# USING *ORDER STATISTICS* TO ESTIMATE PROBABILITIES OF PURCHASE FOR CONSUMER GOODS

# (TOWARD A COMPREHENSIVE MODEL OF THE IMPACT OF ADVERTISING ON DEMAND)

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## Abstract

Much work has been done recently to develop models of individual brand choice. This work has been especially fruitful in the area of "attribute investigation" (e.g., conjoint analysis) for the purpose of uncovering an "ideal" set of product attributes for a given product on a customer-by-customer basis. These techniques have been employed in several strategic areas of marketing, including product specification, pricing, service prioritization, brand image/equity, and satisfaction/loyalty. Similarly, some effort has been made with the issue of optimal advertising strategy. This paper considers the advertising effectiveness function within the context of other interrelated variables such as consumer preference (brand choice) for a brand vis-à-vis its competitors. The model suggests, among other things, that under certain reasonable conditions, the advertising response function may not be "diminishing marginal returns" or 'S-shaped' as is usually assumed, but instead will increase up to a point and then decline. The model also explicitly considers the advertising expenditures of competing brands, as well as intrinsic "liking" for them. The consideration of competitive activity, in the present study may yield a more complete model of advertising response than is found elsewhere. Finally the model provides direction for strategic purposes in its ability to illustrate in a fairly straightforward and graphical sense how advertising and promotion "work" in much the

same way that demand and supply curves illustrate how the various economic inputs work.

**Introduction**

Marketing strategy is effective when it can influence the consumer decision-making process. For most consumer product firms, communications strategy probably accounts for a major portion of the marketing expense. Therefore, this activity is very closely monitored and evaluated (Lamons 1997 and Zufryden 1989).

Advertising is widely accepted as a critical marketing tool by practioners and researchers. (Helgesen1996). While research studies have shown some positive correlation between advertising expenditures and consumer purchase (Ehrenberg, Barnard, and Scriven 1997), it is difficult to identify a direct relationship between advertising expenditures and brand sales (Broadbent 1988). It is not surprising then to find that marketing managers often establish budgets using somewhat trivial models or, more often, some arbitrary rules of thumb. This practice continues despite the advocacy of optimization methods drawn from management science or pooled wisdom that might be applicable to establishing advertising budgets (Piercy 1987). Research has shown that in some instances the use of the Dorfman-Steiner rule by which a firm optimizes its position, can be used to determine an optimal advertising budget (McMeekin 1988/1989). However, important questions regarding the advertising budget remain, "Are we spending the right amount, too much, or too little (Turner 1989)?" How does the advertising process actually "work?" What are the steps "actual and conceptual" that lead from "exposure" to "involvement?"

**Literature review**

The issue of advertising effectiveness is a complex one. Studies have shown that response to advertising depends on a myriad of factors. Some of the factors that may affect the advertising response include nature and strength of competing advertising, the environment in which measurement takes place, and what one chooses to measure (Stewart 1988).

Various studies have demonstrated the effects of advertising on certain performance measures such as attitude shifts and purchase intentions (Axelrod 1963; Zajonc 1980, 1982; Wells 1985; Wansink and Ray 1992; and Dube, Chattopadhyay & Letarte 1996). Though attitude shift may be a necessary condition in the final purchase of a product, by itself, it may not thoroughly explain the variations in purchase by consumers. Similarly, purchase intentions may not always translate into purchase. Wansink and Ray (1992) in their attempt to study advertising impact found that the purchase intention measures are less appropriate when advertising campaigns are developed to encourage consumption.

Some research studies have focused on advertising effectiveness under special circumstances. For example, Dung (1987) attempted to examine the implications of a firm's advertising decisions under conditions of random sales. The analysis suggests that the effect of risk-taking attitudes and that of the random sales response function on the firm's advertising budget are difficult to assess. Wansink and Ray (1996) used a "schema congruity framework" that focused on how advertising can best encourage consumers to use a mature brand in a new situation. Their study suggests that situation comparison ads favorably affect usage attitudes but have no advantage over product comparison ads in enhancing a person's ability to recall the target brand in the target situation.

There are many divergent opinions within the industry regarding the advertising process. Jones and Blair (1996) in their research of the effects of advertising determined that increased advertising alone does not improve results and copy testing alone cannot predict advertising effects on sales. On the other hand, using BEHAVIORSCAN test results, Abraham and Lodish (1990) found that 49% of the electronically controlled tests of increased advertising weight resulted in statistically significant sales increase at the 80% level. Lodish and colleagues (1995) in a seminal study of split cable TV advertising experiments found that, while many of the variables investigated did not differentiate across the different advertising treatments with regard to sales effect, a few others were effective in doing so. Using the same methodology and applying it to the snack food industry advertising, Riskey (1997) was able to replicate the earlier findings of Lodish

and his colleagues. Grunert (1996) in his study of strategic processes in advertising, found that variables such as a consumer's personal relevance lead to less stable effects of advertising on evaluation and attitude, whereas, familiarity leads to more stable effects.

**Objectives**

It would appear from the examination of the literature that, while some models have proposed and often described the implementation of individual brand choices or "attribute" techniques with respect to specific marketing strategies, none as yet appears to have fully applied the techniques in a formal and systematic way with regard to the important question of (optimal) advertising strategy. An attempt is made here to establish a theoretical relationship that would explain under certain conditions the effect of advertising on consumer purchase under realistic competitive environments. The specific objectives of this research paper are:

- To work towards the development of a model that under certain conditions is able to predict consumer brand choice based on brand attribute preferences/liking and advertising expenditures.
- To postulate an alternate advertising response function.
- To provide some general rules for evaluating advertising and promotional strategies in the same sense that demand and supply curves are used for understanding implications of macroeconomic strategies.
- To attempt to illustrate the interrelationships between advertising effort and product liking with a preliminary empirical study.

**The Model**

The model presented here is designed to complement recent models of individual brand choice, and particularly those which have been fruitful in the area of "attribute investigation" for purposes of uncovering "ideal" brands on a customer-by-customer basis (Wittink & Cattin 1989 and Srinivasan & Park 1997). It also may complement the work reported by David Stewart (1989) regarding the advertising response function. In his research, Stewart found that the response function could either represent diminishing

marginal returns to advertising exposures or be the well-known S-shaped curve. In developing the present advertising response model, the advertising expenditures of competing brands are also included in the equation.

**Model assumptions**

1. The model is developed for a class of moderately expensive durable products (small appliances, low-end personal computers, cell phones etc.). The model as it is presented here does not accommodate other types of products such as expensive high-ticket items or frequently purchased package goods. However, it should be noted that some thought has been given to the latter categories and it is strongly suspected at this point that this might be achievable with some modification of the model.

2. For a group of customers who do not currently own a product in the class, and are planning to buy one.

If a consumer does not already own a product within the category of interest and he/she plans to purchase such a product, his/her probability of purchase for a particular product must be related to how much he/she "likes" that product. Suppose that a consumer has been made aware of a product offering (say, product q) and has investigated its attributes. Suppose further, that employing one of the available attribute measurement techniques, we have determined how much he/she likes this product's set of attributes in relation to an array of all "available" products. Let this be given by Lq, where

Lq = percentage of products liked *less* than q

But rationality dictates that the probability that the consumer purchases this product, $P_r(B_q)$ depends on the probability that he/she will *not* be exposed to a better-liked product before actually making the purchase.

