NEW YORK UNIVERSITY
STERN SCHOOL OF BUSINESS
FINANCE DEPARTMENT

Working Paper Series, 1996

"Price Barriers" and the Dynamics of Asset Prices in Equilibrium

Balduzzi, Pierluigi, Silverio Foresi and David Hait

FIN-96-11

"Price Barriers" and the Dynamics of Asset Prices in Equilibrium

Pierluigi Balduzzi, Silverio Foresi, and David J. Hait*

November 18, 1996

Abstract

In a variety of realistic scenarios, some investors trade infrequently rather than continuously, basing their buy or sell decisions on current price levels. A "price barrier" is a price level at which a large number of investors either buy or sell securities. We analyze the dynamics of asset prices in an economy with infrequent traders and price barriers. Our analysis predicts that when price barriers exist, both asset prices and price volatility can jump at the time the price barrier is reached, even if the trade is rationally anticipated. Moreover, the direction of the price jump may very well be the opposite of what one would expect. A price-triggered purchase may generate a downward jump in stock prices and, vice versa, a price-triggered sale may induce stock prices to jump above the price barrier. This is because trades affect prices before they are implemented: the anticipation of a stock purchase inflates stock prices, while the anticipation of a stock sale depresses stock prices. In the case of repeated trades, before-trade prices anticipate not only the next trade, but also the following ones. This may lead to the counterintuitive result that stock prices are inflated, rather than depressed, in the proximity of a stock sale.

---

*The authors wish to thank Jennifer Carpenter, Joel Hasbrouck, and an anonymous referee for useful comments. This work also owes an intellectual debt to Giuseppe Bertola, who contributed to this project at an early stage. Balduzzi and Foresi gratefully acknowledge financial support from NYU Summer Research Grants; Hait gratefully acknowledges financial support from an NYU Doctoral Fellowship. All three authors are with the Finance Department, Stern School of Business, New York University, New York, NY 10012. Corresponding author: Pierluigi Balduzzi; e-mail:pbalduzz@rnd.stern.nyu.edu; tel. (212) 998-0359

1 Introduction

In traditional continuous-time models of asset pricing (*e.g.*, Merton (1971)), investors maximize utility by rebalancing their portfolios continuously. This result depends on the assumption of zero transactions costs. When this assumption is relaxed, investors may achieve higher utility by rebalancing their portfolios *infrequently*. In the case of non-zero transactions costs, for example, the optimal strategy is to rebalance the portfolio only when relative asset prices reach certain extreme levels (Constantinides (1986), Davis and Norman (1990), and Duffie and Sun (1990)), and a similar behavior should arise when investors are subject to costs in gathering and processing information.

When the presence of transaction and information-gathering costs is combined with informational asymmetries, it may be that a large segment of the market trades infrequently, and coordinates to trade at specific price levels.[1] We refer to these triggering price levels as "price barriers." In this paper, we consider a market in which infrequently-trading investors, which we term "buy and hold" investors, coexist with utility-maximizing "market makers" who trade continuously, presumably because they are continuously and costlessly informed and face zero transaction costs. In our market, we assume that sufficient conditions exist to give rise to price barriers,[2] so that the buy-and-hold investors trade a significant amount of assets when the asset's price reaches the barrier. We address the following question: What is the impact of price barriers on the dynamics of asset prices in equilibrium?

We show that our framework can replicate the stylized evidence of Donaldson and Kim (1993), who find that the rise and fall of the Dow Jones Industrial Average index is restrained by support and resistance levels, but that, having breached a barrier, the Dow Jones moves up or down faster than usual. Also, our results are consistent with the empirical evidence on the existence of jumps in asset prices (see, for example, Jorion (1988), Ball and Torous

[1] For example, Admati and Pfleiderer (1991) show that uninformed traders can reduce the costs due to their informational disadvantage by coordinating their behavior to trade only at pre-arranged times. By extension the same advantages should accrue to uninformed traders who coordinate by trading when prices are in the neighborhood of specific values.

[2] The signal triggering trades may be associated with specific price recommendation of market analysts: for example, "Stock XYZ is rated a 'Buy', with a target (sell) price of $30." The empirical relevance of market analysts' recommendations is documented, for example, by Bjerring, Lakonishok, and Vermaelen (1983). Stickel (1995) and Womack (1996) provide evidence that analysts' recommendations do have an impact on stock prices, implying that they generate trading activity. Buying or selling of assets can also occur when a widely-recognized "technical" signal, such as a moving-average crossing, is generated by the price process. Blume, Easley, and O'Hara (1994) show that technical analysis may add value when information gathering is costly, while Neftci (1991) shows that if the price process is nonlinear, simple technical-analysis rules may provide better estimates of future returns than standard econometric techniques.

(1985), and Jarrow and Rosenfeld (1984)) and on the nature of volatility changes (see the extensive GARCH-M literature, and the study by Brown, Harlow, and Tinic (1993)).

Moreover, some of the trading scenarios that we consider can be interpreted as implementations of portfolio-insurance schemes. A number of authors have considered, in different settings, the asset-pricing effects of such strategies. Both Basak (1995) and Grossman and Zhou (1996) present general equilibrium models in which portfolio-insurance strategies arise endogenously as a result of constraints on wealth. A characteristic of these models is that investors sell assets as prices fall. Gennotte and Leland (1990) and Donaldson and Uhlig (1993) also present models in which informational uncertainty combined with the presence of hedgers who sell in response to price declines can lead to "crashes," or jumps in asset prices.

Our analysis allows us to develop several insights about how the presence of buy-and-hold investors affects asset prices and volatility. Specifically, our model predicts that asset prices and price volatility *jump* when buy-and-hold investors enter the market, even if their entrance is rationally anticipated. We show that equilibrium conditions require risk-adjusted prices–the actual asset prices times the market-makers' marginal utility of consumption–not to jump in expectation. At the time of the trade, the market makers' asset allocation is suddenly changed by a finite amount, and their consumption jumps. For risk-adjusted price paths to remain continuous in expectation, the corresponding jump in the marginal utility of consumption needs to be compensated by a jump in actual asset prices. Once the trade has been executed, and if future trades become unlikely, price volatility also jumps reflecting the change in the economic environment. Hence, our model generates jumps in asset prices and volatility *without* jumps in the dividend flow: it is the infrequent trading of the buy-and-hold investors which leads to the occurrence of the jumps. Moreover, the jumps in prices and in volatility are correlated.

Also, we find that the direction of the jump in asset prices does not necessarily correspond to the effect of an exogenous and unexpected "supply shock." For example, when stocks are purchased at an upper price barrier, stock prices may jump downwards, *below* the barrier. The intuition for this result is that the anticipation of a buy order tends to inflate stock prices *before* the trade. When the trade is executed, and if future trades are unlikely, the price pressure disappears, and stock prices suddenly fall. Moreover, in our framework, price jumps are induced by trades which, in turn, are triggered by asset prices. Hence, there is a correlation between asset prices, price jumps (both magnitude and rate), and price volatility.

