Working Paper #05-21

October 2005

Stable and Efficient Electronic Business Networks: Key Players and the Dilemma of Peripheral Firms

Kai Suelzle
Ifo Institute for Economic Research at the University of Munich
& Dresden University of Technology

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Abstract

This paper studies a spatial model of electronic business network formation where firms build links based on a cost-benefit analysis. Benefits result from directly and indirectly connected firms in terms of knowledge flows, which are heterogeneous: a “key-player” (e.g. a firm providing an exchange platform in a business-to-business network) provides a higher level of knowledge flows than “peripheral” firms (e.g. tier 3 suppliers in a vertically differentiated industry). For intermediate cost values of link formation, stable and efficient network structures comprise only a subset of the total set of firms, excluding peripheral firms which are most distantly located to the key player. When link formation implies a certain degree of network congestion, the stable and efficient network size is smaller than in a model with bilateral decisions upon link formation between two firms.

JEL-classification numbers: C70, D85, L22
Keywords: Network Formation, Business-to-Business, Spatial Model

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[*] The author thanks Arun Sundararajan, Tom Kiessl, Klaus Wohlrabe and seminar participants at the Ifo Institute, Munich and the Dresden University of Technology for helpful discussions. Further thanks for comments are due to participants of EARIE, Porto 2005, IEA World Congress, Marrakech 2005, 5th ZEW Conference on the Economics of ICT, Mannheim 2005, and Kiel-Munich Workshop on the Economics of Information and Network Economics, Munich 2005.

Financial support of the NET institute (http://www.NETinst.org) is gratefully acknowledged.

[†] Address: Ifo Institute for Economic Research, Poschingerstr. 5, 81679 Munich, Germany
Phone: +49 (89) 9224 1282; Fax: +49 (89) 9224 1460; e-mail: suelzle@ifo.de

1 Introduction

The design and organization of network structures play an important role in significant economic and social relationships. Informal social networks are often the means for communicating information and allocating goods and services which are not traded on markets. Such goods do not only comprise invitations to parties and other forms of exchanging friendship but also, e.g. in the context of electronic business networks where co-operation is a central competition factor, information about job openings, business opportunities or product development. More than ever firms depend on connecting their abilities and resources with those from external partners. Networks play a fundamental role in providing platforms for research and development and collusive alliances among corporations. Furthermore, they determine how information is exchanged and convey social capital as one of the important determinants in trade.

For several decades, the management of the external environment took a high priority for firms by building stronger relationships with customers and suppliers. Recently, organizations have moved beyond customer/supplier connections to begin to establish alliances even with their competitors, which is - among other reasons - due to the revolution in information technology which has brought organizational changes that modify transaction costs, and thereby affect both the horizontal structure and the vertical configuration of industries. In this context, three lines of research can be distinguished:[1] First, there is a reduction in the frequency of hierarchical coordination, which is due to an increasing fragmentation of value chains. The advances in information technology, which cut coordination and monitoring costs, facilitated codification of knowledge and reduced the importance of geographical distance, at least for some activities. In this way, the new technology has reduced internalization-based advantages and reversed the process of vertical integration. Market-based transactions have squeezed out some of the ones hitherto coordinated hierarchically. The key-players in an increasing number of industries have adopted modular organizational structures. Hierarchically coordinated, vertically integrated organizations have thus given way to network organizations marked by horizontal cooperation, reciprocity and mutual trust, instead of hierarchical supervision of work processes.

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[1] See Szalavetz (2003).

Second, there is a flattening of vertically integrated organizations due to the mounting importance of distributed knowledge. In intellectual capitalism however, a diminishing proportion of the relevant knowledge base remains internal in many industries, and an increasing part is provided by outside experts. The more specialized the knowledge of an actor, the greater the extent to which hierarchical coordination loses its hold. This is especially the case for multi-component, IT-intensive products like aircraft engines, power stations, or office safety systems, which incorporate a plurality of technologies, and firms cannot develop them all inside. The manufacturers of such products and systems integrate the knowledge and coordinate the activity of various external, specialized suppliers and research institutions.

