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Who Benefits from Online Privacy?

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WHO BENEFITS FROM ONLINE PRIVACY? *

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Abstract

When firms can identify their past customers, they may use information about purchase histories in order to price discriminate. We present a model with a monopolist and a continuum of heterogeneous consumers, where consumers can opt out from being identified, possibly at a cost. We find that when consumers can costlessly opt out, they all individually choose privacy, which results in the highest profit for the monopolist. In fact, all consumers are better off when opting out is costly. When valuations are uniformly distributed, social surplus is non-monotonic in the cost of opting out and is highest when opting out is prohibitively costly. We introduce the notion of a *privacy gatekeeper* — a third party that is able to act as a privacy conduit and set the cost of opting out. We prove that the privacy gatekeeper only charges the firm in equilibrium, making privacy costless to consumers.

Keywords: Privacy, price discrimination, anonymity, opt out, e-commerce

JEL Classifications: D02, L12, L50

1 INTRODUCTION

In recent years, revolutionary developments in information technology regarding collection, storage, and retrieval of personal data (Acquisti & Varian, 2005) have brought privacy to the forefront of public awareness and debate.[1] This paper addresses a key component of the emergent concerns regarding electronic privacy, namely the ability of firms to track individual purchasing patterns and to use this information to practice behavior-based price discrimination (Armstrong, 2006; Fudenberg & Villas-Boas, 2006).

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[1] See for example `pcworld.about.com/news/Jan262001id39447.htm`.

The US Census Bureau estimates that $135 billion in online retail sales were conducted in the United States in 2007, and that online retail sales as percentage of total retail sales have steadily increased over the past decade at an average rate of %3 per quarter. Over the last 3 years, online retail sales have increased at an average annual rate of %16. The Census also estimates that the total amount of currency transacted in the online US consumer market in 2006 (including both retail and selected services) exceeded $220 billion (US Census E-Stats, 2008). Records containing the sequence of web sites visited and the online purchases made by individuals provide valuable clues about their personal information, clues that can be used to target tailor-made offers to them (Chen & Zhang, 2008; Wathieu, 2006; Pancras & Sudhir, 2007; Chen, 2006). Such behavior-based advertising and price discrimination are already ubiquitous in electronic commerce (Odlyzko, 2003; Hann *et al.* , 2007). Nevertheless, the economic impact of these practices on individual consumers and on society overall has received little formal study to date. The paucity of rigorous analysis has left policy makers with scant guidance about the appropriate scope or the efficacy of the regulatory instruments at their disposal. Presently, privacy practices in electronic commerce are dictated largely by voluntary compliance with industry standards and recommendations by regulatory agencies.

Although technology has allowed sellers to store and process consumers' online activities with relative ease, consumers still have a choice when it comes to sellers tracking their individual consumer activities. For instance, consumers can exert effort to understand sellers' privacy disclosures and take actions to circumvent being identified by sellers. Such actions can include erasing or blocking browser cookies, making payments using a gift card acquired for cash in a brick-and-mortar store, and using a *privacy gatekeeper*. A privacy gatekeeper is a third party to a transaction that works in the following way: consumers store their sensitive information in the gatekeeper's system, and the gatekeeper in turn allows consumers to make online purchases with enhanced privacy. For example, CitiBank allows their clients to instantly generate one-time-use virtual credit-card numbers, which upon use are charged to a client's actual card. Similarly, Google Checkout gives consumers the option of not sharing their actual email addresses with sellers by using a virtual email address to forward communication. In extreme cases, consumers can also rent a personal postal box to maintain the privacy of their physical address.

In its 2007 guidelines for online privacy (FTC, 2007), the US Federal Trade Commission proposed several guiding principals to govern sellers' online privacy practices:

transparency and consumer control, reasonable security and limited data retention, and consumer consent for changes in policy. In this paper, we focus on the principles of transparency and consumer control. One way to interpret consumer control is that consumers have the ability to opt out from having their information collected, and thus to maintain their privacy.[2] (We note that opting out may still involve using measures such as blocking browser cookies in order to circumvent identification.) The principle of transparency pertains to privacy related disclosures provided by sellers. Increased transparency in disclosures translates to making it easier for consumers to safeguard their information. Essentially, the main implication of the combination of these two principles is that it should not be costly for consumers to maintain their privacy, should they choose to do so.

These currently available set of guiding standards and recommendations, while apparently quite sensible, have little or no basis in formal economic theory or empirical tests. This paper proposes to provide rigorous economic analysis on the impact of privacy regulation with regard to consumer profiling and behavior-based price discrimination. In particular, we consider a monopolist who is able to track consumers' purchase histories. Consumers, however, are able to circumvent being identified as past customers by taking (potentially costly) actions to preserve their anonymity. The cost of these actions can arise from multiple sources, such as exerting effort to understand disclosures, erasing cookies, using virtual credit cards, or masking IP addresses. Interestingly, when consumers can costlessly avoid detection, they all individually choose privacy, which paradoxically results in the highest profit for the monopolist and a lower consumer surplus. In fact, all consumers are better off when opting out is prohibitively costly. When consumers' valuations are uniformly distributed, costless privacy results in the lowest consumer surplus. The situation that arises is a form of a Prisoner's Dilemma: individually, consumers are best-responding by opting out. Collectively, however, they are all worse off. The cost of opting out, when positive, essentially acts as a mitigating factor that reduces this coordination problem among consumers. In fact, consumers are better off overall when opting out is prohibitively costly.

The intuition is as follows. When the cost of opting out is high, consumers hesitate to purchase in the first period, knowing they will pay a premium for doing so in the second period. Anticipating this behavior by consumers, the firm reduces the price it charges in the first period. Additionally, by being able to more effectively price discriminate, the

[2] Consumers may also have the ability to *opt in* to have their information collected, which is especially relevant when there is some benefit to being identified — a setting we do not consider in this paper.

firm can tailor a price to low valuation consumers in the second period — consumers who would otherwise not purchase. Hence, consumers benefit from a high cost of opting out.

We consider a more general framework in which a privacy gatekeeper sets this cost and collects it as a fee. The privacy gatekeeper can also receive offers from the firm for setting the fee at a certain level. We show that the privacy gatekeeper would only charge the firm in equilibrium, making it costless for consumers to opt out. Consequently, the existence of a privacy gatekeeper may hurt consumers. The surprising conclusion is that hard-to-understand disclosures and difficult-to-circumvent identification may actually work to the benefit of consumers and, in some cases, to the benefit of society overall.

Related Literature Work on intertemporal price discrimination and the "ratchet" effect (where consumers who signaled higher willingness to pay receive higher price offers) originates back to the late 1970's. Stokey (1979) and Salant (1989) show that intertemporal price discrimination is never optimal for a monopolist who can commit to future prices. Freixas *et al.* (1985), Weitzman (1980), and Hart & Tirole (1988) study the monopolist's problem in a repeated game under incomplete information and no commitment.

A relatively small economics literature on customer recognition and online privacy has begun to develop over the past several years. Early contributions by Chen (1997), Fudenberg & Tirole (1998), Fudenberg & Tirole (2000), Villas-Boas (1999, 2004), Shaffer & Zhang (2000), Taylor (2003), and Chen & Zhang (2008) introduced the notion of customer recognition and personalized pricing into economic theory, but did not explicitly consider privacy issues in online environments. Fudenberg & Tirole (1998) explore what happens when the ability to identify particular consumers may vary across goods. They consider a model of goods upgrades and buy-backs where customers may be anonymous or "semi-anonymous." Fudenberg & Tirole (2000) analyze a duopoly in which some consumers remain loyal and others defect to the competitor, a phenomenon they refer to as "customer poaching." Villas-Boas (1999) shows that two firms in a duopoly can compete by lowering prices to attract the competitor's previous customers. Villas-Boas (2004) shows that targeted pricing by a monopolist who cannot commit to future prices may make it worse off. Chen & Zhang (2008) analyze a "price for information" strategy, with firms pricing less aggressively in order to learn more about their customers.

Optimal online privacy policies were first studied by Taylor (2004), Acquisti & Varian (2005), Hermalin & Katz (2006), and Calzolari & Pavan (2006). Fudenberg & Villas-Boas (2006) offer a survey of this literature.[3] Taylor (2004) studies the market for customer

[3]For a general discussion of price discrimination see Stole (2007). For a review of the consumer switch-

information and finds that the welfare implications of various regimes depend on the sophistication of consumers. He finds that when consumers are myopic, an “open privacy” regime (where sale of customer information is permitted) works to the benefit of firms. When customers are sophisticated, firms benefit from keeping their customers’ information private. His analysis, however, does not focus on the possibility that the ‘anonymity’ regime can be made endogenous through the consumer’s decision process. Calzolari & Pavan (2006) study contracting environments with two principals (an upstream principal and a downstream principal) that interact sequentially with one common agent (the consumer). Fudenberg & Villas-Boas (2006) show that when the upstream principal expropriates all the rent from the downstream principal, their results coincide with Taylor (2004). However, Taylor (2004) offers a more general environment in that consumers’ valuations need not be constant over time. Acquisti & Varian (2005) explore the possibility of consumers using anonymizing technologies to maintain their privacy. However, they consider a model with discrete consumer types, and only allow for either costless or prohibitively costly anonymizing technologies. In contrast, we allow for continuous consumer types and an arbitrary cost (which we subsequently endogenize) of using anonymizing technologies.

While the above papers provide important insights regarding the basic tensions with respect to consumer privacy and price discrimination, they consider only a small set of policy options available to firms and consumers. This paper proposes to build on the growing privacy literature by exploring a richer environment in which individuals who purchase goods online may (at some cost) choose to remain anonymous; i.e., choose to opt out of a firm’s customer database. Our findings suggest that granting consumers this option may have important consequences for pricing and welfare.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 introduces and solves the benchmarks of no-recognition and full-recognition. Section 4 characterizes the equilibrium with costly privacy. Section 5 fully analyzes the model when valuations are distributed uniformly. Section 6 introduces the notion of a privacy gatekeeper and endogenizes the cost of opting out. Section 7 concludes.

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ing cost literature see Klemperer (1995). For an economic analysis of privacy with respect to lawful search and seizure, see Mialon & Mialon (2008).

2 The Model

2.1 The Consumers

There is a continuum of consumers with total mass normalized to one. All consumers are risk-neutral, possess discount factor $\delta \in [0, 1]$, and maximize their present expected utilities. Each consumer demands at most one unit of a non-durable good in each of two periods. Consumer i's valuation for the good is the same in each period and is determined by the realization of a random variable v_i with support normalized to be the unit interval. Consumer valuations are independently and identically distributed according to the distribution function $F(v)$ with density $f(v)$, which is strictly positive on $[0, 1]$. Consumer i's valuation v_i is initially private information; i.e., known only to him.

2.2 The Firm

There is a monopolist that produces and sells the good in each period. The firm has production cost normalized to zero, possesses discount factor δ, and maximizes its discounted expected profit. It does not observe consumer valuations directly but maintains a database containing purchasing histories. In particular, each consumer is either *anonymous* or *identifiable*. If a consumer is anonymous, then there is no record of his prior purchases; i.e., he is not in the database. If he is identifiable, then the firm knows his purchasing decision in the first period.

