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Abstract

Daily data from intra-dealer government bond brokers is examined for tax and liquidity effects. Utilizing actual trade prices rather than dealer estimated quotes gives us a more accurate measure of market clearing prices. Daily trading volume is also available, which provides us with a robust measure of liquidity. We use two approaches, one of which is new, to create cash flow matching portfolios of similar securities and look for pricing discrepancies associated with liquidity or tax effects. We also look for evidence of tax and liquidity effects by including a liquidity term when fitting a cubic spline to the after-tax yield curve. We find evidence of tax timing options and liquidity effects. However, the effects are much smaller than previously reported and the effects of liquidity are primarily due to high volume bonds with long maturities.

The cash flows of non-callable Treasury securities are fixed and certain, simplifying the pricing of these assets to a present value calculation using the current term structure of interest rates. It is well known, however, that pricing errors exist when government securities are priced by discounting the cash flows by any set of estimated spot rates even for non-flower bonds without option features. A number of theories have been offered to explain these pricing discrepancies. Explanations include economic influences such as liquidity effects, tax regime effects, tax clienteles, tax timing options, and the use of bonds in the overnight repurchase market. Another potential source of pricing errors is data problems that arise from non-synchronous trading and the fact that the prices found in common data sets may be estimates from a model or the best guess of a trader. It is difficult to distinguish between these various explanations because securities rarely exist that are affected by only one of the effects. For example, illiquid securities are likely to be associated with pricing errors due to non-synchronous trading and may also have coupons that would lead to significant tax effects. In addition, it is difficult to sort out the effect of model prices or dealer estimates on pricing errors.

The purpose of this study is to try to separate out the various factors that lead to errors in the pricing of government securities. Our study has three principal advantages over previous work. First, we have access to intra-dealer prices on actual trades, whereas previous studies have examined dealer quotes. As Sarig and Warga (1989) have shown, quotes from a single dealer are often inaccurate because they may be old or model (matrix) prices and not reflective of the actual price that would have cleared the market. Sarig and Warga compare prices found on the quote sheets of two major Treasury dealers, Shearson Lehman and Solomon Brothers (from the CRSP file). They find that more than 20% of the notes and 60% of the bonds have prices that differ by more than 20 basis points. As shown in Section 4, this difference is twice as large as the size of the average pricing error we find using estimated spot rates. Moreover, they show that the inaccuracy is related to variables like liquidity in a way that could seriously bias the results of studies using dealer quotes. In addition to trade prices, we also observe the time to the nearest second at which the trades occur so we can measure any amount of non-synchronousness in the prices. The second strength of our study is that we have access to daily trading volume in the intra-dealer broker market. Trading volume is a more robust measure of asset liquidity than other proxies used in previous studies such as age and type of security. Third, the trades are recorded on a daily basis, which provides us with a large number of observations within the same economic environment. Previous authors have used more limited data, and this has led them to study only one potential source of pricing error. Our much larger data set allows us to distinguish between the effects of data problems and various economic influences, and to differentiate among the economic explanations themselves, such as liquidity, tax effects, and repo specials. Overall, having access to daily trade data from the intra-dealer broker market gives us a unique opportunity to examine the effects of liquidity and taxes on a broad range of maturities.

Our evidence suggests that liquidity is a significant determinant in the relative pricing of Treasury bonds, but its role is much less than previously reported and primarily associated with highly liquid bonds with long maturities. In addition, we confirm the work of Green and Oedegaard (1996) in that we find tax clienteles do not substantially impact bond prices. However, we stop short of declaring that taxes are irrelevant in the Treasury market. Our arbitrage tests provide evidence that tax timing options do have value, and we also discuss the shortcomings of procedures to estimate the tax rate of the marginal investor. Nonetheless, we find the effects of both liquidity and taxes to be quite small, which suggests that a broader sample can be used to estimate empirical term structure models. Practitioners fitting the yield curve commonly restrict their data sets to bonds they believe have small liquidity and tax effects. Our evidence suggests many more bonds can be included, which should reduce estimation error.

The effect of liquidity on the expected return of stocks has been studied by Amihud and Mendelson (1986) and Silber (1991). In the corporate bond market, Fisher (1959) has showed that liquidity was one of the determinants of the yield spread between corporate bonds and Treasury securities. In the Treasury market, Amihud and Mendelson (1991), Warga (1992), Garbade (1996), Garbade and Silber (1979) and Kamara (1994) have all studied aspects of liquidity and expected returns. The effects of tax clienteles and the tax rate of the marginal investor in the government bond market have been examined by Green and Oedegaard (1996), Litzenberger and Rolfo (1984a), and Schaefer (1982). In addition, Ronn and Shin (1993), Jordan and Jordan (1991), Constantinides and Ingersoll (1984), and Litzenberger and Rolfo (1984b), have studied the importance of tax timing options. The effect of repo specialness has been studied by Duffie (1996) and Jordan and Jordan (1996).

The paper is divided into five sections. In the first section we discuss the details of the data. The second section discusses the data used in previous studies and compares our data set to prior data sets. Since we have access to a robust measure of liquidity, the third section examines the reasonableness of the proxies used by others for measuring liquidity. The next two sections examine which factors are important in explaining pricing discrepancies by using arbitrage tests and errors from empirical term structure models. The sixth section reports our conclusions.

I. The Data

The primary data set contains trade prices of Treasury bills, notes, and bonds in the government intra-dealer broker market. According to the Federal Reserve Bulletin, roughly 60% of all Treasury security transactions occur between dealers. Treasury dealers trade with one another through intermediaries called intra-dealer brokers. Dealers use intermediaries rather than trading directly with each other in order to maintain anonymity. Dealers leave firm quotes with brokers along with the largest size at which they are willing to trade. The minimum trade size is one million dollars, and normal units are in millions of dollars. Four of the five brokers representing about 70% of the market, use a computer system managed by GovPX.[1] The GovPX network is tied to each trading desk and displays the highest bid and lowest offer across the four brokers on a terminal screen. When a dealer hits the bid or takes the offer, the broker posting the quote takes a small commission for handling the transaction. In addition to current price quotes, the GovPX terminal reports the last trade timed to the nearest second, as well as the cumulative daily volume for each bond. If the bond has not traded that day, GovPX reports the last day the bond traded.

The data set we examine consists of daily snapshot files provided by GovPX. The daily files contain information on the first trade, the high and low trade, and the last trade (prior to 6:00 EST) stamped to the nearest second, as well as whether the last trade occurred at the bid or offer price. The files also provide daily volume information for each listed security. We have daily data from June 17, 1991 through September 29, 1995. We examine three subsamples of 90 trading days extracted from the full sample. Each subsample is taken from different months in different years so that any calendar effects will affect each subsample differently. In addition to the snapshot files, GovPX provided us with three consecutive days of bid-ask spread information in the intra-dealer market at approximately 10am each day.

II. Comparison with Other Data Sources

Since all previous work that has studied tax and liquidity effects has done so using dealer quotes, either directly from the dealers or indirectly through the Center for Research in Security Prices (CRSP), it is worthwhile examining their origin, their accuracy, and how they compare with the GovPX data.

For much of CRSP history, bond data was taken from the quote sheets of Soloman Brothers. They were also the principal data source used in studies that acquired data directly from a dealer. Soloman Brothers, like Shearson Lehman and other primary dealers actively traded only a portion of the available government bonds (albeit Soloman was the most active dealer). In 1988, CRSP changed the source of their bond data to the Federal Reserve Bank of New York. At the time of the change, they replaced the Soloman data with data from the Fed going back to 1962. The Fed surveys five primary dealers selected at random and creates an equally weighted average the of the five bid and ask quotes. While this method of data collection does average out price noise, it uses data from many dealers who may have little knowledge of actual trades for many of the listed issues, and little incentive to gather more information. Aware of this problem, the Fed has recently switched their method of acquiring quotes to utilizing the electronic feed used in this study.

Two considerations affect whether dealer quotes are reliable indicators of market clearing prices. First, the information set available to traders will help determine whether their quotes reflect market clearing conditions. Second, the incentive structure will also affect whether traders spend time to estimate quotes that are close to prices at which the bonds would actually trade.

The technology was such that until the late 1970's, the information traders received was over the phone resulting from trades they were involved in or quotes from intra-dealer brokers. There was little or no systematic recording of data. In the late 70's and early 80's, CRT screens were introduced and information came across terminal screens placed on trading desks by the intra-dealer brokers, one for each broker. This improvement in technology, along with increased trading in Treasuries, dramatically increased the information set available to traders. However, there remained little systematically recorded data. In June 1991, GovPX was created to supply a consolidated screen for four of the five intra-dealer brokers. This consolidation improved traders ability to process information. Furthermore, this information can be fed into computers which allows for systematic collection. Along with the consolidation of information on Treasury prices, trading volume increased dramatically. The average daily trading volume in January 1970 was \$2.385 billion. It grew to \$17.091 billion in 1980, and by 1990, the daily average was \$117.177 billion. Thus, in recent years all traders are likely to observe current prices.