We examine the sensitivity of our results to changes in the parameters describing preferences and stochastic processes. Interestingly, we find the price-pressure effects from a possible stock purchase on the part of the buy-and-hold investors to be strengthened by high risk aversion, and by high volatility of the dividend process.

We also consider the case of repeated trades and multiple price barriers. In this case, the before-trade pricing functions are affected not only by the anticipation of the next trade, but also by the anticipation of the following ones, and this may lead to counterintuitive results. For example, assume that a lower price barrier where buy-and-hold investors sell stocks, after it is hit, becomes an upper barrier where stocks are *purchased.* In the proximity of the lower barrier, prices incorporate the anticipation of a sell order, as well as of a buy order immediately afterwards. If the second trade is bigger than the first one, prices are *inflated*, rather than depressed, in the proximity of a sale of stocks.

The approach used in this paper is related to that in Balduzzi, Bertola, and Foresi (1995). Here, however, we focus on the possible effects of price barriers, rather than on the effects of feedback trades. In particular, we analyze a variety of economically realistic situations in which multiple price barriers exist at different levels, and in which the direction of the buy-and-hold investor's trade at each barrier is either strategy-dependent (*e.g.*, portfolio insurance), or randomly determined. The second case captures the intuitively appealing idea that there is uncertainty on the part of the rest of the market as to the role of the price barriers. Also, we develop a framework which could be useful to handle quite general assumptions about investors' preferences and random processes for fundamental state variables, as long as the utility function and the parameters of the stochastic process (*e.g.*, drift and diffusion coefficients) are analytic functions of the state variables.

Our approach is also related to that of the noise-trading literature (see Shleifer and Summers (1990) for a review) in that we calculate equilibrium prices taking the asset demands of a segment of the market (the buy-and-hold investors) as given, while explicitly modelling the market makers' utility-maximization problem. Relative to this literature though, our framework has several advantages. First, the bunching of trades around specific price values is quite realistic in light of the existing theoretical and empirical evidence, and is consistent with the findings of Campbell and Kyle (1993) who argue that noise trading needs to be correlated with "fundamentals" for it to contribute to asset-price behavior. Second, focusing on infrequent trade events allows us to conveniently draw on existing techniques for the study of regulated brownian motions (see Dixit (1993) for a useful non-technical introduction to the topic). Third, our analysis allows us to clearly interpret the intertemporal mechanisms underlying dynamic asset-price behavior in the presence of rationally-anticipated trades. Finally, our study can be seen as a first step towards computing the full rational-expectations equilibrium of a transaction-costs economy with informational asymmetries, in which the demand of every trader is derived based on the equilibrium prices, and equilibrium prices, in turn, reflect traders' demands.

In Section 2, we present our framework for describing the behavior of asset prices in the market and derive the partial differential equations and boundary conditions which asset prices must satisfy in equilibrium. In Section 3 we develop an approximate solution technique for solving the equilibrium asset prices, and we solve for the special case in which "buy and hold" investors either buy or sell securities, with pre-specified probabilities, when prices pass through either an upper or a lower price barrier. Section 4 extends the analysis to the case of repeated trades and multiple price barriers. Section 5 concludes.

2 The Pricing Framework

This section describes the general structure of the economy, the optimization problems faced by the buy-and-hold investors and by the market makers, and specifies the features of the equilibrium in the presence of trade. We then specialize the analysis to the case where market makers exhibit constant-relative-risk-aversion (CRRA) preferences, and the dividend flow is driven by a lognormal diffusion process.

2.1 The Economy

There are two assets available for trading in this economy, stocks and infinite-maturity bonds (*i.e.*, consols). Both provide their owners with a continuous stream of cash flows (in terms of the consumption good): in the case of stocks, the dividend rate is denoted ξ and is stochastic; while in the case of bonds, the coupon rate is constant and is denoted r. Both stocks and bonds are in positive supply. The current dividend rate on the stocks, together with the current asset allocation and the structure of trade, comprise the state of the economy. We let Y denote the information available to the market makers regarding the possibility of future trade by the buy-and-hold investors.

There are two representative types of investors in the economy: *market makers* and *buy-and-hold* investors. Market makers are fully informed about the state of the economy, face zero transaction and information-gathering costs, and trade continuously to maximize their utility from consumption. Buy-and-hold investors face non-zero information-gathering and transaction costs, and trade *infrequently* in the proximity of specific values of asset prices (price barriers).

2.2 The Buy-and-Hold Investors' Optimization Problem

While we do not model explicitly the utility-maximization problem for the buy-and-hold investors, one could think of barrier-trading as the optimal policy of utility-maximizing investors facing transaction costs. As in Constantinides (1986) or Davis and Norman (1990), these investors trade only when prices reach levels ("barriers") that cause their portfolio weights to become too far out of line. Moreover, as in Admati and Pfleiderer (1991), an informational disadvantage (relative to the fully-informed market makers) may lead these investors to "bunch" their trades together.

In our applications, we assume that buy-and-hold investors buy or sell assets in response to price movements, much like the "feedback traders" of DeLong, Shleifer, Summers, and Waldmann (1990) and the portfolio insurers of Gennotte and Leland (1990). Hence, the buy-and-hold investors may engage in "contrarian" investment strategies, purchasing assets when their price is low, and selling assets when their price is high. This type of behavior arises quite naturally in the context of optimal portfolio selection: when the price of an asset increases, its weight in the portfolio also increases; hence, the investor may decide to liquidate some of her holdings to bring the portfolio weight back in line. Alternatively, the buy-and-hold investors may follow "trend-chasing" strategies, buying assets when their price is high, and selling assets when their price is low. This type of strategy may be optimal, for example, for investors who do not tolerate falls in wealth below a given fraction of the original wealth (as in Grossman and Zhou (1996)) or declines in consumption (as in Dybvig (1995)).

While we assume that the buy-and-hold investors coordinate their trades at specific price levels, they need not all submit the same market orders. Specifically, some traders may buy, while others may sell at a given price. Hence, the aggregate demands of stocks and bonds presented by the buy-and-hold investors are to be interpreted as *net* demands.

