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Pricing inflation-indexed convertible bonds with credit risk

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Abstract

Issuing convertible bonds has become a popular way of raising capital by corporations in the last few years. An important subgroup is convertibles linked to a price index or exchange rate.

The valuation model of inflation-indexed (or equivalently foreign-currency) convertible bonds derived in this paper considers two sources of uncertainty allowing both the underlying stock and the consumer-price-index to be stochastic and incorporates credit risk in the analysis. We approximate the pricing equations by using a Rubinstein (1994) three-dimensional binomial tree, and we describe the numerical solution. We investigate the sensitivity of the theoretical values with respect to the characteristics of the issuer, the economic environment and the security's characteristics (number of principal payments).

Moreover, we demonstrate the usefulness and the limitations of the pricing model by using inflation-indexed and foreign –currency linked convertibles traded on the Tel- Aviv stock exchange.

1. Introduction

A convertible bond is a hybrid security, part debt and part equity, that while retaining most of the characteristics of straight debt, offers the right to forgo future coupon and principal payments, and instead, receive a pre specified number of the issuer's common stock. In recent years issuing convertible bonds has become a popular financial instrument. Between the years 1995 and 2000, based on dollar volume, the total market has grown at a 53.9% cumulative annual growth rate to $159 billion [1].

In many financial markets convertible contracts as well as straight bonds link the promised payments to a general price index or the price of foreign exchange[2]. Japanese corporations have issued large amounts of convertible bonds with coupon and principal payments denominated in Euros or in U.S. dollars that can be converted to the issuer's stock traded in the domestic currency[3]. In Israel, the coupon and the principal payments of most convertible bonds traded on the Tel-Aviv Stock Exchange (TASE) are linked to inflation as measured by the changes of the consumer-price-index (CPI) or to the Dollar/Shekel exchange rate.

An important factor in the pricing of convertible bonds is credit risk. According to a recent Moody's sample between 1970 and 2000, default rates for rated convertible bond issuers are higher than those without convertible bonds in their capital structures[4]. Clearly credit risk has a crucial effect on convertible bond prices, and should not be ignored. In the last few years practitioners and academics have tried to incorporate credit risk in the pricing of nominal convertible bonds[5].

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[1] See Stumpp (2001).

[2] In Israel virtually all intermediate and long term government bonds are linked to inflation (or the exchange rate); In Great Britain about 20% of government bonds issued in the last decade have been inflation linked; In 1997 the U.S. Treasury started issuing such bonds, called Treasury Inflation Protected Securities (TIPS).

[3] On March 2002 Sony corporation has announced the issue of U.S. Dollar denominated convertible bond with the total amount of 67 million dollar. See: www.sony.jp/en/SonyInfo/News/200203/02-0329Be/.

[4] This high level of default is partly explained by the fact that generally convertibles are issued in the form of junior subordinated debt, which places them low in the priority of payment. Furthermore, the indentures covering convertibles often contain few of the covenants that afford protection to traditional bondholders.

[5] The meaning of "nominal convertible bond " in this paper is a convertible bond that promised a nominal principal and coupon payments that are in the same currency as the underlying stock of the issuer.

The main contribution of this paper is the derivation of a pricing methodology for inflation-indexed convertible bonds where both the underlying stock and the CPI are stochastic and default risk is considered[6].

The Structural Approach for the valuation of risky debt was pioneered by Merton (1974) and is based on the insights of option pricing theory. He focuses on the capital structure of the firm where default occurs when the firm value falls bellow the value of the debt[7]. Relying on this approach, Ingersoll (1977) and Brennan and Schwartz (1977, 1980) take the total value of the firm as a stochastic variable for pricing convertible bonds. The main drawback however of this approach is the need to estimate the total value and the volatility of the firm's assets, parameters that are not observable in the market[8].

McConnell and Schwartz (1986) present a pricing model for a zero coupon, convertible, callable, putable bond (LYON) based on the stock value as the stochastic variable. To incorporate credit risk, they use an interest rate that is "grossed up" to capture the credit risk of the issuer, rather than the risk free rate. However they treat credit spread as constant in their model meaning they do not take into account the fact that the credit risk of the convertible bond varies with respect to its moneyness. For this reason Bardhan et al. (1994) build the standard Cox, Ross and Rubinstein (1979) binomial tree for the underlying asset and consider the probability of conversion at every node. They choose the discount rate to be a weighted average of the risk free rate and the risky discount rate of an identical in quality straight corporate bond. The shortfall of this approach however is its inability to take into account coupon payments or any contingent cash flow occurring due to call and put provisions.

To overcome these drawbacks Tsiveriotis and Fernandes (1998) decompose the convertible bond into two components with different credit quality. The debt only part of the convertible that is generating only cash payments and is exposed to default risk. The

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[6] Of course there are another two sources of randomness- the stochastic behavior of interest rates and the stochastic behavior of the real interest rates or the foreign interest rates, depends on the bond feature. But we prefer to assume that those factors are constant since we want to focus on the influence of the second asset on the convertible price. Brennan and Schwartz (1980) find that the effect of stochastic term structure on convertible prices is insignificant.

[7] Merton (1974) shows that company's equity can be viewed as a European call option on the total value of the firm assets, with a strike price equal to the face value of debt, where default can only occur at debt maturity.

[8] To overcome these problems the reduced form approach has been introduced by Jarrow and Turnball (1995), it uses the stock value as the stochastic component to explain default.

second component is the equity component, which is risk free, since the issuer can always deliver its own stock. They derive two joint PDE one for the "debt only component" and the other for the convertible bond price and approximate the solution by using the explicit finite difference method. Hull (2000) approximates these equations by using the more appealing Cox-Ross-Rubinstein (1979) binomial tree[9].

These single factor models can be adjusted to price inflation-indexed convertible bonds by using Fisher (1978) and Margrabe (1978) closed form solutions for an option to exchange one asset for another, where the underlying stock price and the CPI are both stochastic variables, and in the case of foreign currency convertible bond, the foreign currency replaces the CPI. However since the conversion can take place anytime before maturity and the convertible bond usually has call and put provisions, the closed form solutions fail to price the convertible bond and a numerical method for the dynamics of the two correlated assets should be applied as suggested by Rubinstein (1994) and Boyle (1988) and others[10].

In this paper we derive a model to price convertible bonds with payments that are linked to the CPI yield (the inflation rate). Assuming a bivariate lognormal distribution for the underlying stock price and the CPI, we derive the PDE for pricing the convertible bond and the relevant boundary conditions. As in Tsiveriotis and Fernandes (1998), who price nominal convertible bonds, using a one factor model we incorporate credit risk by presenting two joint PDE, one for the convertible price and one for the artificial security - the "debt only component".

We develop a valuation algorithm for the pricing of inflation indexed convertible bonds where both the underlying stock price and inflation are stochastic. A numerical method for the dynamics of the two correlated assets is used. First we improve upon the one factor model by using the Jarrow and Rudd (1983) binomial tree, which has computational advantage over the traditional Cox-Ross-Rubinstein (1979) binomial tree. In our two-factor model with credit risk we approximate the stochastic behavior of the

[9] Takahashi, Kobayashi and Nakagawa (2001) test the model empirically by using Japanese convertible bonds prices; Ammann, Kind and Wilde (2002) use a broader sample of French convertible bonds to test the pricing model.

[10] A Quanto option is an example of an option on two different assets (foreign currency and equity), however it is a European type option with terms that differ from an inflation indexed convertible and can be priced using a closed form solution, see Derman, Karasinsky, and Wecker (1990)

underlying stock and the CPI by applying Rubinstein (1994) three- dimensional binomial tree. This method enables us to value the convertible by using directly the implied volatility of traded options on the underlying stock and on the CPI without any numerical routines, while also to efficiently add one more dimension- the CPI dimension.

Although we focus on the case of inflation-indexed convertible bonds, the presented solution can be used with little modifications for pricing convertible bonds with coupon and principal payments linked (denominated) to a foreign currency, by using foreign-exchange analogy where the consumer-price-index corresponds to foreign currency.

We furthermore present numerical examples and empirical applications to demonstrate the usefulness of the model and illustrate how the model can be calibrated using market data. We study the convertible bond sensitivity to credit spread, the correlation between the stock and the CPI returns, the CPI volatility and the real interest rates.

The rest of this paper is organized as follows. Section 2 describes the assumptions and derives the theoretical framework for pricing inflation indexed convertible bonds. Section 3 presents the numerical binomial solution for the relevant pricing equations. First generalizing the numerical result of Hull (2000) and extending the analysis for inflation-indexed convertible bond. Section 4 provides a sensitivity analysis of the convertible bond. Section 5 presents empirical applications of the model for the pricing of indexed convertibles trade on the Tel Aviv Stock Exchange. Finally, concluding remarks are presented in section 6.

2. A Model for Pricing Inflation-Indexed Convertible Bonds With Credit Risk

In this section, we develop a valuation algorithm for the pricing of inflation indexed convertible bonds. Unlike a nominal convertible bond that pays known coupons and principal payments the coupon and the principal payments of the inflation indexed convertible bond are linked to the changes of the consumer-price-index (CPI) during the life of the convertible bond.

In order to price this type of convertible the following assumptions are made[11]:

(1) Investors can trade continuously in a complete, frictionless, arbitrage-free financial market. In particular it is assumed that there are no transaction costs, no restriction on short selling, and no differential taxes on coupons versus capital gains income[12]

(2) The uncertainty in the economy is characterized by a probability space (Ω, F, P), where Ω is a state space, F is the set of possible events and P is the objective martingale probability measure on (Ω, F). The stock price S follows the stochastic differential equation

$$\frac{dS}{S} = (\mu_S - \delta)dt + \sigma_S dW_S \tag{1}$$

We also assume that the inflation process follows a geometric Brownian motion, with dynamics given by[13]:

$$\frac{dI}{I} = \mu_I dt + \sigma_I dW_I , \tag{2}$$

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[11] Some of the assumptions could be relaxed. In particular, it would be possible to let the nominal and the real interest rates change over time as in Merton (1973), or to let the covariance and the volatilities to change over time as in Ho, Stapleton and Subrahmanyam (1995). The added complexity would not add significant insights to the present paper.

[12] However, there are recent evidence that differential state taxes on corporate versus government bonds may be important for the determination of corporate bond yields, see Elton, Gruber, Agrawal, and Mann (2001).

[13] The above stochastic process for the dynamics of the CPI can be found in Friend, Landskroner and Losq (1976) and in Benninga, Bjork and Wiener (2001).

where μ_s is the instantaneous expected return on the issuer's common stock, δ is the rate of dividend payout, μ_I is the instantaneous expected inflation rate. It is assumed that σ_s^2 and σ_I^2, which are, respectively the instantaneous variances of the rate of return of the underlying stock, S, and the consumer-price-index, I, are constants. dW_S and dW_I are standard Wiener processes with correlation given by $dW_S dW_I = \rho_{SI} dt$.

