P-E Multiples and Changing Interest Rates*

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Abstract

How should one conceptualize price-earnings multiples (earnings capitalization factors) when interest rates change stochastically? The paper shows that while the multiplier for forthcoming earnings depends on current rates, the multiplier for current earnings depends on lagged rates. With these ideas in place, the paper generalizes Ohlson [1995] model with particular emphasis on the case when earnings provide sufficient accounting information for valuation. Results do not depend on the stochastic behavior of interest rates. The paper further derives the supporting modified information dynamic and shows how earnings persistence depends on both the current and the lagged rate.

1. Introduction

All models relating accounting data to equity value depend on a discount factor. Such a factor is, of course, essential to value sequences of anticipated dividends (or cash flows). In accounting-based valuation, discount factors make their presence felt via price (cum-dividend) to earnings multiples. Practical valuation analyses often focus on price-earnings multiples where the benchmarks depend on (the inverse of) the discount factor.[1] Parsimonious theoretical models such as Ohlson [1995] that relate prices to earnings, dividends, book values, and "other information" have indeed shown that the multiple for current earnings equals R/(R-1) while the multiple for expected earnings equals to 1/(R-1). These models are straightforward because they assume the discount factor does not change over time. The current paper relaxes this assumption, and addresses how one conceptualizes the relation between price and accounting data in settings with stochastic interest rates and risk neutrality.[2]

In broad terms, this paper details a class of valuation functions and the sustaining information dynamics when interest rates are stochastic. Like Ohlson [1995], this class derives from two benchmarks – mark-to-market accounting (i.e., the "balance-sheet approach" where book value provides sufficient information for valuation) and permanent-earnings accounting (i.e., the "income-statement approach" where earnings provide sufficient information for valuation) – and their weighted averaging. As we will clarify, however, the core issue is how one capitalizes expected earnings and, crucially, contemporaneous earnings (realized at the end of the current period). With respect to expected earnings, it seems relatively unambiguous that the multiple for expected earnings depends on the current rate. If one postulates this relation, however, then logical consistency argues that multiple for <u>current</u> earnings should depend on the <u>lagged</u> rate. To be precise, one multiplies current earnings, x_t, by $R_{t-1}/(R_{t-1}-1)$ and not by $R_t/(R_t-1)$. The paper develops the validity of this concept rigorously starting from properties inherent in a savings account. Given this metaphor, it is shown that the results rest on a surprisingly simple yet powerful insight: The concept of an (expected) earnings rate for a period depends on the rate at the beginning of the period, not at the end of the period.

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[1] Liu, Nissim, and Thomas (2000) examine the role of multiples in equity valuation.
[2] Beaver (1999), p 37, questions the assumption of constant discount rates in empirical studies. Liu and Thomas [2000] is an example of an empirical study based on the residual income valuation model that considers the effect of changing discount rates.

Much of the analysis focuses on "permanent-earnings" accounting – defined as the case when current earnings x_t provide sufficient accounting information for the cum-dividend value P_t+d_t, and it identifies the permanent-earnings capitalization as $x_t R_{t-1}/(R_{t-1}-1)$. By combining this setting with mark-to-market accounting, the paper generalizes Ohlson's [1995] model without significant incremental difficulties. The valuation function is the same as the one in Ohlson [1995], except for the use of a lagged (t-1 subscripted) earnings multiple rather than a constant earnings multiple. Given the generalized valuation function, the paper analyzes the properties of the sustaining information dynamics. It is shown that the persistence parameter, conventionally denoted by ω, depends on the current as well as the lagged rate. The analysis pays particular attention to the random-walk hypothesis associated with permanent-earnings accounting. As is well known, with constant interest rates the persistence of residual earnings equals one (ω=1). Stochastic rates result in $\omega_t = (R_t - 1)/(R_{t-1} - 1)$, and thus, under mild regularity conditions about interest rates, ω_t will oscillate around one. However, at any given point ω will not generally equal one, and therefore we obtain a generalized version of the random-walk hypothesis.

Two additional findings should be noted. First, the model does not depend on any specification of how interest rates evolve stochastically. This independence arises from the fact that prices and therefore the accounting numbers subsume expectations of interest rates and the multiple for current earnings depends only on the lagged rate. Second, the analysis yields a testable empirical hypothesis: In the usual returns on earnings regression, the coefficient associated with unexpected earnings should be large when interest rates are low (and conversely). This hypothesis has much intuitive appeal.

We build our analysis by analyzing models with increasing generality and complexity. Section 2 describes the notation and assumptions. Section 3 analyzes the pure mark-to-market model. Section 4 analyzes the pure permanent-earnings model. Section 5 analyzes the weighted average of the two models. Section 6 analyzes the weighted average model with other information. Section 7 summarizes and concludes the paper.

2. Notation and Assumptions

At date t, the "preceding" period refers to the period from date t-1 to date t, and the "forthcoming" period refers to the period from date t to date t+1.

x_t = earnings for the period t-1 to t, i.e., the preceding period

d_t = dividends, net of capital contributions, date t

P_t = ex-dividend market price of equity, date t

b_t = book value, date t

g_t = $P_t - b_t$ = goodwill, date t

r_t = risk free interest rate for the period t to t+1. (At date t, r_t is the current rate and r_{t-1} is the lagged rate.)

R_t = $1 + r_t$

x_t^a = $x_t - r_{t-1}b_{t-1}$ = abnormal or residual earnings for the preceding period.

Assumptions:

1. Risk neutrality,[3] which corresponds to the following:

$$P_t = \frac{E_t(P_{t+1} + d_{t+1})}{R_t} \qquad \text{(RN)}$$

Note that R_t is observed at date t; it is random from the perspective of prior dates.[4]

2. Clean surplus relation:

$$b_{t+1} = b_t + x_{t+1} - d_{t+1} \qquad \text{(CSR)}$$

Subsequent derivations are based on the following goodwill equation (GE), which holds if and only if one assumes risk neutrality and CSR:

$$g_t = \frac{E_t(g_{t+1} + x^a_{t+1})}{R_t} \qquad \text{(GE)}$$

3. The Mark-to-Market Model: The "Balance Sheet" Approach

We start with a simple but important benchmark -- the pure mark-to-market model. In this case, the balance sheet (the book value b_t) provides sufficient accounting information for valuation. Implications of changes in interest rates now pose few problems, but these provide a useful perspective before one considers more complicated valuation/accounting settings.

