# Mortgage Timing* 

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#### Abstract

Mortgages can be broadly classified into adjustable-rate mortgages (ARMs) and fixed-rate mortgages (FRMs). We document a surprising amount of time variation in the fraction of newly-originated mortgages that are of either type in the US and UK. A simple utility framework points to the importance of term structure variables in explaining this variation. In particular, the inflation risk premium, real interest rate risk premium and both the real rate and expected inflation volatility arise as potential determinants. We use a flexible VAR-model to measure these four term structure variables and show that they account for the bulk of variation in the ARM share. Risk premia alone explain sixty percent of the time variation in mortgage choice. Other term structure variables, such as the yield spread, seem only weakly related to the ARM share. We uncover interesting differences between the US and the UK. In the US, the inflation risk premium is most strongly related to the ARM share, while in the UK it is the real rate risk premium. In the US, FRMs contain a prepayment option. We analyze the impact of the prepayment option on optimal mortgage choice. The prepayment option hardly weakens the effects of risk premia on mortgage choice. JEL classification: D14, E43, G11, G12, G21


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## 1 Introduction

One of the most important decisions any household has to make during its lifetime is whether to own a house and, if so, how to finance it. The home ownership rate in the US stands at $68 \%$ and US residential mortgage debt exceeds $\$ 9$ trillion. There are two broad categories of housing finance: adjustable-rate mortgages (ARMs) and fixed-rate mortgages (FRMs). There is a surprisingly large variation in the composition of newly-originated mortgages. Figure 1 plots the share of newlyoriginated mortgages that is of the ARM-type in the US economy between January 1985 and June 2006. This ARM share varies between $10 \%$ and $70 \%$. In this paper we seek to explain this variation.
[Figure 1 about here.]
We claim that a large fraction of the variation in the ARM share can be attributed to timevariation in bond risk premia. Consider a simple homoscedastic economy without inflation in which households have mean-variance preferences over consumption, and consume what is left from income after mortgage payments are made. In such an economy, the choice between an ARM and FRM boils down to comparing expected mortgage payments and their (constant) variability. Ignoring the prepayment option, fixed-rate mortgages are long-term loans whose payments are tied to the long-term nominal interest rate. The adjustable-rate mortgage payments are tied to the short-term nominal interest rate instead. The difference in expected payments on the FRM and ARM equals the nominal bond risk premium. The payments on the FRM are known at origination, while the ARM payments depend on future short rates. The mortgage choice then reduces to a trade-off between bond risk premia and short rate volatility. For the more realistic economy in which inflation erodes nominal mortgage payments, we decompose the nominal bond risk premium into the real rate premium and expected inflation premium. The difference in expected payments between the FRM and the ARM (approximately) equals the sum of the real premium and the expected inflation premium. In this world with constant variances, an increase in bond risk premia also makes the FRM less desirable, and is predicted to increase the share of ARM originations. In sum, time-variation in bond risk premia leads to time-variation in the preferred mortgage type.

Figure 2 plots the ARM share (solid line, measured against the left axis) alongside the five-year expected inflation risk premium (dashed line, measured against the right axis). We obtain the inflation risk premium as the difference between the five-year nominal bond yield and the sum of the five-year real bond yield and the five-year expected inflation. The nominal yield data are from the Federal Reserve Bank of New York and real bond yield data from McCulloch. Real data are available as of January 1997 when the US Treasury introduced treasury inflation-protected securities (TIPS). We use the median long-term inflation forecast of the survey of professional forecasters (SPF) to measure expected inflation. Ang, Bekaert, and Wei (2006) argue that such survey data
provides the best inflation forecasts among a wide array of methods. The contemporaneous correlation between the two series is $80 \%$. This suggests that a large fraction of variation in the ARM share can be understood by time variation in inflation risk premia. To illustrate, in each of the 1998.10-2000.4 and 2003.5-2005.3 periods, the inflation risk premium increased by more than 150 basis points. This made fixed-rate mortgages less desirable, and US households shifted into ARMs. In both episodes, the ARM share tripled.
[Figure 2 about here.]
In Section 2, we formalize the utility-based mortgage choice argument. We distinguish between an investor with money illusion who maximizes utility over nominal consumption, and a rational investor who maximizes utility of real consumption streams instead. ${ }^{1}$ By solving for the determinants of mortgage choice in both models, we illustrate how money illusion potentially affects the financing decision. The latter analysis points to four yield curve determinants of mortgage choice: the expected inflation risk premium, the real rate risk premium, the variability of expected inflation, and the variability of the real rate. We develop a vector auto-regression (VAR) model in Section 3 in order to estimate these four components on US data. The VAR structure readily provides a way to measure expected inflation and expected real rates and is an alternative to the survey data. We then conduct a regression analysis, and find that the four term-structure determinants typically enter with the right sign. The expected inflation risk premium emerges as the dominant explanatory variable for mortgage choice in the US. It alone explains about $60 \%$ of the variation in the ARM share. Adding the other term structure variables does not affect this conclusion.

We compare these results with predictors of the ARM share proposed in the literature. Campbell and Cocco (2003) advocate the spread between the yields on a nominal long-term and shortterm bond, and Campbell (2006) and Vickery (2006) use the spread between a FRM rate and an ARM rate, as a determinant of the ARM share. We find low explanatory power for these variables over the common sample. Our model suggests why. The yield spread is a contaminated measure of bond risk premia because it not only picks up the bond risk premia, but also deviations of expected future nominal short rates from the current nominal short rate. These two components are negatively correlated. For example, when expected inflation is high, the inflation risk premium is high as well, but expected future short rates are below the current one because inflation is expected to revert back to its long-term mean. Vickery (2006) also finds that household-specific characteristics have little explanatory power for mortgage choice. This is an important finding because it suggests that market-wide variables are the relevant variables to study. Theoretically, we show that bond risk premia are the relevant variables and we confirm their importance in our empirical analysis.

[^1]We verify the robustness of our results to (i) alternative definitions of the ARM share, (ii) using a different VAR model to construct long-term expectations and risk premia, (iii) real interest rate data generated by the term structure model of Ang and Bekaert (2005) rather than using TIPS data, (iv) the persistence in the variables included in the regressions. The analysis leads us to conclude that bond risk premia are a robust determinant of aggregate mortgage choice.

In the US, FRMs typically have an embedded prepayment option which allows the mortgage borrower to pay off the loan at will. To understand the impact of the prepayment option on the preference for mortgage types, we value this prepayment option in our model with time-varying interest rates, inflation, risk premia, and volatilities. ${ }^{2}$ We show that the prepayment option reduces the exposures to the underlying risk factors. However, it continues to hold that higher bond risk premia favor ARMs.

We extend our analysis to the UK. If bond risk premia are an important determinant of aggregate mortgage choice, our results should carry over to another country with another interest rate environment. FRM contracts in the UK have much shorter maturities than in the US. This implies that inflation risk, which manifests itself predominantly at long horizons, may be less important for choosing between ARMs and FRMs. In contrast to the US, FRMs do not have a prepayment option. Finally, we have a longer time-series of real interest rate data available than for the US. We find that the real rate and expected inflation premium positively predict the ARM share in the UK, just as they did in the US. However, in sharp contrast to the US, we also find that it is the real rate premium instead of the inflation risk premium that is the dominant predictor of mortgage choice in the UK. The variation in the ARM share explained by these bond risk premia equals $23 \%$ for the 1993-2006 sample with quarterly data and $62 \%$ for the 2002-2006 sample with monthly data.

Our results suggest that households may have an ability to optimally time their mortgage choice. This is certainly no easy task because it requires the ability to calculate inflation and real risk premia. From a normative perspective, time variation in bond risk premia, documented by Fama and French (1989), Campbell and Shiller (1991), Dai and Singleton (2002), Buraschi and Jiltsov (2005), and Cochrane and Piazzesi (2005), certainly has value-added to investors timing bond markets. Indeed, Brandt and Santa-Clara (2006), Campbell, Chan, and Viceira (2003), Sangvinatsos and Wachter (2005), and Koijen, Nijman, and Werker (2006) argue that exploiting time variation in bond risk premia is valuable to (long-term) investors. ${ }^{3}$ Our exercise suggests that

[^2]mortgage choice is another financial decision setting in which households optimally incorporate bond risk premia in their decision making. In addition, we show that bond risk premia are the most important determinants of mortgage choice among a wide variety of yield variables and that they explain most of the variation in mortgage choice. Some have expressed skepticism towards financial sophistication of households (Campbell (2006)). One counter-argument is that mortgage choice is undoubtedly one of the most important financial decisions a household has to make. Many households therefore seek out advice from financial professionals, mostly mortgage lenders. The paper concludes with a discussion which argues that the incentives of mortgage lenders to recommend a particular type of mortgage may be aligned with households' incentives. This strengthens the plausibility of our results.

Finally, our paper also relates to the corporate finance literature on the timing of capital structure decisions. The firm's problem of maturity choice of debt is akin to the household's choice between an ARM and an FRM. Baker, Greenwood, and Wurgler (2003) show that firms are able to time bond markets. The maturity of debt decreases in periods of high bond risk premia. ${ }^{4}$ Our findings suggest that households also have the ability to incorporate information on bond risk premia in their long-term financing decision.

This paper proceeds as follows. Section 2 develops a utility-based framework that identifies the main determinants of mortgage choice. It also defines the term structure variables used in the subsequent empirical analysis, and relates them to the yield spread. In Section 3 we develop the VAR-model that is used to extract long-term expectations and bond risk premia, as well as volatilities of the real rate and expected inflation. We then show how these term structure variables relate to time-variation in mortgage choice in Section 4. Section 5 extends the analysis of Section 3 by modeling the prepayment option embedded in US FRM contracts. To the best of our knowledge, we are the first to value the prepayment option in a model with time-varying risk premia and timevarying volatilities. In Section 6, we repeat the analysis for the UK economy. Section 7 considers the hedging problem that mortgage lenders face and argues that that lenders may have an incentive to recommend ARMs exactly when bond risk premia are high. Section 8 concludes.

## 2 Determinants of Mortgage Choice

This section explores the choice between a fixed-rate (FRM) and an adjustable-rate mortgage (ARM). The model is kept deliberately simple and serves to motivate the use of term structure variables as determinants of mortgage choice in Section 4. We start in a world without inflation
${ }^{4}$ See also Butler, Grullon, and Weston (2006) and Baker, Taliaferro, and Wurgler (2006) for a recent discussion of this result.
(Section 2.1) and subsequently introduce inflation (Section 2.2).

### 2.1 Optimal Mortgage Choice: Nominal Mean-Variance Analysis

We consider a discrete-time setting for an investor with mean-variance preferences over a nominal consumption stream $\left\{C_{t}\right\}$. The preference parameter $\gamma$ summarizes the investor's risk preferences. The subjective time discount factor is 1 . The investor receives an independently identically distributed (i.i.d.) stochastic income stream $\left\{L_{t}\right\}$.

At time 0 , the investor buys a house with a value that is normalized to $\$ 1$. We assume that the house price has a constant nominal value. To finance the house, the investor chooses a mortgage of the ARM or FRM type. The face value of the mortgage equals $\$ 1$ as well; we assume a $100 \%$ loan-to-value ratio. The investment horizon and the maturity of the mortgage contract equal $T$ periods. At times 1 trough $T$ the investor pays interest on the mortgage, but no payments on the principal are due.

Denote the stream of mortgage payments by $\left\{q_{t}\right\}$. To keep the problem as simple as possible, we postulate initially that the investor is liquidity constrained. In each period, she consumes what is left over from income after making the mortgage payment. This seems a plausible assumption because most households are young and not very wealthy at the time of mortgage origination. The mortgage choice at time 0 then boils down to

$$
\begin{align*}
& \max _{h \in\{A R M, F R M\}} \sum_{t=1}^{T} \mathbb{E}_{0}\left(C_{t}^{h}\right)-\gamma \operatorname{Var}_{0}\left(C_{t}^{h}\right),  \tag{1}\\
& \text { s.t. } C_{t}^{h}=L_{t}-q_{t}^{h}, t=1, \cdots, T . \tag{2}
\end{align*}
$$

In the last period, the value of the house and the mortgage balance cancel each other out and do not affect consumption. Because labor income is i.i.d. and uncorrelated with the mortgage payment, the mortgage choice problem simplifies to the following minimization

$$
\begin{equation*}
\min _{h \in\{A R M, F R M\}} \sum_{t=1}^{T} \mathbb{E}_{0}\left(q_{t}^{h}\right)+\gamma \operatorname{Var}_{0}\left(q_{t}^{h}\right) . \tag{3}
\end{equation*}
$$

We denote the nominal price at time $t$ of a nominal $\tau$-period zero-coupon bond by $P_{t}(\tau)$. The yield $y_{t}^{\$}(\tau)$, and the one-period forward rate $f_{t}^{\$}(\tau)$ are given by

$$
\begin{align*}
y_{t}^{\S}(\tau) & \equiv-\frac{1}{\tau} \log \left(P_{t}(\tau)\right)  \tag{4}\\
f_{t}^{\S}(\tau) & \equiv-\log \left(\frac{P(t, \tau+1)}{P_{t}(\tau)}\right) \tag{5}
\end{align*}
$$

We do not impose the Expectations Hypothesis: $f_{t}^{\$}(\tau) \neq \mathbb{E}_{t}\left[y_{t+\tau}^{\S}(1)\right]$.
We think of the FRM investor as paying the time-zero forward rate in each period on forward contracts with delivery dates $1,2, \cdots, T$. This assumption captures the essence of a nominal FRM: future mortgage payments are fixed in nominal terms at the origination time $0 .{ }^{5} \mathrm{By}$ the same token, an ARM investor simply pays the short-rate

$$
\begin{align*}
q_{t}^{F R M} & =f_{0}^{\S}(t-1)  \tag{6}\\
q_{t}^{A R M} & =y_{t-1}^{\$}(1) \tag{7}
\end{align*}
$$

In this world, the crucial difference between an FRM investor and an ARM investor is that the former knows the value of all (nominal) mortgage payments at time 0 , while the latter knows the value of the (nominal) payments only one period in advance.