Networks as a third type of coordination alongside markets and hierarchies are becoming increasingly common in economic activity. Sustainable competitive advantage is determined by factors other than the traditional determinants of corporate competitiveness. Companies now have to capitalize on their internal as well as external knowledge. Alliance business networks also demonstrate sparsity, decentralization and clustering. Interfirm networks tend to be extremely sparse since forming and maintaining alliances has a cost in terms of time and effort. When firms forge relationships with other organizations for information sharing and exchange of knowledge, they face a variety of search, monitoring, and enforcement costs. Monitoring and managing alliances is also complex and costly, causing the firm's effectiveness at managing its alliances to decline with the number of alliances maintained.[2] Thus, due to cost constraints in forging and maintaining links, interfirm networks will tend to have far fewer links than if all pairs of firms were directly connected. Hence, although there is a growing importance of maintaining links to competitors, suppliers or clients in electronic business networks, there are cost factors that prevent many links. Moreover, we often observe networks being formed around certain key-players, often excluding smaller or peripheral firms because those players do not provide sufficient knowledge for the network. This paper therefore analyzes the interplay between network benefits and hindering costs of network formation, when players provide heterogeneous benefits to external partners.

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[2] See Deeds and Hill (1996).

Related literature: An excellent overview on relevant contributions to the theory of network formation is provided by Jackson (2004). Related models of network formation and collaborative networks are found in Bala and Goyal (2000), Jackson and Wolinsky (1996), and Kranton and Minehart (2001). Bala and Goyal (2000) follow a significantly different approach than this paper since they consider a directed network model where individuals are able to connect to others without the consent of the connected individual. Controversially, this paper deals with non-directed networks requiring the consent of both involved individuals to successfully create a link. The work of Kranton and Minehart (2001) deals with networks between vertically related firms. Issues relating to group formation and cooperation have been a central concern in economics and game theory in particular. The traditional approach to these issues has been in terms of coalitions. In recent years, there has been considerable work on coalition formation in games; see e.g. Jackson and Watts (2002), Bloch (1995). One application of this theory is the formation of groups in oligopolies. In this literature, group formation is modeled in terms of a coalition structure which is a partition of the set of firms. The present paper contributes to the theory of network formation by introducing three aspects which are especially observable in electronic business networks. First, we account for the fact that a crucial feature of such electronic business networks is the participation of so-called "key-players", such as crucial value enhancers in value chains or precursors in product development alliances. Accordingly, we account for heterogeneity among firms' information contribution to networks. Key-players provide higher levels of knowledge than "ordinary" firms, which could be tier 2 suppliers in value chains or followers in R&D development consortia. Second, in our model, firms can only connect to their direct neighbors but not to more distant players. This assumption reflects the peculiarities of electronic business networks where it is not necessarily required that every network member has a direct link to all other participants in order to guarantee knowledge exchange flows between all participants. A further intuition behind this assumption is that distances are interpreted in terms of similarities in business activities. That means if a firm intents to join a network, it has to incur costs (e.g. adjustment costs for its database, training of personnel) in order to sample a neighbor which is member of the network. On the other hand, the existing network must incur corresponding adjustment costs. The utility from a connection to a direct industry competitor might be higher than the utility from a very distant network member, say from another industry. Third, we introduce network congestion costs into a model of network formation. A joining member imposes costs on all existing network members in terms of increasing communication costs or adjustment costs, causing a firm's effectiveness at managing its alliances to decline with the number of alliances maintained.[3]

The remainder of the paper is structured as follows: the assumptions of the network model are introduced in section 2. Section 3 determines stability and efficiency in the static model. In section 4 we specify the outcomes of a dynamic network formation process. Section 5 discusses some extensions to the model and section 6 concludes the paper.

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[3] See Deeds and Hill (1996).

2 The Basic Network Model

The present model extends the literature on network formation in various aspects. Closest related is the *"connections model"* by Jackson and Wolinsky (1996).[4] There is a finite set of $N = 1, ..., m, ..., n$ *firms* in a market (with $n \geq 4$) which have a fixed location on a circle and are equidistantly located. This spatial dispersion should be interpreted to represent some diversity in terms of professional distance, differences in industry affiliations, etc. between the firms.[5]

A business *network* g is a list of firms which are linked to each other. Firms are represented by nodes and a link between nodes indicates that two firms are directly connected. This paper focusses on non-directed networks where links are bilateral. Every firm can only connect to (one or both of) its two direct neighbors. This can be interpreted as follows: if a firm wants to join a network, then it would have to adjust its database or its information technology infrastructure in such a way that it is compatible with the existing network. This happens for example by adjusting the database to the network member with the most similar database to the joining firm, which is (because of the spatial dispersion) one of the direct neighbors. We write $ij \in g$ for the link between the firms i and j which are direct neighbors.