The probability of being exposed to a better-liked product than the one being offered (product q) depends among other things, upon the liking "score" of the product being

offered ($L_q$). If we assume randomness of exposure within the appropriate (uniform) distribution, this can be expressed as:

(1)     $p(L) = 1, 0 \le L \le 1$

For example, the probability that a consumer will be "exposed" prior to purchase of a better-liked product than one that is liked better than 90% of available ones  (i.e., whose "liking" is $L_q = .90$) is:

$$\int_{.90}^{1.00} dL = L \Big|_{.90}^{1.00} = .10.$$

But this assumes that the consumer is exposed to only one product other than the one in question. In reality, the consumer could be exposed a number of times to various other products (n) as well as to the one being offered, though it is hardly likely that he would be exposed to all possible products: And the probability that he would like the one being offered best (and consequently purchase it) will be affected (downward) in direct relation to the magnitude of 'n'. (The more products he is exposed to prior to purchase, the less likely that a particular product will be liked best). The question then is: Given that the consumer is exposed to 'n' products, what is the probability that the highest ranking product of those 'n' products -- in terms of intrinsic product preference -- has a liking score *less* than $L_q$?


This is clearly a problem involving "Order Statistics" (Hogg and Craig, 1978).

We have from eq. (1), $p(L) = 1, 0 \le L \le 1$

$\qquad\qquad\qquad\qquad = 0$ elsewhere.

Hence…

(2)     $P(L) = L, 0 \le L \le 1$

…is the cumulative distribution function of the pdf $p(L)$.

Now let $Y_1 < Y_2 < \ldots < Y_n$ denote the order statistics (liking scores) of a random sample of 'n' (the set of 'n' products to which the consumer is exposed) from the above distribution. We want to determine $P_r(Y_n < L_q)$.

Now if $g_k(y_k)$ is the marginal probability density function of the $k^{th}$ order statistic, then it can be shown that…

(3)     $g_k(y_k) = n!/[(k-1)!(n-k)!]\{[P(y_k)^{k-1}[1-P(y_k)]^{n-k}\,p(y_k)]\}$

Since for our problem, $k = n$ (the concern is the $n^{th}$ order statistic), we have…

(4)     $g_n(y_n) = n!/[(n-1)!(n-n)!]\{[P(y_n)^{n-1}[1-P(y_n)]^{n-n}\,p(y_n)]\}$

…which when we substitute the appropriate values and simplify, becomes:

(5)     $g_n(y_n) = n[P(y_n)]^{n-1}p(y_n)$

$$= n(L)^{n-1}(1)$$

$$= n(L)^{n-1}$$

And…

(5a)    $P_r(Y_n < L_q) = n \int_0^{L_q} L^{n-1} dL$

$$= \left[ n(L)^n / n \Big|_0^{L_q} \right] = (L_q)^n$$

Now ignoring all other marketing influences, except product preference for the moment, $(L_q)^n$ can be considered a rough measure of probability of purchase for a product 'q' which has a liking score of $L_q$. For example, if a product 'q' received a liking score of say, $L_q = .75$ (i.e., it is liked better than 75% of all brands in the product category), then after

exposure to product 'q' and three other products (i.e., let n = 3), the probability that the consumer will buy product 'q' -- in this simplified situation is .42. That is, $P_r(B_q) = .75^3$.

**Definition of an "exposure"**

The question now arises as to what exactly is an "exposure?" Strictly speaking, an exposure does not simply mean that the consumer has observed the product or some promotion for it. To be "exposed," he must know (retain) enough about its characteristics to formulate an opinion with respect to how much he likes it vis-à-vis competitive alternatives. Precisely, how many impressions are necessary to trigger such a "showdown" comparison, and how long a period it may be retained is subject to debate (Gibson 1996, D'Souza & Rao 1995, Hitchon & Thorson 1995, McDonald 1994 and Krugman 1972). For some product categories, e.g., food products, "exposure" might even necessitate actual trial of the product[*]. However, for practical purposes, and for many product categories, such awareness can usually be considered synonymous with "exposure" at least in a proportional sense, particularly after the consumer has been "exposed" more than once.

**Effect of Promotional Expenditure on $P_r(B_q)$**

Thus far, it has been assumed that exposure to products within a category is the result of a random pattern, and that the consumer has already been exposed to the product of interest (product q). But in reality, this is not what should be expected. The probability that a particular consumer will be exposed to product 'q' depends importantly on the promotional effort put behind it. The greater the promotional effort for a product, the higher the likelihood of exposure to it. Or putting it another way, (assuming relatively equal efficiencies in making media buys) if product 'q' accounts for say 20% of all promotional expenditures in the category, one would expect it to account for approximately 20% of all category exposures as well. Further, the probability distribution in question will more likely be a discrete one (rather than a continuous case such as that

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[*] Exposure obviously also depends on other factors, like distribution; but one might fairly expect that all of these will be highly related and hence for purposes of this analysis, not measurably distortive.

exercised earlier) since there exists a limited (and countable) number of brands in any category.

To handle the latter situation, let us assume that we have data available on promotional expenditures for each brand in the category, and let …

$f(x)$ = share of promotional expenditures for product x, and

$F(x)$ = cumulative share of promotional expenditures for brand x and all brands ranked below it in terms of intrinsic product preference.

Now suppose that over a period of time, a consumer is exposed (in the sense given earlier) to "n" products (or more precisely, product exposures) in the category (exposures don't have to be unduplicated). As before, the object is to determine the probability that the highest ranking product (in terms of preference) of these n products is liked less than "our" product, 'q'.

Let Y= the brand with the highest preference (liking) score of all brands to which the consumer is exposed, after n exposures.

To determine the distribution function of y, say ...

(6)    $G(y) = P(Y \le y)$

... we note that $Y \le y$ if at least n of the exposures have preference scores <u>less than or equal to y</u>.  Each trial (exposure) has probability of "success" of $F(y)$, and we want at least n successes in n trials.

If we assume that the 'n' exposures are independent of one another – i.e., the probability of being exposed to product 'x' on the $r^{th}$ exposure is essentially unaffected by whether or not the consumer was exposed to 'x' on the $(r-1)^{th}$ exposure, etc., then:

(7)    $G(y) = \binom{n}{n} F(y)^n [1 - F(y)]^0 = F(y)^n$

$F(y)^n$ is analogous to $(L_q)^n$ that was derived previously, except that here we incorporate promotional effort, whereas previously we assumed complete randomness of exposure. If $f(x)$ were a continuous distribution, the pdf of y would be ...

(8)    $g(y) = G'(y) = nF(y)^{n-1}f(y),$

…and this would be a good approximation for situations involving many brands, and where $F(x) - F(x-1)$ was small. For cases where a discrete distribution is deemed appropriate, we note that

(9)    $g(y) = G(y) - G(y-1)$

$\qquad = F(y)^n - F(y-1)^n$

Hence, for a situation where a consumer does not already own a product in the category in question, but plans to purchase such a product, his probability of purchase for a particular product q after 'n' exposures to products in the category can be expressed as:

(10)    $P_r(B_q) = F(Q)^n - F(Q-1)^n$

…where Q is the rank order of product 'q' in terms of this consumer's preference for it relative to all other products in the category, and F(Q) is the cumulative share of promotion expenditures for product 'q' and all brands ranked below it.


**An Example to Demonstrate the concept**

Suppose there are only four products in the category of interest (x = 1,2,3, 4), and promotional expenditures for these four products ranked in (reverse) order of intrinsic product preference are as follows:

| x | Promo. Expenditures |
|---|---|
| 1 | $900 |
| 2 | 25 |
| 3 | 25 |
| 4 | 25 |
| Total | $975 |

(The above should be read: product #4 is ranked above product #3 in terms of intrinsic preference, #3 is ranked above #2 and so on).