The difference between the expected mortgage payments for the FRM and ARM investors equals the bond risk premium

$$
\begin{align*}
\mathbb{E}_{0}\left[\sum_{t=1}^{T} q_{t}^{F R M}\right]-\mathbb{E}_{0}\left[\sum_{t=1}^{T} q_{t}^{A R M}\right] & =\sum_{t=1}^{T} f_{0}^{\S}(t-1)-\sum_{t=1}^{T} \mathbb{E}_{0}\left[y_{t-1}^{\$}(1)\right] \\
& =T\left\{y_{0}^{\$}(T)-\frac{1}{T} \sum_{t=1}^{T} \mathbb{E}_{0}\left[y_{t-1}^{\S}(1)\right]\right\} \\
& \equiv T \phi_{0}^{\$}(T), \tag{8}
\end{align*}
$$

where we used that the yield on a $T$-period zero-coupon bond equals the average forward rate, and where we defined $\phi_{0}^{\S}(T)$ as the risk premium on a $T$-period nominal bond. The FRM investor faces no uncertainty over the nominal mortgage payments, whereas the ARM investor faces nominal interest rate risk. The variability of ARM payments is $\frac{1}{T} \sum_{t=1}^{T} \operatorname{Var}_{0}\left[y_{t-1}^{\$}(1)\right]$. Combining the difference in expected payments and the difference in the variability of the payments, we arrive at equation (9), which states that the investor prefers an ARM if the nominal bond risk premium exceeds the variability of the nominal interest rate multiplied by the risk aversion coefficient

$$
\begin{equation*}
\phi_{0}^{\$}(T)>\frac{\gamma}{T} \sum_{t=1}^{T} \operatorname{Var}_{0}\left[y_{t-1}^{\$}(1)\right] . \tag{9}
\end{equation*}
$$

If the protection that an FRM offers against nominal interest rate volatility to the nominal investor is too expensive, an ARM becomes more attractive.

[^3]
### 2.2 Optimal Mortgage Choice: Real Mean-Variance Analysis

In a world with inflation, a rational investor cares about real consumption streams instead of nominal streams. The only other differences with the previous set-up are that (1) the house price now grows with inflation, and therefore has a constant real value, and (2) the labor income is i.i.d. in real terms. The real payments on the two contracts now equal

$$
\begin{align*}
& q_{t}^{F R M}=\frac{f_{0}^{\S}(t-1)}{\Pi_{t}}=f_{0}^{\S}(t-1) \exp \left(-\sum_{s=1}^{t} \pi_{s}\right),  \tag{10}\\
& q_{t}^{A R M}=\frac{y_{t-1}^{\$}(1)}{\Pi_{t}}=y_{t-1}^{\$}(1) \exp \left(-\sum_{s=1}^{t} \pi_{s}\right), \tag{11}
\end{align*}
$$

where $\Pi_{t}$ denotes the price level at time $t$ and $\pi_{t}=\log \Pi_{t}-\log \Pi_{t-1}$. We need to distinguish between two types of investors: borrowing-constrained and unconstrained. The latter are able to borrow cash to finance mortgage payments. We determine the optimal mortgage choice and its determinants for each of these problems.

### 2.2.1 Borrowing-Constrained Investor

A borrowing-constrained investor maximizes (1) subject to (2), except that $C_{t}^{h}, L_{t}$, and $q_{t}^{h}$, for $h \in\{A R M, F R M\}$ now refer to real quantities, and that the last period consumption satisfies

$$
C_{T}^{h}=L_{T}-q_{T}^{h}+1-\exp \left(-\sum_{s=1}^{T} \pi_{s}\right)
$$

Terminal consumption equals income after the mortgage payment plus the difference between the real value of the house, which is 1 , and the real mortgage balance, which is $\exp \left(-\sum_{s=1}^{T} \pi_{s}\right)$.

Using the fact that real labor income is independent of mortgage payments, the investor prefers the ARM if

$$
\begin{align*}
& \mathbb{E}_{0}\left[\sum_{t=1}^{T} q_{t}^{F R M}\right]+\gamma \sum_{t=1}^{T-1} \operatorname{Var}_{0}\left[q_{t}^{F R M}\right]+\gamma \operatorname{Var}_{0}\left[q_{T}^{F R M}+\exp \left(-\sum_{s=1}^{T} \pi_{s}\right)\right]> \\
& \mathbb{E}_{0}\left[\sum_{t=1}^{T} q_{t}^{A R M}\right]+\gamma \sum_{t=1}^{T-1} \operatorname{Var}_{0}\left[q_{t}^{A R M}\right]+\gamma \operatorname{Var}_{0}\left[q_{T}^{A R M}+\exp \left(-\sum_{s=1}^{T} \pi_{s}\right)\right] . \tag{12}
\end{align*}
$$

To further understand the main determinants of optimal mortgage choice in an inflationary
environment, we make the following -admittedly crude- assumptions:

$$
\begin{align*}
r_{t} \exp \left(-\sum_{s=1}^{t} \pi_{s}\right) & \approx r_{t}\left(1-\sum_{s=1}^{t} \pi_{s}\right) \approx r_{t}  \tag{13}\\
\left(1+r_{t}\right) \exp \left(-\sum_{s=1}^{t} \pi_{s}\right) & \approx\left(1+r_{t}\right)\left(1-\sum_{s=1}^{t} \pi_{s}\right) \approx\left(1+r_{t}-\sum_{s=1}^{t} \pi_{s}\right) \tag{14}
\end{align*}
$$

where $r$ is a generic interest rate. The first approximation is a first-order Taylor expansion. The second approximation says that an interest rate times aggregate inflation is an order of magnitude smaller than the rate itself, if $t$ is not too large. The approximations imply that the real payments at time $t$ on the FRM and ARM equal

$$
\begin{align*}
q_{t}^{F R M} & =f_{0}^{\S}(t-1),  \tag{15}\\
q_{t}^{A R M} & =y_{t-1}^{\$}(1) \tag{16}
\end{align*}
$$

We now use this approximation to simplify the terms in the mortgage choice equation (12).

First, the expected payment differential between the FRM and the ARM in equation (12) is still given by $T \phi_{0}^{\S}(T)$, just as in (8). Under our approximation, the presence of inflation does not affect the expected payments differential between the FRM and the ARM. For future use, we rewrite the nominal bond risk premium as the sum of the inflation risk premium and the real rate risk premium

$$
\begin{equation*}
\phi_{0}^{\S}(T)=\phi_{0}^{x}(T)+\phi_{0}^{y}(T) . \tag{17}
\end{equation*}
$$

Analogous to the nominal risk premium $\phi_{0}^{\$}$ in equation (8), we define the real rate risk premium at time $0, \phi_{0}^{y}$, as the difference between the observed long-term real rate and the expected long-term real rate. The latter is the average of the expected future short real rates

$$
\begin{equation*}
\phi_{0}^{y}(T) \equiv y_{0}(T)-\frac{1}{T} \sum_{t=1}^{T} \mathbb{E}_{0}\left[y_{t-1}(1)\right], \tag{18}
\end{equation*}
$$

where $y_{t}(\tau)$ is the real yield of a $\tau$-period real bond at time $t$. We impose that the yield at time $t$ of an 1-period real bond, $y_{t}(1)$, is the difference between the 1-period nominal yield, $y_{t}^{\$}(1)$, and 1-period expected inflation, $x_{t}=x_{t}(1)$

$$
\begin{equation*}
y_{t}(1)=y_{t}^{\$}(1)-x_{t}(1) . \tag{19}
\end{equation*}
$$

Following Ang and Bekaert (2005), we define the expected inflation premium at time 0 , $\phi_{0}^{x}$, as the difference between long-term nominal yields, long-term real yields, and long-term expected
inflation

$$
\begin{equation*}
\phi_{0}^{x}(T) \equiv y_{0}^{\$}(T)-y_{0}(T)-x_{0}(T) . \tag{20}
\end{equation*}
$$

This uses the decomposition of realized inflation at time $t$ into expected inflation conditional on the time $t-1$ information, $x_{t-1}$, and unexpected inflation, $\varepsilon_{t}$

$$
\begin{equation*}
\pi_{t}=x_{t-1}+\varepsilon_{t} \tag{21}
\end{equation*}
$$

and uses the definition of the long-term expected inflation

$$
x_{t}(T)=\frac{1}{T} \mathbb{E}_{t}\left[\log \Pi_{t+T}-\log \Pi_{t}\right]
$$

Second, the variance of the intermediate FRM payments, at times 1 through $T-1$, is still approximately zero (second term on the left-hand side of 12). The variance of the intermediate ARM payments (second term on the right-hand side) is $\sum_{t=1}^{T-1} \operatorname{Var}_{0}\left[y_{t-1}(1)+x_{t-1}\right]$, where we used equation (19). Intermediate payments on the ARM carry real rate risk and expected inflation risk, while intermediate payments on the FRM carry no risk.

Third, we can rewrite the variance of the terminal payments (third term on left and right of equation (12)) as

$$
\begin{align*}
& \operatorname{Var}_{0}\left[q_{T}^{F R M}+\exp \left(-\sum_{s=1}^{T} \pi_{s}\right)\right]=\left(f_{0}^{F R M}(T-1)+1\right)^{2} \operatorname{Var}_{0}\left[\exp \left(-\sum_{s=1}^{T} \pi_{s}\right)\right]  \tag{22}\\
& \operatorname{Var}_{0}\left[q_{T}^{A R M}+\exp \left(-\sum_{s=1}^{T} \pi_{s}\right)\right]=\operatorname{Var}_{0}\left[\left(1+y_{T-1}\right) \exp \left(-\sum_{s=1}^{T-1} \pi_{s}-\varepsilon_{T}\right)\right] \tag{23}
\end{align*}
$$

where $\varepsilon_{T}$ indicates unexpected inflation from $T-1$ to $T$, using equation (21).
We thus have five possible determinants of mortgage choice: the real rate premium, the expected inflation premium, the real rate variance, the inflation variance, and the covariance of the real rate and expected inflation. First, an increase in either bond risk premium increases the expected payments on the FRM and increases the uncertainty over its terminal payment. Second, an increase in the real rate volatility increases the variance of both intermediate and terminal payments of the ARM contract. The covariance between the real rate and expected inflation increases the variance of the intermediate payments to be made on the ARM. The impact of inflation volatility is more complex. An increase in inflation uncertainty increases the variance of the intermediate payments on the ARM, but not on the FRM. In contrast, the terminal payment of the ARM is hedged against expected inflation from period $T-1$ to $T$, while the terminal payment for the FRM is not. We conjecture that the larger inflation uncertainty over the first $T-1$ periods, associated with the ARM, is likely to dominate the larger inflation uncertainty over the final payment, associated with
the FRM. This makes the ARM contract carry the most inflation risk for a borrowing-constrained investor. In sum, we predict that the ARM share relates positively to the inflation risk premium and the real rate risk premium, but negatively to the real rate volatility and the covariance between the real rate and expected inflation. If households are borrowing constrained, the ARM share relates negatively to inflation volatility.

### 2.2.2 Unconstrained Investor

We now consider an investor that is not borrowing constrained. The availability of a risk-free credit line to borrow against enables the ARM investor to eliminate the expected inflation risk. The reason is that the ARM investor can effectively shift forward the increase in intermediate mortgage payments, arising from increased expected inflation, to time $T$. The additional amount borrowed exactly cancels against the erosion of the nominal mortgage balance due to expected inflation. This greatly reduces the inflation risk of the ARM contract (see also Campbell (2006)). The FRM contract does not admit such a strategy. After all, the intermediate payments are not affected by inflation (to a first-order approximation). Since the terminal payment on the FRM carries inflation risk, it is the FRM contract which carries the most inflation risk. This is the opposite scenario as for a constrained investor, where the ARM contract was the one carrying the most inflation risk. ${ }^{6}$ The prediction for the unconstrained investor is that the ARM share relates positively to inflation volatility.

### 2.3 The Yield Spread as a Predictor of the ARM Share

Campbell and Cocco (2003) and Campbell (2006) have argued that the slope of the yield curve is a key determinant of mortgage choice. They argue that when nominal long-term interest rates are high compared to nominal short-term rates, ARMs seem attractive relative to FRMs.

Condition (24) shows why the yield spread may be an imperfect measure of the relative attractiveness of both mortgage types. Consider the following decomposition of the nominal yield spread into the nominal bond risk premium and the deviations of average expected future short rates and the current nominal short rate,

$$
\begin{equation*}
y_{0}^{\$}(T)-y_{0}^{\$}(1)=\phi_{0}^{\$}(T)+\left(\frac{1}{T} \sum_{t=1}^{T} \mathbb{E}_{0}\left[y_{t-1}^{\$}(1)\right]-y_{0}^{\$}(1)\right) \tag{24}
\end{equation*}
$$

In a homoscedastic world with zero risk premia $\left(\phi_{0}^{\$}(T)=0\right)$, the yield spread equals the difference

[^4]between the average expected future short rates and the current short rate. Since long-term bond rates are the average of current and expected future short rates, both the FRM and the ARM investor will face the same expected payment stream in this world. The yield spread is completely uninformative about mortgage choice. Likewise, in a world with constant risk premia, variations in the yield spread capture variations in deviations between expected future short rates and the current short rate. But again, these variations are priced into both the ARM and FRM contracts. It is only the bond risk premium which affects the mortgage choice for a risk averse investor. A second way of seeing what goes wrong is to think of the current FRM-ARM rate spread as the determinant of mortgage choice. This measure deducts from the current FRM rate (long-term bond rate) the current ARM rate (one-period interest rate). Equation (24) shows that the correct proxy for the bond risk premium, and hence for mortgage choice, subtracts from the FRM rate the average future ARM rate (expected future one-period interest rate). Indeed, the latter is the actual rate that the ARM investor will have to pay over the life of the mortgage.

In our model with time-varying risk premia, estimated below, it turns out that the two terms on the right-hand side of (24) are negatively correlated. This makes the yield spread a poor proxy for the nominal bond risk premium, and as we show empirically below, a weak determinant of mortgage choice.

### 2.4 Variables Predicting Mortgage Choice

We choose the real rate risk premium, expected inflation risk premium, and the variance of both the real rate and expected inflation as the four term-structure predictor variables of mortgage choice in Section 4. There are at least four reasons to consider these four variables separately: aggregation, money illusion, borrowing constraints, and prepayment. We discuss them in turn.

Aggregation The analysis in Section 2.1 and 2.2 pertains to an individual investor's mortgage choice. Since we are interested in explaining the dynamics of the fraction of households that prefers an ARM, we need to aggregate across individuals. This necessitates understanding how heterogeneity within the pool of FRM- and ARM- holders affects the choice of predictor variables in the ARM regressions. For simplicity, we consider mortgage choice in a nominal world. The aggregation argument is similar in a world with inflation.

We consider a cross-section of investors indexed by $j=1, \ldots, J$ that differ in terms of their risk attitudes $\left(\gamma_{j}\right)$ and in terms of the maturities of their FRM mortgage contracts $\left(T_{j}\right)$. Condition (9) implies that household $j$ prefers the ARM if

$$
\phi_{0}^{x}\left(T_{j}\right)+\phi_{0}^{y}\left(T_{j}\right)>\frac{\gamma_{j}}{T_{j}} \sum_{t=1}^{T_{j}} \operatorname{Var}_{0}\left[y_{t-1}(1)+x_{t-1}\right] .
$$

For a single investor, the choice between an FRM and an ARM only depends on the sum of the two risk premia $\phi_{0}^{x}\left(T_{j}\right)+\phi_{0}^{y}\left(T_{j}\right)$ and the variance of the sum of expected inflation and the real rate. Heterogeneity forces us to include all four variables separately however.