[3] For risk aversion, one can replace the expectation operator E by the E* that reflects risk-adjusted probabilities. See Huang and Litzenberger (1988).

1. The pricing equation: $P_t = b_t$.
2. The behavior of abnormal earnings: Since mark-to-market accounting by definition sets goodwill to zero, the goodwill equation (GE) yields $E_t x^a_{t+1} = 0$.
3. The role of the stochastic process underlying interest rates: Interest rates play no role here because the book value subsumes information about interest rates. An analogy to an investment fund is helpful. The prices of securities held by the fund will generally depend on interest rates, but since mark-to-market accounting sets the book value of each security to its market price, the book value reflects variations in market value due to interest rates without having to model stochastic interest rates.
4. The role of current and lagged rates: In mark-to-market accounting, goodwill and expected abnormal earnings are zero. It also follows that $E_t x_{t+1} = r_t b_t = r_t P_t$. Thus, the next-period expected earnings depend only on the current rate.[5] Past rates are irrelevant, except that they may have had an effect on current book value.

4. The Permanent-Earnings Model: The "Income Statement" Approach

We now turn to the other benchmark where the income statement (earnings x_t) provides sufficient information for valuation.[6] In contrast to the mark-to-market model, the permanent-earnings model is more subtle and complex because one needs a capitalization multiple to relate earnings to price. And, as we will establish shortly, it is not a foregone conclusion how one determines the earnings multiple.

As a basic reference point, with a non-stochastic (fixed) interest rate Ohlson (1995) relates price to permanent earnings via the following definitional equation (see also Ryan [1988]):

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[4] Ohlson [1995], and others, assume PVED, i.e., $P_t = \sum R^{-\tau} E_\tau[\tilde{d}_{t+\tau}]$. Constant interest rates (R_t= R), imply that RN, i.e., $P_t = E_t(P_{t+1} + d_{t+1})/R_t$ and PVED are equivalent.

[5] See Nissim and Penman (2000) for an empirical relationship between interest rates and accounting rates of return.

[6] An alternative, but different, definition of permanent-earnings accounting is that it results in earnings that follow a strict random walk, i.e., earnings have a persistence of 1. This is based on the notion that if the underlying economic fundamentals are not expected to change, then we can expect the same earnings next period. This notion makes sense when interest rates are constant. Indeed, with constant interest rates as in Ohlson [1995], the two definitions – earnings being a sufficient statistic and earnings having a persistence of 1 – are equivalent. With stochastic interest rates, however, it unclear why it is appealing to require earnings to have a persistence of 1. For example, the same balance in a savings account will give rise to different earnings as interest rates change. Appendix II shows that requiring earnings to have a persistence of 1 when interest rates are stochastic implies that earnings can no longer be a sufficient statistic and book value must enter the valuation equation.

$$P_t = \frac{R}{r} x_t - d_t$$

where R/r is the capitalization multiple associated with earnings. With full payout (d_t=x_t) one obtains P_t=x_t/r so that x_t simply equals price scaled by the constant r. Without the payout restriction, one can more generally think of x_t as being proportional to P_t+d_t, the cum-dividend value.

To appreciate the above concept of permanent earnings, note that earnings and cum-dividend price both represent the same underlying information. Thus, one can think of earnings as a "sufficient statistic" without any specificity as to the underlying accounting rules.[7] These aspects of the model will be retained in stochastic interest rate setting. But, there is now also the additional complication that the earnings multiplier, virtually by definition, must depend on the interest rates.

We now examine the permanent-earnings model along the same four dimensions along which we analyzed the mark-to-market model in section 3.

4.1 The Pricing Equation under Stochastic Discount Rates

With stochastic interest rates, it is clear that any reasonable representation of permanent earnings must satisfy P_t+d_t = f(R_t, R_{t-1}, …)x_t for some function f(.) that depends at most on the history of discount factors. Given the perspective, a tempting choice of multiple f(.) is R_t/r_t. This specification merely replaces the Ohlson (1995) earnings multiple with a factor based on the current interest rate. However, we argue that it makes more sense to use a factor based on <u>lagged</u> rates, i.e., R_{t-1}/r_{t-1}. In other words, permanent earnings satisfy the following pricing equation:

$$P_t = \frac{R_{t-1}}{r_{t-1}} x_t - d_t$$

There are two reasons for this specification. First, a special case of permanent earnings relates to <u>no earnings uncertainty</u> from one date to the next. Uncertain interest rates do not, <u>per se</u>, imply that next-period earnings are uncertain. A simple savings account can be used to develop this point. If at date t we observe x_t as the earnings for the period t-1 to t, we can infer that the savings account balance at t-1 was x_t/r_{t-1}. Due to the lack of earnings uncertainty, by t the

[7] Ohlson and Zhang [1998] show how one constructs accounting rules converting transactions into permanent-earnings measurements.

balance grows to $x_t + \frac{x_t}{r_{t-1}} = \frac{R_{t-1}}{r_{t-1}} x_t$. The balance after the withdrawal d_t is the price P_t. The earnings rate for the period t-1 to t is the rate prevailing at t-1, not t, so the earnings multiple used to interpret earnings for the preceding period depends on the lagged rate, and not the current rate: one obtains the earnings multiple R_{t-1}/r_{t-1}. Another way of making essentially the same point links current price to forthcoming earnings, assuming that these are certain. To make sense, in such a case permanent earnings requires that $P_t = \frac{x_{t+1}}{r_t}$. Since in addition no arbitrage requires $P_{t-1} = \frac{P_t + d_t}{R_t}$; it follows again that the capitalization rate associated with x_t equals $\frac{R_{t-1}}{r_{t-1}}$, and not $\frac{R_t}{r_t}$. From this simple fact it is hard, if not impossible, to deny the relevance of lagged rates when one conceptualizes the multiple associated with permanent earnings.