2.3 The Market Makers' Optimization Problem

Let c denote the market maker's consumption flow, $U(\cdot)$ an increasing and concave utility function, and ρ the rate of time preference. Also, let s and b denote the market maker's holdings of stocks and bonds, respectively; while P_s and P_b denote the prices of stocks and bonds, respectively. Let $I \equiv I(W)$ denote the value function of the market maker

$$I\, e^{-\rho t} = \max_{c,s,b} \int_t^{\infty} E_t \left\{ U[c(\tau)] \right\} e^{-\rho\tau} d\tau,$$

where the market maker's wealth is defined as

$$W \equiv sP_s + bP_b. \tag{1}$$

The optimal consumption flow of the market maker is found by maximization of the value function, or equivalently by solution of the Hamilton-Jacobi-Bellman equation

$$0 = \max_{c,s,b} \left\{ e^{-\rho t} U(c) + E_t \left[\frac{1}{dt} d\left(I\, e^{-\rho t} \right) \right] \right\}$$

with the constraint

$$-c\, dt = \Delta s\, (P_s + \Delta P_s) + \Delta b\, (P_b + \Delta P_b) - (s\xi + br)\, dt.$$

This formulation of the budget constraint allows for finite changes in both portfolio holdings and asset prices.

The first-order condition for optimal equity holdings is given by

$$\begin{aligned} 0 &= e^{-\rho t} U'(c)\xi + \frac{\partial}{\partial s} E_t \frac{1}{dt} d\left(I\, e^{-\rho t} \right) \\ &= U'(c)\xi + \frac{\partial}{\partial s} E_t \left[\frac{1}{dt} (dI - \rho I\, dt) \right] \\ &= U'(c)\xi - \rho \frac{\partial I}{\partial W} \frac{\partial W}{\partial s} + E_t \left[\frac{1}{dt} d \left(\frac{\partial I}{\partial W} \frac{\partial W}{\partial s} \right) \right]. \end{aligned}$$

Using the envelope condition $\partial I / \partial W = U'(c)$ and the definition of wealth (equation (1)), we obtain the usual asset-pricing equation for the price of equity

$$\rho U'(c) P_s = U'(c)\xi + \frac{1}{dt} E_t \{ d[U'(c) P_s] \}. \tag{2}$$

A similar argument yields

$$\rho U'(c) P_b = U'(c) r + \frac{1}{dt} E_t \{ d[U'(c) P_b] \} \tag{3}$$

for the price of bonds. These equations have a straightforward interpretation: after adjusting by the marginal utility of consumption, the expected rate of return on any asset (the sum of cash flows and expected capital gains divided by the price) is equal to the consumer's rate of time preference. This relationship becomes clearer if we define the risk-adjusted asset prices p_i and cash-flows f_i as

$$p_s \equiv U'(c) P_s, \quad f_s \equiv U'(c)\xi, \quad p_b \equiv U'(c) P_b, \quad f_b \equiv U'(c) r. \tag{4}$$

Now we can rewrite equations (2) and (3) as:

$$\rho p_i = f_i + \frac{1}{dt} E_t(dp_i), \quad i = s,\ b. \tag{5}$$

Equations (5) must be satisfied by the stock and bond price processes at all times, regardless of the likelihood of trade. In addition, risk-adjusted prices must also satisfy the transversality conditions

$$\lim_{\tau \to \infty} E_t(p_i) e^{-\rho(\tau - t)} = 0, \quad i = s,\ b. \tag{6}$$

These conditions, a standard feature of infinite-horizon models, are necessary to rule out the existence of "price bubbles" and the associated "infinite horizon" arbitrage opportunities.

2.4 Equilibrium and Trade

Since market makers only can hold stocks and bonds, their consumption in equilibrium is given by

$$c = S\xi + Br,$$

where S and B denote the economy-wide per-capita holdings of stocks and bonds on the part of the market makers. Note that asset allocations are determined endogenously, given the initial endowments, by the process of trade at equilibrium prices. In fact, the solution for the equilibrium simultaneously determines S, P_s, B, and P_b. Hence, the nature and effects of trade are different from a supply shock which would change exogenously the market maker's asset allocations. Finally, note that since finite amounts of stocks and bonds are traded, the market makers' consumption may jump at trading times.

Given equation (5), $E_t(dp_i)$ cannot be of order larger than dt, and this rules out expected jumps in risk-adjusted price paths. It follows that

$$E_t(\Delta p_i) = 0, \quad i = s,\ b, \tag{7}$$

at trading times. Consequently, even immediately before a trade takes place, the expected risk-adjusted prices remain continuous. This condition hold because any jump in the market maker's marginal utility of consumption (due to a jump in consumption) is offset by a jump in actual asset prices.

We now proceed to calculate the equilibrium price in this economy by first solving for the risk-adjusted prices p_i, and then using the definitions (4) to obtain the true prices P_i.

Given our discussion above, the state of the economy from the market makers' point of view is described by the vector $[\xi, S, B, Y]$, where the variable Y summarizes the probabilistic structure of trade. Hence risk-adjusted and goods-denominated asset prices have the form $p_i = p_i(\xi, S, B, Y)$ and $P_i = P_i(\xi, S, B, Y)$, for $i = s, b$.

To solve equation (5), it is convenient and intuitive to decompose the risk-adjusted prices in the form

$$p_i(\xi, S, B, Y) = g_i(\xi, S, B) + h_i(\xi, S, B, Y), \quad i = s,\ b. \tag{8}$$

The function g_i takes the market makers' portfolio composition as given, and would be the asset's equilibrium value if future trade could be disregarded. Conversely, h_i reflects the effects of trade on the asset's price, and can be viewed as the value of trading the asset at some point in the future. This decomposition corresponds to the particular and homogeneous solutions of the differential equation (5) which describes risk-adjusted asset prices.

Solving for g_i is straightforward; if no future trade is possible, then the risk-adjusted price of an asset is simply the present value of its future dividend flow discounted at the rate ρ. Recognizing that $c = S\xi + Br$, we get

$$g_i = \int_t^{\infty} E_t[f_i(\tau)|\text{no trade}]e^{-\rho(t-\tau)}d\tau, \quad i = s,\ b, \tag{9}$$

where $f_s(\tau)|\text{no trade} \equiv U'(S_t\xi_\tau + B_t r)\xi_\tau$, and $f_b(\tau)|\text{no trade} \equiv U'(S_t\xi_\tau + B_t r)r$.

2.5 A Specialized Framework

Consider a market maker with the CRRA utility function

$$U(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where γ is the relative risk aversion parameter, and assume that dividends follow a lognormal diffusion process

$$d\xi = \mu\xi dt + \sigma\xi dw,$$

with μ and σ positive constants. We further assume that the risk-adjusted stock price p_s is a monotonic function of the dividend process ξ. We will see that these assumptions are simplifying but not essential.

By applying Ito's rule to calculate $E_t(dp_i)$ in (5), we find that in the interior of no-trade regions the prices p_i satisfy the valuation equations

$$\rho p_i = f_i + \mu\xi p_i' + \frac{\sigma^2\xi^2}{2}p_i'', \quad i = s,\ b, \tag{10}$$

with boundary conditions given by the transversality condition (equation (6)) and by the no-expected-jump conditions (equation (7)).