Since we do not observe the individual mortgage maturities $T_{j}$, we use either five-year or ten-year bond risk premia to proxy for the risk premia $\phi_{0}^{x}\left(T_{j}\right)$ and $\phi_{0}^{y}\left(T_{j}\right)$. Bonds with different maturities will have different exposures to the real interest rate and expected inflation. If both risk premia are driven by a single factor, including the nominal bond risk premium $\phi_{0}^{x}(T)+\phi_{0}^{y}(T)$, with $T=5$ or 10, as an explanatory variable in the ARM share regression would be appropriate. However, if two factors are needed to capture the variation in both bond risk premia, then the nominal bond risk premium is no longer the correct explanatory variable for the aggregate mortgage choice. Instead, we must use the real rate premium and expected inflation premium as two separate explanatory variables. As it turns out, a single-factor model does not fit the data well; the correlation between the 5 -year and 10 -year nominal bond risk premium is only $88 \%$. Since most FRM mortgage contracts have a thirty-year maturity, the correlation between the relevant bond risk premium and our five- or ten-year proxies may be even lower.

The same argument applies to the average volatility of the real interest rate and expected inflation. Only their sum matters for a single investor, but their individual components matter in the aggregate if the volatility proxy that we use does not match the maturity of the investor's contract exactly.

Money Illusion and Borrowing Constraints Money illusion, as in Brunnermeier and Julliard (2006), or the presence of borrowing constraints are additional motivations to consider the two volatilities separately. High expected inflation volatility $\left(V_{t}^{x}\right)$ makes the ARM more risky for "nominal" investors as these investors are inapt to disentangle real rates and expected inflation (Section 2.1). The same is true for "real" investors who are borrowing constrained (Section 2.2.1). In contrast, for unconstrained real investors, high expected inflation volatility makes the FRM more risky (Section 2.2.2). This implies that we predict a positive sign in the ARM share regressions on expected inflation volatility if money illusion or borrowing constraints are important for aggregate mortgage choice. We predict a negative sign if rational, unconstrained investors drive aggregate mortgage choice.

Prepayment Third, FRM contracts in the US contain a prepayment option (the details on prepayment are in Section 5). If the four term structure variables affect the option value differently, we need to include them separately in the ARM share regressions. The linear regression can be interpreted as a first-order expansion of the non-linear relationship between between mortgage rates and therefore mortgage choice on the one hand and the two risk premia and volatilities on the other hand.

## 3 VAR model

We set up a VAR model to construct long-term inflation and real interest rate expectations that are needed to estimate real interest rate and expected inflation risk premia. ${ }^{7}$ We allow for heteroscedasticity in the innovations. This structure will turn out to be valuable to understand how exactly the two risk premia and the two conditional volatilities affect mortgage choice, analyzed later in Section 4.

### 3.1 VAR Setup

Our state vector $Y$ contains the one-year $\left(y_{t}^{\S}(12)\right)$, the five-year $\left(y_{t}^{\S}(60)\right)$, and the ten-year nominal yields $\left(y_{t}^{\$}(120)\right)$, as well as realized, one-year $\log$ inflation $\left(\pi_{t}(12)=\log \Pi_{t}-\log \Pi_{t-12}\right)$. On the right-hand side of the $\operatorname{VAR}(1)$ is the 12 -month lag of the state variables. Time $(t)$ is expressed in months and we use overlapping monthly observations. ${ }^{8}$ The law of motion for the state is

$$
\begin{equation*}
Y_{t+12}=\mu+\Gamma Y_{t}+\eta_{t+12}, \quad \text { with } \eta_{t+12} \mid \mathcal{I}_{t} \sim D\left(0, \Sigma_{t}\right) \tag{25}
\end{equation*}
$$

with $\mathcal{I}_{t}$ representing the information at time- $t$. We specify the conditional volatility matrix $\Sigma_{t}$ below.

We start by constructing the 1-year expected inflation series as a function of the state vector

$$
\begin{equation*}
x_{t}(12)=\mathbb{E}_{t}\left[\pi_{t+12}(12)\right]=e_{4}^{\prime} \mu+e_{4}^{\prime} \Gamma Y_{t}, \tag{26}
\end{equation*}
$$

where $e_{4}$ is the fourth unit vector. We construct the 1-year real short rate by subtracting expected inflation from the 1-year nominal rate (see (19))

$$
\begin{equation*}
y_{t}(12)=y_{t}^{\Phi}(12)-x_{t}(12)=-e_{4}^{\prime} \mu+\left(e_{1}^{\prime}-e_{4}^{\prime} \Gamma\right) Y_{t} . \tag{27}
\end{equation*}
$$

Next, we use the VAR structure to determine the $n$-year expectations of the average inflation and the average real rate in terms of the state variables. For expected average inflation this becomes

$$
\begin{equation*}
x_{t}(12 \times n)=\frac{1}{n} \mathbb{E}_{t}\left[\sum_{i=1}^{n} e_{4}^{\prime} Y_{t+(12 \times n)}\right]=\left(\frac{1}{n}\right) e_{4}^{\prime}\left\{\sum_{i=1}^{n}\left(\sum_{j=0}^{i-1} \Gamma^{j} \mu\right)+\sum_{i=1}^{n} \Gamma^{i} Y_{t}\right\} . \tag{28}
\end{equation*}
$$

[^5]The long-run expected average real rate is also a function of the current state

$$
\begin{align*}
y_{t}(12 \times n) & =\frac{1}{n} \mathbb{E}_{t}\left[\sum_{i=0}^{n-1} y_{t+(12 \times i)}(12)\right] \\
& =\left(\frac{1}{n}\right) e_{1}^{\prime}\left\{\sum_{i=1}^{n-1}\left(\sum_{j=0}^{i-1} \Gamma^{j} \mu\right)+\sum_{i=1}^{n-1} \Gamma^{i} Y_{t}\right\}+\frac{e_{1}^{\prime} Y_{t}}{n}-x_{t}(12 \times n) . \tag{29}
\end{align*}
$$

With the long-term expected real rate from (29) in hand, we can form the real risk premium by subtracting this expectation from the observed real rate (as in (18)). Similarly, with the long-term expected inflation from (28) in hand, we form the inflation risk premium as the difference between the observed nominal yield, the observed real yield, and expected inflation (as in (20)).

We now turn to the model for the volatility of the real interest rate and expected inflation. We first estimate the innovations $\left(\hat{\eta}_{t}, t=1, \ldots, T\right)$ from the VAR-model and construct the implied innovations to the real rate and expected inflation according to (30) and (31),

$$
\begin{align*}
\eta_{t+12}^{x} & =x_{t+12}(12)-\mathbb{E}_{t}\left[x_{t+12}(12)\right]=e_{4}^{\prime} \Gamma \eta_{t+12}  \tag{30}\\
\eta_{t+12}^{y} & =y_{t+12}(12)-\mathbb{E}_{t}\left[y_{t+12}(12)\right]=\left(e_{1}^{\prime}-e_{4}^{\prime} \Gamma\right) \eta_{t+12} \tag{31}
\end{align*}
$$

Next, we model both conditional variances as an exponentially affine function in their own level

$$
\begin{align*}
V_{t}^{x} \equiv \operatorname{Var}_{t}\left[x_{t+12}(12)\right] & =\operatorname{Var}_{t}\left[\eta_{t+12}^{x}\right]=\exp \left(\alpha_{x}+\beta_{x} x_{t}(12)\right),  \tag{32}\\
V_{t}^{y} \equiv \operatorname{Var}_{t}\left[y_{t+12}(12)\right] & =\operatorname{Var}_{t}\left[\eta_{t+12}^{y}\right]=\exp \left(\alpha_{y}+\beta_{y} y_{t}(12)\right) \tag{33}
\end{align*}
$$

The coefficients $\alpha_{i}$ and $\beta_{i}, i=x, y$, are estimated consistently via non-linear least squares

$$
\left(\hat{\alpha}_{i}, \hat{\beta}_{i}\right)=\arg \min _{\alpha_{i}, \beta_{i}} \frac{1}{T} \sum_{t=1}^{T}\left(\left[\hat{\eta}_{t \times 12}^{i}\right]^{2}-\exp \left(\alpha_{i}+\beta_{i} i_{t \times 12}(12)\right)\right)^{2} .
$$

### 3.2 VAR Estimation Results

We estimate a VAR-model with monthly observations for the period 1985.1-2006.6. Monthly nominal yield data are from the Federal Reserve Bank of New York. ${ }^{9}$ The inflation rate is based on monthly CPI-U available from the Bureau of Labor Statistics. ${ }^{10}$ We start the model in 1985, near the end of the Volcker deflation. Our stationary, one-regime model would be unfit to estimate the entire post-war history (see Ang and Bekaert (2005)). Estimating the model at monthly frequency gives us a sufficiently many observations (258 months). The VAR(1) structure with the 12-month

[^6]lag on the right-hand side is parsimonious and delivers plausible long-term expectations. ${ }^{11}$
Figure 3 shows the results from the estimation. The top left panel shows the 1-year expected inflation $x_{t}$ as well as the 1-year real rate $y_{t}$, computed from (26) and (27). The bottom two panels show the long-term expectations of the same variables at the five- and ten-year horizons, computed from (28) and (29) respectively. Expected inflation is relatively smooth at all horizons; its values are nearly identical at the five-year and ten-year horizons. It is $2.9 \%$ per year on average; higher at the beginning of the sample ( $3.48 \%$ in 1985.2) and lower near the end of the sample $(2.46 \%$ in 2004.3). Interestingly, the survey data on long-term expected inflation, which we used in the introduction, show a similar pattern. They are also nearly constant albeit at a slightly lower level of $2.5 \%$. Real rate expectations display more variation over time. At the one-year horizon, real yields hover between $-2 \%$ (2004) and $6 \%$ per year (1984). At the ten-year horizon, these expectations are smoother. They hover between $0.5 \%$ and $3.5 \%$, but show the same pattern of fluctuations.

The top right panel plots the conditional volatilities of expected inflation and the real rate (see equations (32) and (33)). Conditional real rate volatility is $1.06 \%$ per year on average, while expected inflation volatility is three times lower at $0.35 \%$ per year on average. There is some time variation in these one-year ahead conditional volatilities. The two conditional volatilities co-move strongly negatively; their correlation is -0.71 . For example, real rate volatility is high in 2004, when the real rate is low, and low in the 1985, when the real rate is high. In contrast, expected inflation volatility is at its highest level in 1991, when expected inflation is high, and low in 2002, when expected inflation is low.
[Figure 3 about here.]
Combining data on nominal and real five-year and ten-year yields, we form the real rate and expected inflation risk premia. The real yield data is from McCulloch. ${ }^{12}$ The left panel of Figure 4 plots the risk premia at a five-year horizon, while the right panel plots the ten-year horizon premia. The figure starts in July of 1997, the first period for which five-year and ten-year real yield data are available in the US. Expected inflation risk premia in both panels are negative until 2004. This negative risk premium not surprising given the fact that the observed spread between nominal and real yields is often below $2 \%$ and inflation expectations are always above $2 \%$. Most of the action in this spread is inherited by the inflation risk premium because expected inflation is estimated to be nearly constant. The ten-year risk premium varies between $-1.65 \%$ in 1998.8 and $+0.35 \%$ in 2004.4. The real rate premium on the other hand is estimated to be positive, and varies between $0.8 \%$ per year in 2005.5 and $2.9 \%$ in 2002.1 at the ten-year horizon.

[^7][Figure 4 about here.]

The two risk premia have a negative correlation of -0.64 and -0.59 at the five-year, and ten-year horizons respectively. Because of this negative correlation, the nominal risk premium, cancels out a lot of interesting variation that is in the component risk premia. Unsurprisingly, this sum will turn out to be less informative for mortgage choice than its components.

### 3.3 Extending the Sample of Bond Risk Premia

The unavailability of real yield data before 1997.7 prevents us from studying mortgage choice in the US before this date using the same methodology. After all, we use the real term structure data to disentangle the real rate and expected inflation risk premium. We now develop a projection method that allows us to extend the sample back to 1985. This exercise is best interpreted as a robustness check.

The data on nominal yields and realized inflation, but also on the nominal bond risk premium (obtained from the VAR) go back to 1985. What we are missing is the decomposition of the nominal bond risk premium into its two components: the inflation risk premium and the real rate risk premium. We construct a long time series for the real interest rate premium by first regressing the real rate risk premium on a set of state variables $z_{t}$ that are observable over the complete sample period. Specifically, we estimate the regression

$$
\begin{equation*}
\phi_{t}^{y}=\alpha+\beta^{\prime} z_{t}+\epsilon_{t}, \tag{34}
\end{equation*}
$$

over the period 1997:7-2006:6, and construct the real rate risk premium for the full sample period using the estimated coefficients $\hat{\phi}_{t}^{y}=\hat{\alpha}+\hat{\beta}^{\prime} z_{t}$. Since the nominal risk premium is available for the entire sample, we back out the inflation rate risk premium as the difference between the nominal risk premium and the projected real rate risk premium. This method gives reliable answers as long as (i) the relationship between risk premia and the state variables $z_{t}$ does not change dramatically over the sample period and (ii) the state variables capture most of the variation in the risk premia.

With these considerations in mind, we select $z_{t}=\left(Y_{t}^{\prime}, Y_{t-1}^{\prime}\right)^{\prime}$, where $Y_{t}$ contains the VAR variables and time measured in years. A regression of the ten-year (five-year) real rate premium on $z$ gives an in-sample $R^{2}$ of $90 \%$ ( $86 \%$ ). Figure 5 shows the observed nominal bond risk premium $\left\{\phi_{t}^{\$}\right\}$ (solid black line) together with its projected components (lines with circles) at the ten-year horizon. It also overlays the risk premia shown in the left panel of Figure 4 for the 1997.7-2006.6 period. The projections are close to these risk premia estimates. Interestingly, the projections indicate that inflation risk premia where higher (and often positive) before 1995. Real rate risk premia came down from $4 \%$ in 1985 to $2 \%$ in 1997.
[Figure 5 about here.]

## 4 ARM Share Regressions

We are interested in explaining time variation in the fraction of all newly-originated mortgages that is of the adjustable-rate type. In this section, we regress the ARM share on the one-period lag of the term structure variables, motivated in Section 2 and computed from the VAR in Section 3. These include the real rate premium, the expected inflation premium, the real rate volatility, and the expected inflation volatility. We lag the predictor variables for one period in order to study what changes in this month's risk premia and volatilities imply for next month's mortgage choice. In addition, the use of lagged regressors mitigates potential endogeneity problems that would arise if mortgage choice affected the term structure of interest rates.