Suppose further, that in the (extreme) case given above, the total category expenditure resulted in a total exposure rate of n = 9.75. (As any media analyst will attest, the

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translation of expenditures into "exposures" – and vice versa – can be accomplished with relative ease in most cases.) Thus using the formulae uncovered earlier, we would have:

| $x$ | Promo. $ | $n_x$ | $f(x)$ | $F(x)$ | $P_r(B_x)$ |
|-----|----------|-------|--------|--------|------------|
| 1 | $900 | 9.00 | .9231 | .9231 | .458 |
| 2 | 25 | 0.25 | .0256 | .9487 | .140[*] |
| 3 | 25 | 0.25 | .0256 | .9743 | .177 |
| 4 | 25 | 0.25 | .0256 | 1.000 | .225 |
| Totals | $975 | 9.75 | 1.000 | | 1.000 |

**Implications**

Share depends not only on product preference per se, but also on exposure rate, which depends in turn on promotional expenditures. Putting it another way, perhaps for the first time, we can clearly depict and *evaluate quantitatively*, what has often been expressed intuitively as the "real' relationship between promotion and sales; namely, that the former provides the opportunity for gaining exposure for the goods in question. Note for our oversimplified (yet arguably meaningful) example, that even though product #1 is liked the least of the four brands, its share is anticipated to be about 46% -- and this by virtue of the fact that it dominates promotional spending in the category and consequently has by far the greatest opportunity for being exposed to prospective consumers. But the above also implies that preference for a product also plays an extremely important role in product purchase. This is illustrated by the fact that product #4 – the best liked product – has an anticipated share of 22% even though it accounts for only 2.5% of promotional expenditures.

Moreover, while these data appear to be in line with what one might have expected intuitively, their real value lies in the fact that we now have the means for analytically quantifying these insights, and, as will be seen shortly, thereby deriving optimal *strategies*. (Note: In the example above, the relationship between promotional

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- $P_r(B_2) = F(2)^n - F(1)^n = (.9487)^{9.75} - (.9231)^{9.75} = .140$

expenditures and exposures ($n_x$) was determined somewhat arbitrarily. If, e.g., $\sum n_x$ were to be say, .975 instead of 9.75, $P_r(B_x)$ for brand 1 would increase to .916 and for 2 it would decrease to .020. The point being that advertising response is not only affected by the proportionate expenditure rate but also by the total amount allocated to advertising by all brands. That is, in addition to intrinsic product liking, and share of the exposure rate, the probability of a brand being purchased is dependent on the magnitude of the total category expenditure).

**Exposure "Wear Out" Rate**

Obviously, the material presented in an advertisement or commercial does not stay indelibly imprinted on one's memory indefinitely; nor does it suddenly and completely disappear from one's consciousness. It tends to wear out gradually over time.

The actual empirical studies required to ascertain such a factor are fairly straightforward; they involve a determination of how long a respondent retains the significant aspects of an ad at various points following exposure (Duke & Carlson, 1993 and Friestad & Thorson, 1993). But it is important to recognize and incorporate into the analysis the appropriate *pattern* of exposure "wear out." This concept is elaborated in Appendix 2, and its result is noted as $n^{/}$ (effective number of exposures) in the analysis that follows.

**Optimal Level of Promotional Expenditures**

Consider the following elements:

- $R_i$ = return on advertising expenditure for the $i^{th}$ consumer,
- $\bar{u}$ = average number of units of product in category of interest bought per person during time period in question,
- $Z_x$ = profit on one unit of brand x, not counting advertising (promotional) expenditures,
- $E_i$ = advertising (promotional) expenditures allotted to the $i_{th}$ consumer = $(n_x)(k)$, where $n_x$ = number of exposures for product x, per person, per day; and k = cost per exposure = (CPP) = CPM/1000.

Then (if time period is say, one year) …

(11)    $R_i = P_r(B_x)_i (\overline{u})(Z_x) - 365(n_x)(k)$

$= [F(x)^n - F(x-1)^n](\overline{u})(Z_x) - 365(n_x)(k),$ and letting …

$n_x$ = number of exposures per person for brand x

$n_1$ = number of exposures per person for all brands ranked *below* brand x
in terms of intrinsic preference (i.e., liked less than x), and

$n_2$ = number of exposures per person for all brands ranked *above* brand x in terms
of preference,

…then recognizing that $n = n_1 + n_2 + n_x$ and reformulating and performing appropriate
substitutions…

(12)   $F(x)^n = [(n_1 + n_x)/n]^n$

(13)   $F(x-1)^n = (n_1/n)^n$

(14)    $R_i = \left[ \left( \dfrac{n_1 + n_x}{n} \right)^n - (n_1/n)^n \right](\overline{u})(Z_x) - 365(n_x)(k)$

Taking the derivative of Ri with respect to $n_x$, gives the following result:

$$dR_i/dn_x = 0$$

$$\Downarrow$$

(15)   $\left( \dfrac{n_1 + n_x}{n} \right)^n \left[ \dfrac{n_2}{n_1 + n_x} + \ln\left( \dfrac{n_1 + n_x}{n} \right) \right] - (n_1/n)^n [\ln(n_1/n) - 1] = 365k / \overline{u}(Z_x)$

And this gives the condition that will be satisfied when the optimal values of $n_i$ are found.


**An Example**

To see how the model might perform within a reasonably practicable structure and to
illustrate a number of the model's (in some cases perhaps, novel) implications a
numerical example is presented utilizing the following hypothetical data:

1.  Assume it is anticipated that the brand in question will generate sales of 11 million
    units for the coming year. Of these 11 million units, 9 million are anticipated for
    replacement sales, and 2 million units are expected to be sold to those who do not
    already own such a product. Further, assume that the total applicable population –

that portion of the population that is effectively reached by advertising -- is 150 million, and that there are an estimated 27.4 million potential buyers among those who do not already own the product within the appropriate population.

2. Also assume total advertising expenditure (E) for all brands in the category is estimated to be $300 MM, $k = \$0.005$, and $n = 1.10$ exposures per day, per person for all brands in category.

3. It is determined that on the average, an exposure is retained for approximately one week (7 days). Hence, $\theta = 1.27$, and …

$$n' = n/(1-e^{-1/\theta}) = 1.10/(1-e^{-1/1.27}) = 2.02 = \text{effective exposure rate for all brands.}$$

4. The brand of interest -- brand q -- spent approximately $100MM on promotion last year, and if this pattern of spending continues…

$$n_q \cong (100/300)(n) = .37$$

…the brand will achieve .37 exposures per day on the average.

5. Profit per unit for the brand of interest – brand q – is $500 (i.e., $Z_q = 500$). Hence,

$$365k / \bar{u} Z_q = (365)(.005)/(.073)(500) \cong .05$$


Now suppose that we consider a single consumer – say consumer #i. And suppose that his product preference pattern is such that $n_1 = .60$, and $n_2 = .13$. That is, the exposure rate for all brands ranked below brand q in preference for this consumer is .60, and the exposure for all brands ranked above brand q is .13. Then ..


$$n_1' = n_1/[1-e^{-(1/1.27)}] = 1.10$$
$$n_2' = n_2/[1-e^{-(1/1.27)}] = 0.24$$
$$n_q' = n_q/[1-e^{-(1/1.27)}] = \underline{0.68}$$
$$n' = 2.02$$

Now if this spending pattern is actually undertaken for the coming year, the anticipated results for this consumer will be:[*]

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[*] To convert this singular response function into a bona fide sales function it is only necessary to sum over the segment of customers, say $C_q$ in number, whose preference ordering of brand q is similar to the one we are considering; then sum over all segments.

$$P_r(B_q)_i = \left(\frac{n_1' + n_q'}{n'}\right)^n - (n_1'/n')^{n'}$$

$$= (1.78/2.02)^{2.02} - (1.10/2.02)^{2.02} = 0.48$$

$$R_i = (\bar{u})\, P_r(B_q)_i (Z_q) - (n_q)(365)(.005)$$

$$= (.073)(.48)(500) - (.37)(365)(.005) = \$16.84$$

Although the above result appears profitable enough, it may not be optimal. Hence, let us examine brand q's strategy for optimality. To do this, we will apply the formulae uncovered earlier against two possible (though hardly all-embracing) situations. First, when only brand q adjusts its advertising expenditure for the coming year, and it is assumed that all other brands maintain theirs (in an absolute sense), and secondly, when all brands attempt to maintain their *proportionate* share of advertising expenditures.

a) <u>Assume that all brands other than brand q maintain spending levels</u> so that no matter what brand q decides to do with respect to $n_q$, $n_1$ and $n_2$ will be maintained at 1.10 and 0.24 respectively.