Each firm $i \in \{1, ..., n\}$ receives a benefit $u_i(g)$ from the network g in terms of communication of information and from the allocation of goods and services which are not traded in markets (information about business opportunities, know-how on information technology, etc.). Although firms may connect only to their immediate neighbors, they also benefit from indirect communication flows with those firms to whom their direct neighbors are linked, and so on. The value of communication or knowledge flows obtained from other firms diminishes in the distance to those players, represented by a spatial depreciation rate $0 < \delta < 1$, which captures the idea that the value that i derives from a connection to j is proportional to the distance between those two firms. Further,

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[4] See also Jackson and Watts (2002) and Jackson (2004), who provide descriptions on the common structures of models on network formation.

[5] Note, that some contributions deal with players that are located on the real line, e.g. Johnson and Gilles (2000). In these models, players located at the end of the line only have one direct neighbor, which would lead to an ex-ante asymmetry of firms. To rule this out, we use a circular model where every firm has two direct neighbors.

there is an "intrinsic value" $w_{ij} \geq 0$, firm i provides to firm j.[6] In what follows it is assumed that all firms are identical except for one so-called "key-player" k, which provides a higher value than all other firms (this could be interpreted as k being a technology leader, or a platform provider in an electronic business network). Without loss of generality, we assume that $w_{ij} = 1, \forall i \neq k$ and $w_{kj} > \frac{1}{1-\delta} - \delta$.[7] For notational purposes, w_{kj} is labeled w_k in the following. The net utility of each firm i from graph g is then given by

$$u_i(g) = \begin{cases} \delta^{l_{ik}} w_k + \sum_{j \neq i} \delta^{l_{ij}}, & \text{for } i \neq k \\ \sum_{j \neq i} \delta^{l_{kj}}, & \text{for } i = k \end{cases} \tag{1}$$

where l_{ij} is the number of links in the shortest path between firm i and firm j (if there is no path between i and j, set $\delta^{l_{ij}} = 0$). The total cost of a link between two firms is c. The value of a link is determined by the benefits the two link establishing players receive through this link.

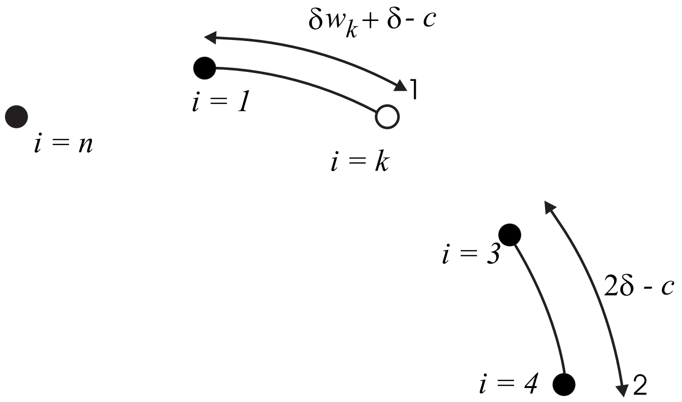

Figure 1: Circular network setup

Figure 1 depicts the setup of the model with a randomly selected position of the key-player k at $i = 2$. All firms are supposed to be initially unconnected. The graph also shows the value of two randomly selected links (given that there are no other links), for the case that those links are the only two existing links in the network. Link 1 between $i = 1$ and the key-player $i = k$ has a total value of $\delta(1 + w_k) - c$, where $\delta w_k (> \delta)$ is the net value to player $i = 1$ and δ is the net value to the key-player k. Link 2 is a link between two non-key-players which has the value $2\delta - c$.

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[6] See Jackson and Wolinsky (1996) for a similar notion.

[7] By $w_{kj} > \frac{1}{1-\delta} - \delta$ it is guaranteed that the increase in utility due to the proximity to the key-player is higher than the increase in utility due to an increase of the network size.

3 Efficiency and Stability in the Static Model

Let us define the efficient network as the graph that maximizes the total surplus function. This is the network g^* that maximizes the sum of each firm's benefit from the network, accordingly $g^* = \arg\max_g \sum_{i=1}^{n} u_i(g)$.[8] Without loss of generality, in the following we assume that n is even. The common notion of *pairwise stability* by Jackson and Wolinsky (1996) is not totally appropriate to network formation in electronic business, since it does not allow for interfirm compensation. Indeed, we often observe firms subsidizing suppliers or customers so that they adopt common e-business solutions, given that the joint profit of both firms is higher than the cost of establishing this connection. Accordingly, the following notion of *joint pairwise stability* describes a network as stable when no pair of adjacent players would benefit by severing an existing link, and no two players would benefit by forming a new link.