Second, it makes more sense to use the lagged rate to identify the capitalization multiple because earnings is a flow variable that presumably ought to depend on the recent change in rates. To develop this somewhat abstract argument, suppose we make the reasonable assumption that price depends only on the current rate but not past rates. If one combines this assumption with a capitalization multiple R_t/r_t, then permanent earnings for a period cannot depend on the change in rates over that period. Such an independence property seems undesirable since earnings is a flow variable which ought to recognize value-relevant information observed during the period. This problem can be avoided if one uses R_{t-1}/r_{t-1} rather than R_t/r_t, and now permanent earnings will generally depend on both the current and lagged rate.

The above points underscore that the concept of permanent earnings based on the multiple R_{t-1}/r_{t-1} is quite general. P_t can depend on the history of interest rates, and so can earnings. The structure of the permanent-earnings model ensures that one can think of earnings as a flow variable and P_t as a stock variable, consistent with the basic economics and accounting for a savings account. Nevertheless, it remains to be seen whether the above earnings capitalization multiple can be incorporated into a full-fledged valuation framework with uncertainty in the spirit of Ohlson's (1995) model.

4.2 The Behavior of Abnormal Earnings and Earnings

Ohlson (1995) shows that in a permanent-earnings model under constant interest rates the abnormal earnings persistence parameter equals one. Specifically, a strict random walk governs the stochastic behavior of abnormal earnings:

$$x^a_{t+1} = x^a_t + \varepsilon_{t+1},$$

where $E_t(\varepsilon_{t+1}) = 0$.

Having stated the above simple equation, one can next ask how it generalizes when one allows for stochastic interest rates and maintains that earnings must be permanent as previously defined. Thus, the issue arises how the persistence parameter depends on interest rates. We hypothesize the following linear information dynamic:

$$x^a_{t+1} = \omega_t x^a_t + \varepsilon_{t+1},$$

where ω_t can depend only on the history of interest rates. The two main questions are: Does ω_t depend on the entire history of interest rates or is a smaller subset sufficient? Does ω_t oscillate around 1, which is its value when interest rates do not change across time?

Proposition 1: Given risk neutrality and clean surplus, $P_t = \frac{R_{t-1}}{r_{t-1}} x_t - d_t$ implies

$$\omega_t = \frac{r_t}{r_{t-1}}.$$

Proof: See Appendix I.

The abnormal earnings persistence parameter depends only on the lagged and current rate, not the entire history of interest rates. It decreases in the lagged rate and increases in the current rate. If the distribution of interest rates satisfies reasonable regularity conditions, then the median abnormal earnings persistence parameter is 1, which is its value when the interest rates are constant.

The intuition underlying the functional form of the earnings persistence parameter can be stated briefly as follows. The current abnormal earnings are first capitalized by dividing by the lagged rate and are then multiplied by the current rate to determine forecasted forthcoming abnormal earnings. Section 4.4 provides further details.

One can of course ask what happens if one requires ω_t=1, that is, residual earnings follow a strict random walk, which can perhaps be viewed as an alternative, but different, definition of permanent earnings (see footnote 6). Appendix II addresses this issue. It is shown that if one assumes ω_t=1, then P_t will depend on not only earnings but also on book value, i.e., earnings can no longer be a sufficient statistic for determining value. Only with constant interest rates (r_t=r), are the two definitions equivalent as ω_t also equals 1.

The Behavior of Earnings: A Modified Random Walk

Ohlson (1995) implies the following stochastic process for earnings (rather than abnormal earnings):

$$E_t x_{t+1} = x_t + r \Delta b_t$$

The first term represents the standard random walk model of earnings and is valid only if there is no new investment and there are no changes in interest rates. The second term represents the adjustment to expected earnings due to changes in investment levels (Δb_t). Since expected earnings depend on the current rate applied to new investments, it is easy to see that r will be replaced by r_t when interest rates are stochastic. The following corollary reveals that changing r to r_t is not enough; stochastic interest rates introduce an additional term in the standard random walk model.

Corollary 1: $E_t x_{t+1} = x_t + r_t \Delta b_t + \%\Delta r_t x_t$ where $\%\Delta r_t = (r_t - r_{t-1}) / r_{t-1}$

Proof: See Appendix I.

The third term, which has not been recognized in prior research, shows that the percentage change in interest rate, nor just the level of interest rates, affects earnings forecasts; an uptick in interest rates lead to higher earnings forecasts, and vice versa.

4.3 The Lack of Need To Specify the Stochastic Process Underlying Interest Rates

The permanent-earnings model requires no specification of the stochastic process underlying interest rates because earnings subsume information about interest rates. In the case of a savings account discussed in section 4.1, the lagged rate is sufficient to infer the savings account balance from observed earnings, and the current rate is sufficient to compute the growth in the balance over the forthcoming period. Expectation of future interest rates is therefore not an issue.

4.4 The Role of Current and Lagged Rates

A key insight of the paper is that the lagged rate alone is needed to capitalize current abnormal earnings and the current rate alone is needed to capitalize expected forthcoming abnormal earnings.

Corollary 2: $g_t = \dfrac{E_t\, x_{t+1}^a}{r_t}$ and $g_t = \dfrac{x_t^a}{r_{t-1}}$.

Proof: See Appendix I.

The corollary brings out the crucial intuition that the earnings rate for a period is the interest rate prevailing at the beginning of that period.

From the corollary, we get $E_t\, x_{t+1}^a = r_t\, g_t = r_t \dfrac{x_t^a}{r_{t-1}}$, i.e., the abnormal earnings persistence parameter $\omega_t = \dfrac{r_t}{r_{t-1}}$. Given current abnormal earnings, the higher the lagged rate, the lower the current goodwill; the higher the current rate, the higher the abnormal earnings that this goodwill is expected to generate.