### 4.1 Data on the ARM Share in the U.S.

Our baseline data series is from the Federal Housing Financing Board. It is based on the Monthly Interest Rate Survey, a survey sent out to mortgage lenders. ${ }^{13}$ These data include loan originations for both newly constructed homes and existing homes. The monthly data start in 1985.1 and run until 2006.6, and we label this series $\left\{A R M_{t}^{1}\right\}$. Our baseline measure of the ARM share includes all adjustable mortgages. In particular, it includes hybrid mortgages which have an initial fixedinterest rate payment period. Starting in 1992, we also know the decomposition of the ARM by initial fixed-rate period. ${ }^{14}$ This allows us to construct two "stricter" measures of the ARM share. The first alternative measure includes only those ARMs with an initial fixed-rate period of five years or less. It omits the ARMs with an intial fixed-rate period of seven and ten years, so called $7 / 1$ and $10 / 1$ hybrids, as well as miscellaneous loans with initial fixed-rate period greater than 5 years. We label this series $\left\{A R M_{t}^{2}\right\}$. The second alternative measure, $\left\{A R M_{t}^{3}\right\}$, contains only ARMs with initial fixed-rate period of 3 years (3/1), one year ( $1 / 1$ ), and miscellaneous loans with initial fixed-rate period less than one year. Finally, there is an alternative source of ARM share data available from Freddie-Mac, which constructs a monthly ARM share based on the Primary Mortgage Market Survey. ${ }^{15}$ This series, which we label $\left\{A R M_{t}^{4}\right\}$, conceptually measures the same

[^8]as $\left\{A R M_{t}^{1}\right\}$, and is available from 1995.1. Figure 6 plots all four series together, starting in 1992.1. The correlation between measure 2 (measure 3) and our benchmark measure 1 is $98.6 \%$ ( $86.3 \%$ ). The correlation between measure 4 and our benchmark is $89.9 \%$.
[Figure 6 about here.]

### 4.2 Regression Results

We start by reporting univariate regressions of the benchmark ARM share on the one-period lag of the term structure variables we identified. Table 1 shows the slope coefficient, its Newey-West t-statistic using 12 lags, and the regression $R^{2}$ for seventeen different explanatory variables. The first panel contains the four term structure variables we propose.

Our main focus is on the 1997.7-2006.6 sample, for which we have real term structure data. ${ }^{16}$ The single strongest explanatory variable of variation in the ARM share is the expected inflation risk premium at the five-year horizon. It has a t-statistic of 8.49 , and explains $63.5 \%$ of the variation in the ARM share. A 1 percentage point, or two-standard deviation, increase in the expected inflation risk premium increases the ARM share by 12.7 percentage points. The inflation risk premium has to be paid by the FRM holder (the investor). An increase in the inflation risk premium makes the FRM relatively less attractive and increases the ARM share. Figure 2 in the introduction confirms that the two variables co-move remarkably. The ten-year inflation risk premium looks very similar to the five-year risk premium (see Figure 4) and has a similar explanatory power of $56.2 \%$. Interestingly, the expected inflation risk premium continues to be strongly related to the ARM share in the full sample 1985.1-2006.6 (left columns). The larger point estimate suggests an even larger sensitivity of the ARM share to the inflation risk premium over the full sample. The t -statistic of $\phi_{t}^{x}(5)$, constructed from the projection exercise in in Section 3.3, equals 5.9, and the regression $R^{2}$ is still $44 \%$.

All other variables explain a much smaller fraction of the variation in the ARM share in the US. First, the real rate risk premium has the right sign in the full sample, but its correlation with the ARM share is lower. Only the real rate premium at the ten-year horizon is statistically significantly related to the ARM share; the $R^{2}$ is $12 \%$. This correlation has the wrong sign in the 1997.7-2006.6 sample. Second, the VAR allows us to compute the 1-year ahead conditional variances $V_{t}^{x}$ and $V_{t}^{y}$, and to include those in the regression. ${ }^{17}$ In contrast to the risk premia, these conditional variances

[^9]are available for the entire 1985-2006 sample (see top right panel of Figure 3). Univariately, we find that a higher volatility of the real rate make the ARM less desirable, consistent with the model. We also find that a higher inflation volatility makes the FRM (ARM) less (more) desirable. As pointed out in Section 2.4, the positive sign on the conditional volatility of expected inflation is consistent with a rational investor who is unconstrained, and not with an investor with money illusion or with binding constraints. We note that the volatility of the nominal interest rate is also significantly negatively related to the ARM share in the full sample (not reported). Most of the conditional variance of the nominal rate is inherited by the conditional variance of the real rate; the two have a correlation of $95 \%$.
[Table 1 about here.]
Turning to the multivariate regressions in Table 2, the importance of the expected inflation risk premium as a determinant of the ARM share remains unchanged. In a first regression, we include both the expected inflation and the real rate risk premium at the ten-year horizon. Column (5) reports the results for the sample for which we have real yield data, while the first column reports the full sample results using risk premia series based on the projection method. ${ }^{18}$ Both variables enter with the right sign, but only the inflation risk premium is significant. Compared to the univariate regression, the $R^{2}$ improves marginally: from 56.2 to $56.8 \%$ for the 1997-206 sample and from 44.6 to $46.3 \%$ for the full sample, respectively. Next, we add the two volatility terms as regressors in columns (2) and (6). They both enter with the right sign in both samples. Inflation volatility makes the ARM more desirable, whereas real rate volatility makes the ARM less attractive. The real rate volatility is significant in the full sample. The $R^{2}$ improves by $7.5 \%$ in the full sample and by $4.1 \%$ in the 1997-2006 sample. The coefficient on the real rate risk premium turns negative, but is insignificant. ${ }^{19}$

Noteworthy is that the coefficient on the inflation risk premium is stable across both samples; it is always around 15. The results with five-year risk premia instead of ten-year risk premia are very similar (not reported). The $R^{2}$ with five-year risk premia is $63.8 \%$ in the $97-06$ sample and also $53.5 \%$ in the full sample. Again, for the 97-06 sample, adding the other three term structure variables barely improves on the fit of the univariate regression. In the US, the expected inflation risk premium turns out to be the most important determinant of mortgage choice.

[^10][Table 2 about here.]

Other Term-Structure Variables The second panel of Table 1 compares the univariate explanatory power of the inflation risk premium to that of other variables. The nominal bond risk premium, which is the sum of the expected inflation and real rate risk premia, is a much weaker determinant than its components. The reason is that its components are negatively correlated. We have argued in Section 2.4 that aggregation of mortgage choice across heterogenous investors necessitates including both risk premia in the regression separately.

Next, we consider the spread of the five- and ten-year yield over the one-year yield. Neither yield spread is significant in either sample. The $R^{2}$ never exceeds $1 \%$. We have argued in Section 2.3 that the yield spread not only measures the nominal risk premium, but also the deviation of the expected future short rate from the current short rate. Our VAR analysis shows that these two components are negatively related. This makes the yield spread a contaminated predictor of mortgage choice. Columns (3) and (7) of Table 2 show that there is no extra information in the yield spread that is useful for predicting the ARM share, and not already present in the other four variables. They add the orthogonal component of the yield spread as an explanatory variable of the ARM share. The $R^{2}$ does not increase and its coefficient is insignificant. Returning to Table 1, the last row of the second panel considers the spread between the FRM and the ARM rate. ${ }^{20}$ The motivation for using this spread is the same as for the one for using the yield spread. Both measure a long maturity - short maturity differential. The FRM-ARM spread has equally little explanatory power in the 1997-2006 sample. But, it is significant in the full sample with an $R^{2}$ of $35 \%$, whereas the government bond yield spread is not. Clearly, the FRM-ARM spread contains additional information unrelated to the term premium. ${ }^{21}$ To get at this additional information, we orthogonalize the FRM-ARM spread to the $10-1$ yield spread and regress the ARM share on the orthogonal component. For the full sample, we find a strongly significant effect. Section 7 suggests one possible explanation for this finding.

The short-term and long-term government bond yield levels, as well as the effective fixed-rate or adjustable-rate mortgage rates are all weak explanatory variables of the ARM share in the 19972006 sample (third panel of Table 1). In the full sample, the $R^{2}$ are higher, but not as high as for the expected inflation risk premium. Furthermore, the explanatory power of the bond yields and mortgage rates seems related to that of the expected inflation risk premium. They are positively correlated with the inflation risk premium in the full sample, but negatively correlated in the later

[^11]sample.

Robustness: Other ARM Share Measures As a robustness exercise, we repeat the analysis in Column (6) of Table 2 for the alternative measures of the ARM share discussed above. The second and third column of Table 3 show that the explanatory power of the term structure variables is very similar without the hybrid mortgages. In the second column we count the hybrid ARM contracts with initial fixed-rate period greater than five years as FRMs. The average fraction of ARMs falls from $21.6 \%$ to $18.1 \%$, but the results remain virtually unchanged. In the third column, we also eliminate the mortgages with an initial fixed-rate period greater than three years. The average fraction of ARMs drops to $11 \%$, half as much as in Column (1). The $R^{2}$ drops by almost 20 percentage points, but the inflation risk premium remains highly significant. The real rate volatility now also becomes significant. We conclude that our results are robust to the classification of the hybrid mortgage contracts. In Column (4), we use the Freddie Mac data instead of the FHFB data. This makes little difference compared to Column 1. The $R^{2}$ is the highest for this measure, and equal to $64 \%$. Finally, we also experiment with splitting up the ARMs in Column (1) into mortgage contracts for newly constructed homes and mortgages for existing structures (not reported in the table). The average fraction of ARM shares is $28 \%$ for the former group and $20 \%$ for the latter. The inflation risk premium is a highly significant determinant of mortgage choice in both groups. The $R^{2}$, however, is twice as high (68\%) for the existing structures group than for the new-construction group (34\%).
[Table 3 about here.]

Robustness: Alternative Interest Rate Models We have studied how the ARM share regressions are affected when we change the underlying term structure model. We have estimated a $\operatorname{VAR}(2)$-model for $Y$ instead of a $\operatorname{VAR}(1)$. The inflation and real rate risk premia in the $\operatorname{VAR}(2)$ model look qualitatively similar to those in Figure 4. The only small difference is that the longterm expected inflation is estimated to be somewhat lower (around $2 \%$ per year), so that the inflation risk premium is higher near the end of the sample, and the increase in the inflation risk premium since 2003 is more pronounced. We then rerun the ARM share regressions, corresponding to Columns (2) and (6) in Table 2. The inflation risk premium is as prominent an explanatory variable as in the benchmark analysis. The $R^{2}$ on the regression further improves to $70.5 \%$ in the second sample (from $61 \%$ ), and to $57.6 \%$ in the full sample (from $53.9 \%$ ).

We also verified the robustness of our results to alternative volatility models for the expected inflation and the real rate in (32) and (34). This comparison showed that the benchmark volatility model has the lowest sum of squared deviation between the squared innovations and the proposed conditional volatility. Further, because the difference between alternative volatility models is rela-
tively small, the ARM share regressions provide similar results for alternative conditional volatility series.

Robustness: Alternative Real Yield Measures Due to the availability of TIPS data, our main results are for the 1997-2006 sample. We have used the projection method as a first robustness exercise to verify whether our results extend to the longer 1985-2006 sample. As a further robustness check, we use the real yield data backed-out from the term structure model of Ang and Bekaert (2005) instead of the TIPS yields. We proceed as in Section 3, forming inflation and real rate expectations in the same way. We treat the real yields as observed, and use them to construct the inflation risk premium and the real rate premium in turn. Since the Ang-Bekaert data are quarterly (1985.IV-2004.IV), we construct the quarterly ARM share as the simple average of the three monthly ARM share observations in that quarter. We then regress the quarterly ARM share on the one-quarter lagged inflation and real rate risk premium. We find that both enter with the correct, positive sign, and both coefficients are statistically significant. The Newey-West t-statistic on the inflation risk premium is 3.9 and the $t$-statistic on the real rate risk premium is 2.12 . The regression $R^{2}$ is $53 \%$. Including the quarterly-sampled conditional volatility terms increases the $R^{2}$ further to $70 \%$, while increasing the importance of the inflation risk premium. Our results are therefore robust to an alternative, model-implied measurement of real yields.

Robustness: Persistence of Regressor In contrast to the inflation risk premium, most term structure variables in Table 1 do not explain much of the variation in the ARM share. This is especially true in the 1997-2006 sample. This suggests that our results for the inflation risk premium are not simply an artifact of regressing a persistent regressand on a persistent regressor, because many of the other term structure variables are at least as persistent. One further robustness check we performed is to regress quarterly changes in the ARM share (between periods $t$ and $t+3)$ on changes in the four term structure variables of the benchmark regression specification (between periods $t-1$ and $t$ ). We continue to find a positive and strongly significant effect of the inflation risk premium on the ARM share. The magnitude of the regression coefficients implies that a one percentage point increase in the inflation risk premium increases the ARM share by 15.5 percentage points in the full sample and 11.6 in the 1997-2006 sample, all else equal. These sensitivities are consistent with our findings for the level regressions. The $R^{2}$ of the regression in changes is obviously lower, but still substantial: $20.8 \%$ in the full sample and $17.0 \%$ in the 1997-2006 sample.

## 5 Prepayment Option

Sofar we have ignored one other potentially important determinant of mortgage choice: the prepayment option. In the US, an FRM contract typically has an embedded prepayment option which allows the mortgage borrower to pay off the loan at will. In this section we first discuss the anatomy of the fixed-rate mortgage with prepayment option and relate it to other, traded, fixed-income securities. Next we show how the presence of the prepayment option affects mortgage choice in a model that accommodates time-varying prices of risk and volatilities.

### 5.1 Anatomy of a Fixed-Rate Mortgage

A fixed-rate mortgage without prepayment option is a coupon-bearing nominal bond, issued by the borrower and held by the lender. ${ }^{22}$ An FRM with prepayment option is a callable bond. The borrower has the right to prepay the outstanding mortgage debt at any point in time; the prepayment option is of the American type. The price sensitivity of a callable bond to interest rate shocks differs from that of a regular bond. Figure 7 plots the price sensitivity of an FRM without prepayment (regular bond) and an FRM with prepayment (callable bond) to changes in the real rate (top panel) and expected inflation (bottom panel). The model that produces this figure is explained below. The regular bond price is decreasing and convex in both the real interest rate and expected inflation. The callable bond price is decreasing in $x$ and $y$, but the relationship becomes concave when the call option is in the money, i.e., when the real rate or expected inflation are low. This implies that the price of a callable bond is less sensitive to interest rate changes. This reduced exposure is most pronounced with respect to expected inflation (bottom panel). In sum, the FRM with prepayment has positive, but diminished exposure to real rate and expected inflation. This implies that the expected payments to the FRM with prepayment increase with the real rate and expected inflation risk premium, but not as much as the FRM without prepayment.
[Figure 7 about here.]