When either $B = 0$ or $S = 0$, the solution for the no-trade component g_i in (8) can be obtained from direct evaluation of the integral in (9). Under these assumptions we have

$$B = 0 \;\Rightarrow\quad g_s = \frac{\xi}{(S\xi)^\gamma d_s}, \quad g_b = \frac{r}{(S\xi)^\gamma d_b}, \tag{11}$$

where $d_s \equiv [\rho + (\gamma - 1)\mu - (\gamma - 1)\gamma\sigma^2/\ 2]$, $d_b \equiv [\rho + \gamma\mu - \gamma(\gamma + 1)\sigma^2/2]$; and

$$S = 0 \Rightarrow \quad g_s = \frac{\xi}{(\rho - \mu)(Br)^\gamma}, \quad g_b = \frac{r}{\rho(Br)^\gamma}. \tag{12}$$

To ensure that prices are positive and finite, parameter values must be such that $d_s > 0$, $d_b > 0$, and $\rho > \mu$. We can immediately verify that solutions (11) and (12) solve the equation (10) and satisfy the transversality condition (6).

The second component of asset prices in (8), h_i, can be found as the homogeneous solution of (10). The solution has the form

$$h_i = Q_i(S, B, Y)\xi^{\lambda_1} + N_i(S, B, Y)\xi^{\lambda_2}, \quad i = s,\ b,$$

where λ_1 and λ_2 solve the characteristic equations associated with equation (10)

$$\lambda_1 \;=\; \frac{-(\mu - \sigma^2/2) + \sqrt{(\mu - \sigma^2/2)^2 + 2\rho\sigma^2}}{\sigma^2} > 0$$

$$\lambda_2 \;=\; \frac{-(\mu - \sigma^2/2) - \sqrt{(\mu - \sigma^2/2)^2 + 2\rho\sigma^2}}{\sigma^2} < 0,$$

and Q_i and N_i are constants of integration.

We can show that the transversality conditions (6) require $Q_i = N_i = 0$ if no trades are expected. First, compute

$$E_t(\xi_\tau^\lambda)e^{-\rho(\tau-t)} = \xi_t^\lambda e^{[(\sigma^2/2)\lambda^2+(\mu-\sigma^2/2)\lambda-\rho](\tau-t)}.$$

Since λ_1 and λ_2 solve $(\sigma^2/2)\lambda^2 + (\mu - \sigma^2/2)\lambda - \rho = 0$, we have

$$\lim_{\tau\to\infty} E_t[h_i(\xi_\tau)]e^{-\rho(\tau-t)} = Q_i\xi_t^{\lambda_1} + N_i\xi_t^{\lambda_2}, \quad i = s,\ b.$$

Hence, h_i satisfies the transversality conditions only if $Q_i = N_i = 0$.

Having solved for the components g_i and h_i, we can compute the risk-adjusted prices p_i. Goods-denominated prices P_i can be computed from the risk-adjusted prices p_i by the relation $P_i = p_i c^\gamma$, with $c = S\xi$ in the $B = 0$ case, and $c = Br$ in the $S = 0$ case.

3 One-Shot Trades

In this section we consider the simple case where trade occurs only once when the stock price hits a lower or an upper price barrier. We lay out the structure of the trades, and we describe the solution technique. The intuition for the effects of price-triggered trades is first developed for and then extended to stochastic trades.

3.1 Trade, Asset Allocations, and Boundary Conditions

We assume buy-and-hold investors to trade only once, at either of two stock price barriers, a lower barrier, P_{sl}, and an upper barrier P_{su}. Since we restrict ourselves to strictly monotonic price correspondences, in equilibrium these price levels uniquely correspond to the dividend levels ξ_l and ξ_u, as implied by the identities $P_{sl} \equiv P_s(\xi_l, S_0, B_0, Y_0)$ and $P_{su} \equiv P_s(\xi_u, S_0, B_0, Y_0)$. Here $S = S_0$ and $B = B_0$ denote the amounts of stocks and bonds held by market makers before the trade, respectively, and $Y = Y_0$ represents the market maker's information that neither price barrier has been reached.

In general, whether the price barriers triggers purchases or sales of stocks is uncertain. For tractability, we assume that the after-trade asset allocations are extreme: after the trade market makers either hold only stocks or only bonds in their portfolios. Specifically, when P_s hits the lower threshold P_{sl}, buy-and-hold traders buy all market makers' bonds with probability π_l, and buy all market makers' stocks with probability $(1 - \pi_l)$. At the upper threshold P_{su}, the same trades occur with probability π_u and $(1 - \pi_u)$, respectively. When either threshold is reached, both barriers disappear forever, and the market maker's information set is updated to $Y = Y_1 = Y_{1u}$, or Y_{1l}. The after-trade market makers' asset allocations are denoted $S = S_1$ and $B = B_1$.

The assumption that after the trade market makers may hold either only stocks ($B_1 = 0$) or only bonds ($S_1 = 0$) is quite convenient: it allows us to use the closed-form solutions (11) and (12) of the previous section to calculate after-trade allocations, and to impose the no-expected-jump boundary conditions (equation (7)) on before-trade prices.

Consider the case where trade takes place at the upper price barrier $P_s = P_{su}$, and all market makers' bonds are exchanged for stocks. Using the market makers' budget constraint and the solutions (11) and (12) we obtain

$$S_1 = S_0 + \frac{P_b(\xi_u, S_1, 0, Y_{1u})}{P_s(\xi_u, S_1, 0, Y_{1u})} B_0 = S_0 + \frac{p_b(\xi_u, S_1, 0, Y_{1u}) c^{-\gamma}}{p_s(\xi_u, S_1, 0, Y_{1u}) c^{-\gamma}} B_0$$

$$= \quad S_0 + \frac{p_b(\xi_u, S_1, 0, Y_{1u})}{p_s(\xi_u, S_1, 0, Y_{1u})} B_0 = S_0 + \frac{r d_s}{\xi_u d_b} B_0, \tag{13}$$

while $B_1 = 0$. Similar expressions hold when all market makers' stocks are exchanged for bonds ($S_1 = 0$), and when trade takes place at the lower price barrier P_{sl}.

Consider now the no-expected-jump conditions (7). At the upper price barrier, we have

$$p_i(\xi_u, S_0, B_0, Y_0) = \pi_u p_i(\xi_u, S_1, 0, Y_{1u}) + (1 - \pi_u) p_i(\xi_u, 0, B_1, Y_{1u}), \quad i = s,\ b; \tag{14}$$

a similar condition holds at the lower price barrier. Given (11) and (12), the right-hand side of (14) is available in closed form.