Definition 1 *A network g is jointly pairwise stable, if*
(i) $\forall ij \in g$, $u_i(g) + u_j(g) - c \geq u_i(g - ij) + u_j(g - ij)$, and
(ii) $\forall ij \notin g$, $u_i(g) + u_j(g) - c < u_i(g - ij) + u_j(g - ij)$.

Definition 1 thus states that a link is *jointly pairwise stable*, so that this link is formed when the sum of both agents' additional values from the link is higher than the sum of their utilities if the link was not formed, where c denotes the total cost of the link. Contrarily, if the sum of the utilities of both link establishing firms is less than without the link, the notion requires that the link is not formed. When a network g is not *jointly pairwise stable* it is said to be *defeated* by g' if either $g' = g + ij$ and (ii) is violated for ij, or if $g' = g - ij$ and (i) is violated for ij. As in the model by Watts (2001) the approval of two firms is required for the formation of a link, but here, those firms have to be adjacent, and the sum of both their utilities minus the cost c of the link creation have to be (weakly) higher than 0. The consideration of the joint utilities of link establishing players introduces the possibility of interfirm compensation. Indeed, in electronic business relationships we observe that many firms subsidize their suppliers to get them connected to a business network.

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[8] See Watts (2001), Jackson and Wolinsky (1996) and Bala and Goyal (2000).

This definition of joint pairwise stability is a relatively weak notion among those which account for link formation, and it is not dependent on any particular formation process. Accordingly, it admits for a relatively large set of stable allocations compared to more restrictive definitions or an explicit formation procedure. But for our purposes, it already narrows the set of graphs substantially, and therefore such a weak definition provides strong results. One obvious strengthening of this stability notion is to allow a decision on the creation of links to be made by coalitions of network members, which include more than two firms (which are connected via the link). This definition of a stable network (i.e. equilibrium) requires that agents have no incentives to sever existing links, or establish fresh ones, or replace existing links with new ones.

Proposition 1 *For all N an efficient and stable network exists. Further,*

(i) if $c \leq 2\delta + \sum_{i=2}^{n-1} \delta^i + \delta^{\frac{n}{2}}(w_k - 1) := \boldsymbol{a}$, then the network g^N, that comprises all n players is efficient and stable.

(ii) If $\boldsymbol{a} < c \leq \delta(1 + w_k + \delta) := \boldsymbol{b}$, then for every c $\exists m$ such that the network is efficient and stable for m firms, with $0 < m < n$ and m being an odd integer. This network comprises k.

(iii) If $c > \boldsymbol{b}$, then the empty network is the only efficient and stable network structure.

Proof. (i). In order to determine the lower threshold value $\mathbf{a}$, we define v_l as the lowest value of a link in the largest possible network. This link connects the last unconnected player, that is most distantly located to the key-player. The value of this link is determined by

$$v_l = \underbrace{2\delta + \sum_{i=2}^{n-1} \delta^i + \delta^{\frac{n}{2}}(w_k - 1)}_{:=\mathbf{a}} - c. \tag{2}$$

The two firms that establish the link each receive δ from their direct connection, and the new network member additionally receives $\delta^{\frac{n}{2}} w_k$ from the indirect connection to the key-player and $\sum_{i=2}^{n-1} \delta^i - \delta^{\frac{n}{2}}$ from all other indirect connections. If $v_l > 0$ (which is the case if $c < \mathbf{a}$), even the last unconnected firm and its neighbor will benefit from creating a link.

(ii). and (iii). The relevant link value to determine the upper threshold value $\mathbf{b}$ is the highest link value in the smallest possible network, which is the network comprising the key-player and its two direct neighbors.[9] This link has the value

$$v_h \;=\; \underbrace{\delta(1 + w_k + \delta)}_{:=\mathbf{b}} - c. \tag{3}$$

The second neighbor who links up to k receives δw_k from the direct link to k and δ^2 from the indirect connection to the other neighbor. k receives δ from the link to the joining member - the cost of this link is c. If c is higher than this value, only the empty network can be efficient.[10] If c lies between the two threshold values $\mathbf{a}$ and $\mathbf{b}$, then the efficient network structure necessarily comprises the key-player k but does not include all n firms but only $m < n$ firms, where m is an odd integer, due to symmetry. ■