The accuracy of bid and ask dealer quotes used in previous studies is also dependent on the motivation of traders to supply accurate estimates. Interviews with Soloman Brothers traders of the 70's and 80's revealed that during this time they only estimated bid prices.[2] Likewise, interviews with traders at other primary dealers indicated they also estimated only bid prices or the midpoint between the bid and ask. At the end of every day, traders estimated prices for all Treasury securities. These prices were used for internal inventory valuation purposes and were also supplied to their customers as a non-binding indication of a price range. The traders we interviewed stated that they devoted effort only when estimating the prices of bonds held in their inventory, along with very active issues where dealers were concerned about supplying prices near those at which they might be willing to trade. Prices for illiquid bonds not in their inventory were quickly recorded at rough premiums or discounts to active issues.

What can be learned from this discussion? First, bid-ask spreads used in studies of liquidity were not estimated by traders and were not used by Soloman Brothers when valuing inventory, but instead were clerically added to the data set afterwards. Second, the illiquid bonds including those with high and low coupons used in tax studies, were priced by the trader often without observing recent trades. Furthermore, these were the bonds where less care was used to estimate the price. Thus, we should expect large estimation error in these prices and that these prices reflect what a trader believes is the impact of tax and liquidity on bond prices. The observed variation in dealer estimates lends support to this argument. As mentioned above, the difference in quotes between Shearson Lehman and Soloman were substantial with over 60% of the bonds differing by 20 basis points. To gage the importance of this difference, it is useful to consider the size of tax and liquidity effects reported in other studies. Evidence of tax option effects comes about because of price differences of about 5 basis points, much less than the differences in prices between the two data sets. The liquidity effects measured later are of a similar order of magnitude.

This discussion highlights one of the strengths of using trade prices. Since we observe prices only for those bonds that have traded during the day, some of the less liquid bonds are excluded from the sample on days they did not trade. Thus, examining trade prices is similar to screening out some of the prices on the quote sheets that are not closely related to market clearing prices.

III. Proxies For Liquidity

One of the most common proxies for liquidity has been the bid-ask spread. The rationale is that dealers will require greater compensation for maintaining inventories of illiquid assets, and this will result in larger bid-ask spreads for illiquid securities. However, as mentioned previously, the bid-ask spreads listed in the CRSP data are not market data but instead merely representative spreads.[3] Thus, the use of bid-ask spreads in previous liquidity studies has been inappropriate. Furthermore, the magnitude, characteristics, and determinants of bid-ask spreads in the Treasury market has not been reliably examined before. Table I provides information on the bid-ask spread for the GovPX data. Although we have data for only three days, the bid-ask spread on one day is highly related to the bid-ask spread on the other two days with a simple correlation of over .96. Thus, the bid-ask spread on any one day seems reflective of general conditions, at least over a short period of time. The average bid-ask spread over the three days varies from four-tenths of a cent for the lowest decile to 12.5 cents per $100 for the highest decile, with an average of 5.5 cents.[4]

The existence of bid-ask spreads will introduce price error in trade data since observed trade prices can be either buyer or seller initiated. If trades occur randomly at bid or ask prices we would expect the size of the average error to be about 2.75 cents when examining trade data. Using our empirical term structure model that adjusts for both taxes and liquidity, the estimated root mean squared error (RMSE) is about 13.6 cents, so that bid-ask spread accounts for about 20% of the RMSE.

The right side of Table I shows the results of two regressions which examine how bid-ask spread varies with security characteristics. The results are reported separately for securities that traded on the day of the analysis and securities that did not trade that day. Several variables are used to explore how bid-ask spreads vary across securities. The variable Bond is a dummy variable that is 1 if the instrument is a bond and 0 if it is a bill. For bonds and bills that did not trade, the variable ln (Days) is the log of the number of days since it last traded. These variables along with log volume and years to maturity explain about 80% of the difference in bid-ask spreads across securities. The bid-ask spread is negatively related to volume and positively related to the length of time since the last trade. Furthermore, the bid-ask spread increases with maturity and is larger for bonds than for bills.[5]

In addition to the bid-ask spread, several other variables have been used to measure liquidity. In all cases the authors selected the measure because of its relationship to volume. Since we can measure volume directly, it is instructive to see how related these variables are to volume. For instance, Amihud and Mendelson (1991) and Kamara (1994) examine Treasuries with less than six months to maturity and use the type of security (bond or bill) as a volume proxy.[6] The GovPX data set allows us to examine the relationship of this variable to volume. Table II contains volume information for bills and bonds with less than six months to maturity. The columns represent average daily trading volumes over one day, five day, and ten day measurement intervals, as well as the percentage of bonds and bills that did not trade. Over a one-day measurement interval, 85% of the different issues of bills traded while 71% of the different issues of bonds traded. When examining a five-day interval, 99.6% of the bills traded while 95% of the bonds traded. Thus, bills did trade more frequently. The lower panel of Table II shows the volume percentiles of bills and bonds (all numbers are in millions of dollars face value). When looking at trading volume over a ten day interval, the median trade size in the bill market was $100 million per day, while the median trade size in the bond market was $17 million per day. However, the relationship was not perfect. The top 10% of bonds in trading volume exceeded the lowest 10% of bills; thus liquid short term bonds traded more frequently than illiquid bills.[7] Overall, Table 2 provides evidence that security type is a reasonable volume proxy for maturities of less than six months.

While security type is one of the most often used proxies for volume, other variables have been used as well. Table III shows the results of a regression of log volume on a series of variables used by others as measures of volume. As mentioned above, the bond-bill classification has been used by Amihud and Mendelson (1991), Kamara (1994), and Garbade (1996). The age of a security has been utilized by Sarig and Warga (1989). Warga (1992) uses whether or not an issue is active as a measure of volume. In addition, since Ederington and Lee (1993) and Harvey and Huang (1991) have results that suggest volume differs over the week, we include dummy variables for each day. The set of variables used by others explains a relatively high proportion of the variation in volume across securities. About 45% of the variation is explained, and all variables except the Monday dummy are significant. However, there is a fair amount of variation in volume left to explain.

To provide a better understanding of how volume varies across the term structure, Figure 1 shows the relationship between daily trading volume and maturity for bonds. There is not a monotonic relationship over the full maturity range. Trading volume increases with maturity from six months to two years. Beyond two years, volume is roughly constant and the same as that of bonds with two years to maturity. Overall, the volume proxies used by others are related to volume, but none are highly correlated with volume across all maturities. It appears that no liquidity proxy is as precise as trading volume, and using other proxies could introduce substantial error in measuring the impact of liquidity on prices.

IV. Pricing Errors in Present Values

Although utilizing trade data provides us with a more accurate measure of market clearing prices, errors still exist when discounting cash flows using estimated spot rates. Non-synchronous trading or the existence of random pricing errors are possible explanations which we will explore again later in this section. However, there are economic influences which could also lead to pricing errors, such as liquidity effects, tax effects, and cross-sectional variation in the demand for assets based on their use as collateral in repurchase agreements.

Theory suggests that illiquid bonds will offer higher returns than similar issues that are easily traded. As Amihud and Mendelson argue, the bid-ask spread is part of the cost of trading. In order to compensate market makers for making a market in illiquid assets, and possibly reflecting a lack of trade information to discern the market clearing price of infrequently traded bonds, bid-ask spreads are larger for illiquid bonds. Since illiquid bonds have higher bid-ask spreads, they must offer a higher return before transaction costs to provide the same return after paying transaction costs. Since our pricing formula does not include transaction costs, illiquid bonds should have trade prices below the price estimated using the present value formula.

Taxes may also affect the relative prices of bonds and lead to errors in estimated prices. One way for this to occur is through the presence of tax clienteles. Investors in different tax brackets may desire bonds with different characteristics (see Schaefer (1982)). If the marginal investors for two different bonds are taxed at different rates, this will affect the relative prices of these bonds. Another way in which taxes can affect bond prices is through tax timing options. Tax timing options are associated with the value of being able to time the sale of a bond to optimize the tax treatment of capital gains or losses (see Constantinides and Ingersoll (1984)). It important to note that even if the ordinary income and capital gains tax rates are the same for all investors, taxes may still enter into the relative prices of bonds. For instance, consider three bonds with different coupons all maturing on the same day. If all three bonds are discount bonds, or all three are premium bonds, then the ratio of bonds 1 and 3 necessary to match the cash flows of bond 2 will be the same regardless of whether the cash flows being matched are before or after tax. However, if bond 1 and 2 are discount bonds and bond 3 is a premium bond, there may be no combination of bonds 1 and 3 that will exactly match the after tax cash flows of bond 2, due to the constant yield method of amortizing the premium of bond 3. Thus, if the tax rate of the marginal investor is positive, taxes may have an effect on the relative prices of bonds.