3.2 Approximate Solutions

In general, both initial asset allocations S_0 and B_0 are different from zero, and hence closed-form expressions for $p_i(\xi, S_0, B_0, Y_0)$, $i = s,\ b$, are elusive. Still, solutions are available in infinite-ser form. For fixed S and B we may write a power-series expansion for f_i as

$$f_i(\xi, S, B) = \sum_{j=0}^{\infty} \frac{1}{j!} f_i^{(j)}(\xi_0, S, B)(\xi - \xi_0)^j, \quad i = s,\ b, \tag{15}$$

where $f_i^{(j)}(\xi_0, S, B)$ is the j^{th} derivative of f_i evaluated at ξ_0. Then solutions to the valuation equations (10) can be written in the form

$$p_i = g_i + h_i = \sum_{j=0}^{\infty} A_{ij}(S, B, \xi_0)(\xi - \xi_0)^j, \quad i = s,\ b, \tag{16}$$

where the A_{ij} functions satisfy a recursion illustrated below.

We rewrite the drift and variance of $d\xi$ as

$$\mu\xi \equiv m_0 + \mu(\xi - \xi_0)$$

and

$$\sigma^2\xi^2 \equiv v_0 + v_1(\xi - \xi_0) + \sigma^2(\xi - \xi_0)^2,$$

where

$$m_0 = \mu\xi_0, \qquad v_0 = \sigma^2\xi_0^2, \qquad v_1 = 2\sigma^2\xi_0.$$

Here, we have expressed the drift and variance in the form of a (exact) Taylor expansion around a particular value of the dividend, ξ_0.

We substitute the approximations (15) and (16) in equation (10). By matching the coefficients of ξ^0 $(=1)$, ξ, ξ^2, etc. we obtain the following recursion for the A_{ij} coefficients:

$$A_{ij} = \frac{f_i^{(j)}/j! + m_0(j+1)A_{ij+1} + \frac{1}{2}\left[v_0(j+2)(j+1)A_{ij+2} + v_1(j+1)jA_{ij+1}\right]}{\rho - \mu j - (\ \sigma^2/2)j(j-1)}, \quad i = s,\ b. \quad (17)$$

In practice, we limit the number of terms in the Taylor expansion (15) to $D+1$. In turn, the recursion (17) produces the coefficients A_{ij}, $j = 0, \ldots, D$ for given $A_{i,D+1}$ and $A_{i,D+2}$. The choice of values for $A_{i,D+1}$ and $A_{i,D+2}$ is arbitrary, and it is equivalent to the choice of values for the constants of integration Q_i and N_i in the homogeneous solution h_i.

In principle, we could use the boundary conditions (14) to determine $A_{i,D+1}$ and $A_{i,D+2}$ directly. Computationally, it is more convenient to take the following, equivalent, route. We set $A_{i,D+1} = A_{i,D+2} = 0$ and then we calculate the $D+1$ coefficients A_{ij} using (17). We then add to the approximate solution $\sum_{j=0}^{D} A_{ij}(S,B,\xi_0)(\xi - \xi_0)^j$ the homogeneous solution $Q_i(S,B,Y)\xi^{\lambda_1} + N_i(S,B,Y)\xi^{\lambda_2}$, and the constants Q_i and N_i are then set to satisfy the boundary conditions (14) at the lower and upper price barrier. Finally, P_i can be computed using the relation $P_i = p_i c^{\gamma}$.

The technique described above starts with the dividend levels ξ_l and ξ_u and solves for the before-trade prices as a function of the dividend level and the initial trade allocations. In order to calculate the equilibrium price in the presence of given barriers, we exploit the equivalence between price and dividend barriers. We start by conjecturing the values of the dividend barriers associated with the price barriers. Given the dividend barriers, we compute the boundary conditions for the risk-adjusted prices and derive the risk-adjusted pricing function. Given the risk-adjusted price and the market makers' marginal utility, we calculate the implied price associated with the dividend barriers. If these prices do not coincide with the postulated price barriers, we update the conjecture about the trigger values of the dividends. The procedure is iterated until we obtain the correct price barriers.

It is worth noting that the illustrated solution technique can be extended to obtain solutions to the ordinary differential equation (10) under general assumptions on preferences and stochastic processes, as long as the fundamentals, f_i, and the drift and diffusion terms of $d\xi$ are real analytic functions of ξ.

3.3 Deterministic Trades

Using the solution techniques of the previous section, we can solve for stock prices in equilibrium under a variety of trading scenarios. We first consider the situation in which the barrier probabilities π_l, π_u are either zero or one; *i.e.*, the market makers know with certainty whether the buy-and-hold investors buy or sell stocks when the barrier is hit.

Two interesting scenarios are depicted in Figure 1: in the left panel, buy-and-hold investors buy all the market maker's bonds when the stock price reaches a lower barrier, *i.e.* they *sell* stocks when stock prices are *low.* They buy all the market maker's stocks when the stock price reaches an upper barrier, *i.e.* they *buy* stocks when stock prices are *high.* This scenario, which we refer to as "Sell Low, Buy High," can be viewed as the buy-and-hold investors chasing an upward trend in stock prices, and taking profits at an upper price target, but selling at a lower price to stop future losses. The complementary strategy, "Buy Low, Sell High," is shown in the right panel.[3]

In both scenarios, a jump in prices occurs when the price barrier is hit. At first glance, the direction of this jump seems counterintuitive: the price jumps downwards when buy-and-hold investors reduce the supply of stocks, and upwards when the supply is increased. In actuality, however, the price pressure associated with the impending purchase or sale is being impounded into the asset's price *before* the trade; consequently, pre-trade asset prices are either inflated or depressed relative to their post-trade values. After the trade, there is no possibility of future trades by the buy-and-hold traders, so price pressure dissipates, and asset prices return to their equilibrium no-trade values.

A number of papers have presented empirical evidence that jumps exist in asset prices (see, for example, Jorion (1988), Ball and Torous (1985), Jarrow and Rosenfeld (1984)), and several authors, starting with Merton (1976), have based option-pricing formulas on jump-diffusion models of asset prices. Naik and Lee (1990) and Ahn and Thompson (1988) have presented a theoretical justification for jump-diffusion models of asset pricing by assuming that the dividend process follows a jump-diffusion process, and showing that this leads to the occurrence of jumps in asset prices in equilibrium. A characteristic of all these models is that the diffusion component of the asset-price process is independent of the jump component. Our model, in contrast, shows that jumps in asset prices can arise even under the assumption of a continuous diffusion process for dividends. It is the infrequent trading of the buy-and-hold investors which leads to the occurrence of the jumps. The jumps are therefore necessarily correlated with the diffusion component of the prices.