The firm wants to maximize the expected discounted value of its profits. Given that there is a continuum of consumers, each of them realizes that his decision does not affect the prices charged by the firm in the next period.

2.3 The Game

At the beginning of the game all consumers are anonymous. Hence, the firm offers the same price p_1 to all of them. Next, each consumer decides whether to buy the good, $q_1^i = 1$, or not to buy it, $q_1^i = 0$. Consumers who elect to buy the good also decide whether to let the firm keep a record of the transaction ($r_1^i = 1$) or to *opt out* and maintain anonymity by deleting the record of the sale ($r_1^i = 0$). The cost to any consumer who opts out is $c \geq 0$. This cost represents the time and effort of maintaining anonymity as well as any monetary expense. A consumer who does not purchase the good continues to be anonymous.

At the beginning of period two, the firm posts a price p_2^0 to the anonymous consumers and a price p_2^1 to the identifiable ones. Consumers may buy the good only at the price offered to them; i.e., no arbitrage is possible. All aspects of the environment, including the distribution of valuations $F(v)$, are common knowledge. The solution concept is perfect Bayesian Nash equilibrium (PBE).[4]

We assume throughout that $v[1 - F(v)]$ is strictly quasi-concave in v (which is the condition necessary for the existence of a unique local maximum in the static monopoly case, and is implied by a monotone hazard rate and $f(v) > 0$ for $v \in [0,1]$). The assumption on the support of the distribution is without loss of generality relative to any compact interval.

3 Benchmarks

3.1 No Customer Recognition

Consider first as a benchmark the case of no customer recognition, in which the monopolist cannot price discriminate in the second period between the consumers that bought and did not buy in the first period. The optimal price charged is the same in both periods and is given by $p^\star = \arg\max_p\{1 - F(p)\}$, generating a profit in each period of $p^\star(1 - F(p^\star))$. Consumer surplus in each period is given by $\int_{p^\star}^1 (v - p^\star)dF(v)$.

Example 1 When valuations are uniformly distributed, $p^\star = 0.5$. Equilibrium present discounted profit is given by $\frac{1+\delta}{4}$, present discounted consumer surplus is given by $(1 + \delta)(1 - \tilde{v})(\frac{1+\tilde{v}}{2} - p^\star) = \frac{1+\delta}{8}$, and present discounted social surplus is $\frac{3(1+\delta)}{8}$.

3.2 Full Customer Recognition

Consider now the case in which the monopolist is able to recognize the previous customers and consumers are unable to opt out, as in Hart & Tirole (1988), Schmidt (1993), Villas-Boas (2004), and Taylor (2004). For example, an internet store may be able to recognize returning customers through cookies installed on their computer, and charge them different prices. In this setting, the monopolist can identify in the second period two different groups of consumers: those who purchased in the first period, and those

[4] The majority of the results go through when there is a finite number of consumers, provided we add the following assumption: the firm does not know (but has beliefs over) how many consumers opted out when setting second period prices. For example, if consumers can opt out by erasing cookies or by using a virtual credit card, then the firm does not know how many consumers actually used these techniques when it sets prices. Having a continuum of consumers allows us to avoid this assumption.

who did not purchase in the first period. In the second period, the monopolist can subsequently charge two different prices, p_2^1 and p_2^0.

Proposition 1 (Fudenberg and Villas-Boas 2006) *In the full-recognition equilibrium, consumers with valuations $v \in [\tilde{v}, 1]$ purchase in both periods and consumers with valuations $v \in [p_2^0, \tilde{v}]$ purchase only in the second period. In the second period, the firm sets $p_2^1 = \tilde{v}$. The cutoff type $\tilde{v}$ and prices p_1 and p_2^0 are determined by solving the following three equations: $\tilde{v}(p_1) = (1-\delta)^{-1}(p_1 - p_2^0(\tilde{v}))$, $\tilde{v}(p_1) = F(p_2^0) + f(p_2^0)p_2^0$, and the first-order condition:*

$$p_1(1-F(\tilde{v}(p_1)))+\delta(p_2^1(\tilde{v}(p_1))(1-F(p_2^1(\tilde{v}(p_1))))+p_2^0(\tilde{v}(p_1))(F(\tilde{v}(p_1))-F(p_2^0(\tilde{v}(p_1)))))$$

Finally, $\tilde{v} \geq p^\star$ holds in equilibrium.

Proof: A consumer of type v decides to buy in the first period if $v - p_1 + \delta \max\{v - p_2^1, 0\} \geq \delta \max\{v - p_2^0, 0\}$. From this inequality one can then obtain directly that if a type $\tilde{v}$ chooses to buy in the first period then all the types $v > \tilde{v}$ also choose to buy in the first period. That is, the consumers that buy for the first time in the second period value the product less than any of the consumers that buy in the first period.

In order to derive the type of the marginal consumer, we first consider the pricing decision of the monopolist with respect to identified consumers in the second period. Recall $p^\star = \arg\max_p p[1 - F(p)]$ is the price that maximizes the profit in one period when consumers do not have any reason to refrain from buying, that is, they buy if their valuation v is greater than the price charged. $p^\star$ is the monopoly price in a one-period game, or the price in the no-recognition equilibrium.

Let $\tilde{v}$ denote the type of the marginal consumer in the first period. If $\tilde{v} > p^\star$, the monopolist sets $p_2^1 = \tilde{v}$. If, on the other hand $\tilde{v} < p^\star$, the monopolist sets $p_2^1 = p^\star$. That is, $p_2^1 = \max\{\tilde{v}, p^\star\}$. The marginal consumer in the first period is then determined by

$$\tilde{v} - p_1 = \delta \max\{\tilde{v} - p_2^0, 0\}$$

which results in $\tilde{v} = p_1$ if $p_1 \leq p_2^0$, and $\tilde{v} = \frac{p_1 - \delta p_2^0}{1-\delta} \geq p_1$ if $p_1 > p_2^0$.

The expression for $\tilde{v}$ shows an important aspect of the market dynamics: If prices are expected to increase, each consumer does not have any reason to behave strategically and buys if his valuation is above the current price. If, on the other hand, prices are expected to decrease, some consumers will behave strategically, choosing not to purchase and be identified in the first period, in order to get a better deal in the second period.

To see that prices cannot increase, assume otherwise. Then $\tilde{v} = p_1$. However, the monopolist then sets $p_2^0(\tilde{v}) = \arg\max_p p(F(p_1) - F(p))$, implying that $p_2^0 < p_1$. Therefore, $p_2^0 \leq p_1$. Thus, the marginal consumer in the first period, $\tilde{v}(p_1)$, is determined by

$$\tilde{v}(p_1) = \frac{p_1 - \delta p_2^0(\tilde{v}(p_1))}{1-\delta}$$

The optimal prices in the second period are $p_2^1(\tilde{v}) = \max\{p^\star, \tilde{v}\}$ and $F(\tilde{v}) = F(p_2^0) + f(p_2^0)p_2^0$ (obtained from $p_2^0(\tilde{v}) = \arg\max_p p(F(\tilde{v}) - F(p))$). Hence, the monopolist sets the first period price, p_1, to maximize

$$p_1(1-F(\tilde{v}(p_1)))+\delta(p_2^1(\tilde{v}(p_1))(1-F(p_2^1(\tilde{v}(p_1))))+p_2^0(\tilde{v}(p_1))(F(\tilde{v}(p_1))-F(p_2^0(\tilde{v}(p_1))))) \tag{1}$$

where the first term represents profit from first-period sales and the second term represents second-period profit. Under the assumption that $\tilde{v} > p^\star$, which is satisfied in equilibrium, we have $p_2^1 = \tilde{v}$. Using the Envelope Theorem on the right-most part of (1), the first-order condition that defines the optimal $p_1^\star$ is given by

$$1 - F(\tilde{v}) - p_1^\star f(\tilde{v})\tilde{v}' + \delta\tilde{v}'(1 - F(\tilde{v}) - f(\tilde{v})\tilde{v} + f(\tilde{v})p_2^0(\tilde{v})) = 0 \tag{2}$$

Note that the marginal consumer buying the product in the first period has a higher valuation than if there were no customer recognition, i.e. $\tilde{v} \geq p^\star$. To see this, note that after substituting for $1 - F(p^\star) - p^\star f(p^\star) = 0$ and $p^\star(1-\delta) = p_1 - \delta p_2^0(p^\star)$, the first-order condition (2) evaluated at $\tilde{v} = p^\star$ is equal to $f(p^\star)p^\star(1-(1-\delta)\tilde{v}')$. Given that $\tilde{v}' = \frac{\partial\tilde{v}}{\partial p_1} = \frac{1}{1-\delta+\delta p_2^{0'}}$ and since $p_2^{0'} = \frac{\partial p_2^0}{\partial \tilde{v}} > 0$ follows from $F(\tilde{v}) = F(p_2^0) + f(p_2^0)p_2^0$ and quasiconcavity of $p(1-F(p))$, that derivative is positive. Hence, the monopolist should increase p_1, which, since $\tilde{v}' > 0$, implies a higher valuation of the marginal consumer than $p^\star$. ■

The monopolist's pricing strategy towards identified consumers in the second period, p_2^1, is of interest. In particular, if the cutoff type for identified consumers, $\tilde{v}$ satisfies $\tilde{v} > p^\star$, the monopolist sets $p_2^1 = \tilde{v}$. If, on the other hand $\tilde{v} < p^\star$, the monopolist sets $p_2^1 = p^\star$. That is, $p_2^1 = \max\{\tilde{v}, p^\star\}$. Hence, the marginal consumer in the first period gets no surplus in second period. This is the "ratchet effect" of consumers who reveal their types (e.g. Freixas *et al.* (1985)).

In equilibrium, a consumer with valuation $\tilde{v}$ is just indifferent between purchasing in both periods and only in the second period. Since $p_2^1 = \tilde{v}$, it follows that $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0)$. One can then simplify the firm's present discounted profit to obtain $\tilde{v}(1 - F(\tilde{v})) + \delta p_2^0(1 - F(p_2^0))$, which is strictly below the present value of profits under no customer recognition, since $p^\star$ uniquely maximizes $p(1 - F(p))$. The intuition is that the marginal consumers refrain from purchasing in the first period because they know that they can get a lower price in the next period. We note that the result of lower profits under full-recognition does not hold if the monopolist's discount factor is sufficiently high relative to the discount factor of consumers.

Finally, we note that if the monopolist is able to commit to second-period prices, the reduced expression for its present-discounted profit is given by

$$p_2^1(1 - F(p_2^1)) + \delta p_2^0(1 - F(p_2^0))$$

which is uniquely maximized when $p_2^1 = p_2^0 = p^\star$. Thus, the firm's profit under commitment coincides with the no-recognition equilibrium, and is higher than in the full-recognition game.

Example 2 When valuations are uniformly distributed, one can obtain $p_2^0(\tilde{v}) = \tilde{v}/2$, $\tilde{v}(p_1) = 2p_1/(2 - \delta)$, and $p_1^\star = (4 - \delta^2)/(8 + 2\delta)$. Simple algebra shows that, as argued above, the present value of profits is lower than in the no customer recognition case for all δ. One can also get that $2/(4 + \delta)$ consumers buy in both periods, while $(2 + \delta)/(8 + 2\delta)$ consumers only buy in the second period. As δ grows larger so that consumers become more strategic, the number of consumers buying in both periods decreases, as consumers wait for future deals. Subsequently, the number of consumers that only buy in the second period increases.