If we substitute 1.10 and 0.24 for $n_1'$ and $n_2'$ into the optimizing equation (15), and vary the values of $n_q'$, we find the value of $n_q'$ which gives $365k/\bar{u}\, Z_q$ is $n_q' \cong 2$.

Hence under the conditions assumed above,

$$n_q'^* = 2 \Rightarrow n_q^*\ 1.09$$

That is, optimality will be reached for this consumer with 1.09 exposures per day or $(365)(1.09) = 398$ exposures (commercial impressions or messages) per year. In addition,

$$P_r(B_q)_i = \left(\frac{n_1' + n_q'}{n'}\right)^{n'} - (n_1'/n')^{n'}$$

$$= (3.10/3.34)^{3.34} - (1.10/3.34)^{3.34}$$

$$= .775 \text{ and}$$

$$R_i = (\bar{u})P_r(B_q)(Z_q) - (n_q)(365)(.005)$$

$$= (.073)(.775)(\$500) - (1.09)(365)(\$.005)$$

$$= \$25.57$$

Note for this consumer, while it is recommended that the exposure be increased from .37 to (an optimal) 1.09, it is also anticipated that the return from these expenditures will be increased from \$16.84 to \$25.57. Again, while this result is for one consumer, with the proper segmentation and summation, an anticipated *sales* outcome may be obtained.

(b) <u>Assume other brands in category do react to brand q in the sense that they attempt to maintain proportionality insofar as advertising expenditures are concerned[*].</u>

If all other brands maintain proportionality, then:

$$n_1{}' = pn_q{}', \text{ and}$$

$$n_2{}' = tn_q{}'; \ p,t \text{ constants.}$$

Now

(16)
$$R_i = \left[ \left( \frac{n_q{}'(p+1)}{n_q{}'(p+t+1)} \right)^{n_q(p+t+1)} - \left( \frac{n_q{}' p}{n_q{}'(p+t+1)} \right)^{n_q(p+t+1)} \right] \bar{u} Z_q - 365 k n_q$$

$$= \left[ \left( \frac{p+1}{p+t+1} \right)^{n_q(p+t+1)} - \left( \frac{p}{p+t+1} \right)^{n_q(p+t+1)} \right] \bar{u} Z_q - 365 k n_q$$

Finally,

$$dR_i/dn_q = 0$$

$$\Downarrow$$

(17)
$$p+t+1 \{ 1n\left( \frac{p+1}{p+t+1} \right) \left[ \left( \frac{p+1}{p+t+1} \right)^{n'_q(p+t+1)} \right] - 1n\left( \frac{p}{p+t+1} \right) \left[ \left( \frac{p}{p+t+1} \right)^{n'_q(p+t+1)} \right] \} = \frac{365 k}{\bar{u} Z_q}$$

For the present example, $p = 1.62$ and $t = .35$, and if we substitute these quantities into the above optimizing formula, the optimal value for $n'_q$ for this consumer becomes.

$$n'_q* \cong 1.00 \Rightarrow n_q* = .54$$

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[*] As suggested earlier, there are of course other ways in which competition can react to brand q's spending, but, given some knowledge of intended strategy (or perhaps using game theory approaches) one may incorporate this into the model in similar fashion.

Further,

$$(18) \quad P_r(Bq)_i^* = \left[ \left( \frac{p+1}{p+t+1} \right)^{n_q'^* (p+t+1)} - \left( \frac{p}{p+t+1} \right)^{n_q'^* (p+t+1)} \right]$$

$$= (2.62/2.97)^{(1)(2.97)} - (1.62/2.97)^{(1)(2.97)}$$

$$= .52, \text{ and}$$

$$R_i^* = (.073)(.52)(\$500) - (.54)(365)(\$.005) \qquad \text{from eq. 11}$$

$$= \$17.99$$

Note that by maintaining proportionality, the competition has in effect forced brand q to accept a lower $P_r(B_q)^*$ and $R^*$ -- though it is still greater than the non-optimal return of $16.84 that was anticipated by simply spending at last year's rate.

## General Implications

These results have general implications that deserve some amplification. From (10) we have the probability that the customer buys brand q given a total of n exposures to products in the category is proportional to $P_r(B_q|n) = F(Q)^n - F(Q-1)^n$.

## A Sales Function

Summing over the segment of customers, say $C_q$ in number, whose preference ordering of brands is similar to the one we are considering, *sales* are proportional to

$$(19) \quad S_q(q|n) = C_q[F(Q)^n - F(Q-1)^n]$$

Finally, summing over *segments*,

Total sales $= \sum S_q(q|n) \cdot P$, where P is a constant reflecting the probability that a purchase of any brand in the category is actually made. For each segment, purchases may not be made because no brand that a customer is exposed to satisfies him or her (for some reason such as price, appropriateness, quality, etc.) or because the need for the purchase has now been reduced, etc.

For ease of analysis let $\rho_q = n_q/n$, $\rho_1 = n_1/n$, and $\rho_2$ be the share of spending by brand q, and the brands that are preferred less or more than it respectively. Hence,

$$(\rho_1 + \rho_q)^n = \left(\frac{n_1 + n_q}{n}\right)^n$$

Again, assuming equal efficiencies in making media buys, the $\rho_i$'s also corresponds to the share of exposures that are delivered. Thus it has been shown that the probability that the customer buys brand q given a total of n exposures is proportional to

(20) $P(q|n) = (\rho_1 + \rho_q)^n - \rho_1^n$

Again, summing over the segment of customers, $C_q$ in number, whose preference ordering of brands is similar to the one we are considering, *sales* are proportionate to

(21) $S_q(q|n) = C_q[(\rho_1 + \rho_q)^n - \rho_1^n]$.


**Product Positioning**

With respect to implications of the model (21), consider a homogeneous segment of consumers – that is, one where all consumers in the segment have roughly the same configuration of brand preference *relative to brand q*.

First, the importance of product positioning is illustrated: Consider two situations for brand q -- when customers would consider it the best and worst brands. In each case suppose the budget and share of expenditure remains the same, say $\rho_q$.

When q is perceived to be the best brand, the probability of purchase is …

(22) $P(q|n) = \alpha(1 - \rho_q + \rho_q)^n - \rho_1^n = 1 - \rho_1^n$

…where $\alpha$ denotes "is proportional to."

When q is the worst brand,

(23) $P(q|n) = \alpha\rho_q^n$.

Clearly, the first case is substantially greater than the last <u>unless</u> $\rho_q = 1$; i.e., the other brands do not advertise. The illustration highlights the importance of <u>product positioning.</u> Sales cannot be equal in the two situations unless competition does not advertise – an unlikely situation. Likewise, the above emphasizes the problem inherent in attempting to use advertising to *change* people's attitudes (vis-à-vis exploiting existing preferences). While it may be recognized that such a strategy is difficult and costly at best, the above suggests that in certain situations –where a brand's share of advertising does not account for a disproportionately great share of the total – that it may be dangerous as well.

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## Alternate Advertising Response Function

Now consider the budgetary implications of the model.