In addition to tax and liquidity effects, there may be shifts in demand or supply for individual bonds that affect their prices relative to other bonds. Duffie (1996) argues that securities that are on special in the repo market (i.e. they have overnight borrowing rates that are below the general collateral rate) will trade at a premium over similar assets that are not on special. Jordan and Jordan (1996) examine repo specials and find that they do significantly impact bond prices. However, their evidence reveals that repo specials alone do not entirely explain the premiums associated with on the run issues, suggesting that the high liquidity of these issues has value in and of itself. Overnight repurchase rates were not reported by GovPX during the time period of our sample. However, specialness is highly correlated with volume, and may be a partial explanation for any volume effects we find.

In this paper we use two types of tests for understanding the determinants of pricing errors in present values--arbitrage tests and an examination of deviations from a term structure fit. We will examine each in turn.

A. Arbitrage Tests

Tests that are based on the principle of no arbitrage are extremely powerful because they do not rely on a valuation model and require only minimal assumptions about preferences. Arbitrage style tests have a long history in examining the determinants of government bond prices (see Litzenberger and Rolfo (1984b), Jordan and Jordan (1991), and Ronn and Shin (1993)). However, these authors examine quite small samples and look exclusively at tax effects. Our daily data and access to trading volume allows us to use triplets to examine both tax timing and liquidity effects.

The arbitrage test commonly used to examine tax timing, tax clientele, and tax regime effects involves the use of bond triplets, three bonds with the same maturity but different coupons. Assuming a zero tax rate for the moment, for each triplet let C_i and P_i be the coupon and price of bond i, where i = 1,2,3 and the bonds are in arranged in ascending order by coupon. The law of one price states that

$$P_2 = x\, P_1 + (1 - x)\, P_3 \qquad (1)$$

where

$$x = \frac{C_2 - C_3}{C_1 - C_3}.$$

Equation 1 must hold, since in the proportions shown the cash flows are the same for bond 2 and the portfolio of bonds 1 and 3. When taxes are present, equation 1 needs to be modified. First, if bondholders are taxed when capital gains and losses are realized, then a tax timing option may be present. If each bond in the portfolio always had price changes in the same direction as bond 2, the portfolio and bond 2 would be equally desirable. However, as long as there are states of the world where the bonds in the portfolio have price changes in the opposite direction, the portfolio of bonds 1 and 3 are more valuable than bond 2, and a tax timing option exists. This is an application of the principal that a portfolio of options is more valuable than the option or the portfolio. With a tax timing option, equation 1 would be an inequality.

To examine the effects of tax timing options, it is necessary to eliminate other tax influences by ensuring that the pre-tax and post-tax cash flows are the same. Since premium and discount bonds are treated differently for tax purposes, the effect of tax timing options is unequivocal only if all three bonds are premium or discount bonds. Because of the lack of a significant number of discount triplet observations, we focus on premium triplets. In addition, the amortization of bond premiums also needs to be considered. The Tax Reform Act of 1986 altered the amortization of bonds. Bonds issued before September 28, 1985 (old bonds) may be amortized using the straight line method, which makes them preferable to bonds issued after that date (new bonds) which must use the constant yield method. Thus, initially we will examine only triplets where all three bonds were issued before or after September 28, 1985. Finally, since the tax timing involves controlling the year of the gain or loss, we do not include bonds with less than one year to maturity.

The measure we use to quantify the tax timing and liquidity effects in bond triplet prices is the difference between the price of bond 2 and the replicating portfolio of bonds 1 and 3. In equation form this difference is:

$$D = P_2 - (X P_1 + (1 - X) P_3).$$

If there is a tax timing option, bond 2 should be less expensive than a portfolio of bonds 1 and 3 and D should be less than zero. Table IV reports our results[8]. When examining triplets consisting of new bonds, the portfolio is more expensive than bond 2 in 83% of the 227 observations with an average price difference of six cents per $100 face value. For triplets which include only old bonds, 60% of the 22 observations have the portfolio being more costly with an average difference of three cents. These results are similar to those reported by others. Our access to superior data and a much larger number of observations (others have 30 to 40 observations) does not refute the sign or magnitude of pricing differences between bond triplets.

However, our much larger sample does allow us to explore whether these results could be due to liquidity rather than tax timing effects. In order to look for evidence of liquidity effects, bonds are separated into high and low volume for groups based on whether the daily volume for each bond was above or below the median volume for all bonds on that day. Less liquid bonds should have lower prices and offer higher returns. If there is a considerable difference in liquidity between bond 2 and the bonds in the portfolio, it should alter the relationship between their prices. When bond 2 is less liquid than the portfolio, then ceteris paribus we would expect bond 2 to be cheaper and D to be more negative. On the other hand, when bond 2 is more liquid than the portfolio we would expect D to be less negative or positive if liquidity effects dominate the tax timing effects. Rows 2 and 3 of Table IV show the results. In both cases, sorting by liquidity affects the relationship in the direction we would theorize. However, D is always negative, indicating both tax timing and liquidity effects are present. The difference caused by liquidity is about 5 cents per $100 face value.[9]

By recognizing the different tax treatment of bonds issued before and after September 29, 1985 (old and new bonds), we can dramatically expand our sample size, which is important for distinguishing between the effects of liquidity and taxes. The type of triplet for which we have a substantial number of observations contains two new bonds and one old, with bond 3 being the old bond. Since old bonds have a tax advantage, examining triplets in which the highest coupon bond is old should result in an increase in the price of bond 3 and a more negative D. Row 5 in Table IV analyzes this case. We have 2,066 observations. The average difference in price between bond 2 and the replicating portfolio is around three cents, with the portfolio being more expensive 69% of the time. Although the average D is negative, it is actually closer to zero than in the all old or all new triplets. Using t-tests, we find that the average D for triplets consisting of two new bonds and one old bond is significantly (at the .01 level) greater than the average D for triplets consisting of all new bonds. This indicates that the difference in tax treatment of old and new bonds is not reflected in market prices. Rows 6 and 7 split the sample by liquidity to see if the results could be due to liquidity differences. Once again, changes in D are consistent with liquidity and tax timing effects. When bond 2 is more liquid than the portfolio, D is less negative, as we would expect. Likewise, when bond 2 is less liquid than the portfolio D increases but is still negative. Evidence from a t-test indicates that the average D for the HLH group is significantly less than the average D for the LHL at the .01 level.

By adjusting the price of the old bond by the estimated value of its preferential tax treatment, ve can compare this adjusted price with the prices of the other two bonds on a common tax basis.[10] The results are shown in rows 8 to 10 of Table IV. As we would expect, D becomes less negative after decreasing the price of bond 3 by the value of the tax advantage. None of the previous results were refuted.

The second to last column provides information on the number of potential arbitrage opportunities. The existence of arbitrage opportunities is interesting because it provides evidence of tax clienteles, or inefficient markets. Table IV reports the number of observations where the three bonds trade within a half hour of each other and the difference in price between bond 2 and the portfolio is greater than 1 cent when using the appropriate bid or ask prices. Specifically, when D is less than (greater than) zero, the ask (bid) prices are examined for bonds 1 and 3 and the bid (ask) price is used for bond 2. When necessary the observed trade prices are adjusted by a conservative bid-ask spread of 5 cents to estimate the other quote. The only way for there to be a substantial number of mispricings between bond 2 and the portfolio is for there to be tax clienteles that value the triplets differently. The number of violations are sufficiently few that there is little support for the existence of tax clienteles or inefficiency. In summary, the bond triplets provide evidence of a liquidity effect and tax timing options. However, examining bond triplets does not provide evidence that the difference in tax treatment of old and new bonds is reflected in market prices or that tax clienteles affect prices.