### 5.2 Valuing the Option in the VAR Framework

We now turn to the valuation the prepayment option. Valuation of this option is based on a numerical dynamic programming algorithm that determines optimal refinancing decisions (see also Pliska (2006)). Due to computational limitations, the algorithm cannot handle the fourdimensional state variable that arises in the $\operatorname{VAR}(1)$ model of Section 3. We use a reduced, three-dimensional model instead for the option valuation, and denote the corresponding variables

[^12]with a ' $\star$ ' superscript. The state vector, $Y_{t}^{\star}$, is comprised of the real rate, expected inflation, and realized inflation, $Y_{t}^{\star}=\left(y_{t}(12), x_{t}(12), \pi_{t}(12)\right)$. The time series for $\left\{x_{t}\right\}$ and $\left\{y_{t}\right\}$ are taken from the large VAR model of Section 3. The law of motion for the state is
\[

$$
\begin{equation*}
Y_{t+12}^{\star}=\mu^{\star}+\Gamma^{\star} Y_{t}^{\star}+\eta_{t+12}^{\star}, \text { where } \eta_{t+12}^{\star} \mid \mathcal{I}_{t} \sim N\left(0, \Sigma_{t}^{\star}\right), \tag{35}
\end{equation*}
$$

\]

where we assume that the innovations are conditionally normally distributed. The conditional variance matrix is given by

$$
\begin{aligned}
\Sigma_{t}^{\star}= & \operatorname{diag}\left(\sqrt{\exp \left(\alpha_{1}^{\star}+\beta_{1}^{\star} Y_{t(1)}\right)}, \ldots, \sqrt{\exp \left(\alpha_{3}^{\star}+\beta_{3}^{\star} Y_{t(3)}^{\star}\right)}\right) \times \Omega^{\star} \times \\
& \operatorname{diag}\left(\sqrt{\exp \left(\alpha_{1}^{\star}+\beta_{1}^{\star} Y_{t(1)}\right)}, \ldots, \sqrt{\exp \left(\alpha_{3}^{\star}+\beta_{3}^{\star} Y_{t(3)}^{\star}\right)}\right)
\end{aligned}
$$

We further assume that the nominal $\log$ pricing kernel $m_{t+1}^{\$}$ reads

$$
\begin{equation*}
-m_{t+12}^{\S}=y_{t}(12)+x_{t}(12)+\frac{1}{2} \Lambda_{t}^{\prime} \Sigma_{t}^{\star} \Lambda_{t}+\Lambda_{t}^{\prime} \eta_{t+12}^{\star} \tag{36}
\end{equation*}
$$

with market prices of risk $\Lambda_{t}$. Following the literature on affine term structure models (e.g., Dai and Singleton (2002) and Duffee (2002)), the market prices of risk $\Lambda_{t}$, are assumed to be affine functions of the state vector

$$
\begin{equation*}
\Lambda_{t}=\lambda_{0}+\Lambda_{1}^{\prime} Y_{t}^{\star} \tag{37}
\end{equation*}
$$

We relegate the discussion of the estimation of the market prices of risk to Appendix A.
The price of the prepayment option is the difference between the rate on a fixed-rate mortgage with prepayment and a (hypothetical) rate on an FRM without prepayment. Let the nominal value to the lender of the former contract be $V_{t}^{p}\left(Y_{t}^{\star}, r_{t}\right)$, and the value of the latter contract $V_{t}^{n p}\left(Y_{t}^{\star}, r_{t}\right)$, where $r_{t}$ is the contractual mortgage rate. The time- $t$ values are determined after the time- $t$ interest payment is made and after the prepayment decision has been made. We assume that there are no costs to prepay and that the borrower prepays optimally. ${ }^{23}$ As before, the face value of the loan is normalized to $\$ 1$. Finally, we assume that there is perfect competition in the mortgage market. Competition implies that the present value of payments must equal the value of the loan at origination. The implied zero-profit mortgage rate at time $t$, denoted $\hat{r}_{t}^{h}$ with $h \in\{p, n p\}$, satisfies

$$
\begin{equation*}
V_{t}^{p}\left(Y_{t}^{\star}, \hat{r}_{t}^{p}\right)=V_{t}^{n p}\left(Y_{t}^{\star}, \hat{r}_{t}^{n p}\right)=1, \forall t=0, \cdots, T . \tag{38}
\end{equation*}
$$

At maturity $T$, this condition simply states that the principal is paid back in full. The zero-

[^13]profit rate will be a function of the state $Y_{t}^{\star}$ and will be different for the FRM with and without prepayment. The contract values can be solved for recursively, working backwards from time $T$. The recursion at time $t$ reads
\[

$$
\begin{align*}
V_{t}^{n p}\left(Y_{t}^{\star}, r_{t}\right) & =E_{t}\left[\exp \left(m_{t+1}^{\$}\right)\left(r_{t}+V_{t+1}^{n p}\left(Y_{t+1}^{\star}, r_{t}\right)\right)\right]  \tag{39}\\
V_{t}^{p}\left(Y_{t}^{\star}, r_{t}\right) & =E_{t}\left[\exp \left(m_{t+1}^{\$}\right)\left(r_{t}+\min \left\{1, V_{t+1}^{n p}\left(Y_{t+1}^{\star}, r_{t}\right)\right\}\right)\right] . \tag{40}
\end{align*}
$$
\]

The minimum operator in equation (40) reflects the prepayment option: the borrower will prepay at time $t+1$ whenever the rate on a new loan is lower than the rate on the existing loan. This is the case when $V_{t+1}^{n p}\left(Y_{t+1}^{\star}, r_{t}\right)>1$. The value of the prepayment option at time 0 is given by the difference in the zero-profit rate between the two contracts, $\hat{r}_{0}^{p}\left(Y_{0}^{\star}\right)-\hat{r}_{0}^{n p}\left(Y_{0}^{\star}\right)$.

Figure 8 displays the model-implied time-series for the prepayment option value, $\hat{r}_{0}^{p}\left(Y_{t}^{\star}\right)-$ $\hat{r}_{0}^{n p}\left(Y_{t}^{\star}\right)$, alongside the real interest rate and expected inflation. The option value tends to increase with the real interest rate and expected inflation. When the real interest rate and expected inflation rate are high, they are likely to revert back to their long-term mean, which increases the likelihood that the option will be in-the-money in the future. To compensate for this prepayment risk, the lender sets a higher contract rate on the FRM with prepayment. In our model, the option value is also affected by time variation in the volatility and the prices of risk.
[Figure 8 about here.]

### 5.3 The ARM Share and the Prepayment Option

In addition to this option value, we also determine the expected real payments stream that the borrower makes on each of the contracts, i.e. $\sum_{t=1}^{T} E_{0}\left[q_{t}^{h}\right]$ for $h \in\{p, n p\}$. The latter computation uses the zero-profit rates and employs a forward simulation technique for the state variables. ${ }^{24}$ Appendix A contains the details. We then regress the sum of expected real payments on a tenyear FRM with and without prepayment on the model-implied ten-year real rate risk premium $\phi^{y \star}$ and expected inflation premium $\phi^{x \star}$ in equations (41) and (42). What is key to our mortgage choice analysis is that the expected payments on both contracts increase with both risk premia. Consistent with Figure 7, we find that the sensitivity of the expected payments on an FRM with prepayment option is smaller than the sensitivity for an FRM option without prepayment. The

[^14]expected payment differential in (43) relates negatively to the risk premia.
\[

$$
\begin{align*}
\sum_{s=1}^{10} E_{t}\left[q_{t+s}^{p}\right] & =1.15+4.80 \times \phi_{t}^{y \star}+2.33 \times \phi_{t}^{x \star}+\varepsilon_{t},  \tag{41}\\
\sum_{s=1}^{10} E_{t}\left[q_{t+s}^{n p}\right] & =1.13+5.74 \times \phi_{t}^{y \star}+2.97 \times \phi_{t}^{x \star}+\varepsilon_{t},  \tag{42}\\
\sum_{s=1}^{10} E_{t}\left[q_{t+s}^{p}\right]-\sum_{s=1}^{10} E_{t}\left[q_{t+s}^{n p}\right] & =0.02-0.94 \times \phi_{t}^{y \star}-0.65 \times \phi_{t}^{x \star}+\varepsilon_{t} . \tag{43}
\end{align*}
$$
\]

Finally, we revisit the ARM share regressions and ask whether the option value contains any additional information. Univariately, we find a positive and significant relationship between the ARM share and the option value. This positive sign survives in the multi-variate regressions in columns (4) and (8) of Table 2. Using the same term structure variables as in columns (2) and (6), we add the orthogonal component of the option value as a fifth regressor. One explanation for the lower preference for an FRM when the prepayment option is expensive (after controlling for risk) is that an expensive prepayment option front-loads the FRM's payment schedule. The option is paid up-front, but the benefits of lower payments arising from exercising the option arise only in the future. This 'tilt' effect is also described in Campbell (2006), and may explain why a borrowing-constrained investor may prefer the ARM in the presence of the option. ${ }^{25}$

## 6 Analysis for the United Kingdom

As a robustness check, we repeat the full analysis for the UK. This is important for at least three reasons. First, to show that the same yield curve variables also explain a substantial fraction of the ARM share in a different country is an important robustness check. Second, to the extent that bond risk premia look different in the UK than in the US and to the extent that these differences are related to variation in the ARM share, the term structure determination theory of mortgage choice gains further credibility. Moreover, our findings may even suggest why the mortgage markets in the UK are so different from those in the US. Third, we have a longer, and potentially better time-series of real bond yields available in the UK than in the US.

[^15]
### 6.1 VAR Results

We repeat the analysis we did for the US. That is, we estimate a monthly VAR with 12-month lagged bond yields of one-, five-, and ten-year maturity and realized inflation on the right-hand side. The nominal yields are from the Bank of England, the inflation rate is the 12-month log difference in Retail Price Index (RPI), the counterpart to the American CPI. As for the US, we estimate the VAR on the period 1985.1-2006.6. After we form long-term expected inflation and long-term expected real rates, we construct the inflation risk premium and real rate risk premium using real yield data on five- or ten-year bonds. The UK has much longer time-series for inflation-linked bond yields; the Bank of England data start in 1985.1.
[Figure 9 about here.]
There are substantial differences between the evolution of the term structure in the US and in the UK. Figures 9 and 10 are the UK counterparts to Figures 3 and 4. Figure 9 shows that longterm expected inflation and real rate trend downwards over the sample, aside from an increase in the late 1980s. The decline in expected inflation after 1991 tracks the decline in realized inflation; likewise, realized inflation was high in the late 1980s. Inflation expectations stabilize around $2 \%$ per year at the end of the sample, lower than in the US. As a result of the bigger nominal-real yield spread and the lower inflation expectations, the inflation risk premium is much larger in the UK. Figure 10 shows that it is mostly positive, and it goes down from $3 \%$ to $1 \%$ per year. Conversely, the real rate premium is negative in the UK; it was positive in the US. After being very negative around 1990 , it stabilizes between $-1 \%$ and $0 \%$ after 1995. Just as in the US, the two risk premia are strongly negatively correlated ( $-76 \%$ at the five- and $-63 \%$ at the ten-year horizon). The bottom two rows zoom in on the 1997.7-2006.6 subsample, the same sample we had in Figure 4 for the US. In the UK, the two risk premia drift away from each other after 2002, whereas in the US, they both seem to converge to zero. Finally, the top right panel of Figure 9 shows that the conditional volatilities of the real rate and expected inflation co-move positively in the UK, whereas they co-move negatively in the US. In short, the term structure determinants of mortgage choice look dramatically different in the UK than in the US.
[Figure 10 about here.]

### 6.2 ARM Share Regression Results

The mortgage market in the UK has some important differences with the US. First, long-term fixed-rate mortgages are a lot less prevalent than in the US. The most prevalent contract is a standard variable rate contract, for which the interest rate is adjusted several times per year. Most fixed-rate contracts have one-to-three-year fixed interest rate periods. Ten-year fixed-rate
contracts are relatively new. ${ }^{26}$ Second, fixed-rate mortgages have no prepayment option. We have data on the mortgage composition in the UK from the Council of Mortgage Lenders starting in 1993. ${ }^{27}$ Despite these differences, Figure 11 shows that there is a lot of variation in mortgage composition in the UK as well. The ARM share varies between $85 \%$ and $25 \%$. The overall fraction of adjustable rate contracts is higher than in the US. The ARM share decreases near the end of the sample because of the increased availability and popularity of longer-term fixed-rate contracts.
[Figure 11 about here.]

We repeat the regressions of the ARM share on the one-month lagged term structure variables in Table 4. The left columns report the results for the 1993.1-2006.6 sample, using a monthly ARM series based on the quarterly ARM share data, whereas the right columns report the results for the sample 2002.1-2006.6, for which we have actual monthly ARM shares. In sharp contrast to the analysis for the US, it is the real rate risk premium that is the key driver of mortgage choice in the UK. In both columns (1) and (5), the ten-year real rate risk premium is significant. The expected inflation risk premium has the right sign, but is insignificant. The $R^{2}$ increases only marginally compared to the univariate regression of the ARM share on the real rate premium. It is $61.9 \%$ for the 2002-2006 sample and $22.6 \%$ for the 1993-2006 sample. The effects are economically large. A 0.35 percentage point, or two-standard deviation, increase in the ten-year real rate risk premium in the 2002-2006 sample increases the ARM share by 16 percentage points, from $52 \%$ to $68 \%$. The $R^{2}$ is on the same order of magnitude as the one we found for the US in the later sample, but this time the real rate premium is the key variable instead of the inflation risk premium. Adding the variance terms in column (2) and (6) improves the explanatory power, especially in the later sample. The $R^{2}$ increases to $77 \%$, all four variables have the right sign, and all are significant. Finally, adding orthogonal information in the yield spread does not improve the $R^{2}$ in the full sample, but it does in the later sample. The $R^{2}$ improves by another $1.6 \%$ in column (7). We do not consider the option value effects because there is no prepayment option in the UK.
[Table 4 about here.]

[^16]
## 7 Discussion

The above results suggest that households seem capable of timing their mortgage choice. They take out ARMs when the FRM contracts are expensive because the expected inflation risk premium or the real rate risk premium is high, or because expected inflation is more volatile compared to the real rate. Arguably, mortgage origination is one of the most important financial decisions a household makes. Since mortgage origination is a rare event which requires an active decision, households seem more likely to devote time and effort to evaluate their choices. In contrast, prepayment decisions may be more prone to inertia. ${ }^{28}$ Most importantly, many households seek advice from professionals or mortgage lenders on mortgage choice. The argument below suggests that there may be reasons why mortgage suppliers may have incentives to recommend the "right" mortgage type at the right time.

