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Tunay I. Tunca

Stanford University

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# Information Precision and Asymptotic Efficiency of Industrial Markets

Tunay I. Tunca*

Graduate School of Business
Stanford University

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**Abstract**

Online market places have an unprecedented power of bringing together a large number of buyers and sellers and aggregating information. Despite its benefits, this scale of aggregation of private information may bring about adverse effects that can cause inefficiencies, which can be ignored by conventional analysis. In this paper, I present a strategic model of a large industrial market with asymmetric information to examine (*i*) the validity of the conjecture of price-taking behavior in such markets as the number of agents becomes large; (*ii*) the effect of the rate that individual information precision decreases with increased number of agents on convergence to price-taking and efficiency. I show that in an industrial market with downstream competition, increasing the number of sellers may make all participants price-takers in the limit, but increasing the number of buyers may not. When the total precision of information in the market is high, price taking and full social efficiency is achieved in the limit with large numbers of buyers and sellers. However, if the total precision of information in the market is poor, large inefficiencies, including full inefficiency, can occur in the limiting outcome. The rate of decrease of individual information precision with increased number of agents determines the rate of convergence to efficiency, and the convergence is slower than that predicted by the single unit trading models in the literature.

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*Graduate School of Business, Stanford University, Stanford, CA 94305-5015. e-mail: tunay@stanford.edu.
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# 1 Introduction

The use of electronic markets for procurement is growing. According to a recent survey done by ISM and Forrester research, in the third quarter of 2003, 33% of the large companies used web-based marketplaces and 84% used the Internet for some form of procurement.[1] Among many benefits that these markets provide, an important one is their power for aggregating information. By enabling a large number of buyers and sellers with diverse characteristics and geography to virtually converge and trade in a centralized marketplace, Internet based markets for both consumer and business transactions have an unprecedented potential for aggregating private information and providing market efficiency. On the other hand, broad usage of private information may also have some undesired effects. Specifically, the presence of private and asymmetric information in a marketplace can create adverse selection, which may reduce the participants' willingness to trade and decrease market liquidity. This can lead to market underperformance and, in the extreme, even to market failure. The goal of this paper is examining the role of private information that is highlighted by the use of centralized electronic procurement markets and exploring the effect of private information on market performance. We also aim to identify a clear and quantitative theoretical measure of price-taking that can be used as a yardstick for market competitiveness and analyze its nature in large markets.

Many models of economies with large number of agents assume price-taking behavior. This assumption usually is based on the intuition that when the agents are small compared to the size of the economy, their individual trade sizes are so small that the price impact is negligible. A conceptually distinct support for the price-taking assumption is that increasing the number of agents in the economy makes them so competitive that even small changes in their "quoted" price causes dramatic declines in the quantities they can sell or buy, i.e., their demand or supply curves as a function of price are nearly vertical and they are nearly true price-takers. In this paper, I present a strategic model of a vertical industrial market with upstream and downstream competition to question the validity of both of these arguments under private information and show that neither of these two has to hold with a large number of agents. In particular, I show that the specific way the precision of total information in the economy changes with the increased number of agents, together with the payoff structure that characterizes such markets can have important consequences for price-taking and efficiency as the number of agents in the market becomes large.

To demonstrate the effect of private information on price-taking behavior in industrial markets, I build and analyze a model of vertical trade in an industrial setting: There are $N$ manufacturers who compete as a Cournot oligopoly in a consumer market with uncertain demand. $M$ upstream suppliers provide a homogenous intermediate good via identical con-

[1] Bartels A., R. Hudson and T. Pohlmann (2003), Report on Technology in Supply Management: Q3 2003. Technical Report, ISM/Forrester.

stant marginal cost technologies. The realization of the marginal cost coefficient is uncertain, reflecting the variation on the price of certain common inputs (for instance oil, energy or raw materials). The agents have private (but correlated) signals about the relevant uncertainty that they are facing. Specifically, each buyer has an imperfect signal about the realization of the consumer demand, and each seller has an imperfect signal about the realization of the marginal cost.

To explore the emergence and nature of price-taking behavior in this market, we have to endogenize price taking through a strategic model. To do this, I employ a common approach and use demand and supply curves as the strategies of the participants (see, e.g., Kyle 1989, Vayanos 1999). This results in each participant's facing a downward sloping residual demand curve or an upward sloping residual supply curve when making her quantity decision. Therefore, for each agent, increasing the quantity he sells (buys) decreases (increases) the market price. The lower the price-impact of the trades for an agent, the closer she is to being a price-taker. This constitutes a concrete measure of price taking, namely, the price impact of the trades, whose magnitude emerges endogenously in equilibrium. As a result, (near) perfectly elastic supply where all agents are price-takers in the limit and (near) perfectly inelastic supply where trading even the smallest quantities can have arbitrarily large price impact in the limit and everything in between can arise endogenously in equilibrium as the number of agents in the market becomes large.

A key issue is the trade-off between two forces that affect the price-taking behavior of the agents: First, consistent with the common wisdom, as the number of sellers increases, competition forces the supply functions of the sellers to become steeper. This effect not only pushes the sellers towards being price-takers, but also, since their residual supply curves (as functions of the price) become steeper, pushes the buyers towards being price-takers. The second and the opposing force is the effect of information aggregation in the market and the agents' reaction to that when determining their optimal quantities conditional on the realization of the market outcome. If a seller puts higher quantity in the market, this signals her willingness to produce and the increased possibility of her having a low cost signal. This increases the remaining sellers' belief that the cost realization will be low. As a consequence, the prices are pushed lower. The higher the magnitude of this adverse selection effect the farther the suppliers will be pushed away from being price-takers. The growth rate of the magnitude of this effect as the number of suppliers increases compared to the rate that the competition pushes the suppliers towards being price-takers determines the price-taking behavior in the limit. A symmetric argument applies for the buyers.

The reaction of the participants to the market outcome is determined by their expectations on the behavior of the other agents in the market. When the private information that each of the remaining agents possesses is relatively accurate, each one of them will respond to her signal by reacting strongly to it. That is when a buyer has an accurate signal about consumer demand, she will increase her quantity to buy by a sizeable amount when she receives a high signal and vice versa, and therefore her trading quantity has a sizable ex-ante variance. On the other hand, when a trader's signal is not very accurate, she does not respond to it strongly, and her trading quantity becomes more predictable to the other agents. Aggregated over all agents, the residual supply or demand curve that each agent is facing will be more predictable if the accuracy of the signals is low. As a consequence, any deviation will signal a strong realization of the underlying variable and result in a strong reaction by the remaining agents. In the limit, this can become so extreme that smallest quantities can have very large impact on prices, and consequently the market performance can suffer dramatically.

Here, the way individual information precision decreases with the number of agents becomes important. Intuitively, one could argue that as the number of competitors in a market increases, their incentives to acquire more precise information along with the gains from using such precise information decreases. Given this premise, depending on the cost function for information acquisition, the total information in the market can increase, stay the same or decrease with increased number of agents.[2] Combined with the specifics of the market and payoff structure in industrial markets (e.g., the effect of downstream competition), this can result in significant changes in the way agents react to information asymmetry as the number of agents becomes large. Consequently, price-taking behavior may be significantly affected and this can influence the efficiency of the market, measured by social welfare in the equilibrium outcome.

Basing on these intuitions, I state and demonstrate three claims:

First, the way information precision decreases with increased number of agents can play an important role in the limiting price-taking behavior as the number of agents increases. If the rate of decline of individual information precision is slow, agents react to their signals with a certain strength. Therefore, a certain variation in the market quantity is expected, and the effect of adverse selection is not strong enough to undo the effect of increased competition. Consequently, in the limit, price taking can be achieved. On the other hand, if individual information precision of the agents declines rapidly as the number of agents grows, they will not react to their signals strongly and the aggregate variation in the market quantity will be low. Consequently, a deviation from the expected quantities would indicate a relatively strong signal received by an agent, which in turn indicates a large shock on the demand or the unit cost. Thus, prices can become very sensitive to quantities traded in the market, i.e., the price impact of quantities traded grows. Therefore the adverse selection effect can become strong enough to withstand the competitive pressure as the number of agents grows and in the limit no agent may become a price-taker.

[2]This can be demonstrated with an approximate argument. In this paper, I do not get into the details of information acquisition and how a certain rate of decrease in individual information precision is achieved. I just start with a given rate and explore the implications. Incorporating the details of information acquisition and the construction of the particular acquisition function, although being interesting, require substantial effort and analysis that does not ultimately add to the insights I aim to present in this paper and increase the length of the paper substantially.

Second, whether the agents become price-takers or not depends on the side of the market on which the number of agents gets large. Specifically, having a large number of sellers and small number of buyers may make all agents price-takers in the limit, but having a small number of sellers and large number of buyers may cause non-price-taking behavior for all agents. The reason for this is the inherent dependence of the buyers' payoffs: in an industrial market where the buyers are competing in the same downstream market, the buyers' payoffs are dependent on the quantities that their competitors buy. In particular, with Cournot style competition, each buyer's desired quantity will be approximately inversely proportional to the number of competitors she has. This inverse proportionality gets multiplied with a scale factor that is negatively correlated with the price of the intermediate good to determine the total quantity. Therefore, the price schedule of each buyer is decreasing in the price of the intermediate good, and the coefficient is in the order of $1/N$. Aggregating over all $N$ agents, one can see that the sensitivity of the inverse demand schedule to quantity can have a positive lower bound. Therefore, because of the interdependence of their payoffs, making the number of buyers large does not necessarily result in price-taking behavior in the limit.

Third, contrary to common perception, increasing the number of agents in the market can, in fact, move the participants arbitrarily away from price-taking and cause large inefficiencies in the market even as the number of agents on both sides of the market gets large. This claim is markedly different from the first claim, but the intuition starts from the same point: As explained above for the first claim, when the rate of decrease of individual information precision is high as the number of agents increases, the adverse selection effect can become strong and withstand the competitive pressure to keep the agents from being price-takers. If the rate is above a certain point, the information effect may become too strong as the number of agents gets large and consequently the price impact of the quantities in the market can go to infinity as the number of agents increases. As a result, the trade and consequently production can significantly decrease causing large inefficiencies. In fact, this effect can be so severe that the production can fully dry up as the number of agents in the market gets very large.

Demand curve equilibrium with private information and common values was first examined by Wilson (1979) in the context of divisible good auctions. In his model, a number of bidders submit demand schedules, and the clearing price is determined by equating the total demand to the fixed number of shares of a good supplied by a seller. Vives (1986) and Klemperer and Meyer (1989) also examine demand curve equilibria in the context of a production oligopoly. Private information, however, is not a part of these models. On the other hand, private information can play a very important role in market outcome. Vives (2002), shows that in a Cournot market, the impact of private information can dominate the impact of market power in determining the behavior of convergence to efficiency. Kyle (1989) analyzes demand curve equilibrium in a financial market setting under private information. Utilizing the normality of value distributions, he derives equilibria in linear demand curves.

The main idea behind the solution is that analyzing each realization of her residual supply curve conditionally, each agent can utilize the other agents' information reflected in the market outcome. Mendelson and Tunca (2004a) utilize demand curve equilibrium to analyze liquidity in an industrial exchange with a monopolistic seller and examine the implications on forward contracting. Tunca (2004) uses demand curve equilibrium with private information to analyze the performance of a market where a single large buyer faces an oligopolistic set of sellers for procurement purposes. Technically, our approach in this paper is similar to Kyle (1989): Making the quantity decision conditional on the realization of the residual demand and supply curves leads to the construction of the ex-ante demand curves from ex-post optimal price-quantity pairs and hence (also as in Klemperer and Meyer 1989), the equilibrium curves are ex-post optimal as well as being ex-ante optimal. Note that demand curve equilibria are indeed implemented today in many markets for procurement, especially for energy. In wholesale electricity markets that operate in many countries, buyers and sellers are required to submit a sequence of price-quantity pairs which form their demand and supply curves and which are aggregated and intersected to reach a market clearing price (see Wilson 2002).

Our market clearing system can also be viewed as a multi-unit (more precisely divisible good) auction and can be implemented as such, i.e., the equilibrium can be implemented through a centralized auction in which a neutral auctioneer collects demand and supply curves from buyers and sellers, aggregates them and sets the market clearing price at the value that equates excess supply (or demand) to zero. From this point of view, our rate of convergence to efficiency results are related to the rate of convergence results in the double auction literature which mostly considers agents having private values and $0-1$ demands (see e.g., Gresik and Satterthwaite 1989, Satterthwaite and Williams 1989 and Rustichini et al. 1994). Vayanos (1999), utilizing the Kyle (1989) framework, considers the more realistic case of multi-unit demands in a dynamic setting and a financial market context. In his model, agents' private information is about individual endowments in contrast to the previous literature in which the private information is about personal valuations. Our model in this paper can be considered to extend on these by letting the agents have multi-unit demands, common values, correlated signals and interdependent utilities, albeit, in a stylized industrial market framework. Comparing our results on rate to efficiency studied in this literature, there are two noteworthy distinctions: First, in our model, the rates of convergence to price taking and efficiency (when there is convergence) are dependent on the rate at which the precision of information changes. Specifically, increasing the rate at which information precision decreases, decreases the rate of convergence. Second, the rate of convergence to efficiency is slower than that given in the double auction literature, that typically finds the convergence rate proportional to $M^{-2}$ (cf., Rustichini et al. 1994).[3] Further, depending on the rate at which information precision decreases with increased number of agents, this convergence can be arbitrarily slow. So our results also demonstrate that such markets that

[3]Note that the specifics of the social efficiency measure differs naturally because of the differences in the setups.

involve multi-unit demands, common values and correlated signals can converge to efficiency much slower than markets that do not have these features.

This paper is also related to the literature that explores strategic justification for price-taking behavior in large exchange economies (see, e.g., Roberts and Postlewaite 1976, Otani and Sicilian, 1982, 1990, Jackson 1992, Jackson and Manelli 1997, and Bonnisseau and Florig 2003). This branch of literature is mainly concerned about finding regularity conditions to guarantee the convergence of strategic equilibria of a sequence of economies to the competitive limit. Our model differs from this literature by examining an economy that has production (rather than pure exchange), and agents' having information asymmetry with correlated signals on underlying uncertainties on which the agents have common values. From this angle, however, we can say that the example family of economies that we give showing that decreasing information precision can cause full inefficiency with a large number of agents, suggests that for these types of extensions, the convergence of strategic equilibria to efficient outcomes is not immediate and requires more attention.

The rest of the paper is organized as follows: Section 2 introduces the model and gives the equilibrium outcome. Section 3 introduces the specifics of information precision decrease rate with increased number of agents, explores its role on price-taking and efficiency, and demonstrates the results supporting our claims about the price-taking behavior. Section 4 offers the concluding remarks. All proofs are in the appendix.

## 2 The Model

There are $N(\geq 2)$ manufacturers (buyers) and $M(\geq 2)$ suppliers (sellers) in the industry.[4] The suppliers provide an intermediate good to the manufacturers' production process. The manufacturers use the intermediate good as a (one-to-one) input and compete in the consumer market with the final product as Cournot oligopolists. All suppliers' products are identical and perfectly substitutable as an input for all the manufacturers.

The manufacturers face the demand curve $p_d = K + d - \sum_{i=1}^{N} q_i$ in the consumer market where $p_d$ is the price in the consumer market, $q_i$ is the quantity produced by the manufacturer $i$, $1 \leq i \leq N$, and $d$ is normally distributed with mean 0 and variance $\sigma_d^2$. The unit production costs are given as $c_0 + h$ where $h$ is normally distributed with mean 0 and variance $\sigma_h^2$. The random variables $d$ and $h$ are independent.

Before trading and subsequent production, the manufacturers have private signals about the state of the demand, $d$. Specifically, manufacturer $i$, $1 \leq i \leq N$, has a signal, $s_i^d = d + \varepsilon_i$ about $d$ where $\varepsilon_i$ are i.i.d. normally distributed with mean 0 and variance $\sigma_\varepsilon^2$. The suppliers'

[4]Throughout the paper I use the terms "manufacturer" and "buyer" interchangeably. Similarly the terms "supplier" and "seller" are used interchangeably.

unit marginal cost is uncertain, but each supplier has a noisy signal about its realization. For $1 \le j \le M$, supplier $j$ has the signal $s_j^c = h + u_j$ where $u_j$ are i.i.d. normally distributed with mean 0 and variance $\sigma_u^2$. The noise terms, $\{\varepsilon_i\}$ and $\{u_j\}$ are independent of $h$, $d$ and each other. For ease of exposition and without loss of generality, I normalize the manufacturers' production costs to zero. Also for notational convenience, define the parameter vector $\mathbf{v} = (\sigma_d^2, \sigma_h^2, \sigma_\varepsilon^2, \sigma_u^2, K, c_0)$.