Using a foreign currency analogy, real prices correspond to foreign prices, nominal prices correspond to the domestic prices in local currency, and the CPI corresponds to the spot exchange rate. Garman and Kohlhagen (1983), assume that the process followed by a foreign currency is the same as that of stock providing a known dividend yield of δ, and therefore the expected change rate under the risk neutral expectation of the foreign currency must be $(r - r_f)$, where r is the nominal domestic risk free rate and r_f is the foreign risk free rate[14]. By analogy, the CPI has a drift rate of $\mu_I = (r - r_r)$, where r_r is the real interest rate [15]. By definition, a real bond provides complete indexation against future movement in price T periods ahead. Although inflation-indexed bonds provide incomplete indexation for the coupon and principal payments, because of reporting lags, Kandel, Ofer and Sarig (1993), show empirically, using Israeli bond data, that differences between expectations of past inflation embedded in bond prices and actual inflation rates are small in magnitude[16].

Let $U(T,S,I)$ be the value at time t of an inflation indexed convertible bond with maturity at date T. The bond can be converted at any time to shares of the underlying stock S, and is paying a principal of F that is linked to changes in the CPI from the issuing date. The convertible bond pays fixed coupon payments, C, are also linked to the CPI changes. To focus on the effects of inflation indexation on the convertible bond value we assume a generic inflation convertible bond that is both non-callable and non-putable.

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[14] Discussion on this relation can be found at Garman and Kohlhagen (1983).

[15] Other approaches that model the expected inflation are Sun (1992), that model this variable as mean reverting, Pennacchi (1991), that assumes that the policy of the government influence the expected inflation and Cox, Ingersoll and Ross (1985) provide two models with exogenous process for the inflation with stochastic inflation expectation.

[16] To cope with this problem, Evans (1998) derives estimates of the real term structure by first estimating the inflation-indexed bonds term structure and then combining these estimates with the nominal term structure to derive the real yields.

In the absence of risk of default by the issuer we can obtain its price dynamics by using Ito's formula for the dynamics of two correlated assets[17]:

$$
\begin{aligned}
rU + f(t) = {} & U_t + U_S(r-\delta)S + U_I(r-r_r)I \\
& + \frac{1}{2}(U_{ss}\sigma_S^2 S^2 + U_{II}\sigma_I^2 I^2 + 2U_{IS}\sigma_S\sigma_I\rho_{IS} IS)
\end{aligned}
\tag{3}
$$

Where $U_S, U_I, U_{SS}, U_{II}, U_{SI}$ and U_t denote the first and second order partial derivatives of the value of the convertible bond with respect to S, I or t respectively and $f(t)$ represents the coupon payment function.

Equation (3) does not account for default risk that is inherent in the convertible bond price. The convertible bond has two components of different credit risk. The underlying equity has no default risk since the issuer can always deliver its own stocks, on the other hand the issuer may fail to pay the coupon and principal payments, and thus introduce default risk. To cope with this problem, Tsiveriotis and Fernandes (1998) define a hypothetical security, which is called the "debt only part of the convertible bond" that generates only cash payments, but no equity that an optimal holder of a convertible would receive. Taking into account that the convertible bond is a derivative security of the underlying stock, they conclude that the "debt only" security is also a contingent claim with the same stock as its single underlying asset, thus the price of the debt only part, V, should follow the Black-Scholes (1973) equation. Since this security involves only cash payments by the convertible bond issuer, the relevant Black-Scholes equation should involve the credit spread of the issuer - the difference between the yield of a straight bond with the same credit quality as the convertible and a Treasury bond, identical in all respects except default risk. On the other hand, $(U-V)$ represents the value of the convertible related to payments in equity, and it should therefore be discounted using the risk free rate. The formulation of the convertible bond dynamics is obtained by the following system of two coupled equations:

[17] See Black and Scholes (1973)

$$r(U-V)+(r+cs)V-f(t)=U_t+\frac{1}{2}U_{SS}\sigma_S^2S^2+U_S(r-\delta)S \tag{4}$$

$$(r+cs)V-f(t)=V_t+\frac{1}{2}V_{SS}\sigma_S^2S^2+V_S(r-\delta)S \tag{5}$$

Equation (4) relates to the convertible bond price, and equation (5) relates to the debt only component, where cs is the credit spread implied by a similar quality non-convertible bond of the same issuer[18].

To expand the model to price inflation-indexed convertible bonds we add the terms which relate to the CPI from equation (3) to equations (4) and (5) and thus we obtain the two PDE that evolve the inflation-indexed convertible bond dynamics:

$$\begin{aligned} r(U-V)+(r+cs)V-f(t)=U_t+U_S(r-\delta)S+U_I(r-r_r)I \\ +\frac{1}{2}(U_{SS}\sigma_S^2S^2+U_{II}\sigma_I^2I^2+2U_{IS}\rho_{IS}\sigma_S\sigma_I SI) \end{aligned} \tag{6}$$

$$\begin{aligned} (r+cs)V-f(t)=V_t+V_S(r-\delta)S+V_I(r-r_r)I \\ +\frac{1}{2}(V_{SS}\sigma_S^2S^2+V_{II}\sigma_I^2I^2+2V_{IS}\rho_{IS}\sigma_S\sigma_I SI) \end{aligned} \tag{7}$$

Next, we characterize the boundary conditions according to the above defined terms of the inflation indexed convertible bond. The final conditions for the convertible bond price, U, and for the debt only component, V, can be written as:

$$U(T,S,I)=\begin{cases} \lambda \mathrm{S} & \lambda \mathrm{S} \geq (F+C)\dfrac{I}{I_0} \\ (\mathrm{F}+\mathrm{C})\dfrac{I}{I_0} & \text{elsewhere} \end{cases} \tag{8}$$

[18] Although Tsiveriotis and Fernandes (1998) assume in their paper that the credit spread is constant it can easily be relaxed and modeled as a time-dependent parameter.

$$V(T,S,I) = \begin{cases} 0 & \lambda S \geq (F+C)\dfrac{I}{I_0} \\ (F+C)\dfrac{I}{I_0} & \text{elsewhere} \end{cases}, \tag{9}$$

where λ is the conversion ratio, i.e., the number of shares of the underlying stock for which the convertible bond can be exchanged and I_0 is the value of the CPI on the issuing date of the convertible bond. Since the bond can be converted at any time prior to maturity we are dealing with an American-type derivative, that has a free boundary conditions, where the upside constrains due to conversion are[19]:

$$U \geq \lambda S \qquad \forall t \in [t,T] \tag{10}$$

$$V = 0 \quad \text{if } U \leq \lambda S \qquad \forall t \in [t,T] \tag{11}$$

The bondholder has the right to convert the bond at any time prior to maturity and thus the exercise policy needs to be known when solving the above partial differential equations (PDE). Since conversion can usually take place anytime before maturity and the convertible bond has usually a call and put provisions no general analytical solution can be used for pricing the convertible bond. In the next section, by using a binomial model, a numerical solution for pricing the inflation indexed convertible bond is presented.

[19] Discussion on the boundary conditions of a convertible bond can be found at Brennan and Schwartz (1977,1980).

3. Numerical Implementation

Since the closed form solution failed to price the convertible bond a numerical method for the dynamics of the two correlated assets is applied. Tsiveriotis and Fernandes (1998) use the finite difference method to approximate the convertible bond price according to the above one factor model. Based on this model, Hull (2000) approximates the convertible bond price by using the more appealing binomial tree of Cox-Ross-Rubinstein (1979). In this section we first generalizes the numerical result of Hull (2000) by using Jarrow and Rudd (1983) binomial tree, then we extend this analysis and demonstrate how to construct a recombining three-dimensional binomial lattice, that approximates the bivariate process of the stock price and the CPI, for pricing inflation-indexed convertible bonds.

We approximate the bivariate diffusion process by using Rubinstein (1994), which builds a three-dimensional tree for two correlated stocks[20]. We incorporate state-dependent credit risk according to Tsiveriotis and Fernandes (1998) model and evaluate the convertible bond and the debt only component at each state according to the defined boundary conditions.

3.1 One factor model for pricing non-callable convertible bond with credit risk

We consider a convertible bond with credit risk, which matures at time T that can be converted at any time to shares of the underlying stock, S, paying a principal of F at expiration if not converted, and an annual fixed coupon of C.

We construct Jarrow and Rudd (1983) binomial tree for the stock price dynamics. Although Hull (2000) uses a standard Cox, Ross and Rubinstein (1979) binomial tree, we preferred to use the former model since by using this model in a multi-dimensional economy the algebra is made simpler relative to the CRR model, as noticed by Chen, Chung and Yang (2001).

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[20] Ruinstein (1994) uses a nonrectangular arrangement of the nodes. Other papers illustrating the implementation of bivariate diffusions are Boyle (1988), who was the first to provide an algorithm for an American option on two correlated assets, Boyle, Evnine and Gibbs (1989), that extend the Boyle model to k factors, Hull and White (1994, 1996), Ho Stapleton and Subrahmanyam (1995), that allow the state variable to have a volatility and covariance that change over time, and Acharya and Carpenter (2001).

By using Ito's Lemma one can write equation (1) in the following way:

$$d\ln(S(t)) = (r - \delta - \frac{\sigma_S^2}{2})dt + \sigma_S dW_S \tag{12}$$

or:

$$dX_S = \alpha_S dt + \sigma_S dW_S \tag{13}$$

where $X = \ln(S(t))$ and $\alpha_S = (r - \delta - \frac{\sigma_S^2}{2})$. As a result $\ln(S(t))$ follows a generalized Wiener process for the time period. In the risk neutral world, dX becomes normally distributed with mean $(r - \delta - \frac{\sigma_2^2}{2})$ and variance $\sigma_S^2 dt$ [21].

The binomial tree is a discrete approximation of the continuous process, which is described in equation (1), for time interval Δt. We set the life of the tree equal to the life of the conversion option, T. This time is divided into short discrete periods of length $\Delta t = T / N$, each of which will be denoted by i, where $i = 0,1,..., N$. After each time interval Δt, the stock price can move from its initial value, S, to one of the two new values, Su and Sd. These two states are defined such that the implied price distribution matches as closely as possible the probability distribution of the underlying continuous state variable. Given the risk neutral standard diffusion process, the values and probabilities of the two states should be restricted in such a way that the expected price return over the next time interval is equal to $\alpha_S \Delta t$ and that its volatility is $\sigma_S \Delta t$. One convenient solution is the Jarrow and Rudd (1983) binomial tree, that defines equal probabilities: $p = (1 - p) = \frac{1}{2}$ for the two state variables Su and Sd, where u and d can be calculated as:

[21] A full explanation of the risk neutrality falls outside the scope of this paper. A clear demonstration of its use can be seen in Benninga and Wiener (1997).

$$u = e^{\alpha_S \Delta t + \sigma_S^2 \sqrt{\Delta t}}, d = e^{\alpha_S \Delta t - \sigma_S^2 \sqrt{\Delta t}}, \tag{14}$$

The no arbitrage conditions are: $u > e^{r(i,i+1)\Delta t} > d$, where $r(i, i+1)$ is the future risk-free rate of interest between period i and period $i+1$. In general, the underlying stock price at each node is set equal to $Su^j d^{i-j}$, where j is the number of up movements of the stock price.