5. A Weighted-Average of the Two Models

We now extend the weighted average of the permanent-earnings model and the mark-to-market model presented in Ohlson (1995) to stochastic interest rates. To facilitate comparison, we continue to study the four aspects discussed in the context of the two benchmark models.

5.1 The Pricing Equation

Ohlson (1995) expresses price as a weighted average of the two models as follows:

$$P_t = k\ (\frac{R}{r} x_t - d_t) + (1 - k\ ) b_t$$

We specify the pricing equation as a weighted average of the permanent-earnings model and the mark-to-market model under stochastic interest rates as follows:

$$P_t = k\ (\frac{R_{t-1}}{r_{t-1}} x_t - d_t) + (1 - k\ ) b_t$$

where $k \in [0,1]$.

Our objective is to derive the linear information dynamic and the modification to the random walk of earnings that support such a representation.

5.2 The Behavior of Abnormal Earnings

Ohlson (1995) shows that the above pricing equation under non-stochastic rates implies the following linear information dynamic:

$$x_{t+1}^{a} = \omega x_{t}^{a} + \varepsilon_{t+1},$$

where $E_t(\varepsilon_{t+1}) = 0$ and

$$\omega = \frac{1+r}{k+r} k .$$

We hypothesize the following linear information dynamic:

$$x_{t+1}^{a} = \omega_t x_{t}^{a} + \varepsilon_{t+1} .$$

As before, ω_t can depend only on the history of interest rates. One can ask whether ω_t continue to increase in the current rate and decrease in the lagged rate, as in the permanent-earnings model.

Proposition 2: Given risk neutrality and clean surplus, $P_t = k\ (\frac{R_{t-1}}{r_{t-1}} x_t - d_t) + (1-k\ ) b_t$ implies $\omega_t = \frac{1+r_t}{k+r_t} r_t \frac{k}{r_{t-1}}$.

Proof: See Appendix I.

Similar to the permanent-earnings model, the abnormal earnings persistence parameter decreases in the lagged rate and increases in the current rate (For k > 0, $\frac{\partial \omega_t}{\partial r_t} > 0$ and $\frac{\partial \omega_t}{\partial r_{t-1}} < 0$.) As the weight assigned to earnings in the pricing equation increases, the abnormal earnings persistence parameter increases ($\frac{\partial \omega_t}{\partial k} > 0$). In the mark-to-market model (k=0), $\omega_t = 0$, which stands in contrast to the permanent-earnings model (k=1), $\omega_t = r_t/r_{t-1}$.

Although the sensitivity of the abnormal earnings persistence parameter to interest rates may be expected, its functional form is not obvious. Rearranging the terms in ω_t highlights the impact of changing interest rates on ω_t.

$$\omega_t = \frac{r_t}{r_{t-1}} \frac{(1+r_t)k}{k+r_t}$$

Though the expression looks somewhat complicated, it has the following interpretation. The first term reflects the "correction" due to the changing interest rates while the second term equals ω under constant interest rates. A further understanding of this relationship requires a specification of how current goodwill relates to current earnings and expected forthcoming earnings. Section 5.4 examines these relationships.

So far, we have assumed that k, the weight assigned to permanent-earnings model, is constant. One can question the extent to which our results depend on this assumption. The robustness of our results is examined in Appendix II, in which k can vary across time. It shows that ω_t continues to increase in the current rate and decrease in the lagged rate when k varies over time but is known at the beginning of a period.

The Behavior of Earnings: A Modified Random Walk

Ohlson [1995] model assumes that abnormal earnings follow a simple auto-regressive process with persistence parameter ω. In terms of expected forthcoming earnings, this dynamic can be expressed as follows:

$$E_t x_{t+1} = \omega(x_t + r\Delta b_t) + (1-\omega) r b_t$$

Two features of the expression are noteworthy. First, expected forthcoming earnings are a weighted average of the expected forthcoming earnings under the two benchmark models. Second, the abnormal earnings persistence parameter (ω) determines the weight assigned to the permanent-earnings part. This structure of the permanent-earnings forecasting is useful because it shows that the earnings dynamic, as well as the valuation function, ultimately rests on the convexification of a permanent-earnings model and a mark-to-market model. Accordingly, it makes sense to ask: Does the above dynamic equation generalize when interest rates are stochastic? The answer is "yes," except for the qualification that the weights attached to the two benchmarks will depend on the current interest rate.

Corollary 3: $E_t x_{t+1} = \theta_t (x_t + r_t \Delta b_t + \%\Delta\ r_t\ x_t) + (1-\theta_t) r_t b_t$ where $\theta_t = \dfrac{1+r_t}{k+r_t} k$.

Proof: See Appendix I.

The corollary shows that the permanent-earnings part of the earnings forecasting equation includes a correction for changes in interest rates, i.e., it includes the term, $\%\Delta r_t x_t$. This aspect is essential and consistent with Corollary 1. The weight θ_t has several interesting properties. It does indeed depend on r_t, yet it differs from ω_t if and only if r_t is stochastic. (In fact, $\theta_t = \dfrac{r_{t-1}}{r_t}\omega_t$, and θ_t depends only on the current rate while ω_t depends on both the current and the lagged rate.) One also sees that θ_t decreases as r_t increases, which means that the permanent-earnings component is relatively more important when interest rates are low. With respect to the coefficient in the valuation function, k, θ_t increases regardless of r_t. The following statement is therefore general: The permanent-earnings component of earnings forecasting is of relative importance if and only if the same is true in the valuation function.

5.3. The Lack of Need to Specify the Stochastic Process of Interest Rates

The weighted-average model does not require that we specify the stochastic process underlying interest rates because the earnings and book value subsume this information. This is not because k is time-independent in our model. Appendix II shows that we do not need a specification of the stochastic process even if k varies through time but is known at the beginning of a period.

