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**Hybrid Advertising Auctions**

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## Hybrid Advertising Auctions

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## **Hybrid Advertising Auctions**

### **Abstract**

Several major websites offer hybrid auctions that allow advertisers to bid on a per-impression or a per-click basis. We present the first analysis of this hybrid advertising auction setting. The conventional wisdom is that brand advertisers (e.g. Coca-Cola) will bid per impression, while direct response advertisers (e.g. Amazon.com) will bid per click. We analyze a theoretical model of advertiser bidding to ask whether this conventional wisdom will hold up in practice. We find the opposite in a static game: brand advertisers bid per click, while direct response advertisers bid per impression. In a more realistic repeated game, we find that direct response advertisers bid per click, but brand advertisers may profitably alternate between bidding for clicks and bidding for impressions. The analysis implies that sellers of online advertising (a) may sometimes prefer not to offer advertisers multiple bidding options, (b) should try to ascertain advertisers' types when they do use hybrid auctions, and (c) should consider advertisers' strategic incentives when forming click-through rate expectations in hybrid auction formats.

**Keywords:** Advertising, Auctions, Internet Marketing, Search Advertising.

Auctions are the dominant sales mechanism to allocate online advertising space. One commonly used auction is the cost per thousand impressions (CPM) auction in which advertisers bid for impressions and make payments each time their ad is displayed on a webpage. A second commonly used auction is the cost per click (CPC) auction in which advertisers bid for clicks and pay only when their ad is clicked.

CPM ad pricing has traditionally been associated with “brand advertisers” who are primarily interested in purchasing advertising exposures. Brand advertisers (e.g., Coca-Cola or Ford) often advertise with the intention of influencing consumers’ product perceptions, for example creating awareness or reinforcing a brand image. They expect this strategy to influence purchase decisions made in offline environments such as retail stores. CPM pricing has been the standard ad price metric in traditional advertising media (e.g., television, newspapers, billboards) for decades, and CPM pricing continues to be standard in online display advertising (Evans 2008).

CPC pricing, by contrast, is relatively new. It was invented by GoTo.com in 1998, in a successful effort to lure advertisers from rival websites (Battelle 200t). It has since been adopted as the standard pricing metric in the search advertising industry. CPC pricing is especially attractive to “direct response advertisers,” for whom clicks are the primary concern. Such firms (e.g., Amazon.com or eBay.com) typically sell products online, and their ads are designed to trigger online sales in the short run. Since these advertisers can usually track consumer profitability at a fine level of granularity, they prefer the CPC model<sup>1</sup>, as it allows them to compare their marginal profits and advertising cost at the level of the individual click.

In principle, it is not clear that any online advertising space should be allocated exclusively via a CPC or a CPM auction model. Several new and high-profile sellers of internet advertising offer the hybrid advertising auction. Google offers it on its content network, which it describes by saying

“The Google content network reaches 80% of global internet users -- making it the world's #1 ad network. Thousands of advertisers use Google to reach users on hundreds of thousands of web sites across all industries, from large, well-known sites to niche sites and audiences. [It has] an audience larger than any other ad network or single web property (even Google.com)...”<sup>2</sup>

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<sup>1</sup> <http://adwords.google.com/support/bin/answer.py?answer=54411&cbid=-1n7xvy51ltxfj&src=cb&lev=answer>. Accessed March 2009.

<sup>2</sup> <http://www.google.com/adwords/contentnetwork>. Accessed March 2009.

Advertisers may choose whether to use CPC bidding, in which they pay per click, or CPM bidding, in which they pay per impression. Bids of both types compete in a hybrid auction for the same advertising space.

Facebook.com is another major seller of internet advertising space. It reached 108 million unique individuals in December 2008, who, cumulatively, used the site for 20.5 billion minutes, up 566% from December 2007 (Nielsen 2009). Facebook advertisers have a prominent option to bid for ad space on the site using either a CPC bid or a CPM bid.

The hybrid auction raises several interesting questions. Does the choice of bid type matter when determining the auction outcome and auction seller revenues? Under what conditions should an advertiser enter a CPC bid or a CPM bid? The conventional wisdom motivating the hybrid auction format is that brand advertisers prefer CPM bidding while direct response advertisers prefer CPC bidding (c.f. Figure 3 below). Does this conventional wisdom obtain in equilibrium?

We are partially motivated to study this hybrid auction market by some related intuition. Advertisers may influence the number of clicks their ads receive by strategically choosing their ad text. As a very simple example, they can choose to encourage consumers to “click here.” Another possible strategy is to include a “hard sell” in their ad, which might be effective in shaping offline behavior but might discourage the consumer from clicking the ad. A third option is to alter the frequency with which new ads are introduced, which in turn influences the likelihood of consumer clicks. Under some conditions, brand advertisers may enter CPC bids while choosing their click-through rates (CTR)<sup>3</sup> to be artificially low. This strategy might be profitable if it lowers advertising costs.

We answer these questions by analyzing a theoretical model of a hybrid advertising auction. We consider a scenario in which a brand advertiser competes with a direct response advertiser for a set of ads sold by a seller of online advertising (which we refer to as the “website”). Each advertiser chooses a bid type (CPC or CPM) and a bid amount. Advertisers may also choose their click-through rates from within a range of feasible values. We show that in a one-shot game, the conventional wisdom is reversed: brand advertisers optimally enter CPC bids and direct response advertisers use CPM bidding (we refer to this result as “bidding against

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<sup>3</sup> The click-through rate is the number of times an ad is clicked divided by the number of times the ad is shown.

type” or “bid reversals”). We then analyze a repeated game, and find that direct response advertisers always use CPC bidding, while brand advertisers may profitably alternate between CPM bids and CPC bids.

In the next section, we describe both some features of the industry, and empirical evidence that motivate our analysis and assumptions. We also review the academic literature to which we contribute. Section 2 presents a static model of a hybrid advertising auction, and section 3 shows that bidding against type persists in a repeated game as well. Section 4 contains a nontechnical discussion of the managerial implications of our results. Section 5 concludes with some limitations and directions for future research. Proofs of all lemmas and propositions are confined to the Appendix.

## **1. Industry Background and Relevant Literature**

While hybrid advertising pricing systems have been used online for more than 10 years, they have received no prior academic study. They were very popular in the early days of internet advertising, but their use has been largely supplanted by pure CPC pricing systems. Hybrid pricing models fell from 48% of advertising revenues in 1998 to 4% in 2008, while performance-based pricing (including CPC pricing) increased from 4% to 47% in the same time period. Impression-based pricing fell slightly from 49% in 1998 to 45% in 2008 (IAB 2000, 2009).<sup>4</sup>

Despite the decrease in hybrid pricing models’ use, they are still offered by sophisticated and new websites that sell advertising. Most early hybrid pricing models were not hybrid auctions, as are being used today. Google allows advertisers to use either CPM or CPC bidding when buying ads on its content network, but not on its organic search results. Figure 1 shows the Google help page that describes competition between CPM and CPC bids, making clear that neither type of bid receives priority placement.

Facebook also offers the choice of CPM or CPC bidding. Figure 2 illustrates that the choice to select bid type is quite prominent in the automated self-service process required to set up an advertisement. Facebook’s help file, shown in Figure 3, reinforces the conventional wisdom regarding the appropriateness of CPM bidding for brand advertisers and CPC bidding for direct response advertisers. It states:

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<sup>4</sup> Due to high internet advertising growth rates, the total amount of advertising dollars sold via hybrid pricing models declined just 14%, from \$990 million to \$850 million.