Arbitrage tests depend on having two portfolios with identical cash flows. Bond triplets are one way to construct these portfolios. However, there are many other possibilities. To further explore the effect of liquidity, we use a new approach in which we construct portfolios with more than three bonds. This allows us to create portfolios with more extreme differences in liquidity. Each day two portfolios are created with an equal number of bonds of consecutive maturities. In each portfolio there is a bond that matures every six months, which enables us to match cash flows at each maturity. One portfolio is constructed from one of each high volume bond, while the other portfolio contains low volume bonds held in (strictly positive) proportions such that they match the cash flows of the high volume portfolio. The law of one price implies they should have the same price if liquidity is unimportant. If liquidity does have value, then the low volume portfolio should have a lower price. Tax timing should not be a consideration, since the portfolios have roughly the same number of bonds.

In order to obtain a large number of observations, we examined bonds that have cash flows in February and August. Whenever possible, we chose bonds with February 15 and August 15 as the cash flow dates. If more than two bonds of a given maturity traded on the same day, the two that had the greatest difference in volume were selected. In some cases two bonds did not exist with these coupons dates; in these cases we included bonds that matured at the end of the month. In Sample 1 there were 30 out of 762 bonds that did not pay on the 15th; in Sample 2, 56 of 608, and in Sample 3, 189 of 626.[11] When the cash flow dates differ, we adjust the cash flows by the forward rate. If one of the portfolio bonds did not pay on the 15th, its cash flows were adjusted to the 15th using the forward rate for that portfolio. The magnitude of this correction was very small compared to the difference of prices of the two portfolios. Furthermore, the frequency of adjustment was roughly the same for the high and low volume group; thus errors in adjusting cash flows should not affect the results. We included as many periods as possible given that cash flows had to match and no payments could differ by more than 16 days. We required there to be at least five bonds in each portfolio. The maturity of the portfolios varied between 2½ and five years, with the median number of years between three and 3½ years.

Table V shows the volume for the high volume portfolio and the low volume portfolio where volume is measured over 1 day in the top panel and over ten days in the lower panel. The average volume difference between the two groups is substantial. To get an idea for the magnitude of this difference, we can consult Table II. Although Table II is restricted to bonds with less than six months to maturity, the high volume shown in Table V would lie in the top decile and the low volume in the lower four deciles. Although there does appear to be a significant volume difference between portfolios, Table V does not support a liquidity effect. The portfolios are normalized so that the high volume portfolio cost $1. The average cost of the low volume portfolio is shown in column 1. It is significantly different from one in Sample 1 and Sample 3, but in opposite directions, and overall it is not significantly different from one.

To compare the value of one portfolio relative to the other, we matched pre-tax cash flows. To test whether tax effects may have driven our results, we use coupon as a proxy for tax effects and regress the ratio of prices on the difference between the weighted average coupons for the high and low volume portfoLos. Difference in coupon was insignificant in explaining the difference in price between the portfolios for both those sorted by 1 day volume and by 10 day volume.

The results may also be influenced by non-synchronous trading. Table V provides information on the weighted average of the number of hours before 6:00 that the last trade occurred. As expected, the low volume trades are older on average by about one hour. Potentially, prices could be falling over the day on average and the low volume prices be an overestimate of the synchronous price. To test for this, on each day we adjusted the earlier price to the later price by using the average return over the day for all bonds adjusted to the appropriate time interval. The prices moved up some days and down other days in our sample, and the adjustment resulted in essentially identical results.

Finally, we were concerned that since we were using trade data, potentially the lack of a liquidity effect came about because the high volume was at the ask and the low volume at the bid. Columns 4 and 5 show the proportion of the trades that were at the bid price. Compared to the high volume portfolio, there is some tendency for the observed trade prices for bonds in the low volume portfolio to occur more often at the bid price. We would expect this to make the low volume portfolios less expensive than the high volume portfolio. However, the evidence does not support this and there seems to be no relation between the difference in portfolio prices and the difference in proportions of bid trades. Thus, bid-ask spread is not an explanation for the price differences in portfolios. Overall, the general arbitrage results provide mixed evidence for liquidity effects in bond prices.

B. Term Structure Tests

In order to study the effects of taxes and liquidity over the entire spectrum of maturities, it is necessary to first specify a model of the term structure. Once a model is selected, we can fit it to the after-tax cash flows of bonds and thus infer the tax rate faced by the marginal investor. If the estimated tax rate is significantly different from zero, we can conclude that taxes do affect the prices of bonds. The after-tax term structure was first estimated by McCulloch (1975) assuming a given set of tax rates. Litzenberger and Rolfo (1984) use a grid search to determine the optimal tax rates implied by the data. More recently, Green and Oedegaard (1996) look for a structural change in the implied taxes before and after the change in tax law in 1986. They find that the tax rate of the marginal investor is positive before 1986, but close to zero afterwards. We use a similar procedure and include an additional parameter to capture the effects of liquidity.

When selecting a model of the term structure, a choice has to be made between two different approaches. One approach estimates the term structure each period using only the information contained in the cross section of bond prices; the other approach constrains the term structure to move with a limited set of state variables, but allows the model to be estimated once over the entire sample. The benefit of cross-sectional models is that they in general provide a better fit than structural models. The cost is that the model has to be estimated each period, and it is not possible to estimate a single tax rate for the entire sample period. Since we are interested in pricing bonds as accurately as possible, and since there exists no structural model that clearly dominates the flexible form method, we use nonlinear least squares to fit Litzenberger and Rolfo's (1984) cubic spline to the after-tax cash flows of bonds in each period.[12]

In order to capture the effects of liquidity on the relative prices of bonds, we add a liquidity term (log volume) to the reduced form price equation. There are four different pricing scenarios for tax purposes: discount bonds issued before and after July 18, 1984, and premium bonds issued before and after September 28, 1985.[13] Under each tax scenario, we solve for price as a nonlinear function of the tax rate and the parameters of the discount function. To these price functions we add log volume to capture the effects of liquidity on prices. This specification of how liquidity affects prices is ad hoc, but is done with tractability in mind. The reduced form price function under the various tax regulations is a nonlinear function of the tax rate and the parameters of the discount function and can be quite complicated. We wanted to allow for the presence of liquidity effects without complicating this function further. We examine maturities up to 10 years. Trade data for long term bonds was relatively sparse, with an average of around five observations a day for maturities greater than 10 years. It has been shown that fitting the curve at long maturities with few observations can lead to spurious results (see Shea (1984)).

Table VI reports the root mean squared errors and the average estimated tax rate and liquidity parameter for three subsamples of 90 days of data as well as the combined sample.[14] Including tax and liquidity terms improves the fit across all maturities except for those less than one year. The average estimated tax rate over the three subsamples is 8%. We find that the tax rate is statistically significant 69 times in the first sample, but only 11 and 18 times in the second and third samples.[15] This is evident in the pattern of estimated tax rates, as shown in Figure 2. Although the estimated tax rates do exhibit considerable volatility, it is evident that the estimated tax rates are small, especially in samples 2 and 3. This confirms the findings of Green and Oedegaard, who find the estimated tax rate during the 1987-1992 period to be close to zero.

Although we find little evidence of tax effects, it is worth noting that the assumptions commonly made to estimate the after-tax term structure imply that differences between capital gains and ordinary income tax rates have little impact on prices. For instance, it is common to assume that all bonds are held to maturity. Thus, the only capital gains are for bonds selling at a discount, and the only losses come from premium bonds. Moreover, the only way for the capital gains tax rate to enter the valuation process is for bonds issued before July 18, 1984 and selling at a discount. If they were issued after this date, all discounts are taxed at the ordinary rate. In all three of our subsamples, we observe no bonds issued before July 18, 1984 that traded at a discount. Thus, we have little hope of distinguishing between the tax rate on capital gains and ordinary income. Since the trade off between these two rates is a major contributor to any tax effects, it is not surprising that we don't find convincing evidence of their existence. In the case of bond triplets, no assumption on the holding period is necessary, and we do find some evidence of tax-timing options, although the magnitude of the effects are quite small.

The liquidity term, on the other hand, appears to carry a higher level of statistical significance. Figure 3 shows the estimated coefficients are also rather volatile, but the liquidity term is significantly positive in 160 of the 270 regressions.[16] The average liquidity coefficient is 2.25 basis points. This value suggests a range of approximately 13 basis points from the lowest to the highest volume deciles.[17] The magnitude of the liquidity effect is much smaller than that found in previous studies. For example, Warga (1992) finds a 40-100 basis point difference in returns between active issues and duration matching portfolios, and Amihud and Mendelson (1991) report a 40 basis point difference in yield on similar notes and bills. Although liquidity does appear to be important, it's value is not near as substantial as previously reported. The large liquidity effects found in earlier work may be due to inaccurate liquidity measures and the lack of precise price data. On the other hand, the increased size of the Treasury market, and the widespread use of empirical term structure models along with a maturing strips market, may also have led to a smaller economic role for liquidity. Whether the diminished role of liquidity we observe is due to the use of more precise data, or is a result of the increased efficiency of the Treasury market, is unclear. In either case, our overall finding is that the current economic role of liquidity in the relative prices of Treasury securities is quite small.