Once we have the tree of future stock prices, we can calculate the value of the convertible bond at each node by starting at maturity, where its value is known with certainty, according to the final conditions, and then moving backwards in time period by period to calculate the value at the earlier nodes.

Applying the final condition (8) and the boundary condition (10), at any time the bondholder has two choices. She can hold (or redeem at maturity) the bond, which has the value at each node of $UH_{i,j}$ or she can convert the bond to stocks and receive $UC_{i,j}$. Summarizing: the value of the convertible bond, $U_{i,j}$, at each node, is worth the maximum of $UC_{i,j}$ and $UH_{i,j}$, which can be written as: $\mathrm{U}_{i,j} = \max[UH_{i,j}, UC_{i,j}]$.

In order to take credit risk into account the convertible bond value is decomposed into two components. The first is the debt only part of the convertible, $V_{i,j}$, generating only cash payments but no equity, that an optimally behaving holder of a convertible would receive. This security is discounted by the risky rate of the issuer, $r^*(i, i+1)$, while the second component, the equity component, $E_{i,j}$, is discounted by the risk free rate, $r(i, i+1)$. At each node the convertible bond value is equal to $\mathrm{U}_{i,j} = V_{i,j} + E_{i,j}$ [22].

We now specify the relationships between the equity and the debt only components, and between the conversion value and the holding value. Starting at the last period of the stock price tree (final node), the holding value, $UH_{N,j}$, and the value received from immediate conversion, $UC_{N,j}$, at each final node, can be calculated as:

[22] Although the equity component could be calculated at each node as the difference between the convertible bond price and the debt only component, we choose to present the above formulation to clarify the model to the reader.

$$UC_{N,j} = \lambda S u^{j} d^{N-j} \qquad UH_{N,j} = F + C \tag{15}$$

Given $UC_{N,j}$ and $UH_{N,j}$, we obtain the value of the equity only component at each of the final nodes as:

$$E_{N,j} = \begin{cases} UC_{N,j} & UC_{N,j} \geq UH_{N,j} \\ 0 & \text{elsewhere} \end{cases} \tag{16}$$

The value of the debt component at each of the final nodes is given by:

$$V_{N,j} = \begin{cases} UH_{N,j} & UH_{N,j} > UC_{N,j} \\ 0 & \text{elsewhere} \end{cases} \tag{17}$$

At any time period prior to maturity, the holding value is calculated at each node by adding the expected value of the debt component one period later, discounted at the appropriate risky rate, and the expected value of the equity component one period later, discounted at the risk free rate:

$$UH_{i,j} = \frac{1}{2} e^{-r^*(i,i+1)\Delta t} (V_{i+1,j+1} + V_{i+1,j}) + \frac{1}{2} e^{-r(i,i+1)\Delta t} (E_{i+1,j+1} + E_{i+1,j}) \tag{18}$$

Applying the boundary condition (10), the value received from immediate conversion at any time period prior to N is calculated as:

$$UC_{i,j} = \lambda S u^{j} d^{i-j} \ . \tag{19}$$

At periods when interest on the debt is paid the coupon value is just added to the holding value of the convertible bond. Given $UC_{i,j}$ and $UH_{i,j}$, we obtain the value of the equity only component at any time period $i \in [0, N-1]$ as:

$$E_{i,j} = \begin{cases} \text{UC}_{\text{i, j}} & \text{UC}_{\text{i, j}} \geq \text{UH}_{\text{i, j}} \\ \frac{1}{2} e^{\text{-r(i,i +1)}} (\text{E}_{\text{i+1, j+1}} + \text{E}_{\text{i+1, j}}) & \text{elsewhere} \end{cases} \quad (20)$$

Applying the free boundary condition from equation (11), the value of the debt component at any of the nodes at periods $i \in [0, N-1]$ is worth zero in cases where the bond had been converted, in cases where the optimal policy is to hold the bond, then the value of the debt component is the expected value of the debt component one time period later, discounted at the risky rate:

$$V_{i,j} = \begin{cases} 0 & \text{UC}_{\text{i, j}} > \text{UH}_{\text{i, j}} \\ \frac{1}{2} e^{-r^*(i,i+1)\Delta t} (V_{i+1,j+1} + V_{i+1,j}) & \text{elsewhere} \end{cases} \quad (21)$$

3.2 Two-factor model for pricing non-callable inflation indexed convertible bond with credit risk.

The valuation model described above is adjusted in this section to price inflation indexed convertible bonds (or foreign currency convertible bonds). Such corporate securities are traded at the Tel-Aviv stock exchange (TASE). The coupon and the face value of those bonds are linked to the consumer-price-index (CPI) return.

At first, to approximate the dynamics of the diffusion processes we construct a Rubinstein (1994) three-dimensional binomial tree, where the underlying stock price and the CPI are the two stochastic variables, next we solve the convertible PDE with a recursive backward algorithm taking into account all the boundary conditions that were derived in section 2.

As in equation (1) and (2), the stock price and the CPI dynamics under risk neutral equilibrium are given as:

$$\frac{dS}{S} = (\mu_S - \delta)dt + \sigma_s dW_S \quad , \frac{dI}{I} = \mu_I dt + \sigma_I dW_I ,$$

where dW_S and dW_I are two Wiener processes with correlation ρ_{SI}. It is assumed that the joined density of the two underlying assets has a bivariate lognormal distribution. The three-dimensional binomial tree is a discrete version of this process for time interval Δt. The time interval $[0,T]$ is again divided into N equal intervals of length Δt, each of which will be denoted by i, where $i = 0,1,...,N$. As in the one factor model, the initial stock price, S, can move up at any period by u or down by d with equal probability, where:

$$u = e^{\alpha_S \Delta t + \sigma_S^2 \sqrt{\Delta t}}, \; d = e^{\alpha_S \Delta t - \sigma_S^2 \sqrt{\Delta t}}.$$

The underlying stock price in each node at any period i is set equal to $Su^j d^{i-j}$, where $j = 0,1,...,i$ is the number of up movements of the stock price.

Inflation uncertainty is introduced via four conditions. If the stock price moves by u, the value of the CPI, I, can move either by A or B with equal probability. If the stock price moves down by d, the CPI value can move by C or D with equal probability, where:

$$A = \exp[\alpha_I \Delta t + \sigma_I \sqrt{\Delta t}(\rho_{S,I} + \sqrt{1-\rho_{SI}^2})]$$

$$B = \exp[\alpha_I \Delta t + \sigma_I \sqrt{\Delta t}(\rho_{S,I} - \sqrt{1-\rho_{SI}^2})]$$

$$C = \exp[\alpha_I \Delta t - \sigma_I \sqrt{\Delta t}(\rho_{S,I} - \sqrt{1-\rho_{SI}^2})]$$

$$D = \exp[\alpha_I \Delta t - \sigma_I \sqrt{\Delta t}(\rho_{S,I} + \sqrt{1-\rho_{SI}^2})] \qquad (22)$$

where $\alpha_I = (r - r_r) - \frac{\sigma_I^2}{2}$.

To make the lattice for each state variable recombine the condition $AD = BC$ is imposed. By setting $A \neq C$ and $B \neq D$, it is possible to construct a nonzero correlation between the underlying stock price and the consumer-price-index. The three–dimensional binomial process converges to the original continouous process as $\Delta t \to 0$.

From any node (i, S, I), the lattice evolves to four nodes, $(i+1, Su, IA)$, $(i+1, Su, IB)$, $(i+1, Sd, IC)$ and $(i+1, Sd, ID)$. Where IA, IB, IC, and ID are the values of the CPI in the different nodes. The CPI in each node, at any time period i and with j up movements of the stock price, is set equal to[23]:

$$I(i, j, k) = I_0 e^{\alpha_I i\Delta t + \sigma_I \sqrt{\Delta t}\left[\rho_{SI}(2j-i) + (\sqrt{1-\rho_{SI}^2})(2k-i)\right]}, \qquad (23)$$

where $k = 0,1,...,i$.

The four nodes have associated risk-neutral probabilities of 0.25. The tree consists of 2^{i+1} distinct nodes at each period and of total $(1+N)^2$ distinct nodes.

Given the value of the stock and the CPI at any node, the value of the convertible bond can be calculated. First the value of the bond is calculated at the final period, according to its final conditions and then, working backwards we discount the convertible bond value while applying the free boundary conditions.

According to the final condition (8) and the boundary condition (10) the value of the convertible bond at each node $U_{i,j,k}$, equals the maximum of $UC_{i,j,k}$ and $UH_{i,j,k}$, which can be written as: $\mathrm{U}_{\mathrm{i,j,k}} = \max[UH_{i,j,k}, UC_{i,j,k}]$.

In order to incorporate credit risk into the pricing model, the convertible bond value is decomposed, as in the one factor model, into two components. The first is the debt only part of the convertible, $V_{i,j,k}$, which is discounted by the risk adjusted rate of the issuer, $r*(i,i+1)$, while the equity component, $E_{i,j,k}$ is discounted by the risk free rate, $r(i,i+1)$. At each node the convertible bond value is equal to $\mathrm{U}_{i,j,k} = V_{i,j,k} + E_{i,j,k}$, that is the sum of the equity component and the bond component at the node.