Figure 1. Google Help Page on CPC/CPM Bid competition



Figure 2. Step 3 of 3 in Facebook’s “Create an Ad” Process

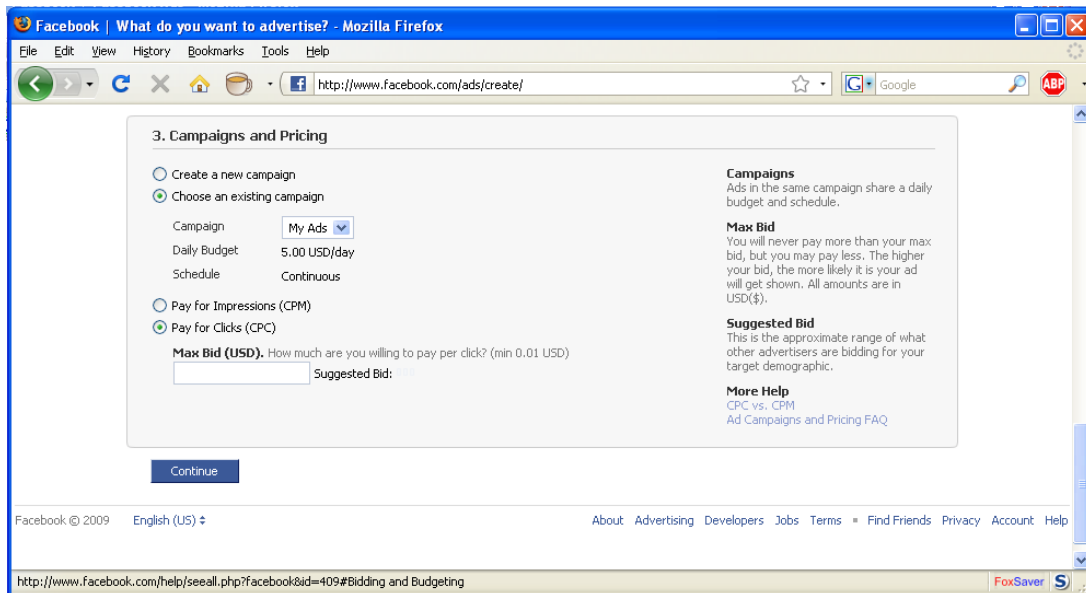
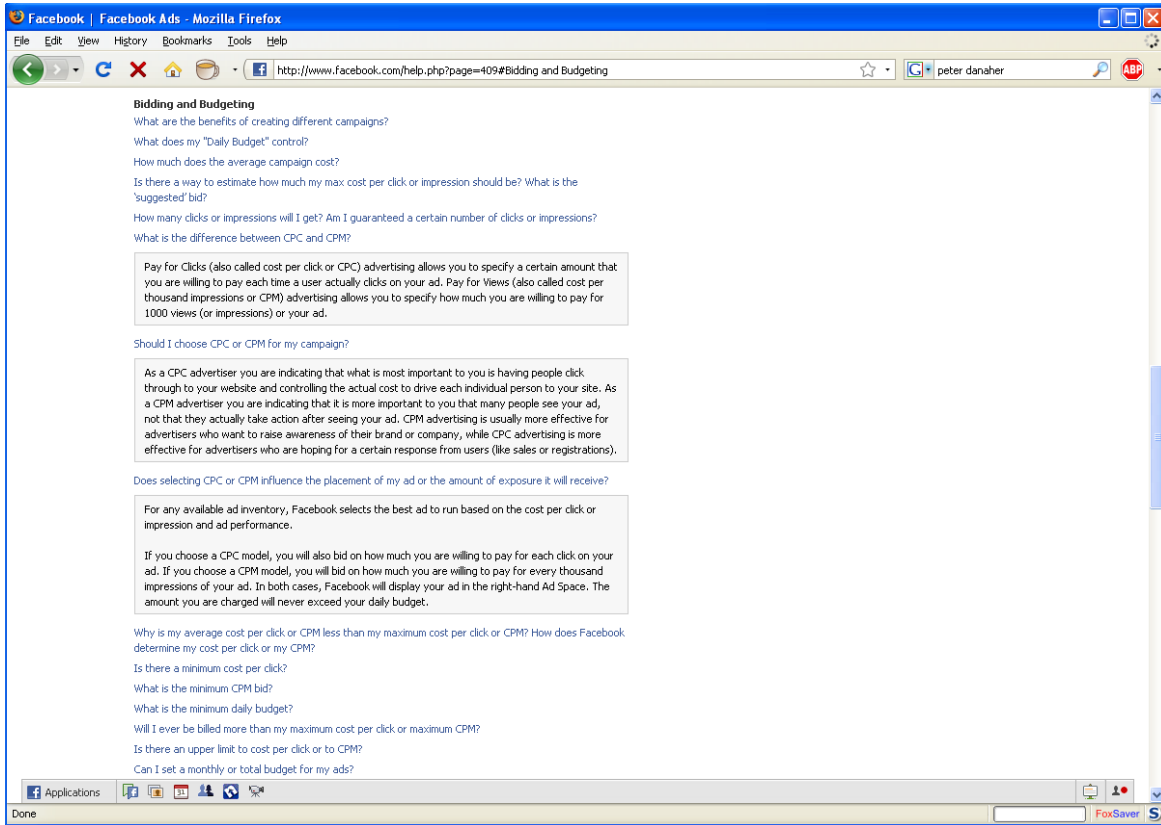


Figure 3. Facebook CPC/CPM Help Answers



“As a CPC advertiser you are indicating that what is most important to you is having people click through to your website and controlling the actual cost to drive each individual person to your site. As a CPM advertiser you are indicating that it is more important to you that many people see your ad, not that they actually take action after seeing your ad. CPM advertising is usually more effective for advertisers who want to raise awareness of their brand or company, while CPC advertising is more effective for advertisers who are hoping for a certain response from users (like sales or registrations).”

While Facebook and Google offer hybrid advertising auctions, other major sellers of advertising such as Yahoo! and MySpace did not offer CPM bidding at the time of writing. In addition, Google did not offer CPM bidding on advertising placed next to its organic search results. As these choices may have strategic consequences, our analysis will shed some light on the pros and cons of offering multiple bid types.

A critical assumption, which has substantial empirical support, in our analysis is that advertisers may influence the click-through rates their ads receive. The primary factors determining click-through rate appear to be ad content, ad familiarity, and user factors. Robinson, Wysocka and Hand (2007) found that increasing the number of words in a banner ad from less than 6 to more than 15, holding other factors constant, can increase the click-through



rate by more than 100%. Mand (1998) finds that interactivity has a substantial effect on banner ad click-through rates. In addition to academic work, sellers of online advertising, such as Google and Facebook, offer numerous tips regarding how to design online ads to maximize click-through rates. They also offer tools that allow advertisers to experiment with a variety of different ads to see which ones generate the highest click-through rates. The advertiser who wants to maximize (or minimize) click-through rates could presumably choose to employ (or deliberately ignore) strategies from a wide range of studies and seller-generated tools.