The inconsequential tax and liquidity effects we find suggests that more bonds can be included in term structure estimation. Market participants commonly narrow the pool of bonds they consider when estimating the term structure in order to use bonds which they believe are unaffected by tax and liquidity effects. Our evidence implies that a larger sample of bonds can be used to estimate the term structure, which could lead to smaller estimation error. The small liquidity effects we observe also has relevance for the tradeoff between the larger bid-ask spreads and higher expected returns of illiquid bonds, which is relevant to investors who must decide whether to hold illiquid bonds.

Table VI and Figure 1 may provide some explanation as to why we did not find convincing evidence of liquidity effects in the arbitrage tests, yet we do observe liquidity effects in the term structure estimation. The arbitrage tests utilize bonds near the short end of the maturity spectrum in order to come up with enough issues to match cash flows. Neither the bond triplet approach nor the cash flow matching approach utilize bonds with more than five years to maturity. However, Figure 1 shows that many of the highly liquid bonds have maturities greater than five years. Moreover, Table VI shows that the reduction in pricing errors by including the liquidity term is strongest for longer maturity bonds. Thus, the term structure approach is more robust in that it allows us to examine liquidity over a broader range of maturities.

The average root mean squared error was .1363 per $100 face value, or about 14 basis points. As discussed previously,.the average bid-ask spread for notes and bonds was .053. Thus, roughly 20% of the root mean squared error can be attributed to the bid-ask spread since we fit the curve to trade data. To examine whether remaining errors are systematically related to certain bond attributes, we regressed pricing errors on a series of bond characteristics. The results are reported in Table VII. The top panel reports the regression results when the tax and liquidity parameter are estimated freely. The lower panel reports the results when the two terms are constrained to be zero. In order to capture the effects of tax timing options, which are not explicitly modeled in the after-tax cash flows, we include two variables that reflect the amount of dollar premium or discount. In addition, we include an age variable to examine whether there are remaining liquidity effects not capture by our liquidity term. We also include a measure of non-synchronousness. The non-synchronous variable is the daily price change in the nearest active times the fraction of the day between the last trade and 6:00 EST. A tax regime dummy variable is included that is 1 if the bond trades at a premium and is issued before 9/28/85, which is meant to capture the preferential amortization rules for old bonds. Another dummy variable is included which is 1 if the issue is on-the-run, which is designed to capture the effects of repo specialness. Finally, to measure the impact of the bid-ask spread, we add a dummy variable that is 1 if the trade occurred at the bid price, and 0 if at the ask. Since we pool the errors across maturities and time, it is highly likely that the errors will be heteroskedastic as well as autocorrelated. In order to obtain robust statistical measures, we need a heteroskedasticy and autocorrelation consistent (HAC) estimate of the covariance matrix. The method we employ is a variant of the Newey-West correction that is discussed in Green and Oedegaard.[18]

We will first look for the presence of tax effects not captured by our estimated tax rate. The tax regime coefficient is insignificant in the combined sample. Since we account for the tax regime when discounting after tax cash flows, we would not expect the tax regime coefficient to be significantly different from zero. It appears for the most part that investors do take advantage of the preferential amortization treatment of bonds issued before September 28, 1985. The coefficient for the premium variable is negative and significant in the combined sample. The coefficient for the discount variable is mixed, although it negative and insignificant overall. The negative estimated coefficients are not consistent with a mis-estimated tax rate, which should result in different signs for the premium and discount variables. Instead, our evidence generally suggests that a significant group of investors is adverse to holding both high discount and high premium bonds. One possible explanation that is consistent with the evidence is the behavior of fiduciaries. Fiduciaries who hold bonds in trust funds face a trade-off between interest income and capital gains. The higher the coupon, the larger the amount of interest income that accrues to the current beneficiary and smaller the capital gains which go to the heirs. Likewise, the lower the coupon, the smaller the amount of interest income that accrues to the current beneficiary and the larger the amount of capital gains for the heirs. Trustees avoid bonds with large discounts or premiums because they are vulnerable to accusations of favoring one beneficiary over the other and there exists a potential for legal action. Our evidence suggests that large premium and large discount bonds sell at a discount, which support this explanation.[19] The effect of premiums and discounts on pricing errors is only important for the most extreme 5% of the observations. For these observations the effect varies from 4 to 60 cents per $100 face value. When we constrain the tax rate to be zero, the tax terms included in the error regressions are still not significant, although the coefficients all change in the expected direction. This is further evidence that tax effects, if any, are not large.

The age variable is not significant, which is not surprising given the inclusion of the liquidity term when estimating the discount function. When the liquidity coefficient is constrained to be zero, the age coefficient is negative and significant in the combined sample, suggesting that the age variable proxies for liquidity in this case. The non-synchronous adjustment variable is negative in every sample. If prices increased steadily throughout the day a bond's last trade occurred several hours before the close, we would expect the observed price to be below its fair closing value and therefore we would observe a negative pricing error. The negative coefficient along with a positive non-synchronous adjustment variable would result in a negative prediction. Furthermore, if prices had decreased over the day, the pricing error should be positive, but since the variable is negative, a negative coefficient predicts the positive error. Thus, the expected sign on the coefficient for our non-synchronous adjustment variable should be negative regardless of how prices move and this is what we observe. The non-synchronous adjustment variable is sizable for only about 20% of the observations. For this part of sample the adjustment for non-synchronous trading is about 2 cents per $100 face value.

The on-the-run dummy variable is positive and significant in two of the three subsamples. Since we have already adjusted for the effects of liquidity, we suggest this variable proxies for the effects of specialness in the overnight repurchase market.[20] It is common for on-the-run issues to be associated with overnight borrowing rates that are lower than the general collateral rate (see Duffie (1996). The size of the coefficient in the combined sample is .1071, which translates to an effect of around 10 basis points for issues on special, which is much smaller than the 20-40 basis point effects reported in Jordan and Jordan (1996). However, the subsample closest to their time period has the largest coefficient, with on-the-run issues commanding premiums of 17 basis points. The average pricing error for the on-the-run issues is about 22 cents when the liquidity term is constrained to be zero. The average pricing error drops to 11 cents when the liquidity term is included. The coefficient on the on-the-run variable, and the reduction in pricing error when freely estimating the liquidity term, suggests that liquidity and repo specials each explain roughly half of the premium associated with on-the-run issues.

Finally, the hit dummy variable is negative and significant in every subsample. The combined coefficient is -.01, which suggests the pricing error due to bid-ask spreads is significant, but quite small.

VI. Conclusion

This paper looks for evidence of tax and liquidity effects in the relative prices of Treasury securities. We utilize two types of arbitrage tests in addition to jointly estimating the tax rate and a liquidity term when fitting a cubic spline to the term structure. We examine trade data from the interdealer market. Our data set provides us with prices that more accurately reflect market clearing conditions than the dealer quotes used in previous studies, and we have access to a more robust measure of liquidity than previously examined. Using more accurate prices, we still find evidence of tax timing options and liquidity effects, although the effects of liquidity are much smaller than previously reported. We find evidence that part of the premium associated with the most liquid bonds, the on-the-run issues, is due to specialness in the overnight repurchase market.

We find evidence that large premium and discount bonds sell at a discount, which is consistent with fiduciaries avoiding these bonds. However, when using trade data all pricing errors are small and the effects of tax timing options, tax regime shifts, and differences in liquidity are measured in pennies. Thus, a significant portion of the liquidity and tax effects found by previous authors appears to be no longer relevant, either because of increased efficiencies in the Treasury market, or perhaps because the original estimates were of influenced by problems with the data.

The lack of substantial tax and liquidity effects in the relative prices of Treasury bonds has important implications for investors deciding which bonds to select and for practitioners determining which bonds to include in their term structure estimations for use by traders.

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Figure 1. 95th Percentile of the Log of Daily Trading Volume Grouped by Maturity. The data points in each maturity range represent the 95th percentile of log volume for all noncallable bonds that fall into that maturity range. Sample 1 is from 10/1/91 - 2/11/92, sample 2 covers 3/1/93 - 7/7/93, and sample 3 covers 5/23/95 - 9/29/95.

Figure 2. Estimated Tax Rates. A cubic spline with a liquidity term is fit to the after-tax term structure. The chart reports the estimated tax rates for three samples. Sample 1 covers 10/1/91 - 2/11/92, Sample 2 covers 3/1/93 - 7/7/93, and Sample 3 covers 5/23/95 - 9/29/95.

