibrium price is not fully revealing even when $\tau_\delta\to\infty$. This result is driven by the fact that the individual signals regarding the fundamental and the individual hedging needs do not cancel out in the aggregate when there is a finite number of investors.

Theorem 8. **(Irrelevance theorem with strategic investors)** *In an economy in which investors are strategic, when investors are ex-ante identical, price informativeness in any equilibrium is independent of the level of trading costs. Formally, the precision of the unbiased signal about the fundamental revealed by the asset price* $\tau_{\hat{p}}^{e}$ *does not depend on* c*.*

Theorem 8 shows that our irrelevance argument does not depend on the assumption of perfect competition. Strategic investors tend to trade more conservatively to limit the price impact and the informational impact of their trades. However, as long as investors are ex-ante identical, changes in the level of trading costs equally affect their trading sensitivities to information and to hedging needs, leaving price informativeness unchanged. Theorem 8 also shares the logic of Theorem 1.

6.5 General utility and signal structure

We have conducted most of our analysis within the CARA-Gaussian framework, which, given its tractability, allows us to provide a full characterization of the equilibrium. Our final irrelevance result relaxes the parametric assumptions on the structure of the private signals and endows investors with more general preferences. This new result allows us to identify which key properties of our benchmark model are crucial for our irrelevance results to hold in a model with ex-ante identical investors, while sidestepping the issues associated with explicitly characterizing the model's equilibrium.[22] The role of this extension is to highlight the importance of ex-post homogeneity in investors' demand sensitivities to information and noise for our irrelevance results. In table 2 in the Appendix, we provide an exact mapping between our benchmark formulation and the general formulation presented in this section.

We start by assuming that investors are heterogeneous, and then proceed to find which specific symmetry conditions are needed for our irrelevance result to hold. In particular, we assume that investors receive a private signal s_i and a hedging need h_i that take the form

$$\begin{aligned} s_i &= f^{si}\left(\theta, \varepsilon\right) \\ h_i &= f^{hi}\left(\delta, \varepsilon\right), \end{aligned}$$

where θ and δ are random variables that represent the fundamental and the aggregate hedging need, and ε corresponds to a vector of errors. We assume that the functions $f^{si}\left(\cdot\right)$ and $f^{hi}\left(\cdot\right)$, which are potentially investor specific, are differentiable.

We now assume that the problem solved by investors can be written as

$$\max_{q_{1i}} U^i\left(q_{1i}, p, s_i, h_i, c\right),$$

where $U^i\left(\cdot\right)$ is a well-behaved function. The variable c represents the magnitude of the trading costs. We do not impose restrictions on the functional form of trading costs. The solution to this problem yields an optimality condition of the form

$$U_q^i\left(q_{1i}, p, s_i, h_i, c\right) = 0, \tag{30}$$

which implicitly defines a demand functions $q_{1i}\left(p, s_i, h_i, c\right)$. Eq. (30) allows for heterogeneity in investor preferences.

[22] A complete characterization of the (set of) equilibrium(a) in this section is beyond the scope of this paper, and would be a significant contribution by itself. We can nonetheless make claims that apply whenever a given equilibrium exists.

The market clearing condition, i.e., $\int \Delta q_{1i}\left(p, s_i, h_i, c\right) di = 0$, implies that we can express the equilibrium price as

$$p\left(\left\{f^{si}\left(\theta, \varepsilon\right)\right\}_i, \left\{f^{hi}\left(\delta, \varepsilon\right)\right\}_i, c\right),$$

where we denote the collections of private signals and hedging needs by $\left\{f^{si}\left(\theta, \varepsilon\right)\right\}_i$ and $\left\{f^{hi}\left(\delta, \varepsilon\right)\right\}_i$.

At this level of generality, it is not possible to find a explicit representation for the conditional variance of the fundamental asset value θ given the asset price. Thus, we use instead a more general measure of price informativeness, defined by

$$\Pi \equiv \frac{\left|\frac{\partial p}{\partial \theta}\right|}{\left|\frac{\partial p}{\partial \delta}\right|}.$$

In our benchmark model, Π exactly corresponds to $\frac{\alpha_s}{\alpha_h}$, which is a monotonic transformation of the conditional variance of the fundamental given the asset price. Intuitively, this definition captures the relative sensitivity of the price to a change in the aggregate fundamental relative to a change in aggregate noise. When Π is high, observing the asset price reveals the value of the fundamental precisely, and vice versa.

Exploiting market clearing, we can express price informativeness as

$$\Pi = \frac{\left|\int \frac{\partial q_{1i}}{\partial s_i} \frac{\partial f^{si}}{\partial \theta} di\right|}{\left|\int \frac{\partial q_{1i}}{\partial h_i} \frac{\partial f^{hi}}{\partial \delta} di\right|}. \tag{31}$$

Therefore, for our irrelevance result to be valid, it is necessary and sufficient that this object remains independent of the trading cost c, for any level of c. We have already established that cross-sectional heterogeneity across investors can break the irrelevance result. In Theorem 9, we identify the key conditions behind our irrelevance result when investors are ex-ante identical.

Theorem 9. (Irrelevance theorem for general utility and signal structure) *In an economy with ex-ante identical investors with general preferences and signal structure, price informativeness is independent of the level of trading costs in a symmetric equilibrium when* Π*, defined in Eq.* (31)*, is independent of the level of trading costs. Under the plausible assumption that* $\frac{\partial f^s}{\partial \theta}$ *and* $\frac{\partial f^h}{\partial \delta}$ *are independent of* c *and constant, the effect of trading costs on price informativeness exclusively depends on the aggregate differential response to trading costs of the demand sensitivities to information and noise, that is,*

$$\operatorname{sgn}\left(\frac{d\Pi}{dc}\right) = \operatorname{sgn}\left(\frac{d\log\left(\left|\int \frac{\partial q_{1i}}{\partial s_i} di\right|\right)}{dc} - \frac{d\log\left(\left|\int \frac{\partial q_{1i}}{\partial h_i} di\right|\right)}{dc}\right). \tag{32}$$

In terms of marginal utilities, $\frac{d\Pi}{dc}$ *is zero locally at any equilibrium, if and only if* $\frac{\partial^2 U^i(q_i;s_i,h_i,p,c)}{\partial q_i^2} = \frac{\partial^2 U^j(q_i;s_i,h_i,p,c)}{\partial q_j^2}$, $\forall i, j$, $\frac{\partial^2 U^i(q_i;s_i,h_i,p,c)}{\partial q_i \partial s_i} = \frac{\partial^2 U^i(q_i;s_i,h_i,p,c)}{\partial q_i \partial s_i}$, $\forall i$, *and* $\frac{\partial^2 U^i(q_i;s_i,h_i,p,c)}{\partial q_i \partial h_i} = \frac{\partial^2 U^i(q_i;s_i,h_i,p,c)}{\partial q_i \partial h_i}$, $\forall i$.

In general, we expect $\frac{\partial f^{si}}{\partial \theta}$ and $\frac{\partial f^{hi}}{\partial \delta}$ to be independent of c: there is no reason to believe that a change in trading costs should affect the structure of the exogenous signals.[23] From Eq. (31) and Eq. (32), it

[23] As shown in Section 5, trading costs affect the structure of the signals when investors have an ex-ante information choice.

becomes clear that the key condition for the irrelevance result is that average *ex-post* demand sensitivities to information and noise respond symmetrically on the aggregate to a change in trading costs. In the CARA-Gaussian case, ex-ante symmetry implies ex-post symmetry, consistent with our new finding. However, more generally, having ex-ante identical investors is not sufficient for the irrelevance result to hold. If ex-ante identical investors are ex-post heterogeneous regarding their demand sensitivities to the fundamental signal and the noise signal, trading costs may affect price informativeness, according to Eq. (32). Hence, Theorem 9 highlights that the key sufficient statistics that determine the response of price informativeness to trading costs are the average ex-post demand sensitivities to information $\frac{\partial q_{1i}}{\partial s_i}$ and hedging $\frac{\partial q_{1i}}{\partial h_i}$.

Because only aggregates matter, one could think of a model in which ex-post demand sensitivities respond differently to the trading costs across the population, but the aggregate effect cancels out. Given that the solution to the mean-variance model is a first-order approximation to more general problems, we conjecture that some form of approximate ex-post symmetry must hold when investors are ex-ante identical.

7 Conclusion

This paper provides a systematic analysis of the effects of trading costs on information aggregation and information acquisition in financial markets. An elementary and intuitive set of irrelevance results emerges from our analysis: when investors are ex-ante identical, changes in trading costs equally discourage trading on both information and hedging needs, leaving price informativeness unchanged. This result holds for different forms of trading costs, alternative formulations of aggregate noise, and competitive and strategic environments. Up to a first-order, they will apply to any model of financial market trading. We have also shown that trading costs discourage the acquisition of information, which tends to reduce price informativeness.

Although we have already explored in this paper how several dimensions of heterogeneity determine the effect of trading costs on the informational role of financial markets, there is scope to study how alternative departures from our symmetric benchmarks better describe the effects of varying trading costs in alternative models of trading in financial markets. This is a fruitful avenue for future research.

Appendix

Information structure for constant conditional covariance

We assume that $\mathbb{C}ov\left(\theta, n_{2i}|\, s_i, h_i, p\right) = h_i$ for all information sets. The formulation in the main text is isomorphic to the following information structure. Let us expand the asset payoff in the following way. Let $v = \theta + z$ be the fundamental value of the asset, where $\theta \sim N\left(\overline{\theta}, \tau_\theta^{-1}\right)$ is the learnable component of asset value and $z \sim N\left(0, \tau_z^{-1}\right)$ is the unlearnable part and θ and z are independent. Let $n_{2i} = h_i z + u_{ni}$. Then, since $\mathbb{C}ov\left(\theta, n_{2i}\right) = 0$ and $\mathbb{C}ov\left(v, n_{2i}\right) = h_i \tau_z^{-1}$, we have

$$\mathbb{C}ov\left(v, n_{2i}|\, s_i, h_i, p\right) = h_i \tau_z^{-1}$$

for all$\{s_i, h_i, p\}$ since there is no information about z in $\{s_i, p\}$.

An investor's problem in this case is

$$\max_{q_{1i}} \left(\mathbb{E}\left[\theta|s_i, h_i, p\right] - \gamma_i h_i \tau_z^{-1} - p\right) q_{1i} - \frac{\gamma_i}{2}\left(\mathbb{V}ar\left[\theta|s_i, h_i, p\right] + \tau_z^{-1}\right) q_{1i}^2 - \frac{c}{2}\left(\Delta q_{1i}\right)^2.$$

In this case, the net demand for the asset for an investor i is

$$\Delta q_{1i} = \frac{\mathbb{E}\left[\theta|s_i, h_i, p\right] - \gamma_i h_i \tau_z^{-1} - p}{\gamma_i \mathbb{V}ar\left[\theta|s_i, h_i, p\right] + \gamma_i \tau_z^{-1} + c} - q_{0i},$$

which is isomorphic to the investor facing a trading cost of $\gamma_i \tau_z^{-1} + c$ while changing the scale of the hedging needs. Consequently, this specification yields the same results as the more parsimonious specification that we adopt in the main body of the paper.

Proofs: Section 3

Investors' portfolio problem

Under the assumptions of CARA utility and normal uncertainty, an investor i solves the following mean-variance problem

$$\max_{q_{1i}} \mathbb{E}\left[w_{2i}\right] - \frac{\gamma_i}{2} \mathbb{V}ar\left[w_{2i}\right],$$

where w_{2i} is given by Eq. (5) in the text. After getting rid of constants, investor i solves Eq. (6) in the text, with an optimality condition given by

$$\begin{aligned} q_{1i} &= \underbrace{\frac{\gamma_i \mathbb{V}ar\left[\theta|s_i, h_i, p\right]}{\gamma_i \mathbb{V}ar\left[\theta|s_i, h_i, p\right] + c}}_{\equiv \omega_i(c)} \underbrace{\frac{\mathbb{E}\left[\theta|s_i, h_i, p\right] - \gamma_i h_i - p}{\gamma_i \mathbb{V}ar\left[\theta|s_i, h_i, p\right]}}_{\equiv \hat{q}_{1i}} + \underbrace{\frac{c}{\gamma_i \mathbb{V}ar\left[\theta|s_i, h_i, p\right] + c}}_{\equiv 1-\omega_i(c)} q_{0i} \\ &= \omega_i\left(c\right) \hat{q}_{1i} + \left(1 - \omega_i\left(c\right)\right) q_{0i}. \end{aligned}$$

The demand elasticity of investor i is given by $\frac{\partial q_{1i}}{\partial p} = -\frac{1}{\gamma_i \mathbb{V}ar[\theta|s_i, h_i, p] + c}$. We can write the net risky asset demand by investor i as

$$\Delta q_{1i} = \omega_i\left(c\right)\left(\hat{q}_{1i} - q_{0i}\right).$$

Equilibrium with ex-ante identical investors

In a symmetric equilibrium in linear strategies in which all investors are ex-ante identical, we guess (and verify) that the optimal net asset demand of investor i takes the form

$$\Delta q_{1i} = \alpha_s s_i - \alpha_h h_i - \alpha_p p + \psi, \tag{A.1}$$

where α_s, α_h and α_p are positive scalars, and ψ can take positive and negative values. The market clearing condition $\int \Delta q_{1i} di = 0$ implies that the equilibrium price takes the form

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}} \theta - \frac{\overline{\alpha_h}}{\overline{\alpha_p}} \delta + \frac{\overline{\psi}}{\overline{\alpha_p}}, \tag{A.2}$$

where we use the notation $\overline{\alpha_s}$, $\overline{\alpha_h}$, $\overline{\alpha_p}$, and $\overline{\psi}$ to emphasize that prices are a function of aggregates. In equilibrium $\overline{\alpha_s} = \alpha_s$, $\overline{\alpha_h} = \alpha_h$, $\overline{\alpha_p} = \alpha_p$, and $\overline{\psi} = \psi$. We assume a Strong Law of Large Numbers, as described in the Appendix of Vives (2008), to be able to write $\int s_i di = \theta$ and $\int h_i di = \delta$ in Eq. (A.2). Hence, using the distributions of θ and δ, defined in Eq. (2) and Eq. (4) in the text, we can write the unconditional distribution of the equilibrium price p as

$$p \sim N\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\overline{\theta} + \frac{\overline{\psi}}{\overline{\alpha_p}}, \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \tau_\theta^{-1} + \left(\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2 \tau_\delta^{-1}\right).$$

While the conditional distribution of the equilibrium price p given the fundamental θ follows

$$p|\theta \sim N\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta + \frac{\overline{\psi}}{\overline{\alpha_p}}, \left(\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2 \tau_\delta^{-1}\right).$$

We denote by $\hat{p} = \frac{\overline{\alpha_p}}{\overline{\alpha_s}}p - \frac{\overline{\psi}}{\overline{\alpha_s}}$ the unbiased signal of θ for a given external observer (denoted by e), which is distributed as follows

$$\hat{p}|\theta \sim N\left(\theta, \left(\tau_{\hat{p}}^e\right)^{-1}\right), \quad \text{where} \quad \tau_{\hat{p}}^e = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 \tau_\delta. \tag{A.3}$$

We define $\tau_{\hat{p}}$ as the precision of the information contained in the price for an individual investor, which incorporates the information conveyed by the hedging realization. Formally $\hat{p} + \frac{\overline{\alpha_h}}{\overline{\alpha_s}}\frac{\tau_h}{\tau_\delta+\tau_h}h_i = \theta - \frac{\alpha_h}{\alpha_s}\left(\delta - \mathbb{E}\left[\delta|h_i\right]\right)$, where

$$\hat{p} + \frac{\overline{\alpha_h}}{\overline{\alpha_s}}\frac{\tau_h}{\tau_\delta + \tau_h}h_i \bigg| \theta \sim N\left(\theta, \tau_{\hat{p}}^{-1}\right), \quad \text{where} \quad \tau_{\hat{p}} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 \left(\tau_\delta + \tau_h\right).$$

Solving the optimal filtering problem – as described in the Online Appendix – from the perspective of investor i allows us to write

$$\mathbb{E}\left[\theta|s_i, h_i, p\right] = \frac{\tau_\theta\overline{\theta} + \tau_s s_i + \tau_{\hat{p}}\left(\hat{p} + \frac{\overline{\alpha_h}}{\overline{\alpha_s}}\mathbb{E}\left[\delta|h_i\right]\right)}{\tau_\theta + \tau_s + \tau_{\hat{p}}}, \text{ where } \mathbb{E}\left[\delta|h_i\right] = \frac{\tau_h}{\tau_\delta + \tau_h}h_i, \tag{A.4}$$

$$\mathbb{V}ar\left[\theta|s_i, h_i, p\right] = \frac{1}{\tau_\theta + \tau_s + \tau_{\hat{p}}}. \tag{A.5}$$

The expected value and the variance of θ, conditional on private signals and equilibrium prices, are the inputs to the portfolio decision of investors, as described in Eq. (7) in the text.