Could an online advertisement that generates a low click-through rate be profitable to a brand advertiser? Several studies support the idea that online advertisements with low click-through rates are still effective in building brands. Dréze and Husherr (2003) show that despite high rates of “ad-blindness” (consumers’ tendency to avoid focusing on the parts of webpages where ads appear), consumers exposed to banner ads exhibit higher rates of aided and unaided brand recall, regardless of whether they clicked the ads or not. Danaher and Mullarkey (2003) demonstrate that time spent viewing a webpage increased the likelihood a consumer would recall a brand whose banner ad appeared on that webpage.

Our primary contribution is to make a first statement about equilibrium strategies in hybrid advertising auctions. Our analysis differs from previous papers’ in several key assumptions. This is the first paper to consider advertiser competition in type of bid as well as bid level. We do so under the realistic assumption of private information about advertisers’ types and profits, whereas most of the literature assumes perfect information. We are not aware of any previous paper that allows for multiple types of advertisers, or accounts for the possibility that advertisers may strategically influence their click through rates. Our analysis is general relative to the bulk of the literature in that we do not limit the number of bidders or ad slots, and we allow for repeated interactions.

While we focus on hybrid auctions with CPC bidding and CPM bidding, a third bid type is the cost per action (CPA) model in which advertisers pay per purchase or lead. CPA bidding is used less frequently than CPC or CPM bidding (Nazerzadeh, Saberi, and Vohra 2008). Our analysis below can be reinterpreted as a CPM/CPA hybrid auction if we assume that advertisers are choosing their conversion rate rather than their click-through rate.

Our paper adds to a quickly growing literature on search advertising auctions. The pioneering treatments on equilibria in such auctions are Edelman, Ostrovsky, and Schwarz

(2007) and Varian (2007), which independently studied aspects of the auction mechanisms used by Google and Yahoo (known as the “Generalized Second Price” auction, or “GSP”). They found that, in general, the GSP does not have a dominant bidding strategy, but under intuitive refinements, advertisers with higher expected valuations per click occupy higher ad positions in equilibrium. Athey and Ellison (2008) and Chen and He (2006) study how advertisers’ bids are affected by interadvertiser competition. Recent analytical work has examined how to incorporate searcher and keyword characteristics into the advertising auction (Even-Dal, Kearns and Wortman, 2008); how CPC advertising auctions affect advertising’s quality-signaling function (Feng and Xie 2007); the interplay between organic and sponsored search links and the website’s optimal choice of paid links (Katona and Sarvary 2008); how to modify the position auction to account for externalities between advertisers at different positions (Kempe and Mahdian 2007); and the effects of “click fraud” on search engine revenues (Wilbur and Zhu 2009), among other topics.

There is also a rapidly expanding collection of empirical studies of search advertising markets. Ghose and Yang (2009) find that click-through and conversion rates decrease with ad position and that search engines account for both current bid price, and prior click-through rates when allocating advertisements to ad slots. Goldfarb and Tucker (2007) find that pricing search advertisements separately across different keywords allows search engines to price discriminate among advertisers. Rutz and Bucklin (2007a) show how to borrow information across a large number of keywords in order to solve the optimal keyword selection and bidding problem. Rutz and Bucklin (2007b) show that while generic keywords (e.g. “hotel los angeles”) are often very expensive, they have spillover effects, as consumers tend to begin shopping with a generic search and later use a branded search to purchase. Yao and Mela (2009) use a structural dynamic Bayesian model to analyze data from a search engine. Among many other findings, their study reports that frequent clickers place a greater emphasis on the position of the sponsored advertising link and that a switch from a first-price to a second-price auction yields advertiser bids that are in line with willingness to pay, but this switch has a small impact on search engine revenue.

## **2. Equilibrium Analysis in a Static Auction**

We consider a set of  $i=1 \dots N$  risk-neutral advertisers bidding for  $k = 1 \dots K \leq N$  ads offered by a website. The website allows advertisers to choose a CPC bid  $b^c$  or a CPM bid  $b^m$ . We normalize the measure of available consumer exposures to 1. We assume that there are two types of advertisers: brand advertisers (“B”) and direct response advertisers (“D”). Type B’s payoff depends on impressions while type D’s depends on clicks. Type B advertisers’ profit per exposure is  $r_{Bi} \in (0, \infty)$ . Type D advertisers’ profit per click is  $r_{Di} \in (0, \infty)$ . Advertisers’ valuations are private information. Each advertiser can choose a click-through rate  $\gamma \in [\underline{\gamma}, \bar{\gamma}]$ .<sup>5</sup>

Following Katona and Sarvary (2008), we assume that an ad appearing in slot  $k$  has a position-dependent click-through multiplier  $X_k$  with  $X_1 \geq X_2 \geq \dots \geq X_K$ . To capture the position effect on brand advertisers’ valuation, we define position dependent exposure multipliers  $Y_1 \geq Y_2 \geq \dots \geq Y_K$ . This allows an ad’s position on the page to determine the likelihood it is seen.

The game is played in two stages. In the first stage, all advertisers simultaneously choose their bid types and their bid amounts. In the second stage, the auction mechanism (described below) allocates advertisers to ad slots, and then the  $K$  advertisers allocated to ad slots simultaneously choose click-through rates. The website anticipates each advertiser  $i$ ’s choice of click through rate; we label this anticipated click-through rate  $\gamma_i^E$ .

### 2.1. The Auction Mechanism

Our goal is to analyze equilibrium bidding strategies in hybrid advertising auctions. However, these auctions have not been studied before, the websites that offer them (e.g. Facebook, Google) typically do not reveal the precise auction mechanisms, and these websites may differ in the hybrid auction mechanisms they use. Websites’ lack of transparency about their auction mechanisms is a strong argument to study these auctions, as it creates uncertainty among advertisers as to what bidding strategies may be optimal. However, it requires us to make some assumptions about how the auction assigns advertisers to slots based on their bids.

*Assumption 1. The website assigns advertisers to slots in order of total expected advertiser willingness to pay.*

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<sup>5</sup> We assume the range of feasible click-through rates are identical for both types of advertiser for simplicity, but our results can be extended to a case in which the range of feasible click-through rates depends on advertiser type.

If advertisers were charged their bids, a CPM bid  $b_i^m$  would represent advertiser  $i$ 's total payment for a slot. If  $i$  instead entered a CPC bid  $b_i^c$ , its total expected payment for slot  $k$  would be  $X_k \gamma_i^E b_i^c$ . Assumption 1 implies that if advertiser  $i$  enters a CPM bid and advertiser  $j$  enters a CPC bid,  $i$  will be allocated to slot  $k$  and  $j$  to a less desirable slot if  $b_i^m > X_k \gamma_i^E b_j^c$ . It is straightforward to show that an auction mechanism which produced a ranking in which this inequality did not hold would produce weakly lower revenue than the mechanism we assume here. This assumption is consistent with Google's statement in Figure 1 that "neither type of ad [CPC or CPM] has a special advantage over the other." It is also consistent with Facebook's statement in Figure 3 that "for any available ad inventory, Facebook selects the best ad to run based on the cost per click or impression and the ad performance." In addition, if the website used any other ranking method, it would systematically bias advertisers toward using one type of bid over the other.

*Assumption 2. Each advertiser is charged the minimum amount necessary to keep its place in the ranking.*

Assumption 2 is in line with the prior literature on CPC auctions (e.g. Edelman et al. 2007, Varian 2007) and the common understanding of Google's pure-CPC keyword auction.<sup>6</sup> Search engines typically use variants of second-price auctions in order to reduce the advertisers' incentive to bid beneath their reservation values. As Google states in Figure 1, "No matter which type of ad [CPC or CPM] wins the position, the Adwords discounter monitors the competition and ensures that the winning ad is charged only what is necessary to maintain its ranking above the next-highest ad."