Accordingly, for each $c \in [\mathbf{a};\ \mathbf{b})$, there is an m such that the network is stable (and efficient) for exactly m but not for $m + 1$ firms. This m is determined by the value $c \approx 2\delta + \sum_{i=2}^{m-1} \delta^i + \delta^{\frac{m-1}{2}}(w_k - 1)$. In every case this network comprises the key-player as a central firm in a symmetric fraction of the circle. The intuition for the definition of the two threshold values $\mathbf{a}$ and $\mathbf{b}$ is as follows: in case of the lower threshold value $\mathbf{a}$ we need to ensure that the smallest possible network is able to adopt without anyone else. For the upper threshold value $\mathbf{b}$ we need to ensure that no departure with everybody else can be beneficial in the biggest possible network (which is the network that comprises all n firms). Figure 2 gives a graphical illustration of the stable and efficient network structures.

This outcome is highly relevant to observations in practice. In the intermediate cost range, peripheral firms which are most distantly located to the key-player cannot be part of a stable and efficient network, since the value added (e.g. in terms of know-how on product development or process

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[9] Note, the smallest network is not one that comprises just the key-player and only one neighbor, since the value of connecting the second neighbor to k given that the other neighbor is already linked to k has a higher value than the link between k an the first (unconnected) neighbor.

[10] This is due to the assumption that $w_{kj} > \frac{1}{1-\delta} - \delta$. Accordingly, the relevant link is the link between k and its second neighbor, given the existing link between k and its first neighbor.

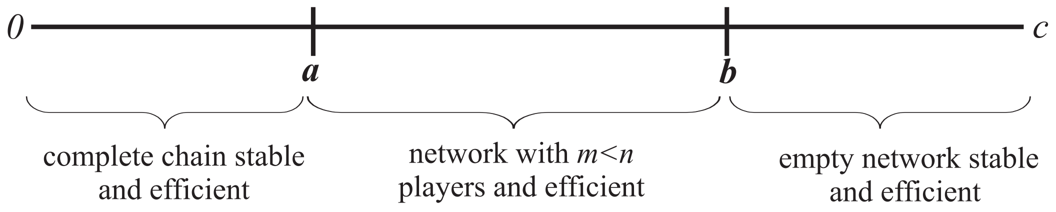

Figure 2: Stability of network structures dependent on c values

data, or industry knowledge) is not enough to compensate for the costs their connection to the network would imply (see the appendix for a numerical and graphical example).

4 Dynamic Network Formation

The n firms are myopic and are supposed to be initially unconnected.[11] Time, T, is divided into periods, being modeled as a countable, infinite set $T = \{1, 2, ..., t, ...\}$. The network that exists at the end of period t is labeled g_t whereas the payoff each firm i receives at the end of t then reads as $u_i(g_t)$. In each period, a (potential) link $i : i \pm 1$ between two neighbored firms is randomly identified to be updated with uniform probability. If the identified link $i : i \pm 1 \in g_{t-1}$, both firms i and $i \pm 1$ decide whether to sever the link or not. Otherwise, if $i : i \pm 1 \notin g_{t-1}$, then firm i and $i \pm 1$ can form link $i : i \pm 1$ requiring that the sum of both firms' utilities from the link is higher than its cost. A *stable state* in the network formation process is reached if after some time period t, no additional links are formed or broken. Accordingly, the resulting network must be a stable (static) network. In proposition 2 we derive what types of network the formation process converges, allowing us to determine whether or not the formation process converges to efficient and stable network structures.

Figure 2 provides a graphical representation of the emerging network structures which are determined in proposition 2.

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[11] See Watts (2001).