4 Opting Out and Partial Recognition

We now consider the setting in which consumers who purchase in the first period can opt out and preserve anonymity at a cost c (c could be the cost of understanding disclosures and taking actions to circumvent detection, such as using a virtual credit card). Consumers who purchase in the first period and do not opt out are recognized by the firm in the second period and will be offered the price p_2^1. All other consumers are offered the price p_2^0 in the second period. As above, let $\tilde{v}$ denote the lowest consumer type that purchases in the first period. Denote by $\alpha(v)$ the (possibly degenerate) probability that a type $v \in [\tilde{v}, 1]$ consumer deletes his sales record and remains anonymous in the second period. Then the distribution of valuations among anonymous consumers is

$$F^0(v) = \begin{cases} \dfrac{F(v)}{F(\tilde{v}) + \int_{\tilde{v}}^1 \alpha(x) f(x)\, dx} & \text{if } v \leq \tilde{v} \\[2ex] \dfrac{F(\tilde{v}) + \int_{\tilde{v}}^v \alpha(x) f(x)\, dx}{F(\tilde{v}) + \int_{\tilde{v}}^1 \alpha(x) f(x)\, dx} & \text{if } v > \tilde{v} \end{cases}$$

and the distribution of valuations among identifiable consumers (for $v \geq \tilde{v}$) is given by

$$F^1(v) = \frac{\int_{\tilde{v}}^{v} (1 - \alpha(x)) f(x)\, dx}{\int_{\tilde{v}}^{1} (1 - \alpha(x)) f(x)\, dx}$$

4.1 Costless Privacy

Proposition 2 (Costless privacy) *If $c = 0$, then the following strategies are part of a PBE: $p_1 = p_2^0 = p^\star$, $p_2^1 \geq p^\star$, and $\tilde{v} = p^\star$. Consumers with valuations $v \in [p^\star, 1]$ purchase in both periods and opt out with probability $\alpha = 1$.*

Proof: If $p_1 = p^\star$, since it is costless to opt out, every consumer with valuation $v \geq p^\star$ would (weakly) prefer to purchase the good in the first period. If $p_2^0 = p^\star$ and $p_2^1 \geq p^\star$, all the consumers who purchased in the first period also purchase in the second period, and other consumers do not. Furthermore, consumers are at most indifferent about opting out and not opting out, hence all consumers opting out is a best response. Consequently, it is a best response for the firm to set $p_2^0 = p^\star$. Similarly, given that all consumers with valuations $v \geq p_1$ purchase in the first period and opt out, the firm is maximizing $(1 - F(p))p + \delta(1 - F(p^\star))p^\star$ in the first period. Hence, setting $p_1 = p^\star$ is a best-response for the firm. Finally, any $p_2^1 \geq p_2^0$ maintains opting out as a best response for consumers (and any such p_2^1 is a best response by the firm, since all consumers opt out). ■

Corollary 1 later shows that under a sensible equilibrium refinement, when $c = 0$, the equilibrium in which $p_1 = p_2^0 = p_2^1 = \tilde{v} = p^\star$ is the unique equilibrium.

This result is alarming from the perspective of consumers. It says that if the cost of preserving anonymity is nil, then it is in the interest of every individual who purchases the good in the first period to delete his record. This, however, leads to an equilibrium outcome that is identical to the profit-maximizing full-commitment sales mechanism. Recall that the firm would like to commit not to use any information it learns about consumer valuations from purchasing histories; i.e., it would like to charge a constant price for the good. When consumers all opt out, they essentially grant the firm the requisite power to commit to this mechanism because it cannot use information it does not possess. Note, however, that if there were no option for preserving anonymity, then the equilibrium derived in the Customer Recognition section would obtain.

4.2 Costly Privacy

The Pooling PBE

In a *pooling* PBE, all consumers who purchase the good in the first period opt out with the same probability α. Hence, second-period beliefs in such an equilibrium are given by

$$F^0(v) = \begin{cases} \frac{F(v)}{F(\tilde{v})+\alpha(1-F(\tilde{v}))} & \text{if } v \leq \tilde{v} \\ \\ \frac{F(\tilde{v})+\alpha(F(v)-F(\tilde{v}))}{F(\tilde{v})+\alpha(1-F(\tilde{v}))} & \text{if } v > \tilde{v} \end{cases}$$

and

$$F^1(v) = \frac{F(v) - F(\tilde{v})}{1 - F(\tilde{v})}$$

In the second period, the firm chooses its prices to maximize profit according to

$$\max_{p_2^r} (1 - F^r(p_2^r))p_2^r \text{ for } r = 0, 1 \tag{3}$$

Lemma 1 *Let $c > 0$ and suppose $\alpha(v) = \alpha$ for all $v \in [\tilde{v}, 1]$. Then in every PBE, $p_2^0 \leq p_1 \leq \tilde{v} \leq p_2^1$.*

Proof: It is straightforward to see that as in the previous section, $p_2^1 = \max\{p^\star, \tilde{v}\}$. Hence, $\tilde{v} \leq p_2^1$. Assume on the contrary that $p_2^0 > p_2^1$. Then no consumer opts out. Hence, the only consumers for whom the firm has no record are those with valuations $v \in [0, \tilde{v}]$. Therefore, $p_2^0 \leq \tilde{v} \leq p_2^1$, which is in contradiction to the premise. Thus, $p_2^0 \leq p_2^1$.

Note that $p_1 \leq \tilde{v}$ must hold for the marginal consumer with valuation $\tilde{v}$ to be willing to purchase in the first period. Thus, it remains to show that $p_2^0 \leq p_1$. Assume on the contrary that $p_2^0 > p_1$. Then no new consumer purchases in the second period, and all consumers with valuations $v \geq p_1$ purchase in the first period. Hence, $\tilde{v} = p_1$. If $p_1 < p^\star$, the firm would set $p_2^1 = p^\star$. If $p_1 > p^\star$, the firm would set $p_2^1 = p_1$. Since no new consumers purchase in the second period, it is optimal for the firm to set $p_1 = p_2^1 = p^\star$. Subsequently, $\tilde{v} = p^\star$. If $p_2^0 > p_1 = \tilde{v}$, the firm has a strictly profitable deviation by setting $p_2^0 = \tilde{v} - \epsilon$, where $\epsilon < c$, because then consumers with valuations $v \in [\tilde{v} - \epsilon, \tilde{v}]$ would purchase in the second period, yet no repeat consumer would opt out. Hence, $p_2^0 \leq p_1$ must hold in equilibrium. ■

The following proposition characterizes the pooling equilibrium for sufficiently small values of c. Proposition 4, which follows immediately, fully specifies the relevant range on c.

Proposition 3 (Pooling equilibrium) *In a pooling equilibrium, consumers with valuations $v \in [\tilde{v}, 1]$ purchase in both periods and opt out with probability α. Consumers with valuations $v \in [p_2^0, \tilde{v}]$ purchase only in the second period. In the second period, the firm sets $p_2^1 = \max\{\tilde{v}, p^\star\}$, and p_2^0 is determined from $F(p_2^0(\tilde{v})) + f(p_2^0(\tilde{v}))p_2^0(\tilde{v}) = F(\tilde{v})(1-\alpha) + \alpha$. For sufficiently small c, the cutoff type $\tilde{v}$, the opting out probability α, and the first period price p_1 are determined by solving the following system of equations:*

$$\tilde{v} = \frac{p_1 - \delta p_2^0(\tilde{v})(1-\alpha) + \delta\alpha c}{1 - \delta(1-\alpha)}$$

$$1 - F(\tilde{v}) - p_1^\star f(\tilde{v})\tilde{v}' + \delta\tilde{v}'(1-\alpha)\big(1 - F(\tilde{v}) - f(\tilde{v})\tilde{v} + f(\tilde{v})p_2^0(\tilde{v})\big) = 0$$

$$\alpha = \frac{(\tilde{v} - c)f(\tilde{v} - c) + F(\tilde{v} - c) - F(\tilde{v})}{1 - F(\tilde{v})}$$

In equilibrium, $\tilde{v} \geq p^\star$, $p_1 = \tilde{v} - \delta c$, $p_2^1 = \tilde{v}$, and $p_2^0 = \tilde{v} - c$.

Proof: It follows from Lemma 1 that the (relevant) solutions to the firm's second period problem are given by

$$p_2^0(\tilde{v}) = \arg\max_p (1 - \frac{F(p)}{F(\tilde{v}) + \alpha(1 - F(\tilde{v}))})p \tag{4}$$

and

$$p_2^1 = \max\{p^\star, \tilde{v}\} \tag{5}$$

To determine the marginal consumer type $\tilde{v}$ that purchases in the first period, equate the expected utility from purchasing the good in the first period to the utility from waiting to purchase until the second period. By Lemma 1, this amounts to

$$\tilde{v} - p_1 + \delta\big(\alpha(\tilde{v} - p_2^0 - c) + (1-\alpha)\max\{0, (\tilde{v} - p_2^1)\}\big) = \delta(\tilde{v} - p_2^0)$$

Assuming $\tilde{v} \geq p^\star$ so that $p_2^1 = \tilde{v}$ (which we show below is satisfied in equilibrium), we obtain

$$\tilde{v} = \frac{p_1 - \delta p_2^0(\tilde{v})(1-\alpha) + \delta\alpha c}{1 - \delta(1-\alpha)} \tag{6}$$

The firm's first-period problem is to choose the marginal type $\tilde{v}$ and the prices p_1, p_2^0 and p_2^1 to solve

$$\max_{p_1} (1 - F(\tilde{v}(p_1)))p_1 + \delta\big((1-\alpha)(1 - F(\tilde{v}(p_1)))\tilde{v}(p_1) +$$
$$+ (F(\tilde{v}(p_1)) + \alpha(1 - F(\tilde{v}(p_1))) - F(p_2^0(\tilde{v}(p_1))))p_2^0(\tilde{v}(p_1))\big)$$

Using the Envelope Theorem on the right-most part, the first-order condition is given by

$$1 - F(\tilde{v}) - p_1^\star f(\tilde{v})\tilde{v}' + \delta\tilde{v}'(1-\alpha)\big(1 - F(\tilde{v}) - f(\tilde{v})\tilde{v} + f(\tilde{v})p_2^0(\tilde{v})\big) = 0 \tag{7}$$

In order for mixing consumers to be indifferent between opting out and not opting out, $p_2^0 + c = p_2^1$ must hold in equilibrium. From the equilibrium condition that $p_2^1 = \tilde{v} = p_2^0 + c$ together with (9), we can obtain

$$\alpha = \frac{(\tilde{v} - c)f(\tilde{v} - c) + F(\tilde{v} - c) - F(\tilde{v})}{1 - F(\tilde{v})} \tag{8}$$

Given a sufficiently small[5] c, equalities (4)-(8) fully characterize the equilibrium. Given a specific distribution F, they can be used to solve for p_2^0, p_1, $\tilde{v}$, and α.