Assume that advertiser  $i$  holds position  $k$  and advertiser  $j$  holds position  $k+1$ . Let  $b_j^{g_j}$  be a bid of type  $g_j \in \{c, m\}$  entered by advertiser  $j$ . If both advertisers entered CPM bids, then  $i$  pays

$b_j^m$ . If both advertisers entered CPC bids, then  $i$  pays  $\frac{\gamma_j^E b_j^c}{\gamma_i^E}$  per click. If advertiser  $i$  entered a

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<sup>6</sup> We follow previous literature in assuming expected click-through rates enter the auction mechanism, rather than the "quality scores" described in Google's help files. Another way to interpret our results is to assume that  $\gamma$  represents the advertiser's quality score rather than its click-through rate.

CPC bid and  $j$  entered a CPM bid, then  $i$  pays  $\frac{b_j^m}{X_k \gamma_i^E}$  per click. If  $i$  entered a CPM bid and  $j$  entered a CPC bid, then  $i$  pays  $X_k \gamma_j^E b_{jk+1}^c$ .

It is notationally convenient to represent the CPM bid or the converted CPC bid of the advertiser in slot  $k+1$  as

$$C_k = \begin{cases} X_k \gamma_{k+1}^E b_{k+1}^c, & \text{if the advertiser who gets slot } k+1 \text{ submits a CPC bid.} \\ b_{k+1}^m, & \text{if the advertiser who gets slot } k+1 \text{ submits a CPM bid.} \end{cases}$$

A direct response advertiser  $i$  who wins ad slot  $k$  with a CPC bid has profits

$$\pi_{ik}^c = X_k \gamma_i r_{Di} - \frac{\gamma_i}{\gamma_i^E} C_k, \quad (1)$$

and when it wins the slot with a CPM bid, it gets

$$\pi_{ik}^m = X_k \gamma_i r_{Di} - C_k. \quad (2)$$

The profit function of a brand advertiser  $i$  who wins ad slot  $k$  with a CPC bid is

$$\pi_{ik}^c = Y_k r_{Bi} - \frac{\gamma_i}{\gamma_i^E} C_k, \quad (3)$$

and when it wins the slot with a CPM bid, it gets

$$\pi_{ik}^m = Y_k r_{Bi} - C_k. \quad (4)$$

Let

$$\delta^{g_i} = \begin{cases} 1 & \text{if } g_i = m \\ \frac{\gamma_i}{\gamma_i^E} & \text{if } g_i = c \end{cases} \quad (5)$$

so advertiser  $i$ 's total costs in position  $k$  are  $\delta_i^{g_i} C_k$ . If we define  $R_{ik}$  as total revenues according to advertiser type and position, profit can be written as  $\pi_{ik}^{g_i} = R_{ik} - \delta_i^{g_i} C_k$ . Finally, we define  $N$ -vectors  $g = (g_i)_{i=1, \dots, N}$ ,  $b = (b_i^{g_i})_{i=1, \dots, N}$ ,  $\gamma^E = (\gamma_i^E)_{i=1, \dots, N}$ , and  $\delta = (\delta_i)_{i=1, \dots, N}$ .

## 2.2. Advertisers' Choice of Click-Through Rates

We solve the game recursively, beginning with advertisers' optimal click-through rates in the second stage of the game.

*Lemma 1. If a direct response advertiser gets an ad slot in the static game, its weakly dominant strategy is to choose  $\gamma = \bar{\gamma}$ .*

*Proof:* See Appendix.

*Lemma 2. If a brand advertiser gets an ad slot in the static game, its weakly dominant strategy is to choose  $\gamma = \underline{\gamma}$ .*

*Proof:* See Appendix.

We assume that the website's expectations about click-through rates in the first stage are in alignment with Lemmas 1 and 2.

### 2.3. Equilibrium Bidding Strategies

We begin by formalizing our notion of equilibrium. In sections 2.4 and 2.5, we characterize advertisers' choice of bid types under two competing assumptions about website information.

Definition 1 formalizes our equilibrium concept.

*Definition 1: Given  $\gamma^E$ , a vector of bids  $b$  and an ordering of advertisers constitute a Nash Equilibrium if the following conditions hold:*

1. *for every advertiser  $i$  in slot  $k \in \{1 \dots K\}$ , for every alternative position  $k' \in \{1 \dots K\}$  and alternate bid type  $g_i'$ ,  $\pi_{ik}^{g_i} = \text{MAX}\{\pi_{ik}^{g_i}, \pi_{ik'}^{g_i}, \pi_{ik}^{g_i'}, \pi_{ik'}^{g_i'}\}$ ,*
2. *for every advertiser  $i$  in slot  $k \in \{1 \dots K\}$ ,  $\pi_{ik}^{g_i} \geq 0$ , and*
3. *for every advertiser  $j$  who is not allocated to any ad slot,  $\pi_{jk}^{g_j} = R_{jk}^{g_j} - \delta^{g_j} C_k \leq 0$  for all  $k \in \{1 \dots K\}$ .*

The first condition is an incentive compatibility constraint, guaranteeing that each advertiser's bid type and bid level are individually optimal. The second condition is a participation constraint, requiring that no advertiser gets negative profits when it wins an advertising slot. The third condition ensures efficient exclusion of advertisers from slots.

Milgrom (2000, Thm. 3) proves that at least one equilibrium in pure strategies always exists in this auction. See Borgers, et al. (2007) for an adaptation of Milgrom's proof in a similar setting.

We now consider how website knowledge of advertisers' types affects advertisers' equilibrium choice of bidding strategies. We begin with the strong assumption that the website has perfect knowledge of advertisers' types, meaning it knows which bidders are brand advertisers and which bidders are direct response advertisers. We then consider what happens in the more realistic case of website uncertainty about bidders' types.

#### 2.4. Bid Type Choice under Perfect Website Knowledge of Advertisers' Types

When the website knows each advertiser's type, it can infer its equilibrium click-through rate, so the website sets  $\gamma_i^E = \bar{\gamma}$  for all direct response advertisers and  $\gamma_i^E = \underline{\gamma}$  for all brand advertisers.

*Proposition 1: If the website has perfect knowledge of advertisers' types, every advertiser is indifferent between entering the best possible CPM bid and the best possible CPC bid in a static game.*

*Proof:* See Appendix.

Proposition 1 states that, when the website knows advertisers' types, the best possible CPM bid and the best possible CPC bid produce identical profits, so advertisers will have no incentive to switch between the two types of bid. This is because every CPM bid has a CPC equivalent that will produce an identical outcome and vice versa. However, when we make the model more realistic by introducing some uncertainty on the part of the website about advertisers' types, this indifference result no longer holds and advertisers will have strict preferences over bid types.

#### 2.5. Bid Type Choice under Website Knowledge of Advertisers' Types

We assume the website believes advertiser  $i$  is a brand advertiser with probability  $p_i$ . The website uses this belief to form its expectation about the bidder's click through rate,

$\gamma_i^E = p_i \underline{\gamma} + (1 - p_i) \bar{\gamma}$ .<sup>7</sup>  $p = (p_i)_{i=1, \dots, N}$  is an  $N$ -vector describing website beliefs about each advertiser's type. Imperfect website information requires that at least one element of  $p$  is strictly between zero and one for some advertiser  $i$ .