We define κ, to simplify notation, as

$$\kappa \equiv \gamma\mathbb{V}ar\left[\theta|s_i, h_i, p\right] + c.$$

Matching coefficients with our initial guess in Eq. (A.1), we show that α_s, α_h, α_p, and ψ must satisfy

$$\alpha_s = \frac{1}{\kappa}\frac{\tau_s}{\tau_\theta + \tau_s + \tau_{\hat{p}}}, \tag{A.6}$$

$$\alpha_h = \frac{1}{\kappa}\left(\gamma - \frac{\tau_h}{\tau_\theta + \tau_s + \tau_{\hat{p}}}\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right), \tag{A.7}$$

$$\alpha_p = \frac{1}{\kappa}\frac{\tau_s}{\tau_s + \tau_{\hat{p}}}, \quad \text{and} \tag{A.8}$$

$$\psi = \alpha_p\left(\frac{\tau_\theta}{\tau_\theta + \tau_s + \tau_{\hat{p}}}\overline{\theta} - \gamma\mathbb{V}ar\left[\theta|s_i, h_i, p\right]q_0\right). \tag{A.9}$$

Combining Eq. (A.6) and Eq. (A.7) allows us to characterize $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$, and consequently $\tau_{\hat{p}}$, $\tau_{\hat{p}}^e$, and $\mathbb{V}ar\left[\theta|s_i, h_i, p\right]$, as a function of primitives. The solution to the following cubic on x determines the equilibrium values of $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$

$$F\left(x\right) := \gamma\left(\tau_\delta + \tau_h\right)x^3 - \tau_h x^2 + \gamma\left(\tau_s + \tau_\theta\right)x - \tau_s = 0. \tag{A.10}$$

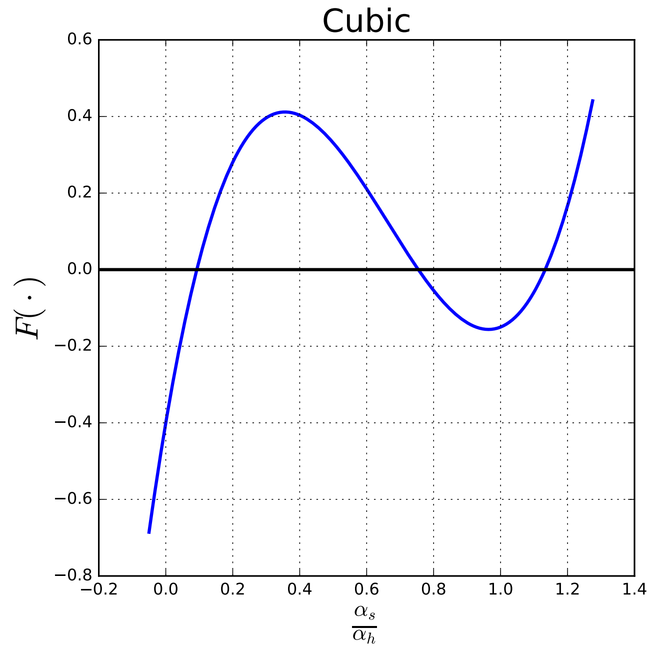

Figure A.1: Illustration of cubic equation (A.10)

Lemma 1. (Existence and multiplicity)

Proof. Because $F(x)$ is continuous, $F(0) < 0$, and $\lim_{x\to\infty} F(x) = \infty$, it follows from the intermediate value theorem that there exists at least one real positive solution to $F(x) = 0$. Using the properties of a cubic function, it is straightforward to show that the slope of the function $F(\cdot)$, $F_x \equiv \frac{dF}{dx} = 3\gamma(\tau_\delta + \tau_h)x^2 - 2\tau_h x + \gamma(\tau_s + \tau_\theta)$, is non-positive only for the middle equilibria in case of multiplicity, but positive otherwise. Figure A.1 illustrates the possible multiple solutions of Eq. (A.10), by plotting $F\left(\frac{\alpha_s}{\alpha_h}\right)$. □

To establish the stability of the solution, the cubic can be expressed in the form of a (change in the) best response as

$$\Delta\left(\frac{\alpha_s}{\alpha_h}\right) := \frac{\alpha_s}{\alpha_h} - \frac{\overline{\alpha_s}}{\overline{\alpha_h}} = \frac{\tau_s}{\gamma(\tau_\delta+\tau_h)\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 - \tau_h\frac{\overline{\alpha_s}}{\overline{\alpha_h}} + \gamma(\tau_\theta+\tau_s)} - \frac{\overline{\alpha_s}}{\overline{\alpha_h}}.$$

It follows that the middle root is always unstable, because $\Delta'(\cdot) > 0$. It also follows that one can find a specific equilibrium convergence process that makes the lower and higher equilibria stable, because $\Delta'(\cdot) < 0$.

The following comparative statics results on τ_θ and τ_δ are valid for any solution

$$\frac{\partial\left(\frac{\alpha_s}{\alpha_h}\right)}{\partial\tau_\theta} = -\frac{\gamma\frac{\alpha_s}{\alpha_h}}{F_x} < 0 \quad \text{and}$$
$$\frac{\partial\left(\frac{\alpha_s}{\alpha_h}\right)}{\partial\tau_\delta} = -\frac{\gamma\left(\frac{\alpha_s}{\alpha_h}\right)^3}{F_x} < 0.$$

The comparative statistics on γ, τ_s, and τ_h are given by

$$\frac{\partial\left(\frac{\alpha_s}{\alpha_h}\right)}{\partial\gamma} = -\frac{(\tau_\delta+\tau_h)\left(\frac{\alpha_s}{\alpha_h}\right)^3 + (\tau_s+\tau_\theta)\frac{\alpha_s}{\alpha_h}}{F_x},$$
$$\frac{\partial\left(\frac{\alpha_s}{\alpha_h}\right)}{\partial\tau_s} = -\frac{\gamma}{F_x}\left(\frac{\alpha_s}{\alpha_h} - \frac{1}{\gamma}\right), \quad \text{and}$$
$$\frac{\partial\left(\frac{\alpha_s}{\alpha_h}\right)}{\partial\tau_h} = -\frac{\gamma}{F_x}\left(\frac{\alpha_s}{\alpha_h} - \frac{1}{\gamma}\right)\left(\frac{\alpha_s}{\alpha_h}\right)^2.$$

It can be easily shown that all solutions to Eq. (A.10) satisfy $\frac{\alpha_s}{\alpha_h} < \frac{1}{\gamma}$, which implies the following sign for the comparative statics in the high and low equilibria

$$\frac{\partial\left(\frac{\alpha_s}{\alpha_h}\right)}{\partial\gamma} < 0, \quad \frac{\partial\left(\frac{\alpha_s}{\alpha_h}\right)}{\partial\tau_s} > 0, \text{ and } \frac{\partial\left(\frac{\alpha_s}{\alpha_h}\right)}{\partial\tau_h} > 0.$$

In the middle equilibrium, these three comparative statics are reversed.

The comparative statistics on price informativeness for an external observer follow from those of $\frac{\alpha_s}{\alpha_h}$, with the exception of $\frac{\partial\tau^e_{\hat{p}}}{\partial\tau_\delta}$, which can be positive or negative

$$\frac{\partial\tau^e_{\hat{p}}}{\partial\tau_\delta} = 2\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\tau_\delta + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 \frac{\partial\left(\frac{\alpha_s}{\alpha_h}\right)}{\partial\tau_\delta} \gtreqless 0.$$

A full characterization of the equilibrium price also requires finding the ratios $\frac{\alpha_s}{\alpha_p}$, $\frac{\alpha_h}{\alpha_p}$, and $\frac{\psi}{\alpha_p}$. These are respectively given by

$$\frac{\alpha_s}{\alpha_p} = \frac{\tau_s + \tau_{\hat{p}}}{\tau_\theta + \tau_s + \tau_{\hat{p}}}, \tag{A.11}$$

$$\frac{\alpha_h}{\alpha_p} = \frac{\gamma - \frac{\frac{\alpha_s}{\alpha_h}\tau_h}{\tau_\theta+\tau_s+\tau_{\hat{p}}}}{\frac{\tau_s}{\tau_s+\tau_{\hat{p}}}}, \quad \text{and} \tag{A.12}$$

$$\frac{\psi}{\alpha_p} = \frac{\tau_\theta}{\tau_\theta + \tau_s + \tau_{\hat{p}}}\overline{\theta} - \gamma\mathbb{V}ar\left[\theta|s_i, h_i, p\right] q_0 \tag{A.13}$$

The first term in the expression for $\frac{\psi}{\alpha_p}$ contains an expected payoff and the second term has a risk premium correction. Although we do not emphasize this result in our statement of Theorem 1, given that $\frac{\psi}{\alpha_p}$ is independent of c, we can conclude that asset prices, not only asset price informativeness and volatility, are invariant to the level of trading costs.

The equilibrium price can thus be written as

$$p = \frac{\tau_s + \tau_{\hat{p}}}{\tau_\theta + \tau_s + \tau_{\hat{p}}}\theta + \frac{\tau_\theta}{\tau_\theta + \tau_s + \tau_{\hat{p}}}\overline{\theta} - \gamma\left(\frac{1}{\frac{\tau_s}{\tau_s+\tau_{\hat{p}}}}\delta + \mathbb{V}ar\left[\theta|s_i, h_i, p\right] q_0\right) + \frac{\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\tau_h}{\frac{\tau_\theta+\tau_s+\tau_{\hat{p}}}{\frac{\tau_s}{\tau_s+\tau_{\hat{p}}}}}\delta,$$

where the unconditional expectation of the price corresponds to $\mathbb{E}[p] = \overline{\theta} - \gamma\mathbb{V}ar\left[\theta|s_i, h_i, p\right] q_0$. We can thus write the asset price in a given equilibrium as

$$p = \mathbb{E}[p] + \frac{\alpha_s}{\alpha_p}\left(\theta - \overline{\theta}\right) - \frac{\alpha_h}{\alpha_p}\delta.$$

Finally, by combining Eq. (A.1) and Eq. (A.2), we can write the net change in investor i's equilibrium portfolio position as

$$\Delta q_{1i} = \alpha_s\left(s_i - \theta\right) - \alpha_h\left(h_i - \delta\right) = \alpha_s\varepsilon_{si} - \alpha_h\varepsilon_{hi}.$$

The equation $\alpha_s\left(s_i - \theta\right) - \alpha_h\left(h_i - \delta\right) = 0$ represents a straight line in the space $s_i \times h_i$, with slope $\frac{dh_i}{ds_i} = \frac{\alpha_s}{\alpha_h}$. It denotes the (measure zero) set of investors who decide not to trade. Investors above this line are sellers of the risky asset, while investors below this line are buyers of the risky asset. Given that the distributions of s_i and h_i are uncorrelated and symmetric, half of the investors will be buyers for any realization of signals and hedging needs, while the other half will be sellers. We can therefore establish that $\Delta q_{1i} \sim N\left(0, \alpha_s^2\tau_s^{-1} + \alpha_h^2\tau_h^{-1}\right)$. The distribution of $|\Delta q_{1i}|$ is a half-normal, with a mean $\mathbb{V}ar\left[\Delta q_{1i}\right]\sqrt{\frac{2}{\pi}}$. Using a Strong Law of Large Numbers, we can write volume exactly in a given equilibrium as

$$\mathcal{V} = \frac{1}{2}\int |\Delta q_{1i}|\, di = \frac{1}{\sqrt{2\pi}}\left(\alpha_s^2\tau_s^{-1} + \alpha_h^2\tau_h^{-1}\right)^{\frac{1}{2}}.$$

Theorem 1. (Irrelevance theorem with ex-ante identical investors)

Proof. It suffices to show that $\frac{\alpha_s}{\alpha_h}$ is independent of c. The solution to Eq. (A.10) does not depend on c, which proves our claim. □

Equilibrium with ex-ante heterogeneous investors

In the equilibrium in linear strategies with ex-ante heterogeneous investors, we guess and verify that the optimal portfolio of investor i takes the form

$$\Delta q_{1i} = \alpha_{si} s_i - \alpha_{hi} h_i - \alpha_{pi} p + \psi_i, \tag{A.14}$$

where α_{si}, α_{hi}, and α_{pi} are positive scalars for every investor and ψ_i can take positive or negative values. The market clearing condition $\int \Delta q_{1i} di = 0$ implies that the equilibrium price takes the form

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\delta + \frac{\overline{\psi}}{\overline{\alpha_p}},$$

where we define

$$\overline{\alpha_s} = \int \alpha_{si} di, \qquad \overline{\alpha_h} = \int \alpha_{hi} di, \qquad \overline{\alpha_p} = \int \alpha_{pi} di, \quad \text{and} \quad \overline{\psi} \equiv \int \psi_i di.$$

A Strong Law of Large Numbers guarantees that $\int \alpha_{si}\varepsilon_{si} di \to 0$ and $\int \alpha_{hi}\varepsilon_{hi} di \to 0$ almost surely, so that we can write $\int \alpha_{si} s_i di = \overline{\alpha_s}\theta$ and $\int \alpha_{hi} h_i di = \overline{\alpha_h}\delta$.

Hence, we can write the distribution of the price p as

$$p \sim N\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\overline{\theta} + \frac{\psi}{\overline{\alpha_p}}, \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \tau_\theta^{-1} + \left(\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2 \tau_\delta^{-1}\right).$$

While the conditional distribution of the equilibrium price p given the fundamental θ follows

$$p|\theta \sim N\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta + \frac{\overline{\psi}}{\overline{\alpha_p}}, \left(\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2 \tau_\delta^{-1}\right).$$

We denote by $\hat{p} = \frac{\overline{\alpha_p}}{\overline{\alpha_s}} p - \frac{\overline{\psi}}{\overline{\alpha_s}}$ the unbiased signal of θ for a given external observer (denoted by e), which is distributed as follows

$$\hat{p}|\theta \sim N\left(\theta, \left(\tau_{\hat{p}}^e\right)^{-1}\right) \quad \text{where} \quad \tau_{\hat{p}}^e = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 \tau_\delta.$$

We define $\tau_{\hat{p}i}$ as the precision of the information contained in the price for an individual investor, which incorporates the information conveyed by the hedging realization. Formally

$$\hat{p} + \frac{\overline{\alpha_h}}{\overline{\alpha_s}} \frac{\tau_{hi}}{\tau_\delta + \tau_{hi}} h_i \bigg| \theta \sim N\left(\theta, \tau_{\hat{p}i}^{-1}\right) \quad \text{where} \quad \tau_{\hat{p}i} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 (\tau_\delta + \tau_{hi}).$$

Solving the optimal filtering problem – as described in the Online Appendix – from the perspective of investor i allows us to write

$$\mathbb{E}[\theta|s_i, h_i, p] = \frac{\tau_\theta \overline{\theta} + \tau_{si} s_i + \tau_{\hat{p}i}\left(\hat{p} + \frac{\overline{\alpha_h}}{\overline{\alpha_s}}\mathbb{E}[\delta|h_i]\right)}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}}, \text{ where } \mathbb{E}[\delta|h_i] = \frac{\tau_{hi}}{\tau_\delta + \tau_{hi}} h_i \tag{A.15}$$

$$\mathbb{V}ar[\theta|s_i, h_i, p] = \frac{1}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}}. \tag{A.16}$$

When needed, we define $\tau_{\theta|s_i,h_i,p} = \frac{1}{\mathbb{V}ar[\theta|s_i,h_i,p]} = \tau_\theta + \tau_{si} + \tau_{\hat{p}i}$. To simplify the notation, we define

$$\kappa_i \equiv \gamma_i \mathbb{V}ar[\theta|s_i, h_i, p] + c = \frac{\gamma_i}{\tau_{\theta|s_i,h_i,p}} + c.$$

Matching coefficients with our guess in Eq. (A.14), we characterize α_{si}, α_{hi}, α_{pi}, and ψ_i as the solution to the following system of equations

$$\alpha_{si} = \frac{1}{\kappa_i}\frac{\tau_{si}}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}}, \quad \alpha_{hi} = \frac{1}{\kappa_i}\left(\gamma_i - \frac{\overline{\alpha_s}}{\overline{\alpha_h}}\frac{\tau_{hi}}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}}\right),$$

$$\alpha_{pi} = \frac{1}{\kappa_i}\left(1 - \frac{\tau_{\hat{p}i}}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}}\frac{\overline{\alpha_p}}{\overline{\alpha_s}}\right), \quad \text{and} \quad \psi_i = \frac{1}{\kappa_i}\left(\frac{1}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}}\left(\tau_\theta\overline{\theta} - \tau_{\hat{p}i}\frac{\overline{\psi}}{\overline{\alpha_s}}\right) - \gamma_i \mathbb{V}ar[\theta|s_i, h_i, p]\, q_{0i}\right).$$

The cross sectional averages, which matter for the determination of demands and prices, are given by

$$\overline{\alpha_s} = \int \frac{1}{\kappa_i} \frac{\tau_{si}}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}} di, \quad \overline{\alpha_h} = \int \frac{1}{\kappa_i} \left(\gamma_i - \frac{\overline{\alpha_s}}{\overline{\alpha_h}} \frac{\tau_{hi}}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}} \right) di,$$

$$\overline{\alpha_p} = \frac{1}{1 + \frac{\int \frac{\tau_{\hat{p}i}}{\tau_{si}} \alpha_{si} di}{\overline{\alpha_s}}} \int \frac{1}{\kappa_i} di, \quad \text{and} \quad \overline{\psi} = \overline{\alpha_p} \int \frac{\frac{\tau_\theta}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}} \overline{\theta} - \gamma_i \mathbb{V}ar\left[\theta | s_i, h_i, p\right] q_{0i}}{\frac{\kappa_i}{\overline{\kappa}}} di$$

where we define $\overline{\kappa} = \left(\int \frac{1}{\kappa_i} di \right)^{-1}$.