Consider a mortgage supplier that has a pool of mortgages outstanding. This pool is comprised of both ARM and FRM contracts. The FRM contracts can be prepaid at each point in time without any costs, and prepayment decisions are made optimally. The mortgage supplier borrows all money via the treasury market to extend the mortgage contracts. We further assume that the ARM contracts are truly single-period contracts so that borrowing cash perfectly hedges the ARM mortgage pool. The FRM position, on the contrary, is hedged by borrowing long-term bonds with maturity $N$. To have the same interest rate risk as the mortgage pool, the mortgage supplier borrows an amount $B_{t}$ so that $B_{t} \times N=D U R_{t} \times F R M_{t} \times C_{t}$, where $C_{t}$ denotes the total value of the mortgage pool at time $t, F R M_{t}$ the fraction of FRM contracts, and $D U R_{t}$ the effective duration of the mortgage pool at time $t$ (Appendix C contains a formal definition of this duration). This guarantees that the outstanding positions in treasury bonds and fixed-rate mortgages have the same interest rate risk. ${ }^{29}$

When bond risk premia increase, holding constant expectations of future short rates, the prepayment probability decreases. For example, expected inflation risk premia are high when expected inflation is high. Investors demand high long-term nominal bond rates to compensate for the inflation risk. With high long-term nominal interest rates come high FRM rates, and this makes prepayment less attractive. Since prepayment becomes less likely, the effective duration of the existing FRM pool increases, and exceeds that of the hedge portfolio. One way to restore the bal-

[^17]ance is to adjust the hedging portfolio $B_{t}$. Alternatively, the mortgage supplier can try to decrease $F R M_{t}$. In other words, the mortgage supplier has the incentive to recommend ARM contracts to new customers. This illustrates that the mortgage lenders may have incentives to recommend ARMs when bond risk premia are high, which is the fact we documented in the data.

## 8 Conclusion

We have shown that the time variation in the risk premium on a long-term nominal bond can explain a large fraction of the variation in the share of newly-originated mortgages that are of the adjustable-rate type. Thinking of fixed-rate mortgages as a short position in long-term bonds and adjustable-rate mortgages as rolling over a short position in short-term bonds implies that fixedrate mortgage holders are paying a nominal bond risk premium, which consists of an expected inflation premium and a real rate premium. In the US, fixed-rate mortgages tend to have long maturities and are therefore very sensitive to inflation risk. We have shown that the inflation risk premium alone can explain more than sixty percent in the variation of the mortgage composition. These results are not sensitive to how expected inflation is measured: we studied both a direct measure from the survey of professional forecasters, and an indirect measure from a VAR model. Other, perhaps more straightforward, term structure variables such as the slope of the yield curve, have much lower explanatory power for the ARM share.

As a robustness check we also study the UK. We use the same VAR model to uncover the term structure variables of interest and find them to be quite different from those in the US. However, the term structure variables are still linked to mortgage choice. In the UK, where FRMs are a lot less prevalent and of much shorter maturity, it is the real rate risk premium that drives the variation in mortgage choice rather than variation in the expected inflation risk premium. This makes sense given that inflation risk is most potent at longer maturities, and the UK simply does not have such long-term mortgage contracts.

In the US, FRMs come with an option to refinance, the prepayment option. To the best of our knowledge, we are the first to value the prepayment option in a VAR model with time-varying expected inflation and real rate risk premia, and time-varying expected inflation and real rate volatility. We show that the FRM with prepayment option continues to have positive exposure to real interest and expected inflation risk, and therefore that higher bond risk premia make it a less attractive compared to the ARM. We find a positive relation between the option value and the ARM share. Expensive prepayment options makes mortgage payments front-loaded, which may make the ARM the more desirable choice for a borrowing-constrained investor.

## References

Acharya, V. V., and J. N. Carpenter (2002): "Corporate Bond Valuation and Hedging with Stochastic Interest Rates and Endogenous Bankruptcy," Review of Financial Studies, 15, 1355-1383.

Ang, A., and G. Bekaert (2005): "The Term Structure of Real Rates and Expected Inflation," Working Paper Columbia University.

Ang, A., G. Bekaert, and M. Wei (2006): "Do Macro Variables, Asset Markets, or Surveys Forecast Inflation Better?," Journal of Monetary Economics, Forthcoming.

Baker, M., R. Greenwood, and J. Wurgler (2003): "The Maturity of Debt Issues and Predictable Variation in Bond Returns," Journal of Financial Economics, 70, 261-291.

Baker, M., R. Taliaferro, and J. Wurgler (2006): "Predicting Returns with Managerial Decision Variables: Is There a Small-sample Bias?," Journal of Finance, 61, 1711-1729.

Boudoukh, J., R. F. Whitelaw, M. Richardson, and R. Stanton (1997): "Pricing MortgageBacked Securities in a Multi-Factor Interest-Rate Environment: A Multivariate Density Approach," Review of Financial Studies, 10, 405-446.

Brandt, M. W., and P. Santa-Clara (2006): "Dynamic Portfolio Selection by Augmenting the Asset Space," The Journal of Finance, 61, 2187-2218.

Brennan, M. J., and Y. Xia (2002): "Dynamic Asset Allocation under Inflation," Journal of Finance, 57, 1201-1238.

Brunnermeier, M., and C. Julliard (2006): "Money Illusion and Housing Frenzies," Working Paper, Princeton University.

Buraschi, A., and A. Jiltsov (2005): "Time-varying inflation risk premia and the expectations hypothesis: A monetary model of the treasury yield curve," Journal of Financial Economics, 75, 429-490.

Butler, A. W., G. Grullon, and J. P. Weston (2006): "Can Managers Successfully Time the Maturity Structure of Their Debt Issues?," Journal of Finance, 61, 1731-1758.

Campbell, J. Y. (2006): "Household Finance," Journal of Finance, 61, 1553-1604.
Campbell, J. Y., Y. L. Chan, and L. Viceira (2003): "A Multivariate Model of Strategic Asset Allocation," Journal of Financial Economics, 67, 41-80.

Campbell, J. Y., and J. Cocco (2003): "Household Risk Management and Optimal Mortgage Choice," Quarterly Journal of Economics, 118, 1449-1494.

Campbell, J. Y., and R. J. Shiller (1991): "Yield Spreads and Interest Rate Movements: A Bird's Eye View," Review of Economic Studies, 58, 495-514.

Campbell, J. Y., and L. Viceira (2001): "Who Should Buy Long-Term Bonds?," American Economic Review, 91, 99-127.

Cochrane, J. H., and M. Piazzesi (2005): "Bond Risk Premia," American Economic Review, 95, 138-160.

Coles, A., and J. Hardt (2000): "Mortgage Markets: Why the US and EU Markets Are So Different," Housing Studies, 15, 775-783.

Dai, Q., and K. J. Singleton (2002): "Expectation Puzzles, Time-varying Risk Premia, and Affine Models of the Term Structure," Journal of Financial Economics, 63, 415-441.

Duffee, G. R. (2002): "Term Premia and Interest Rate Forecasts in Affine Models," Journal of Finance, 57, 405-443.

Dunn, K. B., and J. J. McConnell (1981):"Valuation of Mortgage-Backed Securities," Journal of Finance, 36, 599-617.

Fama, E. F., and K. R. French (1989): "Business Conditions and Expected Returns on Stocks and Bonds," Journal of Financial Economics, 25, 23-49.

Gabaix, X., A. Krishnamurthy, and O. Vigneron (2006): "Limits of Arbitrage: Theory and Evidence from the Mortgage-Backed Securities Market," The Journal of Finance, Forthcoming.

Hansen, L. P., and T. J. Sargent (2004): Recursive Models of Dynamic Linear Economies. Mimeo University of Chicago and New York University, first edn.

Jaffee, D. M. (2006): "Controling the interest rate risk of Fannie Mae and Freddie Mac," Policy Brief 04, Networks Financial Institute, Indiana State University.

Jarrow, R., and Y. Yildirim (2003): "Pricing Treasury Inflation Protected Securities and Related Derivatives Using an HJM Model," Journal of Financial and Quantitative Analysis, 38, 337-358.

Koijen, R. S. J., T. E. Nijman, and B. J. M. Werker (2006): "When Can Life-cycle Investors Benefit from Time-varying Bond Risk Premia?," Working Paper, Tilburg University.

Longstaff, F. A. (2005): "Borrower Credit and the Valuation of Mortgage-Backed Securities," Real Estate Economics, 33, 619661.

Pliska, S. R. (2006): "Optimal Mortgage Refinancing with Endogenous Mortgage Rates: an Intensity Based Equilibrium Approach," Working Paper University of Illinois at Chicago.

Sangvinatsos, A., and J. A. Wachter (2005): "Does the Failure of the Expectations Hypothesis Matter for Long-Term Investors?," Journal of Finance, 60, 179-230.

Schwartz, E. S., and W. N. Torous (1989): "Prepayment and the Valuation of Mortgage-Backed Securities," Journal of Finance, 44, 375-392.

Shen, P., and J. Corning (2001): "Can TIPS Help Identify Long-Term Inflation Expectations," Economic Review, Federal Reserve Bank of Kansas City, Fourth Quarter, 61-87.

Stanton, R. (1995): "Rational Prepayment and the Valuation of Mortgage-Backed Securities," Review of Financial Studies, 8, 677-708.

Tauchen, G., and R. Hussey (1991): "Quadrature-based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," Econometrica, 59, 371-396.
van Hemert, O. (2006): "Lyfe-Cycle Housing and Portfolio Choice with Bond Markets," Working Paper, NYU Stern School of Business.

Vickery, J. (2006): "Interest Rates and Consumer Choice in the Residential Mortgage Market," Working Paper, Federal Reserve Bank of New York.

## A The Prepayment Option in the VAR Model

## A. 1 Estimation of the Prepayment Model

In this appendix, we provide the details on the estimation of the three-dimensional VAR (1)-model in Section 5. The $\operatorname{VAR}(1)$-model in (35) is estimated via OLS per equation, resulting in the following estimates

$$
\hat{\mu}^{\star}=\left(\begin{array}{c}
-0.0283 \\
0.0154 \\
0.0000
\end{array}\right), \hat{\Gamma}^{\star}=\left(\begin{array}{ccc}
0.4935 & 1.8455 & -0.5583 \\
0.0912 & 0.5074 & -0.1049 \\
0.0000 & 1.0000 & 0.0000
\end{array}\right)
$$

Notice that the last element of $\hat{\mu}^{\star}$ is zero and that the last row of $\hat{\Gamma}^{\star}$ equals the second unit vector. This reflects the fact that conditional expectation of the third state variable, realized inflation, equals the second state variable, expected inflation.

Using the residuals $\hat{\eta}_{t}^{\star}$, the parameters $\alpha_{i}^{\star}$ and $\beta_{i}^{\star}$, for $i=1,2,3$ can be estimated consistently via non-linear least squares,

$$
\left(\hat{\alpha}_{i}^{\star}, \hat{\beta}_{i}^{\star}\right)=\arg \min _{\left(\alpha^{\star}, \beta^{\star}\right)} \frac{1}{T} \sum_{t=1}^{T}\left(\hat{\eta}_{t \times 12+12(i)}^{\star 2}-\exp \left(\alpha^{\star}+\beta^{\star} Y_{t \times 12(i)}^{\star}\right)\right)^{2},
$$

and the elements of the correlation matrix $\hat{\Omega}^{\star}$ can be estimated using the standardized residuals:

$$
\left.\operatorname{diag}\left(\sqrt{\exp \left({\hat{\alpha_{1}}}^{\star}+\hat{\beta}_{1}^{\star} Y_{t(1)}^{\star}\right)}, \ldots, \sqrt{\exp \left(\hat{\alpha_{3}}\right.}+\hat{\beta}_{3}{ }^{\star} Y_{t(3)}^{\star}\right)\right)^{-1} \hat{\eta}_{t+12}^{\star}
$$

We find the following values

$$
\begin{array}{rll}
\alpha_{1}^{\star}=-8.8682 & , & \beta_{1}^{\star}=-10.3258 \\
\alpha_{2}^{\star}=-13.1828 & , & \beta_{2}^{\star}=62.8601 \\
\alpha_{3}^{\star}=-9.9898 & , & \beta_{3}^{\star}=22.9146 \tag{46}
\end{array}
$$

and

$$
\Omega^{\star}=\left(\begin{array}{lll}
1.0000 & 0.4894 & 0.3363 \\
0.4894 & 1.0000 & 0.8220 \\
0.3363 & 0.8220 & 1.0000
\end{array}\right)
$$

The only remaining parameters needed are the market prices of risk $\lambda_{0}$ and $\Lambda_{1}$ in (37). They are not pinned down by the VAR. We determine these values by minimizing the squared pricing errors
for the model-implied 5 -year and 10-year nominal bond yield over the period 1985:1-2006:6. The yield of a $\tau$-year bond satifies

$$
\begin{aligned}
y_{t}^{\$}(\tau) & =-\frac{1}{\tau} \log \left(P_{t}(\tau)\right), \text { with } \\
P_{t}(\tau) & =\mathbb{E}_{t} \exp \left(\sum_{s=1}^{\tau} m_{t+s}^{\$}\right) .
\end{aligned}
$$

Specifically, we simulate 100 ten-year antithetic trajectories for the stochastic shocks and minimize the squared pricing error summed over (i) the 258 months in the sample, (ii) the two maturities considered, and (iii) the simulated trajectories. To enhance identification, we assume that the price of realized inflation risk is zero, the price of real interest rate depends on the real interest rate only, and the price of expected inflation risk depends on the real interest and expected inflation rate. This specification of the prices of risk is consistent with Koijen, Nijman, and Werker (2006). This leaves us with five parameters to estimate, for which we find the following values

$$
\lambda_{0}=\left(\begin{array}{c}
-22.45 \\
57.41 \\
0.00
\end{array}\right), \Lambda_{1}=\left(\begin{array}{ccc}
-607.52 & 0.00 & 0.00 \\
775.66 & -386.30 & 0.00 \\
0.000 & 0.00 & 0.00
\end{array}\right) .
$$

## A. 2 Solution Method Valuation Prepayment Option

We solve the model with a $T=10$ year horizon and a time step size of one year. We choose a grid for the state variables in $Y_{t}^{\star}$ and the contract rate $r_{t}$. We use backward induction. At time $t$, for all values on the grid we solve for the lender's valuation, $V_{t}^{p}\left(Y_{t}^{\star}, r_{t}\right)$ and $V_{t}^{n p}\left(Y_{t}^{\star}, r_{t}\right)$, and the zero-profit, prepayment rate $\hat{r}_{t}^{p}\left(Y_{t}^{\star}\right)$. At time 0 we also solve for the zero-profit rate on the FRM without prepayment option $\hat{r}_{0}^{n p}\left(Y_{0}^{\star}\right)$. The conditional expectations in (39) and (40) are approximated using a five-point Gaussian quadrature for the innovations in each of the three state variables (Tauchen and Hussey (1991)). Evaluating the conditional expectation at time- $t$ requires knowing the lender's value at time- $(t+1)$ in-between the chosen grid points. For this we use quadratic interpolation techniques. Similar interpolation techniques are used to determine the zero-profit contract rates, given the lender's value function. Finally, to determine expected real payments for the two contracts we simulate the state variables forward and use the derived prepayment rates to determine the prevailing mortgage rate for the borrower with an FRM with prepayment option.