Let $y_j$ be the quantity sold by supplier $j$, $j = 1, \ldots, M$, $q_i$ be the quantity bought (and produced) by the manufacturer $i$, $i = 1, \ldots, N$, and $p_e$ be the market clearing price for the intermediate good. Then, denoting the payoffs for the manufacturer and the suppliers, respectively, by (the random variables) $\Pi_{mi}$ for $i = 1, \ldots, N$, and $\Pi_{sj}$ for $j = 1, \ldots, M$, we have

$$\Pi_{mi} = q_i(K + d - \sum_{k=1}^{N} q_k - p_e) \tag{1}$$

and

$$\Pi_{sj} = y_j(p_e - (c_0 + h)), \tag{2}$$

for $i = 1, \ldots, N$ and $j = 1, \ldots, M$. Denote the total manufacturer surplus by $\Pi_m$ and total supplier surplus by $\Pi_s$, i.e., $\Pi_m = \sum_{i=1}^{N} \Pi_{mi}$ and $\Pi_s = \sum_{j=1}^{M} \Pi_{sj}$. Finally, denote the total quantity produced by $TQ = \sum_{i=1}^{N} q_i$ and the consumer surplus $(TQ(K + d - p_d)/2 = TQ^2/2)$ by $\Pi_c$.

## 2.1 Equilibrium Conditions

We study the Bayesian Nash equilibrium in this market. The strategies of the traders are demand and supply curves that they submit: Each supplier submits a supply curve indicating the quantity she is willing to sell at each price, and each manufacturer submits a demand curve indicating the quantity she is willing to buy at each price. The market clears at the price that equates the demand and the aggregate supply.

Since the participants have private information, their strategy curves are functions of their signals in addition to price. Denote the equilibrium demand schedule for supplier $j$, $j = 1, \ldots, M$, by $\mathcal{Y}_j : \mathbb{R}^2 \to \mathbb{R}$. That is, if supplier $j$ gets signal $s_j^c$, he will submit the supply curve $\mathcal{Y}_j(s_j^c; \cdot)$, which will indicate that at price $p$, she will supply $\mathcal{Y}_j(h; p)$ units in equilibrium. Similarly, the equilibrium demand schedule for manufacturer $i$ is $Q_i : \mathbb{R}^2 \to \mathbb{R}$. That is, given her signal $s_i^d$, manufacturer $i$ submits the demand curve $Q_i(s_i^d; \cdot)$, where $Q_i(s_i^d; p)$ is the amount she demands at price $p$. Denote the vector of equilibrium supply curves for the suppliers by $\mathcal{Y}$, and for each supplier $j$, denote the vector of equilibrium supply curves of the remaining suppliers by $\mathcal{Y}_{-j}$. Similarly, denote the vector of equilibrium demand curves for the manufacturers by $Q$, and for each manufacturer $i$, denote the equilibrium demand curves for the remaining manufacturers by $Q_{-i}$. Denote the equilibrium price by $p_e(\mathcal{Y}, Q)$, the equilibrium quantity that manufacturer $i$ ends up purchasing by $q_i(\mathcal{Y}, Q)$, and the equilibrium quantity that supplier $j$ ends up selling by $y_j(\mathcal{Y}, Q)$ (all random variables). Also define the *residual demand curve* facing supplier $j$ as the inverse of $\sum_{i=1}^{N} Q_i(s_i^d; p) - \sum_{k \neq j} \mathcal{Y}_k(s_k^c; p)$, and the *residual supply curve* facing the manufacturer as the inverse of $\sum_{j=1}^{M} \mathcal{Y}_j(s_j^c; p) - \sum_{k \neq i} Q_i(s_k^d; p)$ (both with respect to $p$).

The market clearing price, $p_e$, will be determined by solving

$$\sum_{i=1}^{N} Q_i(s_i^d; p_e) - \sum_{i=1}^{M} \mathcal{Y}_j(s_j^c; p_e) = 0, \tag{3}$$

and the allocations will be

$$q_i(\mathcal{Y}, Q) = Q_i(s_i^d; p_e) \text{ and } y_j(\mathcal{Y}, Q) = \mathcal{Y}_j(s_j^c; p_e). \tag{4}$$

Finally, for a manufacturer $i$, define $q_{-i}$ as the vector of quantities produced by the remaining manufacturers.

Given the above definitions, an equilibrium has to satisfy the following properties:

($i$) Given her signal, the trading strategies of the remaining manufacturers and those of the suppliers, each manufacturer's strategy maximizes her expected profit. That is, for each $i$, $1 \leq i \leq N$ and any alternative demand schedule $Q'$,

$$\begin{aligned} &E[\Pi_m(q_i(\mathcal{Y}, Q_i, Q_{-i}), q_{-i}(\mathcal{Y}, Q_i, Q_{-i}), p_e(\mathcal{Y}, Q_i, Q_{-i}))|s_i^d] \\ &\qquad \geq E[\Pi_m(q_i(\mathcal{Y}, Q_i', Q_{-i}), q_{-i}(\mathcal{Y}, Q_i', Q_{-i}), p_e(\mathcal{Y}, Q_i', Q_{-i}))|s_i^d]. \end{aligned} \tag{5}$$

($ii$) Given her signal and the other suppliers' and the manufacturer's strategies, each supplier maximizes her expected profit. That is, for any alternative supply schedule $\mathcal{Y}'$

$$E[\Pi_s(y_j(\mathcal{Y}, \mathcal{Y}_{-j}, Q), p_e(\mathcal{Y}, \mathcal{Y}_{-j}, Q))|s_j^c] \geq E[\Pi_s(y_j(\mathcal{Y}', \mathcal{Y}_{-j}, Q), p_e(\mathcal{Y}', \mathcal{Y}_{-j}, Q))|s_j^c]. \tag{6}$$

Taking advantage of the additive demand structure with normal distributions (hence the fact that the sum of the competitors' signals is a sufficient statistic for the unit marginal cost), an agent's optimization problem can be solved by conditioning on the realization of the residual supply or residual demand and finding the optimum quantity-price pair for each realization. The resulting optimal curve passes through all ex-post optimal price-quantity pairs.[5] This means that agents choose their price-quantity pairs by conditioning on each realization that they may be facing and optimizing point-wise for that realization. As a result, each realization of the residual supply curve will be associated with an optimal quantity and

[5]For more details, see Kyle (1989, pp. 325-332) who first presented a financial market equilibrium solution utilizing this approach.

the price corresponding to that quantity on that curve. The optimal ex-ante demand and supply curves submitted by the agents are constructed by tracing all these pairs, which are ex-post optimal. It follows that the agents will, in effect, be utilizing the information contained in the realization of the residual supply curve *without* actually observing that realization.[6]

Our analysis focuses on symmetric linear equilibria where $Q_i$ has the form

$$Q_i(s_i^d; p) = \alpha_{0i} + \alpha_s s_i^d + \alpha_p p, \tag{7}$$

and $\mathcal{Y}_j$ has the form

$$\mathcal{Y}_j(s_j^c; p) = \beta_{0i} + \beta_s s_j^c + \beta_p p, \tag{8}$$

with $\alpha_s$, $\alpha_p$, being the same for all $i$ and $\beta_s$ and $\beta_p$ being the same for all $j$.[7]

Note that linearity is not a constraint here. In equilibrium, given all other players' strategies, each player endogenously chooses to play a linear strategy. So the strategy space for the players are not restricted to linear curves in the equilibrium we find. Symmetry on $\alpha_s$, $\alpha_p$, $\beta_s$ and $\beta_p$ however, is a constraint for our solution and will be necessary for tractability.

One issue here is that "zero-trading" (i.e., $Q_i = 0$ and $\mathcal{Y}_j = 0$, for all $i$, $j$) is always a trivial symmetric linear equilibrium which is obviously not very interesting. So we would like to have $Q_i \neq 0$ and $\mathcal{Y}_j \neq 0$ and we will distinguish such equilibria as *trading* equilibria. Further, we will call an equilibrium *regular* if the individual demand curves slope down and the individual supply curves slope up. With the linear strategies defined as in (7) and (8), this will correspond to equilibria in which $\alpha_p < 0$ and $\beta_p > 0$ and we will focus on those equilibria.

To summarize, we will be exploring equilibria that satisfy all these conditions, i.e., we will be looking for *Regular Linear Symmetric Trading (Bayesian Nash) Equilibria*, which we will shortly denote as *RLSTE*.

## 2.2 A Measure of Price Taking

An important characteristic of the market and a measure of how close the agents are to being price-takers is the price impact of trades or the slope of the residual supply (or demand) curve: Each unit traded drives the market price *against* a trader, whether she is a buyer or a seller, and the price impact is the marginal price response to this trade. The smaller the market impact of her trades, the closer a participant is to being a price-taker.[8]

[6]Details on this are given in the proof of Proposition 1.

[7]Imposing symmetry on the constant coefficients $\alpha_0$ and $\beta_0$ is not necessary and hence we left them free here. In Proposition 1, we show that in the kind of equilibrium we are looking for, symmetry in these coefficients emerge endogenously. That is, $\alpha_{0i}$ turn out to be equal for all $i$ and $\beta_{0j}$ turn out to be equal for all $j$.

[8]The price impact of trades is also frequently used as a measure of market *illiquidity* in the finance literature (see, e.g., Kyle 1985, O'Hara 1997, Mendelson and Tunca 2004b).

In a linear symmetric trading equilibrium, for manufacturer $i$'s quantity sold $q_i$ and the corresponding price $p$, (3) and (4) imply

$$\sum_{k\neq i}\alpha_{0k}+\alpha_s\sum_{k\neq i}s_k^d+(N-1)\alpha_p p+\hat{q}_i=\sum_{j=1}^{M}\beta_{0j}+\beta_s\sum_{j=1}^{M}s_j^c+M\beta_p p, \tag{9}$$

so manufacturer $i$'s, residual supply curve satisfies

$$p=\lambda_0+\lambda_1 q_i \tag{10}$$

where

$$\lambda_0\triangleq\frac{\sum_{k\neq i}\alpha_{0k}+\alpha_s\sum_{k\neq i}s_k^d-\sum_{j=1}^{M}\beta_{0j}+\beta_s\sum_{j=1}^{M}s_j^c}{M\beta_p-(N-1)\alpha_p}\quad\text{and}\quad\lambda_1\triangleq\frac{1}{M\beta_p-(N-1)\alpha_p}\,. \tag{11}$$

That is, when a manufacturer tries to buy one additional unit, she drives the price up by the manufacturers' price impact coefficient $\lambda_1$. Similarly, for a given supplier $j$, $1\leq j\leq M$, realization of her residual demand curve, and her sold quantity $y_j$

$$(M-1)\beta_0+\beta_s\sum_{k\neq j}s_k^c-(M-1)\beta_p p+y_j=\sum_{i=1}^{N}\alpha_{0i}+\alpha_s\sum_{i=1}^{N}s_k^d+N\alpha_p p\,, \tag{12}$$

and hence, the residual demand curve facing supplier $j$ satisfies

$$p=\Lambda_0+\Lambda_1 y_j \tag{13}$$

where

$$\Lambda_0\triangleq\frac{(M-1)\beta_0+\beta_s\sum_{k\neq j}s_k^c-\sum_{i=1}^{N}\alpha_{0i}-\alpha_s\sum_{i=1}^{N}s_i^d}{N\alpha_p-(M-1)\beta_p}\quad\text{and}\quad\Lambda_1\triangleq\frac{1}{N\alpha_p-(M-1)\beta_p}\,. \tag{14}$$

Parallel to $\lambda_1$, suppliers' price impact coefficient $\Lambda_1$ represents the market impact of the suppliers' orders: For a given realization of her residual demand curve, an additional unit that a supplier decides to sell *decreases* the transaction price by $-\Lambda_1$ dollars.

Note that $\lambda_1>0$ and $\Lambda_1<0$, meaning that for any realization, every unit demanded by the manufacturer increases the clearing price (by $\lambda_1$), and every unit supplied by a supplier decreases the market clearing price (by $\Lambda_1$). Therefore, the absolute value of the corresponding market impact coefficient for each type of trader indicates how difficult it is for that type of trader to trade in this market. For instance, $\lambda_1$ being close to 0 would mean that the manufacturers are almost price-takers. On the other hand, $\lambda_1\to\infty$ means that the participants, are very far away from being price-takers and the trading costs are very large in the market prohibiting each trader from making large trades. Therefore, in this sense, magnitudes of $\lambda_1$ and $\Lambda_1$ measure the degree of the buyers' and the sellers' price-taking behavior respectively.

## 2.3 Bayesian Updates and the Equilibrium

By observing her cost signal and conditional on the realization of her residual demand curve, each supplier can infer the realization of the marginal cost, i.e., $h$. By (12), the information obtained conditional on the realization of the residual supply curve is equivalent to that obtained from the observation of $(\beta_s \sum_{k\neq j} s_k^c - \alpha_s \sum_{i=1}^N s_i^d)$. From the normality of the distributions, for a given supplier $j$, $1 \leq j \leq M$, we then can write the corresponding conditional expectation by the regression equation

$$E[h|\Lambda_0, s_j^c] = \nu_1 s_j^c + \nu_2(\beta_s \sum_{k\neq j} s_k^c - \alpha_s \sum_{i=1}^N s_i^d)\,, \tag{15}$$

where $\nu_1$ and $\nu_2$ are as given in Appendix A, equations (A.3) and (A.4).[9]

Similarly, observing her signal and conditional on the realization of her residual supply curve, manufacturer $i$ can make an inference about the state of the demand, $d$, and about the aggregate quantity bought and produced by all *other* manufacturers, which is identified by $\alpha_s \sum_{k\neq i} s_k^d$ (since the remaining terms in $\sum_{k\neq i} Q_k$ are deterministic for a given $p$). By (9), the information obtained about competitors' actions conditional on a realization of her residual supply curve is equivalent to the information conditional on a realization of $\alpha_s \sum_{k\neq i} s_k^d - \sum_{j=1}^M \beta_s s_j^c$. It again follows that in a linear symmetric trading equilibrium, we can write the corresponding conditional expectations using the regression equations:

$$E[d|s_i^d, \alpha_s \sum_{k\neq i} s_k^d - \sum_{j=1}^M \beta_s s_j^c] = \eta_1 s_i^d + \eta_2(\alpha_s \sum_{k\neq i} s_k^d - \sum_{j=1}^M \beta_s s_j^c)\,, \tag{16}$$

and

$$E[\alpha_s \sum_{k\neq i} s_k^d | s_i^d, \alpha_s \sum_{k\neq i} s_k^d - \sum_{j=1}^M \beta_s s_j^c] = \rho_1 s_i^d + \rho_2(\alpha_s \sum_{k\neq i} s_k^d - \sum_{j=1}^M \beta_s s_j^c)\,, \tag{17}$$

where, $\eta_1$, $\eta_2$, $\rho_1$ and $\rho_2$ are given in Appendix A, (A.1), (A.2), (A.5) and (A.6).

As can be seen from (15), each supplier has two pieces of information to use when deciding her quantity, namely signal and the realization of her residual demand curve. The coefficients $\nu_1$ and $\nu_2$ determine the relative weight that each supplier puts on those two sources when determining the optimal quantity to choose. Particularly, a high $\beta_s \sum_{k\neq j} s_k^c - \alpha_s \sum_{i=1}^N s_i^d$ referred from the market outcome, signals a high realization of the marginal cost and a low value referred signals the opposite. Notice that the accuracy of these two information sources are related and determined by the "noisiness" of the suppliers' signals, i.e., the relative magnitudes of $\sigma_u^2$ and $\sigma_h^2$. If $\sigma_u^2$ is small relative to $\sigma_h^2$, then the suppliers' signals are relatively

[9] Detailed derivations for these coefficients (as well as the corresponding ones for the manufacturers) are given in the proof of Proposition 1.

accurate and $\nu_1$ is high. This gets reflected on $\beta_s$, i.e., in such a case, the suppliers' quantities sold are strongly related to their signals and $\beta_s$ is high. Aggregated over all competitors' trades, this relation gets reflected to residual demand curve that a supplier is facing, or equivalently $\beta_s \sum_{k\neq j} s_k^c - \alpha_s \sum_{i=1}^{N} s_i^d$. That is, when suppliers' signals are informative, the residual supply curve each one of them faces will be more informative as well and the opposite will be true when their signals are not very accurate. This naturally affects the way the suppliers react to the realization of the residual demand. The coefficient $\nu_2$, which determines how sensitive they are towards unexpected quantity movements in the market adjusts accordingly determining how large $1/\beta_p$ is, which, in turn, determines how close the participants are to being price-takers.

This fundamental intuition will form the foundation of our arguments for the main results in the paper and based on that, the specific way information precision decreases as the number of agents increases plays a key role in determining the price-taking behavior in the limit. We will discuss these effects further in Section 3 when we get into the analysis of how the way information precision decreases with increased number of agents specifically determines the asymptotic market behavior.

We will now proceed to the solution for the equilibrium. The following proposition summarizes the necessary and sufficient conditions for an equilibrium for a given set of parameters.