As in the symmetric case, a full characterization of the equilibrium hinges on finding the equilibrium value of $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$. In this case, it is given by the solution to the following nonlinear equation in $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$

$$\frac{1}{\frac{\overline{\alpha_s}}{\overline{\alpha_h}}} = \frac{\int \frac{1}{\kappa_i} \left(\gamma_i - \frac{\overline{\alpha_s}}{\overline{\alpha_h}} \frac{\tau_{hi}}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}} \right) di}{\int \frac{1}{\kappa_i} \frac{\tau_{si}}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}} di} = \frac{\int \frac{\gamma_i - \frac{\overline{\alpha_s}}{\overline{\alpha_h}} \frac{\tau_{hi}}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}}}{\gamma_i \mathbb{V}ar[\theta | s_i, h_i, p] + c} di}{\int \frac{1}{\gamma_i \mathbb{V}ar[\theta | s_i, h_i, p] + c} \frac{\tau_{si}}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}} di} = \frac{\int \frac{\gamma_i - \frac{\overline{\alpha_s}}{\overline{\alpha_h}} \frac{\tau_{hi}}{\tau_\theta + \tau_{si} + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 (\tau_\delta + \tau_{hi})}}{\frac{\gamma_i}{\tau_\theta + \tau_{si} + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 (\tau_\delta + \tau_{hi})} + c} di}{\int \frac{1}{\frac{\gamma_i}{\tau_\theta + \tau_{si} + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 \tau_\delta} + c} \frac{\tau_{si}}{\tau_\theta + \tau_{si} + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 (\tau_\delta + \tau_{hi})} di}.$$

From our analysis of the symmetric case, we conjecture and find numerically that this equation may have multiple solutions – in our simulations, we choose values of γ sufficiently high/low so that there exists a unique solution. Once the equilibrium value of $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$ is determined, $\tau_{\hat{p}i}$ and $\tau_{\theta|s_i,h_i,p}$ are uniquely pinned down. It follows immediately that, if κ_i is constant, $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$ is independent of c for any value of c. The reverse result is also true: only when $\kappa_i = \kappa$, $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$ is independent of c for any value of c. Therefore, $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$ is independent of c if and only if $\kappa_i = \kappa$, $\forall i$.

Theorem 2. (One-dimensional heterogeneity)

Proof. From Theorem 3b, we know that

$$\text{sgn}\left(\frac{d\tau^e_{\hat{p}}}{dc}\right) = -\,\text{sgn}\left(\mathbb{C}ov_i\left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}, \frac{1}{\kappa_i}\right]\right).$$

With two groups of investors with measures μ_A and μ_B, the relevant cross-sectional covariance corresponds to

$$\mathbb{C}ov_i\left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}, \frac{1}{\kappa_i}\right] = \frac{\mu_A}{\kappa_A} \frac{\mu_B}{\kappa_B} \frac{\alpha_{hA}\alpha_{hB}}{\overline{\alpha_s \alpha_h}} \left(\frac{\alpha_{sA}}{\alpha_{hA}} - \frac{\alpha_{sB}}{\alpha_{hB}}\right)(\kappa_B - \kappa_A).$$

So finding the sgn $\left(\frac{d\tau^e_{\hat{p}}}{dc}\right)$ is identical to finding sgn $\left(\left(\frac{\alpha_{sA}}{\alpha_{hA}} - \frac{\alpha_{sB}}{\alpha_{hB}}\right)(\kappa_B - \kappa_A)\right)$. Without loss of generality we can assume that $\kappa_B > \kappa_A$. This is the same as assuming one directional deviations in the following ways for three different possibilities of one-dimensional heterogeneity: i) $\gamma_B > \gamma_A$, $\tau_{sA} = \tau_{sB} = \tau_s$ and $\tau_{hA} = \tau_{hB} = \tau_h$, ii) $\tau_{sA} > \tau_{sB}$, $\gamma_A = \gamma_B = \gamma$ and $\tau_{hA} = \tau_{hB} = \tau_h$, or iii) $\tau_{hA} > \tau_{hB}$, $\gamma_A = \gamma_B = \gamma$ and $\tau_{sA} = \tau_{sB} = \tau_s$. So finding the sgn $\left(\frac{d\tau^e_{\hat{p}}}{dc}\right)$ becomes identical to finding sgn $\left(\frac{\alpha_{sA}}{\alpha_{hA}} - \frac{\alpha_{sB}}{\alpha_{hB}}\right)$.

For each of the three cases involving one directional deviations, it is the case that sgn $\left(\frac{\alpha_{sA}}{\alpha_{hA}} - \frac{\alpha_{sB}}{\alpha_{hB}}\right)$ respectively corresponds to: i) $-$ sgn $\left(\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial\gamma_i}\right)$, ii) sgn $\left(\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial\tau_{si}}\right)$, and iii) sgn $\left(\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial\tau_{hi}}\right)$. We establish in the Online Appendix that sgn $\left(\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial\gamma_i}\right) < 0$, sgn $\left(\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial\tau_{si}}\right) > 0$, sgn $\left(\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial\tau_{si}}\right) > 0$, which implies that sgn $\left(\frac{d\tau^e_{\hat{p}}}{dc}\right) < 0$. ☐

Theorem 3. (Two-dimensional heterogeneity and general directional effects of trading costs with ex-ante heterogeneous investors)

Proof. a) From Theorem 3b, we know that

$$\text{sgn}\left(\frac{d\tau^e_{\hat{p}}}{dc}\right) = -\,\text{sgn}\left(\mathbb{C}ov_i\left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}, \frac{1}{\kappa_i}\right]\right).$$

☐

With two groups of investors with measures μ_A and μ_B, the relevant cross-sectional covariance corresponds to

$$\mathbb{C}ov_i\left[\frac{\alpha_{si}}{\overline{\alpha_s}}-\frac{\alpha_{hi}}{\overline{\alpha_h}},\frac{1}{\kappa_i}\right]=\frac{\mu_A}{\kappa_A}\frac{\mu_B}{\kappa_B}\frac{\alpha_{hA}\alpha_{hB}}{\overline{\alpha_s}\overline{\alpha_h}}\left(\frac{\alpha_{sA}}{\alpha_{hA}}-\frac{\alpha_{sB}}{\alpha_{hB}}\right)(\kappa_B-\kappa_A).$$

So finding the sgn $\left(\frac{d\tau_{\hat{p}}^e}{dc}\right)$ is identical to finding sgn $\left(\left(\frac{\alpha_{sA}}{\alpha_{hA}}-\frac{\alpha_{sB}}{\alpha_{hB}}\right)(\kappa_B-\kappa_A)\right)$. Since $\kappa_i=\gamma_i\mathbb{V}ar\left[\theta|s_i,h_i,p\right]+c$ we have

$$\frac{\partial\kappa_i}{\partial\gamma_i}>0,\quad\frac{\partial\kappa_i}{\partial\tau_{si}}<0,\quad\frac{\partial\kappa_i}{\partial\tau_{hi}}<0.$$

Moreover, we establish in the Online Appendix that sgn $\left(\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial\gamma_i}\right)<0$, sgn $\left(\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial\tau_{si}}\right)>0$, sgn $\left(\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial\tau_{si}}\right)>0$. Then, for all economies in which

$$(\gamma_A-\gamma_B)(\tau_{sA}-\tau_{sB})<0\quad\text{and}\quad\tau_{hA}=\tau_{hB}$$

or

$$(\gamma_A-\gamma_B)(\tau_{hA}-\tau_{hB})<0\quad\text{and}\quad\tau_{sA}=\tau_{sB}$$

or

$$(\tau_{hA}-\tau_{hB})(\tau_{sA}-\tau_{sB})>0\quad\text{and}\quad\gamma_A=\gamma_B$$

we have sgn $\left(\frac{d\tau_{\hat{p}}^e}{dc}\right)<0$. Since $\mathbb{C}ov_i\left[\frac{\alpha_{si}}{\overline{\alpha_s}}-\frac{\alpha_{hi}}{\overline{\alpha_h}},\frac{1}{\kappa_i}\right]$ is continuous in τ_{si}, τ_{hi}, and γ_i, sgn $\left(\frac{d\tau_{\hat{p}}^e}{dc}\right)<0$ for over half of the parameter space when there is two-dimensional heterogeneity.

Proof. b) We show in the Online Appendix that $\frac{d\log\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}$ is given by

$$\frac{d\log\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}=\frac{\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}}{\frac{\overline{\alpha_s}}{\overline{\alpha_h}}}=\frac{\mathbb{C}ov_i\left[\frac{\alpha_{si}}{\overline{\alpha_s}}-\frac{\alpha_{hi}}{\overline{\alpha_h}},-\frac{1}{\kappa_i}\right]}{1+2\int\frac{1}{\kappa_i}\left(\frac{\gamma_i}{\kappa_i}\frac{1}{\tau_{\theta|s_i,h_i,p}}+c\frac{\alpha_{si}}{\overline{\alpha_s}}\right)\frac{\tau_{\hat{p}}}{\tau_{\theta|s_i,h_i,p}}di}.$$

Eq. (19) follows immediately. □

Proofs: Section 5

In this section, we focus on the equilibria in which $\pi\in\{0,1\}$ to simplify the notation and the analysis. All the arguments extend for the cases in which $\pi\in(0,1)$. We make these arguments explicit as necessary.

Endogenous precision of the signal about the fundamental

Investor i chooses τ_{si} solves $\max_{\tau_{si}}\mathbb{E}\left[v_i\right]-\lambda\left(\tau_{si}\right)$, where $\mathbb{E}\left[v_i\right]$ is given by

$$\begin{aligned}\mathbb{E}\left[v_i\right]&=\mathbb{E}\left[\left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p\right)\right]\mathbb{E}\left[q_{1i}^*\right]+\mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]-\frac{1}{2}\left(\gamma\mathbb{V}ar\left[\theta|s_i,h_i,p\right]+c\right)\mathbb{E}\left[\left(q_{1i}^*\right)^2\right]\\&=\mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]-\frac{1}{2}\left(\gamma\mathbb{V}ar\left[\theta|s_i,h_i,p\right]+c\right)\mathbb{E}\left[\left(q_{1i}^*\right)^2\right],\end{aligned}$$

where we use the fact that $\mathbb{E}\left[\left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p\right)\right]=\left(\overline{\theta}-\mathbb{E}\left[p\right]\right)\mathbb{E}\left[q_{1i}^*\right]=0$, given the assumption that $q_{0i}=0$.[24]

The optimal precision choice τ_{si}^* is given by the solution to $H\left(\tau_{si}^*\right)=0$, where

$$H\left(\tau_{si}\right)\equiv\frac{\partial\mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]}{\partial\tau_{si}}+\frac{\gamma}{2}\mathbb{V}ar\left[\theta|s_i,h_i,p\right]^2\mathbb{V}ar\left[q_{1i}^*\right]-\frac{1}{2}\left(\gamma\mathbb{V}ar\left[\theta|s_i,h_i,p\right]+c\right)\frac{\partial\mathbb{V}ar\left[q_{1i}^*\right]}{\partial\tau_{si}}-\lambda'\left(\tau_{si}\right).$$

The expression $H\left(\tau_{si}\right)$ can be rewritten as

$$H\left(\tau_{si}\right)=\frac{1}{2}\frac{\partial\mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]}{\partial\tau_{si}}-\lambda'\left(\tau_{si}\right),$$

[24] As described in Veldkamp (2009), we are assuming that investors' preferences correspond to $\mathbb{E}\left[u_i\left(\mathbb{E}\left[U_i\left(w_{2i}\right)|s_i,p,h_i\right]\right)\right]$, where $U_i\left(w_{2i}\right)=-e^{-\gamma_i w_{2i}}$ and $u_i(x)=-\ln(-x)$.

where we use the following two relations

$$\frac{\partial \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]}{\partial \tau_{si}}=\left(\gamma \mathbb{V}ar\left[\theta|s_i,h_i,p\right]+c\right)\frac{\partial \mathbb{V}ar\left[q_{1i}^*\right]}{\partial \tau_{si}}-\gamma \mathbb{V}ar\left[\theta|s_i,h_i,p\right]^2 \mathbb{V}ar\left[q_{1i}^*\right],$$
$$\frac{\partial \mathbb{V}ar\left[\theta|s_i,h_i,p\right]}{\partial \tau_{si}}=\mathbb{V}ar\left[\theta|s_i,h_i,p\right]^2.$$

The second order condition of the information choice problem is given by

$$\frac{\partial H\left(\tau_{si}\right)}{\partial \tau_{si}}=\frac{\partial^2 \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]}{\partial \tau_{si}^2}-\lambda''\left(\tau_{si}\right)<0,$$

which is strictly negative, guaranteeing that the first order condition is necessary and sufficient for optimality, since

$$\frac{\partial^2 \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]}{\partial \tau_{si}^2}=-2\frac{\mathbb{V}ar\left[\theta|s_i,h_i,p\right]}{\gamma \mathbb{V}ar\left[\theta|s_i,h_i,p\right]+c}\frac{\partial \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|h_i,s_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]}{\partial \tau_{si}}c<0,$$

which uses the fact that

$$\frac{\partial \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|h_i,s_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]}{\partial \tau_{si}}=\frac{1}{\gamma}\omega_i^2\left[\mathbb{V}ar\left[\mathbb{E}\left[\theta|h_i,s_i,p\right]-\gamma h_i-p\right]+\mathbb{V}ar\left[\theta|s_i,h_i,p\right]+\frac{1}{\gamma}c\right]>0,$$

where ω_i is defined in Eq. (9) in the text.

Equilibrium

Lemma 2. (Existence and symmetry of equilibrium)

Proof. Since all investors are infinitesimal, any two investors face the same first order condition, given the choices of all other investors. Because the objective function is strictly concave in the precision of the investor's private information in all its domain, there is at most one solution to the first order condition. Therefore, any two investors make the same precision choice and any equilibrium has to be symmetric. This argument establishes that any equilibrium must be symmetric.

For a given equilibrium of the trading stage we know that $H\left(\tau_{si}\right)$ is continuous in $\tau_{si}\in(0,\infty)$, $\lim_{\tau_{si}\to 0} H\left(\tau_{si}\right)=\infty$, and $\lim_{\tau_{si}\to\infty} H\left(\tau_{si}\right)=-\infty$. If there is a unique equilibrium in the trading game it follows from the intermediate value theorem that there is always a solution to the first order condition $H\left(\tau_{si}\right)=0$. If there are multiple equilibria, there always exists a probability π such that

$$\pi\overline{H}\left(\tau_{si}\right)+(1-\pi)\underline{H}\left(\tau_{si}\right)=0,$$

where $\overline{H}\left(\cdot\right)$ and $\underline{H}\left(\cdot\right)$ are the first order conditions of the investor when the high and low equilibria in the trading game are played with probability 1, respectively. ☐

Finally, in a symmetric equilibrium,

$$\left.\frac{\partial \tau_{si}^*}{\partial \tau_s}\right|_{\tau_{si}^*=\tau_s}=\frac{\left.\frac{\partial H\left(\tau_{si}\right)}{\partial \tau_s}\right|_{\tau_{si}^*=\tau_s}}{-\left.\frac{\partial H\left(\tau_{si}\right)}{\partial \tau_{si}}\right|_{\tau_{si}^*=\tau_s}}<0,$$

which implies that the equilibrium is unique. It is thus sufficient to show that $\left.\frac{\partial H\left(\tau_{si}\right)}{\partial \tau_s}\right|_{\tau_{si}^*=\tau_s}<0$. From the definition of $H\left(\cdot\right)$

$$\frac{\partial H\left(\tau_{si}\right)}{\partial \tau_s}=\frac{\partial^2 \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|h_i,s_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]}{\partial \tau_{si}\partial \tau_s}.$$

In a symmetric equilibrium,

$$\left.\frac{\partial^2 \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|h_i,s_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]}{\partial \tau_{si}\partial \tau_s}\right|_{\tau_{si}^*=\tau_s}=$$
$$=-2\frac{\gamma \mathbb{V}ar\left[\theta|s_i,h_i,p\right]^3}{\left(\gamma \mathbb{V}ar\left[\theta|s_i,h_i,p\right]+c\right)^2}\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\left(\begin{array}{c}2\left(\mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|h_i,s_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]+\frac{1}{\gamma_i}\right)c\\+\mathbb{V}ar\left[\theta|s_i,h_i,p\right]\tau_{\hat{p}i}+\mathbb{V}ar\left[\theta|h_i,s_i,p\right]\frac{\tau_s}{\tau_\theta+\tau_s+\tau_{\hat{p}}}\end{array}\right)\frac{\partial\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{\partial \tau_s}.$$