5.4 The Role of Current and Lagged Rates

With respect to permanent earnings, prior analysis has shown that (i) one uses the lagged rate to capitalize current abnormal earnings, and (ii) one uses the current rate to capitalize expected forthcoming abnormal earnings. Corollary 2 provided the precise result. The corollary below extends this result to the more general weighted-average setting.

Corollary 4: $g_t = \dfrac{k+r_t}{1+r_t}\dfrac{E_t x^a_{t+1}}{r_t}$ and $g_t = k\ \dfrac{x^a_t}{r_{t-1}}$.

Proof: See Appendix I.

Corollary 4 also brings out a difference between an interior weighted-average model (0 < k < 1) as compared to the permanent-earnings model and the mark-to-market model. At both extremes, $E_t x_{t+1}^a = r_t g_t$ and $E_t x_{t+1} = r_t P_t$. These two simple relations, however, do not hold in the interior (0 < k < 1).

6. The Role of Other Value Relevant Information

Up to this point, we have generalized the Ohlson (1995) model without "other" information. We have related stock prices and forecasts of forthcoming earnings to accounting numbers alone in the presence of stochastic interest rates. Main insights from the preceding analysis relate to the concept that both the lagged and current rates are needed to forecast forthcoming earnings based on current earnings. Current earnings are first divided by the lagged rate to capitalize them and are then multiplied by the current rate to arrive at the forecast of forthcoming earnings.

The idea that one needs lagged rates to capitalize current earnings is as unique as it is important. To underscore and develop this idea, one can introduce "other information" in the spirit of the Ohlson (1995) model. It is now central whether (i) lagged rates still determine the multiple for current earnings, and (ii) lagged rates do not influence how other information enters into the forthcoming expected earnings equation. To develop the case that includes "other information" and stochastic rates, consider first the Ohlson (1995) model without stochastic rates. The valuation function satisfies the following equation:

$$P_t = k\left(\frac{R}{r} x_t - d_t\right) + (1-k)b_t + \beta\upsilon_t \qquad k \in [0,1]$$

and, the linear information dynamic is specified by the following equations:

$$x_{t+1}^a = \omega x_t^a + \upsilon_t + \varepsilon_{1,t+1}$$
$$\upsilon_{t+1} = \qquad + \gamma\upsilon_t + \varepsilon_{2,t+1}$$

where $E_t(\varepsilon_{1,t+1}) = 0$, $E_t(\varepsilon_{2,t+1}) = 0$. One interprets ν_t as "other information."

Ohlson (1995) shows that these information dynamics imply the valuation function stated above if and only if $\omega = \frac{kR}{k+r}$ and $\gamma = R - \frac{k+r}{r\beta}$.

Given previous generalizations of the Ohlson (1995) model, it is clear that the time dependence of parameters will enter only via the information dynamic not via the valuation function. In other words, to allow stochastic interest rates, we specify the valuation function and the linear information dynamic as follows:

$$P_t = k\ (\frac{R_{t-1}}{r_{t-1}} x_t - d_t) + (1-k\ ) b_t + \beta\ \upsilon_t$$

$$x_{t+1}^a = \omega_t\ x_t^a + \upsilon_t + \varepsilon_{1,t+1}$$
$$\upsilon_{t+1} = \qquad \gamma_t\, \upsilon_t + \varepsilon_{2,t+1}$$

where $E_t(\varepsilon_{1,t+1}) = 0$ and $E_t(\varepsilon_{2,t+1}) = 0$. Note that ω_t and γ_t are the only parameters that may depend on the history of interest rates.

One can now ask whether the introduction of "other" information changes the functional form of ω_t, and whether γ_t depends on the lagged rate. Again, ω_t ought to depend on the lagged rate because the lagged rate is needed to interpret current earnings; γ_t, on the other hand, should depend only on the current rate.

Proposition 3: Given risk neutrality and clean surplus,

$P_t = k\ (\frac{R_{t-1}}{r_{t-1}} x_t - d_t) + (1-k\ ) b_t + \beta\, \upsilon_t$ implies $\omega_t = \frac{r_t + 1}{r_t + k}\, r_t \frac{k}{r_{t-1}}$ and $\gamma_t = R_t - \frac{r_t + k}{r_t \beta}$.

Proof: See Appendix I.

The proposition shows that the functional form of abnormal earnings persistence (ω_t) is unaffected by the introduction of "other" information. The persistence of other information (γ_t) depends only on the current rate, not the lagged rate.

Two additional implications of the above model generalize Ohlson [1995]. First, one can explain the unexpected equity returns in terms of the uncertainty resolution variables ε_{1t} and ε_{2t}. Specifically, as shown in the Appendix,

$$\frac{P_{t+1} + d_{t+1}}{P_t} - R_t = (1 + \alpha_{1t}) \frac{\varepsilon_{1,t+1}}{P_t} + \alpha_{2t} \frac{\varepsilon_{2,t+1}}{P_t}$$

where

$$\alpha_{1t} = \frac{k}{r_t}$$
$$\alpha_{2t} = \beta R_t - \frac{k}{r_t} - 1$$

The price-normalized response coefficients, α_{1t} and α_{2t}, are known at the beginning of the return interval (date t), and hence one can write α_{nt} as opposed to $\tilde{\alpha}_{nt}$. This model attribute follows because the capitalization multiple depends only on the lagged rate. The specification also yields an interesting empirically testable hypothesis: Unexpected earnings, which equal $\varepsilon_{1,t+1}$, have a larger response coefficient when interest rates are low. This claim follows because for a fixed k α_{1t} increases as r_t decreases.