[3]Analogously, one could consider two more scenarios, a "Buy Low, Buy High" and a "Sell Low, Sell High" scenario (not shown in the figure), where buy or sell trades occur as the stock price exits from a prespecified range. One can view the "Buy Low, Buy High" trading pattern as the buy-and-hold investors perceiving the passage through the upper barrier as a signal of a bullish trend, while perceiving the lower barrier as a "support level" below which the stock is undervalued. Similarly, the "Sell Low, Sell High" pattern can be interpreted as the holders of the stock taking profits at the high barrier and cutting potential future losses at the lower barrier.

The mechanism of the price jump arises from the interaction of market makers and buy-and-hold investors in equilibrium. Consider the "Sell Low, Buy High" (left) case. When prices reach the upper barrier, a large purchase of stocks occurs. Exchange at equilibrium prices suddenly reduces the market makers' consumption, and consequently leads to an increase in the market maker's marginal utility. Since risk-adjusted prices cannot jump (note that in this case the trade is certain) the increase in the market makers' marginal utility translates into a fall in market prices.

Our model predicts another important effect of trade by buy-and-hold investors. Referring back to Figure 1, it is apparent that the volatility of stock prices, deriving from the volatility of the dividend stream, is greater before the price hits either barrier, and thereafter reverts to its no-trade value. We can explain this effect in terms of the uncertainty within the market before the trade with regard to the future supply of stocks. After the trade, the asset supply is stable and thus the uncertainty is resolved, so the excess volatility disappears.

The relationship between price levels and volatility, dubbed the "leverage effect," has been observed repeatedly in asset prices, particularly within the GARCH-M econometric modeling framework.[4] This feature is consistent with an economy similar to ours, where large trades are submitted infrequently, and where the occurrence of a large trade reduces the likelihood of another large trade taking place in the near future.

3.4 Stochastic Trades

We now consider the effect of adding uncertainty. Let us modify the two scenarios of Figure 1 in the following way: at each price barrier, if the buy-and-hold investor is destined to buy, we now assume that the market makers believe he buys bonds with a probability of 80% and sells bonds with a probability of 20% (and conversely for sell barriers). These new scenarios represent situations in which market-makers know that buy-and-hold traders are going to trade, but are not certain of the net asset demands that will be generated when the barrier is breached.

These two new scenarios are illustrated in Figure 2. When uncertainty is introduced,

[4]See Bollerslev, Chou, and Kroner (1992) for a survey. The GARCH-M family of models can be viewed as discrete-time approximations to "stochastic volatility" diffusion processes for asset prices (see Nelson (1990)). Our model suggests that a more appropriate econometric framework would be one in which the limiting continuous-time process has volatility following a jump-diffusion process rather than a diffusion process. Brown, Harlow, and Tinic (1993), for example, find that a significant proportion of stock price jumps are followed by lower volatility, and, when volatility does drop after a large price movement, it tends not to revert to its pre-jump value.

there is still the effect on price volatility that was observed in the certainty case. However, in this scenario stock prices jump upwards when stocks are bought, and downwards when stocks are sold. In contrast to the certainty case, there is now price pressure in two opposite directions, because of the possibility of either a buy or sell trade to be executed. It is the relative importance of these two effects which determines the magnitude and direction of the price jump.[5]

3.5 Sensitivity Analysis

Obviously, the before-trade and after-trade prices are affected by the choice of parameters which characterize the market maker's preferences. For example, a higher value of ρ implies that the market maker discounts more future consumption (*i.e.*, savings) relative to current consumption, so that for any dividend level the prices of all assets, both before and after trade, are lower. Conversely, a lower value of ρ corresponds to higher utility from future consumption, and thus higher asset prices. Changes in the parameter γ, which regulates relative risk aversion and the elasticity of intertemporal substitution, have the following effect: An increase in γ induces a relative strengthening of the price pressure from a possible purchase of stocks on the part of the buy-and-hold investors. An increase in γ can be thought of as a reduction in the "depth" of the market: when $\gamma = 0$, buy-and-hold trades are absorbed with no effect on price. As a result, the before-trade price function may lie above both after-trade functions for the whole range of values of ξ. When γ is reduced, the relative strength of the price pressure from a possible stock purchase is smaller, and the before-trade price function falls in between the two after-trade functions.

Similarly, prices are affected by the choice of parameters which characterize the stochastic process for ξ. A higher drift parameter, μ, corresponds to higher expected cash-flows over time, which leads to higher prices for stocks, while lower dividend growth leads to lower stock prices. The effect of changing σ^2 is similar to that of changing γ. An increase in σ^2 strengthens the relative price pressure of a possible stock purchase, while a decrease in σ^2 weakens such price pressure.

[5]Similar arguments hold for two other scenarios not shown in the figure: In a "Buy Low, Sell Low" scenario when a sell trade is possible, the price pressure from the buy trade is reduced and the price function is shifted downwards. On the other hand, in a "Sell Low, Sell High" scenario the possible buy trade reduces the sell-trade price pressure, and moves up the before-trade price function.

4 Repeated Trades

So far, we have developed the intuition for the effects of price barriers by using a stylized framework in which trade occurs only once, when a barrier is hit. In order to introduce additional elements of realism, we now consider the case of *repeated* trades and *multiple* price barriers. Specifically, we assume a scenario of two rounds of trade, which combines the deterministic trades of Section 3.3 (in the first round) with the stochastic trades of Section 3.4 (in the second round).

In all the exercises of this section, we assume the market makers' initial inventories of stocks and bonds to be $S = S_0 = 1$ and $B = B_0 = 1$. Before the first barrier is hit, there are two stock-price barriers, a lower barrier, P_{sl}, and an upper barrier P_{su}. After the first barrier is hit, buy-and-hold investors trade again when the price reaches two new stock-price barriers. If the first barrier breached is the lower one, P_{sl}, the new lower and upper barriers become $P_{s,ll}$ and $P_{s,lu}$, respectively. If the upper barrier P_{su} is breached first, buy-and-hold traders trade again at the new lower barrier $P_{s,ul}$ and at the new upper barrier $P_{s,uu}$. After the second trade, no more trades take place. As in the analysis of Section 3.3, we assume the first trade to be deterministic, but to lead to *strictly positive* new allocations of stocks and bonds: $S_1, B_1 > 0$. The second trade, on the other hand, is stochastic as in the analysis of Section 3.4, and leads to *extreme* after-trade asset allocations: either $S_2 = 0$ or $B_2 = 0$. We denote with π the probability that buy-and-hold traders purchase all the bonds in the hands of the market makers (and $S_2 = 0$) at the time of the second trade. Specifically, we have the four probabilities π_{ll}, π_{lu}, π_{ul}, π_{uu} at the corresponding four price barriers.