**Proposition 1**
*Let $M$, $N$ and a parameter vector $\mathbf{v}$ be given and let $a(r)$, $b(r)$, $c(r)$, $d(r)$, $e(r)$, $f(r)$, $g(r)$ and $h(r)$ be as defined in Appendix A. A coefficient vector $((\alpha_{01},\ldots,\alpha_{0N}),(\beta_{01},\ldots,\beta_{0M}),\alpha_s,\beta_s,\alpha_p,\beta_p)$ constitutes the coefficients of a regular linear symmetric trading equilibrium strategy profile $\{Q,\mathcal{Y}\}$ as given in (7) and (8) if and only if*

*(a) There exists a root $r\in\mathbb{R}$, of the following polynomial equation*

$$\frac{(a(r)-c(r)-e(r))(h(r)(b(r)-(N-1))-d(r))}{(2-d(r)+b(r)+(N-1)h(r))(a(r)h(r)+e(r)-c(r))}=1 \tag{18}$$

*on the region*

$$r_l \triangleq -\sqrt{\frac{M(M-1)\sigma_u^2}{(M-2)N(N\sigma_d^2+\sigma_\varepsilon^2)}} < r < -\sqrt{\frac{(M-1)\sigma_u^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)}} \triangleq r_h\,, \tag{19}$$

*that satisfies*

$$a(r)-c(r)-e(r)>0\,. \tag{20}$$

*(b) For a root $r \in \mathbb{R}$ that satisfies the conditions in (a),*

$$\alpha_s = \frac{a(r) - c(r) - e(r)}{2 - d(r) + b(r) + (N-1)h(r)}, \tag{21}$$

$$\beta_s = \alpha_s / r, \tag{22}$$

$$\alpha_p = \frac{((M-1)g(r) + 1)\beta_s}{N f(r)}, \tag{23}$$

$$\beta_p = \frac{g(r)\beta_s}{f(r)}. \tag{24}$$

*Moreover,*

$$\Lambda_1 = f(r)/\beta_s \text{ and } \lambda_1 = e(r)/\alpha_s, \tag{25}$$

*and the constant coefficients satisfy*

$$\alpha_{0i} = \frac{K(M\nu_2 + \Lambda_1) + Mc_0(\eta_2 - \rho_2)}{(M\nu_2 + \Lambda_1)(N + 1 + \lambda_1((N-1)\alpha_p + 1)) + N\Lambda_1(\eta_2 - \rho_2)} \triangleq \alpha_0, \; 1 \le i \le N, \tag{26}$$

*and*

$$\beta_{0j} = \frac{N\alpha_0\nu_2 + c_0}{\Lambda_1 + M\nu_2} \triangleq \beta_0, \; 1 \le j \le M, \tag{27}$$

*where $\eta_2$ and $\rho_2$ are as given in Appendix A.*

This result will be the basis of our analysis and we will use it extensively when we derive our main conclusions in the next section.

# 3 Information Precision and Efficiency

We now consider the effect of division of information among the agents on market competitiveness as the number of agents becomes large. As the number of buyers or sellers increases, one would expect that the incentives to acquire information would decrease, and consequently the quality of information each agent has would decrease. To quantify this, I modify and extend the notion of *information precision* introduced in Kyle (1989). Formally, for a given signal (or collection of signals) $S$ on an underlying information variable $v$, define the *precision* of $S$ as

$$\tau_v(S) = \frac{1}{Var[v|S]}. \tag{28}$$

Notice that, by the signal structure given in Section 2 and the normality assumptions, we have

$$\tau_d(s_i^d) = \frac{\sigma_d^2 + \sigma_\varepsilon^2}{\sigma_d^2 \sigma_\varepsilon^2} \text{ and } \tau_h(s_j^c) = \frac{\sigma_h^2 + \sigma_u^2}{\sigma_h^2 \sigma_u^2}. \tag{29}$$

Further, the precision of the total information in the market for $d$ and $h$ will be

$$\tau_d(\{s_1^d,\ldots,s_N^d\}) = \frac{1}{\sigma_d^2} + \frac{N}{\sigma_\varepsilon^2} \text{ and } \tau_h(\{s_1^c,\ldots,s_M^c\}) = \frac{1}{\sigma_h^2} + \frac{M}{\sigma_u^2}, \tag{30}$$

respectively.

The question then becomes how individual signal precisions change with an increased number of sellers and buyers. Since the increased number of traders reduces incentives to acquire information by reducing the relative market power and hence the relative expected benefits of having information, the precision in each signal should decrease. This means increased noise in each trader's signal as the number of traders increases. When considering the change in precision of information as the number of agents increases, Kyle (1989) examines the special case where the total amount of private information in the market, stays constant and is divided equally among the participants. This implies that the individual noise variance in each signal is proportional to the number of agents that have a signal about the underlying variable (cf. Kyle 1989, p. 339).

I extend this notion by generalizing the way the total amount of information precision decreases with an increased number of agents in the market. Let

$$\sigma_\varepsilon^2(N) = \varphi_m(N)\sigma_{\varepsilon o}^2 \text{ and } \sigma_u^2(M) = \varphi_s(M)\sigma_{uo}^2, \tag{31}$$

where $\varphi_m$, $\varphi_s : \mathbb{N}_+ \to \mathbb{R}_+$ with

$$\lim_{N\to\infty} \varphi_m(N) \neq 0, \tag{32}$$

and

$$\lim_{M\to\infty} \varphi_s(M) \neq 0. \tag{33}$$

Define

$$\gamma_m = \inf\{\gamma : \lim_{N\to\infty} \frac{\varphi_m(N)}{N^\gamma} < \infty,\ \gamma \geq 0\}. \tag{34}$$

That is, for instance, if $\varphi_m(N) = N$, then the precision of signals scale proportionally as the number of manufacturers increases and $\gamma_m = 1$. Alternatively, if $\varphi_m(N) = 1$, then the signal precision of the individual manufacturers does not change as their number increases and $\gamma_m = 0$.

Similarly, define

$$\gamma_s = \inf\{\gamma : \lim_{M\to\infty} \frac{\varphi_s(M)}{M^\gamma} < \infty,\ \gamma \geq 0\}. \tag{35}$$

Given this information structure, I will examine the conjecture of price taking for three main cases. In Case I, the number of sellers will get large while the number of buyers will stay fixed. In Case II, the number of sellers will be fixed while the number of buyers will get large. Finally, in Case III, the number of buyers and sellers will both get large. For each case, I will examine three subcases of information division: The cases of the total information about the relevant uncertainty in the economy increasing ($0 \leq \gamma_s < 1$ and/or $0 \leq \gamma_m < 1$); staying the same ($\gamma_s = 1$ and/or $\gamma_m = 1$); and decreasing ($\gamma_s > 1$ and/or $\gamma_m > 1$) as the number of agents gets large.

### 3.1 Case I: Fixed number of Buyers as the Number of Sellers gets Large

I start by examining the case when the number of buyers is fixed and the number of sellers gets large. That is, I will be examining the equilibria in a sequence of economies when $N$ is fixed and as $M \to \infty$. For this case, by our construction, the amount of demand information in the economy will remain constant while the total precision of cost information will change with the number of suppliers, $M$, in one of the three ways described above. The following proposition presents the results for all three subcases:

**Proposition 2** *For a given* $\mathbf{v}$*,*

*(i) If* $0 \leq \gamma_s < 1$*, then there exists an* $\underline{M} \in \mathbb{N}$ *such that if* $M > \underline{M}$*, there exists a unique RLSTE for the economy. For any such sequence of equilibria as* $M \to \infty$*,* $\lim_{M\to\infty} \lambda_1 = 0$ *and* $\lim_{M\to\infty} \Lambda_1 = 0$*. That is, all participants in the market become price-takers in the limit. Further,* $\lim_{M\to\infty} \lambda_1 \cdot M^{\frac{\gamma_s - 1}{2}}$ *and* $\lim_{M\to\infty} -\Lambda_1 \cdot M^{\frac{\gamma_s - 1}{2}}$ *are positive constants, i.e., the agents become price-takers at a rate* $M^{\frac{\gamma_s - 1}{2}}$*.*

*(ii) If* $\gamma_s = 1$ *and* $\sigma_{uo}^2/\sigma_h^2$ *is sufficiently small, then there exists an* $\underline{M} \in \mathbb{N}$ *such that if* $M > \underline{M}$ *there exists a RLSTE. For any such sequence of equilibria as* $M \to \infty$*, there exist* $\lambda^* > 0$ *and* $\Lambda^* < 0$ *such that* $\lim_{M\to\infty} \lambda_1 = \lambda^*$ *and* $\lim_{M\to\infty} \Lambda_1 = \Lambda^*$*, i.e., no participant becomes a price-taker in the limit.*

*(iii) If* $\gamma_s > 1$*, then there exists an* $\overline{M} \in \mathbb{N}$ *such that if* $M > \overline{M}$ *an RLSTE does not exist.*

Part $(i)$ of Proposition 2 states that when the amount of information in the economy increases with the number of sellers, the conjecture of price-taking is supported in equilibrium. This is because, when $0 \leq \gamma_s < 1$, as the number of suppliers gets large, the signals of the sellers are still informative beyond a threshold and they react to their signals with a certain strength (i.e., $\beta_s$ converges to zero slowly). Aggregated over a large number of agents, this effect results in a certain variation in the market outcome being expected by the agents. Therefore, increasing the quantity traded has a limited signal value for the realization of costs and consequently a limited impact on the clearing price. As a result, in this case, the pressure from increased number of agents takes over. This manifests itself in the sum of the slopes of the supply curves submitted by the sellers and pushes towards price-taking, making the agents price-takers in the limit.

However, when the total amount of information stays the same, (i.e., when $\gamma_s = 1$), the price-taking conjecture fails. As stated in part (*ii*), increasing the number of sellers does not lead to price-taking. The reason for this is that as the number of sellers increases, this time each unexpected unit has a sizeable informational effect pushing the sellers away from becoming price-takers, and this effect is comparable to the effect of the increased number of sellers that pushes the sellers towards price-taking. As a result, the two effects balance each other in the limit, and no agent becomes a price-taker asymptotically.

Finally, part (*iii*) says that when the total precision of information in the market is decreasing with the number of sellers and when the number of sellers is beyond a certain threshold, a regular linear symmetric trading equilibrium does not exist. This is because, as the number of sellers increases in this case, the prices become very sensitive to the quantities traded, and beyond a certain point, an equilibrium of the type we are looking for ceases to exist. This does not mean, however, that no other equilibrium exists. This only means that the specific type of equilibrium we are looking for is silent for these market specifications. We will discuss this inexistence issue further in Section 3.3, where we will also obtain insights into what the equilibrium outcome might look like for such regions based on examples where a RLSTE exists in the limit as the number of agents becomes large.

Note that as given in part (*i*), as the number of sellers increases, the agents become price-takers at a rate $M^{\frac{\gamma_s-1}{2}}$, and the convergence can be arbitrarily small as $\gamma_s \to 1$. That is reducing the relative informativeness of the signals of the agents slows down the convergence to efficiency. We discuss this issue further in Section 3.3.

The contrast between parts (*i*) and (*ii*) of Proposition 2 also confirms our first claim: Whether the agents become price-takers or not depends on the specific way the information precision decreases as the number of agents gets large. Notice that when the number of sellers increases, agents from both sides become price-takers.

The results from this case will further provide us with a contrast point to the mirror-image case where the number of suppliers is fixed and the number of buyers gets large. We will see that the conjecture of price-taking is sensitive to which side of the market (buyer or seller side) has a large number of agents.

### 3.2 Case II: Fixed number of Sellers as the Number of Buyers gets Large

We now examine the case when $M$ is fixed and $N$ gets large. The following proposition presents the outcome:

**Proposition 3**

*(i) If* $0 \leq \gamma_m \leq 1$ *and* $\sigma_d^2$ *is sufficiently large, then there exists* $\underline{N} \in \mathbb{N}$ *such that when*

*$N > \underline{N}$, there exists a unique RLSTE. For any sequence of equilibria as $N \to \infty$, there exist $\lambda^* > 0$ and $\Lambda^* < 0$ such that $\lim_{N\to\infty} \lambda_1 = \lambda^*$ and $\lim_{N\to\infty} \Lambda_1 = \Lambda^*$. That is, as the number of buyers gets large, the participants do not become price-takers.*

*(ii) If $\gamma_m > 1$, then there exists an $\overline{N} > 0$ such that if $N > \overline{N} > 0$ a RLSTE does not exist.*

As can be seen from part (*i*) of Proposition 3, when the number of sellers is fixed and the number of buyers becomes large, even when the total amount of information in the market increases with increased number of buyers (i.e., when $0 \le \gamma_m < 1$), the participants do not become price-takers. This result is in contrast to the corresponding case with fixed number of buyers and number of sellers going to infinity, given in part (*i*) of Proposition 2, in Section 3.1. The reason why we observe such a difference here is that each buyer has a share in the order of $N^{-1}$ in the consumer market, and each one's optimal quantity as a function of $p_e$ is linear with a coefficient approximately proportional to $-1/N$ as $N$ gets large. As a result, the rate of change of the optimal total quantity for the manufacturers with the price of the intermediate good will asymptotically be a constant. Consequently, the participants will not be price-takers in the limit.

When the total precision of information is increasing with the number of buyers, i.e., when $\gamma > 1$, similar to the case stated in part (*iii*) of Proposition 2, the particular kind of equilibrium we are looking for is again silent. Again, this issue is discussed further in Section 3.3.

The contrast between part (*i*) of Proposition 2 and part (*i*) of Proposition 3 confirms our second claim that limiting price-taking behavior may depend on the side of the market, on which the number of participants becomes large. Notice that the interdependence of payoffs of the oligopolist manufacturers was crucial for this result. This suggests that in industrial markets with downstream competition that makes the buyers' payoffs naturally dependent on each other, having a large number of buyers does not necessarily imply that the participants will be price-takers. Finally, this also suggests that in order to have limiting price-taking behavior in such a market, the number of sellers may have to get large. In the next section, we examine the case with large numbers of buyers and sellers and explore the effects of increasing the number of agents on both sides of the market.

## 3.3 Case III: Both the Number of Sellers and the Number of Buyers get Large

Finally, we examine the case where the number of agents on both sides of the market get large. For brevity, we will focus on the case where the precision of information changes at the same rate in the limit for both the buyers and the sellers, and define $\gamma_o \triangleq \gamma_m = \gamma_s$. We will also take the limit for $M$ and $N$ when they lie on any given ray in the interior of $\mathbb{R}^2_+$.

This will allow us to demonstrate the necessary results for our purposes while keeping the analysis concise.

The following proposition presents the outcome:

**Proposition 4** *Let* $\mathbf{v}$ *be given and consider any path in* $\mathbb{N}_+^2$ *where* $\lim_{N,M\to\infty} \frac{N}{M} = k$ *for some* $k > 0$. *Then,*

*(i) If* $0 \leq \gamma_o < 1$, *then there exist* $\underline{N}$, $\underline{M} \in \mathbb{N}$ *such that if* $M > \underline{M}$, *there exists a unique RLSTE for the economy. For any such sequence of equilibria as* $M\to\infty$, $\lim_{N,M\to\infty} \lambda_1 = 0$ *and* $\lim_{N,M\to\infty} \Lambda_1 = 0$. *That is, all participants in the market become price-takers in the limit. Further,* $\lim_{N,M\to\infty} \lambda_1 \cdot M^{\frac{\gamma_o - 1}{2}}$ *and* $\lim_{N,M\to\infty} -\Lambda_1 \cdot M^{\frac{\gamma_o - 1}{2}}$ *are positive constants, i.e., the agents become price-takers at a rate* $M^{\frac{\gamma_o - 1}{2}}$.

*(ii) If* $\gamma_s = 1$ *and* $\sigma_{uo}^2/\sigma_h^2$ *is sufficiently small, then there exists an* $\underline{M} \in \mathbb{N}$ *such that if* $M > \underline{M}$, *there exists a RLSTE. For any such sequence of equilibria as* $N, M\to\infty$, *there exist* $\lambda^* > 0$ *and* $\Lambda^* < 0$ *such that* $\lim_{M\to\infty} \lambda_1 = \lambda^*$ *and* $\lim_{M\to\infty} \Lambda_1 = \Lambda^*$, *i.e., no participant becomes a price-taker in the limit.*

*(iii) When* $\gamma_o > 1$ *then*

*(a) If*

$$\frac{\sigma_d^2}{\sigma_h^2} = \sqrt{\frac{\sigma_{\varepsilon o}^2}{\sigma_{uo}^2} k^{\gamma - 1}}, \tag{36}$$

*and* $\sigma_{uo}^2/\sigma_h^2$ *is sufficiently small, then there exists an* $\underline{M} \in \mathbb{N}$ *such that if* $M > \underline{M}$, *there exists a RLSTE. For any such sequence of equilibria, as* $N, M\to\infty$, $\lim_{N,M\to\infty} \lambda_1 = \lim_{N,M\to\infty} -\Lambda_1 = \infty$.