We now show that $\tilde{v} \geq p^\star$ is indeed satisfied in equilibrium. From (4), $p_2^0(\tilde{v})$ is implicitly defined by the following first-order condition:

$$F(p_2^0(\tilde{v})) + f(p_2^0(\tilde{v}))p_2^0(\tilde{v}) = F(\tilde{v})(1 - \alpha) + \alpha \tag{9}$$

Since $F(\tilde{v})(1 - \alpha) + \alpha \leq 1$, it follows from (9) that $p_2^0 \leq p^\star$ (recall $p^\star$ satisfies $F(p^\star) + f(p^\star)p^\star = 1$). Quasiconcavity of $p(1 - F(p))$ implies that $2f(p) + f'(p)p \geq 0$ for $p \leq p^\star$. In addition, by implicit function theorem, $\partial p_2^0/\partial \tilde{v} = p_2^{0'} = \frac{f(\tilde{v})(1-\alpha)}{2f(p_2^0)+f'(p_2^0)p_2^0}$. Since $p_2^0 \leq p^\star$, it follows that $p_2^{0'} \geq 0$.

Now, after substituting for $1 - F(p^\star) - p^\star f(p^\star) = 0$ and $p^\star(1 - \delta(1 - \alpha)) = p_1 - \delta(1 - \alpha)p_2^0(p^\star) - \delta\alpha c$, the first-order condition (7) evaluated at $\tilde{v} = p^\star$ reduces to

$$f(p^\star)\big(p^\star - \tilde{v}'(p^\star(1 - \delta(1 - \alpha)) - \delta\alpha c)\big)$$

Using (6), we can substitute for p_1, and given that $\tilde{v}' = \frac{1}{1-\delta(1-\alpha)+\delta(1-\alpha)p_2^{0'}}$ and $p_2^{0'} \geq 0$, the above derivative reduces to $f(p^\star)\big(p^\star(1 - \frac{1-\delta(1-\alpha)}{1-\delta(1-\alpha)+\delta(1-\alpha)p_2^{0'}}) + \frac{\delta\alpha c}{1-\delta(1-\alpha)+\delta(1-\alpha)p_2^{0'}}\big)$, which is non-negative and strictly positive if $\delta > 0$. Hence, the monopolist would increase p_1. Since $\tilde{v}' > 0$, this entails $\tilde{v} \geq p^\star$. (We note that when $c = 0$ or $\delta = 0$, $\tilde{v} = p^\star$.)

When $\alpha \in (0, 1)$, $p_2^0 + c = p_2^1$ must hold in equilibrium for consumers to be willing to mix. Since $p_2^1 = \tilde{v}$, this further implies that $p_2^0 + c = \tilde{v}$. Subsequently, we have $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0)$. In other words, after purchasing in the first period, the marginal consumer anticipates zero payoff from the second period. Thus, the marginal consumer is just indifferent between purchasing only in the first period and purchasing only in the second period. Therefore, when $\alpha \in (0, 1)$, $p_1 = \tilde{v} - \delta c$, $p_2^0 = \tilde{v} - c$, and $p_2^1 = \tilde{v}$. ■

It follows from Proposition 3 that when c is sufficiently small, $p_1 = \tilde{v} - \delta c$, $p_2^0 = \tilde{v} - c$, and $p_2^1 = \tilde{v}$. Essentially, new customers receive an "introductory" offer in both periods. The following proposition formally characterizes equilibrium as a function of c, and provides the relevant range on c.

[5]The relevant range of c is formally described in Proposition 4.

Proposition 4 (Scope of pooling equilibrium) *There exists $\bar{c} > 0$ such that $\alpha(c) > 0$ for all $c \in [0, \bar{c})$ and $\alpha(c) = 0$ for all $c \geq \bar{c}$. When $c \geq \bar{c}$, the pooling equilibrium coincides with the full-recognition equilibrium, and when $c = 0$, the pooling equilibrium coincides with the no-recognition equilibrium.*

Proof: Consider the firm's pricing strategy in the second period of the full-recognition game:

$$p_2^{0\star} = \arg\max_{p_2^0} (F(\tilde{v}) - F(p_2^0))p_2^0$$

and

$$p_2^{1\star} = \max\{\tilde{v}, p^\star\}$$

Let $\bar{c}$ denote the first $c > 0$ such that $\alpha(\bar{c}) = 0$. Such $\bar{c}$ exists for the following reason. Consider $\hat{c} = p_2^{1,FR} - p_2^{0,FR}$, where the latter are the second period prices from the equilibrium of the full-recognition game. In the partial-privacy game, given $c = \hat{c}$, it is equilibrium for $\alpha(\hat{c}) = 0$, and for all other variables to coincide with the full-recognition outcome. Hence, $\hat{c}$ is a candidate for $\bar{c}$, and indeed satisfies $\alpha(\hat{c}) = 0$. To see that $\bar{c} = \hat{c}$, suppose otherwise. Then there exists $c < \hat{c}$ such that $p_2^1(c) - p_2^0(c) = c$ and $\alpha(c) = 0$. However, that implies that there is another solution to the full-recognition game, which is a contradiction. Thus, $\bar{c} = \hat{c} = p_2^{1,FR} - p_2^{0,FR}$.

Now, assume on the contrary that for some $c > \bar{c}$, α increases (above 0). First, note that since $c > \bar{c}$, the full recognition equilibrium is feasible at c, and the firm can obtain it by setting $p_1 = p_1^{FR}$. From $p_2^1 - p_2^0 = c$, we have $p_2^0 = \tilde{v} - c$. Taking a derivative with respect to c, we obtain $\frac{\partial p_2^0}{\partial c} = \frac{\partial \tilde{v}}{\partial c} - 1$. Additionally, from (9),

$$\frac{\partial p_2^0}{\partial c} = \frac{\partial p_2^0}{\partial \tilde{v}} \frac{\partial \tilde{v}}{\partial c} + \frac{\partial p_2^0}{\partial \alpha} \frac{\partial \alpha}{\partial c}$$

Combining the equalities and simplifying, we obtain

$$\frac{\partial \tilde{v}}{\partial c}(p_2^{0'} - 1) + \frac{\partial p_2^0}{\partial \alpha} \frac{\partial \alpha}{\partial c} = -1 \tag{10}$$

It is straightforward to check that $\frac{\partial p_2^0}{\partial \alpha} > 0$. Since α increases and $p_2^{0'} \in (0, 1)$, we have $\tilde{v} > \tilde{v}^{FR}$. Furthermore, $\frac{\partial \tilde{v}}{\partial c} > 1$ must hold, so that both p_1 and p_2^0 are higher than in the full-recognition equilibrium.

Now, because $\tilde{v}^{FR} < \tilde{v}(c)$, the firm is able to set $p_2^0 = p_2^0(c)$ and $p_2^1 = p_2^1(c)$ under $\bar{c}$. Furthermore, the firm's profit would be higher than under c since $\alpha(\bar{c}) = 0$ and $\alpha(c) > 0$. Thus, the firm's profit is strictly higher under the full-recognition outcome than the outcome under the equilibrium when $c > \bar{c}$. Finally, since the full-recognition equilibrium outcome is feasible at c, this is a contradiction. Therefore, $\alpha(c) = 0$ for all $c \geq \bar{c}$.

When $c \in [0, \bar{c}]$, the firm's first-order condition in the first period is given by

$$1 - F(\tilde{v}) - p_1^\star f(\tilde{v})\tilde{v}' + \delta\tilde{v}'(1-\alpha)\big(1 - F(\tilde{v}) - f(\tilde{v})\tilde{v} + f(\tilde{v})p_2^0(\tilde{v})\big) = 0$$

we can substitute in the equilibrium equalities $p_1 = \tilde{v} - \delta c$ and $p_2^0 = \tilde{v} - c$ to obtain

$$1 - F(\tilde{v}) - (\tilde{v} - \delta c) f(\tilde{v})\tilde{v}' + \delta\tilde{v}'(1-\alpha)\big(1 - F(\tilde{v}) - f(\tilde{v})\tilde{v} + f(\tilde{v})(\tilde{v} - c)\big) = 0 \quad (11)$$

Substituting in $1/\tilde{v} = 1 - \delta(1-\alpha) + \delta(1-\alpha)p_2^{0'}$ and rearranging, we obtain

$$\alpha\delta c = \tilde{v} - \frac{1 - F(\tilde{v})}{f(\tilde{v})}(1 + \delta(1-\alpha)p_2^{0'}) \quad (12)$$

Clearly when $\alpha = 0$, $\tilde{v}$ is independent of c and is equal to v^{FR}. Thus, since $\alpha(c) = 0$ for all $c \geq \bar{c}$, $\tilde{v}(c) = v^{FR}$ for $c \geq \bar{c}$ holds as well.

To see that $\alpha(c) \in [0, 1)$ for any $c > 0$, assume otherwise. Then, $\alpha(c) = 1$ for some $c > 0$. Thus, all consumers with valuations $v \in [\tilde{v}(c), 1]$ who purchase in the first period choose to opt out. For this to hold, it must be that $p_2^1 - p_2^0 > c$. However, in the second period, the firm possesses a profitable deviation: by setting $p_2^1 = p_2^0 + c - \epsilon$, for some small $\epsilon > 0$, the firm's profit increases by at least $(1 - F(\tilde{v}(c)))(c - \epsilon)$, which is strictly positive as $\epsilon \to 0$ since $c > 0$. Hence, $\alpha(c) = 1$ cannot occur when $c > 0$. ■

One can obtain that for $c \in [0, \bar{c}]$, the present value of profit under partial recognition is given by

$$\Pi(c) = (\tilde{v}(c) - \delta\alpha(c)c)(1 - F(\tilde{v}(c))) + \delta(\tilde{v}(c) - c)(1 - F(\tilde{v}(c) - c))$$

which may be above or below profits under full recognition, depending on the value of c (see uniform example in Section 5). Note that the firm can always obtain the second period profit in the no-recognition equilibrium by setting $p_2^0 = p_2^1 = p^\star$. Since $\tilde{v} > p^\star$, first period profit is lower than under no recognition, but second period profit is higher because the firm is able to price discriminate. When $c = 0$, the firm obtains the no-recognition equilibrium profit. When $c > \bar{c}$, $\alpha(c) = 0$, and the firm obtains the full-recognition equilibrium profit. It turns out that the firm's profit is always higher under no-recognition, which we prove in the following result.

Proposition 5 (Firm profit) *The firm's profit is highest when $c = 0$.*

Proof: First, we note that the firm's profit in a pooling equilibrium under any $c \in (0, \bar{c}]$ is no greater (but potentially less) than its profit when it collects the opting out fee, which is given by

$$\Pi(c) = \tilde{v}(c)(1 - F(\tilde{v}(c))) + \delta p_2^0(1 - F(p_2^0))$$

The above expression is uniquely maximized when $\tilde{v}(c) = p_2^0 = p^\star$. However, $\tilde{v} > p^\star$ for all $c > 0$. Thus, the firm's profit is lower when $c > 0$ than under the no-recognition equilibrium when $c = 0$. ■

An intuitive explanation for this result is that when $c = 0$, the firm is able to obtain the full-commitment profit, since it is effectively able to commit to constant prices $p^\star$ in both periods. Essentially, this allows the firm to commit not to use information about consumers (because the firm in fact does not have this information in equilibrium).