At each final node (N, j, k) the holding value, $UH_{N,j,k}$, can be calculated by multiplying the promised final payment by the CPI yield, which is calculated in equation (23). The

[23] A similar expression for the correlated asset price for Rubinstein three-dimensional binomial tree can be found in Haug (1997).

value received from immediate conversion, $UC_{N,j,k}$, is calculated in a similar manner to the one factor model, according to equation (15):

$$UC_{N,j,k} = \lambda S u^{j} d^{N-j},$$

$$UH_{N,j,k} = (F+C)e^{\alpha_I i\Delta t+\sigma_I\sqrt{\Delta t}\left[\rho_{SI}(2j-i)+(\sqrt{1-\rho_{SI}^2})(2k-i)\right]} \quad (24)$$

As in the one-factor model, given $UC_{N,j,k}$ and $UH_{N,j,k}$, we obtain the value of the equity and debt components at each final node:

$$E_{N,j,k} = \begin{cases} \mathrm{UC}_{\mathrm{N,j,k}} & \mathrm{UC}_{\mathrm{N,j,k}} \geq \mathrm{UH}_{\mathrm{N,j,k}} \\ 0 & \text{elsewhere} \end{cases} \quad (25)$$

$$V_{N,j,k} = \begin{cases} \mathrm{UH}_{\mathrm{N,j,k}} & \mathrm{UH}_{\mathrm{N,j,k}} > \mathrm{UC}_{\mathrm{N,j,k}} \\ 0 & \text{elsewhere} \end{cases} \quad (26)$$

At any time period prior to maturity, the holding value is calculated at each node by adding the expected value of the debt component of the four leading nodes one time step later, multiplied by 0.25 and discounted at the appropriate risky rate, to the expected value of the equity component one time step later discounted at the risk free rate:

$$\begin{aligned} UH_{i,j,k} &= \frac{1}{4}e^{-r^*(i,i+1)\Delta t}(V_{i,j,k}+V_{i,j,k+1}+V_{i,j+1,k}+V_{i,j+1,k+1}) \\ &+\frac{1}{4}e^{-r(i,i+1)\Delta t}(E_{i,j,k}+E_{i,j,k+1}+E_{i,j+1,k}+E_{i,j+1,k+1}) \end{aligned} \quad (27)$$

Applying the free boundary condition (10), the value received from immediate conversion at any time period prior to N is calculated as:

$$UC_{i,j,k} = \lambda S u^{j} d^{i-j}. \quad (28)$$

At periods where interest on the debt is paid the coupon value is multiplied by the CPI yield and added to the holding value of the convertible bond. Given $UC_{i,j,k}$ and $UH_{i,j,k}$, we obtain the value of the equity only component at any time period $i \in [0, N-1]$ as:

$$E_{i,j,k} = \begin{cases} \mathrm{UC}_{\mathrm{i,j,k}} & \mathrm{UC}_{\mathrm{i,j,k}} \geq \mathrm{UH}_{\mathrm{i,j,k}} \\ \frac{1}{4} e^{-r(i,i+1)\Delta t} (E_{i,j,k} + E_{i,j,k+1} + E_{i,j+1,k} + E_{i,j+1,k+1}) & \text{elsewhere} \end{cases} \tag{29}$$

Applying the free boundary condition from equation (11), the value of the debt component at any node at period $i \in [0, N-1]$ is worth zero in cases where the bond has been converted, in cases that the optimal policy is to hold the bond, the value of the debt component is the expected value of the debt component one time step later, discounted at the risky rate:

$$V_{i,j,k} = \begin{cases} 0 & \mathrm{UC}_{\mathrm{i,j,k}} \geq \mathrm{UH}_{\mathrm{i,j,k}} \\ \frac{1}{4} e^{-r^*(i,i+1)\Delta t} (V_{i,j,k} + V_{i,j,k+1} + V_{i,j+1,k} + V_{i,j+1,k+1}) & \text{elsewhere} \end{cases} \tag{30}$$

4. Comparative Statics Analysis

In this section we perform a sensitivity analysis of the theoretical Inflation-indexed convertible bond and the nominal convertible bond values with respect to a variety of parameters (tables and figures 1 through 4), later we demonstrate the effect of installments on the convertible bond value (tables and figures 5 through 6).

4.1 Credit spread and CPI parameters

To focus on the effects of indexing the CPI coupon and principal payments of the convertible, we choose to value two convertible bonds under the same set of market data (nominal interest rate, stock volatility, credit spread and stock price) with similar conditions (principal payment, coupon rate and frequency of payment, debt maturity, and conversion ratio), except for the fact that one of the bonds is linked to the CPI yield and the other is not (nominal convertible bond). To emphasize the impact of the CPI stochastic behavior on the bond value we assume that the expected nominal interest rate is equal to the expected real interest rate and thus the drift term is equal to zero[24]. In Table 1 (and figure 1) the CPI-volatility is set equal to 5% and 15%, the low level of volatility is similar to the actual volatility of the CPI returns in countries with low inflation rates, and the higher volatility is appropriate to the volatility level that exists in the currency markets. When the correlation between the stock price returns and the CPI returns are high and positive (+0.5 in our example) the inflation-indexed convertible bond value is lower than the nominal convertible value by 0.77% and 1.40% for equity value that equals the bond face value (100 in our example) and volatility levels of 5% and 15%, respectively. Since the two assets returns are positively correlated there is a good chance for the linked principal payment to be lower than the nominal principal payment and thus we observe a discount on the inflation-indexed convertible price. When the correlation between the two assets returns is negative (-0.5) we find the opposite phenomena, the inflation-indexed bond value is higher than the nominal bond by 0.91% and 3.27% for CPI volatility of 5% and 15% respectively.

[24] In most real world cases the inflation expectation is positive and thus the drift is positive and does not equal zero as in the chosen example.

Table 2 (and Figure 2) presents the convertible bond values for combination of CPI volatility and credit spread. As expected, the convertible value increases with CPI volatility and decreases with credit spread for negative correlation (-0.5). Interestingly, when the correlation between the assets is positive (0.5) the relationship between convertible bond value and the CPI volatility is U-Shaped. Using a one period binomial tree and assuming a unit correlation between the two assets we can intuitively explain this result. At expiration there are two possible states, up movement of the stock and the CPI, and down movement of these two assets. If the optimal policy in the up state is to convert the bond and the optimal policy in the down state is to redeem the bond, then the convertible bond value would decrease with the CPI volatility. On the other hand, when the optimal policy is to redeem the bond in the up state and to convert it in the down state, then the convertible value increases with the CPI volatility[25].

Table 3 (and figure 3) provides the values of a convertible bond for a combination of issuer's credit spreads and correlations. As the issuer's credit spread increases the convertible's value decreases until it converges to its lower boundary- the conversion value that equals λS . As the correlation decreases the convertible bond value converges more slowly to its lower boundary.

Table 4 (and figure 4) provides the value of the convertible for combinations of stock price, real interest rate, and the initial level of the CPI (i.e., the cumulative change of the CPI yield from the issuing date till the current pricing date). Having in mind our foreign currency analogy we choose two levels of the CPI at the pricing date. In the first the CPI is equal to 1.2, and thus the accumulated inflation rate until the pricing date is equal 20% and in the second case the CPI level is 0.8 (decline of 20%) where in both cases the stock price is equal to 100 (at the money). In the first case, the conversion option is out-of-the-money and as a result the drift term of the CPI is $(r - r_r)dt$, has a relatively large effect on the convertible value. When the real interest rate is equal to the nominal interest rate (6%), and thus the drift term is equal to zero, the convertible is worth 124.7. A decrease of the real interest rate to zero would increase the convertible price to 129.9. In the second case, we assume a decrease in the CPI from the issuing date until the pricing date

[25] A discussion of the optimal exercise regions of American options on multiple assets can be found in Broadie and Detemple (1997).

is –20%, and using. In this case the conversion option is in the money, and thus the convertible bond value increases from only 105 to 106.5.

4.2 Installment Payments

The convertible bonds that are traded on the TASE have the feature that the principal is paid in several payments (installments).). In appendix 1 we expand our model to price amortized convertible bond. Dividing the principal payment of a straight bond to several payments, without changing other conditions, decreases the duration of a straight bond and increases its value. In the case of a convertible bond the effects are more puzzling.

Table 5 (and figure 5) shows the effects of the installments on the convertible bond values. When the conversion value is very low compared to the principal payment, it is obvious that conversion will not take place at any state and the convertible value is identical to the value of a straight bond. In that region, the convertible bond value increases with the number of principal payments, similar to a straight bond.

However, when the stock price is high relative to the promised principal payment, the installment has two opposing effects on the convertible price. The security promises earlier principal payments compared to a standard convertible bond, which is identical in all features except for the installments, and thus the increase in the number of principal payments has a positive influence on the convertible price. On the other hand, a standard convertible bondholder may find that the optimal policy is to hold the bond till maturity, but the holder of the convertible with installments is forced, at each non-final partial principal payment, to decide whether to receive the cash payment or to convert the entire bond to stocks. Ingersoll (1977), shows that the optimal policy of a convertible bondholder in the absence of dividends is to hold the bond till maturity, thus in states that the bondholder decides to convert the bond to the underlying stocks, the installments convertible bond value decreases compared to the value of a standard convertible bond. The last effect becomes crucial as the stock price increases. As can be seen in table 5, when the stock price equals 50, the value of the convertible bond, with four years to maturity, annual coupon rate of 4% and one principal payment is equal to the value of a similar convertible bond with four principal payments (the principal payments are paid annually). Above this price, the value of the convertible with a single principal payment is higher than the value of a convertible with four principal payments, and below this price the result is inverted.

Table 6 (and figure 6) shows the relationship between the convertible value and the coupon rate, C. The right side of the table provides the convertible values where there is just one principal payment. As expected the value of the convertible increases with C. Since the optimal policy is to hold the bond till maturity, the convertible value converges to the conversion value plus the discount value of the accumulated coupon payments. However, when there are several principal payments, as explained above, early conversion may be the optimal policy at relative high stock prices and therefore the convertible bond values converge to the conversion ratio as the stock price increases.

5. Empirical Applications of the Inflation-indexed Convertible Bond Model

To demonstrate the Inflation-Indexed CB pricing model and to better understand it we present here two applications of the model for the valuation of Ezorim Inc inflation-indexed convertible bond and Machteshim-Agan Inc foreign-currency linked convertible bond traded on the TASE (see table 7).

On December 3, 2001, Machteshim-Agan convertible bond was traded at a price of 92.3 Agorot (Agorot100= 1 Israeli Shekel) per 1.00 Shekel par value of the bond. The market capitalization of the bonds was USD 67.4 million. The closing price of Machteshim-Agan common stock was 840.1 Agorot

According to the indenture agreement, each Machteshim-Agan foreign-currency linked CB has a face value of 1.00 Shekel and matures on November 20, 2007. If the security has not been converted prior to this date and if the issuer does not default, the investor receives 1 Shekel that is linked to the change of the Dollar/Shekel exchange rate during this period. The convertible bond pays a fixed annual coupon rate of 2.5% that is also linked to the exchange rate. At anytime before maturity the investor may elect to convert the bond into 0.0936 shares of Machteshim-Agan common stock (See Table 7).

To apply the Inflation-Indexed CB pricing model to Machteshim-Agan Inc it was necessary to calculate the local and foreign risk free interest rates[26], which were 6.66% and 4.73% respectively. Besides these observable input parameters, the pricing model requires estimation of unobservable parameters inputs.

These inputs include, the company's common stock volatility, the Dollar/Shekel foreign exchange volatility, the correlation between these two underlying assets and the appropriate credit spread and dividend yield. The common stock volatility and the exchange rate volatility used were the historic standard deviation of daily returns over the 100 trading days prior to the issue date of the foreign currency-indexed CB. The estimated stock and exchange rate annualized volatilities on the issue date were 26.5% and 4% respectively. The chosen credit spread is expressed in basis points over the government yield. Since the issuer had not issued straight debt in the market, the credit spread is calculated on the basis of credit risk rating. Machteshim-Agan foreign-currency CB was rated by the Israeli rating agency "Maalot" as AA, which is somehow parallel to the Baa rating of Moody's international rating agency, so the credit spreads were calculated as the difference between Moody's Seasoned Baa index and the yield on US Treasury notes.[27] The dividend yield was estimated based on the stock's historical dividends during the last 12 months.