*(b) Otherwise, there exists an* $\overline{M} \in \mathbb{N}$ *such that if* $M > \overline{M}$, *a RLSTE does not exist.*

Parts (*i*) and (*ii*) of Proposition 4 further demonstrate the sensitivity of the price-taking conjecture to the particular way the information precision decreases as the number of agents increases: As it was for the case with large number of sellers and fixed number of buyers, when the market becomes more informative with the addition of new agents, the agents become price-takers in the limit; however, when the total information in the market stays constant with the increased number of agents, the price-taking conjecture does not survive.

Part (*iii*)(*a*) of Proposition 4 confirms our third claim: When the total amount of information decreases with the increased number of agents, increasing the number of agents in the market can move the traders away from price taking. In fact, this part tells us that there exist sequences of economies where, as the number of agents in the market increases, the opposite extreme of price-taking happens in equilibrium, and the price impact of each unit traded in the market goes to infinity. This is noteworthy, and we will see shortly how this effect can cause significant inefficiencies in the market.

Part $(iii)(b)$ of Proposition 4 says that when the parameters do not satisfy condition (36), a linear, symmetric trading equilibrium does not exist when the number of agents in the market goes beyond a certain point. Part $(iii)(a)$ provides some insights for this: When the total precision of information in the market decreases with the number of agents, the agents start reacting strongly to unexpected quantities in the market, and the price becomes too sensitive to the quantity, making it very costly to trade in the limit. Unless a certain balance among the informational parameters is satisfied (i.e., equation (36)), there can be no regular linear symmetric trading equilibrium (note that zero-trading is always a trivial equilibrium). Once again, this is more a statement about the limitations of the specific class of equilibria (namely linear symmetric trading equilibria) that we are examining. Unfortunately, outside of the class of linear symmetric equilibria, we lose tractability. But it should be kept in mind that there may be other trading equilibria outside of this class as $N, M \to \infty$. The family of cases specified in part $(iii)(a)$ of Proposition 4, besides providing a class of examples in which increasing the number of agents in the market can lead to non-price taking behavior (and in which the market performance may suffer as we shall see), may also provide some insights of the characteristics of the non-linear equilibria in that region.

The effect of the failure of price taking in the market requires further attention. In particular, the effect can be severe for the case when $\lambda_1 \to \infty$ and $\Lambda_1 \to -\infty$, i.e., for $\gamma_o > 1$. The following proposition presents the results on this:

**Proposition 5** *Maintaining the existence of a sequence of RLSTE as* $M, N \to \infty$ *and* $\lim_{M,N\to\infty} N/M = k$,

*(i) If* $\gamma_o < 2$, *then* $E[p_e] \to c_0$ *and* $E[TQ] \to K - c_0$ *as* $N, M \to \infty$.

*(ii) If* $\gamma_o = 2$, *then for any* $k > 0$ *and* $p$ *satisfying* $c_0 < p < (kK + c_0)/(k+1)$, *one can find* $(\sigma_d^2, \sigma_h^2, \sigma_{uo}^2, \sigma_{\varepsilon o}^2) \in \mathbb{R}_+^4$, *such that* $E[p_e] \to p$, *as* $N, M \to \infty$. *When* $E[p_e] \to p$, $E[TQ] \to K - p - \frac{1}{k}(p - c_0)$. *Particularly, as* $p \to c_0$, $E[TQ] \to K - c_0$ *and as* $p \to (kK + c_0)/(k+1)$, $E[TQ] \to 0$.

*(iii) If* $\gamma_o > 2$, *then* $E[p_e] \to (kK + c_0)/(k+1)$ *and* $E[TQ] \to 0$ *as* $N, M \to \infty$.

Part $(i)$ of Proposition 5 tells that when the total information in the market increases or stays the same with the increased number of agents, the market is efficient. Further, it says that even when the total information decreases with the number of sellers in the market at a slow rate, the efficiency is preserved, although the participants move arbitrarily away from price-taking (i.e., even though $\lambda_1 \to \infty$ and $\Lambda_1 \to -\infty$). Parts $(ii)$ and $(iii)$ of Proposition 5 are noteworthy: When the total information in the market decreases sufficiently fast with increased number of sellers (i.e., when $\gamma_o \geq 2$), the increase in the price impact of the trades in the market can affect the market performance dramatically. Notice that in the limiting economy with no information, the only equilibrium that can be supported under price-taking is one that has $p_e = c_0$ and $TQ = K - c_0$. However, Proposition 5 demonstrates that not only the price-taking conjecture can fail for markets with private information, but also assuming price-taking behavior can lead to gross errors in predicting market performance and the allocation of surplus.

So how is the rate of convergence to efficiency affected by the rate at which information precision decreases? To see this, we examine the rate of convergence to efficiency for the cases in which the equilibrium outcome does converge to efficiency, i.e., for $\gamma_o < 2$. But first, we define the notion of *efficiency loss* in our context. For this, consider the first-best social optimum, in which, for any realization of $h$ and $d$, $TQ_{eff} = K + d - c_0 - h$. Then, denoting total social welfare (the sum of expected seller, buyer and consumer surplus) obtained with that quantity by $W_{eff}$, we have

$$E[W_{eff}] = E[\Pi_s + \Pi_m + \Pi_c] = E[TQ_{eff}(K + d - c_0 - h - \frac{1}{2}TQ_{eff})] = \frac{1}{2}((K - c_0)^2 + \sigma_d^2 + \sigma_h^2)\,. \tag{37}$$

Further, define the *efficiency loss* as

$$\mathcal{L} \triangleq E[W_{eff}] - E[TQ(K + d - c_0 - h - \frac{1}{2}TQ)]\,. \tag{38}$$

Now we can look at the rate of convergence to efficiency when there is such convergence, i.e., when $0 \leq \gamma_o < 1$. The following proposition presents this result.

**Proposition 6** *For $0 \leq \gamma_o < 1$ on any path such that $M \to \infty$ and $\lim_{M,N\to\infty} N/M = k$ for a $k > 0$, there exists a positive constant $\kappa$ such that*

$$\lim_{M,N\to\infty} \mathcal{L} \cdot M^{1-\gamma_o} = \kappa\,. \tag{39}$$

*That is, the efficiency loss is in the order of $M^{\gamma_o - 1}$.*

Proposition 6 states that the rate of decrease of the efficiency loss as the number of sellers and buyers gets large depends on the rate at which information precision decreases as the number of agents increases. The proposition states that when the rate of individual information precision decrease is low (i.e. when $\gamma_o$ is low), the equilibrium outcome converges to efficiency faster. This is also in alignment with the rate of convergence to price-taking, i.e., $M^{\frac{\gamma_o - 1}{2}}$, as given in part (*i*) of Proposition 4. On the other hand, as $\gamma_o$ approaches 1, convergence slows to a halt. This is further noteworthy because, considering the equivalence of the market clearing mechanism to a multi-unit double auction, this result states that with common values and correlated signals, the convergence rate can be significantly lower than that found in the unit-demand double auction literature with private values (e.g., Gresik and Satterthwaite 1989, Satterthwaite and Williams 1989 and Rustichini et al. 1994) and that found in multi-unit models with uncorrelated signals (e.g., see Vayanos 1999). Therefore, this result suggests that in multi-unit auctions with common values and correlated signals, the rate of convergence to efficiency can be very slow (in fact, arbitrarily slow as $\gamma_o$ converges to 1).

The convergence behavior is also consistent with the behavior of the market beyond $\gamma_o = 1$ and as $N$ and $M$ get large. When $\gamma_o$ is equal to or greater than 1, there will always be efficiency loss, since even when all the information that the agents have in the economy is aggregated, the true values of $d$ and $h$ (or that of $d - h$) cannot be known and the efficient total production quantity will not be achieved. Specifically, when $\gamma_o > 1$, even if the expected prices and quantities converge to the efficient levels as stated in Proposition 5 (as they do for $1 < \gamma_o < 2$), there is always a constant loss due to the total loss of information in the economy in the limit. At $\gamma_o = 2$, as stated in Proposition 5, prices and the quantities can be at any efficiency level depending on the parameters. Consequently, the limiting efficiency of the market can be at any level from maximum efficiency, ignoring the mandatory loss due to the informational loss, to full inefficiency.

Figure 1 demonstrates how the limiting equilibrium outcome changes with the "noise-to-signal ratio" $\sigma_{uo}^2/\sigma_h^2$ for the case $\gamma_o = 2$ for two different values of $k$. As can be seen from panel (a), as $\sigma_{uo}^2/\sigma_h^2$ varies, the expected price of the intermediate good moves from the fully efficient level (i.e., $c_0 = 5$) to less efficient levels and ultimately to the fully inefficient level specified by part ($b$) of Proposition 5. The near full price efficiency is obtained for low levels of base noise-to-signal ratio, and the price increases as the suppliers' information becomes less precise. Similarly, the expected quantity produced is at the efficient level when $\sigma_{uo}^2/\sigma_h^2$ is low and decreases rapidly as the ratio increases. Panel ($c$) shows that despite their large numbers both on the supplier and the buyer side, the industry participants make profits as a group in the limit. Considering in this case that the maximum total welfare is 113.5 (from equation (37)), it is noteworthy that the industry may capture a large percentage of the maximum possible surplus even if the number of competitors in each layer is very large, as can be seen from panel ($c$). Finally, in part ($d$), we can see that, there is a substantial welfare loss in the economy despite the large number of buyers and the sellers, which again is in sharp contrast to the outcome that would be predicted by a price-taking model in the limiting economy. Also note that increasing the relative concentration of the buyers compared to that of sellers (i.e., moving $k$ from 1 to 10 in the figure) tends to increase the clearing price of the intermediate good and the quantity produced as intuition would suggest. This turns out to decrease the total industry profits, as can be seen in panel ($c$), with most of the loss accounting for the reduced manufacturer profits. The effect on welfare of the relative concentration of buyers and sellers, on the other hand, is ambiguous as seen in panel ($d$). Overall, however, Figure 1 also highlights the fact that the relative concentration of buyers and sellers, as their numbers get large, can be an important determinant of market efficiency. Note that when $\gamma_o > 2$ however, as can be seen from part ($iii$) of Proposition 5, the only possible outcome in the

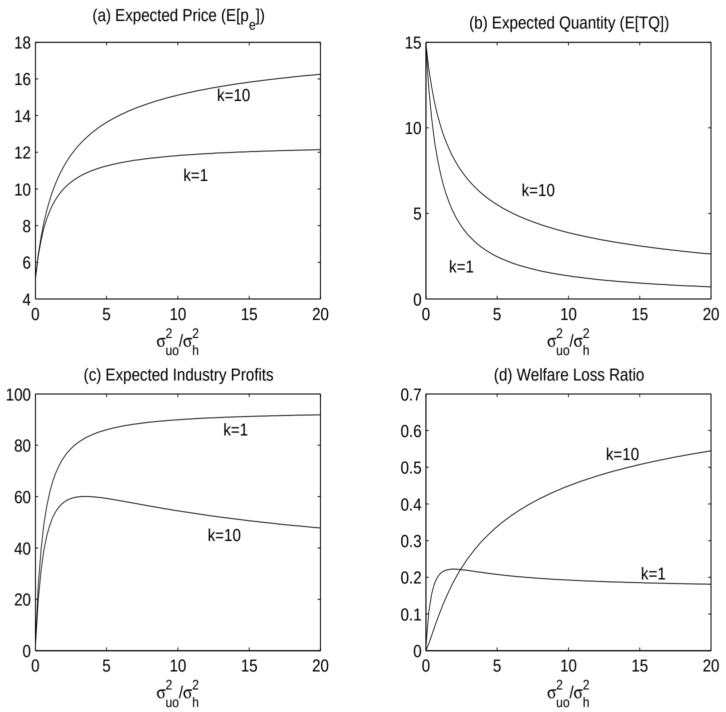


Figure 1: The numerically calculated equilibrium outcome for $\gamma_o = 2$ and $M = 10^6$ obtained by varying $\sigma^2_{uo}$ and $\sigma^2_{\varepsilon o}$ on the surface defined by $\frac{\sigma^2_d}{\sigma^2_h} = \sqrt{\frac{\sigma^2_{\varepsilon o}}{\sigma^2_{uo}} k^{\gamma_o - 1}}$, keeping $\sigma^2_d$ and $\sigma^2_h$ constant. The parameter values are $\sigma^2_d = \sigma^2_h = 1$, $K = 20$, $c_0 = 5$ and $N = M{\cdot}k$. Panel $(a)$ shows the expected price $(E[p_e])$, panel $(b)$ shows the expected total quantity $(E[TQ])$, panel $(c)$ shows the expected total industry profits $(E[\Pi_m + \Pi_s])$ and panel $(d)$ shows the welfare loss ratio $(\mathcal{L}/E[W_{eff}])$.

limit is full loss of efficiency.

An important point here is that these observations are in sharp contrast to what would have been predicted by price-taking assumptions in the limiting model in which the agents' signals have no information content: Consider a sequence of economies in our setting where the agents' signals are completely uninformative and the numbers of suppliers and manufacturers become large. In the unique price-taking equilibrium of each economy in the sequence, since the marginal cost of production is constant, the clearing price for the intermediate good has to be $c_0$. Further, as $N \to \infty$, the production quantity converges to $K - c_0$, the social surplus is maximized, and the suppliers and the manufacturers make zero profits. That is, an economy with a large number of agents behaving strategically and having very little information about the uncertainty can be arbitrarily close to being fully inefficient while such an economy as predicted by a price-taking analysis should achieve (near) full efficiency. Perhaps more strikingly, this deviation gets even larger as the economy moves informationally closer to the benchmark no-information case (i.e., as $\gamma_o$ increases). A critical observation here is that individual information precision can be the key determinant in this outcome and particularly, agents' having very inaccurate private information (compared to their having no information or accurate information) can result in very inefficient outcomes.

## 4 Concluding Remarks

In this paper, I explored the role of information precision on price-taking and efficiency of large industrial markets with downstream competition. I stated three claims and demonstrated support for them using a model of an industrial market with private information. First, whether the agents become price-takers in the limit depends on how the total precision of the information in the economy changes with increased number of agents. Second, in industrial markets with downstream competition, whether the agents become price-takers or not depends on the type of the agents (buyers or sellers) whose number is getting large. In particular, if the number of buyers becomes large, the interdependence of the utilities of the buyers can prevent the agents from being price-takers even when the number of buyers approaches infinity. Third, I claim that, with information effects, increasing the number of agents can result in agents' moving further away from price-taking. In fact, I demonstrated that this effect can be extremely severe to make even the smallest quantities have a very large impact on the market price and this can affect the market performance dramatically. Our results also support the claim that in multi-unit double-auctions with common values and correlated signals, the rate of convergence to efficiency depends on the precision of the agents' information and can be significantly slower than that is predicted by the studies on the single unit double auctions with private values.

In my analysis, I focused on symmetric linear equilibria and normal distributions. Further, I assumed a linear end consumer demand curve. These assumptions are necessary for tractability and are common in analysis of markets with information asymmetry with agents having correlated signals and multi-unit demands.[10] The in-depth analysis of such markets under private information is nearly impossible under more general distributions and demand functions. Importantly, although I believe that the main nature of the results will be sustained for more general distributions and demand curves as well, my main goal in this paper is to demonstrate how precision of private information in large industrial markets can affect price-taking behavior and consequently how the performance of the market can be affected

[10]See, e.g., Novshek and Sonnenschein (1982), Vives (1984), Gal-Or (1985), (1986), Li (1985), Shapiro (1986), Raith (1996) and Jin (2000) for a sequence of papers that utilize similar assumptions to analyze the information sharing problem in oligopoly as well as Vayanos (1999) and O'Hara (1997) for such models relating to financial markets among numerous other sources.

substantially. Future work that further explores and extends on these results can help shed more light on the price and surplus formation in large industrial markets with information asymmetry.