Figure 3. Estimated Liquidity Parameter. A cubic spline with a liquidity term is fit to the after-tax term structure. The chart reports the estimated liquidity parameter for three samples. Sample 1 covers 10/1/91 - 2/11/92, Sample 2 covers 3/1/93 - 7/7/93, and Sample 3 covers 5/23/95 - 9/29/95.

1 The brokers monitored by GovPX are Garban, Hilliard Farber, Liberty, and RMJ. The one exception is Cantor Fitzgerald, which provides its own direct feed to dealers.

2 Coleman, Fisher, and Ibbotson (1992) report that until around 1979, prices on dealers quotations sheets were honored until noon the next day for small transactions. After that, quotes were indicative and although bid prices were used for internal purposes, ask prices were arbitrary. In addition, they state that during this time period the Fed survey data also used non-binding quotes. The bid price was an average of the surveyed bid quotes, but the ask price was the bid plus a "representative" spread.

3 See Coleman, Fisher and Ibbotson (1992) and our discussion in the previous section.

4 Quote observations are examined if both a bid and ask price are reported. Some of the reported prices for bonds that did not trade are indicative quotes. Removing these observations had little effect on the results.

5 The bond dummy variable is not significant in the sample of bonds that traded. In the sample of bonds that did not trade, the bond variable may be proxying for volume, thus, its importance is unclear.

6 Amihud and Mendelson use transaction costs as a measure of liquidity. They find that bills have lower transaction costs than notes or bonds and this leads them to use instrument type as an indirect proxy for liquidity.

7 The data we examine is from the intra-dealer market. Other investigators have used data in the retail market. While the volume patterns need not be the same, they should be closely related.

8 The hypothesis tested is that the percentage of triplet observations with D less than 0 is equal to ½ using the property that $2(\sin^{-1}\sqrt{p} - \sin^{-1}\sqrt{.5}) / \sqrt{n}$ is distributed standard normal in the limit, where p is the sample proportion and n is the number of observations (Litzenberger and Rolfo 1984b).

9 At the suggestion of the referee, we also pooled the triplet observations together and regressed D on dummy variables for the 3 cases we consider and a liquidity parameter that is the weighted average of volumes for bonds 1 and 3 over the volume for bond 2. We found that CCC and CCS are significantly less than zero, and the magnitude of the coefficients are similar to the average D's listed in the table. The liquidity term was not found to be significantly different from zero.

10 The adjustment was made by calculating the amoratization schedule for old and new bonds for all maturities and premiums, and discounting these differences by the estimated spot rates.

11 Each sample contains 50 days where at least five bonds where available for each portfolio. Sample 1 is taken from 10/2/91 - 1/8/92, Sample 2 covers 3/1/93 - 6/14/93, and Sample 3 covers 5/31/95 - 9/28/95.

12 There is an abundant literature on splines. See Shea (1984) for a discussion of the issues. Using simulated data, Beim (1992) finds that cubic splines perform as well or better than other estimation techniques. The methodology we use is along the lines of Litzenberger and Rolfo (1984a). Breakpoints for the spline functions are chosen at 1, 2, and 4 years to maturity. Although it is common to use more breakpoints, we use relatively few to guard against overfitting the data. We use nonlinear regressions to allow for the nonlinear interaction between the tax rate and the parameters of the discount function (see Langetieg and Smoot (1989) for evidence of the advantage of nonlinear methods over the instrumental variable approach used in Litzenberger and Rolfo (1984)).

13 See Green and Oedegaard (1996), Ronn and Shin (1993), or Fabozzi and Nirenberg (1991) for the precise treatment discount and premium bonds under the different tax regulations.

14 Sample 1 covers 10/1/91-2/11/92, Sample 2 covers 3/1/93-7/7/93, and Sample 3 covers 5/23/95-9/29/95.

15 Standard errors are derived from a heteroskedasticity-consistent estimate of the covariance matrix (see Mackinnon and White (1985)).

16 Assuming the liquidity parameters are independent draws from a Normal distribution, the mean liquidity parameter is significantly different from zero at any level of significance. The level of autocorrelation in the liquidity estimates is small, and does not materially alter this result.

17 Daily trading volume (in logs) for notes and bonds with less than ten years to maturity ranges from a bottom decile of 0 to a 95th percentile of 5.8.

18 The approach uses errors lagged across time and across the term structure. For a discussion of HAC estimators, see chapter 17 of Davidson and MacKinnon (1993).

19 The authors thank Ken Garbade for this insight.

[20] Alternatively, the significance of the on-the-run dummy could be associated with some aspect of liquidity not captured by the volume proxy.

Table I

Bid-Ask Spreads in the Interdealer Market for Treasury Securities

Data on bid-ask spreads and trading volume is obtained from screen output provided by GoxPX Inc. Information on bills and bonds is aggregated over three consecutive days in June, 1996. Bond is a dummy variable that is 1 if the issue is a bond, 0 otherwise. Ln(Vol) is the natural log of the daily trading volume. Ln(Day) is the natural log of the number of days since the bond last traded.

Mean Bid-Ask Spread
0.052945

Percentiles

10th	0.003906
20th	0.007813
30th	0.015625
40th	0.031250
50th	0.062500
60th	0.062500
70th	0.062500
80th	0.078125
90th	0.125000
100th	0.125000

Regressions of Bid-Ask Spread on Security Characteristics

Securities that Traded

Obs. 190 R^2 0.8131

	Coef.	T-Stat	P-value
Constant	0.0244	5.5803	0.0000
Maturity	0.0044	25.8330	0.0000
Bond	0.0029	0.8217	0.4123
Ln(Vol)	-0.0046	-6.7327	0.0000

Securities that did not Trade

Obs. 397 R^2 0.7944

	Coef.	T-Stat	P-value
Constant	0.0049	1.6114	0.1079
Maturity	0.003[illegible]	23.4397	0.0000
Bond	0.0313	9.4562	0.0000
Ln(Day)	0.0014	2.5643	0.0107

Table II

Volume Data for Treasury Bills and Bonds with Less than Six Months to Maturity

The reported numbers are for daily volume of all listed non-callable Treasury securities with less than six months to maturity. The trading percentages and percentiles for the 5 and 10 day intervals are obtained from overlapping observations of 5 and 10 trading days. The sample covers 6/17/1991 - 9/29/1995.

Trading Percentages

Measurement Interval	Total Observations	Percent Traded	Total Observations	Percent Traded
1 Day	30871	85.34	20666	70.8
5 Days	25766	99.6	19998	94.74
10 Days	23327	99.97	19162	96.91

Distribution of Volume (Millions)

	Bills			Bonds		
Percentile	1 Day	5 Day Average	10 Day Average	1 Day	5 Day Average	10 Day Average
10th	0	19.6	25.6	0	1	2.2
20th	9	37.8	42.6	0	3.8	5.6
30th	27	57.4	63.4	1	6.8	8.9
40th	51	79.6	86	2	10.4	12.4
50th	82	104.4	108.6	6	14.8	16.6
60th	124	133.2	135.6	11	20	21.7
70th	186	170.6	168.2	20	26.8	27.7
80th	293	231.8	218.3	34	36.4	35.9
90th	607	399	333.2	63	52.4	49.3
95th	1124	752.2	565.3	96	69.2	63.8
100th	8215	2756.2	1776.1	4499	925.4	504.8

Table III

Regression Results of Volume on Liquidity Proxies

The results are from an ordinary least squares regression of the natural log of trading volume on the independent variables.

$$Ln(vol) = b_0 + b_1 Bill + b_2 Active + b_3 Age + b_4 Monday + b_5 Tuesday + b_6 Wednesday + b_7 Thursday + \varepsilon.$$

The sample contains information on all noncallable Treasury securities. The bill dummy is 1 if the security is a bill, 0 otherwise. The active dummy is 1 if the issue is on-the-run, 0 otherwise. Age is the number of years since issuance. Sample 1 covers 10/1/91 - 2/11/92, sample 2 covers 3/1/93 - 7/7/93, and sample 3 covers 5/23/95 - 9/29/95. The results reported are for the combined sample.