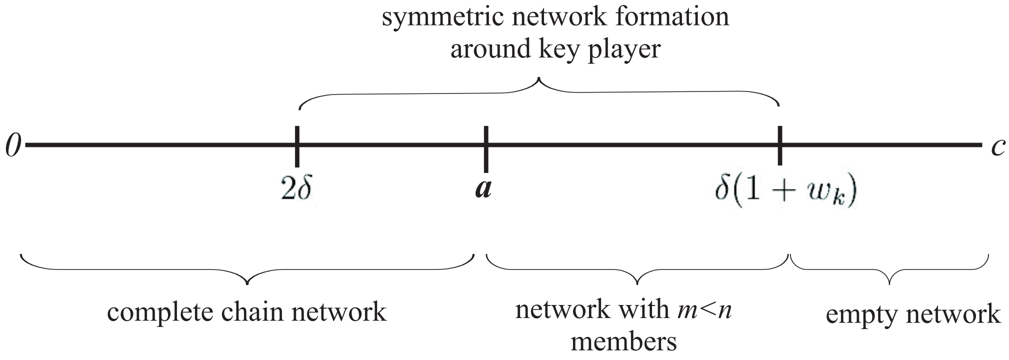

Figure 3: Outcomes of the dynamic network formation process

Proposition 2 *The dynamic network formation process converges to the following network structures:*

(i) *If $c \leq 2\delta$, then every link forms (as soon as possible) and remains. The network converges to the complete ring graph g^N.*

(ii) *If $2\delta < c \leq \delta(1 + w_k)$, links form symmetrically around the key-player (starting with a link between k and one of its two neighbors):*
a) in case of $2\delta < c \leq \boldsymbol{a}$, the network converges to the complete ring g^N,
b) in case of $\boldsymbol{a} < c \leq \delta(1 + w_k)$, the network size reaches $m < n$ members.

(iii) *If $c > \delta(1 + w_k)$, then no links ever form.*

Proof. The proof takes into account the results from the proof to Proposition 1. Dependent on the values of c different links may form. Note that the lowest net value of a link is 2δ, which is a link between two firms which are not the key-player. Further,

- if $c < 2\delta$ then even this link forms immediately when those two neighbors are matched. Accordingly, with such a low value for c every link forms.
- If $2\delta < c \leq \mathbf{a}$, then only a link between the key-player and one of its neighbors is valuable in the first period. Because $c > 2\delta$, no other link will be formed in the first period. Due to this argumentation, in subsequent periods only links to the already existing network, including the key-player, can be valuable. Since $c \leq \mathbf{a}$, the cost for a link is low enough that the network converges to the complete chain.

- If $\mathbf{a} < c \leq \delta(1+w_k)$, due to the same argumentation as above, only a link involving the key-player k and one of its neighbors can be valuable in the first period. Again, due to $c > 2\delta$, in subsequent periods only links to the already existing network, including the key-player, can be valuable. But now, since $\mathbf{a} < c$ the network will not converge to the complete ring graph g^N but only to a network including just $m < n$, determined by $2\delta + \sum_{i=2}^{m-1} \delta^i + \delta^{\frac{m-1}{2}}(w_k - 1) \approx c$.

- If $c > \delta(1 + w_k)$, then no links can ever form. This is because $\delta(1 + w_k)$ is the highest possible value of a first link in the network. This must be a link between the key-player and one of its direct neighbors. If this value is lower than the cost c of establishing a link, there is no incentive to form any link. ■

Proposition 3 tells us what type of networks the formation process converges to. This information allows us to determine whether or not the formation process converges to an efficient network. Each agent prefers a direct link to any indirect link. In each period, two agents, say i and $i + 1$, meet. If players i and $i+1$ are not yet connected, then they will each gain at least from forming a direct link; if $c < 2\delta$ and so the connection will take place. Using the same reasoning as above, if an agent ever breaks a direct link, his payoff will strictly decrease. Therefore, no direct links are ever broken. Proposition 3 says that if $0 \leq c < \mathbf{a}$, then the network formation process always converges to the complete ring network, which is the unique efficient network according to Proposition 1. This network is also the unique stable network.

5 Extensions

The basic network model can be extended in various ways. Most common in the literature is a distinction between two-sided and one-sided knowledge flows yielding quantitatively slightly differentiated results (see e.g. Bala and Goyal, 2000). Here, we want to focus on extensions, namely the impact of changes in the bearer of the link cost, of a network congestion cost and of the number of firms which are involved in the decision of forming a link between two firms.

5.1 Differentiated Distributions of Link Costs

In order to allow for the possibility of interfirm compensation payments, we thus far considered the joined payoff of the two firms that establish the link as relevant for the creation of a link. But we also observe other forms of distributing the cost burden of link creation among players.