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# Appendix

## A Mathematical Preliminaries and Definitions

For given $\alpha_s$, $\beta_s$, let

$$\eta_1(\alpha_s,\beta_s) = \frac{\sigma_d^2(\alpha_s^2(N-1)\sigma_\varepsilon^2 + M(M\sigma_h^2+\sigma_u^2)\beta_s^2)}{(N-1)(N\sigma_d^2+\sigma_\varepsilon^2)\sigma_\varepsilon^2\alpha_s^2 + M(M\sigma_h^2+\sigma_u^2)(\sigma_d^2+\sigma_\varepsilon^2)\beta_s^2}\,, \tag{A.1}$$

$$\eta_2(\alpha_s,\beta_s) = \frac{(N-1)\sigma_d^2\sigma_\varepsilon^2\alpha_s}{(N-1)(N\sigma_d^2+\sigma_\varepsilon^2)\sigma_\varepsilon^2\alpha_s^2 + M(M\sigma_h^2+\sigma_u^2)(\sigma_d^2+\sigma_\varepsilon^2)\beta_s^2}\,, \tag{A.2}$$

$$\nu_1(\alpha_s,\beta_s) = \frac{\sigma_h^2(\beta_s^2(M-1)\sigma_u^2 + N(N\sigma_d^2+\sigma_\varepsilon^2)\alpha_s^2)}{(M-1)(M\sigma_h^2+\sigma_u^2)\sigma_u^2\beta_s^2 + N(N\sigma_d^2+\sigma_\varepsilon^2)(\sigma_h^2+\sigma_u^2)\alpha_s^2}\,, \tag{A.3}$$

$$\nu_2(\alpha_s,\beta_s) = \frac{(M-1)\sigma_h^2\sigma_u^2\beta_s}{(M-1)(M\sigma_h^2+\sigma_u^2)\sigma_u^2\beta_s^2 + N(N\sigma_d^2+\sigma_\varepsilon^2)(\sigma_h^2+\sigma_u^2)\alpha_s^2}\,, \tag{A.4}$$

$$\rho_1(\alpha_s,\beta_s) = \frac{\alpha_s\beta_s^2 M(M\sigma_h^2+\sigma_u^2)(N-1)\sigma_d^2}{(N-1)(N\sigma_d^2+\sigma_\varepsilon^2)\sigma_\varepsilon^2\alpha_s^2 + M(M\sigma_h^2+\sigma_u^2)(\sigma_d^2+\sigma_\varepsilon^2)\beta_s^2}\,, \tag{A.5}$$

and

$$\rho_2(\alpha_s,\beta_s) = \frac{\alpha_s^2(N-1)\sigma_\varepsilon^2(N\sigma_d^2+\sigma_\varepsilon^2)}{(N-1)(N\sigma_d^2+\sigma_\varepsilon^2)\sigma_\varepsilon^2\alpha_s^2 + M(M\sigma_h^2+\sigma_u^2)(\sigma_d^2+\sigma_\varepsilon^2)\beta_s^2}\,. \tag{A.6}$$

Also define

$$a(r) = \frac{\sigma_d^2(r^2(N-1)\sigma_\varepsilon^2 + M(M\sigma_h^2+\sigma_u^2))}{(N-1)(N\sigma_d^2+\sigma_\varepsilon^2)\sigma_\varepsilon^2 r^2 + M(M\sigma_h^2+\sigma_u^2)(\sigma_d^2+\sigma_\varepsilon^2)}\,, \tag{A.7}$$

$$b(r) = \frac{M(M\sigma_h^2+\sigma_u^2)(N-1)\sigma_d^2}{(N-1)(N\sigma_d^2+\sigma_\varepsilon^2)\sigma_\varepsilon^2 r^2 + M(M\sigma_h^2+\sigma_u^2)(\sigma_d^2+\sigma_\varepsilon^2)}\,, \tag{A.8}$$

$$c(r) = \frac{r^2(N-1)\sigma_d^2\sigma_\varepsilon^2}{(N-1)(N\sigma_d^2+\sigma_\varepsilon^2)\sigma_\varepsilon^2 r^2 + M(M\sigma_h^2+\sigma_u^2)(\sigma_d^2+\sigma_\varepsilon^2)}\,, \tag{A.9}$$

$$d(r) = \frac{r^2(N-1)\sigma_\varepsilon^2(N\sigma_d^2+\sigma_\varepsilon^2)}{(N-1)(N\sigma_d^2+\sigma_\varepsilon^2)\sigma_\varepsilon^2 r^2 + M(M\sigma_h^2+\sigma_u^2)(\sigma_d^2+\sigma_\varepsilon^2)}\,, \tag{A.10}$$

$$f(r) = \frac{\sigma_h^2 N(N\sigma_d^2+\sigma_\varepsilon^2)r^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)(\sigma_h^2+\sigma_u^2)r^2 + (M-1)\sigma_u^2(M\sigma_h^2+\sigma_u^2)}\,, \tag{A.11}$$

$$g(r) = \frac{2(M-1)\sigma_u^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)r^2 + (M-1)\sigma_u^2} - 1\,, \tag{A.12}$$

$$e(r) = \frac{Nf(r)r}{(M+N-1)g(r) - (N-1)}\,, \tag{A.13}$$

$$h(r) = \frac{(M-1)g(r)+1}{(M+N-1)g(r) - (N-1)}\,. \tag{A.14}$$

Finally, define the relationship "$\sim$" as follows.

**Definition:** Given two functions, $\omega$ and $\zeta$ mapping vectors of variables $(v, Z) = (v, Z_1, ..., Z_k)$, where $v \in \mathbb{R}^l$ and $l, k, Z_i \in \mathbb{N}^+$, to $\mathbb{R}$, denote

$$\omega \sim \zeta \text{ as } Z_1, \ldots, Z_k \to \infty \text{ if } \lim_{Z_1,\ldots,Z_k \to \infty} \frac{\omega(v, Z)}{\zeta(v, Z)} = 1. \tag{A.15}$$

# B Proofs of Propositions

**Proof of Proposition 1:** We start with necessity. Given other players' strategies, manufacturer $i$'s problem can be written as

$$\max_{Q_i} E[\Pi(Q_i, Q_{-i}, \mathcal{Y})|s_i^d] \equiv \max_{Q} E[E[\Pi(Q_i, Q_{-i}, \mathcal{Y})|\lambda_0, s_i^d]] \,, \tag{B.1}$$

where $\lambda_0$ is as given in (11). Given (9) and this problem can be solved pointwise. That is for any given realization of her residual supply curve, manufacturer $i$ solves

$$\max_{q_i} E[q_i(K + d - \sum_{k \neq i} (\alpha_{0i} + \alpha_s s_k^d + \alpha_p(\lambda_0 + \lambda_1 q_i)) - q_i - (\lambda_0 + \lambda_1 q_i))|\lambda_0, s_i^d] \,, \tag{B.2}$$

where $\lambda_1$ is again as given in (11). The first order condition is:

$$K + E[d|\lambda_0, s_i^d] - \sum_{k \neq i} \alpha_{0k} - E[\alpha_s \sum_{k \neq i} s_k^d|\lambda_0, s_i^d] - (N-1)\alpha_p \lambda_0 \\ - 2(N-1)\lambda_1 \alpha_p q_i - 2q_i \lambda_0 - 2\lambda_1 q_i = 0 \,, \tag{B.3}$$

and the second order condition is $1 + ((N-1)\alpha_p + 1)\lambda_1 > 0$. Notice that, by (9) and (11), the information contained in $\lambda_0$ is equivalent to $\alpha_s \sum_{k \neq i} s_k^d - \sum_{j=1}^M \beta_s s_j^c$. Now $d$, $s_i^d$ and $\alpha_s \sum_{k \neq i} s_k^d - \sum_{j=1}^M \beta_s s_j^c$ are jointly normally distributed with means zero and the covariance matrix

$$\begin{bmatrix} \sigma_d^2 & & \\ \sigma_d^2 & \sigma_d^2 + \sigma_\varepsilon^2 & \\ \alpha_s(N-1)\sigma_d^2 & \alpha_s(N-1)\sigma_d^2 & \alpha_s^2(N-1)((N-1)\sigma_d^2 + \sigma_\varepsilon^2) + M\beta_s^2(M\sigma_h^2 + \sigma_u^2) \end{bmatrix} . \tag{B.4}$$

Similarly, $\alpha_s \sum_{k \neq i} s_k^d$, $s_i^d$ and $\alpha_s \sum_{k \neq i} s_k^d - \sum_{j=1}^M \beta_s s_j^c$ are jointly normally distributed with means zero and the covariance matrix

$$\begin{bmatrix} \alpha_s^2(N-1)((N-1)\sigma_d^2 + \sigma_\varepsilon^2) & & \\ \alpha_s(N-1)\sigma_d^2 & \sigma_d^2 + \sigma_\varepsilon^2 & \\ \alpha_s^2(N-1)((N-1)\sigma_d^2 + \sigma_\varepsilon^2) & \alpha_s(N-1)\sigma_d^2 & \alpha_s^2(N-1)((N-1)\sigma_d^2 + \sigma_\varepsilon^2) + M\beta_s^2(M\sigma_h^2 + \sigma_u^2) \end{bmatrix} . \tag{B.5}$$

From this, it follows that

$$E[d|s_i^d, \alpha_s \sum_{k\neq i} s_k^d - \sum_{j=1}^{M} \beta_s s_j^c] = \eta_1 s_i^d + \eta_2(\alpha_s \sum_{k\neq i} s_k^d - \sum_{j=1}^{M} \beta_s s_j^c)\,, \tag{B.6}$$

where

$$\begin{aligned}
\begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix} &= \begin{bmatrix}\sigma_d^2\\ \sigma_d^2\end{bmatrix} \cdot \begin{bmatrix}\sigma_d^2+\sigma_\varepsilon^2 & \alpha_s(N-1)\sigma_d^2\\ \alpha_s(N-1)\sigma_d^2 & \alpha_s^2(N-1)((N-1)\sigma_d^2+\sigma_\varepsilon^2)+M\beta_s^2(M\sigma_h^2+\sigma_u^2)\end{bmatrix}^{-1}\\
&= \frac{1}{(N-1)(N\sigma_d^2+\sigma_\varepsilon^2)\sigma_\varepsilon^2\alpha_s^2+M\beta_s^2(M\sigma_h^2+\sigma_u^2)(\sigma_d^2+\sigma_\varepsilon^2)}\\
&\qquad \cdot \begin{bmatrix}\sigma_d^2(\alpha_s^2(N-1)\sigma_\varepsilon^2+M\beta_s^2(M\sigma_h^2+\sigma_u^2))\\ (N-1)\sigma_d^2\sigma_\varepsilon^2\alpha_s\end{bmatrix}, 
\end{aligned} \tag{B.7}$$

and

$$E[\alpha_s \sum_{k\neq i} s_k^d | s_i^d, \alpha_s \sum_{k\neq i} s_k^d - \sum_{j=1}^{M} \beta_s s_j^c] = \rho_1 s_i^d + \rho_2(\alpha_s \sum_{k\neq i} s_k^d - \sum_{j=1}^{M} \beta_s s_j^c)\,, \tag{B.8}$$

where

$$\begin{aligned}
\begin{bmatrix}\rho_1\\ \rho_2\end{bmatrix} &= \begin{bmatrix}\alpha_s^2(N-1)((N-1)\sigma_d^2+\sigma_\varepsilon^2)\\ \alpha_s(N-1)\sigma_d^2\end{bmatrix}\\
&\qquad \cdot \begin{bmatrix}\sigma_d^2+\sigma_\varepsilon^2 & \alpha_s(N-1)\sigma_d^2\\ \alpha_s(N-1)\sigma_d^2 & \alpha_s^2(N-1)((N-1)\sigma_d^2+\sigma_\varepsilon^2)+M\beta_s^2(M\sigma_h^2+\sigma_u^2)\end{bmatrix}^{-1}\\
&= \frac{1}{(N-1)(N\sigma_d^2+\sigma_\varepsilon^2)\sigma_\varepsilon^2\alpha_s^2+M\beta_s^2(M\sigma_h^2+\sigma_u^2)(\sigma_d^2+\sigma_\varepsilon^2)}\\
&\qquad \cdot \begin{bmatrix}\alpha_s M\beta_s^2(M\sigma_h^2+\sigma_u^2)(N-1)\sigma_d^2\\ \alpha_s^2(N-1)\sigma_\varepsilon^2(N\sigma_d^2+\sigma_\varepsilon^2)\end{bmatrix}, 
\end{aligned} \tag{B.9}$$

as given in (A.1), (A.2), (A.5) and (A.6). Therefore, by (B.3), it follows that

$$\begin{aligned}
K + (\eta_1-\rho_1)s_i^d + (\eta_2-\rho_2)(\sum_{j=1}^{M}\beta_{0j} - \sum_{k\neq i}\alpha_{0k} + p(M\beta_p-(N-1)\alpha_p) - x_i)\\
-\sum_{k\neq i}\alpha_{0k} - (N-1)\alpha_p p - (N-1)\lambda_1\alpha_p q_i - 2x_i - \lambda_0 - 2\lambda_1 x_i = 0\,.
\end{aligned} \tag{B.10}$$

From (10) and (B.10), we then have

$$\begin{aligned}
\alpha_{0i} &= \frac{K+(\eta_2-\rho_2)\sum_{j=1}^{M}\beta_{0j} - ((\eta_2-\rho_2)+1)\sum_{k\neq i}\alpha_{0k}}{2+\eta_2-\rho_2+\lambda_1(1+(N-1)\alpha_p)},\ 1\le i\le N, && \text{(B.11)}\\
\alpha_s &= \frac{\eta_1-\rho_1}{2+\eta_2-\rho_2+\lambda_1(1+(N-1)\alpha_p)}\,, && \text{(B.12)}
\end{aligned}$$

and

$$\alpha_p = \frac{M(\eta_2 - \rho_2)\beta_p - ((\eta_2 - \rho_2) + 1)(N-1)\alpha_p - 1}{2 + \eta_2 - \rho_2 + \lambda_1(1 + (N-1)\alpha_p)}\,. \tag{B.13}$$

We next solve the suppliers' problem. Similar to the manufacturers, suppliers' problem can also be solved pointwise. Hence, for a given realization of her residual demand curve, supplier $j$ solves

$$\max_{y_j} E[y_j(\Lambda_0 + \Lambda_1 y_j - c_0 - h)|\Lambda_0, s_j^h] \tag{B.14}$$

where $\Lambda_0$ and $\Lambda_1$ are given as in (14). The first order condition is

$$\Lambda_0 + 2\Lambda_1 y_j - c_0 - E[h|\Lambda_0, s_j^c] = 0\,, \tag{B.15}$$

and the second order condition is $\Lambda_1 < 0$. Now notice that, by (12) and (14), the information contained in $\Lambda_0$ is equivalent to $\beta_s \sum_{k\neq i} s_k^c - \alpha_s \sum_{i=1}^N s_i^d$. Further, $h$, $s_j^c$ and $\beta_s \sum_{k\neq i} s_k^c - \alpha_s \sum_{i=1}^N s_i^d$ are jointly normally distributed with means zero and the covariance matrix

$$\begin{bmatrix} \sigma_h^2 & & \\ \sigma_h^2 & \sigma_h^2 + \sigma_u^2 & \\ \beta_s(M-1)\sigma_h^2 & \beta_s(M-1)\sigma_h^2 & \beta_s^2(M-1)((M-1)\sigma_h^2 + \sigma_u^2) + \alpha_s^2\sigma_d^2 \end{bmatrix}. \tag{B.16}$$

Hence

$$E[h|\Lambda_0, s_j^c] = \nu_1 s_j^c + \nu_2(\beta_s \sum_{k\neq j} s_k^c - \alpha_s \sum_{i=1}^N s_i^d)\,, \tag{B.17}$$

where

$$\begin{aligned} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} &= \begin{bmatrix} \sigma_h^2 \\ \sigma_h^2 \end{bmatrix} \cdot \begin{bmatrix} \sigma_h^2 + \sigma_u^2 & \beta_s(M-1)\sigma_h^2 \\ \beta_s(M-1)\sigma_h^2 & \beta_s^2(M-1)((M-1)\sigma_h^2 + \sigma_u^2) + N\alpha_s^2(N\sigma_d^2 + \sigma_\varepsilon^2) \end{bmatrix}^{-1} \\ &= \frac{1}{(M-1)(M\sigma_h^2 + \sigma_u^2)\sigma_u^2\beta_s^2 + N\alpha_s^2(N\sigma_d^2 + \sigma_\varepsilon^2)(\sigma_h^2 + \sigma_u^2)} \\ &\qquad \cdot \begin{bmatrix} \sigma_h^2(\beta_s^2(M-1)\sigma_u^2 + N\alpha_s^2(N\sigma_d^2 + \sigma_\varepsilon^2)) \\ (M-1)\sigma_h^2\sigma_u^2\beta_s \end{bmatrix}. \end{aligned} \tag{B.18}$$

That is, $\nu_1$ and $\nu_2$ are as given in (A.3) and (A.4). Therefore, from (B.15) and (B.17) we have

$$\Lambda_0 + 2\Lambda_1 y_j - c_0 - \nu_1 s_j^c - \nu_2(\sum_{i=1}^N \alpha_{0i} - \sum_{k\neq j} \beta_{0k} + p(N\alpha_p - (M-1)\beta_p) - y_j) = 0. \tag{B.19}$$

Then by (13) and (B.19) we obtain

$$\beta_{0j} = \frac{(\sum_{i=1}^N \alpha_{0i} - \sum_{k\neq j} \beta_{0k})\nu_2 + c_0}{\Lambda_1 + \nu_2},\ 1 \leq j \leq M, \tag{B.20}$$

$$\beta_s = \frac{\nu_1}{\Lambda_1 + \nu_2} \tag{B.21}$$

and

$$\beta_p = \frac{(N\alpha_p - (M-1)\beta_p)\nu_2 - 1}{\Lambda_1 + \nu_2}\,. \tag{B.22}$$