- Suppose first, that total product class expenditure remains constant but q's share increases. S(q|n) increases at an increasing rate as q captures the entire market, because:

$$\frac{dS(q\,|\,n)}{d\rho_q} = C_q n(\rho_1 + \rho_q)^{n-1} \text{ and}$$

$$\frac{d^2 S}{d\rho_q^{\,2}} = C_q n(n-1)(\rho_1 + \rho_q)^{n-2}$$

are both strictly positive for $0 \le \rho_q \le 1$

- Now, to investigate the implications of the model in perhaps a more likely scenario, when total budgets of brands perceived superior and inferior to brand q are held constant, while the budget for brand q increases,
  Let $b_1$, $b_2$, and $b_q$ be the respective budgets. Then

$$\rho_1 = \frac{b_1}{b_1 + b_2 + b_q} \quad \text{and} \quad \rho_q = \frac{b_q}{b_1 + b_2 + b_q}$$

$n = \eta(b_1+b_2+b_q)$, where $\eta$ is a constant that translates dollar budgets into exposures. Thus,

$$(24)\; S(q\,|\,n) = PC_q \left[ \left( \frac{b_1 + b_q}{b_1 + b_2 + b_q} \right)^{\eta(b_1+b_2+b_q)} - \left( \frac{b_1}{b_1 + b_2 + b_q} \right)^{\eta(b_1+b_2+b_q)} \right]$$

It is evident from 29 that as $b_q$ increases, $b_1$ $b_2$ remaining constant, S(q|n) also increases from 0 for $b_q = 0$ to $PC_q$ when $b_q \to \infty$

In order to derive the shape of this advertising response, the first two derivatives of S(q|n) need to be taken with respect to $b_q$. They are rather complicated and will not be reproduced here. However, one can show that,

$$\frac{dS(q\,|\,n)}{db_q} > 0 \text{ for } b_q = 0; \to 0 \text{ as } b_q \to \infty$$

$$\frac{d^2S(q\mid n)}{db^2{}_q} \le 0 \quad \text{when } b_q = 0, \text{ depending on the relative values of } b_1 \text{ and } b_2.$$

Specifically, if $\ldots b_2 = 0$ and $b_1 > 0$, $\quad \dfrac{d^2S(q\mid n)}{db_q{}^2}\mid\left(b_q = 0\right) < 0$

$\ldots$ if $\quad b_1 = 0$ and $b_2 > 0$, $\quad \dfrac{d^2S(q\mid n)}{db_q{}^2}\mid\left(b_q = 0\right) = 0$

But for several combinations of values of $b_1$ and $b_2$, e.g., $b_1 = b_2$, the second derivative is positive.

**Thus, within this "competitive-constant" scenario, the response to advertising follows a family of more or less generally accepted response curves, from S-shaped to strictly concave, all leveling off as the budget for brand q increases indefinitely.** Moreover, the model shows that advertising response for a brand depends importantly on its preference rank and the expenditure by competing brands; in all cases where the brand's expenditure increases, while the other brands spending remains constant, sales reach an asymptotic level. However, the portion of the response function at lower levels of expenditure can be S-shaped or not depending on competitive budgets.

However, when speculating on possible response functions, there is one other reasonably likely circumstance that should be considered that has some interesting implications.

- Consider the budgetary implications of the model when total advertising expenditure for the product class increases, but the various brands maintain their shares; that is, n grows but $\rho_1$, $\rho_2$, and $\rho_3$ remain constant. What is being suggested is that competition will react to increases in brand q's advertising expenditures, and in fact, will seek to maintain proportionality.

$$(25) \quad S^{/}(q\mid n) = \frac{dS}{dn} = \{(\rho_1 + \rho_q)^n ln(\rho_1 + \rho_q) - \rho_1{}^n ln\rho_1\}C_q P$$
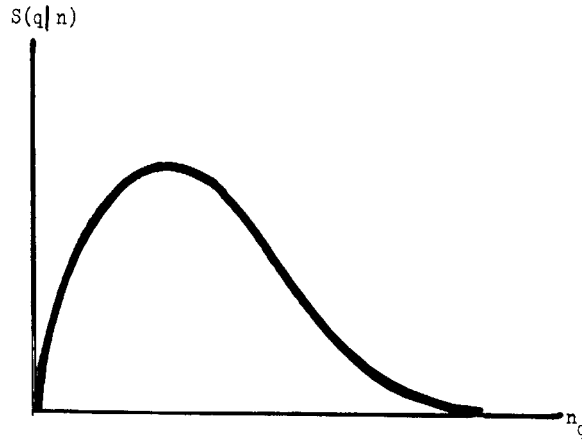
Thus, $S'(q\mid n) > 0$ when $n = 0$, and $\to 0$ as $n \to \infty$.

Also, $S(q\mid n) = 0$ when $n = 0$, and $\to 0$ as $n \to \infty$.

Finally, it can be shown that $S^{//}(q\mid n) < 0$ when $S^{/}(q\mid n) = 0$.

$n_q{}^{/}$

$n_q{}^{/*}$

**Thus, within a "competitive proportionate" environment, S (q|n) will increase as n increases up to a point, and then decline, eventually tending to zero.** See Figure 2.

Figure 2



Sales as a function of advertising, assuming advertising shares are maintained constant.

An example of this phenomenon might be the case of the automobile industry in the US. In the sixties General Motors (GM) had a sizable market share, in excess of 70%, but the entry of Japanese and European cars with more apparent desirable attributes and the awareness of American consumers to this fact (in large part due to promotion of these attributes) resulted in GM's share plummeting in spite of a significant increase in their own advertising expenditures over that period.

Since for many product categories, there exists a sort of "kinked" competitive advertising relationship in the sense that if you increase expenditures, others will follow, whereas if you decrease expenditures, you do so alone, this could become a rather plausible scenario. Within this setting, it may well be that increasing expenditures beyond a certain point can actually damage or destroy a brand not perceived to be the best. Or, putting it another way, increasing expenditures in a product class as a whole can have a deleterious effect on a brand not perceived to be the best, (at least to a sufficiently large segment) <u>even though it maintains its share of spending.</u>

To maintain or increase share in such a situation, either product quality (liking score) has to be improved, or a highly segmented strategy should be followed. If a segmentation strategy is followed, the brand should be advertised to groups likely to rate it highly, making sure that the copy concentrates on those brand attributes that are responsible for the high liking score. In fact, profound justification (indeed, obligation) for a strategy of "market segmentation" is given here.

Putting it another way, in certain situations heavy advertising alone cannot overcome product deficiencies – it can only augment intrinsic product appeal. In other situations, however, it can.

**Thus the model reveals important implications for managing brands other than the best one. The second-best brand as well as other inferior brands cannot compete by enhancing promotion spending alone, as they will be driven out of the market because of the low rank-order in preferences. To compete, the model argues, each brand has to *create* a unique dimension on which it is the perceived leader. Therefore, for product managers to succeed, they need to differentiate on *anything,* even by introducing negative attributes like nuisance and hassle** (Chu, Gerstner, and Hess 1995**).**

While it is granted that this insight is not necessarily new, the introduction of some mathematical justification as well as quantification has been attempted here.

**DISCUSSION**

As discussed earlier, the model presented is couched in terms of a prospective customer for an *inexpensive durable product*. However, as also previously noted, it can be extended to other product types and other categories of consumers. Two such cases (requiring little or no modification) follow:

1. **A new brand**. When setting the budget for a new product, it is a relatively simple matter to obtain the preference ranking based on product attributes (or conceptual

ones) for the new brand compared to its competitors on a segment by segment basis. Then, given estimates of competitive spending, and of "C" (customers whose preference ordering of brands is similar to the one we are considering) one can determine the impact of alternative spending patterns on a segment by segment basis.

2.  **A repositioned brand**. The argument is much the same here as in (1) although "C" is likely to be greater. In addition, the model can be extended to cover those who already own a product in the category in question. In this case, probabilities *can be developed* on whether or not any product in the category will be purchased. Also, such things as effect of a trade-in allowance on probability of purchase can be effectively studied.