Second, following Ohlson (1999), an extension of Ohlson (1995), one can express price in terms of expected next-period earnings in lieu of ν_t. As shown in the Appendix,

$$P_t = w_{1,t} b_t + w_{2,t}\left(\frac{R_{t-1}}{r_{t-1}} x_t - d_t\right) + w_{3,t}\frac{E_t x_{t+1}}{r_t}$$
$$w_{1,t} = (1 - k - \beta r_t + \beta \omega_t r_{t-1})$$
$$w_{2,t} = (k - \beta \omega_t r_{t-1})$$
$$w_{3,t} = \beta r_t$$
$$\sum_m w_{m,t} = 1$$

The expression has a straightforward interpretation. One conceptualizes value as a weighted average of these "pure" valuation models: (i) mark-to-market accounting (b_t), (ii) permanent-earnings accounting ($\frac{R_{t-1}}{r_{t-1}} x_t - d_t$), and (iii) capitalized expected earnings ($\frac{E_t x_{t+1}}{r_t}$). Consistent with previous observation in this paper, further note that the lagged rate determines the multiplier for permanent earnings while the current rate determines the multiplier for expected earnings. This fundamental message concerning the conceptualization of earnings multiples apitalization rates has not changed through the model and is now considerably more complex than mark-to-market accounting or permanent-earnings accounting.

7. Summary and Implications

The analysis in this paper yields a number of striking observations. First, the generalization of Ohlson [1995] hinges on a thorough understanding of how the benchmark settings – mark-to-market and permanent-earnings accounting – can allow for stochastic interest rates.[8] Neither of these two cases leaves any choice as to how one models value as it relates to book value and earnings, respectively, when interest rates change. In particular, with respect permanent earnings it is clear that the capitalization depends solely on the lagged interest rate. Second, given the two benchmarks it is reasonably straightforward to expand the modeling to weighted-average settings, and to include so-called "other information". Third, in all of these cases the lagged interest rates serves the critical role of scaling current earnings so one can infer how current value relates to current earnings. Fourth, current interest rates enter the analysis by influencing the forecast of forthcoming expected earnings. Whether one considers current book value or current capitalized earnings, the current interest rate thus determines the earnings rate in a traditional sense.

It may at first glance seem unsatisfactory, or at least surprising, that the lagged rate rather than the current rate specifies the capitalization multiple. After all, it is generally agreed upon that equity values should reflect the current rate and that price changes should reflect changes in interest rates. Nevertheless, the analysis is consistent with these stylized facts even though the current rate has no explicit presence in the valuation function. This point is important because it serves as a reminder that current earnings can depend on the current rate. The same is of course true for other information. Similarly, unexpected changes in interest rates, $R_t - E_{t-1}[\tilde{R}_t]$, will correlate with unexpected returns, $\frac{P_t + d_t}{P_{t-1}} - R_{t-1}$, as long as unexpected changes in interest rates correlate with ε_{1t} or ε_{2t}. No assumption in this paper precludes this correlation, and thus the use of the multiple based on lagged rates does not preclude the dependence of price on expected interest rates.

[8] It is unclear how stochastic interest rates will affect valuation under conservative accounting as examined in Feltham and Ohlson [1995] and Zhang [2000].

Appendix I: Proofs

Proof of Proposition 1

We can restate the expression for P_t as:

$$P_t = b_t + \frac{x_t^a}{r_{t-1}}$$

That is:

$$g_t = \frac{x_t^a}{r_{t-1}}$$

From the goodwill equation (GE) we get,

$\frac{R_t x_t^a}{r_{t-1}} = E_t(\frac{x_{t+1}^a}{r_t} + x_{t+1}^a)$, which simplifies to

$E_t\, x_{t+1}^a = \frac{r_t}{r_{t-1}} x_t^a$. Thus, $\omega_t = \frac{r_t}{r_{t-1}}$ QED.

Proof of Corollary 1

From Proposition 1 we get, $E_t\, x_{t+1}^a = \frac{r_t}{r_{t-1}} x_t^a$. Substituting the expression for abnormal earnings, we get

$E_t\, x_{t+1} - r_t\, b_t = \frac{r_t}{r_{t-1}}(x_t - r_{t-1} b_{t-1})$, which simplifies to

$E_t\, x_{t+1} = x_t + r_t(b_t - b_{t-1}) + (r_t - r_{t-1})\frac{x_t}{r_{t-1}}$,or

$$E_t\, x_{t+1} = x_t + r_t\, \Delta\; b_t + \%\Delta\; r_t\, x_t$$

Proof of Corollary 2

From Proposition 1 we get, $E_t\, x_{t+1}^a = \frac{r_t}{r_{t-1}} x_t^a$. From the proof of proposition 1, we get $g_t = \frac{x_t^a}{r_{t-1}}$.

Substituting, we get $E_t\, x_{t+1}^a = r_t\, g_t$. QED

It is interesting to examine the relationship between expected forthcoming earnings and current stock price. Substituting the expression for abnormal earnings in $E_t\, x_{t+1}^a = \frac{r_t}{r_{t-1}} x_t^a$ we get, $E_t\, x_{t+1} - r_t\, b_t = \frac{r_t}{r_{t-1}}(x_t - r_{t-1} b_{t-1})$. Using CSR, we can restate this as

$$E_t\, x_{t+1} = r_t\,(\frac{x_t}{r_{t-1}} + x_t - d_t) = r_t\,(\frac{R_{t-1}}{r_{t-1}} x_t - d_t) = r_t\, P_t\,.$$

An analogy to the savings account brings out the relationship between prices and expected earnings. The earnings x_t for the period (t-1, t) imply that the savings account balance at t-1 was x_t/r_{t-1}. The balance at t equals the balance at t-1 plus the earnings over the period (t-1, t) minus the withdrawals over that period (x_t-d_t). The earnings rate for the period (t, t+1) is r_t.