Equilibrium consumer surplus is given by

$$\int_{\tilde{v}}^{1} v f(v)dv - (1-F(\tilde{v}))(\tilde{v}-\delta c) + \delta(\int_{\tilde{v}-c}^{1} v f(v)dv + (F(\tilde{v})-F(\tilde{v}-c))c - (1-F(\tilde{v}-c))\tilde{v}) \tag{13}$$

The first and second terms in (13) represent the surplus from period 1 transactions: consumers with valuations $v \in [\tilde{v}, 1]$ purchase the good and pay a price $\tilde{v} - \delta c$. The remaining terms represent the surplus from period 2 transactions. Specifically, consumers with valuations $v \in [\tilde{v} - c, 1]$ are repeat customers and end up paying $\tilde{v}$ (including the cost of opting out). Consumers with valuations $v \in [\tilde{v} - c, \tilde{v}]$ are first time shoppers and they receive a price discount of c. Hence, as mentioned before, new customers effectively receive an "introductory" offer in both periods.

Lemma 2 *When δ is close to 1 and $c > 0$, $p_1(\bar{c}) < p^\star$.*

Proof: Consider the marginal consumer with type $\tilde{v}$ who is just indifferent between purchasing in both periods and only in the second period. For $\tilde{v}$, we have $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0)$. Recall that p_2^0 targets consumers in $[0, \tilde{v}]$, and is implicitly defined by the equality

$$F(p_2^0(\tilde{v})) + f(p_2^0(\tilde{v}))p_2^0(\tilde{v}) = F(\tilde{v})$$

Also recall that $\bar{c}$ and $\tilde{v}(\bar{c})$ satisfy

$$F(\tilde{v} - \bar{c}) + f(\tilde{v} - \bar{c})(\tilde{v} - \bar{c}) = F(\tilde{v})$$

Since $p^\star$ uniquely satisfies $F(p^\star) + f(p^\star)p^\star = 1$, if $\tilde{v}(\bar{c}) = 1$, no consumer would purchase in the first period, so that $p_2^0 = p^\star$. If this is the case, the monopolist possesses a profitable deviation by lowering p_1, thus selling to some consumers in the first period, while still being able to set $p_2^0 = p_2^1 = p^\star$ and obtain the same second period profit as before. Therefore, $\tilde{v}(\bar{c}) < 1$, so that $p_2^0 < p^\star$.

Let $k > 0$ such that $p_2^0 = p^\star - k$. Then in equilibrium we have $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0) = \delta(\tilde{v} - (p^\star + k))$. Substituting $\tilde{v} = p_1 + \delta c$ and rearranging, we obtain

$$p^\star - p_1 = k - c(1 - \delta)$$

Therefore, for sufficiently high δ (e.g., $\delta = 1$), we have $p_1(\bar{c}) < p^\star$. ■

Proposition 6 (Consumer surplus) *When δ is close to 1 and consumer valuations are distributed according to $F_x(v) = v^x$ for $x \in [0,1]$, consumer surplus is higher under full-recognition ($c = \bar{c}$) than under no-recognition ($c = 0$).*

Proof: The proof proceeds as follows. First, it is immediately apparent that consumers with valuations $v \in [p_2^0, p^\star]$ are able to purchase in the second period under full-recognition ($c = \bar{c}$), whereas under no-recognition ($c = 0$) they do not purchase at all. Hence, such consumers are better off under full-recognition. Additionally, consumers with valuations $v \in [0, p_2^0(\bar{c})]$ do not purchase at all in both settings, and so they are indifferent. Next, we prove that consumers who purchase the good in both periods under full-recognition are better off than under no-recognition. Finally, we will show that consumers with valuations $v \in [p^\star, \tilde{v}(\bar{c}))$ (who purchase only in the second period under $\bar{c}$) are also better off under full-recognition.

In the proof of Proposition 3, we showed that $\tilde{v} > p^\star$. Under $c = 0$, consumers with valuations $v \in [\tilde{v}(\bar{c}), 1]$ obtain a net utility of $(1+\delta)(v - p^\star)$. Under $\bar{c}$, their utility is $v - p_1 + \delta(v - \tilde{v})$. The difference in utilities is given by $p^\star - p_1 - \delta(\tilde{v} - p^\star)$. Hence, for these consumers to be at least as well off under $\bar{c}$, we must have $(1+\delta)p^\star - p_1 - \delta\tilde{v} \geq 0$. Since $\tilde{v} = p_1 + \delta\bar{c}$ holds in equilibrium, it suffices to show that

$$\frac{p_1 + \delta\tilde{v}}{1+\delta} = p_1 + \frac{\delta^2}{1+\delta}\bar{c} \leq p^\star$$

Suppose on the contrary that this condition is not satisfied, i.e. for some $k > 0$, $p_1 = p^\star - \frac{\delta^2}{1+\delta}\bar{c} + k$, so that $\tilde{v} = p^\star + \frac{\delta}{1+\delta}\bar{c} + k$. Recall that p_2^0 targets consumers in $[0, \tilde{v}]$, and is implicitly defined by the equality

$$F(p_2^0(\tilde{v})) + f(p_2^0(\tilde{v}))p_2^0(\tilde{v}) = F(\tilde{v})$$

Since F is continuous, there exists $w < 1$ such that $p_2^0 < w\tilde{v}$. In equilibrium, we have $p_2^0 = \tilde{v} - c$. Since $p_1 = \tilde{v} - \delta c$, we have $p_1 = p_2^0 + (1-\delta)c$. Thus, $p_1 \leq w\tilde{v} + (1-\delta)c$. Substituting for p_1 and $\tilde{v}$, we obtain:

$$p^\star - \frac{\delta^2}{1+\delta}\bar{c} + k \leq w(p^\star + \frac{\delta}{1+\delta}\bar{c} + k) + (1-\delta)c$$

Rearranging, we obtain

$$p^\star(1-w) - (\delta + w)\frac{\delta}{1+\delta}\bar{c} + (k(1-w) - (1-\delta)c) \leq 0 \tag{14}$$

When δ is close to 1, the right-most term $k(1-w)-(1-\delta)c$ is non-negative. Noting that $\delta \le 1$ and $\delta/(1+\delta)$ is increasing in δ (and highest at $1/2$ when $\delta = 1$), in order to violate Inequality (14) (and thus obtain a contradiction), it is sufficient to show that

$$2p^\star \ge \frac{1+w}{1-w}\bar{c} \tag{15}$$

Consider the class of cumulative distribution functions $F_x(v) = v^x$ on $[0,1]$, where $x \le 1$. For such distributions, $w = p_x^\star = \frac{1}{(1+x)^{1/x}}$. Let $\bar{c}_x$ denote the minimum opting out cost where the full-recognition equilibrium is obtained for a given x and some fixed δ. Note that $p_x^\star = \frac{1}{(1+x)^{1/x}}$. Using the equilibrium characterization in Proposition 3 and substituting $F(v) = v$, one can obtain $\bar{c}_1 = (2+\delta)(8+2\delta)^{-1}$, which is highest when $\delta = 1$ at 0.3. Thus, $\bar{c}_1 \le 0.3$. Recall the first-order condition in the full-recognition game, i.e. when $c = \bar{c}$:

$$1 - F(\tilde{v}) - p_1^\star f(\tilde{v})\tilde{v}' + \delta\tilde{v}'(1 - F(\tilde{v}) - f(\tilde{v})\tilde{v} + f(\tilde{v})p_2^0(\tilde{v})) = 0 \tag{16}$$

where $\tilde{v}' = \frac{1}{1-\delta+\delta p_2^{0'}}$. Substituting for F_x in the equality that defines p_2^0,

$$F_x(p_2^0(\tilde{v})) + f_x(p_2^0(\tilde{v}))p_2^0(\tilde{v}) = F_x(\tilde{v})$$

we obtain $p_2^0 = \frac{\tilde{v}}{(1+x)^{1/x}}$, so that $p_2^{0'} = p^\star < 1$. Thus, $\tilde{v}' = \frac{1}{1-\delta(1-p^\star)} > 1$. Furthermore, $\tilde{v}'$ increases as x decreases. Substituting $p_2^0 = \tilde{v} - \bar{c}$ in (16) and rearranging, we obtain

$$\frac{1-F_x(\tilde{v})}{f_x(\tilde{v})} - \tilde{v}_x'(p_1 + \delta\bar{c}_x - \delta\frac{1-F_x(\tilde{v})}{f_x(\tilde{v})}) = 0 \tag{17}$$

Suppose for a moment that $\bar{c}_x$ is fixed. The term on the left hand-side in (17) is positive and decreases as x decreases, while the bracketed term on the right hand-side is positive (since $\tilde{v}' > 0$) and increases as x decreases. Since $\tilde{v}'$ also increases as x decreases, p_1 (and subsequently, $\tilde{v}$) would have to be lower. Since $\tilde{v} = p_1 + \delta\bar{c}$ holds in equilibrium and $\tilde{v}' > 1$, $\bar{c}_x$ must decrease as well. Therefore, as x decreases, $\bar{c}_x$ decreases, so that for all $x < 1$, $\bar{c}_x \le \bar{c}_1 \le 0.3$.

Combining the above observations, in order for Inequality (15) to hold for all $x \le 1$, it is sufficient to have

$$p_x^\star \frac{1-p_x^\star}{1+p_x^\star} > 0.15 \tag{18}$$

Simple algebra can be used to check that the left hand-side of Inequality (18) is smallest when $x = 1$ and $p_1^\star = 0.5$, at which point Inequality (18) is satisfied. Thus, Inequality (14) is violated, which implies that consumers with valuations $v \in [\tilde{v}(\bar{c}), 1]$ are indeed better off under full-recognition than under no-recognition.

From the above analysis, we have $p^\star \leq \tilde{v}(\bar{c}) \leq p^\star + \frac{\delta}{1+\delta}\bar{c}$ and $p_1 \leq p^\star - \frac{\delta^2}{1+\delta}\bar{c}$. It remains to show that consumers with valuations $v \in [p^\star, \tilde{v}(\bar{c}))$ are also better off under full-recognition than under no-recognition. Under no-recognition, a consumer with valuation $v \in [p^\star, \tilde{v}(\bar{c}))$ purchases in both periods, and their present discounted utilities is given by $v - p^\star + \delta(v - p^\star)$. Under full-recognition, such a consumer is able to purchase in the first period and by Lemma 2 obtain a non-negative utility of $v - p_1 \geq v - p^\star + \frac{\delta^2}{1+\delta}\bar{c}$. Since $\delta(v - p^\star) \leq \delta(\tilde{v} - p^\star) \leq \frac{\delta^2}{1+\delta}\bar{c}$, it follows that all consumers with valuations $v \in [p^\star, \tilde{v}(\bar{c})]$ are also better off under full-recognition, which completes the proof. ■

In Section 5, we provide sharper comparative static results using a specific distribution. In particular, we show that when valuations are distributed uniformly and $\delta > 0$, p_1, p_2^0, and α decrease in c for $c \in [0, \bar{c})$, while $\tilde{v}$ and p_2^1 increase. Additionally, consumer surplus is increasing in c, while firm profit and social surplus are non-monotonic.

4.3 Purification Refinement

In this subsection, we present an equilibrium refinement based on taking the limit of a sequence of games where all consumers' costs of opting out are perturbed. More formally, let G denote the original game. Let d_i, $i \in \mathbb{N}$, denote a sequence of *continuous* distributions over the cost of opting out (which is conditionally independent from F), such that $\lim_{i\to\infty} d_i$ is the degenerate distribution on c. Let G^{d_i} denote the perturbed game where consumers realize their cost of opting out after they make their first period purchasing decisions.