From (14) and (B.22) we have

$$\beta_p = \frac{\nu_2 - \Lambda_1}{\Lambda_1(\Lambda_1 + \nu_2)}. \tag{B.23}$$

Combining (B.23) with (B.21), defining $r = \alpha_s/\beta_s$, and using the definitions in Appendix A, we find that

$$\Lambda_1 = \frac{\nu_1 - \beta_s \nu_2}{\beta_s} = \frac{f(r)}{\beta_s}\,, \tag{B.24}$$

from which it follows that

$$\beta_p = \frac{g(r)}{\Lambda_1} = \frac{g(r)\beta_s}{f(r)}. \tag{B.25}$$

Substituting the definition of $\Lambda_1$ into (B.25) we obtain

$$\alpha_p = \frac{((M-1)g(r)+1)\beta_s}{Nf(r)} \tag{B.26}$$

which, in turn, yields $\lambda_1 = e(r)/\alpha_s$. Substituting this into (B.12) and solving for $\alpha_s$ we obtain

$$\alpha_s = \frac{a(r)h(r) - c(r) + e(r)}{(b(r) - (N-1))h(r) - d(r)}. \tag{B.27}$$

Similarly from (B.13), we obtain

$$\alpha_s = \frac{a(r) - e(r) - c(r)}{2 + b(r) - d(r) + (N-1)h(r)}. \tag{B.28}$$

Hence, we find that equilibrium $r$ must solve

$$\phi(r) = 1\,, \tag{B.29}$$

where $\phi(r)$ is given by the left hand side of (18). In addition, from (B.25), it follows that for $\beta_p > 0$, $g(r) < 0$ must hold, which implies

$$r^2 > \frac{(M-1)\sigma_u^2}{N(N\sigma_d^2 + \sigma_\varepsilon^2)}\,. \tag{B.30}$$

The condition $\alpha_p < 0$ implies $(M-1)g(r) + 1 > 0$, which in turn yields

$$r^2 < \frac{M(M-1)\sigma_u^2}{(M-2)N(N\sigma_d^2 + \sigma_\varepsilon^2)}. \tag{B.31}$$

Since $\alpha_s > 0$ and $\beta_s < 0$ has to hold, $r < 0$ and we conclude that $r$ has to be in the region specified by (19). Further, $(M-1)g(r) + 1 > 0$ implies $-1/(M-1) < g(r) < 0$ which in turn implies $-1/(N-1) < h(r) < 0$. Combining this with (B.28) and since $d(r) < 1$, it follows that for $\alpha_s > 0$ (20) must be satisfied. Finally, solving the system of $M + N$ linear equations in as many variables described by (B.11) and (B.20) gives (26) and (27). This proves necessity.

Conversely, suppose a set of coefficients satisfies the conditions described in the proposition. Then, since (19) is satisfied, $r < 0$ holds. By (20), $\alpha_s > 0$ and hence $\beta_s < 0$. Again by (19), $g(r) < 0$ and $(M-1)g(r) + 1 > 0$, we obtain $\beta_p > 0$ and $\alpha_p < 0$ and $e(r) > 0$. Since $\alpha_s > 0$, we then have $\lambda_1 > 0$. Further, $-1/(N-1) < h(r) = \alpha_p \lambda_1 < 0$, which implies $(N-1)\alpha_p\lambda_1 + 1 > 0$. Therefore the second order condition for the manufacturers is satisfied. Also, $\beta_s < 0$ implies $\Lambda_1 < 0$, which means that the second order condition for the suppliers is also satisfied. Following along the lines given above for the proof of necessity, the candidate solution also satisfies the first order conditions. This shows sufficiency and completes the proof . ■

**Proof of Proposition 2:** To see part ($i$), we first examine the behavior of (18) over the relevant region specified by (19) as $M \to \infty$. Plugging in $\sigma_u^2 = M^{\gamma_s}\sigma_{uo}^2$ and taking the limit as $M \to \infty$ in (19), we see that for any $r$ that satisfies (19)

$$r \sim -M^{\frac{1+\gamma_s}{2}} \sqrt{\frac{\sigma_{uo}^2}{N(N\sigma_d^2 + \sigma_\varepsilon^2)}} \quad \text{as } M \to \infty\,, \tag{B.32}$$

where the relation "$\sim$" is as defined in Appendix A. Therefore, taking the limit as $M \to \infty$, we then can see that

$$a(r) \sim \frac{\sigma_d^2}{\sigma_d^2 + \sigma_\varepsilon^2}\,, \tag{B.33}$$

$$b(r) \sim \frac{(N-1)\sigma_d^2}{\sigma_d^2 + \sigma_\varepsilon^2}\,, \tag{B.34}$$

$$c(r) \sim \frac{\sigma_{uo}^2 (N-1)\sigma_d^2\sigma_\varepsilon^2}{N(N\sigma_d^2 + \sigma_\varepsilon^2)\sigma_h^2(\sigma_d^2 + \sigma_\varepsilon^2)} M^{\gamma_s - 1}\,, \tag{B.35}$$

$$d(r) \sim \frac{\sigma_{uo}^2 (N-1)\sigma_\varepsilon^2}{N\sigma_h^2(\sigma_d^2 + \sigma_\varepsilon^2)} M^{\gamma_s - 1}\,, \tag{B.36}$$

and

$$f(r) \sim M^{-1}\,, \quad \text{as } M \to \infty. \tag{B.37}$$

Also, on the range specified by (19), $g(r)$ moves from $-1/(M-1)$ to 0 and $h(r)$ moves from 0 to $-1/(N-1)$ monotonically. Further, note that $-\frac{M}{M-1} < Mg(r) < 0$, which implies that

$$-1 < \lim_{M \to \infty} Mg(r) < 0\,. \tag{B.38}$$

It follows that

$$e(r) \sim \sigma_{uo}^2 \sqrt{\frac{\sigma_{uo}^2}{N(N\sigma_d^2 + \sigma_\varepsilon^2)}} M^{\frac{\gamma_s - 1}{2}} \quad \text{as } M \to \infty. \tag{B.39}$$

Combining (B.33)-(B.37) and (B.39) we then obtain

$$\frac{\sigma_d^2}{(N+1)\sigma_d^2+\sigma_\varepsilon^2} \le \lim_{M\to\infty} \frac{a(r)-c(r)-e(r)}{2-d(r)+b(r)+(N-1)h(r)} \le \frac{\sigma_d^2}{N\sigma_d^2+\sigma_\varepsilon^2}. \tag{B.40}$$

From (18), (B.35), (B.36) and (B.39) we then have, $\phi(r_l)\to 0$ as $M\to\infty$. Moreover, by (B.33), (B.35), (B.39) and since $h(r)\to -1/(N-1)$ there exists $r^* \in (r_l, r_h)$ such that $a(r^*)h(r^*) + e(r^*) - c(r^*) = 0$ for sufficiently large $M$ and $h(r^*)(b(r^*) - (N-1)) - d(r^*) > 0$. Therefore, there exists an $r_0 \in (r_l, r^*)$ such that $\phi(r_0) = 1$. Further, again by (B.33), (B.35) and (B.39), (20) is satisfied. Therefore, by Proposition 1, there exists a regular linear symmetric trading equilibrium. By (B.33)-(B.39) and since $h(r)$ is monotonically increasing on $(r_l, r_h)$, $r_0$ that satisfies $\phi(r_0) = 1$ is unique on $(r_l, r^*)$. Finally, again by monotonicity of $h(r)$, when $r > r_0$, $\phi(r) < 0$ and consequently, the solution to $\phi(r_0) = 1$ is unique on $(r_l, r_h)$.

Now, plugging (B.33)-(B.35) and the fact that $-1/(N-1) < h(r) < 0$ in (21), we see that $\alpha_s$ converges to a positive constant as $M\to\infty$. By (22) and (B.32), we then have $\beta_s$ being in the order of $M^{\frac{\gamma_s-1}{2}}$ as $M\to\infty$. Combining this with (22) and by (25) and (B.37), we then see that $\Lambda_1$ converges to zero as $M^{\frac{\gamma_s-1}{2}}$ as $M\to\infty$. Further, since $\alpha_s$ converges to a constant and again by (25) and (B.39), we also have $\lambda_1$ converging to zero as $M^{\frac{\gamma_s-1}{2}}$ as $M\to\infty$. This completes the proof of part $(i)$.

For part $(ii)$, we will only show part $(ii)(a)$. The proof of part $(ii)(b)$ will be similar in nature. For $\gamma_s = 1$ again by (19) first notice that

$$r \sim -M\sqrt{\frac{\sigma_{uo}^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)}} \quad \text{as } M\to\infty. \tag{B.41}$$

From (B.41), taking the limit as $M\to\infty$, it follows that

$$a(r) \sim \frac{\sigma_d^2((N-1)\sigma_\varepsilon^2\frac{\sigma_{uo}^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)} + \sigma_h^2+\sigma_{uo}^2)}{\frac{N-1}{N}\sigma_\varepsilon^2\sigma_{uo}^2 + (\sigma_h^2+\sigma_{uo}^2)(\sigma_d^2+\sigma_\varepsilon^2)}, \tag{B.42}$$

$$b(r) \sim \frac{\sigma_d^2(N-1)(\sigma_h^2+\sigma_{uo}^2)}{\frac{N-1}{N}\sigma_\varepsilon^2\sigma_{uo}^2 + (\sigma_h^2+\sigma_{uo}^2)(\sigma_d^2+\sigma_\varepsilon^2)}, \tag{B.43}$$

$$c(r) \sim \frac{\sigma_{uo}^2(N-1)\sigma_d^2\sigma_\varepsilon^2\frac{\sigma_{uo}^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)}}{\frac{N-1}{N}\sigma_\varepsilon^2\sigma_{uo}^2 + (\sigma_h^2+\sigma_{uo}^2)(\sigma_d^2+\sigma_\varepsilon^2)}, \tag{B.44}$$

$$d(r) \sim \frac{\sigma_{uo}^2\frac{N-1}{N}\sigma_\varepsilon^2}{\frac{N-1}{N}\sigma_\varepsilon^2\sigma_{uo}^2 + (\sigma_h^2+\sigma_{uo}^2)(\sigma_d^2+\sigma_\varepsilon^2)}, \tag{B.45}$$

and

$$f(r) \sim \frac{\sigma_h^2}{\sigma_h^2+2\sigma_{uo}^2}M^{-1}, \text{ as } M\to\infty. \tag{B.46}$$

Also, again, on the range specified by (19), $g(r)$ moves from $-1/(M-1)$ to 0 and $h(r)$ moves from 0 to $-1/(N-1)$ monotonically. Further,

$$e(r_l) \sim \frac{\sigma_h^2\sqrt{\frac{\sigma_{uo}^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)}}}{\sigma_h^2+2\sigma_{uo}^2}\,,\ and\ \frac{e(r_h)}{e(r_l)} \sim \frac{N}{N-1}\ \text{as}\ M\to\infty\,. \tag{B.47}$$

Then

$$a(r)h(r)+e(r)-c(r)\Big|_{r=r_l} \sim \sqrt{\frac{\sigma_{uo}^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)}}\left(\frac{\sigma_h^2}{\sigma_h^2+2\sigma_{uo}^2}-\frac{(N-1)\sigma_d^2\sigma_\varepsilon^2}{\frac{N-1}{N}\sigma_\varepsilon^2\sigma_{uo}^2+(\sigma_d^2+\sigma_\varepsilon^2)(\sigma_h^2+\sigma_{uo}^2)}\right), \tag{B.48}$$

and

$$a(r)h(r)+e(r)-c(r)\Big|_{r=r_h} \sim$$
$$\frac{1}{N-1}\left(\frac{N\sigma_h^2\sqrt{\frac{\sigma_{uo}^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)}}}{\sigma_h^2+2\sigma_{uo}^2}-\frac{(N-1)^2\sigma_d^2\sigma_\varepsilon\sqrt{\frac{\sigma_{uo}^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)}}-\sigma_d^2(\sigma_h^2+\sigma_{uo}^2+\frac{(N-1)\sigma_\varepsilon^2\sigma_{uo}^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)})}{\frac{N-1}{N}\sigma_\varepsilon^2\sigma_{uo}^2+(\sigma_d^2+\sigma_\varepsilon^2)(\sigma_h^2+\sigma_{uo}^2)}\right). \tag{B.49}$$

Now by (B.48), as $\sigma_{uo}^2\to 0$, $a(r)h(r)+e(r)-c(r)|_{r=r_l}\sim 0$ and if $\sigma_h^2 > \frac{(N-1)\sigma_d^2\sigma_\varepsilon^2}{\sigma_d^2+\sigma_\varepsilon^2}$, $a(r)h(r)+e(r)-c(r)>0|_{r=r_l}$. Moreover, $a(r)h(r)+e(r)-c(r)|_{r=r_h}\sim -\frac{\sigma_d^2}{\sigma_d^2+\sigma_\varepsilon^2}$. Therefore, if $\sigma_h^2$ is sufficiently large, there exists $r^*$ in (19), for which $a(r^*)h(r^*)+e(r^*)-c(r^*)=0$. Noting that $h(r)(b(r)-(N-1))-d(r)$ is monotonic on (19), there are two cases: If, $h(r^*)(b(r^*)-(N-1))-d(r^*)>0$, then there exists an $r_0\in(r_l,r^*)$ such that $\phi(r_0)=1$. If $h(r^*)(b(r^*)-(N-1))-d(r^*)<0$ on the other hand, by (B.42)-(B.47) $\phi(r_h)\sim -\frac{(N-1)\sigma_\varepsilon^2}{N\sigma_d^2+\sigma_\varepsilon^2}<0$ there exists an $r_0\in(r^*,r_h)$ such that $\phi(r_0)=1$. In both cases, since $a-e-c$ is increasing on (19) and since by (B.42)-(B.47) $a(r_l)-e(r_l)-c(r_l)\sim\frac{\sigma_d^2}{\sigma_d^2+\sigma_\varepsilon^2}>0$ as $M\to\infty$, so (20) is satisfied at $r_0$. Therefore, by Proposition 1, if $\sigma_{uo}^2$ is small enough and $\sigma_h^2$ is large enough, there exists a regular symmetric trading equilibrium.

Now take any sequence of regular symmetric trading equilibria as $M\to\infty$ and with $\gamma_s=1$. Then, by (21), (B.42)-(B.47) and since (18) is polynomial, we have $\alpha_s$ converge to a constant as $M\to\infty$. Therefore by (25) and (B.46), we then see that $\Lambda_1$ converges to a constant as $M\to\infty$. Further, since $\alpha_s$ converges to a constant and again by (25) and (B.47), we also have $\lambda_1$ converging a constant as $M\to\infty$. This completes the proof of part (*ii*).