*Proposition 2: Any direct response advertiser whose type is not known with certainty by the website has a strictly dominant strategy to enter a CPM bid. Any brand advertiser whose type is not known with certainty by the website has a strictly dominant strategy to enter a CPC bid.*

*Proof:* See Appendix.

Next we consider how the website's revenue is affected by its uncertainty about advertisers' types.

*Proposition 3. An equilibrium exists under imperfect website information that produces the same assignment of advertisers to slots as the equilibrium under perfect website information.*

*Comparing the first equilibrium with the second, the website's advertising revenues fall by*

$$\frac{(1 - p_i)(\bar{\gamma} - \underline{\gamma})}{p_i \underline{\gamma} + (1 - p_i) \bar{\gamma}}$$

*for any brand advertiser  $i$  whose type is not known with certainty. Revenues*

*received from direct response advertisers do not fall in comparison with the perfect information benchmark.*

*Proof:* See Appendix.

Imperfect website information about advertiser types opens the door to strategic bid reversals by both brand advertisers and direct response advertisers, but it only lowers website revenues (compared with the perfect information benchmark) received from brand advertisers. This is because the direct response advertisers actually pay more using CPC bidding under imperfect website information—the switch to CPM bidding makes them better off but does not lower their total cost compared to the perfect website information case. The harm to website revenues associated with uncertainty and strategic bidding by brand advertisers is increasing in the degree of website uncertainty about advertisers' types, at an increasing rate.

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<sup>7</sup> This specific assumption is a useful simplification of the form the website's belief could take, but the results below will go through under any prior formulation such that  $pr\{\gamma_i^E = \bar{\gamma}\} < 1$  when  $i$  is a direct response advertiser and  $pr\{\gamma_i^E = \underline{\gamma}\} < 1$  when  $i$  is a brand advertiser.



### 3. Equilibrium Analysis in a Repeated Game

Online advertising auctions are repeated frequently. Our analysis, to this point, relies on an assumption of private information about advertiser types and click-through rates. Given frequent repetitions of a static game, it may be the case that players can infer each other's types. Private information may be revealed through equilibrium strategies. It is an open question whether the bidding reversal result of Proposition 2 would hold in a more realistic dynamic setting. It also may be that the website learns about each advertiser's click-through rate over time, so a large amount of data on historical click-through rates could reduce website uncertainty about advertiser types. To investigate, we generalize the model beyond the assumed one-shot game to show that bidding against type can also occur in repeated games.

In section 3.1, we show that brand advertisers may still use bid reversals as an equilibrium strategy in a repeated game, under the assumption that the website uses only the prior period to form its expected click-through rate in the current period. This simplification is useful for setting up the result in section 3.2, which shows that allowing the website access to the full history of click-through rates still does not remove the possibility of equilibrium bid reversals.

#### 3.1. A Dynamic Hybrid Advertising Auction with Simple Website Expectations

We add the following assumptions to our previous framework. We assume that the static game described in section 2 is played within each of  $t=1, \dots, T$  time periods. For simplicity, we assume the range of possible click-through rates does not change over time, but our analysis below is robust to modifications to this assumption.

Within a dynamic game, the website faces the problem that it must anticipate what each advertiser's click-through rate will be prior to allocating advertisers to slots. It therefore still faces the possibility that advertisers may strategically deviate from anticipated click-through rates. We begin by assuming a simplistic structure for website beliefs about advertisers' click-through rates and generalize it substantially in section 3.2. For now, we assume that the website anticipates advertiser  $i$ 's click-through rate in period  $t$  to be  $\gamma_{it}^E = \gamma_{it-1}$ , the realization of the advertiser's click-through rate in the previous period.

Each advertiser chooses  $T$  bid types  $(g_{i1}, \dots, g_{iT})$ ,  $T$  bids  $(b_{i1}^{g_{i1}}, \dots, b_{iT}^{g_{iT}})$  and  $T$  click-through rates  $(\gamma_{i1}, \dots, \gamma_{iT})$  to solve the profit maximization problem

$$\Pi_i = \max_{\{g_{it}\}, \{\gamma_{it}\}, \{b_{it}^{g_{it}}\}} \sum_{t=0}^T \beta^t \pi_{ikt}(\gamma_{it}, b_{it}^{g_{it}} | \gamma_{it}^E, b_{-it}^{g_{-it}}) \quad (6)$$

where  $\beta$  is the discount factor and  $b_{-it}^{g_{-it}}$  is an  $(N-1)$ -vector of other advertisers' bids in period  $t$ .

We characterize our notion of equilibrium in Definition 2.

*Definition 2: Based on a set of website expectations about advertisers' click-through rates*

*$(\gamma_{it}^E)_{i=1, \dots, N, t=1, \dots, T}$ , a set of bids  $(b_{it}^{g_{it}})_{i=1, \dots, N, t=1, \dots, T}$  and a set of click-through rates  $(\gamma_{it})_{i=1, \dots, N, t=1, \dots, T}$  is a Subgame Perfect Nash Equilibrium (SPNE) if there is a sequence of orders of bidders such that for every advertiser  $i$  we have the following:*

1. *The choice sequence of click-through rates  $(\gamma_{i1}, \dots, \gamma_{iT})$  and bids  $(b_{i1}^{g_{i1}}, \dots, b_{iT}^{g_{iT}})$  maximize advertiser  $i$ 's total profits in equation (6).*
2. *For every period  $t$ , if advertiser  $i$  wins slot  $k$  with bid  $b_{it}^{g_{it}}$ , then for every other position  $k' = 1, \dots, K$ ,  $\pi_{ikt}^{g_{it}} = \text{MAX}\{\pi_{ikt}^{g_{it}}, \pi_{ik't}^{g_{it}}, \pi_{ikt}^{g_{i't}}, \pi_{ik't}^{g_{i't}}\}$ ;*
3. *For any advertiser  $i$  who wins slot  $k$  in period  $t$ ,  $\pi_{ikt}^{g_{it}} \geq 0$ ;*
4. *For any advertiser  $j$  who doesn't win a slot in any period  $t$ ,  $\pi_{jkt}^{g_{jt}} = R_{jkt}^{g_{jt}} - \delta^{g_{jt}} C_{kt} \leq 0$  for all  $k = 1, \dots, K$ .*

The first condition ensures that advertiser choices maximize dynamic profits, rather than static profits. The other conditions are similar to definition 1, ensuring incentive compatibility, participation, efficient exclusion, and subgame perfection in each time period. We now make it clear how a brand advertiser can exploit the auction mechanism within the dynamic game. We first define the strategy the advertiser would follow and then show that it may be used in equilibrium.

*Definition 3: A Lattice Strategy is a repeated two-stage strategy profile in which an advertiser submits a CPM bid with a maximal click-through rate  $\bar{\gamma}$  in the first period, then enters an equivalent CPC bid with a minimal click-through rate  $\underline{\gamma}$  in the second period.*

The Lattice Strategy is essentially a bait-and-switch tactic: the advertiser sets a high click-through rate to calibrate the website's expectation in the next period. When the next period comes, it switches to a minimal click-through rate and uses the CPC bidding strategy to reduce

its advertising costs. To show this strategy may be played in equilibrium, we employ the One-Stage Deviation Principle of Blackwell (1965), a method of testing whether a sequential strategy profile in a dynamic game is subgame perfect. This principle states that a multi-stage strategy is subgame perfect in a dynamic game if and only if no player has incentive to deviate from this strategy profile in exactly one stage. Fudenberg and Tirole (1991) provide a detailed discussion. This substantially simplifies the process of showing a strategy is subgame perfect, since it is not necessary to rule out deviations in every single stage of the game.