To solve for stock prices when there are multiple trades, the solution technique developed in the previous sections must be extended. In the single-trade case, the amount of bonds traded in exchange for stocks is easily computed, given the size of the sell/buy order of the buy-and-hold traders. In fact, the relative price of stocks and bonds is determined by the after-trade pricing functions, which, given our assumption of extreme final asset allocations, are available in closed-form and *do not depend* on the asset allocation. However, in the case of multiple trades, the number of stocks and bonds exchanged at the intermediate barriers and the price levels at the barriers are interdependent, so we must use an iterative technique. Assuming that the number of stocks traded by buy-and-hold investors at the first barrier is given, we start by conjecturing the number of bonds exchanged for stocks at the first barrier. Using this value, we find the pricing function for the second stage as in the previous sections. This pricing function, evaluated at the first barrier, allows us to determine the relative prices for stocks and bonds at the first barrier. Using the relative prices and the initial allocation, we update our conjecture about the number of bonds traded at the first barrier. The procedure is iterated until convergence. With this in hand, we solve for the pricing function prior to the first barrier.

Scenario I, illustrated in Table 1, uses the solution technique above and assumes the first lower price barrier to become the new upper price barrier: $P_{sl} = P_{s,lu}$. Similarly, we have $P_{su} = P_{s,ul}$. Also, we assume buy-and-hold investors to engage in trend-chasing trades: "Sell Low, Buy High." In fact, they purchase stocks with certainty (high probability) at the first (second) upper barrier, and they sell stocks with certainty (high probability) at the first (second) lower barrier. The resulting pricing functions are presented in Figure 5.

Table 1: Repeated Trades, Scenario I

		1st trade, deterministic		2nd trade, stochastic, extreme alloc.
				$P_{s,uu} = 100$, buy stocks, prob. = 0.8
			↗	
	↗	$P_{su} = 75$, buy 0.1 stocks	→	$P_{s,ul} = 75$, sell stocks, prob. = 0.8
P_{s0}, $S_0 = B_0 = 1$				
	↘	$P_{sl} = 50$, sell 0.1 stocks	→	$P_{s,lu} = 50$, buy stocks, prob. = 0.8
			↘	
				$P_{s,ll} = 25$, sell stocks, prob. = 0.8

One interesting feature of the equilibrium is that even if buy-and-hold investors follow trend-chasing strategies, prices react less to dividends before the first round of trade (first panel from the left) than before the second round of trade (second and third panels). This is for the following reason. Approaching the lower barrier, for example, not only signals that stocks sales on the part of buy-and-hold traders are imminent, but also that there may be an immediate reverse trade, since the barrier may be crossed back soon prompting a larger trade in the opposite direction. Since first-round trades are small relative to the initial allocations, portfolio holdings and consumption of the market makers is little affected by the first trade. This implies "small" price jumps at the first barriers. Hence, the before-first-trade pricing function essentially joins the after-first-trade functions, and to do this it must be *flatter* than the steep after-first-trade pricing functions. As a result, this sequence of trend-chasing trades leads to a before-first-trade pricing function, which is quite similar to that of the one-shot *contrarian-strategy* scenario "Buy Low, Sell High" of Section 3.3. Moreover, we have the counterintuitive result that stock prices are inflated, rather than depressed, in the proximity of a stock sale.

This theoretical evidence provides useful insights on the empirical findings of Donaldson and Kim (1993) and Ley and Varian (1994) regarding the behavior of the Dow Jones Industrial Average index. Both studies find that the distribution of the Dow-Jones' digits is decidedly non-uniform; specifically, Donaldson and Kim (1993) find that the Dow-Jones's rise and fall is restrained by barrier support and resistance levels, but that, having breached a barrier, the Dow Jones moves up or down more than usual. This evidence is consistent, for example, with the scenario illustrated in Figure 5, where prices move little in response to cash flows before either barrier is breached (the upper and lower barrier are a resistance and a support level, respectively), while they react strongly after the barrier is breached (prices move up or down more than usual).

We can contrast the previous result with that of a second scenario, Scenario II, which represents a situation in which a drop in the stock price signals investors to pursue a contrarian strategy. This situation is entirely analogous to Scenario I, but for the fact that buy-and-hold investors engage in contrarian trades if the first trade takes place at the lower barrier P_{sl}: they buy stocks with high probability at the second lower barrier $P_{s,ll}$, and they sell stocks with high probability at the second upper barrier $P_{s,lu}$. The scenario is illustrated in Table 2, and the corresponding pricing functions are presented in Figure 6.

Table 2: Repeated Trades, Scenario II

		1st trade, deterministic		2nd trade, stochastic, extreme alloc.
				$P_{s,uu} = 100$, buy stocks, prob. = 0.9
			↗	
	↗	$P_{su} = 75$, buy 0.1 stocks	→	$P_{s,ul} = 75$, sell stocks, prob. = 0.6
P_{s0}, $S_0 = B_0 = 1$				
	↘	$P_{sl} = 50$, sell 0.1 stocks	→	$P_{s,lu} = 50$, sell stocks, prob. = 0.8
			↘	
				$P_{s,ll} = 25$, buy stocks, prob. = 0.8

Now the pricing function before the first round of trade (first panel from the left) has an average slope which is intermediate relative to the two pricing functions after the first round of trade (second and third panel). This is because approaching the lower barrier is not only a signal of an imminent sale of stocks, but also of a likely larger trade in the opposite direction. As a result, the shape of the before-first-trade pricing function is more similar (relative to the previous scenario) to that of the one-shot trend-chasing scenario "Sell Low, Buy High," of Section 3.3. This highlights the ambiguous effects of "trend-chasing" and "contrarian" strategies when the old barriers, after being breached, become *new* barriers for future price-triggered trades.

These results depend strongly on the distance between the barriers and the probability of an imminent breach. For example, different implications arise in the next scenario, Scenario III, where buy-and-hold investors engage in a sequence of trend-chasing trades similar to that of Figure 5, but where the old barriers *disappear* after being breached, and where the new barriers arise at *different* price levels. The structure of trades is described in Table 3 and the corresponding pricing functions are presented in Figure 7.

Table 3: Repeated Trades, Scenario III

		1st trade, deterministic		2nd trade, stochastic, extreme alloc.
				$P_{s,uu} = 100$, buy stocks, prob. = 0.8
			$\nearrow$	
	$\nearrow$	$P_{su} = 80$, buy 0.1 stocks	$\searrow$	$P_{s,ul} = 60$, sell stocks, prob. = 0.8
P_{s0}, $S_0 = B_0 = 1$				
	$\searrow$	$P_{sl} = 40$, sell 0.1 stocks	$\nearrow$	$P_{s,lu} = 60$, buy stocks, prob. = 0.8
			$\searrow$	
				$P_{s,ll} = 20$, sell stocks, prob. = 0.8

In this scenario, when prices approach the first barrier and the first round of trade, there is no presumption of an immediate (possibly reverse) future trade, since the second-round trades are "far" (in the ξ dimension) from the first-round trades. Hence, the before-first-trade pricing function resembles that of the one-shot "Sell Low, Buy High" scenario of Section 3.3.