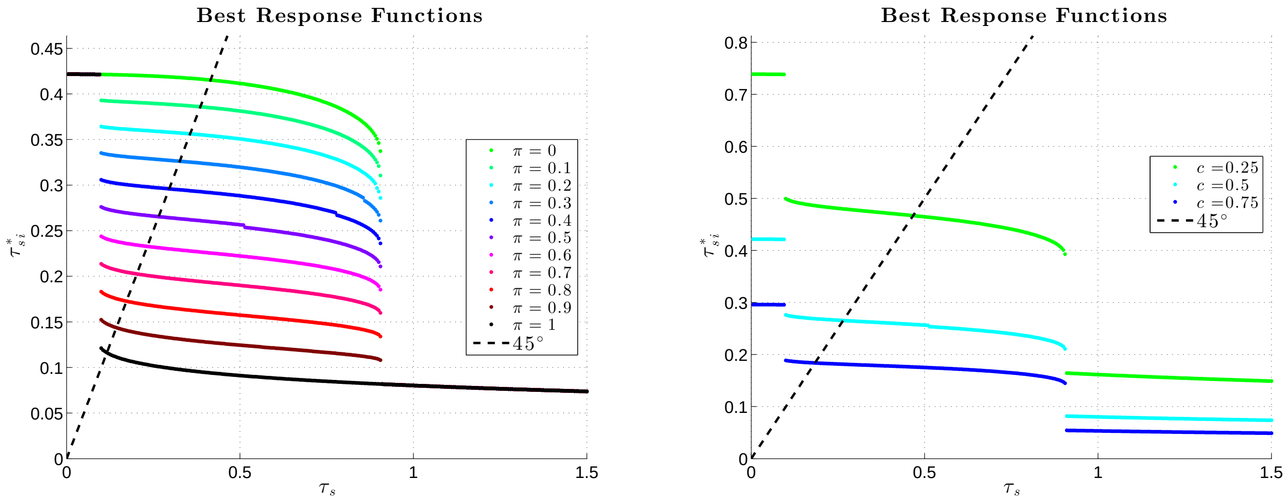

Figure A.2: Best responses for different sunspot values π/trading costs c

The sign of $\frac{\partial\left(\frac{\overline{\alpha_s}}{\alpha_h}\right)}{\partial\tau_s}$ determines whether information acquisition choices are strategic complements or substitutes. Since in both stable equilibria $\frac{\partial\left(\frac{\overline{\alpha_s}}{\alpha_h}\right)}{\partial\tau_s} > 0$, there is a unique equilibrium in the information choice game given the equilibrium sunspot. This highlights that the multiplicity of equilibria in the information acquisition game comes directly from the multiplicity in the trading game and not from strategic complementarities in information acquisition.

Theorem 4. (Effect of trading costs with endogenous precision of the signal on the fundamental)

Proof. The implicit function theorem implies that for any equilibrium in the trading stage

$$\frac{d\tau_{si}^*}{dc} = \frac{\frac{\partial H(\tau_{si})}{\partial c}}{-\frac{\partial H(\tau_{si})}{\partial \tau_{si}}} < 0,$$

because

$$\frac{\partial H\left(\tau_{si}\right)}{\partial c} = \frac{\partial^2 \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|h_i, s_i, p\right] - \gamma h_i - p\right), q_{1i}^*\right]}{\partial\tau_{si}\partial c} = -\frac{\omega_i^2}{\gamma}\left(2\mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|h_i, s_i, p\right] - \gamma h_i - p\right), q_{1i}^*\right] + \frac{1}{\gamma}\right) < 0.$$

Since in any sunspot equilibria the first order condition is a linear combination of the first order condition for the case in which each equilibria is played with probability one, the first result follows. The second result follows directly from the fact that $\frac{\partial\left(\frac{\overline{\alpha_s}}{\alpha_h}\right)}{\partial\tau_s} > 0$ in both stable equilibria. □

The left plot of Figure A.2 illustrates best responses for a given c, while varying the sunspot probability π. The right plot of Figure A.2 illustrates best responses for a given sunspot probability π, while varying the level of trading costs.

Proofs: Section 6

Linear trading costs

Investor i wealth is given by $w_{2i} = n_{2i} + q_{1i}\theta + q_{0i}p - q_{1i}p - \phi\left|\Delta q_{1i}\right|$. Investor i solves the well-behaved problem stated in Eq. (22), with the necessary and sufficient condition for optimality given in Eq. (23). Note that the optimal portfolio demand can be written as

$$q_{1i} = \underbrace{\frac{\mathbb{E}\left[\theta|s_i, h_i, p\right] - \gamma h_i - p - \phi\,\mathrm{sgn}\left(\Delta q_{1i}\right)}{\mathbb{E}\left[\theta|s_i, h_i, p\right] - \gamma h_i - p}}_{\equiv w_{1i}}\underbrace{\frac{\mathbb{E}\left[\theta|s_i, h_i, p\right] - \gamma h_i - p}{\gamma\mathbb{V}ar\left[\theta|s_i, h_i, p\right]}}_{\equiv \hat{q}_{1i}}.$$

The distributions of p, $p|\theta$, and $\hat{p}|\theta$, as well as the conditional moments $\mathbb{E}[\theta|s_i,h_i,p]$ and $\mathbb{V}ar[\theta|s_i,h_i,p]$ take identical expressions as in the benchmark model. Matching coefficients with our guess in Eq. (24), we show that α_s, α_h, α_p, ψ^+, and ψ^- must satisfy

$$\alpha_s = \frac{1}{\kappa}\frac{\tau_s}{\tau_\theta+\tau_s+\tau_{\hat{p}}}, \quad \alpha_h = \frac{1}{\kappa}\left(\gamma - \frac{\tau_h}{\tau_\theta+\tau_s+\tau_{\hat{p}}}\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right), \quad \alpha_p = \frac{1}{\kappa}\frac{\tau_s}{\tau_s+\tau_{\hat{p}}},$$
$$\psi^+ = \alpha_p\left(\frac{\tau_\theta}{\tau_\theta+\tau_s+\tau_{\hat{p}}}\overline{\theta} - \gamma\mathbb{V}ar[\theta|s_i,h_i,p]\,q_0 - \phi\right), \quad \text{and}$$
$$\psi^- = \alpha_p\left(\frac{\tau_\theta}{\tau_\theta+\tau_s+\tau_{\hat{p}}}\overline{\theta} - \gamma\mathbb{V}ar[\theta|s_i,h_i,p]\,q_0 + \phi\right),$$

where $\kappa \equiv \gamma\mathbb{V}ar[\theta|s_i,h_i,p]$. It follows that

$$\frac{\psi^-+\psi^+}{2} = \alpha_p\left(\frac{\tau_\theta}{\tau_\theta+\tau_s+\tau_{\hat{p}}}\overline{\theta} - \gamma\mathbb{V}ar[\theta|s_i,h_i,p]\,q_0\right).$$

Note that asset prices behave identically in the models with linear and quadratic costs. The cubic equation in (A.10) characterizes again the equilibrium.

Theorem 5. (Irrelevance theorem with linear trading costs)

Proof. It suffices to show that $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$ is independent of c. The proof is identical to the one of Theorem 1. □

Fixed trading costs

When trading, investors' problem is identical to the problem solved in the baseline case. The cubic equation in (A.10) characterizes again the equilibrium of the model. In this case, in a given equilibrium, investor i's indirect utility satisfies

$$W = \max_{q_{1i}}\left(\mathbb{E}[\theta|s_i,h_i,p] - \gamma h_i - p\right)q_{1i} - \frac{\gamma}{2}\mathbb{V}ar[\theta|s_i,h_i,p]\,q_{1i}^2 - \Phi\cdot\mathbf{1}[\Delta q_{1i}\neq 0].$$
$$= \frac{\gamma}{2}\mathbb{V}ar[\theta|s_i,h_i,p]\,(q_{1i}^\star)^2 - \Phi\cdot\mathbf{1}[\Delta q_{1i}\neq 0],$$

where $q_{1i}^\star$ satisfies $q_{1i}^* = \alpha_s\varepsilon_{si} - \alpha_h\varepsilon_{hi}$ if non-zero. The set of active investors is given by

$$|\alpha_s\varepsilon_{si} - \alpha_h\varepsilon_{hi}| \geq \sqrt{\left(\tau_\theta+\tau_s+\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2(\tau_\delta+\tau_h)\right)\frac{2\Phi}{\gamma}}.$$

Theorem 6. (Irrelevance theorem with fixed trading costs)

Proof. It suffices to show that $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$ is independent of c. The proof is identical to the one of Theorem 1. □

Random heterogeneous priors

Given the realization of his prior, investor i solves

$$\max_{q_{1i}}\left(\mathbb{E}\left[\theta|\overline{\theta}_i,s_i,h_i,p\right] - \gamma h_i - p\right)q_{1i} - \frac{\gamma}{2}\mathbb{V}ar_i\left[\theta|\overline{\theta}_i,s_i,h_i,p\right]q_{1i}^2 - \frac{c}{2}(\Delta q_{1i})^2.$$

Investor i optimal net portfolio demand is given by

$$\Delta q_{1i} = \frac{\mathbb{E}\left[\theta|\overline{\theta}_i,s_i,h_i,p\right] - p - \gamma h_i - \gamma\mathbb{V}ar_i\left[\theta|\overline{\theta}_i,s_i,h_i,p\right]q_0}{\gamma\mathbb{V}ar_i\left[\theta|\overline{\theta}_i,s_i,h_i,p\right] + c}.$$

In a symmetric equilibrium in linear strategies in which all investors are ex-ante identical, we guess (and verify) that the optimal net asset demand of investor i takes the form

$$\Delta q_{1i} = \alpha_\theta\overline{\theta_i} + \alpha_s s_i - \alpha_h h_i - \alpha_p p + \psi.$$

Market clearing implies an equilibrium price of the form

$$p = \frac{\overline{\alpha_\theta}}{\overline{\alpha_p}}\overline{\theta} + \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\delta + \frac{\overline{\psi}}{\overline{\alpha_p}}.$$

We denote by $\hat{p} = \frac{\overline{\alpha_p}}{\overline{\alpha_s}}p - \frac{\overline{\alpha_\theta}}{\overline{\alpha_s}}\mu_{\overline{\theta}} - \frac{\overline{\psi}}{\overline{\alpha_s}}$ the unbiased signal of θ for a given external observer (denoted by e), which is distributed as follows

$$\hat{p}|\theta \sim N\left(\theta, \left(\tau_{\hat{p}}^e\right)^{-1}\right) \quad \text{where} \quad \left(\tau_{\hat{p}}^e\right)^{-1} = \left(\frac{\overline{\alpha_\theta}}{\overline{\alpha_s}}\right)^2 \tau_{\overline{\theta}}^{-1} + \left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)^2 \tau_\delta^{-1}.$$

The solution to the optimal filtering problem implies that

$$\mathbb{E}\left[\theta|\overline{\theta}_i, s_i, h_i, \hat{p}\right] = \frac{\tau_\theta\overline{\theta}_i + \tau_s s_i + \tau_{\hat{p}}\left(\hat{p} + \frac{\alpha_h}{\alpha_s}\frac{\tau_h}{\tau_h+\tau_\delta}h_i\right)}{\tau_\theta + \tau_s + \tau_{\hat{p}}} \quad \text{and} \quad \mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, h_i, \hat{p}\right] = \frac{1}{\tau_\theta + \tau_s + \tau_{\hat{p}}},$$

where

$$\tau_{\hat{p}}^{-1} = \left(\frac{\overline{\alpha_\theta}}{\overline{\alpha_s}}\right)^2 \tau_{\overline{\theta}}^{-1} + \left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)^2 \left(\tau_\delta + \tau_h\right)^{-1}.$$

Matching coefficients with our initial conjecture, we show that α_θ, α_s, α_p, α_h, and ψ must satisfy

$$\alpha_\theta = \frac{1}{\kappa}\frac{\tau_\theta}{\tau_\theta + \tau_s + \tau_{\hat{p}}}, \quad \alpha_h = \frac{1}{\kappa}\left(\gamma - \frac{\tau_{\hat{p}}}{\tau_\theta + \tau_s + \tau_{\hat{p}}}\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\frac{\tau_h}{\tau_h + \tau_\delta}\right)$$
$$\alpha_s = \frac{1}{\kappa}\frac{\tau_s}{\tau_\theta + \tau_s + \tau_{\hat{p}}}, \quad \alpha_p = \frac{1}{\kappa}\frac{\tau_s}{\tau_s + \tau_{\hat{p}}}, \quad \text{and} \quad \psi = \alpha_p\left(\frac{\tau_{\hat{p}}}{\tau_s}\frac{\tau_\theta}{\tau_\theta + \tau_s + \tau_{\hat{p}}}\mu_{\overline{\theta}} - \gamma\mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, p\right]q_0\right),$$

where we define $\kappa = \gamma\mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, h_i, \hat{p}\right] + c$.

Theorem 7. (Irrelevance theorem with random heterogeneous priors)

Proof. It suffices to show that $\frac{\overline{\alpha_\theta}}{\overline{\alpha_s}}$ and $\frac{\overline{\alpha_h}}{\overline{\alpha_s}}$ are independent of c. We can write

$$\frac{\overline{\alpha_\theta}}{\overline{\alpha_s}} = \frac{\tau_\theta}{\tau_s} \qquad \text{and} \qquad \frac{\overline{\alpha_h}}{\overline{\alpha_s}} = \frac{\gamma - \frac{\tau_{\hat{p}}}{\tau_\theta+\tau_s+\tau_{\hat{p}}}\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\frac{\tau_h}{\tau_h+\tau_\delta}}{\frac{\tau_s}{\tau_\theta+\tau_s+\tau_{\hat{p}}}}.$$

Both ratios are independent of the trading cost c, because $\tau_{\hat{p}}$, which is a function of $\frac{\overline{\alpha_h}}{\overline{\alpha_s}}$, is independent of c. □

Strategic investors

Investor i solves the well-behaved problem stated in Eq. (27), with the necessary and sufficient condition for optimality given in Eq. (28)

In a symmetric equilibrium in linear strategies, investors portfolio demands take the form

$$\Delta q_{1i} = \alpha_s s_i - \alpha_h h_i - \alpha_p p + \psi.$$

The equilibrium price implied by market clearing is given by

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\delta + \frac{\overline{\psi}}{\overline{\alpha_p}} + \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\frac{\sum_{j=1}^N \varepsilon_{sj}^M}{N} - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\frac{\sum_{j=1}^N \varepsilon_{hj}^M}{N},$$

where we use $\overline{\alpha_s}$, $\overline{\alpha_h}$, $\overline{\alpha_p}$, and $\overline{\psi}$ to denote the equilibrium choices of other investors. In this case, the residual demand for investor i is given by

$$p_{-i} = \frac{\sum_{j\neq i}\overline{\alpha_s}s_j}{\sum_{j\neq i}\overline{\alpha_p}} - \frac{\sum_{j\neq i}\overline{\alpha_h}h_j}{\sum_{j\neq i}\overline{\alpha_p}} + \frac{\sum_{j\neq i}\overline{\psi}}{\sum_{j\neq i}\overline{\alpha_p}} + \frac{\Delta q_i}{\sum_{j\neq i}\overline{\alpha_p}}.$$

which allows us to write the price impact term for investor i as

$$\frac{\partial p_{-i}}{\partial q_{1i}}=\frac{1}{\sum_{j\neq i}\alpha_p}=\frac{1}{(N-1)\,\alpha_p}.$$

The unconditional distribution of the equilibrium price p is given by

$$p\sim N\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\overline{\theta}+\frac{\overline{\psi}}{\overline{\alpha_p}},\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\tau_\theta^{-1}+\left(\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2\tau_\delta^{-1}+\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\frac{\tau_s^{-1}}{N}+\left(\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2\frac{\tau_h^{-1}}{N}\right).$$

We denote by $\hat{p}=\frac{\overline{\alpha_p}}{\overline{\alpha_s}}p-\frac{\overline{\psi}}{\overline{\alpha_s}}$ the unbiased signal of θ for a given external observer (denoted by e), which is distributed as follows

$$\hat{p}|\theta\sim N\left(\theta,\left(\tau_{\hat{p}}^e\right)^{-1}\right),\quad \text{where}\quad \left(\tau_{\hat{p}}^e\right)^{-1}=\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)^2\left(\tau_\delta^{-1}+\frac{\tau_h^{-1}}{N}\right)+\frac{\tau_s^{-1}}{N}.$$

Solving the filtering problem of strategic investors involves an adjustment to account for investor i's own signal. We the denote the unbiased signal in prices from the perspective of investor i, $\hat{p}_i$, by

$$\hat{p}_i=\frac{\overline{\alpha_p}}{\overline{\alpha_s}}p-\frac{\overline{\psi}}{\overline{\alpha_s}}-\frac{1}{\overline{\alpha_s}}\frac{\Delta q_{1i}}{N-1}=\sum_{j\neq i}s_j-\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\sum_{j\neq i}h_j.$$

The information contained in the price for an investor i also corrects for the fact that h_i contains information about δ. The following signal is given by

$$\hat{p}_i+\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\frac{\tau_h}{\tau_h+\tau_\delta}h_i\,\bigg|\,\theta\sim N\left(\theta,\tau_{\hat{p}i}^{-1}\right),$$

where the price informativeness from the perspective of an investor i is given by

$$\left(\tau_{\hat{p}i}\right)^{-1}=\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)^2\left((\tau_h+\tau_\delta)^{-1}+\frac{\tau_h^{-1}}{N-1}\right)+\frac{\tau_s^{-1}}{N-1}$$

Matching coefficients with the guess we get the following system of equations for the set of parameters we conjectured:

$$\alpha_s=\frac{1}{\hat{\kappa}}\frac{\tau_s}{\tau_\theta+\tau_s+\tau_{\hat{p}}},\quad \alpha_h=\frac{1}{\hat{\kappa}}\left(\gamma-\tau_{\hat{p}i}\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\frac{\tau_h}{\tau_h+\tau_\delta}\right),\quad \alpha_p=\frac{1}{\hat{\kappa}}\left(1-\frac{\tau_{\hat{p}i}}{\tau_\theta+\tau_s+\tau_{\hat{p}i}}\frac{\overline{\alpha_p}}{\overline{\alpha_s}}\right),\ \text{and}$$
$$\psi=\frac{1}{\hat{\kappa}}\left(\frac{1}{\tau_\theta+\tau_s+\tau_{\hat{p}i}}\left(\tau_\theta\overline{\theta}-\tau_{\hat{p}i}\frac{\overline{\psi}}{\overline{\alpha_s}}\right)-\gamma\mathbb{V}ar\left[\theta|s_i,h_i,p\right]q_0\right),$$

where we define

$$\hat{\kappa}=\gamma\mathbb{V}ar\left[\theta|s_i,h_i,p\right]+c+\frac{1}{N-1}\frac{1}{\overline{\alpha_s}}\frac{\tau_{\hat{p}i}}{\tau_\theta+\tau_s+\tau_{\hat{p}i}}+\frac{1}{(N-1)\,\overline{\alpha_p}}.$$

Compared to the competitive case, the scale effect $\hat{\kappa}$ is dampened by the pecuniary price impact $\frac{\partial p_{-i}}{\partial q_{1i}}=\frac{1}{\sum_{j\neq i}\alpha_{pj}}=\frac{1}{(N-1)\overline{\alpha_p}}>0$ and the informational price impact $\frac{1}{N-1}\frac{1}{\overline{\alpha_s}}\frac{\tau_{\hat{p}i}}{\tau_\theta+\tau_s+\tau_{\hat{p}i}}$. Formally, $\hat{\kappa}>\kappa$, which makes strategic investors less reluctant to trade.