## B Monthly ARM Share Data in the UK

We observe the ARM share data for the UK only at a quarterly frequency in the 1993-2001 period. Since all models are specified at monthly frequency, we want to estimate the data points in between. We employ a Kalman filter together with a specification for the ARM dynamics that is motivated by the US ARM share dynamics. The method is discussed in Hansen and Sargent (2004), Chapter 9.13. The goal is to improve upon linear interpolation by postulating reasonable dynamics for the ARM share dynamics. Linear interpolation may be sub-optimal because the average over month 1 to 3 may have very little to with the average over month 4 to 6 . Linearly interpolation introduces dependencies that are not present in the underlying data. The current approach explicitly incorporates the aggregation.

## B. 1 Kalman Filter

Denote the monthly fraction of ARM mortgages at time $t$ by $x_{t}$. We assume that $x_{t}$ follows an AR(1) model:

$$
\begin{equation*}
x_{t+1}=a+b x_{t}+\epsilon_{t+1}, \epsilon_{t+1} \sim N\left(0, \sigma^{2}\right) . \tag{47}
\end{equation*}
$$

The distributional assumption is required to be able to use the Kalman filter. To justify this time series process, we estimate an $\mathrm{AR}(1)$ on the ARM share in the $U S$. The $R^{2}$ of this $\mathrm{AR}(1)$ process is $93 \%$ over the sample 1985:1-2005:12. The autoregressive parameter equals $\hat{b}=0.96$.

For the Kalman filter, we need a state transition equation and an observation equation. Towards this end, we introduce the state vector $y_{t}=\left(x_{t}, x_{t-1}, x_{t-2}\right)^{\prime}$. The state transition equation for the state vector is given by:

$$
\begin{align*}
y_{t} & =\left(\begin{array}{l}
a \\
0 \\
0
\end{array}\right)+\left(\begin{array}{lll}
b & 0 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0
\end{array}\right) y_{t-1}+\left(\begin{array}{c}
\epsilon_{t} \\
0 \\
0
\end{array}\right)  \tag{48}\\
& \equiv c+D y_{t-1}+u_{t} . \tag{49}
\end{align*}
$$

Since we observe the average mortgage choice over three months, we lag the system for two addition periods:

$$
\begin{align*}
y_{t} & =\left(I+D+D^{2}\right) c+D^{3} y_{t-3}+u_{t}+D u_{t-1}+D^{2} u_{t-2}  \tag{50}\\
& \equiv e+F y_{t-3}+\xi_{t}, \tag{51}
\end{align*}
$$

which results in the transition equation at a quarterly frequency. We observe the average mortgage choice over a quarter, i.e.:

$$
\begin{equation*}
z_{t}=\iota_{3 \times 1}^{\prime} y_{t} / 3 \tag{52}
\end{equation*}
$$

which constitutes the observation equation. Finally, we initialize the Kalman filter assuming that the vector of starting values is drawn from the unconditional distribution.

## B. 2 Empirical Results

We estimates the coefficients $a, b$, and $\sigma$ by maximum likelihood. We find $\hat{a}=2.9828$ (1.7566), $\hat{b}=0.9457$ ( 0.0315 ), and $\sigma=5.0174$ ( 0.4751 ), where the numbers in parentheses are standard errors, computed from the outer product gradient. The solid line in Figure 11 shows the resulting monthly ARM share for the UK.

To verify that the Kalman filter does work properly, we verify that the average ARM shares over each three month-period coincides with the quarterly data. We also compare the monthly ARM share that arises from the Kalman filter to the one that arises from a Kalman smoother, and find them to be very similar. Finally, we verify that the monthly data obtained from the Kalman filter are close to the actual monthly data over the 2002.1-2006.6 period, for which we have the monthly data. The red circles in Figure 11 correspond to the actual monthly data; the solid line cuts through these data points.

## C Details on the Supply-Side Argument

The FRM pool at time $t$ is characterized by the effective duration, which is defined as:

$$
\begin{equation*}
D U R_{t}=\frac{\sum_{j=1}^{J_{t}} \sum_{n=1}^{n_{t}^{j}} p_{t}^{j}(n) c_{t}^{j} n}{\sum_{j=1}^{J_{t}} \sum_{n=1}^{n_{t}^{j}} p_{t}^{j}(n) c_{t}^{j}}, \tag{53}
\end{equation*}
$$

where $j=1, \ldots, J_{t}$ indicates the individual FRM contracts in the pool, $n_{t}^{j}$ the remaining number of payments to be made by the FRM borrower $j$, and $p_{t}^{j}(n)$ the prepayment probability probability at time $t$ that household $j$ will prepay $n$ periods from now given a current contract rate $c_{t}^{j}$. This definition of the effective duration assumes that the contract is terminated in case of prepayment. If the household chooses to refinance with the same mortgage provider, it is considered a new borrower with a new contract rate and new prepayment probabilities. Likewise, the mortgage supplier only hedges the outstanding contracts.

Table 1: Univariate Regression Analysis of the ARM Share for the US.
This table reports slope coefficients, Newey-West t-statistics (12 lags), and $R^{2}$ statistics for univariate regressions of the ARM share on a constant and one regressor, reported in the first column. The regressors are the following variables. The $\tau$-year inflation risk premium $\phi_{t}^{x}(\tau)$, the $\tau$-year real rate risk premium $\phi_{t}^{y}(\tau)$, the conditional variance of expected inflation $V_{t}^{x}$, and the conditional variance in the real rate $V_{t}^{y}$. The $\tau$-year nominal bond risk premium $\phi_{t}^{\$}(\tau)$ is the sum of the $\tau$-year real interest rate and $\tau$-year expected inflation premium. The $\tau$-year nominal yield is given by $y_{t}^{\$}(\tau)$. The $\tau$-one-year yield spread is $y_{t}^{\$}(\tau)-y_{t}^{\$}(1) . y_{t}^{\$}(F R M)-y_{t}^{\$}(A R M)$ denotes the difference between the FRM rate $y_{t}^{\$}(F R M)$, and the ARM rate $y_{t}^{\$}(A R M)$. The regressor is lagged by one period, relative to the ARM share. The left panel is for the longest sample from 1985.1-2006.6; the right panel is for the sample over which we have data for both the 5 -year and the 10-year treasury inflation-protected security: 1997.7-2006.6. All variables have been multiplied by 100 .

| Sample | 1985.1-2006.6 |  |  | 1997.7-2006.6 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | slope | t-stat | $R^{2}$ | slope | t-stat | $R^{2}$ |
| $\phi_{t}^{x}(5)$ | 16.77 | 5.91 | 44.12 | 12.76 | 8.49 | 63.52 |
| $\phi_{t}^{y}(5)$ | 3.68 | 0.87 | 2.19 | -8.88 | $-3.53$ | 28.51 |
| $\phi_{t}^{x}(10)$ | 17.19 | 4.91 | 44.59 | 14.46 | 6.77 | 56.24 |
| $\phi_{t}^{y}(10)$ | 6.88 | 2.20 | 12.21 | -8.59 | -3.01 | 25.49 |
| $V_{t}^{x}$ | 169.20 | 2.87 | 25.67 | -17.19 | $-0.25$ | 0.18 |
| $V_{t}^{y}$ | -28.91 | -2.92 | 22.84 | 3.31 | 0.34 | 0.76 |
| $\phi_{t}^{\phi}(5) \equiv \phi_{t}^{y}(5)+\phi_{t}^{x}(5)$ | 9.99 | 4.16 | 32.21 | 6.47 | 1.63 | 11.45 |
| $\phi_{t}^{\$}(10) \equiv \phi_{t}^{y}(10)+\phi_{t}^{x}(10)$ | 8.04 | 3.91 | 35.13 | 3.62 | 0.76 | 3.33 |
| $y_{t}^{8}(5)-y_{t}^{8}(1)$ | 0.60 | 0.21 | 0.11 | 0.80 | 0.35 | 0.64 |
| $y_{t}^{8}(10)-y_{t}^{\text {¢ }}(1)$ | -0.59 | -0.32 | 0.23 | 0.34 | 0.22 | 0.27 |
| $y_{t}^{¢}(F R M)-y_{t}^{\$}(A R M)$ | 16.03 | 3.17 | 35.31 | 0.36 | 0.05 | 0.01 |
| $y_{t}^{\$}(F R M)-y_{t}^{\$}(A R M)$ orth. | 18.25 | 3.85 | 41.23 | 0.21 | 0.03 | 0.00 |
| $y_{t}^{\text {¢ }}$ (1) | 3.47 | 3.27 | 28.73 | -0.41 | -0.34 | 0.73 |
| $y_{t}^{\text {¢ }}$ (5) | 4.46 | 3.76 | 37.76 | -0.58 | -0.29 | 0.57 |
| $y_{t}^{\text {s }}$ (10) | 4.98 | 3.85 | 39.26 | -1.27 | -0.41 | 1.25 |
| $y_{t}^{\text {¢ }}(A R M)$ | 4.08 | 3.28 | 22.33 | -3.45 | -1.03 | 7.73 |
| $y_{t}^{¢}(F R M)$ | 4.28 | 3.71 | 32.87 | -2.47 | -0.82 | 5.45 |

## Table 2: Multivariate Regression Analysis for ARM Share in US.

This table reports slope coefficients, Newey-West t-statistics, and $R^{2}$ statistics for multivariate regressions of the ARM share on a constant and the variables listed in the first column. The regressors are lagged by one period, relative to the ARM share. The righthand side variables are demeaned so that the constant gives the average ARM share in the sample. The regressors are the ten-year inflation risk premium $\phi_{t}^{x}(10)$, the ten-year real rate risk premium $\phi_{t}^{y}(10)$, the conditional variance of expected inflation $V_{t}^{x}$, and the conditional variance in the real rate $V_{t}^{y}$. The ten-one-year yield spread $y_{t}^{\$}(10)-y_{t}^{\$}(1)$ and the prepayment option value $\hat{r}_{t}^{p}-\hat{r}_{t}^{n p}$ are orthogonalized to the other four term structure variables. We run an auxiliary regression of these variables on the first four variables and include the regression residual as a fifth explanatory variable. The left panel is for the longest sample from 1985.1-2006.5; the right panel is for the sample over which we have reliable real yield data: 1997.7-2006.6. Newey-West t-statistics (12 lags) are in parentheses.

| $\mathbf{1 9 8 5 . 1 - 2 0 0 6 . 6}$ | $\mathbf{1 9 9 7 . 7 - 2 0 0 6 . 6}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ | $(6)$ | $(7)$ | $(8)$ |
|  | 28.66 | 28.66 | 28.66 | 28.66 | 21.61 | 21.61 | 21.61 | 21.61 |
|  | $(14.72)$ | $(16.39)$ | $(16.53)$ | $(17.09)$ | $(14.24)$ | $(16.10)$ | $(16.08)$ | $(16.50)$ |
| $\phi_{t}^{x}(10)$ | 15.97 | 14.27 | 14.27 | 14.27 | 13.38 | 15.13 | 15.13 | 15.13 |
|  | $(4.81)$ | $(5.15)$ | $(5.14)$ | $(5.14)$ | $(5.86)$ | $(5.09)$ | $(5.10)$ | $(5.29)$ |
| $\phi_{t}^{y}(10)$ | 2.72 | -0.96 | -0.96 | -0.96 | -1.62 | -1.32 | -1.32 | -1.32 |
|  | $(1.15)$ | $(-0.36)$ | $(-0.34)$ | $(-0.34)$ | $(-0.53)$ | $(-0.46)$ | $(-0.46)$ | $(-0.53)$ |
| $V_{t}^{x}$ |  | 43.89 | 43.89 | 43.89 |  | 32.50 | 32.50 | 32.50 |
|  |  | $(0.73)$ | $(0.76)$ | $(0.76)$ |  | $(0.62)$ | $(0.63)$ | $(0.65)$ |
| $V_{t}^{y}$ |  | -15.11 | -15.11 | -15.11 |  | -6.19 | -6.19 | -6.19 |
|  |  | $(-2.08)$ | $(-2.03)$ | $(-2.03)$ |  | $(-0.89)$ | $(-0.90)$ | $(-0.84)$ |
| $y_{t}^{\$}(10)-y_{t}^{\$}(1)$ |  |  | 3.89 |  |  |  | -0.43 |  |
| $\hat{r}_{t}^{p}-\hat{r}_{t}^{n p}$ |  |  | $(0.65)$ |  |  |  | $(-0.08)$ |  |
| $R^{2}$ | 46.27 | 53.93 | 54.30 | 55.92 | 56.82 | 60.98 | 60.99 | 62.24 |

## Table 3: Alternative ARM Share Measures in US.

This table reports slope coefficients, Newey-West t-statistics (12 lags) in parentheses, and $R^{2}$ statistics for multivariate regressions of the ARM share on a constant and the variables listed in the first column. The right-hand side variables are demeaned so that the constant gives the average ARM share in the sample. The regressors are the ten-year inflation risk premium $\phi_{t}^{x}(10)$, the ten-year real rate risk premium $\phi_{t}^{y}(10)$, the conditional variance of expected inflation $V_{t}^{x}$, and the conditional variance in the real rate $V_{t}^{y}$. The first column is our benchmark measure; the other three ARM share measures are defined in Section 4.1. The regressors are lagged by one period, relative to the ARM share. The sample is 1997.7-2006.6.

| RHS variables | $A R M^{1}$ | $A R M^{2}$ | $A R M^{3}$ | $A R M^{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| constant | 21.61 | 18.10 | 10.92 | 21.54 |
|  | $(16.10)$ | $(16.25)$ | $(15.95)$ | $(15.76)$ |
| $\phi_{t}^{x}(10)$ | 15.13 | 12.88 | 6.18 | 14.98 |
|  | $(5.09)$ | $(5.27)$ | $(4.50)$ | $(5.12)$ |
| $\phi_{t}^{y}(10)$ | -1.32 | -0.39 | 0.91 | -2.93 |
|  | $(-0.46)$ | $(-0.16)$ | $(0.59)$ | $(-0.97)$ |
| $V_{t}^{x}$ | 32.50 | 23.45 | 5.72 | 36.86 |
|  | $(0.62)$ | $(0.56)$ | $(0.23)$ | $(0.61)$ |
| $V_{t}^{y}$ | -6.19 | -3.55 | -5.40 | -4.91 |
|  | $(-0.89)$ | $(-0.64)$ | $(-2.02)$ | $(-0.62)$ |
| $R^{2}$ | 60.98 | 60.51 | 41.59 | 63.98 |

Table 4: Multivariate Regression Analysis for ARM Share in UK.