5 Summary and Conclusions

In this paper we present a model of asset prices in which "buy and hold" investors enter the market only when prices reach either a lower or an upper price barrier. Our analysis shows that the risk-neutral asset prices in this economy must follow a partial differential equation with a specific set of boundary conditions. We derive a solution technique based on an infinite-series approach, and use it to solve for asset prices under several scenarios, which incorporate certainty and uncertainty of trading at price barriers, and repeated trades at multiple price barriers.

Our model shows that the presence of infrequent buy-and-hold investors in a market can influence the dynamics of asset prices in several important ways. First, when buy-and-hold investors enter the market, asset prices exhibit jumps whose magnitude and direction depends on the probabilities with which different trades are expected to occur at the price barrier. Depending on the stochastic structure of trade, the jumps may be in a direction counter to that of the trade; *e.g.*, a large buy order could actually cause prices to jump downwards. A second implication of our model is that the volatility of the price process can also jump at the time of the trade, and jumps in prices and in volatility are correlated with each other and with the level of asset prices. Hence, our model can explain how discontinuous price and volatility behavior can arise even when the flow of payoffs is continuous. Third, prices are affected by the presence of buy-and-hold investors before they enter the market: prices tend to be inflated if buy orders are expected, while they tend to be depressed if sell orders are expected. Fourth, in the case of repeated trades and multiple price barriers, the before-first-trade pricing functions are affected not only by the anticipation of the next trade, but also by the anticipation of the following ones. This may lead to the counterintuitive result that stock prices are inflated, rather than depressed, in the proximity of a stock sale.

These features of the modeled equilibrium are empirically testable, and are in fact consistent with many empirical studies of the distributional characteristics of asset prices. Our analysis motivates the econometric consideration of a jump-diffusion model of asset prices in which both the jump component and the instantaneous volatility are correlated with the price. Such an approach towards the specification of the asset price process could have important implications for the study of derivative securities, for example.

References

Admati, A.R., and P. Pfleiderer, 1991, Sunshine trading and financial market equilibrium, *Review of Financial Studies* 4, 443-481.

Ahn, C. M., and H. E. Thompson, 1988, Jump-diffusion processes and the term-structure of interest rates, *Journal of Finance* 43, 155-174.

Balduzzi, P., G. Bertola, and S. Foresi, 1995, Asset price dynamics and infrequent feedback trades, *Journal of Finance* 50, 1747-1766.

Ball, C., and W. Torous, 1985, On jumps in common stock prices and their impact on call pricing, *Journal of Finance* 40, 155-173.

Basak, S., 1995, A General equilibrium model of portfolio insurance, *Review of Financial Studies* 8, 1059-1090.

Bjerring, J.H., J. Lakonishok, and T. Vermaelen 1983, Stock prices and financial analysts's recommendations, *Journal of Finance* 38, 187-204.

Blume, L., D. Easley, and M. O'Hara, 1994, Market statistics and technical analysis: The role of volume, *Journal of Finance* 49, 153-181.

Bollerslev, T., R. Y. Chou, and K. F. Kroner, 1992, ARCH modeling in finance: a review of the theory and empirical evidence, *Journal of Econometrics* 52, 5-60.

Brown, K. C., W. V. Harlow, and S. M. Tinic, 1993, The risk and required return of common stock following major price innovations, *Journal of Financial and Quantitative Analysis* 28, 101-116.

Campbell, J.Y., and A.S. Kyle, 1993, Smart money, noise trading, and stock price behavior, *Review of Economic Studies* 60, 1-34.

Constantinides, G. M., 1986, Capital market equilibrium with transactions costs, *Journal of Political Economy* 94, 842-862.

Davis, M. H. A., and A. R. Norman, 1990, Portfolio selection with transactions costs, *Mathematics of Operations Research* 15, 677-713.

DeLong, J.B., A. Shleifer, L.H. Summers, and R.J. Waldmann, 1990, Positive feedback investment strategies and destabilizing rational speculation, *Journal of Finance* 45, 379-396.

Dixit, A., 1993, The art of smooth pasting, Harwood Academic Publishers, Chur, Switzerland.

Donaldson, R. G., and H. Y. Kim, 1993, Price barriers in the Dow Jones Industrial Average, *Journal of Financial and Quantitative Analysis* 28, 313-330.

Donaldson, R. G., and H. Uhlig, 1993, The impact of large portfolio insurers on asset prices, *Journal of Finance* 48, 1943-1955.

Duffie, D. and T. Sun, 1990, Transactions costs and portfolio choice in a discrete-continuous-time setting, *Journal of Economic Dynamics and Control* 14, 35-51.

Dybvig, P.H., 1995, Ratcheting, consumption and portfolios, *Review of Economic Studies* 62, 287-313.

Gennotte, G., and H. Leland, 1990, Market liquidity, hedging, and crashes, *American Economic Review* 80, 999-1021.

Grossman, S. J., and Z. Zhou, 1996, Equilibrium analysis of portfolio insurance, *Journal of Finance* 51, 1379-1403.

Jarrow, R., and E. Rosenfeld, 1984, Jump risks and the intertemporal capital asset pricing model, *Journal of Business* 57, 337-351.

Jorion, P., 1988, On jump processes in the foreign exchange and stock markets, *Review of Financial Studies* 1, 427-445.

Ley, E., and H. R. Varian, 1994, Are there psychological barriers in the Dow-Jones index?, *Applied Financial Economics* 4, 217-224.

Merton, R. C., 1971, Optimum consumption and portfolio rules in a continuous time model, *Journal of Economic Theory* 3, 373-413.

Merton, R. C., 1976, Option pricing when underlying stock returns are discontinuous, *Journal of Financial Economics* 3, 125-144.

Naik, V., and M. Lee, 1990, General equilibrium pricing of options on the market portfolio with discontinuous returns, *Review of Financial Studies* 3, 493-522.

Nelson, D. B., 1990, ARCH models as diffusion approximations, *Journal of Econometrics* 45, 7-38.

Neftci, S.N., 1991, Naive trading rules in financial markets and Wiener-Kolmogorov prediction theory: A study of "technical analysis." *Journal of Business* 64, 549-571.

Stickel, S. E., 1995, The influence of brokerage house buy and sell recommendations on stock prices, Working paper, La Salle University.

Shleifer, A., and L.H. Summers, 1990, The noise trader approach in finance, *Journal of Economic Perspectives*, 4, 19-33.

Womack, K. L., 1996, Do brokerage analysts' recommendations have investment value?, *Journal of Finance 51*, 137-167.