Theorem 8. (Irrelevance theorem with strategic investors)

Proof. It suffices to show that $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$ is independent of c. We can write

$$\frac{\overline{\alpha_s}}{\overline{\alpha_h}}=\frac{\frac{\tau_s}{\tau_\theta+\tau_s+\tau_{\hat{p}i}}}{\gamma-\tau_{\hat{p}i}\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\frac{\tau_h}{\tau_h+\tau_\delta}},$$

which is independent of c, because $\tau_{\hat{p}i}$, which depends on $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$ as shown in Eq. (29), is not a function of c. □

General utility and signals

From the expression for the equilibrium price, we show that

$$\frac{\partial p}{\partial \theta} = \frac{\int \frac{\partial q_{1i}}{\partial s_i} \frac{\partial f^{si}}{\partial \theta} di}{\int \frac{\partial q_{1i}}{\partial p} di} \quad \text{and} \quad \frac{\partial p}{\partial \delta} = \frac{\int \frac{\partial q_{1i}}{\partial h_i} \frac{\partial f^{hi}}{\partial \delta} di}{\int \frac{\partial q_{1i}}{\partial p} di},$$

which implies Eq. (31) in the text. Table 2 provides a comparison between the general case and the benchmark model with ex-ante identical investors.

Table 2: Equivalence between general and benchmark models

General model	Benchmark model
$U(p, s_i, h_i, c, q_{1i})$	$(\mathbb{E}[\theta \mid s_i, h_i, p] - \gamma_i h_i - p)\, q_{1i} - \frac{\gamma_i}{2} \mathbb{V}ar[\theta \mid s_i, h_i, p]\, q_{1i}^2 - \frac{c}{2}(\Delta q_{1i})^2$
$\Delta q_{1i}(p, s_i, h_i, c)$	$\Delta q_{1i} = \alpha_s s_i - \alpha_h h_i - \alpha_p p + \psi$
$p(\{s_i\}, \{h_i\}, c)$	$p = \frac{\alpha_s}{\alpha_p}\theta - \frac{\alpha_h}{\alpha_p}\delta + \frac{\psi}{\alpha_p}$
$\frac{\partial p}{\partial \theta}, \frac{\partial p}{\partial \delta}$	$\frac{\alpha_s}{\alpha_p}, -\frac{\alpha_h}{\alpha_p}$
$\frac{\left\lvert \frac{\partial p}{\partial \theta} \right\rvert}{\left\lvert \frac{\partial p}{\partial \delta} \right\rvert}$	$\frac{\alpha_s}{\alpha_h}$

Theorem 9. (Irrelevance theorem for general utility and signal structure)

Proof. Because investors may be ex-post heterogeneous, we cannot conclude in general that $\int \frac{\partial q_{1i}}{\partial s_i} di = \frac{\partial q_{1i}}{\partial s_i}$. When $\frac{\partial f^{si}}{\partial \theta} = \frac{\partial f^{hi}}{\partial \delta} = 1$, we can find instead

$$\frac{d \log \Pi}{dc} = \frac{d \log \left(\left\lvert \int \frac{\partial q_{1i}}{\partial s_i} di \right\rvert \right)}{dc} - \frac{d \log \left(\left\lvert \int \frac{\partial q_{1i}}{\partial h_i} di \right\rvert \right)}{dc} = \left(\frac{\left\lvert \int \frac{d \frac{\partial q_{1i}}{\partial s_i}}{dc} di \right\rvert}{\left\lvert \int \frac{\partial q_{1i}}{\partial s_i} di \right\rvert} - \frac{\left\lvert \int \frac{d \frac{\partial q_{1i}}{\partial h_i}}{dc} di \right\rvert}{\left\lvert \int \frac{\partial q_{1i}}{\partial h_i} di \right\rvert} \right),$$

which corresponds to Eq. (32) in the text.

Note that we can express Π in a given equilibrium as

$$\Pi = \frac{\left\lvert \frac{\partial p}{\partial \theta} \right\rvert}{\left\lvert \frac{\partial p}{\partial \delta} \right\rvert} = \frac{\int \left(\frac{\frac{\partial^2 U^i(q_i; s_i, h_i, p, c)}{\partial q_i \partial s_i}}{\frac{\partial^2 U^i(q_i; s_i, h_i, p, c)}{\partial q_i^2}} \right) \frac{\partial s_i}{\partial \theta} di}{\int \left(\frac{\frac{\partial^2 U^i(q_i; s_i, h_i, p, c)}{\partial q_i \partial h_i}}{\frac{\partial^2 U^i(q_i; s_i, h_i, p, c)}{\partial q_i^2}} \right) \frac{\partial h_i}{\partial \delta} d_i}, \tag{A.17}$$

so the relevant conditions on primitives that guarantee at a given equilibrium that price informativeness is invariant to the level of trading costs are

$$\frac{\partial^2 U^i(q_i; s_i, h_i, p, c)}{\partial q_i^2} = \frac{\partial^2 U^j(q_j; s_j, h_j, p, c)}{\partial q_j^2}, \ \forall i, j$$
$$\frac{\partial^2 U^i(q_i; s_i, h_i, p, c)}{\partial q_i \partial s_i} = \frac{\partial^2 U^i(q_i; s_i, h_i, p, c)}{\partial q_i \partial s_i}, \ \forall i$$
$$\frac{\partial^2 U^i(q_i; s_i, h_i, p, c)}{\partial q_i \partial h_i} = \frac{\partial^2 U^i(q_i; s_i, h_i, p, c)}{\partial q_i \partial h_i}, \ \forall i.$$

In that case, $\frac{\left\lvert \frac{\partial p}{\partial \theta} \right\rvert}{\left\lvert \frac{\partial p}{\partial \delta} \right\rvert}$ is independent of the level of trading costs in the economy since the signal and hedging needs structures are independent of the trading cost. In particular, these conditions hold when investors' optimality conditions satisfy

$$\frac{\partial \hat{U}^i(q_i; s_i, h_i, p)}{\partial q_i} - \frac{\partial C(q_i)}{\partial q_i} = 0 \quad \text{and} \quad \frac{\partial \hat{U}^{2i}(q_i; s_i, h_i, p)}{\partial q_i^2} - \frac{\partial^2 C(q_i)}{\partial q_i^2} = \frac{\partial \hat{U}^{2i}(q_i; s_j, h_j, p)}{\partial q_j^2} - \frac{\partial^2 C(q_j)}{\partial q_j^2} \quad \forall i, j.$$

□

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Online Appendix (not for publication)

A Filtering

Investors observe two pieces of information about the fundamental θ, the private signal s_i and the public signal p. Moreover, the realization of their individual hedging need reveals information about the aggregate hedging need in the economy δ and, thus, about the noise contained in the price. In the equilibrium in linear strategies, the unbiased signal of the fundamental contained in the price can be summarized in $\hat{p} = \theta - \frac{\overline{\alpha_h}}{\overline{\alpha_s}}\delta$. The linear system that characterizes the unknown fundamentals and the information observed by an individual investor is the following

$$\begin{bmatrix} s_i \\ h_i \\ \hat{p} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -\frac{\overline{\alpha_h}}{\overline{\alpha_s}} \end{bmatrix} \begin{bmatrix} \theta \\ \delta \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{si} \\ \varepsilon_{hi} \end{bmatrix}$$

where

$$\begin{bmatrix} \theta \\ \delta \end{bmatrix} \sim N\left(\begin{bmatrix} \overline{\theta} \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_\theta^{-1} & 0 \\ 0 & \tau_\delta^{-1} \end{bmatrix} \right)$$

and

$$\begin{bmatrix} \varepsilon_{si} \\ \varepsilon_{hi} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{si}^{-1} & 0 \\ 0 & \tau_{hi}^{-1} \end{bmatrix} \right).$$

A standard application of the Kalman filter yields

$$\mathbb{E}\left[\begin{bmatrix} \theta \\ \delta \end{bmatrix} \middle| s_i, h_i, p \right] = \frac{1}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}} \begin{bmatrix} \tau_\theta \overline{\theta} + \tau_{si} s_i + \tau_{\hat{p}i} \hat{p} + \frac{\overline{\alpha_s}}{\overline{\alpha_h}} \tau_{hi} h_i \\ \tau_{hi} h_i - \frac{\overline{\alpha_h}}{\overline{\alpha_s}} \left(\tau_{si} + \tau_\theta \right) \hat{p} + \frac{\overline{\alpha_h}}{\overline{\alpha_s}} \tau_{si} s_i + \frac{\overline{\alpha_h}}{\overline{\alpha_s}} \tau_\theta \overline{\theta} \end{bmatrix}$$

and

$$\mathbb{V}ar\left[\begin{bmatrix} \theta \\ \delta \end{bmatrix} \middle| s_i, h_i, p \right] = \frac{1}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}} \begin{bmatrix} 1 & \frac{\overline{\alpha_s}}{\overline{\alpha_h}} \\ \frac{\overline{\alpha_s}}{\overline{\alpha_h}} & \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}} \right)^2 \end{bmatrix}$$

where

$$\tau_{\hat{p}i} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}} \right)^2 \left(\tau_\delta + \tau_{hi} \right) \quad \text{and} \quad \tau_{\hat{p}} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}} \right)^2 \tau_\delta.$$

Note that we can write $\mathbb{E}\left[\theta | s_i, h_i, p\right]$ as follows

$$\mathbb{E}\left[\theta | s_i, h_i, p\right] = \frac{\tau_\theta \overline{\theta} + \tau_{si} s_i + \tau_{\hat{p}i} \hat{p} + \frac{\overline{\alpha_s}}{\overline{\alpha_h}} \tau_{hi} h_i}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}} = \frac{\tau_\theta \overline{\theta} + \tau_{si} s_i + \tau_{\hat{p}i} \left(\hat{p} + \frac{1}{\frac{\overline{\alpha_s}}{\overline{\alpha_h}}} \frac{\tau_{hi}}{\tau_\delta + \tau_{hi}} h_i \right)}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}},$$

where $\mathbb{E}\left[\delta | h_i\right] = \frac{\tau_{hi}}{\tau_\delta + \tau_{hi}} h_i$.

B Proof of Theorem 3: auxiliary results

B.1 Derivation of $\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}$

The sign of $\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}$ can be determined as follows. We can write

$$\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}=\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\left[\frac{d\log\overline{\alpha_s}}{dc}-\frac{d\log\overline{\alpha_h}}{dc}\right]=\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\left[\frac{\int\frac{d\alpha_{si}}{dc}di}{\overline{\alpha_s}}-\frac{\int\frac{d\alpha_{hi}}{dc}di}{\overline{\alpha_h}}\right]$$

$$=\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\left[\int\left(\frac{\alpha_{hi}}{\overline{\alpha_h}}-\frac{\alpha_{si}}{\overline{\alpha_s}}\right)\frac{\frac{d\kappa_i}{\kappa_i}}{dc}di-\int\frac{\alpha_{si}}{\overline{\alpha_s}}\frac{\frac{d\tau_{\theta|s_i,h_i,p}}{\tau_{\theta|s_i,h_i,p}}}{dc}di+\int\frac{1}{\kappa_i}\frac{\tau_{hi}}{\tau_\theta+\tau_{si}+\tau_{\hat{p}i}}\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\left(2\frac{\tau_{\hat{p}}}{\tau_{\theta|s_i,h_i,p}}+1\right)\frac{\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}}{\frac{\overline{\alpha_s}}{\overline{\alpha_h}}}di\right]$$

$$=\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\left[\int\left(\frac{\alpha_{hi}}{\overline{\alpha_h}}-\frac{\alpha_{si}}{\overline{\alpha_s}}\right)\frac{1}{\kappa_i}di-\int\left(\frac{\alpha_{hi}}{\overline{\alpha_h}}-\frac{\alpha_{si}}{\overline{\alpha_s}}\right)\left(\frac{1}{\kappa_i}\frac{\gamma_i}{\tau_{\theta|s_i,h_i,p}}\frac{\frac{d\tau_{\theta|s_i,h_i,p}}{\tau_{\theta|s_i,h_i,p}}}{dc}\right)di-\int\frac{\alpha_{si}}{\overline{\alpha_s}}\frac{\frac{d\tau_{\theta|s_i,h_i,p}}{\tau_{\theta|s_i,h_i,p}}}{dc}di\right]$$

$$=\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\left[\int\left(\frac{\alpha_{hi}}{\overline{\alpha_h}}-\frac{\alpha_{si}}{\overline{\alpha_s}}\right)\frac{1}{\kappa_i}di-\int\frac{\alpha_{hi}}{\overline{\alpha_h}}\left(\frac{1}{\kappa_i}\frac{\gamma_i}{\tau_{\theta|s_i,h_i,p}}\frac{\frac{d\tau_{\theta|s_i,h_i,p}}{\tau_{\theta|s_i,h_i,p}}}{dc}\right)di+\int\frac{\alpha_{si}}{\overline{\alpha_s}}\left(\frac{1}{\kappa_i}\frac{\gamma_i}{\tau_{\theta|s_i,h_i,p}}-1\right)\frac{\frac{d\tau_{\theta|s_i,h_i,p}}{\tau_{\theta|s_i,h_i,p}}}{dc}\right]$$

$$=\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\left[\int\left(\frac{\alpha_{hi}}{\overline{\alpha_h}}-\frac{\alpha_{si}}{\overline{\alpha_s}}\right)\frac{1}{\kappa_i}di-\int\frac{\alpha_{hi}}{\overline{\alpha_h}}\frac{1}{\kappa_i}\frac{\gamma_i}{\tau_{\theta|s_i,h_i,p}}\frac{\frac{d\tau_{\theta|s_i,h_i,p}}{\tau_{\theta|s_i,h_i,p}}}{dc}di-c\int\frac{\alpha_{si}}{\overline{\alpha_s}}\frac{1}{\kappa_i}\frac{\frac{d\tau_{\theta|s_i,h_i,p}}{\tau_{\theta|s_i,h_i,p}}}{dc}\right]$$

Therefore

$$\frac{\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}}{\frac{\overline{\alpha_s}}{\overline{\alpha_h}}}=\left[\int\left(\frac{\alpha_{hi}}{\overline{\alpha_h}}-\frac{\alpha_{si}}{\overline{\alpha_s}}\right)\frac{1}{\kappa_i}di-\int\left(\frac{\gamma_i}{\kappa_i}\frac{1}{\tau_{\theta|s_i,h_i,p}}+c\frac{\alpha_{si}}{\overline{\alpha_s}}\frac{1}{\kappa_i}\right)2\frac{\tau_{\hat{p}}}{\tau_{\theta|s_i,h_i,p}}di\frac{\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}}{\frac{\overline{\alpha_s}}{\overline{\alpha_h}}}\right]$$

$$=\frac{-\int\left(\frac{\alpha_{si}}{\overline{\alpha_s}}-\frac{\alpha_{hi}}{\overline{\alpha_h}}\right)\frac{1}{\kappa_i}di}{1+2\int\frac{1}{\kappa_i}\left(\frac{\gamma_i}{\kappa_i}\frac{1}{\tau_{\theta|s_i,h_i,p}}+c\frac{\alpha_{si}}{\overline{\alpha_s}}\right)\frac{\tau_{\hat{p}i}}{\tau_{\theta|s_i,h_i,p}}di}$$