Definition 1 (Refined Equilibrium) *An equilibrium of the original game is a* refined equilibrium *if it is the limit of a sequence of equilibria of conditionally independent (continuously) cost-perturbed games.*

The following proposition shows that the pooling equilibrium is the limit of G^{d_i} as $i \to \infty$. Moreover, it is the only equilibrium that satisfies this refinement.

Proposition 7 (Refinement) *The pooling equilibrium is the unique refined equilibrium.*

Proof: Since consumers realize their cost of opting out after they make their first period purchasing decisions, we need only focus on the monopolist's price setting problem and consumer's purchasing decisions in the second period. Let $\tilde{v}$ denote the marginal consumer who is just indifferent between purchasing in the first period and possibly in the second period, and purchasing only in the second period. First, we point out that $p_2^0 \le p_2^1$. To see this, note that p_2^0 targets consumers with valuations in $[0,1]$, while p_2^1 targets consumers in $[\hat{v},1]$, where $\hat{v} \ge \tilde{v}$. Since G^{d_i} is conditionally independent from $F(v)$, the proportion of consumers in $[\hat{v},1]$ who do not opt out is independent of their valuation. Combined, these observations imply that $p_2^0 \le p_2^1$. Consequently, a consumer whose realized opting out cost is less than $p_2^1 - p_2^0$ would prefer to opt out. In setting period 2 prices, the firm's problem is the following:

$$\max_{p_2^0, p_2^1} (F(\tilde{v}) - F(p_2^0))p_2^0 + (1 - F(\tilde{v}))(p_2^1 - d_i(p_2^1 - p_2^0)(p_2^1 - p_2^0))$$

Here, $d_i(p_2^1 - p_2^0)$ is the proportion of consumers with valuations $v \in [\tilde{v},1]$ who pay the "discounted price" p_2^0 in the second period. Hence, out of consumers with valuations $v \in [\tilde{v},1]$, a proportion of $d_i(p_2^{1\star} - p_2^{0\star})$ will opt out. Suppose d_i is distributed over $[c_L, c_H]$. Note that if $p_2^{1\star} - p_2^{0\star} > c_H$, all consumers will opt out (which is suboptimal in the second period if $c_H > 0$, because the firm can capture (some of) the opting out cost paid by some consumers by slightly increasing p_2^1). On the other hand, if $p_2^{1\star} - p_2^{0\star} < c_L$, since $p_2^1 \ge \tilde{v}$, we have $p_2^0 + c_L > \tilde{v}$. Hence, no consumer would opt out. Subect to $p_2^0 + c_L > \tilde{v}$, the firm's second period problem becomes

$$\max_{p_2^0, p_2^1} (F(\tilde{v}) - F(p_2^0))p_2^0 + (1 - F(\tilde{v}))p_2^1$$

The firm's optimal pricing strategy in this case is the same as in the full recognition equilibrium in the original game when $c \ge \bar{c}$. Hence, when $c_L \ge \bar{c}$, the equilibria of the original game and of the perturbed game are strategically equivalent, whereby $p_2^1 - p_2^0 = \bar{c}$.

It remains to consider the equilibrium of the perturbed game when $c_L < \bar{c}$. When $c_L < \bar{c}$, $p_2^0 + c_L > \tilde{v}$ can no longer be part of an equilibrium, or we would have a contradiction with $c = c_L$ and $\alpha = d_i(p_2^{1\star} - p_2^{0\star})$ in the original game by Proposition 2. Hence, under d_i, $p_2^{1\star} - p_2^{0\star} \in [c_L, c_H]$. Therefore, as $i \to \infty$, $p_2^{1\star} - p_2^{0\star} = c$. Additionally, under $d_{i\to\infty}$, the opting out decision of a consumer who purchases the good in both periods behave independently of their valuation.

The pooling equilibrium of the original game satisfies $p_2^1 - p_2^0 = c$. Additionally, it is the only equilibrium where all consumer types who purchase the good in both periods have the same opting out strategy. Hence, it is the only refined equilibrium. ■

Combining Proposition 7 with the result in Proposition 2, we obtain the following corollary.

Corollary 1 (Costless Privacy) *If $c = 0$, then every* refined *PBE outcome involves $p_1 = p_2^0 = p_2^1 = \tilde{v} = p^\star$ and $\alpha(v) = 1 \ \forall \ v \in [p^\star, 1]$.*

Proof: By Proposition 7, the pooling equilibrium is the unique refined equilibrium. Taking $c \to 0$ in the pooling equilibrium yields $p_1 = p_2^0 = \tilde{v} = p^\star$. From Proposition 2, any $p_2^1 \geq p^\star$ can be a part of a PBE when $c = 0$. However, a slight perturbation in cost yields $p_2^1 = p^\star$. Hence, p_2^1 must equal $p^\star$ in a refined equilibrium. The result follows. ■

4.4 Myopic Consumers

So far, we have not emphasized the fact that $\tilde{v}$, α, and $\bar{c}$ are functions of the parameter δ. For example, when $\delta = 0$, all consumers with valuations $v \in [p_1, 1]$ purchase in the first period. Consequently, when $\delta = 0$, the firm is able to obtain its no-recognition (full-commitment) profit by setting $p_1 = p^\star$. Mathematically, the first-order condition (12) shows that $\tilde{v} = p^\star$ when $\delta = 0$, and $p_1 = \tilde{v} - \delta c = p^\star$. Importantly, if $\delta = 0$, we have from $p_2^1 - p_2^0 = c$ that

$$\bar{c}_{\delta=0} = p^\star - \frac{F(p^\star) - F(p^\star - c)}{f(p^\star - c)}$$

When valuations are distributed uniformly and $\delta = 0$, we have $\alpha = 1 - 4c$ for $c \in [0, 1/4]$. Effectively, consumers "wake up" in the second period and realize that although they made a purchase in the first period, they can still opt out to try and get a better deal. However, if the firm were not myopic, it may choose to set $p_1 > p^\star$ in order to be able to better price discriminate against consumers with high valuations in the second period.

We will now consider this case. Let δ_F denote the discount factor of the firm and δ_A that of agents. Note that $\tilde{v}$ is defined by

$$\tilde{v} = \frac{p_1 - \delta_A p_2^0 + \alpha \delta_A c}{1 - \delta_A(1 - \alpha)}$$

It is evident that as $\delta_A \to 0$, $\tilde{v}$ approaches p_1. The firm's first-order condition with respect to p_1 is now given by

$$1 - F(\tilde{v}) - p_1^\star f(\tilde{v})\tilde{v}' + \delta_F \tilde{v}'(1 - \alpha)\big(1 - F(\tilde{v}) - f(\tilde{v})\tilde{v} + f(\tilde{v}) p_2^0(\tilde{v})\big) = 0$$

where $\tilde{v}' = 1/(1 - \delta_A(1 - \alpha)(1 - p_2^{0'}))$. When $\delta_A = 0$, we have $\tilde{v} = p_1$ and $\tilde{v}' = 1$. The firm's first-order condition becomes

$$1 - F(p_1) - p_1 f(p_1) + \delta_F(1 - \alpha)\big(1 - F(p_1) - f(p_1)p_1 + f(p_1)p_2^0\big) = 0 \qquad (19)$$

When $p_1 = p^\star$ and $\delta_F > 0$, the left-hand side of (19) is positive since $1 - F(p^\star) - p^\star f(p^\star) = 0$. Thus, if $\delta_F > 0$, $p_1 > p^\star$. The intuition is that the firm takes advantage of consumers being myopic and purchasing in the first period in order to further price discriminate against them in the second period. Although the firm initially loses some profit as a result of raising p_1 above $p^\star$, it more than makes up for it in the second period.

We further note that under the circumstances of δ_F being relatively large in comparison to δ_A, the firm's profit can surpass the no-recognition equilibrium profit. This is straightforward to see when $\delta_A = 0$: the no-recognition equilibrium profit is feasible by setting $p_1 = p_2^0 = p_2^1 = p^\star$ (as a result of $\tilde{v} = p_1$). However, given $\delta_F > 0$, the firm prefers to set $p_1 > p^\star$.

4.5 A More General Cost Function

So far, we have assumed that the cost of opting out, c, is expended in the second period. More generally, the cost of opting out can be described by $C = c_1 + \delta c_2$, where c_1 is the part of the cost expended in the first period (e.g. using a virtual credit card when making a purchase, or acquiring a gift certificate from a brick and mortar store), and c_2 is the part of the cost expended in the second period (e.g. calling a service in order to opt out or to cancel a service before purchasing again). The assumption underlying this cost function is that both costs c_1 and c_2 must be expended in order to successfully opt out. For example, if a consumer used a credit card as opposed to a gift certificate to make a purchase in the first period, but did call to cancel a service before purchasing again, this consumer would still be recognized.

Fortunately, all of the previous analysis goes through by setting $c = c_1/\delta + c_2$. Hence, if $c_1/\delta + c_2 \geq \bar{c}$, the full recognition equilibrium results. It is straightforward to see that if there is a cost associated with opting out in the first period (i.e., $c_1 > 0$) and δ is sufficiently close to 0 (e.g., $\delta \in (0, c_1]$), then no consumer opts out and the full-recognition equilibrium results. (If $c_1 > 0$ and $\delta = 0$, then no consumer opts out, but all consumers with valuations $v \in [p_1, 1]$ purchase in the first period, and so the no-recognition equilibrium results.)

5 Uniformly Distributed Valuations

In order to better illustrate the partial-recognition pooling equilibrium, we use this section to provide its full characterization when valuations are uniformly distributed. We are also able to provide sharper comparative static results.