Table 7 presents all the necessary data for pricing the convertible bond, the observed price of the convertible bond and the model's theoretical price. The difference between the theoretical and the market price is 2.1 Agorot, which is 2.2% of the bond price. If we drop the terms that relate to the stochastic behavior of the exchange rate from equation (2), the model becomes a one-factor model, and thus the one-factor model price equals 93.4 Agorot. Figure 7 presents the theoretical price and the market price of Machteshim-Agan convertible bond during the period between 3/12/01 and 1/8/02 (dd/mm/yy).

From the beginning of April 2002 the price difference between the one-factor model and the two-factor model is significant. On 10/6/02 the convertible bond price, according to

[26] The local yields were calculated as the average of the intermediate Israeli government bonds yields at each pricing date (named "Shahar"). The foreign yields are the average yields on the 5 and 7 years constant maturity treasury bonds indexes, as is daily published by the U.S. Treasury.

[27] We assume that all of the corporate- Treasury yield spread is due to credit risk; however taking a smaller credit spread would not affect our results significantly.

the two-factor model, is equal 103.9 Agorot while the one- factor model bond price is equal only 101.4 Agorot. The increase of the price difference between the two models can be explained mainly by the increase in the Dollar/Shekel exchange rate volatility from 4% to almost 10% (Figure 8). During the sample period the theoretical equity component value ranged from 19.2% to 54% of the total convertible bond price, the average equity component equals to 37%.

As the data in table 8 indicates, over the first four weeks following issuance, the theoretical prices closely track the reported market closing prices. To evaluate our model more analytically and to compare it to the one-factor model we calculate the *model error ratio*, which is defined as the deviation of the theoretical from the observed price divided by the observed price. The two-factor model average error ratio of -0.52% indicates an observed under pricing, while the one-factor model indicates a greater average error ratio of –0.91%.

The *absolute error ratio* is defined as the absolute value of the deviation of the theoretical from the observed market price divided by the observed market price of each of the 154 observations. The two-factor-model average absolute error ratio is equal to 1.42% while the one-factor-model average absolute error ratio is equal to 1.56%.

To better understand the source of the absolute value error of the suggested two-factor model we have regressed the absolute value error against the following: common stock volatility, the exchange rate volatility, the difference between the daily high and low price, the ratio between the equity component, and the convertible bond price and the exchange rate yield. Results are shown in table 9. All regression coefficients except that of the exchange rate volatility are statistically significant at the 5% level. The common stock volatility is also significant at the 1% level. The regression Adjusted-R^2 is equal to 0.13. The common stock volatility has a significant and a negative coefficient, which indicates that improving the volatility estimating methods might improve the model results. The negative coefficient of the equity component indicates a negative relationship between the convertible moneyness and the model predicting power.

In pricing the convertible bond, we used the “representative” exchange rate published by the Bank of Israel each business day. This exchange rate is calculated according to a random sample of trades that took place between the hours 14:00 and 15:00. This exchange rate might be different from the actual exchange rate that was observed at the closing time of the TASE. As expected, we find that the coefficient of the difference between the high and the low daily prices divided by the closing price has a significant positive influence on the absolute model error ratio.

A second regression (table 10) was run to understand the factors that influence the *model error ratio*, i. e. to understand the model overpricing or mispricing. The Adjusted R^2 is equal 0.18. The exchange rate yield and the equity component have negative significant coefficients, and thus as the moneyness increases the model underpricing increases, the common stock volatility has the opposite effect.

A second pricing example is the inflation-indexed convertible bond issued by Ezorim Inc. The needed data for valuing the convertible and the pricing results are shown at the bottom of table 7. Because the CPI is published monthly, the CPI volatility was the standard deviation of the returns over the last 36 months prior to the pricing date of the inflation-indexed convertible bond value, and its value is 2.45%. At this low level of volatility, ignoring the stochastic component of the CPI process has almost no impact on the value of the inflation-indexed convertible bond, and the value is equal 1.13 Shekel (while the two-factor model price equal 1.128). These results emphasis that at low level of the CPI volatility the indexed convertible bond can be priced by using the one factor model without much loss in the model accuracy.

6. Concluding Remarks

Convertible bonds with coupon and principal payments that are linked to the yields of foreign currency or consumer-price-index are traded in numerous capital markets. These corporate securities are exposed to credit risk since the issuer can default on coupons or principal payments. Previous attempts to price this type of convertible bonds have not incorporated all these features and sources of risk. In this paper, we provide a valuation method for inflation-indexed convertible bonds that allows for both, the underlying stock and the consumer-price-index, to be stochastic and incorporates exogenous credit spread. We approximate the pricing equations by using a Rubinstein (1994) three-dimensional binomial tree. Credit risk is introduced by extending Tsiveriotis and Fernandes (1998) convertible pricing model.

In our study of the convertible bond's sensitivity to the different risk factors we show first that positive correlation between the returns of the underlying stock and the CPI has a negative effect on the value of an inflation-indexed convertible bond. Second, when the correlation is negative the convertible bond price increases with CPI volatility, but when the correlation is positive the convertible bond price curve has a U-shape with respect to CPI volatility. Third, the higher the correlation, the faster will the convertible price converge to the conversion value as the credit spread increases.

To further the understanding of the convertible bonds that are traded on the TASE, we value convertible bonds with installment payments. These convertible bonds have the feature that the principal is repaid in installments. Although installments are not a rare feature in straight debt agreement, it is still not common when dealing with convertible bonds. We obtain, the counterintuitive result that at relative high stock prices the value of the convertible bond decreases with the number of principal payments for a given maturity.

In the empirical application of the model we estimated the theoretical values of two convertible bonds traded on the TASE. One indexed to foreign exchange and the second to the CPI. Our results indicate that for relatively high volatility of the indexing asset (foreign currency in our case) the difference between the one factor model and the two-factor model, that considers uncertainty of the underlying index, is significant.

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Appendix 1: Pricing convertible bonds with installment payments

The convertible bonds traded on the TASE have the additional feature that the principal is repaid in several payments (installments). In this section we adjust the valuation model to this feature. For the sake of simplicity we make the installment adjustment to a nominal convertible bond, where the extension to inflation-indexed convertible bonds is straightforward. We assume that there are g principal payment dates, where $g = 1,2,...,M$ and in each of those dates the convertible bondholder is entitled to $\frac{F}{M}$ of the principal if she decides to hold the bond at the payment date.

Thus, The holding value, $UH_{N,j}$, and the value received from immediate conversion, $UC_{N,j}$, at each final node can now be calculated as:

$$UH_{N,j} = \frac{F + C}{M} = \frac{F}{M} + \frac{C(M - g + 1)}{M} \tag{A1}$$

$$UC_{N,j} = \lambda S u^{j} d^{i-j} \frac{(M - k + 1)}{M} = S u^{j} d^{N-j} \frac{(M - g + 1)}{M} \tag{A2}$$

At the final nodes, the equations for calculating the value of the equity only component and the debt only component of convertible bonds, that pay principal in several payments, are identical to the equations for calculating the value of a nominal convertible bond, as was shown at equations (16) and (17) respectively

In earlier periods, where the principal is paid, the holding value of the convertible is the sum of the periodic principal payment, the coupon payments on the remaining principal payment and the expected value, in a risk neutral world, of the debt component and the equity component, each of them is discounted at it’s appropriate discount rates. Consequently the holding value can be calculated by adding equation (A1) and equation (18):

$$UH_{i,j} = \left(\frac{F}{M} + \frac{C(M-g+1)}{M}\right) In_{\{(i \le N) \cap (i \in g)\}} + \frac{1}{2} e^{-r^*(i,i+1)\Delta t} (V_{i+1,j+1} + V_{i+1,j})$$
$$+ \frac{1}{2} e^{-r(i,i+1)\Delta t} (E_{i+1,j+1} + E_{i+1,j})$$

(A3)

where In is simply an indicator function equal to one if principal payment is supposed to take place at period i.

The value received from immediate conversion at any time period prior to N is calculated according to equation (32). The value of the equity only component at any principal payment date can be calculated in an identical way as for the nominal convertible bonds (see equation 20). The value of the debt component is calculated in cases where the holding value is greater than the conversion value as the sum of the periodic principal payment, the coupon payments on the remaining principal payment and the expected value in a risk neutral world of the debt component discounted at the risky rate. The value of $V_{i,j}$ can be written as:

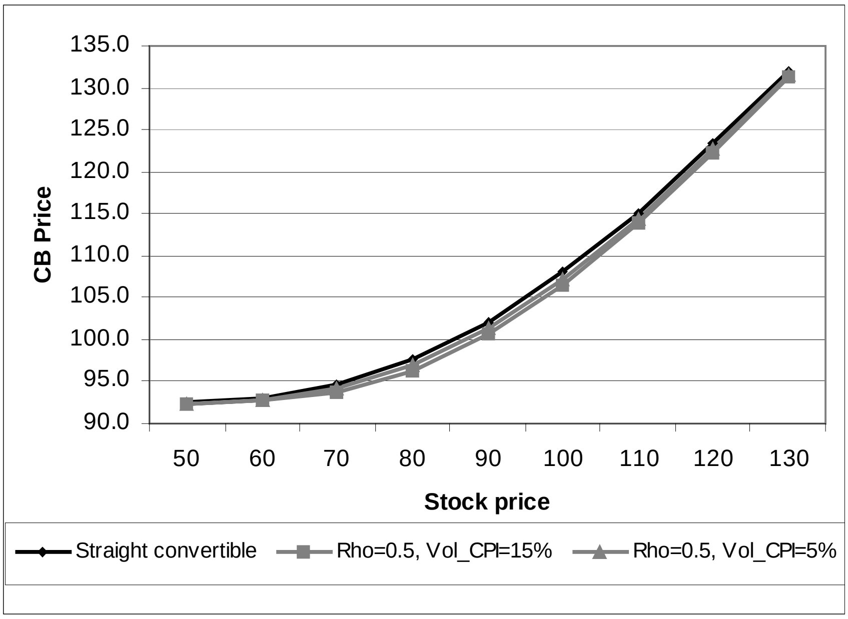
$$V_{i,j} = \begin{cases} 0 & UC_{i,j} \ge UH_{i,j} \\ \left[\frac{F}{M} + \frac{C(M-g+1)}{M}\right] In_{\{(i \le N) \cap (i \in g)\}} & \\ + \frac{1}{2} e^{-r^*(i,i+1)\Delta t} (V_{i+1,j+1} + V_{i+1,j}) & \text{elsewhere} \end{cases} \quad \text{(A4)}$$

Table 1
The values of the convertible bond for a combination of stock price, CPI volatility and correlation.