R-Squared	No. of Obs.
0.453	40631

	Coefficient	T-Statistic	P-Value
Constant	3.115	175.158	0.000
Bill Dummy	0.868	46.315	0.000
Active Dummy	3.847	128.928	0.000
Age	-0.182	-68.317	0.000
Monday	0.023	1.006	0.314
Tuesday	0.236	10.994	0.000
Wednesday	0.304	14.053	0.000
Thursday	0.265	12.159	0.000

Table IV

Evidence of Tax and Liquidity Effects in Bond Triplet Prices

Bond triplets consist of three bonds with differing coupon rates but the same maturity. Tax type S denotes bonds issued before 9/28/85, for which premiums may be amortized using the straight line method. Tax type C denotes bonds issued after 9/27/85, for which the constant yield method must be used. CCS represents a triplet in which the two bonds with smaller coupons are "new", while the bond with the highest coupon is "old." Volume type H denotes volume observations greater than the median and L denotes observations less than the median. D represents the price deviation between the price of bond 2 and the replicating combination of bonds 1 and 3.

$$D = P2 - (XP1 + (1 - X) P3).$$

The hypothesis tested is that the percentage of triplet observations with D less than 0 is equal to 1/2 using the property that:

$$2(\sin^{-1}\sqrt{p} - \sin^{-1}\sqrt{.5}) / \sqrt{n}$$

is distributed standard normal in the limit, where p is the sample proportion and n is the number of observations (Litzenberger and Rolfo 1984b). "Num Arb. Opp." attempts to capture the number of arbitrage opportunities, which are defined as observations in which the triplet trades are within a half our of each other and $|D| > .01$. When D is less than (greater than) 0, bid (ask) prices are used for bonds 1 and 3 and the ask (bid) price is used for bond 2. When these are not available, we adjust the observed trade price by a conservative bid-ask spread of .05. Also reported are the statistics when the prices of "old" bonds are adjusted for the average additional value of being able to amortize the premium using the straight line method over the constant yield method.

Bond Triplet Type

	Tax Type	Volume Type	Average D	Percent D < 0	T-stat	P-val	Num. Arb. Opp.	obs
					All New Bonds			
1	CCC		-0.0556	82.82	-10.788	0.0000	5	227
2	CCC	LHL	-0.0142	50.00	0.000	0.5000	0	6
3	CCC	HLH	-0.0679	86.84	-5.107	0.0000	1	38
					All Old Bonds			
4	SSS		-0.0285	59.09	0.858	0.1956	0	22
					Unadjusted Mixture of Old and New Bonds			
5	CCS		-0.0230	68.88	-17.597	0.0000	31	2066
6	CCS	LHL	-0.0141	63.05	-3.763	0.0001	8	203
7	CCS	HLH	-0.0270	70.91	-6.399	0.0000	2	220
					Adusted for Tax Regime			
8	CCS		-0.0218	68.15	-16.886	0.0000	31	2066
9	CCS	LHL	-0.0135	62.56	-3.618	0.0002	8	203
10	CCS	HLH	-0.0257	70.46	-6.251	0.0000	2	220

Table V

Pricing Differences Between Cash Flow Matching Portfolios Grouped by Volume

Notes: Each day two portfolios are created from bonds that mature in February and August. The two portfolios contain an equal number of bonds of consecutive maturities, i.e., in each portfolio there is a bond that matures on every cash flow date. The portfolios are sorted by volume. In the top panel bonds are separated by daily volume, in the lower panel bonds are separated by average volume over the past ten days. Portfolio weights are found for the low volume portfolio to match the cash flows of a high volume portfolio which contains one of each high volume bond. Each sample contains 50 days where at least five bonds where available for each portfolio. The reported statistics are averages over each sample. Low Volume Price is the ratio of the cost of the low volume portfolio over the cost of the high volume portfolio. The T-statistic and P-value are for a test whether this ratio is different from 1. Proportional high and low volume are the weighted averages of the daily trading volumes of each portfolio. The number of pairs is the average number of bonds in each portfolio. High and low volume coupon are the weighted averages of the coupons in each portfolio. High and low volume time is the weighted average of the number of hours from the last possible trade time (6:00) that the trades for each portfolio occured. High and Low volume SL is the weighted average of the number bonds which were issued before 9/28/85, which allows any premium to be amortized using the straight line method.

High and Low Volume Portfolios Sorted on Daily Volume

	Low Volume Price	T- Stat	P - Val	Propor. High Volume	Propor. Low Volume	High Volume Ave Hts	Low Volume Ave Hts	# Prs	High Volume Coupon	Low Volume Coupon	High Volume Time	Low Volume Time	High Volume SL Bnd	Low Volume SL Bnd
Sample 1	0.99947	-2.875	0.006	131.534	12.652	0.435	0.480	7.62	7.902	8.596	2.941	3.993	0.055	0.229
Sample 2	1.00009	0.440	0.662	328.844	12.173	0.438	0.485	6.08	6.557	8.077	2.868	4.110	0.079	0.125
Sample 3	1.00047	2.436	0.019	314.778	17.357	0.500	0.519	6.26	6.440	6.524	2.908	4.197	0.016	0.028
Combined	1.00001	0.061	0.951	258.385	14.061	0.458	0.495	6.65	6.966	7.732	2.906	4.100	0.050	0.127

High and Low Volume Portfolios Sorted on Ten Day Average Volume

	Low Volume Price	T- Stat	P - Val	Propor. High Volume	Propor. Low Volume	High Volume Ave Hts	Low Volume Ave Hts	# Prs	High Volume Coupon	Low Volume Coupon	High Volume Time	Low Volume Time	High Volume SL Bnd	Low Volume SL Bnd
Sample 1	0.99935	-4.820	0.000	134.476	13.115	0.428	0.486	7.62	7.732	8.768	3.063	3.872	0.019	0.265
Sample 2	1.00020	0.967	0.338	343.395	13.112	0.468	0.455	6.08	6.216	8.416	2.827	4.153	0.023	0.180
Sample 3	1.00065	3.585	0.001	295.558	19.270	0.500	0.519	6.26	6.251	6.711	3.138	3.967	0.012	0.031
Combined	1.00007	0.354	0.725	257.810	15.166	0.466	0.487	6.65	6.733	7.965	3.009	3.997	0.018	0.159

Table VI

Estimated Tax and Liquidity Parameters

The after-tax term structure is fitted with a cubic spline. Log of volume is added to the reduced form price equation for all bonds to capture the effects of liquidity. The tax and liquidity terms are estimated simultaneously with the spline parameters in a nonlinear regression. The top panel reports the root mean squared errors (pooled over the combined sample) when the tax rate and liquidity terms are estimated freely or constrained to be zero. The lower panel reports the mean estimated tax and liquidity parameters for the three samples. Also reported is the number of instances where the parameters are significant at the .05 level, using heteroskedasticity-consistent standard errors (Mackinnon and White (1985)).

Root Mean Squared Errors

Maturity	Tax = 0 Liq. = 0	Tax = 0	Liq. = 0	Both Est. Freely
< 1	0.0528	0.0902	0.0694	0.0983
1 - 3	0.1483	0.1356	0.1471	0.1320
3 - 5	0.1623	0.1422	0.1476	0.1294
5 - 10	0.2373	0.2088	0.2132	0.1892
All mat.	0.1566	0.1441	0.1480	0.1363

Parameter Means

	Tax Rate	Number Significant	Liquidity	Number Significant
Sample 1	0.127	69/90	0.0328	62/90
Sample 2	0.032	11/90	0.0225	55/90
Sample 3	0.085	18/90	0.0121	43/90
Combined	0.081	98/270	0.0225	160/270

Table VII

Results of Regressions of Spline Errors on Bond Characteristics

Each day a cubic spline is used to estimate the after tax term structure for noncallable bonds with less than ten years to maturity. When estimating the spline, the log of trading volume is included to capture the importance of liquidity. The resulting pricing errors are regressed on various bond characteristics. T-statistics are created using a heteroskedasticity- and autocorrelation-consistent (HAC) estimate of the covariance matrix. Discount (Premium) is the dollar amount of discount (premium), zero otherwise, for each bond assuming a face value of $100. Age is the number of years since issuance. Nonsync is an adjustment for nonsynchronous trading in which the daily price changes of active issues are assumed to occur in a linear fashion throughout the day. For each bond the change in the nearest active issue is used to approximate the change in price that occured from the last trade to the close of trading. Tax Reg is a dummy variable that is 1 if the bond was issued before 9/28/85, the cuttoff date for the Tax Reform Act of 1986. Hit is 1 if the trade occurred at the bid price, 0 if at the ask. Sample 1 contains daily observations from 10/1/91-2/11/92, Sample 2 covers 3/1/93-7/7/93, and Sample 3 covers 5/23/95-9/29/95. The results reported are for the combined sample.