Where we use the following results

$$\frac{d\log\overline{\alpha_s}}{dc}=\frac{\frac{d\overline{\alpha_s}}{dc}}{\overline{\alpha_s}}=\frac{\int\frac{d\alpha_{si}}{dc}di}{\overline{\alpha_s}}\quad\text{and}\quad\frac{d\log\overline{\alpha_h}}{dc}=\frac{\frac{d\overline{\alpha_h}}{dc}}{\overline{\alpha_h}}=\frac{\int\frac{d\alpha_{hi}}{dc}di}{\overline{\alpha_h}}$$

$$\frac{d\alpha_{si}}{dc}=-\frac{1}{\kappa_i^2}\frac{d\kappa_i}{dc}\frac{\tau_{si}}{\tau_\theta+\tau_{si}+\tau_{\hat{p}i}}-\frac{1}{\kappa_i}\frac{\tau_{si}}{\left(\tau_\theta+\tau_{si}+\tau_{\hat{p}i}\right)^2}\frac{d\tau_{\tau_{\theta|s_i,h_i,p}}}{dc}=-\alpha_{si}\left[\frac{\frac{d\kappa_i}{\kappa_i}}{dc}+\frac{\frac{d\tau_{\theta|s_i,h_i,p}}{\tau_{\theta|s_i,h_i,p}}}{dc}\right]$$

$$\frac{d\alpha_{hi}}{dc}=-\frac{1}{\kappa_i^2}\frac{d\kappa_i}{dc}\left(\gamma_i-\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\frac{\tau_{hi}}{\tau_\theta+\tau_{si}+\tau_{\hat{p}i}}\right)-\frac{1}{\kappa_i}\left(\frac{d\tau_{\theta|s_i,h_i,p}}{dc}\tau_{hi}\frac{\overline{\alpha_s}}{\overline{\alpha_h}}+\frac{\tau_{hi}}{\tau_\theta+\tau_{si}+\tau_{\hat{p}i}}\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}\right)$$

$$=-\alpha_{hi}\frac{\frac{d\kappa_i}{\kappa_i}}{dc}-\frac{1}{\kappa_i}\frac{\tau_{hi}}{\tau_\theta+\tau_{si}+\tau_{\hat{p}i}}\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\left(\frac{\frac{d\tau_{\theta|s_i,h_i,p}}{\tau_{\theta|s_i,h_i,p}}}{dc}+\frac{\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}}{\frac{\overline{\alpha_s}}{\overline{\alpha_h}}}\right)$$

$$\frac{d\kappa_i}{dc}=1-\frac{\gamma_i}{\left(\tau_{\theta|s_i,h_i,p}\right)^2}\frac{d\tau_{\theta|s_i,h_i,p}}{dc}=1-\frac{\gamma_i}{\tau_{\theta|s_i,h_i,p}}\frac{\frac{d\tau_{\theta|s_i,h_i,p}}{\tau_{\theta|s_i,h_i,p}}}{dc}\Rightarrow\frac{\frac{d\kappa_i}{dc}}{\kappa_i}=\frac{1}{\kappa_i}\left(1-\frac{\gamma_i}{\tau_{\theta|s_i,h_i,p}}\frac{\frac{d\tau_{\theta|s_i,h_i,p}}{\tau_{\theta|s_i,h_i,p}}}{dc}\right)$$

$$\frac{d\tau_{\theta|s_i,h_i,p}}{dc}=\frac{d\tau_{\hat{p}i}}{dc}=2\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}\left(\tau_\delta+\tau_{hi}\right)\Rightarrow\frac{\frac{d\tau_{\theta|s_i,h_i,p}}{\tau_{\theta|s_i,h_i,p}}}{dc}=2\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}\left(\tau_\delta+\tau_{hi}\right)=2\frac{\tau_{\hat{p}i}}{\tau_{\theta|s_i,h_i,p}}\frac{\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}}{\frac{\overline{\alpha_s}}{\overline{\alpha_h}}}$$

Finally, we can write

$$\frac{\int \frac{\alpha_{si}}{\kappa_i} di}{\overline{\alpha_s}} - \frac{\int \frac{\alpha_{hi}}{\kappa_i} di}{\overline{\alpha_h}} = \frac{\mathbb{E}_i \left[\alpha_{si} \frac{1}{\kappa_i}\right]}{\overline{\alpha_s}} - \frac{\mathbb{E}_i \left[\alpha_{hi} \frac{1}{\kappa_i}\right]}{\overline{\alpha_h}} = \frac{\mathbb{E}_i \left[\alpha_{si}\right] \mathbb{E}_i \left[\frac{1}{\kappa_i}\right] + \mathbb{C}ov_i \left[\alpha_{si}, \frac{1}{\kappa_i}\right]}{\overline{\alpha_s}} - \frac{\mathbb{E}_i \left[\alpha_{hi}\right] \mathbb{E}_i \left[\frac{1}{\kappa_i}\right] + \mathbb{C}ov_i \left[\alpha_{hi}, \frac{1}{\kappa_i}\right]}{\overline{\alpha_h}}$$

$$= \mathbb{C}ov_i \left[\frac{\alpha_{si}}{\overline{\alpha_s}}, \frac{1}{\kappa_i}\right] - \mathbb{C}ov_i \left[\frac{\alpha_{hi}}{\overline{\alpha_h}}, \frac{1}{\kappa_i}\right] = \mathbb{C}ov_i \left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}, \frac{1}{\kappa_i}\right]$$

So the sign of $\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}$ is determined by

$$\operatorname{sgn}\left(\frac{d\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)}{dc}\right) = -\operatorname{sgn}\left(\frac{\int \frac{\alpha_{si}}{\kappa_i} di}{\overline{\alpha_s}} - \frac{\int \frac{\alpha_{hi}}{\kappa_i} di}{\overline{\alpha_h}}\right) = \operatorname{sgn}\left(\mathbb{C}ov_i \left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}, -\frac{1}{\kappa_i}\right]\right)$$

Because

$$\frac{d \log\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc} = \frac{\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{dc}}{\frac{\overline{\alpha_s}}{\overline{\alpha_h}}} = \frac{\mathbb{C}ov_i \left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}, -\frac{1}{\kappa_i}\right]}{1 + 2 \int \frac{1}{\kappa_i} \left(\frac{\gamma_i}{\kappa_i} \frac{1}{\tau_{\theta|s_i,h_i,p}} + c \frac{\alpha_{si}}{\overline{\alpha_s}}\right) \frac{\tau_{\hat{p}}}{\tau_{\theta|s_i,h_i,p}} di}$$

B.2 Auxiliary results for part b)

First, for case i), it is straightforward to establish that $\frac{\alpha_{si}}{\alpha_{hi}}$ is a decreasing monotone function of γ_i. Formally

$$\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial \gamma_i} = -\frac{\tau_{si}\left(\tau_\theta + \tau_{si} + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 (\tau_\delta + \tau_{hi})\right)}{\left(\gamma_i \left(\tau_\theta + \tau_{si} + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 (\tau_\delta + \tau_{hi})\right) - \tau_{hi} \frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2} < 0 \quad \forall \gamma_i.$$

Therefore $\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial \gamma_i} < 0$.

Second, for case ii), $\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial \tau_{si}}$ formally corresponds to

$$\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial \tau_{si}} = \frac{\gamma\left(\tau_\theta + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 (\tau_\delta + \tau_h)\right) - \tau_h \frac{\overline{\alpha_s}}{\overline{\alpha_h}}}{\left(\gamma \tau_{si} + \gamma\left(\tau_\theta + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 (\tau_\delta + \tau_h)\right) - \tau_h \frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2},$$

which implies that

$$\operatorname{sgn}\left(\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial \tau_{si}}\right) = \operatorname{sgn}\left(\gamma\left(\tau_\theta + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 (\tau_\delta + \tau_h)\right) - \tau_h \frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right) = \operatorname{sgn}\left(\frac{1}{\gamma} - \frac{\alpha_{si}}{\alpha_{hi}}\right),$$

where the last equality follows from the fact that

$$\gamma\left(\tau_\theta + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 (\tau_\delta + \tau_h)\right) - \tau_h \frac{\overline{\alpha_s}}{\overline{\alpha_h}} = \mathbb{V}ar_i \left[\theta | s_i, h_i, p\right]^{-1} \left(\gamma - \left(\tau_h \frac{\overline{\alpha_s}}{\overline{\alpha_h}} + \gamma \tau_{si}\right) \mathbb{V}ar_i \left[\theta | s_i, h_i, p\right]\right)$$

$$= \mathbb{V}ar_i \left[\theta | s_i, h_i, p\right]^{-1} \gamma \kappa_i \alpha_{hi} \left(\frac{1}{\gamma} - \frac{\alpha_{si}}{\alpha_{hi}}\right).$$

Since we show in Claim 2 that $\frac{1}{\gamma} - \frac{\alpha_{si}}{\alpha_{hi}} > 0$, this establishes that $\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial \tau_{si}} > 0$.

Third, for case iii), $\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial\tau_{hi}}$ formally corresponds to

$$\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial\tau_{hi}} = -\frac{\gamma\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\frac{\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}-\frac{1}{\gamma}\right)}{\tau_{si}}}{\left(\gamma+\frac{\gamma\left(\tau_\theta+\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2(\tau_\delta+\tau_{hi})\right)-\tau_{hi}\frac{\overline{\alpha_s}}{\overline{\alpha_h}}}{\tau_{sA}}\right)^2}$$
$$= -\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)^2\frac{\gamma}{\tau_{si}}\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}-\frac{1}{\gamma}\right),$$

which implies that

$$\text{sgn}\left(\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial\tau_{hi}}\right) = \text{sgn}\left(\frac{1}{\gamma}-\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right).$$

Since we show in Claim 1 that $\frac{1}{\gamma}-\frac{\overline{\alpha_s}}{\overline{\alpha_h}}>0$, this establishes that $\frac{\partial\left(\frac{\alpha_{si}}{\alpha_{hi}}\right)}{\partial\tau_{hi}}>0$.

Two final claims complete our argument. Note first that

$$\frac{\overline{\alpha_s}}{\overline{\alpha_h}} = \frac{\mu_A\alpha_{sA}+\mu_B\alpha_{sB}}{\mu_A\alpha_{hA}+\mu_B\alpha_{hB}} = w_A\frac{\alpha_{sA}}{\alpha_{hA}}+w_B\frac{\alpha_{sB}}{\alpha_{hB}} \tag{A.18}$$

where

$$w_i = \frac{\mu_i\alpha_{hi}}{\mu_A\alpha_{hA}+\mu_B\alpha_{hB}} \quad \text{with} \quad w_A+w_B=1.$$

Claim 1. Let $\gamma_A=\gamma_B=\gamma$. Then, $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}<\frac{1}{\gamma}$.

Proof. Using the definition of α_{si} and α_{hi} we know that

$$\frac{1}{\gamma}>\frac{\alpha_{si}}{\alpha_{hi}} \tag{A.19}$$

if and only if

$$\tau_\theta+\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2(\tau_\delta)+\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\tau_{hi}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}-\frac{1}{\gamma}\right)>0.$$

Assume that $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}>\frac{1}{\gamma}$. Then, Eq. (A.19) holds for $i=A,B$ and Eq. (A.18) implies $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}<\frac{1}{\gamma_i}$ which is a contradiction. □

Claim 2. Let $\gamma_A=\gamma_B=\gamma$, $\tau_{hA}=\tau_{hB}=\tau_h$ and $\tau_{sA}>\tau_{sB}$. Then, $\frac{\alpha_{sA}}{\alpha_{hA}}<\frac{1}{\gamma}$.

Proof. We know that

$$\text{sgn}\left(\frac{\partial\left(\frac{\alpha_{sA}}{\alpha_{hA}}\right)}{\partial\tau_{sA}}\right) = \text{sgn}\left(\frac{1}{\gamma_A}-\frac{\alpha_{sA}}{\alpha_{hA}}\right) = \text{sgn}\left(\tau_\theta+\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2(\tau_\delta)+\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\tau_h\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}-\frac{1}{\gamma}\right)\right)$$

which is independent of τ_{sA}. Suppose $\frac{\alpha_{sA}}{\alpha_{hA}}>\frac{1}{\gamma}$. Then, $\frac{\partial\left(\frac{\alpha_{sA}}{\alpha_{hA}}\right)}{\partial\tau_{sA}}<0$ and since $\tau_{sA}>\tau_{sB}$, this implies

$$\frac{\alpha_{sB}}{\alpha_{hB}}>\frac{1}{\gamma}$$

and using Eq. (A.18) we would have

$$\frac{\overline{\alpha_s}}{\overline{\alpha_h}}>\frac{1}{\gamma}$$

which contradicts Claim 1. □

C Equilibrium with classic noise trading

For the question we study, it is important that we introduce aggregate hedging needs to have a meaningful filtering problem, as opposed to modeling directly some form of "noise demand". In particular, what matters for our irrelevance result is that the source of noise that makes the filtering problem non trivial affects the primitives of the portfolio problem solved by investors.

Here, we eliminate the aggregate uncertainty arising from hedging needs and solve our model using the more standard stochastic noisy demand for the risky asset. We specifically work with the symmetric competitive benchmark model, and we further assume that $\tau_{hi} = \infty$ and $\delta = 0$. We introduce noise traders, modeled as a random variable x, such that

$$x \sim N\left(0, \tau_x^{-1}\right)$$

These assumptions prevent the equilibrium from being fully revealing. We guess and verify that investors' portfolio demands take the form

$$\Delta q_{1i} = \alpha_s s_i - \alpha_p p + \psi, \tag{A.20}$$

where α_s and α_p are positive scalars and ψ can take positive or negative values. The market clearing condition $\int \Delta q_{1i} di + x = 0$ implies an equilibrium price of the form

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta + \frac{\overline{\psi}}{\overline{\alpha_p}} + \frac{x}{\overline{\alpha_p}}$$

We can write the distribution of the price p as

$$p \sim N\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\overline{\theta} + \frac{\overline{\psi}}{\overline{\alpha_p}}, \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \tau_\theta^{-1} + \left(\frac{1}{\overline{\alpha_p}}\right)^2 \tau_x^{-1}\right)$$

While the conditional distribution of the equilibrium price p given the fundamental θ follows

$$p|\theta \sim N\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta + \frac{\overline{\psi}}{\overline{\alpha_p}}, \left(\frac{1}{\overline{\alpha_p}}\right)^2 \sigma_x^2\right)$$

We again denote by $\hat{p} = \frac{\overline{\alpha_p}}{\overline{\alpha_s}}p - \frac{\overline{\psi}}{\overline{\alpha_s}}$ the unbiased signal of θ, which is distributed as follows

$$\hat{p}|\theta \sim N\left(\theta, \tau_{\hat{p}}^{-1}\right), \quad \text{where} \quad \tau_{\hat{p}} = \left(\overline{\alpha_p}\right)^2 \tau_x \tag{A.21}$$

As in our benchmark model

$$\mathbb{E}\left[\theta|s_i, p\right] = \mathbb{E}\left[\theta|s_i, \hat{p}\right] = \frac{\tau_\theta\overline{\theta} + \tau_s s_i + \tau_{\hat{p}}\hat{p}}{\tau_\theta + \tau_s + \tau_{\hat{p}}} \quad \text{and} \quad \mathbb{V}ar\left[\theta|s_i, p\right] = \mathbb{V}ar\left[\theta|s_i, \hat{p}\right] = \frac{1}{\tau_\theta + \tau_s + \tau_{\hat{p}}}$$

Substituting these expressions in investors' demand functions, given by Eq. (7), we can write q_{1i} as

$$\Delta q_{1i} = \frac{\left(\tau_\theta\overline{\theta} + \tau_s s_i + \tau_{\hat{p}}\left(\frac{\overline{\alpha_p}}{\overline{\alpha_s}}p - \frac{\overline{\psi}}{\overline{\alpha_s}}\right)\right)\mathbb{V}ar\left[\theta|s_i, p\right] - p - \gamma\mathbb{V}ar\left[\theta|s_i, p\right] q_0}{\gamma\mathbb{V}ar\left[\theta|s_i, p\right] + c},$$

where we define $\kappa \equiv \gamma\mathbb{V}ar\left[\theta|s_i, p\right] + c$. As in our benchmark model, matching coefficients with our initial guess in Eq. (A.20), we are able to characterize α_s, α_p, and ψ as the solution to a system of equations. It is clear from Eq. (A.21) that $\frac{d\tau_{\hat{p}}}{dc}$ is negative, because α_p is a decreasing function of c, that is

$$\frac{d\tau_{\hat{p}}}{dc} < 0$$

Remark. The model with exogenously given noise trading demand spuriously concludes that high trading costs decrease price informativeness and increase price volatility. It implicitly models the behavior of a group of investors in the economy as if they were fully inelastic to trading costs.