Finally for part (*iii*), notice that

$$r\sim -\sqrt{\frac{\sigma_{uo}^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)}}M^{1+\gamma_s}\,, \tag{B.50}$$

as $M\to\infty$. Then, again taking the limit as $M\to\infty$, we have

$$a(r)\sim\frac{\sigma_d^2(1+\frac{(N-1)\sigma_\varepsilon^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)})}{\sigma_d^2+\sigma_\varepsilon^2(1+\frac{N-1}{N})}\,,\;b(r)\sim\frac{(N-1)\sigma_d^2}{\sigma_d^2+\sigma_\varepsilon^2(1+\frac{N-1}{N})}\,, \tag{B.51}$$

$$c(r)\sim\frac{\sigma_d^2\frac{(N-1)\sigma_\varepsilon^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)}}{\sigma_d^2+\sigma_\varepsilon^2(1+\frac{N-1}{N})}\;\text{ and }\;d(r)\sim\frac{(N-1)\sigma_\varepsilon^2}{\sigma_d^2+\sigma_\varepsilon^2(1+\frac{N-1}{N})}\,, \tag{B.52}$$

as $M\to\infty$. Further, $f(r)\sim\sigma_h^2/(2\sigma_u^2)$,

$$e(r_l)\sim f(r)\sqrt{\frac{\sigma_{uo}^2}{N(N\sigma_d^2+\sigma_\varepsilon^2)}}M^{\frac{1+\gamma_s}{2}}\;\text{ and }\;e(r_h)\sim(N/(N-1))e(r_l) \tag{B.53}$$

as $M\to\infty$. Combining these with the fact that on the range specified by (19), $g(r)$ moves from $-1/(M-1)$ to 0 and $h(r)$ moves from 0 to $-1/(N-1)$ monotonically, we obtain

$$\phi(r_l)\sim\frac{d(r)}{2-d(r)+b(r)}<1\;\text{ and }\;\phi(r_l)\sim-\frac{1-\frac{b(r)}{N-1}-d(r)}{1+b(r)-d(r)}<0\,, \tag{B.54}$$

and since by (18) and (B.53), $\phi$ is monotonically increasing with $h$ and consequently decreasing with $r$ on (19), we conclude that for large enough $M$, (18) cannot have a solution on (19) and hence a regular linear symmetric trading equilibrium cannot exist. This concludes the proof. ■

**Proof of Proposition 3:** We again start by examining the behavior of (18) over the relevant region given in (19). For simplicity in exposition, we also start by first assuming $0<\gamma_m<1$. Later, we will complete the case for $\gamma_m=0$ where there is a difference. Since $\sigma_\varepsilon^2=\varphi(N)\sigma_{\varepsilon o}^2\sim N^\gamma\sigma_{\varepsilon o}^2$ as $N\to\infty$ and taking the limit as $N\to\infty$ in (19), we see that for any $r$ in (19)

$$r\sim-N^{-1}\sqrt{\frac{(M-1)\sigma_u^2}{\sigma_d^2}}\varrho(r)\quad\text{as } M\to\infty, \tag{B.55}$$

where

$$\varrho(r)=\frac{1-\sqrt{\frac{M}{M-2}}}{r_h(N)-r_l(N)}(r-r_l(N))+\sqrt{\frac{M}{M-2}}\,. \tag{B.56}$$

Now,

$$a(r)\sim\frac{M\sigma_d^2(M\sigma_h^2+\sigma_u^2)}{\sigma_{\varepsilon o}^2(M^2\sigma_h^2+(M+(M-1)\varrho^2(r))\sigma_u^2)}N^{-\gamma_m}\,, \tag{B.57}$$

$$b(r)\sim\frac{M\sigma_d^2(M\sigma_h^2+\sigma_u^2)}{\sigma_{\varepsilon o}^2(M^2\sigma_h^2+(M+(M-1)\varrho^2(r))\sigma_u^2)}N^{1-\gamma_m}\,, \tag{B.58}$$

$$c(r)\sim\frac{\sigma_u^2\sigma_{\varepsilon o}^2\varrho^2(r)(M-1)}{\sigma_{\varepsilon o}^2(M^2\sigma_h^2+(M+(M-1)\varrho^2(r))\sigma_u^2)}N^{-1}\,, \tag{B.59}$$

$$d(r)\sim\frac{(M-1)\sigma_u^2\sigma_{\varepsilon o}^2\varrho^2(r)}{\sigma_{\varepsilon o}^2(M^2\sigma_h^2+(M+(M-1)\varrho^2(r))\sigma_u^2)}\,, \tag{B.60}$$

and

$$f(r) \sim \frac{\sigma_h^2 \varrho(r)}{(\sigma_h^2 + \sigma_u^2)\varrho^2(r) + M\sigma_h^2 + \sigma_u^2}\,, \text{ as } N \to \infty. \tag{B.61}$$

Also, again, on the range specified by (19), $g(r)$ moves from $-1/(M-1)$ to 0 and $h(r)$ moves from 0 to $-1/(N-1)$ monotonically. It then follows that

$$e(r_l) \sim \frac{M(M-1)\sigma_h^2\sqrt{\frac{(M-1)\sigma_u^2}{\sigma_d^2}}}{M(2M-1)\sigma_h^2 + (M^2+M-2)\sigma_u^2} N^{-1}, \tag{B.62}$$

and

$$e(r_h) \sim \frac{\sigma_h^2\sqrt{\frac{(M-1)\sigma_u^2}{\sigma_d^2}}}{(M+1)\sigma_h^2 + 2\sigma_u^2} N^{-1} \text{ as } N \to \infty. \tag{B.63}$$

Combining (B.57)-(B.61), (B.62) and (B.63) we then obtain

$$\lim_{N\to\infty} \frac{a(r) - c(r) - e(r)}{2 - d(r) + b(r) + (N-1)h(r)} = 1. \tag{B.64}$$

By multiplying the numerator and denominator of left hand side of (18) by $N^{\gamma_m}$ and by (B.57)-(B.63) we then have

$$\phi(r_l) \sim \frac{(M\sigma_h^2 + 2\sigma_u^2)\sqrt{\sigma_d^2(M-1)\sigma_u^2}}{(M\sigma_h^2 + 2\sigma_u^2)\sqrt{\sigma_d^2(M-1)\sigma_u^2} - (M(M-2)\sigma_h^2 + (2M-3)\sigma_u^2)\sqrt{\frac{M}{M-2}}}\,, \tag{B.65}$$

and

$$\phi(r_h) \sim \frac{M(M\sigma_h^2 + \sigma_u^2)(M+1)\sigma_h^2 + 2\sigma_u^2}{(M^2\sigma_h^2 + (2M-1)\sigma_u^2)\sigma_h^2\sqrt{\frac{(M-1)\sigma_u^2}{\sigma_d^2}} - (M-1)\sigma_u^2((M+1)\sigma_h^2 + 2\sigma_u^2)} \text{ as } N \to \infty. \tag{B.66}$$

From (B.65) and (B.66) we can then see that as $\sigma_d^2$ gets large, $\phi(r_l) \to 1^+$ and $\phi(r_h) \to -\frac{M(M\sigma_h^2+su)}{(M-1)\sigma_u^2}$ with $\phi$ having no singularities on (19) for $0 < \gamma_m < 1$. On the other hand, for $\gamma_m = 0$ we have

$$c(r) \sim \frac{\sigma_u^2\sigma_{\varepsilon o}^2\varrho^2(r)(M-1)}{\sigma_{\varepsilon o}^2(M^2\sigma_h^2 + (M + (M-1)\varrho^2(r))\sigma_u^2)} N^{-1}\,, \tag{B.67}$$

and

$$d(r) \sim \frac{\sigma_u^2\sigma_{\varepsilon o}^2\varrho^2(r)(M-1)}{\sigma_{\varepsilon o}^2(M^2\sigma_h^2 + (M + (M-1)\varrho^2(r))\sigma_u^2)}\,. \tag{B.68}$$

Therefore $\phi(r_l) \to$ and $\phi(r_h) \to 0$ as $N \to \infty$. Further, as $\sigma_d^2$ gets large, by (B.62) and (B.67), we see that $e(r) - c(r) \to 0^+$ on (19). On the other hand,

$$a(r_l)h(r_l) = 0 \quad \text{and} \quad a(r_h)h(r_h) \sim -\frac{M(M\sigma_h^2 + \sigma_u^2)\sigma_d^2}{(N-1)(M(M\sigma_h^2 + \sigma_u^2)\sigma_d^2 + (M^2\sigma_h^2 + (2M-1)\sigma_u^2)\sigma_{\varepsilon o}^2)}\,, \tag{B.69}$$

as $N\to\infty$. Therefore $\phi$ has a singularity on (19) and hence a solution to $\phi(z)=1$.

Therefore, in both cases, we conclude that there exists $\underline{\sigma_d^2}>0$ such that when $\sigma_d^2>\underline{\sigma_d^2}$, (18) has a solution on (19). Further, by (B.57), (B.57), (B.62) and (B.63), (20) is satisfied and therefore a regular linear symmetric trading equilibrium exists.

Now let any such equilibrium be given. By (21), and (B.57)-(B.63) we then have $\alpha_s\sim N^{-1}$. Further, then by (22) and (B.55), $\beta_s$ converges to a constant as $N\to\infty$. From (A.13), (B.61) and (25), we then conclude that $\exists\ \lambda^*>0$ and $\Lambda^*<0$ such that $\lim_{N\to\infty}\lambda_1=\lambda^*$ and $\lim_{N\to\infty}\Lambda_1=\Lambda^*$.

For the case with $\gamma_m=1$, plugging in $\sigma_\varepsilon^2=N^\gamma\sigma_{\varepsilon o}^2$ and taking the limit as $N\to\infty$ in (19), we see that for any $r$ in (19)

$$r\sim-N^{-1}\sqrt{\frac{(M-1)\sigma_u^2}{\sigma_d^2+\sigma_{\varepsilon o}^2}}\varrho(r)\quad\text{as } M\to\infty, \tag{B.70}$$

where $\varrho(r)$ is as defined in (B.56). We then have

$$a(r)\ \sim\ \frac{\sigma_d^2\big((M-1)\sigma_u^2\frac{\sigma_d^2}{\sigma_d^2+\sigma_{\varepsilon o}^2}+M(M\sigma_h^2+\sigma_u^2)\big)}{\sigma_{\varepsilon o}^2(M^2\sigma_h^2+(M+(M-1)\varrho^2(r))\sigma_u^2)}N^{-1}, \tag{B.71}$$

$$b(r)\ \sim\ \frac{M\sigma_d^2(M\sigma_h^2+\sigma_u^2)}{\sigma_{\varepsilon o}^2(M^2\sigma_h^2+(M+(M-1)\varrho^2(r))\sigma_u^2)}, \tag{B.72}$$

and

$$c(r)\sim\frac{\sigma_u^2\sigma_{\varepsilon o}^2\varrho^2(r)(M-1)\frac{\sigma_d^2}{\sigma_d^2+\sigma_{\varepsilon o}^2}}{\sigma_{\varepsilon o}^2(M^2\sigma_h^2+(M+(M-1)\varrho^2(r))\sigma_u^2)}N^{-1}. \tag{B.73}$$

$d(r)$ and $f(r)$ still satisfy (B.60) and (B.61) and $e(r_l)$ and $e(r_h)$ are scaled by $\sqrt{\sigma_d^2/(\sigma_d^2+\sigma_\varepsilon^2)}$ compared to (B.62) and (B.63). Keeping in mind that on the range specified by (19), $g(r)$ moves from $-1/(M-1)$ to 0 and $h(r)$ moves from 0 to $-1/(N-1)$ monotonically. Combining these with (B.61), (B.71) and (B.72) we then obtain

$$\lim_{N\to\infty}\frac{a(r)-c(r)-e(r)}{2-d(r)+b(r)+(N-1)h(r)}=1+\frac{(M-1)\sigma_u^2\sigma_d^2}{M(M\sigma_h^2+\sigma_u^2)(\sigma_d^2+\sigma_{\varepsilon o}^2)}. \tag{B.74}$$

Given these, the rest of the proof of existence is similar to the $0<\gamma<1$ case. Further, the existence of $\lambda^*>0$ and $\Lambda^*<0$ such that $\lim_{N\to\infty}\lambda_1=\lambda^*$ and $\lim_{N\to\infty}\Lambda_1=\Lambda^*$ follows similarly. This completes the proof of part $(i)$.

To see part $(ii)$, note that for $\gamma>1$

$$r\sim-\sqrt{\frac{(M-1)\sigma_u^2}{N^{1+\gamma_m}\sigma_{\varepsilon o}^2}}\varrho(r)\qquad\text{as } N\to\infty. \tag{B.75}$$

It follows that

$$a(r) \sim \frac{\sigma_d^2(M-1)\sigma_u^2\varrho^2(r)}{\sigma_{\varepsilon o}^2(M^2\sigma_h^2+(M+(M-1)\varrho^2(r))\sigma_u^2)}N^{-\gamma_m}\,, \tag{B.76}$$

$$c(r) \sim \frac{\sigma_u^2(M-1)\sigma_d^2\varrho^2(r)}{\sigma_{\varepsilon o}^2(M^2\sigma_h^2+(M+(M-1)\varrho^2(r))\sigma_u^2)}N^{-\gamma_m}\,, \tag{B.77}$$

and $f(r)$ still satisfies (B.61). Then, by (A.13), on (19), we have

$$\lim_{N\to\infty}\frac{M(M-1)\sigma_h^2\sqrt{\frac{(M-1)\sigma_u^2}{\sigma_{\varepsilon o}^2}}}{e(r)(M(2M-1)\sigma_h^2+(M^2+M-2)\sigma_u^2)}N^{-\frac{1+\gamma_m}{2}}\geq 1\,. \tag{B.78}$$

By (B.76)-(B.78), it then follows that (20) is violated on (19) as $N\to\infty$ and therefore, there cannot be a regular symmetric linear trading equilibrium. This completes the proof. ■

**Proof of Proposition 4:** The proof of parts (*i*) and (*ii*) are similar in nature to those for Propositions 2, and 3 and therefore skipped here.

To see part (*iii*), first note that since $\sigma_\varepsilon^2=\varphi_m(N)\sigma_{\varepsilon o}^2\sim N^{\gamma_o}\sigma_{\varepsilon o}^2$ as $N\to\infty$, $\sigma_u^2=\varphi_s(M)\sigma_{uo}^2\sim M^{\gamma_o}\sigma_{uo}^2$ and $\lim_{N,M\to\infty}\frac{N}{M}=k$, taking the limit as $N,M\to\infty$ in (19), we see that for any $r$ in (19)

$$r\sim-\sqrt{\frac{\sigma_{uo}^2}{k^{1+\gamma_o}\sigma_{\varepsilon o}^2}}\quad\text{as } M\to\infty, \tag{B.79}$$

Therefore, and taking the limit as $N$ and $M\to\infty$, we then can see that, for $r$ in (19) $a(r)\sim\frac{\sigma_d^2}{\sigma_{\varepsilon o}^2k^{\gamma_o}}M^{-\gamma_o}$, $b(r)\sim\frac{\sigma_d^2}{2\sigma_{\varepsilon o}^2k^{\gamma_o-1}}M^{1-\gamma_o}$, $c(r)\sim\frac{\sigma_d^2}{2\sigma_{\varepsilon o}^2k^{\gamma_o}}M^{-\gamma_o}$, $d(r)\sim\frac{1}{2}$ and $f(r)\sim\frac{\sigma_h^2}{2\sigma_{uo}^2}M^{-\gamma_o}$. Further, $e(r)\sim\frac{\sigma_h^2}{2\sigma_{uo}^2}\sqrt{\frac{\sigma_{uo}^2}{k^{1+\gamma_o}\sigma_{\varepsilon o}^2}}M^{-\gamma_o}$. This means that

$$\phi(r)\sim\frac{(a(r)-e(r)-c(r))(-h(r)(N-1)-\frac{1}{2})}{(\frac{3}{2}+(N-1)h(r))(a(r)h(r)+e(r)-c(r))} \tag{B.80}$$

Now, first suppose (36) does not hold. Then by (B.80)

$$\phi(r_l)\sim-\frac{\frac{\sigma_d^2}{2\sigma_{\varepsilon o}^2k^{\gamma_o}}-\frac{\sigma_h^2}{2\sigma_{uo}^2}\sqrt{\frac{\sigma_{uo}^2}{k^{1+\gamma_o}\sigma_{\varepsilon o}^2}}}{3(\frac{\sigma_h^2}{2\sigma_{uo}^2}\sqrt{\frac{\sigma_{uo}^2}{k^{1+\gamma_o}\sigma_{\varepsilon o}^2}}-\frac{\sigma_d^2}{2\sigma_{\varepsilon o}^2k^{\gamma_o}})}\sim\frac{1}{3}\,, \tag{B.81}$$

and similarly, $\phi(r_h)\sim-1$ as $N\to\infty$. Further, by (B.80), $\phi$ is increasing over (19) in $h(r)$, which means it is decreasing in $r$ and therefore there cannot be a solution to (18) and hence no regular linear symmetric trading equilibrium exists.

Now suppose (36) holds. Then $\lim_{M\to\infty}M^{\gamma_o}(e(r)-c(r))=0$ and $\lim_{M\to\infty}M^{\gamma_o}(a(r)-e(r)-c(r))=0$. Further, carrying out the algebra and imposing (36) we have

$$\frac{e(r)-c(r)}{a(r)}\sim\begin{cases}\frac{1}{4}\Big(\frac{(\sigma_h^2)^2}{\sigma_d^2\sigma_{uo}^2}-\frac{x(r)}{k}\Big)M^{-(\gamma_o-1)}, & 1<\gamma_o<2;\\ \frac{1}{4}\Big(\frac{(\sigma_h^2)^2}{\sigma_d^2\sigma_{uo}^2}+\frac{3-x(r)}{k}\Big)M^{-1}, & \gamma_o=2;\\ \frac{3-x(r)}{4k}M^{-1}, & 2<\gamma_o;\end{cases} \tag{B.82}$$

and

$$\frac{a(r)-e(r)-c(r)}{a(r)} \sim \begin{cases} \frac{1}{4}\big(\frac{\sigma_h^2(2\sigma_d^2+\sigma_h^2)}{\sigma_d^2\sigma_{uo}^2} - (2k-1)\frac{x(r)}{k}\big)M^{-(\gamma_o-1)}, & 1<\gamma_o<2; \\ \frac{1}{4}\big(\frac{\sigma_h^2(2\sigma_d^2+\sigma_h^2)}{\sigma_d^2\sigma_{uo}^2} + (2k-1)\frac{1-x(r)}{k}\big)M^{-1}, & \gamma_o=2; \\ (2k-1)\frac{1-x(r)}{4k}M^{-1}, & 2<\gamma_o; \end{cases} \tag{B.83}$$

as $M, N\to\infty$, where

$$x(r) = \frac{2(r_h^2-r^2)}{r_h^2-r_l^2}, \; r_l<r<r_h. \tag{B.84}$$

Using (B.82) and (B.83) we can then analyze (18) as $M\to\infty$. Here, we will only demonstrate the case for $1<\gamma_o<2$. The other cases will be similar. First, by (A.13), (A.14) and (B.84) we have

$$h(r)(N-1) \sim -\frac{2-x(r)}{2-\frac{2k+1}{(N-1)x(r)}} \sim \frac{x(r)}{2}-1\,, \tag{B.85}$$

as $M, N\to\infty$. By (B.82), (B.83) and (B.85) we have

$$\phi(r) \sim \frac{\big(\frac{\sigma_h^2(2\sigma_d^2+\sigma_h^2)}{\sigma_d^2\sigma_{uo}^2} - \frac{2k-1}{4k}x(r)\big)M^{-(\gamma_o-1)}(1-x(r))}{(1+x(r))(h(r)+\big(\frac{(\sigma_h^2)^2}{\sigma_d^2\sigma_{uo}^2}-\frac{x(r)}{4k}\big)M^{-(\gamma_o-1)})}\,. \tag{B.86}$$

as $M, N\to\infty$. Then (excluding the case $x(r)\neq 4k(\sigma_h^2)^2/\sigma_d^2\sigma_{uo}^2$ which can be ignored for our purpose) (18) tends to

$$\frac{k}{2}x^2(r) - (\frac{k-1}{2} + k\frac{2\sigma_h^2(\sigma_d^2+\sigma_h^2)}{\sigma_d^2\sigma_{uo}^2})x(r) + k\frac{\sigma_h^2\sigma_d^2}{\sigma_d^2\sigma_{uo}^2} = 0\,. \tag{B.87}$$

It is easy to show that $\exists\; \underline{\sigma_h^2}>0$ and $\overline{\sigma_{uo}^2}>0$ such that when $\sigma_h^2>\underline{\sigma_h^2}$ or $0<\sigma_{uo}^2<\overline{\sigma_{uo}^2}$, there exists a (unique) solution $x(r)\in(0,2)$ to (B.87). Combining this with (B.83) and (B.84), it then follows that there exists a (unique) solution to (18) on (19) that satisfies (20) and consequently, by Proposition (1), a linear symmetric trading equilibrium exists for $M$, $N$ large enough.