Joining firm bears total linking cost

In electronic business relationships, often the joining firm has to bear the total cost burden to get linked to a network alone. Accordingly, the most distant firm to k will only join the network if its net payoff is positive. Compared to the scenario where the two connecting firms share the cost, the identified threshold values **a** and **b** now change to **a'** and **b'** as follows:

$$\mathbf{a'} = \delta + \sum_{i=2}^{n-1} \delta^i + \delta^{\frac{n}{2}}(w_k - 1) \tag{4}$$

$$\mathbf{b'} = \delta(w_k + \delta). \tag{5}$$

Since **a'**<**a** and **b'**< **b** it follows that when the joining firm has to fully bear c, the crucial cost value to ensure a stable and efficient network where all n firms participate has to be lower than in the basic model. Further, for the intermediate cost values $c \in [\mathbf{a'};\mathbf{b'}]$ the number of m firms for which the network is stable and efficient tends to be lower than in the basic model. This outcome is due to the fact that in this scenario there is no possibility of interfirm compensation between link-establishing firms.

Joint pairwise stability vs. pairwise stability

The common stability concept in the extant literature is that of *pairwise stability* by Jackson and Wolinsky (1996), which is a more narrow concept than joint pairwise stability since it does not allow for the possibility of compensation payments between link establishing partners. Pairwise stability is defined as follows:

Definition 2 *(Jackson and Wolinsky, 1996)*
A network g is pairwise stable if
(i) $\forall ij \in g$, $u_i(g) \geq u_i(g - ij)$ and $u_j(g) \geq u_j(g - ij)$
(ii) $\forall ij \notin g$, if $u_i(g) < u_i(g + ij)$ then $u_j(g) > u_j(g + ij)$.

This definition considers only the individual benefits of two link-establishing firms. In can easily be shown that in comparison to the notion of joint pairwise stability, the threshold cost value for stable networks is lower when we apply the notion of pairwise stability. That means, with pairwise stability some networks are efficient but not stable, whereas they are stable and efficient under joint pairwise stability.

5.2 Network Congestion

Instead of modeling a link creation cost that has to be incurred by (at most) the two firms between which the link is created, we now introduce a network congestion cost. That is, with every joining member there arises a cost c' to all existing network members in terms adjustment costs to the new member or increased administrative effort, for example. This network congestion cost is modeled as an alternative cost to the link-establishing cost c from above, such that the utility of a player i from the network g denotes as:

$$u_i(g) = \begin{cases} \delta^{l_{ik}} w_k + \sum_{j \neq i} \delta^{l_{ij}} - (n-1)c', & \text{for } i \neq k \\ \sum_{j \neq i} \delta^{l_{kj}} - (n-1)c', & \text{for } i = k \end{cases} \tag{6}$$

where n represents the cardinality of g. Accordingly, the higher the number of network members, the greater becomes the interest of network members to prevent further firms to join. Furthermore, the decision to accept a link between a network member and a firm outside the network could also be influenced by all existing members of the network. Especially when network congestion costs are present, such a scenario is highly relevant to practice. In such a case the stable and efficient network size is smaller than in the case with only link-establishing firms being involved in bearing the cost burden of a new link.

6 Conclusion

The recent advances of information technology have brought about many organizational changes for firms in always faster changing markets. Together with a reduction in the frequency of hierarchical coordination and increasing fragmentation of value chains, we observe a flattening of vertically integrated organizations. Furthermore, networks as a form of coordination alongside markets have become increasingly common, especially in the electronic business. Recently, organizations have moved beyond customer/supplier relationships to begin to establish alliances with their direct and closely related industry competitors. Typically, these inter-firm alliances take the form of formal organizational partnerships, which are of growing importance in the context of electronic business networks. Such competitor alliances formerly focused exclusively on specific joint product development efforts, but tend increasingly to long-term basic research and development collaborations.

The present paper contributes to the theory of network formation by introducing three aspects which are especially observable in electronic business networks. First, we account for the fact that a crucial feature of such electronic business networks is the participation of so-called “key-players”, which are crucial value enhancers in value chains or precursors in product development alliances, for example. Accordingly, we account for heterogeneity among firms’ information contribution to networks. Key-players provide higher levels of knowledge than ordinary firms, which could be tier 2 suppliers in value chains or followers in R&D development consortia. Second, in our model, firms can only connect to their direct neighbors but not to more distant players. This assumption reflects the peculiarities of electronic business networks where it is not necessarily required that every network member has a direct link to all other participants in order to guarantee knowledge exchange flows between all participants. A further intuition behind this assumption is that distances are interpreted in terms of similarities in business activities. That means if a firm intents to join a network it has to incur costs (e.g. adjustment costs for its database, training of personnel) in order to sample a neighbor which is member of the network. On the other hand, the existing network has to incur corresponding adjustment costs. The utility from a connection to a direct industry competitor might be higher than the utility from a very distant network member say from another industry. Third, we introduce network congestion costs into a model of network formation. A joining member imposes costs on all existing network members in terms of increasing communication costs or adjustment costs causing a firm's effectiveness at managing its alliances to decline with the number of alliances maintained.