D Equilibrium without learning

For reference, we characterize as a benchmark the equilibrium of the competitive economy when there is no learning. To ease the notation, we use $\mathbb{E}_i[\theta]$ for $\mathbb{E}[\theta|s_i]$ and $\mathbb{V}ar_i[\theta]$ for $\mathbb{V}ar_i[\theta|s_i]$. For reference, we derive market clearing in the case without learning as follows:

$$\int \omega_i \Delta \hat{q}_{1i} di = \int \omega_i \left(\frac{\mathbb{E}_i[\theta] - \gamma_i h_i - p}{\gamma_i \mathbb{V}ar_i[\theta]} - q_{0i} \right) di = \int \Gamma_i \left(\mathbb{E}_i[\theta] - \gamma_i h_i - p - \gamma_i \mathbb{V}ar_i[\theta] q_{0i} \right) di = 0,$$

where $\Gamma_i = \frac{\omega_i}{\gamma_i \mathbb{V}ar_i[\theta]} = \frac{1}{\gamma_i \mathbb{V}ar[\theta]+c}$ and $\int \Gamma_i di = \int \frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c} di$. We can write the equilibrium price as

$$p = \int g_i \left(\mathbb{E}_i[\theta] - \gamma_i h_i - \gamma_i \mathbb{V}ar_i[\theta] q_{0i} \right) di,$$

where $g_i = \frac{\Gamma_i}{\int \Gamma_i di} = \frac{\frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c}}{\int \frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c} di}$. g_i is the contribution of investor i to the harmonic average of demand sensitivities. When $\gamma_i = \gamma$, we can write $g_i = \frac{\frac{1}{\gamma \mathbb{V}ar_i[\theta]+c}}{\int \frac{1}{\gamma \mathbb{V}ar_i[\theta]+c} di} = 1$. In the general case,

$$\frac{dp}{dc} = \int \frac{dg_i}{dc} \left(\mathbb{E}_i[\theta] - \gamma_i h_i - \gamma_i \mathbb{V}ar_i[\theta] q_{0i} \right) di,$$

where

$$\frac{dg_i}{dc} = \frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c} \frac{-\frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c} \int \frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c} di + \int \frac{1}{(\gamma_i \mathbb{V}ar_i[\theta]+c)^2} di}{\left(\int \frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c} di \right)^2}$$

So $\frac{dg_i}{dc} \gtreqless 0$ if $\int \frac{1}{(\gamma_i \mathbb{V}ar_i[\theta]+c)^2} di \gtreqless \frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c} \int \frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c} di$. Note that

$$\int \frac{dg_i}{dc} di = 0$$

So we can write

$$\frac{dp}{dc} = \mathbb{C}ov_i \left[\frac{dg_i}{dc}, \mathbb{E}_i[\theta] - \gamma_i h_i - \gamma_i \mathbb{V}ar_i[\theta] q_{0i} \right]$$

Therefore, the price goes up or down when c increases depending on the cross-sectional covariance of $\frac{dg_i}{dc}$, which captures the change induces in demand elasticities, with $\mathbb{E}_i[\theta] - \gamma_i h_i - \gamma_i \mathbb{V}ar_i[\theta] q_{0i}$, which captures the desire for trading unrelated to prices. The main takeaway of this analysis is the following.

Remark. In the model without learning, the equilibrium price is independent of the level of trading costs as long as $\gamma_i \mathbb{V}ar_i[\theta]$ is constant.

E Welfare of external investor and price informativeness

The choice of price informativeness as the variable of interest is justified by the fact that it corresponds to the welfare of an external investor who must make a choice based on its expectation about θ.

Formally, assume that there exists an external investor who solves

$$\min_x \mathbb{E} \left[(x-\theta)^2 \middle| \mathcal{I} \right],$$

where $\mathcal{I}$ is the investor's information set. We assume that the external investor has the same prior over θ as all the other investors in our economy, and only observes the asset price p, so $\mathcal{I} = p$. It is optimal for the external investor to choose

$$x = \mathbb{E}[\theta | p] = \frac{\tau_\theta \overline{\theta} + \tau_{\hat{p}}^e \hat{p}}{\tau_\theta + \tau_{\hat{p}}^e},$$

where $\hat{p}$ and $\tau_{\hat{p}}^e$ are given in Eq. (A.3). The welfare $W(\tau_{\hat{p}}^e)$ of this external investor is given by

$$W(\tau_{\hat{p}}^e) = -\mathbb{V}ar[\theta | \hat{p}] = -\frac{1}{\tau_\theta + \tau_{\hat{p}}^e},$$

which is an increasing function of price informativeness.

F Endogenous precision of the signal about the aggregate hedging needs

We now explore the possibility that investors may acquire information about the aggregate hedging component – the noise component of asset prices – and show that an increase in trading costs decreases the information acquired about the aggregate hedging component in equilibrium.

To have a meaningful precision choice separate from investors' hedging motives, we extend the benchmark model with ex-ante identical investors by assuming that, in addition to the private signal about the fundamental, investors receive a private signal about the aggregate hedging need given by

$$\eta_i = \delta + \varepsilon_{\eta i},$$

where

$$\varepsilon_{\eta i} \sim N\left(0, \tau_{\eta i}^{-1}\right)$$

and $\varepsilon_{\eta i}$ is independent of all other random variables in the economy. Investors choose the precision of this signal at a cost $\lambda_\eta(\tau_\eta)$, where $\lambda_\eta(\cdot)$ is continuous and twice differentiable and it satisfies, $\lambda'_\eta(\cdot) > 0$, $\lambda''_\eta(\cdot) \geq 0$ and the Inada condition $\lim_{\tau_{\eta i}\to\infty} \lambda'_\eta(\tau_{\eta i}) = \infty$.

Equilibrium of the trading stage In a symmetric equilibrium in linear strategies of the trading stage, we conjecture and verify that investors follow net trading demands given by

$$\Delta q_{1i} = \alpha_s s_i - \alpha_h h_i + \alpha_\eta \eta_i - \alpha_p p + \psi,$$

which imply an equilibrium price that takes the form

$$p = \frac{\alpha_s}{\alpha_p}\theta - \frac{\alpha_h - \alpha_\eta}{\alpha_p}\delta + \frac{\psi}{\alpha_p}.$$

The unbiased signal contained in the price is $\hat{p} = \frac{\alpha_p}{\alpha_s}p - \frac{\psi}{\alpha_s}$ which, from an external observer's point of view, is distributed as follows.

$$\hat{p}|\theta \sim N\left(\theta, \left(\tau_{\hat{p}}^e\right)^{-1}\right), \quad \text{where} \quad \tau_{\hat{p}}^e = \left(\frac{\alpha_s}{\alpha_h - \alpha_\eta}\right)^2 \tau_\delta. \tag{A.22}$$

As in the model presented in Section 2, the relevant measure of price informativeness is $\tau_{\hat{p}}^e$. As can be seen from Eq. (A.22), price informativeness is higher the more sensitive investors are to their private signals, either about the fundamental θ or about the aggregate hedging needs δ, and the less sensitive investors are to their own hedging needs. Intuitively, the more weight investors put on their information, the higher the informational content of prices.

Lemma 3. (Existence and multiplicity) *An equilibrium of the trading stage always exists. There are at most three equilibria.*

The results from Lemma 3 follow from the cubic equation that characterizes $\frac{\alpha_s}{\alpha_h - \alpha_\eta}$, which is analogous to Eq. (15). In fact, from the analysis of Eq. (15) in the Appendix, it can be seen that the model analyzed here has either one or three equilibria. Also, if there are multiple equilibria, only the higher and lower equilibria can be made stable under plausible assumptions on equilibrium convergence. Finally, it can be seen that in the two stable equilibria

$$\frac{\partial\left(\frac{\alpha_s}{\alpha_h - \alpha_\eta}\right)}{\partial \tau_s} > 0 \quad \text{and} \quad \frac{\partial\left(\frac{\alpha_s}{\alpha_h - \alpha_\eta}\right)}{\partial \tau_\eta} > 0.$$

Intuitively, price informativeness always increases with the amount of information in the economy. This is true regardless of whether the information is about the fundamental or about the aggregate hedging needs.

Investors' information choice The equilibrium of the model with information acquisition about the aggregate hedging need δ is defined analogously to the equilibrium of the model with information acquisition about the fundamental θ. The equilibrium of both models with information acquisition takes into account the equilibrium in linear strategies played in the trading stage. Since the equilibrium in the trading stage may not be unique, the probability with which each equilibrium is played at date 1 is also an equilibrium outcome. Again, we focus on equilibria with a degenerate distribution $\pi \in \{0, 1\}$ in what follows.

Investors optimally choose $\tau_{\eta i}$ by solving

$$\max_{\tau_{\eta i}} V\left(\tau_{\eta i};\{\tau_{\eta j}\}_{j\neq i}\right), \quad \text{where} \quad V\left(\tau_{\eta i};\{\tau_{\eta j}\}_{j\neq i}\right)=\mathbb{E}\left[v_i\right]-\lambda\left(\tau_{\eta i}\right),$$

and $\mathbb{E}\left[v_i\right]$ is given by

$$\mathbb{E}\left[v_i\right]=\mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|s_i,h_i,\eta_i,p\right]-\gamma_i h_i-p\right),q_{1i}^*\right]-\frac{1}{2}\left(\gamma_i\mathbb{V}ar\left[\theta|s_i,h_i,\eta_i,p\right]+c\right)\mathbb{E}\left[\left(q_{1i}^*\right)^2\right].$$

The first order condition of this problem, formally given in the Appendix, is analogous Eq. (21) in the previous subsection. The following lemma shows that there is an equilibrium in the information acquisition stage and that all equilibria are symmetric.

Lemma 4. (Existence and symmetry of equilibrium) *There always exists an equilibrium in the information acquisition stage. Any equilibrium is symmetric.*

The economic forces at play when investors acquire information about the aggregate hedging needs are analogous to those that are present when investors acquire information about the fundamental. A higher precision of the signal about the aggregate hedging needs increases the accuracy of the investor's demand. A higher precision $\tau_{\eta i}$ allows the investor to predict the amount of noise contained in the price better, effectively increasing the informativeness of the price for the investor. When trading costs increase, the benefit of trading more accurately decrease and so does the information acquired in equilibrium. Theorem 10 formalizes this argument.

Theorem 10. (Effect of trading costs with endogenous precision of the signal on the aggregate hedging need) *When investors are ex-ante identical, an increase in trading costs decreases the information acquired about the fundamental in equilibrium, i.e.,*

$$\frac{d\tau_{\eta i}^*}{dc}<0.$$

In the two well-behaved equilibria, this reduction in information acquisition also generates a reduction in price informativeness, hence $\frac{d\tau_{\hat{p}}^e}{dc}<0$.

As in the case in which investors can acquire information about the fundamental, an increase in transaction costs decreases the amount of information acquired by investors and leads to a less informative price if investors coordinate on the stable equilibria, as described in Eq. (16). Figure A.1 illustrates how the precision about the aggregate hedging need and price informativeness in equilibrium change as a function of the level of trading cost c.

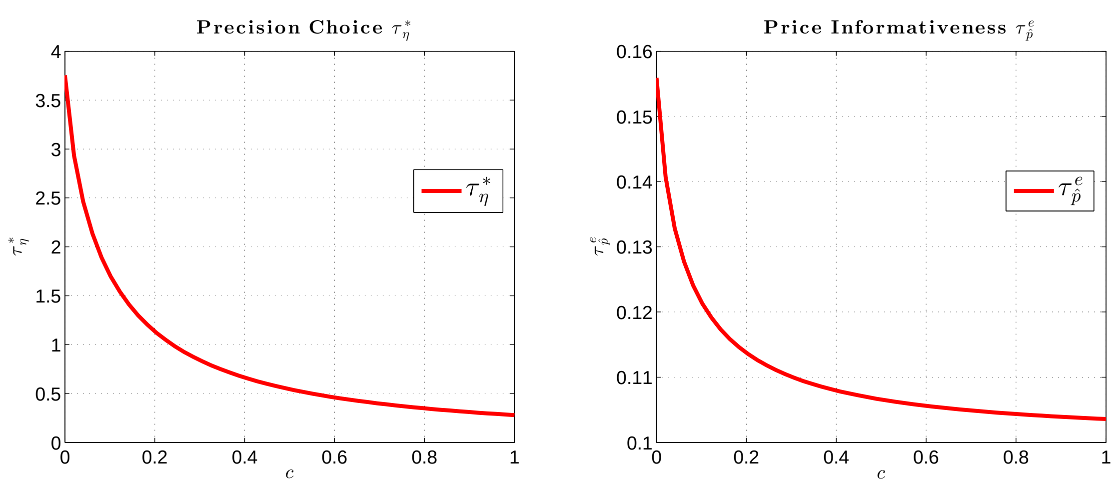

Figure A.1: Equilibrium comparative statics

F.1 Proofs

Consider the benchmark model with symmetric investors with the only difference that investors also receive a private signal about the aggregate hedging need. More specifically, consider that

$$\eta_i = \delta + \varepsilon_{\eta i}, \quad \text{where} \quad \varepsilon_{\eta i} \sim N\left(0, \tau_{\eta i}^{-1}\right).$$

and $\varepsilon_{\eta i}$ is independent of all other random variables in the economy.

Also, before any information is revealed, we allow investors to choose the precision of this signal at a cost $\lambda_\eta\left(\tau_{\eta i}\right)$, where $\lambda_\eta\left(\cdot\right)$ is continuous and twice differentiable and it satisfies, $\lambda_\eta'\left(\cdot\right) > 0$, $\lambda_\eta''\left(\cdot\right) \geq 0$ and the Inada condition $\lim_{\tau_{\eta i}\to\infty} \lambda_\eta'\left(\tau_{\eta i}\right) = \infty$.

Then, in a symmetric equilibrium in linear strategies

$$\Delta q_{1i} = \alpha_s s_i - \alpha_h h_i + \alpha_\eta \eta_i - \alpha_p p + \psi,$$

where market clearing implies an equilibrium price

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta - \frac{\overline{\alpha_h} - \overline{\alpha_\eta}}{\overline{\alpha_p}}\delta + \frac{\psi}{\alpha_p}.$$

Then, the unbiased signal of θ contained in the price is $\hat{p} = \frac{\overline{\alpha_p}}{\overline{\alpha_s}}\left(p - \frac{\psi}{\overline{\alpha_p}}\right) = \theta - \frac{\overline{\alpha_h} - \overline{\alpha_\eta}}{\overline{\alpha_s}}\delta$. Solving the optimal filtering problem from the perspective of investor i allows us to write

$$\mathbb{E}\left[\theta | h_i, s_i, \eta_i, p\right] = \frac{\tau_\theta \overline{\theta} + \tau_s s_i + \tau_{\hat{p}i}\left(\hat{p} + \frac{\overline{\alpha_s}}{\overline{\alpha_h} - \overline{\alpha_\eta}}\frac{\tau_h h_i + \tau_{\eta i}\eta_i}{\tau_{\hat{p}i}}\right)}{\tau_\theta + \tau_s + \tau_{\hat{p}i}}$$

and

$$\mathbb{V}ar\left[\theta | h_i, s_i, \eta_i, p\right] = \frac{1}{\tau_\theta + \tau_{si} + \tau_{\hat{p}i}}, \quad \text{where} \quad \tau_{\hat{p}i} = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_h} - \overline{\alpha_\eta}}\right)^2 \left(\tau_\delta + \tau_h + \tau_{\eta i}\right).$$

The first order condition for the investors in the trading stage is

$$\Delta q_{1i}^* = \frac{\mathbb{E}\left[\theta | s_i, h_i, \eta_i, p\right] - \gamma_i h_i - p}{\gamma_i \mathbb{V}ar\left[\theta | h_i, s_i, \eta_i, p\right] + c}.$$

In a symmetric equilibrium,

$$\alpha_s = \frac{1}{\kappa}\frac{\tau_s}{\tau_\theta + \tau_s + \tau_{\hat{p}}}, \quad \alpha_h = \frac{1}{\kappa}\left(\gamma - \frac{\alpha_s}{\alpha_h - \alpha_\eta}\frac{\tau_h}{\tau_\theta + \tau_s + \tau_{\hat{p}}}\right),$$

$$\alpha_\eta = \frac{1}{\kappa}\frac{\alpha_s}{\alpha_h - \alpha_\eta}\frac{\tau_\eta}{\tau_\theta + \tau_s + \tau_{\hat{p}}}, \quad \alpha_p = \frac{1}{\kappa}\frac{\tau_s}{\tau_s + \tau_{\hat{p}}}, \quad \text{and} \quad \psi = \alpha_p \frac{\tau_\theta \overline{\theta}}{\tau_\theta + \tau_s + \tau_{\hat{p}}},$$

where $\frac{\alpha_s}{\alpha_h - \alpha_\eta}$ is given by the solution to

$$\gamma\left(\frac{\alpha_s}{\alpha_h - \alpha_\eta}\right)^3 \left(\tau_\delta + \tau_h + \tau_\eta\right) - \left(\frac{\alpha_s}{\alpha_h - \alpha_\eta}\right)^2 \left(\tau_h + \tau_\eta\right) + \gamma\left(\tau_\theta + \tau_s\right)\frac{\alpha_s}{\alpha_h - \alpha_\eta} - \tau_s = 0.$$

Investors' information choice An investor i chooses $\tau_{\eta i}$ to solve $\max_{\tau_{hi}} \mathbb{E}\left[v_i\right] - \lambda_\eta\left(\tau_{hi}\right)$ where $\mathbb{E}\left[v_i\right]$ is given by

$$\begin{aligned}\mathbb{E}\left[v_i\right] &= \mathbb{E}\left[\left(\mathbb{E}\left[\theta | s_i, h_i, p\right] - \gamma h_i - p\right)\right]\mathbb{E}\left[q_{1i}^*\right] + \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta | s_i, h_i, \eta_i, p\right] - \gamma h_i - p\right), q_{1i}^*\right] - \frac{1}{2}\left(\gamma \mathbb{V}ar\left[\theta | s_i, h_i, p\right] + c\right)\mathbb{E}\left[\left(q_{1i}^*\right)^2\right] \\ &= \left(\overline{\theta} - \mathbb{E}\left[p\right]\right)\mathbb{E}\left[q_{1i}^*\right] + \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta | s_i, h_i, \eta_i, p\right] - \gamma h_i - p\right), q_{1i}^*\right] - \frac{1}{2}\left(\gamma \mathbb{V}ar\left[\theta | s_i, h_i, \eta_i, p\right] + c\right)\mathbb{E}\left[\left(q_{1i}^*\right)^2\right],\end{aligned}$$

where we use the fact that $\mathbb{E}\left[\left(\mathbb{E}\left[\theta | s_i, h_i, p\right] - \gamma h_i - p\right)\right] = \left(\overline{\theta} - \mathbb{E}\left[p\right]\right)\mathbb{E}\left[q_{1i}^*\right] = 0$, given the assumption that $q_{0i} = 0$.