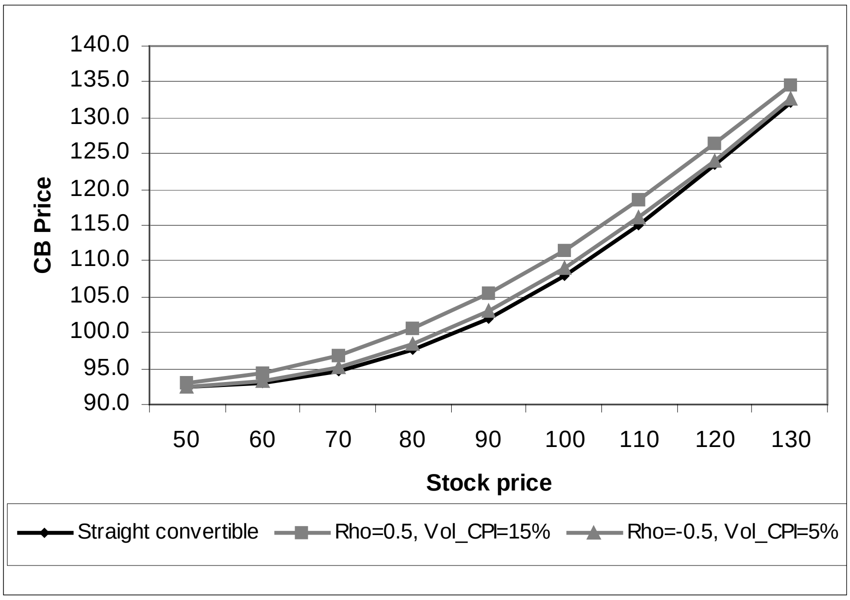
Convertible bond values									
	Straight convertible	**Inflation-indexed convertible bond**							
		Rho=-0.5				**Rho=0.5**			
CPI Volatility		**5%**		**15%**		**5%**		**15%**	
Stock price									
50	92.5	92.6	(0.12)	93.0	(0.59)	92.4	(-0.06)	92.4	(-0.10)
60	93.0	93.4	(0.35)	94.4	(1.41)	92.8	(-0.21)	92.7	(-0.38)
70	94.6	95.2	(0.61)	96.8	(2.32)	94.2	(-0.48)	93.8	(-0.85)
80	97.6	98.4	(0.86)	100.6	(3.08)	96.9	(-0.68)	96.3	(-1.24)
90	102.1	103.0	(0.96)	105.5	(3.37)	101.3	(-0.78)	100.6	(-1.44)
100	108.0	109.0	(0.91)	111.6	(3.27)	107.2	(-0.77)	106.5	(-1.40)
110	115.1	116.1	(0.87)	118.6	(2.97)	114.5	(-0.58)	113.9	(-1.10)
120	123.3	124.1	(0.63)	126.3	(2.42)	122.7	(-0.51)	122.2	(-0.90)
130	132.0	132.7	(0.51)	134.6	(1.94)	131.6	(-0.35)	131.2	(-0.62)
140	141.2	141.8	(0.39)	143.4	(1.51)	140.9	(-0.22)	140.7	(-0.40)
150	150.8	151.2	(0.25)	152.5	(1.11)	150.5	(-0.17)	150.4	(-0.28)

Parameters in this table are: $F = 100$, $\lambda = 1$, $C = 0\%$, $\delta = 0$, $cs = 3\%$, $r = r_r = 6\%$, $\sigma_S = 30\%$, $T = 1$, $N = 100$, $I = 1$.
The percent of change between the inflation-index convertible bond value and the same but straight convertible bond value appear on barracks.

Figure 1
The values of the convertible bond for a combination of stock price, CPI volatility and correlation.



Rho=0.5



Rho=-0.5

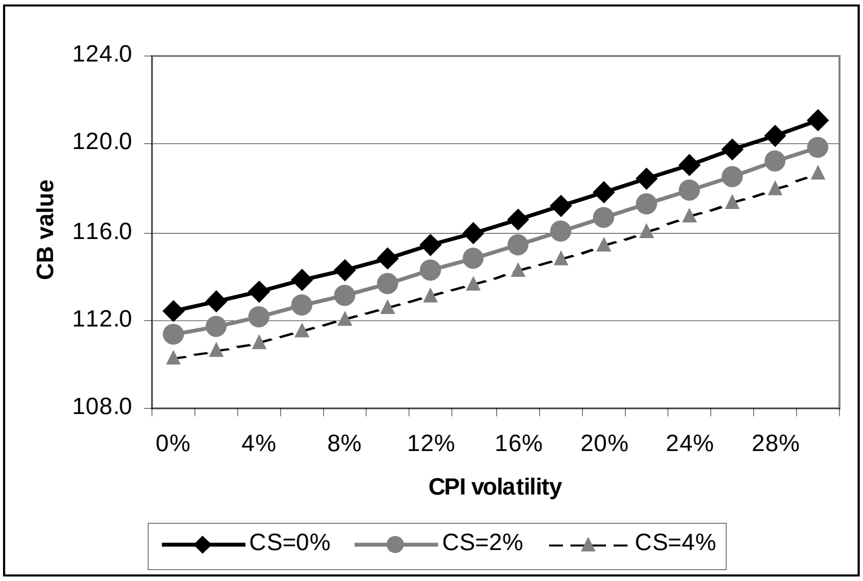
Parameters: See table 1.

Table 2
The values of the convertible bond for a combination of CPI's volatility credit spread and correlation.

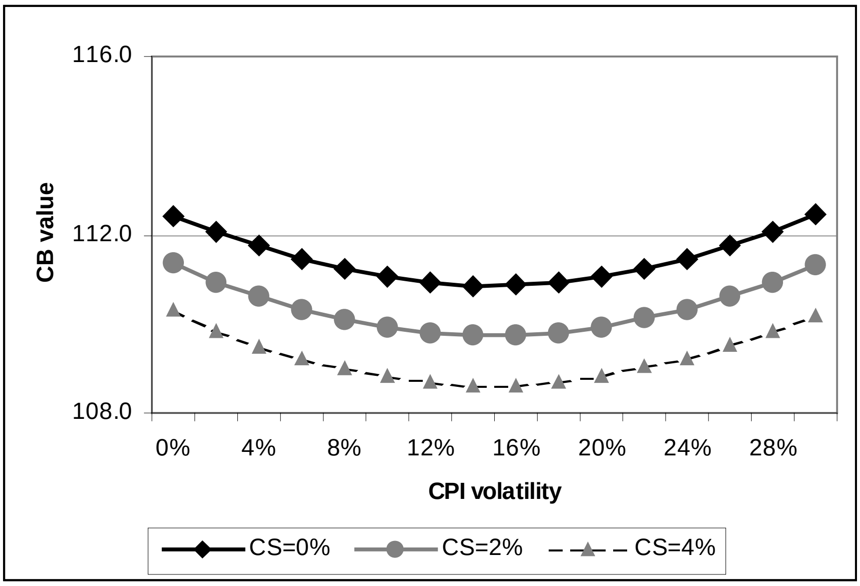
	Convertible bond values					
	Rho=0.5			**Rho=-0.5**		
Credit spread	**0%**	**2%**	**4%**	**0%**	**2%**	**4%**
CPI Volatility						
0%	112.4	111.4	110.3	112.4	111.4	110.3
2%	112.1	110.9	109.8	112.9	111.7	110.6
4%	111.7	110.6	109.5	113.3	112.2	111.0
6%	111.5	110.3	109.2	113.8	112.6	111.5
8%	111.2	110.1	109.0	114.3	113.2	112.0
10%	111.0	109.9	108.8	114.8	113.7	112.6
12%	110.9	109.8	108.7	115.4	114.2	113.1
14%	110.9	109.7	108.6	116.0	114.8	113.7
16%	110.9	109.7	108.6	116.6	115.4	114.2
18%	110.9	109.8	108.7	117.2	116.0	114.8
20%	111.0	109.9	108.8	117.8	116.6	115.5
22%	111.2	110.1	109.0	118.4	117.3	116.1
24%	111.5	110.3	109.2	119.1	117.9	116.7
26%	111.7	110.6	109.5	119.7	118.5	117.4
28%	112.1	110.9	109.8	120.4	119.2	118.0
30%	112.5	111.3	110.2	121.1	119.9	118.7

Parameters in this table are:
$S = F = 100$, $\lambda = 1$, C = 4% paid annualy, $\delta = 0$, r = 6%, $r_r = 3\%$, $\sigma_S = 30\%$, T = 1, N = 100, I = 1

Figure 2
The values of the convertible bond for a combination of CPI volatility, credit spread and correlation.



Rho=-0.5



Rho=0.5

Parameters: See Table 2.

Table 3
The values of the convertible bond for a combination of the issuer's credit spread and the correlation between the stock and the CPI returns.

The correlation coefficient / The issuer's credit spread	The convertible bond values -0.5	0	0.5
0.0%	114.8	113.1	111.0
1.0%	114.3	112.5	110.5
2.0%	113.7	111.9	109.9
3.0%	113.1	111.4	109.4
4.0%	112.6	110.8	108.8
5.0%	112.0	110.3	108.3
6.0%	111.5	109.7	107.7
7.0%	110.9	109.2	107.2
8.0%	110.4	108.6	106.7
9.0%	109.8	108.1	106.1
10.0%	109.3	107.6	105.6
11.0%	108.8	107.1	105.1
12.0%	108.3	106.5	104.6
13.0%	107.8	106.0	104.1
14.0%	107.3	105.5	103.6
15.0%	106.8	105.0	103.1
16.0%	106.3	104.5	102.7
17.0%	105.8	104.0	102.2
18.0%	105.3	103.6	101.8
19.0%	104.8	103.1	101.4
20.0%	104.3	102.6	101.0
21.0%	103.9	102.2	100.7
22.0%	103.4	101.8	100.5
23.0%	103.0	101.4	100.3
24.0%	102.5	101.1	100.1

Parameters in this table are:
$S = F = 100$, $\lambda = 1$, C = 4% paid annualy, $\delta = 0$, r = 6%, $r_r = 3\%$, $\sigma_S = 30\%$, $\sigma_I = 10\%$, T = 1, N = 100, I = 1

Figure 3
The values of the convertible bond for a combination of the issuer's credit spread and the correlation between the stock and the CPI returns.