This table reports slope coefficients, Newey-West t-statistics, and $R^{2}$ statistics for multivariate regressions of the ARM share on a constant and the variables listed in the first column. The regressors are lagged by one period, relative to the ARM share, and are demeaned. The regressors are the ten-year inflation risk premium $\phi_{t}^{x}(10)$, the ten-year real rate risk premium $\phi_{t}^{y}(10)$, the conditional variance of expected inflation $V_{t}^{x}$, and the conditional variance in the real rate $V_{t}^{y}$. We also consider the ten-year minus one-year yield spread, orthogonalized to the other four term structure variables. The left panel is for the longest sample from 1993.1-2006.6. It uses the monthly ARM share obtained through the Kalman filter, see Appendix B. The right panel is for the sample over which we have monthly data for the ARM share: 2002.1-2006.6. Newey-West t-statistics (12 lags) are in parentheses.

| Regressors | 1993.1-2006.6 |  |  | 2002.1-2006.6 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(1)$ | $(2)$ | $(3)$ | $(5)$ | $(6)$ | $(7)$ |
|  | 57.53 | 57.53 | 57.53 | 52.48 | 52.48 | 52.48 |
|  | $(20.13)$ | $(20.35)$ | $(20.21)$ | $(20.77)$ | $(51.56)$ | $(55.92)$ |
| $\phi_{t}^{x}(10)$ | 2.05 | 0.64 | -0.77 | 12.81 | 59.96 | 67.40 |
|  | $(0.40)$ | $(0.10)$ | $(-0.10)$ | $(1.21)$ | $(4.39)$ | $(4.63)$ |
| $\phi_{t}^{y}(10)$ | 22.40 | 20.59 | 17.15 | 45.62 | 98.07 | 87.50 |
|  | $(2.48)$ | $(2.07)$ | $(1.29)$ | $(5.40)$ | $(11.01)$ | $(8.48)$ |
| $V_{t}^{x}$ |  | 0.79 | -16.23 |  | 595.96 | 506.82 |
|  |  | $(0.02)$ | $(-0.31)$ |  | $(7.50)$ | $(6.56)$ |
| $V_{t}^{y}$ |  | 15.36 | 41.69 |  | -74.72 | 91.58 |
|  |  | $(0.69)$ | $(0.87)$ |  | $(-2.37)$ | $(1.61)$ |
| $y_{t}^{\$}(10)-y_{t}^{\$}(1)$ |  |  | 21.02 |  |  | 86.65 |
|  |  |  | $(0.58)$ |  |  | $(4.27)$ |
| $R^{2}$ | 22.55 | 23.90 | 24.48 | 61.86 | 76.87 | 78.59 |

Figure 1: The Share of Adjustable Rate Mortgages in the US.

The figure plots the fraction of all newly originated mortgages that are of the adjustable-rate type. The complementary fraction are fixed-rate mortgages. The data are from the Federal Housing Financing Board and are based on the Monthly Interest Rate Survey sent out to mortgage lenders. It covers all property types: newly constructed homes, and existing homes. ARMs include hybrid mortgages, which may have an initial fixed-interest rate payment period of up to ten years.


Figure 2: The Inflation Risk Premium and the ARM Share in the US.

The figure plots the fraction of all mortgages that are of the adjustable-rate type against the left axis, and the inflation risk premium against the right axis. The inflation risk premium is computed as the difference between the 5 -year nominal bond yield, the 5 -year real bond yield and the expected inflation. The nominal and real 5-year bond yields are from McCulloch and start in January 1997. The inflation expectation is the median long-term (10-year) inflation forecast from the Survey of Professional Forecasters (SPF).


Figure 3: VAR Estimation for the US.

The figure plots long-term risk premia. The 5 -year risk premia for expected inflation risk and for real rate risk are plotted in the left panel. The 10-year risk premia, formed by subtracting the 10-year real rate yield data from the VAR-implied 10-year real rate.



Five-Year Long-Term Expectations


Ten-Year Long-Term Expectations


Figure 4: Inflation and Real Rate Risk Premia in the US.
The figure plots long-term risk premia. The five-year risk premia for expected inflation risk and for real rate risk are plotted in the left panel. The ten-year risk premia, formed by subtracting the ten-year real rate yield data from the VAR-implied five-year real rate.



Figure 5: Backfilling Inflation and Real Rate Risk Premia for the US.

The figure plots ten-year risk premia. The solid black line is the ten-year nominal risk premium. It is computed as the difference between the observed nominal ten-year yield $\left\{y_{t}^{\$}(120)\right\}$ and the average expected nominal ten-year yield. These expectations are readily obtained from the VAR for the entire 1985.1-2006.6 period. The circled purple line is the projected ten-year real rate risk premium, $\phi_{t}^{y}(10)$, formed as the product of the state variables $z$ and the regression coefficients in equation (34). These loadings are estimated on the 1997.7-2006.6 sub-sample. The circled turquoise line is the expected inflation risk premium, $\phi_{t}^{x}(10)$. It is formed as the difference between the nominal risk premium and the inflation risk premium according to equation (17). For the 1997.7-2006.6 sample, we overlay the actual risk premia (reported earlier in Figure 4) on the projected risk premia.


Figure 6: The ARM Share for the US in More Detail.

The figure plots the fraction of all newly originated mortgages that are of the adjustable-rate type between 1992.1 and 2006.6. The first three series are from the Federal Housing Financing Board and are based on the Monthly Interest Rate Survey sent out to mortgage lenders. The first series includes all hybrid mortgages. The second series excludes hybrids with an initial fixed-rate period of more than five years, and the third series excludes hybrids with an initial fixed-rate period of more than three years. The last series is from Freddie Mac and is based on the Primary Mortgage Market Survey. Like the first measure, it contains all ARM originations.


Figure 7: Price Sensitivity to Changes in the Real Rate and Expected Inflation for the US.

The figure plots the price sensitivities of the FRM contract with and without prepayment to the real interest rate $y$ (top panel) and expected inflation $x$ (bottom panel). The mortgage values are determined within the model of Section 5.2. The analogous fixed-income securities are a regular bond (FRM without prepayment) and a callable bond (FRM with prepayment).


Figure 8: Value of the Prepayment Option for the US.

The figure plots the value of the prepayment option along with the real interest rate and expected inflation between 1985.1 and 2006.6. The prepayment option is defined as the mortgage rate differential between an FRM with and without prepayment. The mortgage rates are determined within the model of Section 5.


Figure 9: VAR Estimation for the UK.

The figure plots long-term risk premia. The 5 -year risk premia for expected inflation risk and for real rate risk are plotted in the left panel. The 10-year risk premia, formed by subtracting the 10-year real rate yield data from the VAR-implied 10-year real rate.


Figure 10: Inflation and Real Rate Risk Premia in the UK.

The figure plots long-term risk premia. The 5 -year risk premia for expected inflation risk and for real rate risk are plotted in the left panel. The 10-year risk premia, formed by subtracting the 10-year real rate yield data from the VAR-implied 10-year real rate.

Five-Year Risk Premia


5-Year Risk Premia from 1997.7


Ten-Year Risk Premia


10-Year Risk Premia from 1997.7


Figure 11: The Share of Adjustable Rate Mortgages in the UK.

The figure plots the fraction of all newly originated mortgages that are of the adjustable-rate type. The complementary fraction are fixed-rate mortgages. The data are from the Council of Mortgage Lenders (sheet ML5) and are based on the Survey of Mortgage Lenders before April 2005 and based on Product Sales Data reported to the CML after April 2005. It covers both house purchases and remortgages. Adjustable rate mortgages are the sum of standard variable rate contracts (SVR), discounted (variable rate) mortgages and trackers. Fixed rate mortgages are the sum of fixed contracts and capped contracts. The red-circled line plots the monthly data, which are only available from January 2002 onwards. The solid blue line is a monthly time-series, which we generate from quarterly temporally aggregated data that start in 1993.I. See Appendix B for details on this procedure.



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[^1]:    ${ }^{1}$ Brunnermeier and Julliard (2006) argue that money illusion is prevalent in the housing market and can explain a large part of the recent run-up in house prices.

[^2]:    ${ }^{2}$ There exists a large literature on prepayment models which either assume optimal prepayment (e.g., Dunn and McConnell (1981) and Pliska (2006)) or empirical prepayment behavior (e.g., Schwartz and Torous (1989) and Boudoukh, Whitelaw, Richardson, and Stanton (1997)). We consider a rational prepayment model and abstract from refinancing costs. Longstaff (2005) and Stanton (1995) model refinancing costs explicitly.
    ${ }^{3}$ Campbell and Viceira (2001), Brennan and Xia (2002), and van Hemert (2006) derive the optimal portfolio strategy for long-term investors in the presence of stochastic real interest rates and inflation, but these papers

[^3]:    ${ }^{5}$ For ease of exposition we do not impose that the FRM interest payments are equal over time, only that they are known at time 0 . Constant mortgage payments would be the harmonic mean of all forward rates of maturities $1, \cdots, T$. We comment further on this assumption in Section 2.4.

[^4]:    ${ }^{6}$ Note that an unconstrained FRM investor could hedge inflation risk by borrowing cash and investing in longterm nominal bonds. This would effectively transform the FRM into an ARM so that the investor might as well opt for the ARM to begin with.

[^5]:    ${ }^{7}$ The VAR offers an alternative way to form inflation expectations to the professional analyst survey data, used in the introduction. In addition, it allows us to form real rate risk premia.
    ${ }^{8}$ We have also estimated the model on quarterly data and found very similar results.

[^6]:    ${ }^{9}$ The nominal yield data is available at http://www.federalreserve.gov/pubs/feds/2006.
    ${ }^{10}$ The inflation data is available at http://www.bls.gov.

[^7]:    ${ }^{11}$ As a robustness check, we also considered a VAR(2)-model. Below we redo the ARM share regressions for the term structure variables arising from that model.
    ${ }^{12}$ The real yield data is available at http://www.econ.ohio-state.edu/jhm/ts/ts.html. As a robustness check, we perform our analysis with real yield data generated by the term structure model of Ang and Bekaert (2005). We show below that our main conclusions are unaffected.

[^8]:    ${ }^{13}$ Major lenders are asked to report the terms and conditions on all conventional, single-family, fully-amortizing, purchase-money loans closed the last five working days of the month. The data thus excludes FHA-insured and VA-guaranteed mortgages, refinancing loans, and balloon loans. The data for our last sample month, June 2006, is based on 21,801 reported loans from 74 lenders, representing savings associations, mortgage companies, commercial banks, and mutual savings banks. The data is weighted to reflect the shares of mortgage lending by lender size and lender type as reported in the latest release of the Federal Reserve Board's Home Mortgage Disclosure Act data.
    ${ }^{14}$ We are very grateful to James Vickery for making these detailed data available to us.
    ${ }^{15}$ This survey goes out to 125 lenders. The share is constructed based on the dollar volume of conventional mortgage originations within the 1-unit Freddie Mac loan limit as reported under the Home Mortgage Disclosure

[^9]:    Act (HMDA) for 2004.
    ${ }^{16}$ We do not use the first six months of 1997, in which only a five-year TIPS was available. As a robustness check, we have also repeated all regressions starting in 1999.1, because the TIPS market may have suffered from liquidity problems early on (see Shen and Corning (2001), Jarrow and Yildirim (2003), and Ang and Bekaert (2005)). The regression results starting in 1999 are very similar to the ones reported here.
    ${ }^{17}$ Equation (12) calls for the average of the 1-period- to T-period-ahead conditional variances instead. Because these long-term average variances increase are positively correlated with the 1-period-ahead conditional variance,

[^10]:    the sign on $V_{t}^{x}$ and $V_{t}^{y}$ should be the same. The reason is that the term-structure of volatilities is upward sloping. It converges to the unconditional variance, which is higher than the 1-period-ahead conditional variance.
    ${ }^{18}$ We use the projection for the entire 1985-2006 sample.
    ${ }^{19}$ We also studied a regression with the two risk premia and the conditional variance of the nominal interest rate. The regression coefficient is negative and significant in the full sample, and negative but not significant in the later sample. Finally, we also studied a regression which adds the conditional covariance between the real rate and expected inflation to the four determinants in column. The explanatory power is the same. We do not report these results because the signs on the variance terms loose their independent interpretation. The reason is that there is no variation in the conditional covariance that is not already in the conditional variances because the correlation between $x$ and $y$ is assumed to be constant.

[^11]:    ${ }^{20}$ We use the effective rate data from the Federal Housing Financing Board, Table 23. The effective rate adjusts the contractual rate for the discounted value of initial fees and charges. The FRM-ARM spreads with and without fees have a correlation of .998.
    ${ }^{21}$ The correlation between the FRM-ARM spread and the ten-one-year government bond yield spread is only $32 \%$ over the full sample, and $11 \%$ over the 1997-2006 sample. This spread also captures the value of the prepayment option (see below), as well as the lenders' profit margin differential on the FRM and ARM contracts.

[^12]:    ${ }^{22}$ This analogy is exact for an interest-only mortgage. When the mortgage balance is paid off during the contractual period (amortizing), the loan can be thought of as a portfolio of bonds with maturities equal to the dates on which the downpayments occur. Acharya and Carpenter (2002) discuss the valuation of callable, defaultable bonds.

[^13]:    ${ }^{23}$ Under this assumption, prepayment behavior is fully driven by the dynamics of the term structure of interest rates. We do not consider sub-optimal prepayment behavior and the premium associated with it (see Gabaix, Krishnamurthy, and Vigneron (2006)).

[^14]:    ${ }^{24}$ This computation takes into account all effects of inflation on intermediate and terminal payments. We do not make any of the approximations of Section 2.2 in computing the real payment streams.

[^15]:    ${ }^{25}$ That same relationship between the ARM share and the option value holds when we use the ten-year risk premia that are generated inside the three-dimensional VAR. These risk premia are computed inside the model, based on the dynamics of $Y^{\star}$ and the market prices of risk. They do not use real yield data, unlike the risk premia used in Section 4. All signs are the same, and the regression $R^{2}$ using these model-generated premia is $41.8 \%$.

[^16]:    ${ }^{26}$ The Miles report (2004) contains a detailed overview of the UK mortgage market, and its recommendation is to deepen the long-term fixed-rate mortgage market. Coles and Hardt (2000) discuss differences between US and European mortgage markets.
    ${ }^{27}$ The data are monthly from 2002.1 onwards, and quarterly from 1993.1 onwards. The quarterly data are averages of the three months of the quarter. We use a Kalman filtering procedure, suggested by Hansen and Sargent (2004), to undo the temporal aggregation, and to obtain a monthly time-series that starts in 1993.1. Appendix B contains the details.

[^17]:    ${ }^{28}$ A large literature documents suboptimally slow prepayment decisions by households, see for instance Schwartz and Torous (1989), Stanton (1995), and Boudoukh, Whitelaw, Richardson, and Stanton (1997).
    ${ }^{29}$ A large fraction of FRM debt, in the conforming loan market, is purchased by Freddie Mac and Fannie Mae, who exchange the mortgages for mortgage-backed securities (MBS). Many depository institutions do not resell all MBS to the secondary markets, and end up holding a substantial fraction on their balance sheet. The Aggregated Thrift Financial Report, available from the FDIC, suggests that the US financial sector had direct mortgage holdings of $\$ 1$ trillion and MBS holdings of $\$ 175$ billion in June 2006. In addition, Freddie Mac and Fannie Mae held a combined total of $\$ 1.4$ trillion of mortgage-backed securities on their own books at the end of 2005 . Some doubt whether the associated interest rate risk is appropriately hedged (Jaffee (2006)).