Now take any such sequence of equilibria as $M, N\to\infty$. In that sequence, $x(r^*)\to x*$, where $x\in(0,2)$ is a root of (B.87). Then

$$\alpha_s \sim \frac{\big(\frac{\sigma_h^2(2\sigma_d^2+\sigma_h^2)}{\sigma_d^2\sigma_{uo}^2} - \frac{2k-1}{4k}x^*\big)M^{-2\gamma_o+1}}{2(1+x^*)}\,. \tag{B.88}$$

By (22) and (B.79) we also have $\beta_s \sim -\big(\frac{\sigma_h^2(2\sigma_d^2+\sigma_h^2)}{\sigma_d^2\sigma_{uo}^2} - \frac{2k-1}{4k}x^*\big)\sqrt{\frac{\sigma_{\varepsilon o}^2 k^{\gamma_o+1}}{\sigma_{uo}^2}}(2(1+x^*))^{-1}M^{-2\gamma_o+1}$. Then $\Lambda_1 = f(r^*)/\beta_s \sim (\sigma_h^2/2\sigma_{uo}^2\beta_s)M^{-\gamma_o}\to-\infty$, and $\lambda_1 = e(r^*)/\alpha_s \sim (\sigma_h^2/2\sigma_{uo}^2\alpha_s)M^{-\gamma_o}\to\infty$ as $M, N\to\infty$. This completes the proof. ■

**Proof of Proposition 5:** We start with the case $1 < \gamma_o < 2$. First notice that, by (3), (7), (8), (26) and (27), we have

$$E[p_e] = \frac{\sum_{i=1}^{N} \alpha_{0i} - \sum_{j=1}^{M} \beta_{0j}}{M\beta_p - N\alpha_p} = \frac{\Lambda_1 N\alpha_0 - Mc_0}{(M\beta_p - N\alpha_p)(\Lambda_1 + M\nu_2)} \,. \tag{B.89}$$

By (B.83) and (B.85) $\alpha_s \sim \alpha_s^* M^{-2\gamma_o+1}$, $\beta_s \sim -\alpha_s^* \sqrt{\frac{k^{1+\gamma_o}\sigma_{uo}^2}{\sigma_{\varepsilon o}^2}} M^{-2\gamma_o+1}$, $\Lambda_1 \sim -\frac{\sigma_h^2}{2\sigma_{uo}^2\alpha_s^*}\sqrt{\frac{\sigma_{\varepsilon o}^2 k^{1+\gamma_o}}{\sigma_{uo}^2}} M^{\gamma_o-1}$ and $\lambda_1 \sim \frac{\sigma_h^2}{2\sigma_{uo}^2\alpha_s^*}\sqrt{\frac{\sigma_{\varepsilon o}^2}{k^{1+\gamma_o}\sigma_{uo}^2}} M^{\gamma_o-1}$, as $M, N \to \infty$ where $x^*$ is the solution to (B.87). Then, by (26), we have

$$N\alpha_0 = \frac{K + Mc_0 \frac{\eta_2 - \rho_2}{\Lambda_1 + M\nu_2}}{1 + \Lambda_1 \frac{\eta_2 - \rho_2}{\Lambda_1 + M\nu_2}} \,. \tag{B.90}$$

Now, by (A.2), (A.4) and (A.6), we also have

$$\eta_2 \sim \frac{\sigma_d^2 r_0^2}{\alpha_s^*(\sigma_{\varepsilon o}^2 + \sigma_{uo}^2 r_0^2)} M^{\gamma_o - 1} \,, \tag{B.91}$$

$$\rho_2 \sim \frac{\sigma_{\varepsilon o}^2 k^{\gamma_o}}{\sigma_{\varepsilon o}^2 + \sigma_{uo}^2 r_0^2} \,, \tag{B.92}$$

and

$$\nu_2 \sim -\frac{\sigma_h^2}{2\sigma_{uo}^2\alpha_s^*}\sqrt{\frac{\sigma_{\varepsilon o}^2 k^{1+\gamma_o}}{\sigma_{uo}^2}} M^{\gamma_o-1} \,, \tag{B.93}$$

as $M, N \to \infty$, where $r_0 = \frac{\sigma_{\varepsilon o}^2 k^{1+\gamma_o}}{\sigma_{uo}^2}$. Therefore, by (B.89) and (B.90), and since

$$M\beta_p - N\alpha_p \sim -\frac{1}{\Lambda_1} \,, \tag{B.94}$$

we have

$$E[p_e] \sim \frac{-\frac{\sigma_h^2}{2\sigma_{uo}^2\alpha_s^*}\sqrt{\frac{\sigma_{\varepsilon o}^2 k^{1+\gamma_o}}{\sigma_{uo}^2}} M^{\gamma_o-1} K - Mc_0}{-M} \sim c_0 \,, \tag{B.95}$$

as $M, N \to \infty$. This completes the proof of part (i).

To see part (*ii*), following similar steps as given for $\gamma_o < 2$ in part (*i*) above and part (*iii*) of Proposition 4, we can find that $\alpha_s \sim \alpha_s^* M^{-3}$, $\beta_s \sim -\frac{\alpha_s^*}{r_0} M^{-3}$, $\Lambda_1 \sim -\frac{\sigma_h^2 r_0}{2\sigma_{uo}^2\alpha_s^*} M$, $\lambda_1 \sim \frac{\sigma_h^2 r_0}{2\sigma_{uo}^2\alpha_s^*} M$, as $M, N \to \infty$, where

$$\alpha_s^* = \frac{2(\frac{\sigma_h^2(2\sigma_d^2+\sigma_h^2)}{4\sigma_d^2\sigma_{uo}^2} + \frac{2k-1}{4k}(1 - x^*))}{1 + x^*} \,, \tag{B.96}$$

and $x^*$ is the solution to the quadratic equation

$$\frac{k}{2}x^2 + (1 - k(1 + \frac{\sigma_h^2(\sigma_d^2 + \sigma_h^2)}{2\sigma_d^2\sigma_{uo}^2}))x + \frac{k}{2}(1 - \frac{(\sigma_h^2)^2}{2\sigma_d^2\sigma_{uo}^2}) = 0 \tag{B.97}$$

subject to $\frac{\sigma_h^2(2\sigma_d^2+\sigma_h^2)}{4\sigma_d^2\sigma_{uo}^2}+\frac{2k-1}{4k}(1-x^*)>0$. It can be shown that for small enough $\sigma_{uo}^2/\sigma_h^2$, such a solution always exists and (20) is satisfied. Further, note that calling $\lambda_1^*=\frac{\sigma_h^2 r_0}{2\sigma_{uo}^2\alpha_s^*}$, and utilizing (36), as $\sigma_{uo}^2/\sigma_h^2$ goes to an upper limit on the extended positive real line, $\lambda_1^*$ converges to $\infty$ and as $\sigma_{uo}^2/S\to 0$, $\lambda_1^*$ converges to 0.

Taking the limits, we have $M\nu_2 \sim -\frac{\sigma_h^2 r_0}{2\sigma_{uo}^2\alpha_s^*}M^2$, $\eta_2 \sim \frac{\sigma_d^2}{2\alpha_s^* k^2\sigma_{\varepsilon o}^2}M$ and $\rho_2$ converges to a constant as $M,N\to\infty$. Noting that $\sigma_d^2/(k^2\sigma_{\varepsilon o}^2)=\sigma_h^2 r_0\sigma_{uo}^2$ since (36), it follows that

$$\frac{\eta_2-\rho_2}{\Lambda_1+M\nu_2}\sim -\frac{1}{M} \text{ as } M,N\to\infty. \tag{B.98}$$

By (B.89), (B.90) and calling $\lambda_1^*=\frac{\sigma_h^2 r_0}{2\sigma_{uo}^2\alpha_s^*}$, we then have

$$\sum_{i=1}^{N}\alpha_{0i}\sim\frac{k(K-c_0)}{\lambda_1^*(k+1)+k}, \tag{B.99}$$

and $E[p_e]\sim\tau K+(1-\tau)c_0$ as $M,N\to\infty$, where

$$\tau=\frac{\lambda_1^* k}{\lambda_1^*(k+1)+k}. \tag{B.100}$$

Combining this with the fact that $\lambda_1^*$ ranges from 0 to $\infty$ for different values of $(\sigma_d^2,\sigma_h^2,\sigma_{uo}^2,\sigma_{\varepsilon o}^2)$, it follows that for any $p\in(c_0,\frac{kK+c_0}{k+1})$, there exists a $(\sigma_d^2,\sigma_h^2,\sigma_{uo}^2,\sigma_{\varepsilon o}^2)\in\mathbb{R}_+^4$ such that $E[p_e]\to p$. Now

$$E[TQ]=\sum_{i=1}^{N}\alpha_{0i}+N\alpha_p E[p_e]. \tag{B.101}$$

Notice that $MN\alpha_p$ converges to a constant as $M,N\to\infty$. Then, for any given $p$, by writing $\tau$ and $\lambda_1^*$ in terms of $E[p_e]$, substituting in (B.99), we obtain $E[TQ]\to K-p-\frac{1}{k}(p-c_0)$, as $E[p_e]\to p$.

Finally, to see part (*iii*), first, by following similar steps as given in the proof of part (*iii*) of Proposition (4) this time for $\gamma_o>2$, we obtain the quadratic equation

$$2(x(r^*)-1)(k-1-kx(r^*))=0 \tag{B.102}$$

as the analogue of (B.87). Further, by (18), (21) (B.82) and (B.83),

$$\alpha_s^*\sim\frac{(2k-1)(1-x(r^*))}{2(1+x(r^*))} \tag{B.103}$$

and when $x(r^*)\neq 1$

$$\frac{(2k-1)(1-x(r^*))}{2(1+x(r^*))}\sim-\frac{1}{2} \tag{B.104}$$

has to hold as $M,N\to\infty$. Now for $(k-1)/k$ to be a valid solution to (B.103), $k\geq 1$ has to hold. On the other hand, when $k\geq 1$, (B.104) cannot be satisfied for $x(r^*)=(k-1)/k<1$.

Therefore, $x(r^*) = 1$ can be the only valid solution to (B.102). By (B.103), it then follows that $\alpha_s \sim \alpha_s^* M^{-(\gamma_o+1+\delta)}$, $\beta_s \sim -\frac{\alpha_s^*}{r_0} M^{-(\gamma_o+1+\delta)}$, $\Lambda_1 \sim -\frac{\sigma_h^2 r_0}{2\sigma_{uo}^2 \alpha_s^*} M^{1+\delta}$, $\lambda_1 \sim \frac{\sigma_h^2 r_0}{2\sigma_{uo}^2 \alpha_s^*} M^{1+\delta}$, as $M, N \to \infty$, where $\delta, \alpha_s^* > 0$. Again taking the limits, we have $M\nu_2 \sim -\frac{\sigma_h^2 r_0}{2\sigma_{uo}^2 \alpha_s^*} M^{2+\delta}$, $\eta_2 \sim \frac{\sigma_d^2}{2\alpha_s^* k^2 \sigma_{\varepsilon o}^2} M^{1+\delta}$ and $\rho_2 \sim \frac{1}{2}$ as $M, N \to \infty$. Noting again that $\sigma_d^2/(k^2\sigma_{\varepsilon o}^2) = \sigma_h^2 r_0/\sigma_{uo}^2$ by (36), it follows that $\frac{\eta_2 - \rho_2}{\Lambda_1 + M\nu_2} \sim -\frac{1}{M}$ as $M, N \to \infty$. Further, substituting this into (B.89) and (B.90)

$$\sum_{i=1}^{N} \alpha_{0i} \sim \frac{k(K - c_0)}{\lambda_1^*(k+1) + k} M^{-\delta}, \tag{B.105}$$

and

$$E[p_e] \sim \frac{kK + c_0}{k+1} \tag{B.106}$$

follow, as $M, N \to \infty$,. The fact that $E[TQ] \to 0$, as $M, N \to \infty$, then follows by (B.101), (B.105) and since $M^{1+\delta} N \alpha_p$ converges to a constant as , as $M, N \to \infty$. This completes the proof. ■

**Proof of Proposition 6:** First, notice that for any $r$ in (19),

$$r \sim -\sqrt{\frac{\sigma_{uo}^2}{k^2 \sigma_d^2}}, \tag{B.107}$$

as $M, N \to \infty$. From this, it follows that

$$a(r) \sim \frac{\sigma_d^2}{k_o^\gamma} M^{-\gamma_o} \text{ and } b(r) \sim \frac{k^{1-\gamma_o}\sigma_d^2}{\sigma_{\varepsilon o}^2} M^{1-\gamma_o}, \tag{B.108}$$

and the other terms in (21) are of lower order as $M, N \to \infty$. From this, and again by taking the limit as $N, M \to \infty$, it follows that

$$\alpha_s \sim \frac{1}{k} M^{-1}, \quad \beta_s \sim -\frac{1}{kr_0} M^{-\frac{\gamma_o+1}{2}}, \tag{B.109}$$

$$\Lambda_1 \sim -kr_o M^{\frac{\gamma_o-1}{2}} \text{ and } \lambda_1 \sim kr_0 M^{\frac{\gamma_o-1}{2}}, \tag{B.110}$$

where $r_0 = \sqrt{\sigma_{uo}^2/(k^2\sigma_d^2)}$. By taking the limits as $N, M \to \infty$, we also have

$$M\nu_2 \sim -kr_0 M^{\frac{1+\gamma_o}{2}}, \text{ and } \eta_2 \sim \rho_2 \sim \frac{\sigma_{uo}^2}{\sigma_h^2} M^{\gamma_o - 1}. \tag{B.111}$$

By (3), (7), (8), (26), (27), (B.90) and by Taylor expansion we then have

$$p_e \sim c_0 + kKr_0 M^{\frac{\gamma_o-3}{2}} + (d - h + \frac{1}{Mk}\sum_{i=1}^{N} \varepsilon_i - \frac{1}{M}\sum_{j=1}^{M} u_j) kr_0 M^{\frac{\gamma_o-1}{2}}, \tag{B.112}$$

as $M, N \to \infty$. Again by (7), (B.90) and Taylor expansion it then follows that

$$TQ \sim K + d - (c_0 + h) - \frac{K}{N}(d - h) k r_0 M^{\frac{\gamma_o - 1}{2}} - k K r_0 M^{\frac{\gamma_o - 3}{2}} + \left(\frac{1}{Mk}\sum_{i=1}^{N} \varepsilon_i - \frac{1}{M}\sum_{j=1}^{M} u_j\right)\left(1 - k r_0 M^{\frac{\gamma_o - 1}{2}}\right). \tag{B.113}$$

Using the definition of the welfare loss, and plugging in (B.113) we then have

$$\mathcal{L} \sim M^{\gamma_o - 1}\left(k r_o(\sigma_d^2 + \sigma_h^2) + \sigma_{\varepsilon o}^2 k^{\gamma_o - 1} + \sigma_{uo}^2\right), \tag{B.114}$$

which completes the proof. ■