The optimal precision choice $\tau_{\eta i}^*$ is given by the solution to $\hat{H}\left(\tau_{\eta i}^*\right) = 0$ where

$$\hat{H}\left(\tau_{\eta i}\right) = \frac{1}{2}\frac{\partial \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta | s_i, h_i, \eta_i, p\right] - \gamma h_i - p\right), q_{1i}^*\right]}{\partial \tau_{\eta i}} - \lambda_\eta'\left(\tau_{\eta i}\right).$$

where we use that

$$\frac{\partial \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]}{\partial \tau_{\eta i}} = \left(\gamma \mathbb{V}ar\left[\theta|s_i,h_i,p\right]+c\right)\frac{\partial \mathbb{V}ar\left[q_{1i}^*\right]}{\partial \tau_{\eta i}} - \gamma \frac{\partial \mathbb{V}ar\left[\theta|s_i,h_i,\eta_i,p\right]}{\partial \tau_{\eta i}}\mathbb{V}ar\left[q_{1i}^*\right].$$

The second order condition of the information choice problem is given by

$$\begin{aligned}\frac{\partial \hat{H}\left(\tau_{\eta i}\right)}{\partial \tau_{\eta i}} &= \frac{\partial^2 \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]}{\partial \tau_{\eta i}^2} - \hat{\lambda}''\left(\tau_{\eta i}\right)\\ &= -2\frac{1}{\kappa_i}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}+\overline{\alpha_\eta}}\right)^2 \mathbb{V}ar\left[\theta|s_i,h_i,p\right]^3 \frac{\partial \mathbb{C}ov\left[\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p,q_{1i}^*\right]}{\partial \tau_{\eta i}}c - \hat{\lambda}''\left(\tau_{\eta i}\right) < 0.\end{aligned}$$

Equilibrium

Lemma 4. (Existence and symmetry of equilibrium)

Proof. The proof is analogous to the proof of Lemma 2, since for any given equilibrium of the trading game the first order condition of the information acquisition game is

$$\hat{H}\left(\tau_{\eta i}^*\right)=0,$$

where $\hat{H}\left(\tau_{\eta i}\right)$ is continuous in $\tau_{\eta i}\in(0,\infty)$, $\lim_{\tau_{\eta i}\to 0}\hat{H}\left(\tau_{\eta i}\right)=\infty$, and $\lim_{\tau_{\eta i}\to\infty}\hat{H}\left(\tau_{\eta i}\right)=-\infty$. □

Theorem 10. (Effect of trading costs with endogenous precision of the signal on the aggregate hedging need)

Proof. The implicit function theorem implies

$$\frac{\partial \tau_{\eta i}^*}{\partial c} = -\frac{\frac{\partial \hat{H}(\tau_{\eta i})}{\partial c}}{\frac{\partial \hat{H}(\tau_{\eta i})}{\partial \tau_{\eta i}}} < 0,$$

because

$$\frac{\partial \hat{H}\left(\tau_{\eta i}\right)}{\partial c} = -\left(\frac{\gamma \mathbb{V}ar\left[\theta|s_i,h_i,p\right]^2\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}+\overline{\alpha_\eta}}\right)^2}{\kappa_i^2}\mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]+\frac{1}{\kappa_i}\frac{\partial \mathbb{C}ov\left[\left(\mathbb{E}\left[\theta|s_i,h_i,p\right]-\gamma h_i-p\right),q_{1i}^*\right]}{\partial \tau_{\eta i}}\right) < 0.$$

Since in any sunspot equilibria the first order condition is a linear combination of the first order condition for the case in which each equilibria is played with probability one, the first result follows. The second result follows directly from the fact that $\frac{\partial\left(\frac{\alpha_s}{\alpha_h - a_\eta}\right)}{\partial \tau_{\eta i}} > 0$ in both stable equilibria. □

G Dynamics

In this extension, we add an additional round of trading to capture dynamic trading considerations. Forward-looking investors who buy and sell over time are more sensitive to the presence of trading costs, because they anticipate facing trading costs twice. To tractably allow for multiple trading rounds within our framework, we assume that investors start with asset holdings q_{-1}, and have the opportunity to choose portfolios both at dates 0 and 1, as illustrated in Figure A.2.

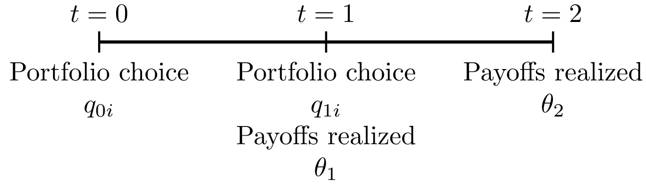

Figure A.2: Timeline dynamic model

We further assume a) that investors maximize expected utility of consumption at the final date 2 and b) that the risky asset pays dividends at dates 1 and 2, respectively denoted by θ_1 and θ_2. The structure of the model at each date is identical to the one in the benchmark model with ex-ante identical investors, assuming that all variables have i.i.d. realizations. To simplify the analysis, we focus on the case in which the sunspot equilibrium at date 2 is degenerate if there are multiple equilibria.

In this environment, investors' net worth at dates 1 and 2 are respectively are given by

$$w_{2i} = n_{2i} + q_{1i}\theta_2 + w_{1i} - q_{1i}p - \frac{c}{2}\left(\Delta q_{1i}\right)^2 \quad \text{and}$$
$$w_{1i} = n_{1i} + q_{0i}\left(\theta_1 + p_1\right) + q_{-1}p_0 - q_{0i}p_0 - \frac{c}{2}\left(\Delta q_{0i}\right)^2.$$

The solution to the problem from date 1 onward is identical to our benchmark model.[25] Hence, we focus our attention on characterizing the equilibrium of the economy at date 0. Investor's objective function at date 0 corresponds to

$$\max_{q_{0i}} \left(\mathbb{E}\left[\theta_1|s_{0i},h_{0i},p_0\right] - \gamma h_{1i}\right) q_{0i} - p_0\Delta q_{0i} - \frac{\gamma}{2}\mathbb{E}\left[\theta_1|s_{0i},h_{0i},p_0\right] q_{0i}^2 - \frac{c}{2}\left(\Delta q_{0i}\right)^2 + \underbrace{\mathbb{E}\left[p\right] q_{0i} - \frac{c}{2}\left(q_{0i}\right)^2}_{\text{Forward-Looking Term}}.$$

The investors' objective function at date 0 incorporates a new term that accounts for the future benefits and costs associated with risky asset holdings. The additional benefit from holding an additional unit of q_{0i} is given by its expected sale price at date 1. The additional cost is determined by the trading cost level c in a quadratic way.

The first order condition to the investors problem yields the following demand for the risky asset at date 0

$$q_{0i} = \frac{\mathbb{E}\left[p\right] + \mathbb{E}\left[\theta_1|s_{0i},h_{0i},p_0\right] - \gamma h_{0i} - p_0 - cq_{-1i}}{\gamma\mathbb{E}\left[\theta_1|s_{0i},h_{0i},p_0\right] + 2c}.$$

This expression is almost identical to the optimal demand at date 1, with the exception that now the level of trading costs in the denominator is effectively doubled: because investors are forward-looking, they trade less in the risky asset, because they internalize the effect of future trading costs when they have to further buy or sell assets.

In the equilibrium in linear strategies that we study, we guess (and subsequently verify) that the optimal portfolio of investor i takes the form

$$\Delta q_{0i} = \alpha_{s0}s_i - \alpha_{h0}h_i - \alpha_{p0}p + \psi_0,$$

where α_s, α_h, and α_p are positive scalars and ψ can take any sign. As in our benchmark model, market clearing implies that the equilibrium price takes the form

$$p_0 = \frac{\alpha_{s0}}{\alpha_{p0}}\theta_1 - \frac{\alpha_{h0}}{\alpha_{p0}}\delta_1 + \frac{\psi_0}{\alpha_{p0}},$$

We again defined by $\hat{p}$ the unbiased signal of θ_1 in equilibrium. Therefore, the relevant measure of price informativeness in this context is given by $\tau_{\hat{p}0}$, defined by

$$\tau_{\hat{p}0} = \left(\frac{\alpha_{s0}}{\alpha_{h0}}\right)^2 \tau_{\delta 1}.$$

We can thus prove a new irrelevance theorem in the context of this dynamic model.

Theorem 11. (Irrelevance theorem in dynamic environment) *In an economy in which investors trade at multiple dates, when investors are ex-ante identical, price informativeness in any equilibrium is independent of the level of trading costs. Formally, the precision of the unbiased signal about the fundamental revealed by the asset price at every date $\tau^e_{\hat{p}t}$ does not depend on c.*

Theorem 11 shows that our irrelevance argument also applies when investors trade over time. As we have shown, trading demands vary depending on investors' trading horizons. In particular, it is well known that small trading costs can have very large effects on trading volume when investors trade a high frequencies. However, as long as the reduction in trading after a trading costs increase is symmetric across investors, both information and hedging trading are reduced at the same rate, leaving price informativeness unchanged. Although, for clarity, we derive our results in a two date model, it is straightforward to extend our result to multi-period dynamic economies.

[25] Investors hold different levels of asset holdings at date 1. As shown in Section 3, the irrelevance result applies to that case.

G.1 Proofs

The wealth accumulation equations for investors at dates 1 and 2 are

$$w_{2i} = n_{2i} + q_{1i}\theta_2 + w_{1i} - q_{1i}p_1 - \frac{c}{2}(\Delta q_{1i})^2$$
$$w_{1i} = n_{1i} + q_{0i}(\theta_1 + p_1) + w_{0i} - (\Delta q_{0i})p_0 - \frac{c}{2}(\Delta q_{0i})^2.$$

The indirect utility of investor i at date 1 is a function of his initial asset holdings q_{0i}. Formally, we can write $V(q_{0i})$ as

$$V(q_{0i}) = \mathbb{E}[U_i(w_{2i})|s_{0i}, h_{0i}, p_0] = -\mathbb{E}\left[e^{-\gamma w_{2i}}|s_{0i}, h_{0i}, p_0\right] = -e^{-\gamma\left[\mathbb{E}[w_{2i}|s_{0i},h_{0i},p_0] - \frac{\gamma}{2}\mathbb{V}ar[w_{2i}|s_{0i},h_{0i},p_0]\right]}.$$

Let $\mathbb{E}_{0i}[\cdot] \equiv \mathbb{E}[\cdot|s_{0i}, h_{0i}, p_0]$, $\mathbb{V}ar_{0i}[\cdot] \equiv \mathbb{V}ar[\cdot|s_{0i}, h_{0i}, p_0]$ and $\mathbb{C}ov_{0i}[\cdot] = \mathbb{C}ov_{0i}[\cdot|s_{0i}, h_{0i}, p_0]$. Then, investor i's portfolio choice in period 0 can be written as

$$\max_{q_{0i}} \mathbb{E}_{0i}[w_{2i}] - \frac{\gamma}{2}\mathbb{V}ar_{0i}[w_{2i}],$$

which can be written as

$$\max\left(\mathbb{E}_{0i}[\theta_1 + p_1^*] - p_0\right)q_{0i} - \gamma h_{1i}q_{0i} - \frac{c}{2}(\Delta q_{0i})^2 - \frac{c}{2}\mathbb{E}_{0i}\left[(\Delta q_{1i}^*)^2\right]$$
$$- \frac{\gamma}{2}\mathbb{V}ar_{0i}[\theta_1 + p_1^* + cq_{1i}^*]q_{0i}^2 - \gamma(1+c)\mathbb{C}ov_{0i}[n_{2i} + q_{1i}^*(\theta_2 - p_1), q_{1i}^*]q_{0i}.$$

The first order condition for this problem is

$$q_{0i}^* = \frac{\mathbb{E}_{0i}[\theta_1 + p_1^* + cq_{1i}^*] - p_0 + cq_{-1i} - \gamma h_{1i} - \gamma(1+c)\mathbb{C}ov_{0i}[n_{2i} + q_{1i}^*(\theta_2 - p_1), q_{1i}^*]}{\gamma(\mathbb{V}ar_{0i}[\theta_1 + p_1^* + cq_{1i}^*]) + 2c}.$$

In a symmetric equilibrium in linear strategies $q_{0i}^* = \alpha_{s0}s_{0i} - \alpha_{h0}h_{0i} - \alpha_{p0}p_0 + \psi_0$, where

$$\alpha_{s0} = \frac{1}{\kappa}\frac{\tau_{s0}}{\tau_{s0} + \tau_{\theta 0} + \tau_{\hat{p}0}}, \quad \alpha_{h0} = \frac{1}{\kappa}\left(\gamma - \frac{\tau_{\hat{p}0}\frac{\overline{\alpha_{s0}}}{\overline{\alpha_{h0}}}\frac{\tau_{h0}}{\tau_{h0}+\tau_{\delta 0}}}{\tau_{s0} + \tau_{\theta 0} + \tau_{\hat{p}0}}\right), \quad \alpha_{p0} = \frac{1}{\kappa}\left(1 - \frac{\tau_{\hat{p}0}}{\tau_{s0} + \tau_{\theta 0} + \tau_{\hat{p}0}}\right), \text{ and}$$
$$\psi_0 = \frac{1}{\kappa}\left(\frac{-\tau_{\hat{p}0}\frac{\overline{\alpha_{p0}}}{\overline{\alpha_{s0}}}\overline{\psi_0}}{\tau_{s0} + \tau_{\theta 0} + \tau_{\hat{p}0}} + \mathbb{E}_{0i}[p_1^* + cq_{1i}^*] + cq_{-1i} - \gamma(1+c)\mathbb{C}ov_{0i}[n_{2i} + q_{1i}^*(\theta_2 - p_1), q_{1i}^*]\right),$$

where we define $\overline{\kappa} = \gamma(\mathbb{V}ar_{0i}[\theta_1 + p_1^* + cq_{1i}^*]) + 2c$.

The measure of price informativeness is

$$\tau_{\hat{p}0} = \left(\frac{\overline{\alpha_{s0}}}{\overline{\alpha_{h0}}}\right)^2(\tau_{\delta 0} + \tau_{h0}).$$

Theorem 11. (Irrelevance theorem in dynamic environment)

Proof. It is sufficient to show that $\frac{\overline{\alpha_{s0}}}{\overline{\alpha_{h0}}}$ is independent of c, since we know from the benchmark model that $\frac{\overline{\alpha_{s1}}}{\overline{\alpha_{h1}}}$ is independent of c. $\frac{\overline{\alpha_{s0}}}{\overline{\alpha_{h0}}}$ is the solution to the following system which is independent of c, thus, the proposition holds

$$\frac{\overline{\alpha_{s0}}}{\overline{\alpha_{h0}}} = \frac{\frac{\tau_{s0}}{\tau_{s0}+\tau_{\theta 0}+\tau_{\hat{p}0}}}{\gamma - \frac{\tau_{\hat{p}0}\frac{\overline{\alpha_{s0}}}{\overline{\alpha_{h0}}}\frac{\tau_{h0}}{\tau_{h0}+\tau_{\delta 0}}}{\tau_{s0}+\tau_{\theta 0}+\tau_{\hat{p}0}}}.$$

□