*Proposition 4: A SPNE exists in which brand advertisers employ the Lattice Strategy under simple website expectations, and direct response advertisers enter their CPC bids from the static game.*

*Proof:* See Appendix.

When direct response advertisers enter CPC bids with maximal click-through rates, brand advertisers' best strategy is to use the Lattice Strategy. Even if this does not change the equilibrium ranking among advertisers, it gives brand advertisers a cost advantage by allowing them to take advantage of the website's backward-looking formation of click-through rate expectations. Note, however, that direct response advertisers now bid according to type.

Next, we consider whether the Lattice Strategy is still profitable if the website employs more data to form expected click-through rates.

### 3.2. A Dynamic Hybrid Advertising Auction with Sophisticated Website Expectations

Sellers of online advertising typically say they form expectations about advertisement performance based on historical click-through rates. We now show that simplistic analysis of historical click-through rates, regardless of how much data they use, would not eliminate the profitability of the Lattice Strategy.

We consider a family of click-through rate expectation functions  $\gamma_{it}^E = G_t(\gamma_{i0}, \dots, \gamma_{it-1})$ , with the properties (i)  $\frac{\partial G_t}{\partial \gamma_{it-s}} \geq 0$  for all  $s \in (0, 1, \dots, t)$   $0 < s < t$ , (ii)  $G_t < \bar{\gamma}$  if and only if  $\gamma_{it-s} < \bar{\gamma}$  for at least one  $s \in (0, 1, \dots, t)$ , and (iii)  $G_t > \underline{\gamma}$  if and only if  $\gamma_{it-s} > \underline{\gamma}$  for at least one  $s \in (0, 1, \dots, t)$ . This function nests intuitive formulations such as a simple average of historical

click-through rates  $\gamma_{it}^E = t^{-1} \sum_{s=1}^t \gamma_{it-s}$ , or a weighted average  $\gamma_{it}^E = \sum_{s=1}^t \gamma_{it-s} \beta_s$  with positive weights  $\beta_s > 0$ .

*Proposition 5. If the website sets  $\gamma_{it}^E = G_t(\gamma_{i0}, \dots, \gamma_{it-1})$ , a SPNE exists in which brand advertisers employ the Lattice Strategy, and direct response advertisers enter their CPC bids from the static game.*

*Proof:* See Appendix.

We have shown that if the website uses some average or weighted average of prior click-through rates, it cannot remove the profitability of the Lattice Strategy. Yet, if brand advertiser  $i$  is simply alternating between high and low click-through rates in every period as prescribed by the Lattice Strategy, the website could form its expectation using a non-averaging technique and predict the advertiser's click-through rate exactly in every period. However, there are more sophisticated extensions of the Lattice Strategy that the website would be unable to predict. For example, the advertiser could draw a random positive integer  $s$  from a distribution  $f(s)$  and only deviate to CPC bidding with minimal click-through rates after  $s$  periods, then draw another  $s$ , and repeat. Even if the website knew  $f(s)$ , it could not perfectly predict the website's pattern of click-through rates due to the inherent randomness of the strategy, leaving some periods in which the Lattice Strategy extension would be profitable. In addition, given that realized click-through rates will likely be noisy, it would be difficult for the website to know the true bounds of an advertiser's click-through rate. The only way the website could completely remove the profitability of the Lattice Strategy is if its click-through rate expectations were based only on periods in which the advertiser used CPC bidding. This is one of the key implications of our analysis that we discuss in the next section.

#### **4. Managerial Implications**

Our analysis has produced several results that could influence websites' and advertisers' business practices.

Our results imply that *brand advertisers*, rather than using a default CPM bidding strategy, ought to think carefully about which type of bid to enter. They may often be better off using a CPC bidding strategy while simultaneously discouraging clicks on their ads. They may

also profit from alternating high-click ads in conjunction with CPM bids in some periods interspersed with low-click ads purchased with CPC bids in other periods.

Our results imply that *direct response advertisers* might benefit from taking a CPM bidding strategy with a maximal click-through rate in one-off advertising auctions. However, they do not suggest that these firms should alternate high-click and low-click ads in repeated auction settings.

For *advertising sellers such as websites, search engines, and other media*, we have three implications. Most importantly, our results suggest that offering clients multiple bid types may backfire. While there may be non-negligible cognitive costs associated with either requiring direct response advertisers to bid on a per-impression basis, or requiring brand advertisers to bid on a per-click basis, these must be traded off against the potential revenue damage associated with strategic choice of bid types. If the advertising seller is not cognizant of the results presented in this paper, it could find the opposite of what it expects when it offers a hybrid advertising auction: namely, brand advertisers bidding on a per-click basis. Sites where direct response advertisers predominate (such as search engines) might find it optimal to exclude CPM bidding, and sites where brand advertisers abound (such as news sites) might find it best to exclude CPC bidding. This recommendation should appeal not only to sites currently offering the hybrid auction format, but also to sites that might consider doing so in the future.

Second, our analysis highlights how website uncertainty about advertisers' types opens the door to strategic bid choices. The greater the website's uncertainty, the more strategic bidding against type will harm website revenues. Websites using hybrid advertising auctions might consider instituting policies that induce advertisers to reveal their true types. For example, a website might offer an add-on service that would be valued differentially by brand advertisers and direct response advertisers and then use observations regarding which advertisers use that service to infer advertisers' types. The website might also manually investigate advertisers to try to glean more information about their types. The website can then use any information it gains about advertisers' types to set expected click-through rates.

Third, the profitability of the lattice strategy in the dynamic setting suggests that the website should be careful in choosing expected click-through rates. Advertisers' potential use of this strategy suggests that if an advertiser's click-through rate is observed to vary up and down, it may be desirable to use the lower bound of the observed click-through rates as the expected

click-through rate in future periods. When formulating expected click-through rates, another possibility would be to ignore a bidder's observed click-through rates realized in periods when the bidder used CPM bids, though this might reduce the amount of data available for use in the expectation formation process.

## **5. Discussion**

We have presented the first analysis of equilibrium bidding strategies in a hybrid advertising auction. We have shown that if the seller of advertising knows which bidders are direct response advertisers and which bidders are brand advertisers, it can make advertisers indifferent between CPM and CPC bidding. However, if the website is uncertain about the type of any advertiser, that advertiser will have the incentive to bid against type in a static game: brand advertisers will enter CPC bids and direct response advertisers will enter CPM bids. This bid reversal hurts the seller's profits, and the harm increases in the level of uncertainty about bidder type. The strategic bid reversal result holds up for brand advertisers in a repeated game, but not for direct response advertisers. Our analysis is remarkably general compared with most of the literature: it does not rely on assumptions about the number of ad slots, bidders, or time periods; public information; a single type of advertiser; or fixed click-through rates.