First, we note that the no-recognition equilibrium results when $c = 0$, where the firm sets prices $p = p^\star = 1/2$. The full-recognition equilibrium results when $c \geq \bar{c}$ (so that $\alpha = 0$). From the definition of α, we have that $\alpha = 0$ implies

$$\tilde{v}(\bar{c}) - \bar{c} = \frac{F(\tilde{v}(\bar{c})) - F(\tilde{v}(\bar{c}) - \bar{c})}{f(\tilde{v}(\bar{c}) - \bar{c})}$$

Thus, with uniformly distributed valuations we have $\tilde{v}(\bar{c}) = 2\bar{c}$. The solution to the firm's second period problem is given by

$$p_2^0 = \frac{1}{2}\left(\tilde{v} + \alpha(1 - \tilde{v})\right) \tag{20}$$

and

$$p_2^1 = \tilde{v} \tag{21}$$

To determine the marginal consumer type $\tilde{v}$ that purchases in the first period, we equate the expected utility from purchasing the good in the first period to the utility from waiting to purchase until the second period:

$$\tilde{v} - p_1 + \alpha\delta(\tilde{v} - p_2^0 - c) + (1 - \alpha)\delta(\tilde{v} - p_2^1) = \delta(\tilde{v} - p_2^0)$$

Substituting the second-period prices and rearranging yields

$$p_1 = \frac{1}{2}(\delta\alpha(1 - \alpha) + \tilde{v}(2 - \delta + \delta\alpha^2)) - \delta\alpha c \tag{22}$$

The firm's first-period problem is to choose the marginal type $\tilde{v}$ and the prices p_1, p_2^0 and p_2^1 to solve

$$\max_{\tilde{v}, p_1, p_2^0, p_2^1} (1 - F(\tilde{v}))(p_1 + (1 - \alpha)p_2^1 + \alpha p_2^0) + (F(\tilde{v}) - F(p_2^0))p_2^0 \tag{23}$$

subject to (20), (21), and (22). Eliminating the prices by substituting the constraints into the objective, differentiating with respect to $\tilde{v}$, and rearranging the resulting first-order condition yields

$$\tilde{v} = \frac{2 + \delta((1 - \alpha)^2 + 2\alpha c)}{4 + \delta(1 - \alpha)^2} \tag{24}$$

Finally, for a consumer to be willing to randomize about opting out, he must be indifferent about doing so; i.e., the benefit from remaining anonymous must equal the cost

$$p_2^1 - p_2^0 = c$$

Substituting for the prices from (20) and (21) and rearranging yields

$$\alpha = \frac{\tilde{v} - 2c}{1 - \tilde{v}} \tag{25}$$

The pooling PBE outcome is found by solving (24) and (25). In particular, let

$$\phi(c) = \sqrt{1 + \delta c(2 - \delta(4 - 9c))}$$

Then

$$\alpha = \frac{1}{\delta}(1 + \delta(1 - 3c) - \phi(c)) \tag{26}$$

and

$$\tilde{v} = \frac{1}{4(1 + \delta)}(3 + \delta(2 + 3c) - \phi(c)) \tag{27}$$

Additionally, we have $p_1 = \tilde{v} - \delta c$, $p_2^0 = \tilde{v} - c$, and $p_2^1 = \tilde{v}$. It is worth noting that the expression for α is not well-defined when $\delta = 0$. However, by taking the limit as δ goes to 0, one can obtain $\lim_{\delta \to 0} \alpha(c, \delta) = 1 - 4c$. Finally, by setting $\alpha = 0$, one can obtain $\bar{c} = (2 + \delta)(8 + 2\delta)^{-1}$.

The following proposition provides comparative statics results.

Proposition 8 (Uniform valuations) *When valuations are distributed uniformly, the pooling equilibrium satisfies the following properties:*

- $\partial \tilde{v}/\partial c \in (0, \delta)$ *for* $c \in [0, \bar{c})$.
- p_1, p_2^0, *and* α *are non-increasing in* c, *and strictly decreasing for* $c \in [0, \bar{c})$.
- *Consumer surplus is strictly increasing for* $c \in [0, \bar{c})$, *so that it is highest at* $\bar{c}$. *Moreover, all consumers experience a non-negative increase in utility as* c *increases.*

Proof: The derivative of $\tilde{v}$ with respect to c is given by

$$\frac{\partial \tilde{v}}{\partial c} = \delta\left(\frac{3 + \frac{(2-9c)\delta - 1}{\sqrt{1+c\delta(2-(4-9c)\delta)}}}{4(1 + \delta)}\right) \tag{28}$$

It can be verified that $\partial \tilde{v}/\partial c$ is maximized when $c = 0$, at which point it is equal to

$$\frac{7\delta}{8(1 + \delta)} \leq \delta$$

Since $p_1 = \tilde{v} - \delta c$ in equilibrium, it follows that as c increases, p_1 decreases. In the proof of Proposition 4, we showed that the following equality holds in equilibrium:

$$\frac{\partial \tilde{v}}{\partial c}(p_2^{0'} - 1) + \frac{\partial p_2^0}{\partial \alpha}\frac{\partial \alpha}{\partial c} = -1 \tag{29}$$

We have $\frac{\partial p_2^0}{\partial \alpha} = (1-\tilde{v})/2 > 0$, $\frac{\partial \tilde{v}}{\partial c} > 0$, and $p_2^{0'} = (1-\alpha)/2 < 1$. Hence, it follows from (29) that $\frac{\partial \alpha}{\partial c} < 0$. Thus, as c increases, fewer and fewer consumers opt out. Now, since $p_2^0 = \tilde{v} - c$ in equilibrium, it follows from $\frac{\partial \tilde{v}}{\partial c} < \delta$ that p_2^0 decreases as c decreases.

We will now prove the final results regarding consumer surplus. First, note that consumers who purchase in both periods always pay $p_2^1 = p_2^0 + c = \tilde{v}$ in the second period, independent of whether or not they opt out. Consider a small (marginal) increase in c, denoted by Δ_c. As shown by (28), the highest increase in $\tilde{v}$ as a result of an increase in c is bounded above by $\delta(1+\delta)^{-1}\Delta_c$. Since $p_1 = \tilde{v} - \delta c$, as c increases, p_1 decreases by at least $(\delta - \delta(1+\delta)^{-1})\Delta_c = \delta^2(1+\delta)^{-1}\Delta_c$. A consumer with valuation v who purchases in both periods before and after the increase in c has utility $v - p_1 + \delta(v - \tilde{v})$ before the increase. After the increase, the consumer's utility is at least $v - (p_1 - \delta^2(1+\delta)^{-1}\Delta_c) + \delta(v - (\tilde{v} + \delta(1+\delta)^{-1}\Delta_c))$. Hence, the consumer is no worse off after the increase in c, but potentially better off. Now, we note that some consumers with valuations $v \in [\tilde{v}, \tilde{v} + \delta(1+\delta)^{-1}\Delta_c]$ used to purchase in both periods before the increase in c, but after the increase in c they do not. However, these consumers, whether or not they now purchase in both periods, are still better off, as the decrease in p_1 more than offsets the present-discounted increase in second period prices. Finally, since p_2^0 is strictly decreasing in c for $c \in [0, \bar{c})$, increasingly more consumers find it beneficial to purchase the good in the second period as c increases, and all consumers who only purchase in the second period now pay a lower price. Hence, every consumer experiences a non-negative gain in utility as c increases. Consequently, consumer surplus is highest at $\bar{c}$. ∎

The intuition for the results in Proposition 8 is the following. As c increases, the wedge between $\tilde{v}$ and p_1 increases, as more consumers prefer to wait to purchase in the second period due to the higher cost of opting out. To mitigate the effect of losing first period consumers, the firm responds by reducing the first period price. Now, although the firm also benefits from the increase in $\tilde{v}$, as it is better able to price discriminate against high valuation consumers in the second period, the loss in profit due to losing first period customers dominates the increase in profit due to better price targeting in the second period. Hence, the firm responds by slightly decreasing p_1 as c increases. Additionally, although as c increases, more consumers prefer to wait until the second period to purchase the good, a lot fewer consumers opt out, making it optimal for the firm to reduce p_2^0.

The intuition behind the increase in consumer surplus is the following: when $c = 0$, a Prisoner's Dilemma situation arises among consumers. Specifically, it is in the best interest of every consumer to opt out, but doing so makes all of them worse off. As c increases, fewer and fewer consumers opt out. Effectively, the cost of opting out mitigates the coordination failure among consumers. When c is high, the firm and consumers anticipate that very few consumers will opt out. This in turn forces the firm to lower p_1 in order to attract first period consumers, but allows it to reduce p_2^0, thus serving a larger segment of the market in the second period.

Proposition 9 (Non-monotonicity) *When valuations are uniformly distributed, the firm's profit is non-monotonic in the cost of opting out, c.*

Proof: The present value of the firm's profit is given by

$$\Pi(c) = (\tilde{v}(c) - \delta\alpha(c)c)(1 - F(\tilde{v}(c))) + \delta(\tilde{v}(c) - c)(1 - F(\tilde{v}(c) - c))$$

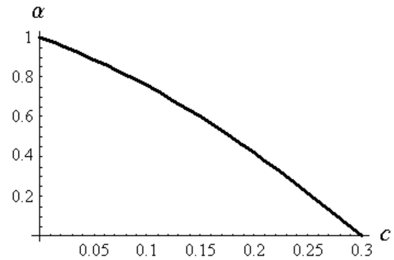
Substituting for $\tilde{v}$ and α, one can obtain

$$\Pi(c) = \frac{1}{8(1+\delta)}\left(1 + \phi(c) + \delta(2(2+\delta) - c(6 + 2\delta - c(8 + 5\delta) - \phi(c)))\right)$$

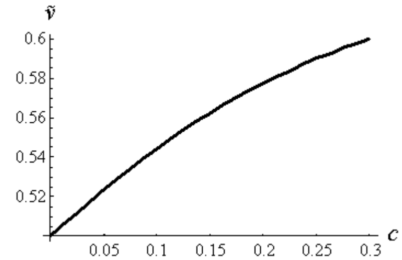
Taking the derivative of $\Pi(c)$ with respect to c, simple algebra shows that $d\Pi(c)/dc$ evaluated at $\bar{c} = \frac{2+\delta}{2(4+\delta)}$ is strictly positive for all $\delta > 0$. Continuity of $\Pi(c)$ in c implies that when $\delta > 0$, for c in a close neighborhood $\bar{c}$, the firm's profit is increasing in c. By Proposition 5, the firm's profit is maximized at $c = 0$. It follows that the firm's profit must decrease before it increases, directly implying that the firm's profit is non-monotonic in c. ■

In order to provide some intuition for the non-monotonicity result in Proposition 9, we now consider the case where $\delta = 1$. When $\delta = 1$, we have $\phi(c) = \sqrt{9c^2 - 2c + 1}$, $\alpha = 2 - 3c - \phi(c)$, and $\tilde{v} = \frac{1}{8}(5 + 3c - \phi(c))$. Figures 1(a)-(c) show how α, $\tilde{v}$, and p_1 are affected by changes in c. We note that if $c = 0$, then this solution yields the no-recognition outcome, in which half the consumers purchase the good in each period ($\tilde{v} = p_2^0 = \frac{1}{2}$) and they all opt to remain anonymous ($\alpha = 1$). At the other extreme, if $c = \frac{2+\delta}{8+2\delta} = \frac{3}{10}$, then the full-recognition outcome obtains in which 40% of the consumers purchase in the first period ($\tilde{v} = \frac{3}{5}$), 70% purchase in the second period ($p_2^0 = \frac{3}{10}$ and $p_2^1 = \frac{3}{5}$), and no consumer opts out ($\alpha = 0$).