With respect to ascertaining brand rank, it should be noted that whatever technique is used for determining brand rank, it is important to realize that an understanding of how *product attributes* affect preference will provide much greater meaning to the analysis.

It will, for example, ensure that a ranking is obtained for all possible products in the category, even those with which the consumer may be currently unfamiliar but with which he might become familiar and buy if he is exposed to it. In addition, a data gathering scheme involving appropriate attributes will provide a means for evaluating purchase probabilities and promotional strategies for products which do not as yet exist, but which might be under consideration for future introduction. Finally, an attribute analysis is much more sensitive than a simple brand ranking. It not only provides insight into how brands are ranked per se, but also on which attributes are responsible for liking (or disliking) the brand in question. Consequently, it indicates what should be said and to whom with respect to a brand "exposure".  (A brand may lose sales while increasing "n" as indicated earlier because the "wrong" attributes have been stressed to the "wrong" people.)

**CONCLUSION**

A model has been presented that relates the individual's sales response to advertising to his/her ranking of the various brands in the product class. The model is found to be mathematical logical, has passed the test of both a numerical example and an experimental study (the experimental study is presented as Appendix 1).

It is shown that by aggregating the individual response function, an aggregated sales response to advertising can be developed. This sales response function can have a variety of shapes, including S-shaped, and one where brand sales actually decline as a function of brand advertising. The parameters of the response is shown to be determined by the total product class spending, brand ranking, and expenditure by brands ranked above and below the brand in question, and brand spending itself.

The model suggests that unless a brand can be positioned so that it is favorably ranked by at least some segment, advertising is likely not only to be ineffective, but could actually have a deleterious effect on sales of the product. While methods are not presented for changing the perception of a brand's image and thereby its ranking, it might be speculated that price, as an easily manipulated variable, can serve to change consumer perceptions of a brand in some instances. Thus, in a market where a brand is perceived to be inferior, frequent promotions, and advertisements of low price might be an effective policy.

While emphasis has been placed on the descriptive value of the model, implicit in all of the above, and perhaps quite consequential, is the fact that within the framework of the model as it is now formulated is the means for deriving strategies regarding the determination of the optimum size of promotional budgets. That is, for finding the optimum level of $b_q$. However, practical use of the model for deriving actual budgets (predictive purposes) will gain greater credulity upon empirical validation (beyond that given here in Appendix1) of its structure and estimation of its parameters.

An essential value of the model, as it is presented here, is the direction it provides for strategic purposes. In particular, it possesses the ability to illustrate in a fairly straightforward and graphical sense, how advertising and promotion "work" - in much the same way that demand and supply curves illustrate how the various economic inputs work. Hence while the model, as presented, may require some fairly elaborate undertakings to "predict" with precise specificity, it does give some important (apparently heretofore-unpublicized) "rules" for developing competitive advertising strategy. Additionally, it seems to provide mathematical support for some existing strategies that may have had only intuitive exegesis up till now. This is because it is a model whose essential "character" is not based simply on judgment, but on a sequence of what appears to reflect the rational steps in the consumer buying process. It is one, moreover, that takes into account not only the impact of promotional effort per se in the selling process (i.e., that of gaining impressions and hence "exposures" for the product in question), but also of the interrelation of this with intrinsic liking for the product's attributes vis-à-vis competitors' brands. Finally, in determining promotional response, the model takes into account not only the level of promotional expenditures for a given brand, but also that of competition.

**Limitations**

From a "predictive" standpoint this model has several limitations. As indicated earlier, the model presented above makes the assumption that one exposure to a brand's advertising is all that is required to produce a response. This assumption may be unduly simplistic. To repeat, the number of exposures necessary to trigger a response would vary from product class to product class, and might very well be an increasing function of total advertising exposures for the product class—that is, more exposures may be necessary to cut through the "noise level." *However, the model structure can easily be extended to the*

$$P(q \mid n) = a(\rho_1 + \rho_q)^n - \{\rho_1^n + \binom{n}{1}\rho_1^{n-1}\rho_q \ldots + \binom{n}{\varepsilon-1}\rho_1^{n-\varepsilon+1}\rho_q^{\varepsilon-1}\}$$

*case when at least $\varepsilon$ exposures are necessary.*

Another assumption that the model makes is that the customers receive exactly 'n'

exposures to total advertising for the product class. When we consider a homogeneous segment of customers, the total number of exposures received by any customer must be a random variable so that we can define $P_n$ to be the probability that exactly 'n' exposures are received. Incorporating this modification, the probability of buying q becomes…

$$P(q \mid n) = aP_n \sum P(q \mid n)a \sum_n P_n \left[ (\rho_1 + \rho_q)^n - \rho_1^n \right]$$

*This modification does not change any of the qualitative observations about the shape of the response function discussed in the last section.*

While the distribution of 'n' is a consequence of the media plans employed by the competing brands, and can be estimated based on these plans, the value of "C" can only be established empirically.

In contrast to other models that consider competition indirectly, for example through the use of indices of copy effectiveness  (Batra & Holbrook 1990 and Wansink & Ray 1992), or media efficiency (Aaker & Brown 1972, Dyer, Forman & Mustafa 1992, and Zufryden & Pedrick 1993) *this model considers the effect of competition directly*. This leads to another possible limitation: copy effectiveness conceivably could contribute to some degree to the potential consumer preference for a brand[*]. Similarly, media efficiency will lead a brand to direct its advertising at the segment that ranks it highest. Since the analysis is at an individual/segment level, one needs to determine the optimum media strategy on an individual segment level as well.

Probably this analysis' major limitation for prediction purposes is its lack of conclusive real world "empirical validation." [**]On the other hand, for purposes of implementation, there would seem to be no question that the model's parameters are readily accessible and available. Segmentation of the type recommended here has been accomplished via

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[*] "Copy effectiveness," a rather nebulous term at best, would include in this instance, the proper choice of product attributes to be tested in the message.
[**] Though an empirical study has been undertaken (Appendix 1) and appears to support the model's inferences, "conclusive validation" would require a much more ambitious enterprise conducted over what would amount to a prohibitively lengthy time frame.

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generally accepted techniques (some already noted) and does not appear to pose any significant roadblock to the model's implementation. Brand advertising weight is certainly available (medium by medium) via such sources as A.C. Nielsen, Starch INRA

generally accepted techniques (some already noted) and does not appear to pose any significant roadblock to the model's implementation. Brand advertising weight is certainly available (medium by medium) via such sources as A.C. Nielsen, Starch INRA Hooper, and Simmons Market Research. Bureau. Additionally, there are other sources that provide data that can be used to determine those attributes that are being promoted brand-by-brand (Haley 1985; Turk and Katz 1992). While it has been admitted that exposure can come from a variety of sources in addition to advertising and promotion, it has also been suggested that this factor would not seem to materially negate the basic structure of the model, nor is its essential implications.

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**Testing the model**

While the model as described above would appear to be intuitively and mathematically logical, a preliminary test of the model was conducted employing a convenience sample of 120 subjects from undergraduate students at a private university. Subjects were randomly assigned to each experimental condition. The experiment used was 2*2*2 factorial design. The three variables used in the experiment were:

- Brand – Two different brands

- Number of significant attributes – Two levels (one attribute and four attributes)

- Number of exposures – Two different numbers of exposures. The first group received one exposure and the second group received 3 exposures of the same print advertisement.

*Brand*. Two fictional brands of cellular phones were chosen. A pretest showed that the product was familiar to most of the subjects and was widely used by many of them. The brands were given fictional names (E-tech and Tone-a-phone) as opposed to calling them brand "A" and brand "B" to lend an element of realism.