Tax and Liquidity Parameters Freely Estimated

Obs.	Mean Error	R^2
31696	-0.0019	0.0480

	Coeff.	T-Stat.	P-val.
Constant	0.0171	3.811	0.000
Premium	-0.0035	-4.406	0.000
Discount	-0.0057	-1.323	0.186
Age	-0.0002	-0.135	0.893
Nonsync	-0.3900	-10.435	0.000
Tax Reg	-0.0114	-0.402	0.688
Hit	-0.0102	-5.719	0.000
Active	0.1071	7.034	0.000

Tax and Liquidity Parameters Constrained to be Zero

Obs.	Mean Error	R^2
31696	0.0015	0.1210

	Coeff.	T-Stat.	P-val.
Constant	0.0356	7.455	0.000
Premium	-0.0051	-6.801	0.000
Discount	-0.0026	-0.529	0.597
Age	-0.0036	-2.510	0.012
Nonsync	-0.5826	-13.439	0.000
Tax Reg	-0.0464	-1.722	0.085
Hit	-0.0114	-6.013	0.000
Active	0.2011	10.842	0.000

Figure 1

95th Percentile of the Log of Daily Trading Volume Grouped by Maturity

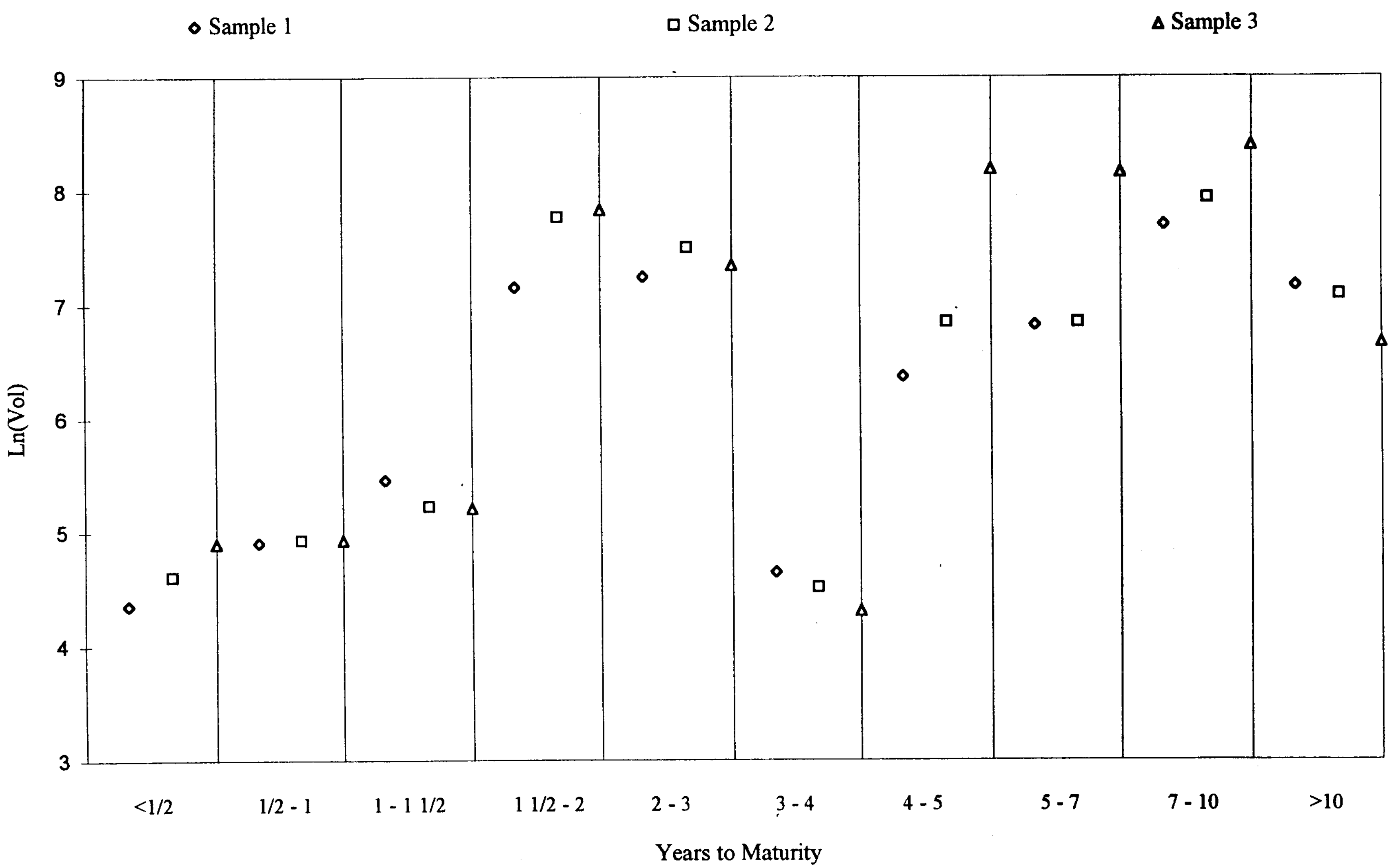

Figure 2

Estimated Tax Rates

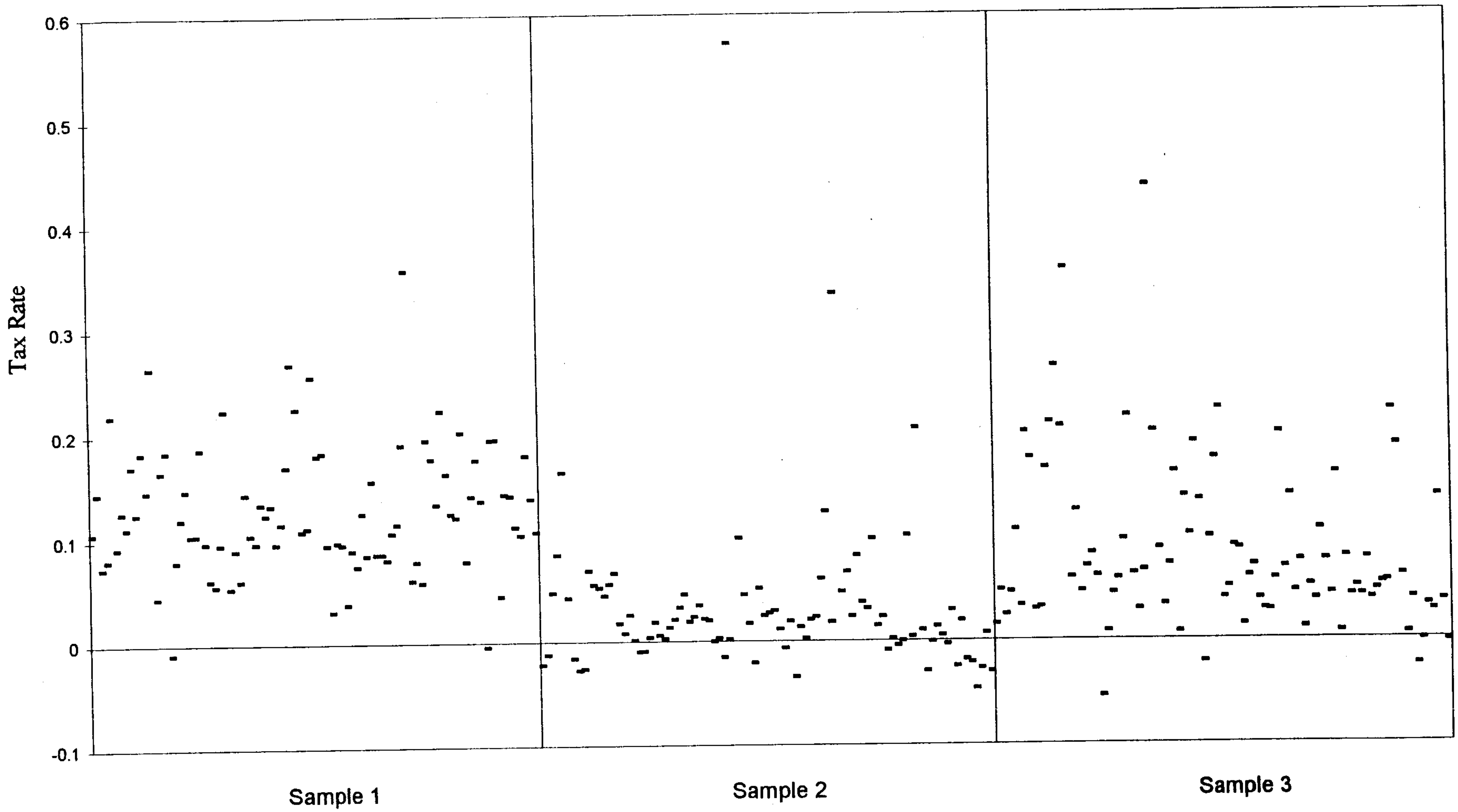

Figure 3

Estimated Liquidity Parameter