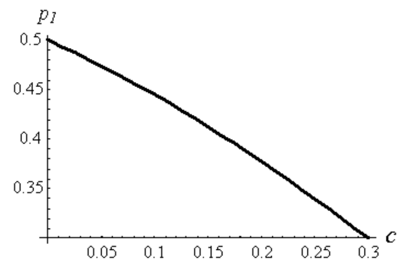
It is straightforward to use the above solutions to derive equilibrium profit and consumer surplus as a function of the cost parameter c (Figures 2(a)-(c) present the case when $\delta = 1$). Equilibrium profit to the firm initially decreases in c and then increases. Equilibrium consumer surplus is monotonically increasing in c. When c is deadweight



(a) The probability of first-period consumers opting out as a function of c

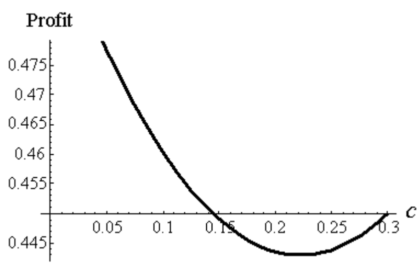


(b) First-period consumer cutoff type as a function of c

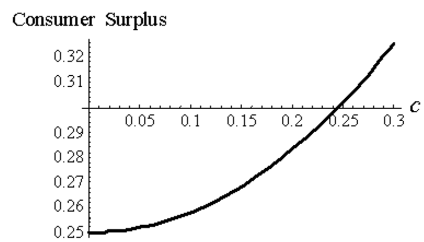


(c) First-period price as a function of c

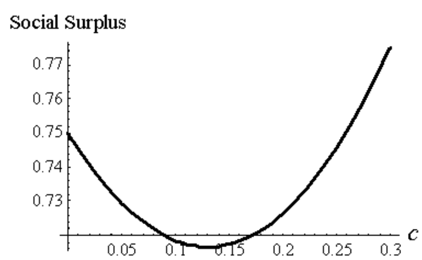
Figure 1: Comparative statics in the pooling equilibrium when c is deadweight loss



(a) Firm profit as a function of opting out cost c



(b) Consumer surplus as a function of opting out cost c



(c) Total surplus as a function of c

Figure 2: Comparative statics in the pooling equilibrium when c is deadweight loss

loss (e.g., the time and effort of setting up a new account), equilibrium social surplus initially decreases and then increases, achieving a global maximum at $c = \frac{3}{10}$ when no consumers opt out. The intuition is as follows. When $c = 0$, all consumers who purchase in the first period choose anonymity. As c begins to rise, consumers must pay a non-trivial resource cost in order to remain anonymous. Since in equilibrium, consumers who purchased the good in the first period must be indifferent between anonymity and identification, the cost of preserving privacy is passed on to the firm, resulting in lower profits and lower social surplus. As c continues to rise, fewer consumers opt out and deadweight loss eventually falls, causing profit to rise. Additionally, the decline in the price charged to anonymous consumers in the second period, p_2^0, results in more sales and even higher consumer surplus. When c is not deadweight loss (e.g., a fee charged by a third party), equilibrium social surplus is monotonically increasing in c.

6 Regulation

In this section, we consider settings where the cost of opting out is revenue set and collected either by a third party or by the firm itself.

6.1 In-House Privacy

We will begin by considering the case where the firm itself sets the cost of opting out. The following proposition characterizes the firm's behavior.

Proposition 10 (In-house privacy) *If the firm is able to commit to the fee of opting out before consumers' purchasing decisions in the first period, it would set $c = 0$ and the no-recognition outcome results. If the firm cannot commit, it would set $c \geq \bar{c}$, and the full-recognition outcome results. This is independent of whether or not the firm collects the fee of opting out.*

Proof: First, we note that for a given c, the firm's profit when it collects the fee of opting out is no less than its profit when it does not. The firm's profit when it collects the fee is given by

$$\tilde{v}(c)(1 - F(\tilde{v}(c))) + \delta p_2^0(1 - F(p_2^0))$$

which is maximized when $\tilde{v} = p_2^0 = p^\star$, or when $c = 0$. Hence, if the firm could commit to the fee of opting out before consumers' first-period purchasing decisions, it would set $c = 0$.

If the firm cannot commit to a certain opting out fee before consumers' first-period purchasing decisions, it would set $c \geq \bar{c}$. To see this, note that if the firm does not collect the fee, it would never want consumers to opt out, and so it would set the fee prohibitively high. Consumers would anticipate this when making their first period choices, and so the full-recognition equilibrium results. If the firm collects the fee, it would set $p_2^1 - p_2^0 \leq c$. To see this, note that if $p_2^1 - p_2^0 > c$, all identified consumers would opt out, and the firm would reap a profit of $p_2^0 + c$ from each of these consumers. However, if the firm sets the opting out fee to be $c' > c$ such that $p_2^1 - p_2^0 > c'$ is still satisfied, identified consumers would still strictly prefer to opt out, but the firms profit from each of them would now be $p_2^0 + c'$. Hence, $p_2^1 - p_2^0 \leq c$ holds. Additionally, $p_2^1 = \max\{p^\star, \tilde{v}\} \geq \tilde{v}$ continues to hold. Thus, consumers anticipate that they cannot increase their utility in the second period by opting out, and so consumers behave in the first period as in the first period of the full-recognition game. Subsequently, the firm sets prices as in the full-recognition game and sets $c \geq \bar{c}$. Hence, the full-recognition equilibrium outcome results. ■

The result of Proposition 10 implies that when the firm can commit to setting the opting out fee in the first period, it can effectively commit to prices. When valuations are uniformly distributed, social surplus is monotonically increasing in the cost of opting out, and is maximized when $c = \bar{c}$. Therefore, with uniformly distributed valuations, social surplus is maximized when the firm sets the opting out fee and does not have commitment power.

6.2 Privacy Gatekeeper

Suppose there is a third party that acts as a privacy gatekeeper, operates at no variable cost, and is able to commit to an opting out fee before first period purchases. The gatekeeper can set an opting out fee for consumers, c, and it can negotiate a price with the firm for setting c at a certain level (throughout, we maintain the assumption that the gatekeeper holds all the bargaining power when negotiating with the firm). The gatekeeper's equilibrium actions in each case are stated in the following proposition.

Proposition 11 (Privacy gatekeeper) *When the privacy gatekeeper is only able to charge consumers, it would set $c = c^\star \in (0, \bar{c})$, where $c^\star = \arg\max_c \alpha(c)(1 - F(\tilde{v}(c)))c$. When the privacy gatekeeper is able to charge both consumers and firm, it would set $c = 0$ and charge the firm $\Pi(c = 0) - \Pi(c^\star)$.*

Proof: When the gatekeeper does not negotiate with the firm, its optimal fee is given by $c^\star \in (0, \bar{c})$, such that

$$c^\star = \arg\max_c \;\; \delta\alpha(c)(1 - F(\tilde{v}(c)))c$$

We note that $c^\star \in (0, \bar{c})$ holds because when $c = 0$ or $c = \bar{c}$, the gatekeeper's profit is zero. However, when the gatekeeper is able to negotiate with the firm, the result is very different. Recall that the present value of the firm's profit is given by

$$\Pi(c) = (\tilde{v}(c) - \delta\alpha(c)c)(1 - F(\tilde{v}(c))) + \delta(\tilde{v}(c) - c)(1 - F(\tilde{v}(c) - c))$$

Thus, given any positive level of c, the firm would be willing to pay the gatekeeper the following amount in order for the gatekeeper to set $c = 0$:

$$\Pi(c = 0) - (\tilde{v}(c) - \delta\alpha c)(1 - F(\tilde{v}(c))) + \delta p_2^0(1 - F(p_2^0))$$

where $\Pi(c = 0) = (1 + \delta)p^\star(1 - F(p^\star))$. By Proposition 5, the firm's profit is highest at $c = 0$. Moreover, for all $c > 0$, we have

$$\Pi(c = 0) > \tilde{v}(c)(1 - F(\tilde{v}(c))) + \delta p_2^0(1 - F(p_2^0)) \tag{30}$$

This holds because the right-hand side of (30) is uniquely maximized when $\tilde{v} = p_2^0 = p^\star$, which only occurs at $c = 0$, while $\Pi(c = 0) = (1 + \delta)p^\star(1 - F(p^\star))$. Now, since $\Pi(c) = \big(\tilde{v}(c)(1 - F(\tilde{v}(c))) + \delta p_2^0(1 - F(p_2^0))\big) - \delta\alpha(1 - F(\tilde{v}(c)))c$, it follows that for all $c > 0$

$$\Pi(c = 0) - \Pi(c) > \delta\alpha(1 - F(\tilde{v}(c)))c$$

Therefore, the privacy gatekeeper is always better off charging the firm $\Pi(c = 0) - \Pi(c^\star)$ and setting $c = 0$ than setting any other $c > 0$ and extracting $\max\{\Pi(c) - \Pi(c^\star), 0\}$ from the firm. ∎

The intuition for this result is as follows. When $c > 0$, the monopolist has to compensate consumers who purchase in the first period by lowering the first period price. In addition, the monopolist loses some first period customers (specifically, those whose valuations are close to the first period price). Thus, the amount of money the monopolist is willing to pay the gatekeeper to set $c = 0$ is composed not only from its loss from compensating consumers who opt out, but also from losing customers. Hence, the gatekeeper is always better off setting $c = 0$ and charging the monopolist $\Pi(c = 0) - \Pi(c^\star)$.

Example 3 With uniformly distributed valuations and $\delta = 1$, social surplus is monotonically increasing in c when the cost of opting out is not deadweight loss (in the setting of this example it is a fee collected by the gatekeeper). When the gatekeeper is only able to charge consumers, it sets $c^\star \approx 1.5$. The resulting profit for the firm is $\Pi(c^\star) \approx .45$. The privacy gatekeeper's profit is ≈ 0.04. Interestingly, the firm's profit in this case is lower than its profit in the full-recognition equilibrium.

When the gatekeeper is able to set prices to both firm and consumers, it sets $c = 0$, and (when $\delta = 1$) charges the firm approximately .05. Both consumers and firm are worse off than in the full-recognition equilibrium, and social surplus is at its lowest level. Furthermore, consumer surplus is lower than the situation where the privacy gatekeeper is only able to charge consumers and sets $c = c^\star$.

7 Conclusions

This paper provides an explanation for why certain aspects of online consumer privacy may be misjudged by policymakers. We present a game theoretic analysis of an environment in which firms are able to recognize their previous customers, and may use information about consumers' purchase histories in order to price discriminate.

Specifically, we analyzed a model with a monopolist and a continuum of heterogeneous consumers, where consumers were able to circumvent being identified at some cost. We showed that when consumers can costlessly opt out, they all individually choose privacy, which paradoxically results in the highest profit for the monopolist. Under some conditions on the distribution of valuations, we also showed that consumers are better off overall when opting out is prohibitively costly. We considered a general equilibrium framework, where a privacy gatekeeper (with commitment power) is able to act as a privacy conduit. We proved that this privacy gatekeeper would only charge the firm in equilibrium, making privacy costless to consumers. With uniformly distributed valuations, we showed that firm profit and social surplus are non-monotonic in the cost of opting out, and social surplus is highest when opting out is prohibitively costly. Overall, it appears that hard-to-understand disclosures and difficult-to-circumvent detection (i.e. a high c) may actually work to the benefit of strategic consumers and, in some cases, to the benefit of society overall.

There are several important directions in which the current work can be extended. First, our comparative statics results can be extended to more general distributions over consumer valuations. Additionally, competition among both firms should be considered. Another interesting direction is to analyze a setting with multiple privacy gatekeepers that are competing for contracts from firms and for business from consumers. An additional important direction is to model the cost of opting out as private information to each consumer, which is potentially correlated with their valuation (extending the equilibrium refinement setting). Also, settings where consumers may obtain some benefit from being identified, such as smaller search costs or better technical support should be considered (notably, even in such environments some consumers stand to lose from being identified — for instance, consumers who shop relatively infrequently may receive poorer technical support, *etc*). Settings where consumers may obtain some benefit from being identified also beg for an opt-in policy to be considered. Yet another direction is to consider firms collecting data other than consumers' purchase histories and using it to price discriminate (e.g., the amount of time taken from a consumer's initial browsing of a store's webpage until purchase may be indicative of the consumer's search cost, which may affect their willingness to pay for future purchases).

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