Furthermore, through the definition of jointly pairwise stability, we extend the common notion of pairwise stability (Jackson and Wolinsky, 1996) by providing a concept that allows for interfirm compensation payments. Additionally, this concepts eliminates the shortfall of pairwise stability with regard to the existence of efficient but non-stable networks. Indeed, in electronic as well as in non-electronic business relationships, we observe that many (big) firms subsidize their suppliers (or as well customers) to get them connected to a network or R&D cooperation.

The present paper can by extended by generalizing the concept of joint pairwise stability to a concept of joint m-wise stability, meaning that we consider not only the payoffs of two link-establishing players but of all the players which are already in the existing network. A further extension is to generalize the model for arbitrary network structures, where the basic concept of jointly pairwise stability should also hold when we introduce a key-player. This will be taken up in future research.

Appendix

Example: A Network with $n = 12$ potential Members

For illustrative purposes consider an exemplary network with $n = 12$ potential members. Let $\delta = 0,300$ and $w_k = 1,730$ then it follows, that in this setup, $\mathbf{a} = 2\delta + \sum_{i=2}^{n-1} \delta^i + \delta^{\frac{n}{2}}(w_k - 1) = 0,909$ and $\mathbf{b} = \delta(1 + w_k + \delta) = 0,729$. The

If $c \in$	g stable & efficient for m firms, with $m =$
$[0,909; \infty)$	0
$[0,791; 0,909)$	3
$[0,748; 0,791)$	5
$[0,734; 0,748)$	7
$[0,730; 0,734)$	9
$[0,729; 0,730)$	11
$[0,000; 0,729)$	$n = 12$

Table 1: $c-$ranges for stable and efficient networks with m firms

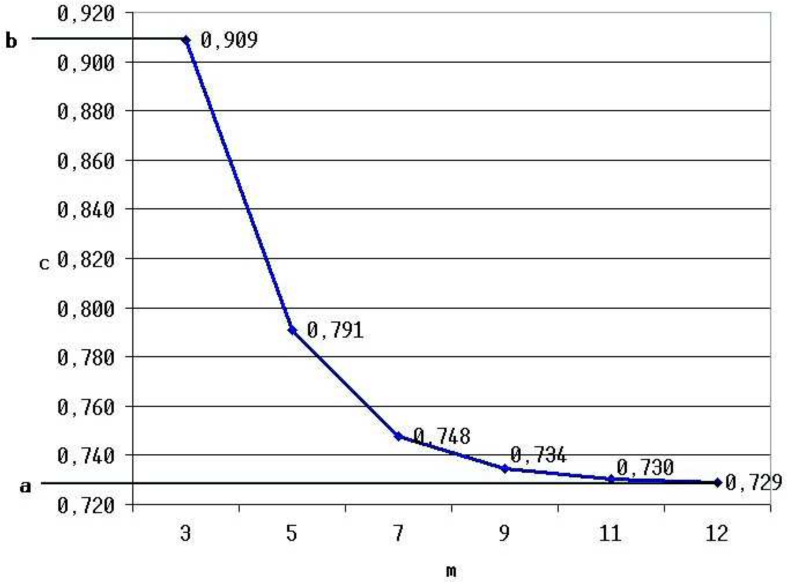

Figure 4: # of m firms for which the network is stable and efficient, given c

calculated ranges for c so that the network is only stable for $m < n$ firms are shown in table 1 above. Accordingly, in the middle ranges for c, i.e. between **a** and **b**, the network is stable and efficient for only $m < n$ firms, leaving peripheral firms out of the network. Figure 4 shows the relationship between the cost to establish a link between two neighbored firms and the number of firms for which the network is stable and efficient. The vertical axis shows the ranges for c in which the network is stable for exactly m firms, e.g. for $c \in [0,791; 0,909)$; this is the case for $m = 3$ firms. If $c > \mathbf{b} = 0,909$ the empty network is the only stable and efficient network; if $c < \mathbf{a} = 0,729$, this is the case for all $n = 12$ firms.

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