Despite this generality, we have made several simplifying assumptions. Two assumptions in particular suggest directions for future research. The first assumption that might be relaxed is that, like other papers in this literature, we have analyzed a monopoly seller of advertising. Expanding the analysis to multiple websites could introduce elements of two-sided market competition. Advertisers might have a chance to choose among different websites. Also, websites might choose their policies based on the possibility of advertiser or consumer defection to competitors. In this paper, we have not considered that advertiser adoption of a website's platforms could be a function of its business model.

A second dimension that might be profitably relaxed is in the direction of richer strategic interactions between advertisers. For example, we do not allow for the possibility of advertiser collusion, which may be sustainable under a small number of advertisers, a large number of time periods, and/or relatively flat distributions of slot-dependent click-through and impression rates. Another possibility would be to model the "bid jamming" argument of Ganchev et al. (2007), in which advertisers increase their bids without changing their positions in order to exhaust rivals'

budgets faster without increasing their own advertising costs. These effects have been ignored by most of the theoretical literature, but may yield important new insights.

## Appendix. Proofs of all Lemmas and Propositions.

*Lemma 1. If a direct response advertiser gets an ad slot in the static game, its weakly dominant strategy is to choose  $\gamma = \bar{\gamma}$ .*

*Proof:* When a direct response advertiser enters a CPC bid, its profit is  $\pi_{ik}^c = X_k \gamma_i r_{Di} - \frac{\gamma_i}{\gamma_i^E} C_k$ ,

which is weakly increasing in  $\gamma_i$  so long as  $r_i \geq \frac{C_k}{X_k \gamma_i^E}$ , which must be true if the advertiser bid

rationally and won the auction. If the D advertiser enters a CPM bid, the D advertiser's profit is

$\pi_{ik}^m = X_k \gamma_i r_{Di} - C_k$ , which is strictly increasing in  $\gamma_i$ . *Q.E.D.*

*Lemma 2. If a brand advertiser gets an ad slot in the static game, its weakly dominant strategy is to choose  $\gamma = \underline{\gamma}$ .*

*Proof:* If a brand advertiser enters a CPC bid, its profit is  $\pi_{ik}^c = Y_k r_{B,i} - \frac{\gamma_i}{\gamma_i^E} C_k$ , which is strictly

decreasing in  $\gamma_i$ . If it enters a CPM bid, its profit is  $\pi_{ik}^m = Y_k r_{Bi} - C_k$ , which is not a function of  $\gamma_i$ . *Q.E.D.*

*Proposition 1: If the website has perfect knowledge of advertisers' types, every advertiser is indifferent between entering the best possible CPM bid and the best possible CPC bid in a static game.*

*Proof:* Let  $b_{-i}$  be a vector describing the bids of the  $N-1$  advertisers other than advertiser  $i$ . If advertiser  $i$  chooses CPC bid  $b_i^{c*}$ , it must be the case that  $b_i^{c*} = \arg \max E\pi_i(b_i^c, b_{-i})$ ; in other words, if it chooses any CPC bid, it will choose the best available CPC bid available. Denote  $i$ 's equilibrium position given  $b_i^{c*}$  and  $b_{-i}$  as  $k$ . Its profit is then  $\pi_{ik}^c = R_{ik} - \delta^c C_k$ .

We now prove that advertiser  $i$  has no incentive to submit a CPM bid by contradiction. Assume there exists a CPM bid  $b_i^{m*}$  yielding position  $k'$  (which may be the same as position  $k$ ) which can increase  $i$ 's profit. Under this assumption,  $\pi_{ik}^m = R_{i,k'} - \delta^m C_k$ . Advertiser  $i$  could submit a CPC bid  $b_i^{c**} = \frac{b_i^{m*}}{X_k \gamma_i^E}$  that wins position  $k'$  and earns profit  $\pi_{ik}^m$  since  $\delta^m = \delta^c = 1$ . However this produces a contradiction in the definition of  $b_i^{c*} = \arg \max E\pi_i(b_i^c, b_{-i})$ . The proof that an advertiser entering a CPM bid has no incentive to switch to CPC bidding is symmetric. *Q.E.D.*

*Proposition 2: Any direct response advertiser whose type is not known with certainty by the website has a strictly dominant strategy to enter a CPM bid. Any brand advertiser whose type is not known with certainty by the website has a strictly dominant strategy to enter a CPC bid.*

*Proof:* A direct response advertiser's payoff when submitting a CPC bid is

$$\pi_{ik}^c = R_{ik} - \frac{\bar{\gamma}}{p_i \underline{\gamma} + (1-p_i)\bar{\gamma}} C_k. \text{ If it submits a CPM bid, it gets } \pi_{ik}^m = R_{ik} - C_k. \text{ CPM bidding strictly dominates since } \frac{\bar{\gamma}}{p_i \underline{\gamma} + (1-p_i)\bar{\gamma}} > 1.$$

The opposite thing happens for brand advertisers. A B type bidder's payoff when submitting a CPC bid is  $\pi_{ik}^c = R_{ik} - \frac{\underline{\gamma}}{p_i \underline{\gamma} + (1-p_i)\bar{\gamma}} C_k$ , while it gets  $\pi_{ik}^m = R_{ik} - C_k$  for submitting a CPM bid. Since  $\frac{\underline{\gamma}}{p_i \underline{\gamma} + (1-p_i)\bar{\gamma}} < 1$ , the CPC bidding strategy is strictly dominant. *Q.E.D.*

*Proposition 3. An equilibrium exists under imperfect website information that produces the same assignment of advertisers to slots as the equilibrium under perfect website information.*

*Comparing the first equilibrium with the second, the website's advertising revenues fall by*

$$\frac{(1-p_i)(\bar{\gamma}-\underline{\gamma})}{p_i \underline{\gamma} + (1-p_i)\bar{\gamma}} \text{ for any brand advertiser } i \text{ whose type is not known with certainty. Revenues}$$

*received from direct response advertisers do not fall in comparison with the perfect information benchmark.*



*Proof:* We start by first proving that bid reversals do not give any advertiser an incentive to deviate from the assignment of advertisers to slots found under perfect website information about advertiser types. Considering the equilibrium in pure strategies under perfect website information, assume that advertiser  $i$  gets position  $k$  with a CPM bid (or a CPC equivalent bid), with  $\pi_{ik}^{g_i} = \text{MAX}\{\pi_{ik}^{g_i}, \pi_{ik'}^{g_i}, \pi_{ik}^{g_i'}, \pi_{ik'}^{g_i'}\}$  for  $k' \neq k$  and  $g_i' \neq g_i$ . So advertiser  $i$ 's profit is  $\pi_{ik}^c = R_{ik} - C_k$  and  $R_{ik} - C_k \geq R_{ik'} - C_{k'}$  for any slot  $k'$ . Under imperfect website information about advertiser types, we know from proposition 2 that brand advertiser  $i$  will use its CPC bid equivalent  $b_i^{c^*} = \frac{b_i^m}{X_k(p_i\underline{\gamma} + (1-p_i)\bar{\gamma})}$ . Given no change in rivals' equivalent bids, advertiser  $i$

can certainly get slot  $k$ , and  $i$ 's profits are highest in slot  $k$  because

$$R_{ik} - \frac{\underline{\gamma}}{p_i\underline{\gamma} + (1-p_i)\bar{\gamma}} C_k > R_{ik'} - \frac{\underline{\gamma}}{p_i\underline{\gamma} + (1-p_i)\bar{\gamma}} C_{k'}, \text{ since } \frac{\underline{\gamma}}{p_i\underline{\gamma} + (1-p_i)\bar{\gamma}} < 1. \text{ So advertiser } i$$

would not switch to any other slot  $k'$ . The proof for a type D advertiser is symmetric.