Proof of Proposition 2

The pricing equation $P_t = k\ (\frac{R_{t-1}}{r_{t-1}} x_t - d_t) + (1-k\ ) b_t$ can be restated as follows:

$$P_t = k\ (\frac{R_{t-1}}{r_{t-1}} x_t - d_t - b_t) + b_t\,.$$

From the clean surplus relation, we get $b_t + d_t = x_t + b_{t-1}$. Substituting for $b_t + d_t$ in the expression above, we get

$$P_t = k\ (\frac{R_{t-1}}{r_{t-1}} x_t - x_t - b_{t-1}) + b_t = k\ (\frac{x_t}{r_{t-1}} - b_{t-1}) + b_t$$

Substituting for the expression of abnormal earnings, we get $P_t = k\ \frac{x_t^a}{r_{t-1}} + b_t$, which implies

$$g_t = k\ \frac{x_t^a}{r_{t-1}}\,.$$

Using the goodwill equation (GE) we get,

$$R_t\, k\ \frac{x_t^a}{r_{t-1}} = E_t(k\ \frac{x_{t+1}^a}{r_t} + x_{t+1}^a)$$

$$E_t x_{t+1}^a = \frac{1+r_t}{r_t+k} r_t\, k\ \frac{x_t^a}{r_{t-1}}$$, which implies $\omega_t = \frac{1+r_t}{r_t+k} r_t \frac{k}{r_{t-1}}$ QED

Proof of Corollary 3

From Proposition 2 we get, $E_t\, x^a_{t+1} = \frac{1+r_t}{r_t+k} r_t \frac{k}{r_{t-1}} x^a_t$. Substituting for abnormal earnings we get,

$$E_t x_{t+1} = \frac{1+r_t}{r_t+k} r_t k \frac{(x_t - r_{t-1} b_{t-1})}{r_{t-1}} + r_t b_t$$

Define $\theta_t = \frac{1+r_t}{k+r_t} k$

Thus, $E_t x_{t+1} = \theta_t \left(\frac{r_t}{r_{t-1}} (x_t - r_{t-1} b_{t-1}) + r_t b_t \right) + (1-\theta_t) r_t b_t$, which can be restated as follows:

$E_t x_{t+1} = \theta_t (x_t + r_t \Delta b_t + \%\Delta\ r_t\ x_t) + (1-\theta_t) r_t b_t$ QED.

Proof of Corollary 4

From Proposition 2 we get, $E_t\, x^a_{t+1} = \frac{1+r_t}{r_t+k} r_t \frac{k}{r_{t-1}} x^a_t$. From the proof of proposition 2, we get

$g_t = k \frac{x_t^a}{r_{t-1}}$. Substituting we get, $E_t x^a_{t+1} = \frac{1+r_t}{r_t+k} r_t\, g_t$. QED

Substituting for abnormal earnings and goodwill in the equation above, we get:

$$E_t x_{t+1} - r_t b_t = \frac{r_t}{r_t+k} R_t (P_t - b_t)$$

Upon simplification, we get:

$$E_t x_{t+1} = \frac{r_t}{r_t+k} (R_t P_t - (1-k) b_t)$$

Proof of Proposition 3

$$g_t = P_t - b_t = k\ \left(\frac{R_{t-1}}{r_{t-1}} x_t - d_t - b_t\right) + \beta \upsilon_t$$

Substituting for b_t from the clean surplus relation, $b_t + d_t = x_t + b_{t-1}$, and using the definition of abnormal earnings we get:

$$g_t = k\ \frac{x_t^a}{r_{t-1}} + \beta \upsilon_t$$

Using the goodwill equation (GE) we get,

$$R_t k \frac{x_t^a}{r_{t-1}} + R_t \beta \upsilon_t = E_t\left(k \frac{x_{t+1}^a}{r_t} + \beta \upsilon_{t+1} + x_{t+1}^a\right)$$

Since $\upsilon_{t+1} = \gamma_t \upsilon_t + \varepsilon_{2,t+1}$, we get

$$E_t x_{t+1}^a = \frac{r_t + 1}{r_t + k} r_t k \frac{x_t^a}{r_{t-1}} + \frac{r_t}{r_t + k} \beta \left(R_t - \gamma_t\right)\upsilon_t$$

This implies,

$$\frac{r_t}{r_t + k} \beta \left(R_t - \gamma_t\right) = 1$$

$$\gamma_t = R_t - \frac{r_t + k}{r_t \beta} \text{ QED}$$

Derivation of the Coefficients

$$P_t = b_t + \frac{k}{r_{t-1}} x_t^a + \beta v_t$$

$$P_{t+1} = b_{t+1} + \frac{k}{r_t} x_{t+1}^a + \beta v_{t+1}$$

$$P_{t+1} + d_{t+1} = b_{t+1} + d_{t+1} + \frac{k}{r_t} x_{t+1}^a + \beta v_{t+1}$$

From CSR

$$b_{t+1} + d_{t+1} = b_t + x_{t+1}$$

$$P_{t+1} + d_{t+1} = b_t + x_{t+1} + \frac{k}{r_t} x_{t+1}^a + \beta v_{t+1}$$

$$= b_t + r_t b_t + x_{t+1} - r_t b_t + \frac{k}{r_t} x_{t+1}^a + \beta v_{t+1}$$

$$= R_t b_t + \frac{k + r_t}{r_t} x_{t+1}^a + \beta v_{t+1}$$

$$= R_t b_t + \frac{k + r_t}{r_t} (\omega_t x_t^a + v_t + \varepsilon_{1,t+1}) + \beta(\gamma_t v_t + \varepsilon_{2,t+1})$$