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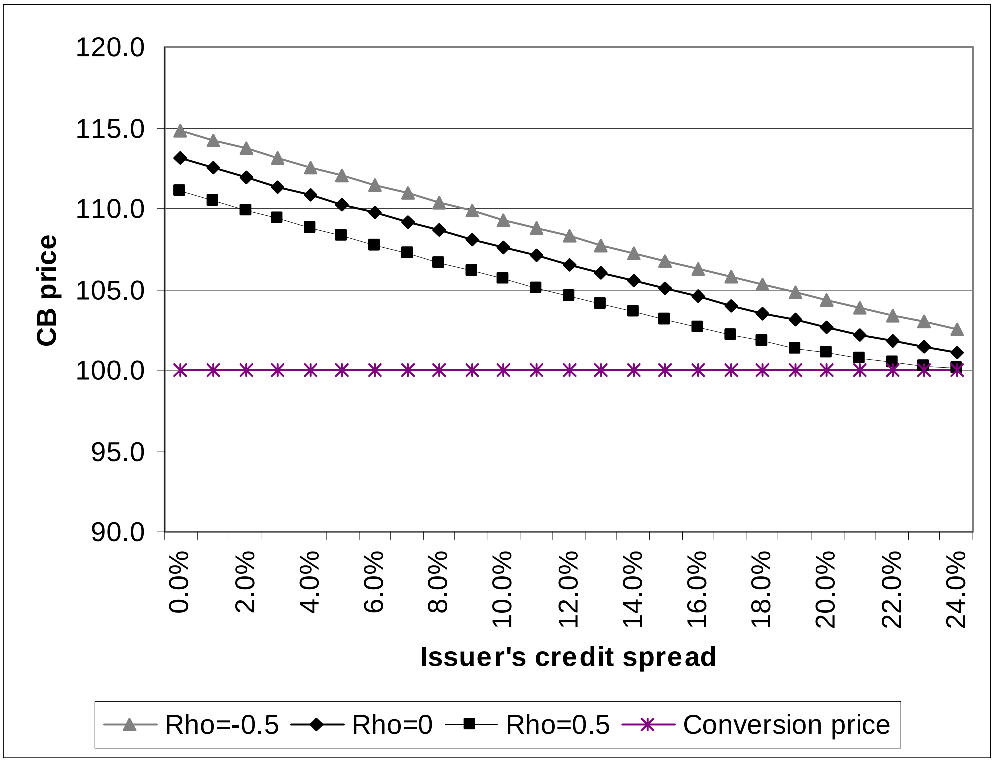


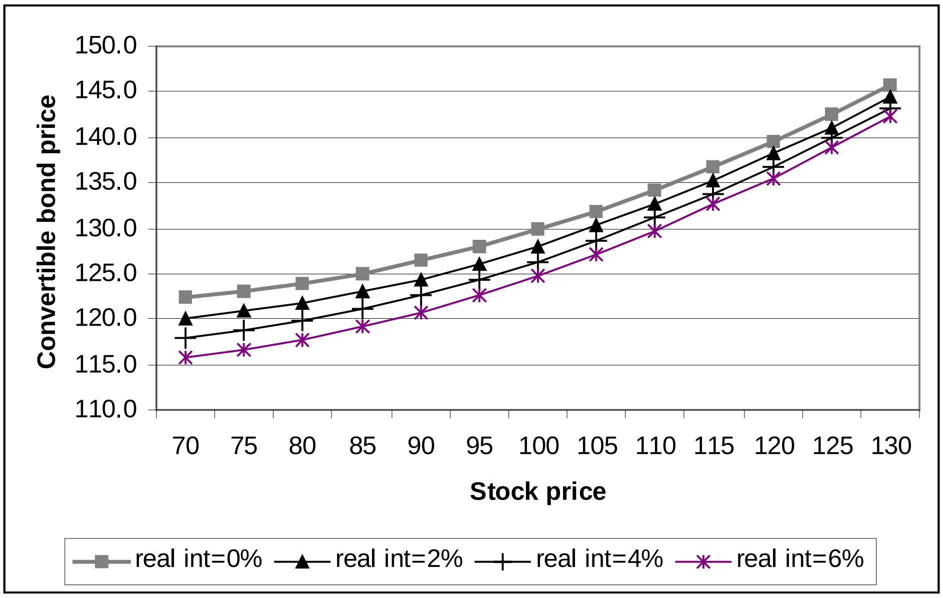
Parameters: See Table 3.

Table 4
The values of the convertible bond for a combination of stock price, real interest rate and CPI level.

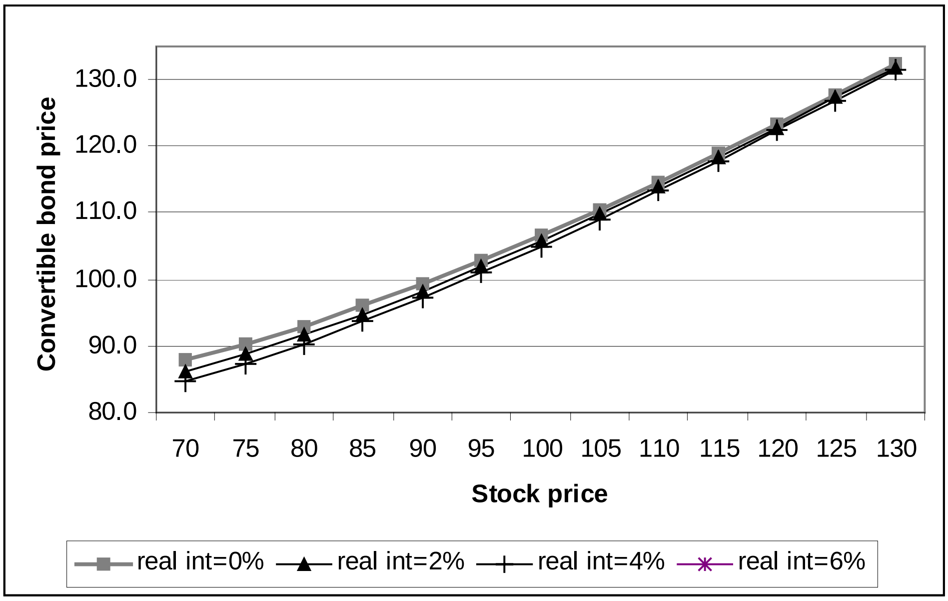
	Convertible bond values					
	CPI yield=-20% (I=0.8)			**CPI yield=20% (I=1.2)**		
Real interest	**0%**	**3%**	**6%**	**0%**	**3%**	**6%**
Stock price						
70	87.9	86.3	84.7	122.5	119.1	115.9
75	90.3	88.8	87.4	123.1	119.9	116.7
80	93.0	91.6	90.4	124.0	120.9	117.8
85	96.0	94.8	93.6	125.1	122.1	119.2
90	99.3	98.2	97.2	126.5	123.5	120.8
95	102.8	101.8	101.0	128.0	125.2	122.6
100	106.5	105.7	105.0	129.9	127.2	124.7
105	110.5	109.7	109.1	131.9	129.4	127.1
110	114.6	113.9	113.4	134.2	131.9	129.7
115	118.8	118.3	117.8	136.7	134.5	132.5
120	123.2	122.7	122.3	139.5	137.4	135.5
125	127.7	127.2	126.9	142.4	140.5	138.8
130	132.2	131.9	131.6	145.6	143.8	142.2

Parameters in this table are:
$F = 100$, $\lambda = 1$, C = 4% paid annualy, $\delta = 0$, r = 6%, $\sigma_S = 30\%$, $\sigma_I = 15\%$, T = 1, $\rho = 0.5$, cs = 3%, N = 100,

Figure 4
The values of the convertible bond for a combination of stock price, real interest rate and CPI level.



I=1.2



I=0.8

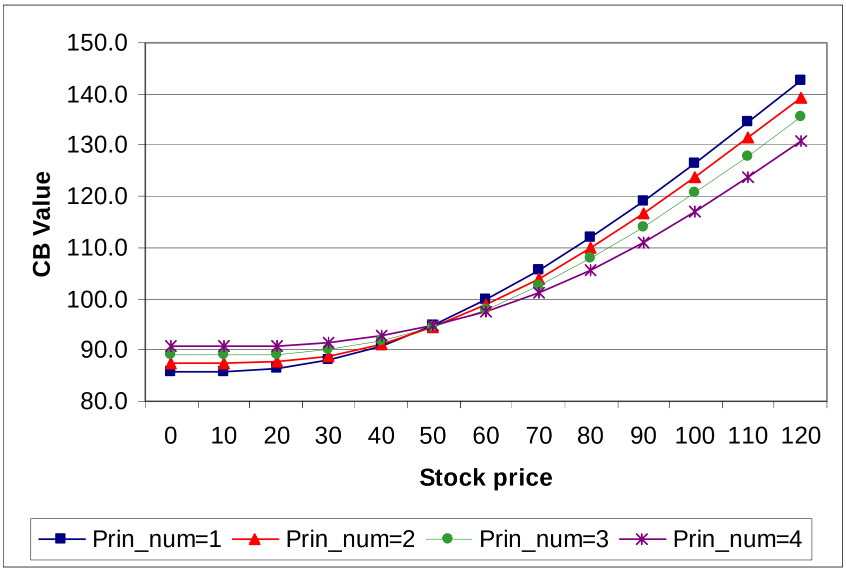
Parameters: See Table 4.

Table 5
The convertible bond values for combination of stock price and number of installment payments.

The number of principal payments / Stock price	Convertible bond values 1	2	3	4
0	85.8	87.3	89.0	90.7
10	85.8	87.4	89.0	90.8
20	86.3	87.7	89.2	90.9
30	87.9	88.8	90.0	91.5
40	90.7	91.0	91.7	92.8
50	94.8	94.4	94.3	94.8
60	99.8	98.8	97.9	97.6
70	105.4	103.9	102.4	101.2
80	112.1	110.0	107.9	105.7
90	119.1	116.6	113.9	111.0
100	126.6	123.7	120.6	117.0
110	134.5	131.3	127.9	123.7
120	142.7	139.4	135.7	131.0

Parameters in this table are: $F = 100$, C = 4% paid annualy, $\lambda = 1$, $\delta = 0$, r = 5%, σ_s = 40%, T = 4, cs = 3%, N = 300.

Figure 5
The convertible bond values for combination of stock price and number of installment payments.



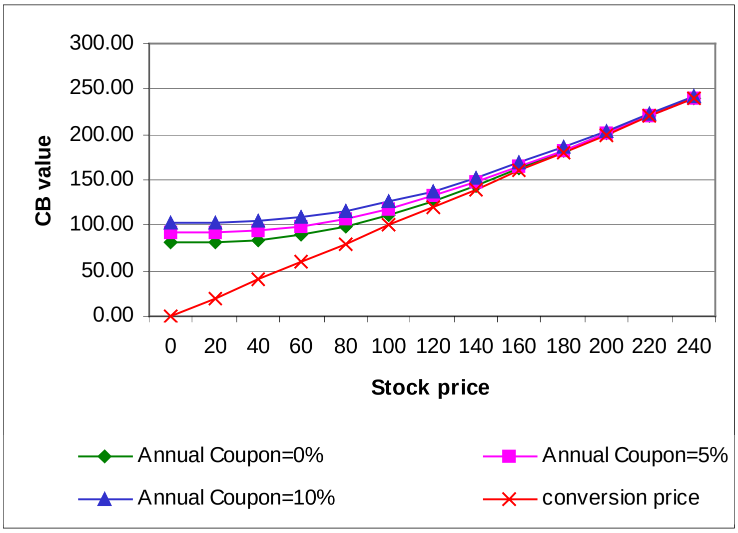
Parameters: See Table 5.

Table 6
Convertible bond values for a combination of stock price, number of installment payments and coupon rate.