To compare the website's revenue in two cases, first we know under perfect website information about advertiser types, brand advertiser  $i$  in slot  $k$  pays  $C_k$ . Under imperfect website information, it pays  $\frac{\underline{\gamma}}{p_i\underline{\gamma} + (1-p_i)\bar{\gamma}} C_k$ , which is lower by a fraction of  $\frac{(1-p_i)(\bar{\gamma} - \underline{\gamma})}{p_i\underline{\gamma} + (1-p_i)\bar{\gamma}}$ . Under imperfect website information, the direct response advertiser in slot  $k$  pays the same amount as it does under perfect website information,  $C_k$ . *Q.E.D.*

*Proposition 4: A SPNE exists in which brand advertisers employ the Lattice Strategy under simple website expectations, and direct response advertisers enter their CPC bids from the static game.*

*Proof:* We start by considering the equilibrium in pure strategies of the static game under perfect website information about advertisers' types (the setting analyzed in section 2.4). We assume that advertiser  $i$  gets position  $k$  with a CPM bid (or a CPC equivalent bid)  $b_i^{m^*}$  in the static equilibrium. Since the Lattice Strategy is a two-stage strategy, then by the One-Stage Deviation Principle, we can prove it is a SPNE strategy if we can show that  $i$  has no incentive to deviate from it in either stage, given other advertisers' actions. We will do this by first proving that

repeated interactions do not give any advertiser an incentive to deviate from the assignment of advertisers to slots found in the static game. Second, we prove that, for advertiser  $i$ , the Lattice Strategy strictly dominates other options in the first period (stage  $t-1$ ). Finally, we show that it is also strictly dominant in the second period ( $t$ ).

The advertising cost in position  $k$  in period  $t$  is

$$C_{kt} = \begin{cases} X_k \gamma_{i't}^E b_{i't}^{c*}, & \text{where } i' \text{ denotes the advertiser in slot } k+1 \text{ and } g_{i't} = c. \\ b_{i't}^{m*}, & \text{where } i' \text{ denotes the advertiser in slot } k+1 \text{ and } g_{i't} = m. \end{cases}$$

It can be shown that when brand advertisers employ the Lattice Strategy,  $C_{kt}$  is equal to the cost structure in the static game and does not vary over  $t$ . This is straightforward to see for direct response advertisers, who do not change their bids from the equilibrium in the static game. For brand advertiser  $i'$  in slot  $k'$  employing the Lattice Strategy,  $C_{k't} = b_{i't}^{m*}$  for  $t$  in the set of periods that  $i'$  enters a CPM bid. For the set of periods that  $i'$  enters a CPC bid,  $C_{k't} = X_{k'} \gamma_{i't}^E b_{i't}^{c*}$ . Since  $\gamma_{i't}^E = \underline{\gamma}$  by the Lattice Strategy and  $b_{i't}^m = X_{k'} \underline{\gamma} b_{i't}^{c*}$  by the definition of CPC/CPM equivalence in section 2, we have shown that  $C_{kt}$  does not vary over  $t$ . It therefore follows that, for each direct response advertiser and for brand advertiser entering CPM bids, the assignment of advertisers to slots that was optimal in the static game remains optimal in every period of the repeated game in the proposed equilibrium. However, brand advertisers entering CPC bids get a cost advantage, so we must consider whether this induces them to optimally switch slots. We can rule this out by

noting that when brand advertiser  $i$  gets slot  $k$  with a CPC bid, it gets  $\pi_{ikt}^c = R_{ik} - \frac{\gamma}{\bar{\gamma}} C_k$  in period  $t$ .

Since definition 1 gives us  $R_{ik} - C_k \geq R_{ik'} - C_{k'}$  for all  $k'$ , it must be the case that

$$R_{ik} - \frac{\gamma}{\bar{\gamma}} C_k \geq R_{ik'} - \frac{\gamma}{\bar{\gamma}} C_{k'}, \text{ for all } k', \text{ since } \frac{\gamma}{\bar{\gamma}} < 1. \text{ For Direct Response advertisers, from Lemma 1}$$

we know that DR advertisers always benefit from a higher click-through rate, so it is not optimal for DR advertisers to take the Lattice Strategy. As a result, DR advertisers always maximize their click-through rates.

We now show that the Lattice Strategy is strictly dominant for brand advertisers. In period  $t-1$ ,  $i$  can vary its click-through rate or its bid type. Any  $\gamma_{i,t-1} < \bar{\gamma}$  with a CPM bid would leave  $i$ 's payoff in period  $t-1$  unchanged, but would weakly decrease its payoff in period  $t$ . Since

$\gamma_{it}^E = \gamma_{it-1}$ ,  $\pi_{ikt}^c = R_{ik} - \frac{\gamma_{it}}{\gamma_{it}^E} C_{kt}$  is maximized at  $\gamma_{it-1} = \bar{\gamma}$ . The other possible deviation in period  $t-1$

is switching to a CPC bid and possibly also changing its click-through rate. Under the Lattice Strategy,  $\gamma_{it-2} = \underline{\gamma}$ , so  $\gamma_{it-1}^E = \underline{\gamma}$  and  $i$ 's profits from a CPC bid in  $t-1$  would be

$\pi_{ikt-1}^c = R_{ik} - \frac{\gamma_{it-1}}{\underline{\gamma}} C_{kt}$ . Yet for any click-through rate  $\gamma_{it-1}$ ,  $\pi_{ikt-1}^c$  is weakly less than  $i$ 's profits

under a CPM bid,  $\pi_{ikt-1}^m = R_{ik} - C_{kt}$ , and  $\pi_{ikt}^c$  would be strictly diminished, lowering total profits in equation (6). Therefore the Lattice Strategy is strictly preferred to any other in period  $t-1$ .

In period  $t$ ,  $i$  can again vary its click-through rate or its bid type. We have

$\pi_{ikt}^c = R_{ik} - \frac{\gamma_{it}}{\underline{\gamma}} C_{kt}$ , so any  $\gamma_{it} < \bar{\gamma}$  would reduce  $\pi_{ikt}^c$ . If  $i$  switched to a CPM bid, its profit is

$R_{ik} - C_{kt}$ , strictly less than  $\pi_{ikt}^c$ . So the Lattice Strategy is also strictly preferred in period  $t$ .

*Q.E.D.*

*Proposition 5. If the website sets  $\gamma_{it}^E = G_t(\gamma_{i0}, \dots, \gamma_{it-1})$ , a SPNE exists in which brand advertisers employ the Lattice Strategy, and direct response advertisers enter their CPC bids from the static game.*

*Proof:* Again we used the One-Stage Deviation Principle. The key point is that now, in period  $t$ ,

advertiser  $i$  gets profit  $\pi_{ikt}^c = R_{ik} - \frac{\gamma_{it}}{G_t(\gamma_{i0}, \dots, \gamma_{it})} C_{kt}$ . Since  $\frac{\partial G_t}{\partial \gamma_{it-1}} > 0$ , the advertiser has a

dominant strategy to maximize its click-through rate in period  $t-1$  and minimize it in period  $t$ .

The rest of the proof is parallel to that of Proposition 4, so it is omitted for brevity. *Q.E.D.*

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