*Number of brand attributes*. Fourteen known cellular phone attributes as listed by consumer reports were pretested with a sample group of 80 on a five-point scale. The 4 attributes with the highest mean scores were used for the experiment. One group of subjects was shown print ads listing four top attributes selected through the pre-test process. The second group of subjects was shown print ads listing one top attribute for

cell phones and three "non-significant" attributes -- those previously identified to be the ones having very little or negligible importance (total number of attributes in both cases was kept at four to reduce the bias due to varying attribute numbers in the print ads).

Half of the subjects were exposed to only one print advertisement during the experiment and the other half were exposed to the same ad three times over a 6-day period.

The two advertising appeals were manipulated by varying the copy headline of the advertisement. For E-tech brand that was presented as the more desirable brand, the headline read, "Top rated brand by Consumer Reports". For Tone-a-phone brand, considered less desirable, the headline read, "Rated as the 'value' brand by Consumer Reports".

**Experimental procedure**

Subjects in the one-exposure group were randomly assigned to four subgroups. Each group received one of the four versions of the print ads (print ad #1 = E-tech with copy headline "top rated" and one significant attribute and three non- significant attributes; print ad #2 = E-tech with copy headline "top rated" and four significant attributes; print ad #3 = Tone-a-phone with copy headline "value brand" and one significant attribute and three non-significant attributes; print ad #4 = Tone-a-phone with copy headline "value brand" and two significant attributes and two non-significant attributes. After each exposure, subjects were asked to complete a questionnaire that provided information on the demographics, cellular phone usage, attitude towards brand, and intentions to buy that particular brand.

Subjects in the three exposure groups were also randomly assigned to four subgroups. Each subject was assigned a randomly generated ID number. As mentioned earlier, experimental treatment and data collection for this group was spread over 6 to 8 days. Each of the four print ads mentioned earlier was shown once to each subgroup, with demographic information gathered as follow up. After an interval of 2 to 3 days the same ads were presented a second time to the same subjects by matching their ID's, with cellular phone usage information gathered thereafter. Once again, after an interval of 2 to 3 days the same ad was shown to the same subjects by again matching their ID's, and then ascertaining information on study variables (i.e., ID #1 subject saw the print ad #1 and answered demographic questions; after 2 to 3 days ID #1 saw the print ad #1 again and cellular phone usage information was now gathered from this subject; and again after 2 to 3 days ID #1 saw print ad #1 one more time and completed a questionnaire that contained experimental variables).

*Attitude towards the brand.* Brand attitude was measured with a three-item, nine-point semantic differential (unlikable-likeable, bad-good, and not nice-nice) scale (-4 to +4). The total of the multiple ratings served as the measure of attitude towards the brand. The scale had a reliability coefficient of .91 (alpha).

*Brand choice (intention to buy)*. Intention to buy variable was measured with a three-item (unlikely-likely, improbable-probable, and impossible-possible) scale (-4 to +4). The total

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score of the intention to buy was used as the dependent measure. The scale had a reliability coefficient of .93 (alpha).

**Analysis and Results**

Univariate analysis of variance with intentions to purchase and attitudes towards the brand as dependent variables, and the three factors (brand, number of attributes, and number of exposures) as the dependent variables, was carried out to test the model. Table 1 presents the mean value for the two dependent variables across different experimental groups.

Table 1

Mean Values for the Dependent Variable Across Experimental Groups

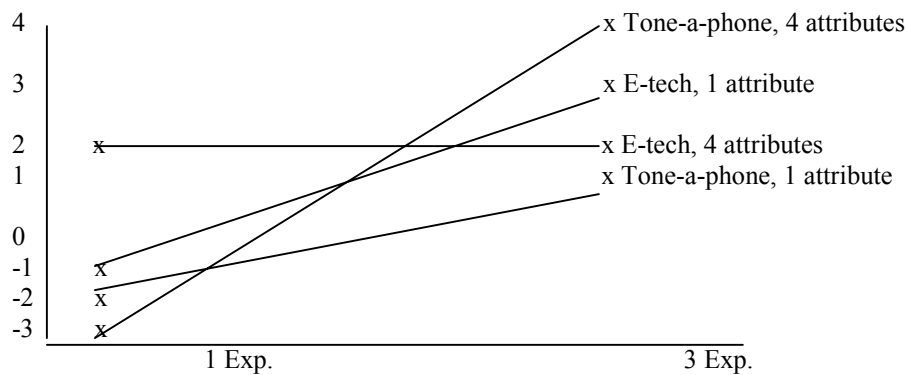| Brand | # of Attributes | Attitude toward brand | | Intention to purchase | |
|---|---|---|---|---|---|
| | | #of Exposures | | #of Exposures | |
| | | One | Three | One | Three |
| E-tech | One | 2.286 | 2.267 | -0.933 | 2.533 |
| | Four | 4.357 | 5.000 | 2.286 | 1.933 |
| Tone-a-phone | One | 1.385 | 1.600 | -1.286 | 0.200 |
| | Four | -2.600 | 3.000 | -2.467 | 4.000 |

**Intentions to buy**

In 3 out of 4 cases the intention to purchase increased when the number of exposures was increased from 1 to 3 (Table 1). For the top E-tech brand with one strong attribute, the mean value increased from –0.933 with one exposure to 2.533 with three exposures. For the less desirable Tone-a-phone brand with one attribute, the mean value increased from –1.286 to 0.200, and for the same brand with four attributes the increase was more

dramatic – the mean value increased from –2.467 to 4.000 with increase in exposure from one to the three. The only exception was the top rated E-tech brand, its intention to purchase value showed no significant difference from an already very high level of purchase intention. Figure 1 shows these results more clearly.

Figure 1
Graphic Presentation of Intention to Purchase and Exposure Rate

Intention To Purchase

```
4                                          x Tone-a-phone, 4 attributes

3                                          x E-tech, 1 attribute

2    x                                     x E-tech, 4 attributes
1                                          x Tone-a-phone, 1 attribute

0
-1   x
-2   x
-3   x
         1 Exp.                    3 Exp.
```

This appears to demonstrate that a brand with fewer strong attributes can overcome its weaknesses in the minds of the consumer provided it gains greater awareness.

**Attitude toward brand**

Closely related to the intention to purchase is the variable "attribute toward brand". Here too, in 3 out of 4 cases, "attitude toward brand" increased when the number of exposures was increased from 1 to 3. For the top E-tech brand with four strong attributes, the mean value increased from purchase 4.357 with one exposure to .5000 with three exposures. For the less desirable Tone-a-phone brand with one attribute, the mean value increased from 1.385 to 1.6000, and for the same brand with four attributes the increase was again

more dramatic – the mean value increased from –2.600 to 3.000 with increase in exposure from one to three. The only exception was the top rated E-tech brand with one attribute, attitude toward it remained essentially constant. It is suspected that the top rated brand was considered so superior that one exposure was sufficient to effect "attitude", so that multiple exposures did not add to conviction in this experimental format. Figure 2 shows these results more clearly.