Substituting for ω_t and γ_t, we get,

$$= R_t b_t + \frac{R_t k}{r_{t-1}} x_t^a + [\frac{k+r_t}{r_t} + \beta(R_t - \frac{k+r_t}{r_t \beta})]\nu_t + (\frac{k+r_t}{r_t})\varepsilon_{1,t+1} + \beta(R_t - \frac{k+r_t}{r_t \beta})\varepsilon_{2,t+1}$$

$$= R_t b_t + \frac{R_t k}{r_{t-1}} x_t^a + \beta R_t \nu_t + \frac{k+r_t}{r_t}\varepsilon_{1,t+1} + (\beta R_t - \frac{k+r_t}{r_t})\varepsilon_{2,t+1})$$

$$= R_t P_t + \frac{k+r_t}{r_t}\varepsilon_{1,t+1} + (\beta R_t - \frac{k+r_t}{r_t})\varepsilon_{2,t+1})$$

$$\frac{P_{t+1} + d_{t+1}}{P_t} - R_t = (1+\alpha_{1t})\varepsilon_{1,t+1} / P_t + \alpha_{2t} / P_t$$

$$\alpha_{1t} = \frac{k}{r_t}$$

$$\alpha_{2t} = \beta R_t - \frac{k}{r_t} - 1$$

Derivation of the Triple Weighted Average

$$P_t = (1-k)b_t + k\left(\frac{R_{t-1}}{r_{t-1}} x_t - d_t\right) + \beta \nu_t$$

We can express ν_t in terms of expected earnings as follows:

$$E_t x_{t+1}^a = \omega_t x_t^a + \nu_t$$

$$\nu_t = E_t x_{t+1}^a - \omega_t x_t^a$$

$$= E_t x_{t+1} - r_t b_t - \omega_t (x_t - r_{t-1} b_{t-1})$$

$$= E_t x_{t+1} - r_t b_t - \omega_t (x_t - r_{t-1}(b_t - x_t + d_t))$$

$$= E_t x_{t+1} - r_t b_t - \omega_t (R_{t-1} x_t - r_{t-1}(b_t + d_t))$$

$$= E_t x_{t+1} - r_t b_t - \omega_t (R_{t-1} x_t - r_{t-1}(b_t + d_t))$$

Substituting for ν_t in the pricing equation we get the following:

$$P_t = (1-k)b_t + k\left(\frac{R_{t-1}}{r_{t-1}} x_t - d_t\right) + \beta\left(E_t x_{t+1} - r_t b_t - \omega_t [R_{t-1} x_t - r_{t-1}(b_t + d_t)]\right)$$

$$= (1 - k - \beta r_t + \beta \omega_t r_{t-1}) b_t + (k - \beta \omega_t r_{t-1})\left(\frac{R_{t-1}}{r_{t-1}} x_t - d_t\right) + \beta r_t \frac{E_t x_{t+1}}{r_t}$$

From proposition 3, we know $\omega_t = \frac{r_t + 1}{r_t + k} r_t \frac{k}{r_{t-1}}$. Thus, $\omega_t r_{t-1} = \frac{r_t + 1}{r_t + k} r_t k$, which depends only on r_t.

Thus, we can express price as follows:

$$P_t = w_{1,t} b_t + w_{2,t}\left(\frac{R_{t-1}}{r_{t-1}} x_t - d_t\right) + w_{3,t}\frac{E_t x_{t+1}}{r_t}$$

$$w_{1,t} = (1 - k - \beta r_t + \beta \omega_t r_{t-1})$$

$$w_{2,t} = (k - \beta \omega_t r_{t-1})$$

$$w_{3,t} = \beta r_t$$

$$\sum_m w_{m,t} = 1$$

Appendix II: The Weighted Average Model with Variable but Known Weights

We now examine a setting where the weights can vary over time, but are known at the beginning of a period. Thus, price is expressed as follows:

$$P_t = k_{t-1}(\frac{R_{t-1}}{r_{t-1}} x_t - d_t) + (1 - k_{t-1}) b_t$$

Rearranging the terms in the above equation and applying the CSR, it follows that:

$$R_t k_{t-1} \frac{x_t^a}{r_{t-1}} = E_t(k_t \frac{x_{t+1}^a}{r_t} + x_{t+1}^a)$$

Inserting the last equation into the goodwill equation (GE) yields,

$$R_t k_{t-1} \frac{x_t^a}{r_{t-1}} = E_t(k_t \frac{x_{t+1}^a}{r_t} + x_{t+1}^a)$$

Since k_t and r_t are known at time t, the RHS equals $\frac{k_t}{r_t} E_t x_{t+1}^a + E_t x_{t+1}^a$ and one obtains the following:[9]

$$E_t x_{t+1}^a = \frac{r_t + 1}{r_t + k_t} r_t \frac{k_{t-1}}{r_{t-1}} x_t^a$$

The abnormal earnings persistence parameter therefore is represented by

$$\omega_t = \frac{r_t + 1}{r_t + k_t} r_t \frac{k_{t-1}}{r_{t-1}}$$

In this expression, one can think of ω_t as being the endogenous result of realization of interest rates and k_t's where the k_t's follow some exogenous stochastic process (though, as noted, the weights are determined at the beginning of a period). More important, as shown below, we can rearrange the terms to state k_t in terms ω_t, k_{t-1}, r_t, and r_{t-1}, i.e., we can think of ω_t as being exogenous.

$$k_t = \frac{r_t}{\omega_t} R_t \frac{k_{t-1}}{r_{t-1}} - r_t$$

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[9] If k_t, instead of k_{t-1}, is the weight in the pricing equation, i.e., the weight is determined at the end of the period rather than at the beginning, then we would need to know the covariance of k_{t+1} and $\frac{x_{t+1}^a}{r_t}$.

If one substitutes recursively, it follows that k_t is some function of the history of interest rates and the history of ω_t; i.e., one can write $k_{t+1} = f(r_t, r_{t-1}, \ldots, \omega_t, \omega_{t-1}, \ldots)$ where the ω_t are determined by some exogenous process.

One can now ask the following question: What happens if ω_t is simply a constant, such as $\omega_t = 1$? That is, what happens if abnormal earnings follow a random walk and interest rates can change over time? The answer is clear: P_t will generally depend on book value as well as capitalized earnings (adjusted for dividends). This is because, in contrast to the setting in which r_t is constant, k_t need not be 1 when $\omega_t = 1$, i.e., the weight on book value $(1-k_t)$ can be non-zero even when $\omega_t = 1$. One cannot, therefore, view earnings as a sufficient statistic when abnormal earnings follow a random walk as book value still generally enters the valuation function.

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