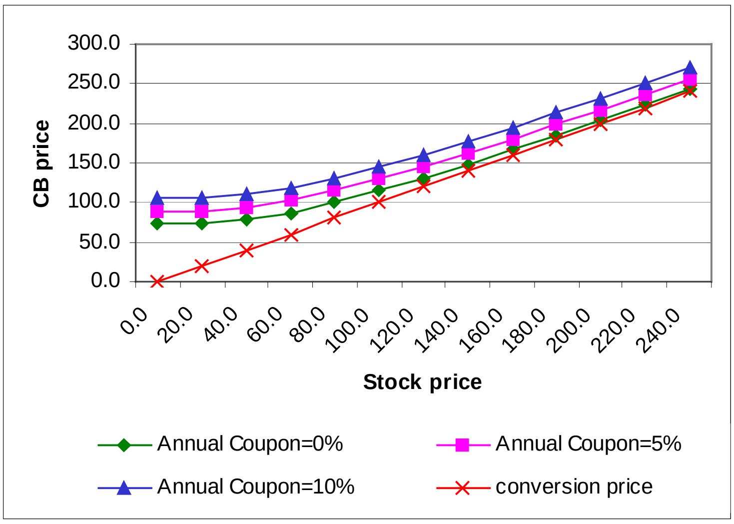
Convertible bonds prices						
	One principal payment			4 principal payments		
Coupon rate	0%	5%	10%	0%	5%	10%
Stock price						
0.0	72.6	89.1	105.5	82.2	92.9	103.6
20.0	73.2	89.6	106.0	82.4	93.1	103.7
40.0	78.0	93.9	110.1	84.5	94.8	105.3
60.0	87.3	102.6	118.4	90.0	99.5	109.3
80.0	99.9	115.2	130.1	99.3	107.4	116.2
100.0	115.0	129.5	144.0	111.9	118.3	125.8
120.0	131.0	145.6	159.8	127.1	132.0	138.1
140.0	148.6	162.9	176.8	144.3	147.7	152.4
160.0	166.9	180.6	194.7	162.5	164.8	168.4
180.0	185.4	198.9	212.8	181.4	183.1	185.6
200.0	204.3	217.6	231.4	200.8	202.0	203.7
220.0	223.4	237.0	250.3	220.5	221.3	222.4
240.0	242.8	256.0	269.5	240.3	240.8	241.6
260.0	262.3	275.4	288.9	260.2	260.5	261.1

Parameters in this table are: $F = 100$, $\lambda = 1, \delta = 0$, $\mathrm{r} = 5\%, \sigma_s = 40\%, \mathrm{T} = 4, \mathrm{cs} = 3\%, \mathrm{N} = 300.$

Figure 6
Convertible bond values for a combination of stock price, number of installment payments and coupon rate.



4 Principal payments



One Principal payment

Parameters: See Table 6.

Table 7: Pricing Applications

1. Foreign-Currency linked Convertible bond: **Machteshim-Agan on December 3 2001** (N=100).

Relevant Input:

Stock price	840.1	Local risk free yield	6.66%
Conversion ratio	100/1068	Foreign risk free rate	4.73%
Initial Exchange rate	4.22	Issuer's credit spread	3.9%
Current Exchange rate	4.24	Time to expiry	5.97
Stock volatility	26.4%	Exchange rate volatility	4%
Assets correlation	-0.19	Bond face value	100
Dividend yield	1.5%	Coupon rate	2.75
Num of principal payments	1		

Pricing results of the two factor model:

Convertible bond model price	94.3	Convertible bond market price:	92.3
Equity component	44.1	Bond component	50.3
Straight bond price	71.2	Option price	22.7

2. Inflation-indexed convertible bonds: Ezorim **on June 10 2001**(N=100).

Relevant Input:

Stock price	4,420	Nominal risk free yield	6.4%
Conversion ratio	1/45	Real risk free rate	4.6%
Initial CPI	91.3	Issuer's credit spread	2%
Current CPI	99.7	Time to expiry	4.74
Stock volatility	23.8%	CPI volatility	2.45%
Assets correlation	0.19	Bond face value	100
Dividend yield	5%	Coupon rate	4.2
Num of principal payments	5		

Pricing results of the two factor model:

Convertible bond model price	112.8	Convertible bond market price	111.6
Equity component	34.3	Bond component	78.5
Straight bond price	104	Option price	8.8

Table 8

Machteshim-Agan Common stock price, Dollar/Shekel exchange rate yield, theoretical convertible bond price and equity component value, and reported convertible bond price from December 3, 2001 through January 5, 2002.

Date	Exchange rate yield	Closing stock price	CB Closing market price	High market price	Low market price	Convertible theoretical price	% of Equity component
Dec 3	0.5%	840	92.3	93.4	101.6	94.4	47%
4	0.7%	853	92.8	94.4	103.2	95.4	47%
5	0.4%	856	92.7	94.7	104.0	95.0	48%
6	0.1%	858	92.3	95.2	105.3	95.5	49%
9	0.1%	855	92.6	94.6	105.3	94.9	50%
10	0.0%	852	92.6	94.5	104.7	94.8	49%
11	0.1%	855	93.1	94.8	104.9	95.0	49%
12	0.1%	856	93.5	95.2	100.8	95.4	49%
13	0.2%	845	93.5	94.0	101.3	94.9	48%
16	0.4%	843	93.5	94.1	100.1	94.8	48%
17	0.3%	841	93.4	93.9	98.8	94.2	49%
18	0.4%	834	93.4	93.7	97.2	94.6	48%
19	0.9%	831	93.4	94.0	96.2	94.9	47%
20	1.2%	828	93.7	94.0	96.6	94.8	46%
23	2.4%	862	95.4	96.6	94.0	97.5	48%
24	2.7%	852	95.4	96.2	94.0	97.1	48%
25	2.7%	865	96.5	97.2	93.7	98.0	48%
26	3.6%	877	97.4	98.8	93.9	99.2	49%
27	3.6%	893	97.9	100.1	94.1	100.5	50%
30	3.9%	907	99	101.3	94.0	101.7	51%
31	4.6%	902	99.2	100.8	95.2	101.3	50%
Jan 2	5.1%	965	102.2	104.9	94.8	105.7	54%
3	6.0%	955	102.4	104.7	94.5	105.3	53%
6	6.4%	959	105	105.3	94.6	105.9	53%
7	6.9%	951	102.8	105.3	95.2	105.9	51%
8	6.4%	931	100.9	104.0	94.7	104.5	50%
9	5.7%	923	100.9	103.2	94.4	103.5	50%
10	5.9%	902	101.5	101.6	93.4	102.5	48%

Table 9

The impact of the common stock volatility, Exchange rate volatility, Exchange rate yield, equity component and the gap between daily high and low price on the absolute two-factor model error

The table records the results of the regression analysis of the following model:

$$|\%Error_i| = a_0 + a_1\sigma_I + a_2\sigma_S + a_3 E/V + a_4(P_H - P_L)/P_C + a_5(\frac{I}{I_0} - 1)$$

$$|\%Error_i| = \left|\frac{(P_{MODEL} - P_{MARKET})}{P_{MARKET}}\right|$$

Where σ_s is the historical volatility of Machteshim-Agan common stock over the last 100 business days, σ_I is the Dollar/Shekel volatility over the same time period, E/V is the equity component divided by the theoretical price of the convertible bond, $(P_H - P_L)/P_C$ is the difference between the daily observed high market price and the low market price divided by the closing price and $\ln(\frac{I}{I_0})$ is the Exchange rate yield from the issuing date till the pricing date. There are 154 observations, from Dec 3 2001 till August 1 2002.

Variable	Constant	σ_S	σ_I	E/V	$(P_H - P_L)/P_C$	$\ln(\frac{I}{I_0})$	Adj. R^2
Regression Coefficient	0.075	-0.224	-0.148	-0.297	0.079	0.032	0.13
(p-value)	(0.003)	(0.006)	(0.322)	(0.045)	(0.021)	(0.047)	

Table 10

The impact of the stock volatility, Exchange rate volatility, Exchange rate yield, equity component and the gap between daily high and low price on the model error

The table records the results of the regression analysis of the following model:

$$\%Error = a_0 + a_1\sigma_I + a_2\sigma_S + a_3 E/V + a_4(P_H - P_L)/P_C + a_5(\frac{I}{I_0} - 1)$$

$$\%Error = \frac{P_{MODEL} - P_{MARKET}}{P_{MARKET}}$$

Variable	Constant	σ_S	σ_I	E/V	$(P_H - P_L)/P_C$	$\ln(\frac{I}{I_0})$	Adj. R^2
Regression Coefficient	0.003	0.022	0.209	-0.668	-0.056	-0.057	0.18
(p-value)	(0.944)	(0.000)	(0.134)	(0.009)	(0.340)	(0.042)	

Figure 7: Machteshim-Agan Inc convertible bond market price, the two-factor model price and the one-factor model price for the period 2/12/01-1/8/02 (dd/mm/yy)

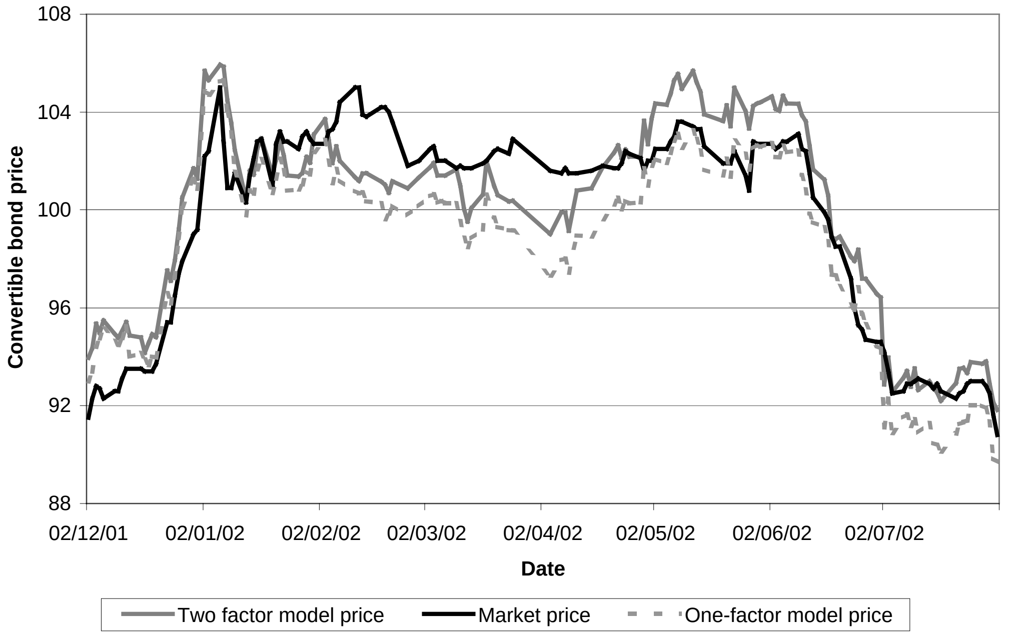


Figure 8: The price spread between the two-factors and the one-factor model for Machteshim-Agan foreign currency linked convertible bond and the exchange rate volatility for the period 2/12/01-1/8/02 (dd/mm/yy)