<u>Figure 2</u>
<u>Graphic Presentation of Attitude Toward Brand and Exposure Rate</u>

Attitude Toward Brand
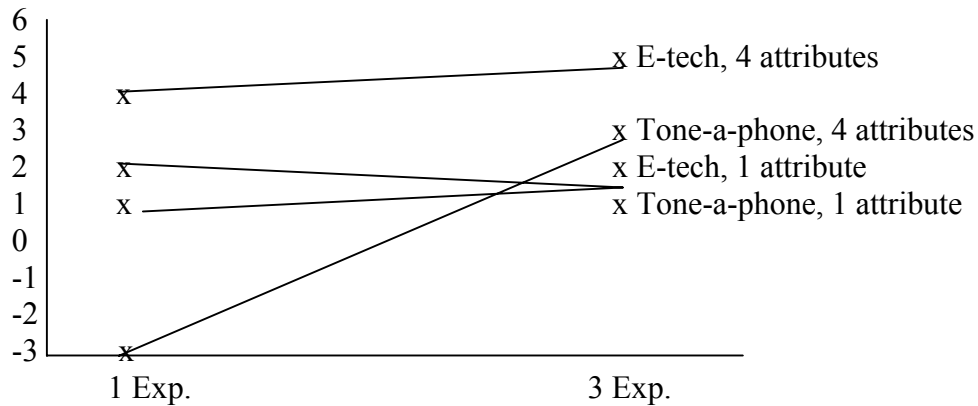


## ANOVA Results

Three-way interaction between the independent variables had a significant effect on intentions to purchase ($F = 6.329$, $p < .05$). Examining the influence of individual variables and interaction between 2 variables at a time, we notice that brand name (E-tech versus Tone-a-phone) did not influence intention to purchase. One-way interaction ANOVA results for the two variables are given in Table 2.

## Table 2
## Tests Between Subject Effects

| | Attitude | | Intention | |
|---|---|---|---|---|
| Variables | F | Sig. | F | Sig. |
| Corrected Model | 4.625 | .000 | 3.297 | .003 |
| Intercept | 31.717 | .000 | 3.209 | .076 |
| Brand | 11.747 | .001 | 2.358 | .128 |
| Attribute | 11.961 | .001 | 9.133 | .003 |
| Exposure | 0.408 | .524 | 2.686 | .104 |
| Brand * Attribute | 0.108 | .743 | 2.325 | .130 |
| Brand * Exposure | 1.092 | .298 | .020 | .888 |
| Attribute * Exposure | 3.877 | .052 | .110 | .740 |
| Brand * Attribute * Exposure | 2.365 | .127 | 6.329 | .013 |

The experiment seems to support the notion that in the short run at least, if consumers are exposed more to a "second best" brand, they would tend to prefer it over the "best brand" (with the most desirable attributes) provided all other marketing variables remain the same. That is the research seems to support the notion promulgated (quantitatively) by the model that share depends not only on product preference per say, but also on the exposure rate.

Appendix 2

**Exposure Wear Out Rate**

In practice, advertisements are known to wear out gradually over a period of time. But, one would hardly expect the nature of this phenomenon to be linear -- where after a certain period of time all memory of the exposure is suddenly and abruptly lost. It would more likely follow a pattern such as that shown below (Figure 1),

Figure 1
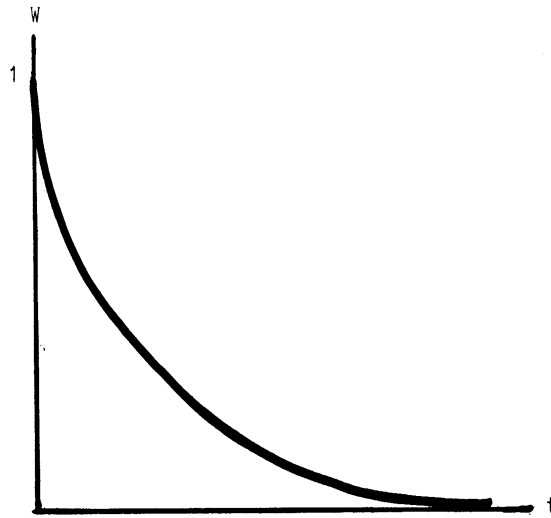


where t = time following exposure, and W = wear-out rate for any exposure, or "the proportion of an exposure" retained or recalled.

A function which "fits" the above concept of wear-out would be:

(1) $w(t) = e^{-t/\theta}$

…where $\theta$ can be adjusted to accommodate a *particular* pattern of exposure wear-out. For example, suppose the appropriate research indicates that, on the average, consumers "retain" the essentials of an exposure for about a week (7 days). Then set…

(2) $e^{-7/\theta} = 00 \Rightarrow \theta = 1.27$.

This function can then be used for determining the *effective* number of exposures experienced by a consumer at any given point in time.

Over the years, exposure "wear-out" has been studied in various contexts. Depending upon the particular approach taken, "wear-out" has been called "the sales decay effect", "the holdover effect", the distribution lag effect", and "the carry-over effect" (Appel 1971; Calder & Sternthal 1980; Craig, Sternthal, & Leavitt 1976; Darral 1977; Duke & Carlson 1993; Parsons 1976; Scott & Solomon 1988; and Stansell & Wilder 1976.) The particular response pattern used above is a generally accepted one based on theoretical considerations as well as practical ones.

For example,

$$w(t) = e^{-t/1.27}$$

…achieves approximately the same result as a Koyck lag scheme with $\lambda = .5$ (Koyck 1954). Consequently, the same rationale could apply. Also, it is virtually identical to Vidale and Wolfe's (1957) "sales decay constant," $e^{-\lambda(t-T)}$.

**Effective Number of Exposures**

During the course of a year, a consumer is exposed to a constant stream of product messages (ads or commercials); and while he/she is exposed to a "new" ad or commercial, the effect of a previous one is being eroded by time. The question is: How many exposures (or more properly perhaps, what *weight* of exposure) are actually in effect at a given point in time?

For example, if we let t= days, and use a wear-out rate of one week, with an exposure rate of one ad or commercial per day, we could envision the following pattern of *effective* exposures (or exposure weight) for say, a one week period commencing with the start of advertising.

| t | ad#1 | ad#2 | ad#3 | ad#4 | ad#5 | ad#6 | ad#7.... | | Effective No of exposures |
|---|------|------|------|------|------|------|------|------|---------------------------|
| 0 | 1.00 | -    | -    | -    | -    | -    | -    | -    | 1.00 |
| 1 | 0.46 | 1.00 | -    | -    | -    | -    | -    | -    | 1.46 |
| 2 | 0.21 | 0.46 | 1.00 | -    | -    | -    | -    | -    | 1.67 |
| 3 | 0.09 | 0.21 | 0.46 | 1.00 | -    | -    | -    | -    | 1.76 |
| 4 | 0.04 | 0.09 | 0.21 | 0.46 | 1.00 | -    | -    | -    | 1.80 |
| 5 | 0.02 | 0.04 | 0.09 | 0.21 | 0.46 | 1.00 | -    | -    | 1.82 |
| 6 | 0.01 | 0.02 | 0.04 | 0.09 | 0.21 | 0.46 | 1.00 | -    | 1.83 |
| 7 | 0.00 | 0.01 | 0.02 | 0.04 | 0.09 | 0.21 | 0.46 | 1.00 | 1.83 |
| . | | | | | | | | | . |
| . | | | | | | | | | . |
| . | | | | | | | | | . |

Or, the same result could have been obtained by:

Effective exposure rate = $1/(1-e^{-1/1.27}) = 1.83$

And, in general

$$(3) \quad n^{/} = n/(1-e^{-1/\theta}),$$

where, $n^{/}$ = effective number of exposures at a point in time,

     $n$ = actual daily exposure rate, and

     $\theta$ = a factor embodying the approximate length of time an exposure is retained

**A Further Point on the Magnitude of $\theta$**

Some additional factors to be considered with respect to the determination of the magnitude of $\theta$ are:

- To some extent the product class may determine the magnitude of $\theta$ in the sense that ads for big-ticket items in all likelihood would be "retained" longer than those for less costly, less significant items.

- Also, the medium used must be considered in the sense that one would expect print ads, once read, to be retained longer than broadcast commercials; and 60 second commercials to be retained longer than 10 second spots in most cases, etc. (Singh and Cole 1993 and Maclachlan and Logan 1992).

- Estimates of the relative magnitudes of $\theta$ (recall decays) have been obtained with fairly good success over the years using conventional research techniques, e.g., Markov Decision Models (Desai and Gupta 1996).